NUMERICAL ANALYSIS OF LAMINAR
FORCED CONVECTION IN A SPHERICAL ANNULUS

Dean B. Tuft

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I. ABSTRACT

Calculations of steady laminar incompressible fluid-flow and heat transfer in a spherical annulus are presented. Steady pressures, temperatures, velocities, and heat transfer coefficients are calculated for an insulated outer sphere and a 0°C isothermal inner sphere with 50°C heated water flowing in the annulus. The inner sphere radius is 13.97 cm, the outer sphere radius is 16.83 cm and the radius ratio is 1.2.

The transient axisymmetric equations of heat, mass, and momentum conservation are solved numerically in spherical coordinates. The transient solution is carried out in time until steady state is achieved. A variable mesh is used to improve resolution near the inner sphere where temperature and velocity gradients are steep.

It is believed that this is the first fully two-dimensional analysis of forced flow in a spherical annulus. Local and bulk Nusselt numbers are presented for Reynolds numbers from 4.4 to 440. Computed bulk Nusselt Numbers ranged from 2-50 and are compared to experimental results from the literature.

Inlet flow jetting off the inner sphere and flow separation are predicted by the analysis. The location of wall jet separation was found to be a function of Reynolds number, indicating the location of separation depends upon the ratio of inertia to viscous forces. Wall jet separation has a pronounced effect on the distribution of local heat flux. The area between inlet and separation was found to be the most significant area for heat transfer.

Radial distributions of azimuthal velocity and temperature are presented for various angles beginning at the inlet. Inner sphere pressure distribution is presented and the effect on flow separation is discussed.
II. NOMENCLATURE

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>A</td>
<td>area</td>
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<tr>
<td>a</td>
<td>thermal diffusivity</td>
</tr>
<tr>
<td>c</td>
<td>specific heat</td>
</tr>
<tr>
<td>GFR</td>
<td>radial mesh stretching factor</td>
</tr>
<tr>
<td>H</td>
<td>bulk heat transfer coefficient</td>
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<tr>
<td>h</td>
<td>local heat transfer coefficient</td>
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<tr>
<td>k</td>
<td>thermal conductivity</td>
</tr>
<tr>
<td>( \dot{m} )</td>
<td>mass flow rate</td>
</tr>
<tr>
<td>Nu</td>
<td>bulk Nusselt number</td>
</tr>
<tr>
<td>Nu</td>
<td>local Nusselt number</td>
</tr>
<tr>
<td>( \bar{Q} )</td>
<td>total heat transfer</td>
</tr>
<tr>
<td>q</td>
<td>local heat transfer</td>
</tr>
<tr>
<td>Re</td>
<td>Reynolds number</td>
</tr>
<tr>
<td>RD</td>
<td>radius ratio ( R_2/R_1 )</td>
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<tr>
<td>r</td>
<td>radial coordinate</td>
</tr>
<tr>
<td>( R_1 )</td>
<td>inner sphere radius</td>
</tr>
<tr>
<td>( R_2 )</td>
<td>outer sphere radius</td>
</tr>
<tr>
<td>t</td>
<td>time</td>
</tr>
<tr>
<td>T</td>
<td>temperature</td>
</tr>
<tr>
<td>( \bar{T}_{w} )</td>
<td>average inner sphere temperature</td>
</tr>
<tr>
<td>u</td>
<td>radial velocity</td>
</tr>
<tr>
<td>( \bar{u} )</td>
<td>tentative radial velocity</td>
</tr>
<tr>
<td>v</td>
<td>azimuthal velocity</td>
</tr>
<tr>
<td>( \bar{v} )</td>
<td>tentative azimuthal velocity</td>
</tr>
<tr>
<td>( \bar{w} )</td>
<td>average velocity at equator</td>
</tr>
<tr>
<td>( \partial )</td>
<td>partial derivative</td>
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$\Delta$ - interval of change
$\Theta$ - azimuthal coordinate
$\eta$ - kinematic viscosity
$p$ - density
$\phi$ - longitudinal coordinate

Subscripts

$B$ - bulk
$D$ - inner sphere diameter
$g$ - gap (R2-R1)
$in$ - inlet
$i$ - radial cell index
$j$ - azimuthal cell index
$m$ - mean value
$out$ - outlet
$r$ - radial direction
$\Theta$ - azimuthal direction
$S$ - separation
$w$ - inner sphere wall
III. INTRODUCTION

The spherical annulus is defined as the region between concentric spheres. The sphere provides maximum volume per unit surface area and is advantageous in heat transfer applications, such as cryogenic storage systems and guard heating systems. In these applications, a material inside the inner sphere is maintained at constant temperature by fluid flowing in the annulus. In addition, gyroscopic symbols and spherical fuel elements in homogeneous nuclear reactors are cooled by spherical annulus flow.

