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ORNL-TM-25

COPY NO. - 49

DATE - Oct. 17, 1961

SUBJECT: The Cyclotron Resonance Instability with Negative Mass Ions

TO: Distribution

FROM: T. K. Fowler and E. G. Harris

Abstract

Both the effect of resonant coupling of ion cyclotron motion to electron plasma oscillations, as previously discussed by Harris, and the effect of a negative radial gradient in a magnetic mirror field, the so-called "negative mass" effect, have been combined in an approximate calculation for the threshold for electrostatic instability in DCX. The two mechanisms for instability are found to act more or less independently.

The plasma instability due to resonance between ion cyclotron motion and electron plasma oscillations[1] has been recalculated including the "negative mass" effect which approximates the action of the negative radial gradient of a magnetic mirror field.[2]

As we shall see, it turns out that the resonance instability and the instability due to negative mass alone occur more or less independently. Which dominates depends on the direction of propagation of the disturbance. Because resonance calls for strong coupling to electron oscillations, this mechanism dominates when the perturbed electric field points almost parallel to the direction of electron motion, the z-direction. Because the negative mass instability requires strong coupling between ion bunches at different θ-positions, this mechanism dominates for propagation perpendicular to the field. Since both modes of propagation are possible, both instability mechanisms are active. To obtain thresholds, one may calculate their effects independently.

We have chosen as a simple model an infinitely long cylindrical shell of plasma concentric to the magnetic axis, the z-direction. Electrons, assumed cold, move only in this direction. Ions move in z and also in θ, around the cylinder. Their θ-motion is approximated as a linear motion with periodicity accounting for its actual circular nature. Radial motion of the ions is accounted for by taking as their mass in θ-motion - M/n, where M is their

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1. E. G. Harris, J. Nucl. Energy, Part C: Plasma Physics 2, 138 (1961).

2. T. K. Fowler, "Calculation of the 'Negative Mass' Instability for DCX-1", ORNL CF 61-7-1, July 3, 1961.

true mass and n is the field index, about 0.1 at the beam position in DCX-1 and defined positive for a negative radial gradient. Then, letting the y-direction be equivalent to θ-motion, we obtain the linearized Boltzmann equations for perturbations f_+ and f_- of the ion and electron distributions, f_o^+ and f_o^- in equilibrium:

$$\frac{\partial f^+}{\partial t} + v_y \frac{\partial f^+}{\partial y} + v_z \frac{\partial f^+}{\partial z} - e \frac{n}{M} \frac{\partial f_o^+}{\partial v_y} E_y + \frac{e}{M} \frac{\partial f_o^+}{\partial v_z} E_z = 0 \qquad (1)$$

$$\frac{\partial f^-}{\partial t} + v_z \frac{\partial f^-}{\partial z} - \frac{e}{m} \frac{\partial f_o^-}{\partial v_z} E_z = 0. \qquad (2)$$

Here e is the electronic charge, m the electron mass, and E_y and E_z are components of the perturbed electric field. Magnetic perturbations are ignored.

The resonance instability always has a density threshold. To contrast with the negative mass instability, we choose as the ion distribution a mono-energetic beam, velocity v_o, which is unstable with negative mass for any density N. Thus

$$f_o^+ = N\, \delta(v_y - v_o)\, \delta(v_z)$$

$$f_o^- = N\, \delta(v_z). \qquad (3)$$

With this simple choice, coupling (1), (2) with Poisson's equation, and, as usual, taking the Laplace transform in time ($\exp - i\omega t$) and Fourier transforms in y and z $[\exp i(k_y y + k_z z)]$, we obtain the dispersion relation

$$F(\omega) = N^{-1} \qquad (4)$$

with

$$F(\omega) = \left[\frac{m}{M} \frac{k_z^2 - nk_y^2}{k^2(\omega - k_y v_o)^2} + \frac{k_z^2}{k^2 \omega^2} \right] \frac{4\pi e^2}{m} . \qquad (5)$$

To approximate circular motion, we require

$$k_y v_o = \ell \omega_c, \quad \ell = 0, \pm 1, \ . \ . \ . \ , \qquad (6)$$

ω_c being the ion cyclotron frequency.