One of the first analytical treatments of fluid flow in a spherical shell was by Cobble (1). Cobble assumed a simplified tangential velocity distribution and then calculated heat transfer based on the energy equation. Bird, Stewart and Lightfoot (2), presented the solution to isothermal creeping flow in a spherical annulus. Ward (3) provided a flow visualization study of isothermal flow in a spherical annulus between 60 and 120 degrees downstream of the entrance. Rundell et al (4) measured the temperature profiles between inner and outer spheres and the bulk heat transfer coefficient for steady flow in a spherical shell heat exchanger and obtained a heat transfer correlation for two sets of sphere sizes. Bozeman et al (5) added to the isothermal flow visualization work by Ward by focusing on the entrance region. They state that the most significant heat transfer occurs upstream of separation in the region near the inlet where the flow impinges on the inner sphere and jets off tangentially.
Rundell observed a flow rate independent separation point located between 45 and 50 degrees downstream of the entrance and Bozeman added that upstream of the separation point the flow is characterized by a high velocity jet of fluid near the inner sphere with a relatively low velocity return flow near the outer wall. The high velocity jet near the inner sphere upstream of the separation point makes this an area of significant heat transfer. Beyond the separation point, the fluid is moving slowly near the inner sphere and as a result a lower heat transfer rate is expected in that region.

The problem of laminar natural convection flow in a closed spherical annulus was solved numerically by Brown (6). Brown solved the vorticity and temperature equations in spherical coordinates by an explicit finite-difference technique coupled to an iterative solution of the vorticity-stream function relation. The majority of the calculations were for air while some calculations for water and mercury were included. Astill (7) applied a boundary-layer order of magnitude analysis to the steady incompressible Navier-Stokes equations in spherical coordinates to reduce them to a set of parabolic differential equations. The solution was obtained by a finite-difference method that marches forward in the azimuthal angle. Astill's solution technique was limited because of the simplifying boundary layer assumptions. These assumptions are only valid in regions of nearly parallel flow and thus the entrance and exit regions as well as separated and recirculating flows could not be handled. He, therefore, used simplified inlet and outlet flow conditions at specified angles and neglected the important jetting action of the inlet flow against the inner sphere. Solutions were obtained for isothermal spheres with air flowing in the annulus at low Reynolds numbers.
Newton (8) investigated spherical annulus convective heat transfer experimentally. Local velocity and temperature profiles and mean heat transfer coefficients were measured with air as the working fluid. A correlation was presented based on bulk heat transfer measurements.

A numerical analysis of the transient filling of a spherical annulus was presented by Tuft (9). A modified form of the Marker-and-Cell method was used to handle the free surface aspects of the problem. Since the full Navier-Stokes equations were solved, recirculation and separation were included and the jetting action of the inlet flow was evident in the results. Transient temperature contours and inner sphere heat flux at various times were presented.

This study presents the numerical calculation of steady convective heat transfer in a spherical annulus with an isothermal inner sphere, an insulated outer sphere, and heated water flowing in the annulus. Laminar, incompressible, axially symmetric flow is assumed and the transient equations of mass, momentum, and energy conservation are solved numerically in spherical coordinates. A variable mesh is used to improve resolution near the inner sphere where velocity and temperature gradients are large. The transient equations are solved numerically with the same basic algorithm (minus the free surface treatment), presented earlier by Tuft (9).

In the numerical scheme, the momentum equations are first solved explicitly for the radial and azimuthal velocity components and these velocities are iteratively adjusted along with pressure until mass conservation is satisfied. Temperatures are then obtained explicitly from the energy conservation equation which is coupled to the momentum.
equation by the fluid velocity. Results are obtained by allowing the transient solution to reach steady state. A sketch illustrating the basic flow problem is shown in Fig. 1.
Outlet flow

Isothermal inner sphere $T_w$

Insulated outer sphere

Flow separation

Uniform inlet flow $T_{in}$

Fig. 1 Basic spherical annulus flow configuration.
The governing equations are the conservation equations of mass, radial momentum, azimuthal momentum and energy. The primitive variables of pressure, velocity, and temperature are used and the equations are solved in the spherical \( r - \theta \) plane shown in Fig. 2. The azimuthal coordinate \( \theta \), is measured from the lower pole and the radius is measured from the common center. The sphere radii are denoted as \( R_1 \) for the inner sphere and \( R_2 \) for the outer sphere.

**Assumptions**

For all equations, transient laminar incompressible axially symmetric flow is assumed. The assumption of flow symmetry has been shown to be valid experimentally by others \((3,4)\). In the momentum equations, buoyancy forces are neglected and in the energy equation, viscous dissipation is neglected. No other simplifying assumptions are made.