From (5), it is easy to see the two propagation limits discussed above. For $k_z^2 \gg nk_y^2$, $F(\omega)$ is approximately independent of n, whatever its sign, and the resonance instability dominates. For $nk_y^2 \gg k_z^2$, the sign of the first term of F depends on the sign of the mass. The two cases are plotted in Fig. 1. Equation (4), quartic in ω, has four roots. For positive mass (Fig. 1A), if $N < N_o$ the curve N^{-1} intersects $F(\omega)$ four times. All roots of (4) are real, corresponding to stability. For negative mass (Fig. 1B), the singularity at $\omega = \ell \omega_c$ has been inverted in sign and only two roots can be real for any positive N. The other roots are now complex, and one must be a growing mode.

Thus, as claimed, resonance dominates if $k_z^2 \gg nk_y^2$ and the negative mass dominates if $k_z^2 \ll nk_y^2$. It will be noticed that the latter condition requires a rather long wavelength in the z-direction. If R is the orbit radius, $k_y = \ell \omega_c / v_o = \ell / R$. Thus, in order for the negative mass to dominate, $k_z < n\ (\ell/R)$, or $\lambda_z > \frac{2\pi}{\ell n} R = \frac{20}{\ell} R$ for DCX-1. Since in DCX-1 the plasma is thin in the z-direction, one might be tempted to say the negative mass effect is thereby suppressed. However, by means of charge accumulation at the surface modes of long wavelength in z ought to be able to exist.

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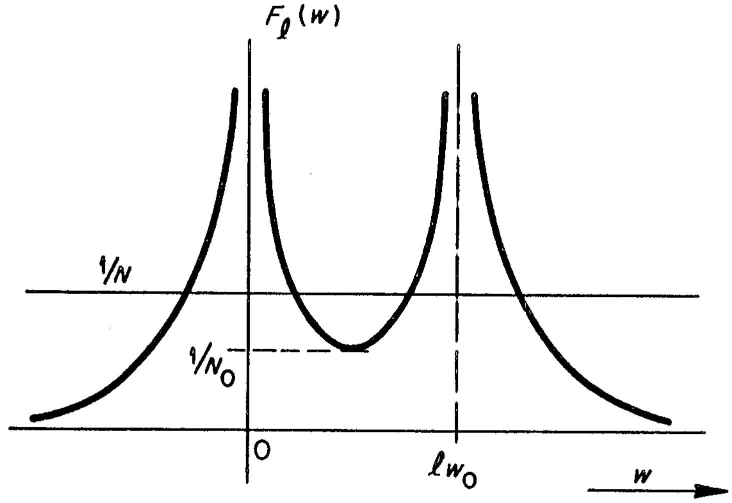
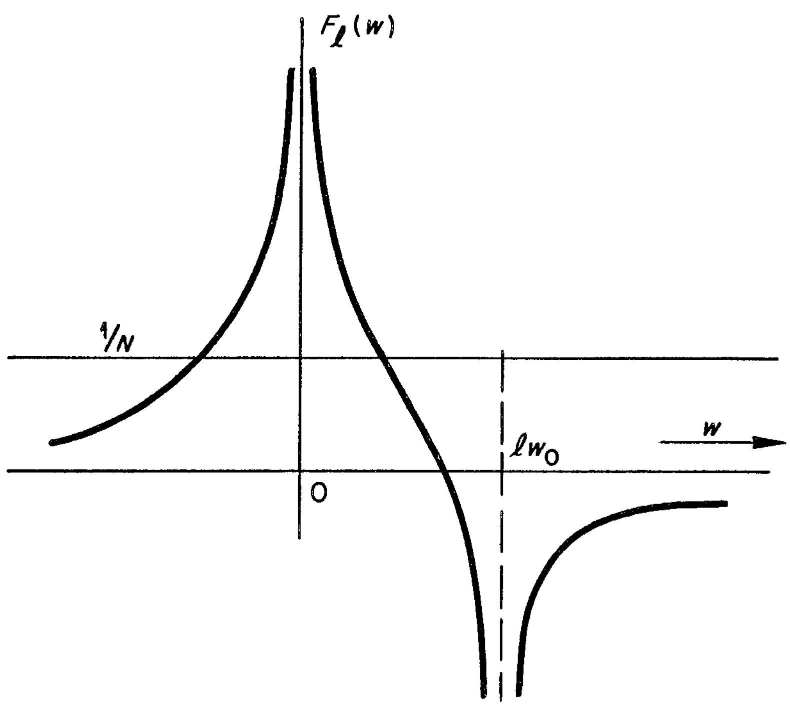


(a) POSITIVE MASS



(b) NEGATIVE MASS

Fig. 1. The Function $F_\ell(w)$ for $k_z^2 << nk_y^2$.

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