**Differential Equations**

The governing momentum equations for this problem are the two-dimensional Navier-Stokes equations in spherical coordinates. The momentum and continuity equations are

\[
\frac{\partial u}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 u^2) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (r \sin \theta v) - \frac{v^2}{r} = -\frac{1}{\rho} \frac{\partial p}{\partial r} \\
+ \frac{1}{r^2 \sin \theta} \left( \frac{\partial}{\partial \theta} \left( \nu \sin \theta \frac{\partial u}{\partial \theta} \right) - \frac{\partial}{\partial \theta} \left( \nu \sin \theta \frac{\partial (r v)}{\partial r} \right) \right)
\]

\[(1)\]
Fig. 2 Spherical coordinate system.
Azimuthal Momentum

\[
\begin{align*}
\frac{\partial v}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 uv) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (v^2 \sin \theta) + \frac{uv}{r} &= - \frac{1}{pr} \frac{\partial p}{\partial \theta} + \\
&+ \frac{1}{r} \left\{ \frac{\partial}{\partial r} \left( \frac{\nu}{\partial r} (rv) \right) - \frac{\partial}{\partial r} \left( \frac{v}{\partial \theta} \right) \right\}
\end{align*}
\]

Continuity

\[
\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 u) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta v) = 0
\]

and the differential form of the energy equation is

\[
\frac{\partial T}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 uT) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (vT \sin \theta) =
\]

\[
\left[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( ar^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} (a \sin \theta) \frac{\partial T}{\partial \theta} \right]
\]

Equations 1, 2, 3, and 4, are the differential equations governing this problem. They are four differential equations written in terms of four independent unknowns. The unknowns are, radial velocity \(u\), azimuthal velocity \(v\), pressure \(P\), and temperature \(T\). The momentum and energy equations are nonlinear, parabolic partial differential equations. The
energy equation is coupled to the momentum equations by the velocity, and the momentum equations are uncoupled from the energy equation.

**BOUNDARY CONDITIONS**

The governing equations require boundary conditions on velocity, pressure and temperature at each physical boundary ($r = R_1$, and $r = R_2$) and along symmetry lines at the sphere poles ($\theta = 0$ and $\theta = \pi$). Both normal and tangential velocity components are specified at sphere walls. The normal velocity is zero at a wall except at the inlet and outlet where it is constant. The no-slip condition of zero tangential velocity is applied at sphere walls and the symmetry lines at $\theta = 0$ and $\theta = \pi$ are treated as free-slip boundaries.

Application of explicit boundary conditions on pressure at the walls is not required because a slope boundary condition is implicitly applied by the solution technique. In addition, singularities in the equations at $\theta = 0$ and $\theta = \pi$ are avoided by the placement of velocities in the finite-difference mesh.

The boundary conditions are summarized as follows:

**Velocity:**

\[
\begin{align*}
\text{at } & 0 \leq \theta \leq \theta_{in}, & r = R_2 \\
\text{at } & 0 \leq \theta \leq \pi, & r = R_1 \text{ or } r = R_2 \\
& \frac{\partial u}{\partial \theta} = 0, & \theta = 0 \text{ or } \theta = \pi, & R_1 \leq r \leq R_2 \\
v = 0 \text{ or } & \frac{\partial (v/r)}{\partial r} = 0, & 0 \leq \theta \leq \pi, & r = R_1 \text{ or } r = R_2
\end{align*}
\]

-9-
Temperature:

\[ T = T_{in} \]

\[ \frac{\partial T}{\partial r} = 0 \text{ or } T = T_0 \]

\[ \frac{\partial T}{\partial \theta} = 0 \]

\[ 0 \leq \theta \leq \theta_{in} \quad r = R_2 \]

\[ 0 \leq \theta \leq \pi \quad r = R_1, \quad r = R_2 \]

\[ \theta = \alpha, \pi \quad R_1 \leq r \leq R_2 \]
V. NUMERICAL SOLUTION

FINITE-DIFFERENCE EQUATIONS

For a numerical solution of the governing equations, variables are placed in the mesh as shown in Fig. 3. Half cell notation is used to indicate variables located on cell faces. Applying a variable mesh forward-time centered-space (10) finite-difference approximation to equations 1 and 2, results in equations 5 and 6 shown in Figs. 4 and 5. The numerical time increment is represented by $\Delta t$, fluid viscosity by $\nu_{ij}$ and the time level by superscript n. In this study, constant laminar viscosity is used although the equations are derived and coded for variable fluid properties. Variable donor cell differencing is used in the convective terms of the momentum equation.

Applying variable donor cell differencing to the convective terms in the energy equation and forward-time centered-space finite-differences to all other terms, results in the finite-difference energy equation shown in Fig. 6. The term $a_{i,j}$ represents the thermal diffusivity of the fluid.

Terms such as VT, VB, UT, UB, TT, TB, etc. in equations 5, 6, and 7 are velocities and temperatures that do not lie on the usual mesh point locations shown in Fig. 3. These terms are calculated by flow weighted averages and are discussed in detail in Ref. 9.

Spatially centered finite-differences are used in the finite-difference form of the continuity equation.
Fig. 3 Location of variables in finite-difference mesh.
\[ u_{i+1/2,j}^{n+1} = u_{i+1/2,j}^{n} - \frac{\Delta t}{r_{i+1/2}^2 (r_{i+1} - r_i)} \left\{ \frac{1}{r_{i+1}^2} \left| \text{UR}^2 + \frac{\alpha}{\text{UR}} \left( \frac{u_{i+1/2,j}^n - u_{i+1/2,j}^{n+3/2}}{2} \right) \right| - r_i^2 \left| \text{UL}^2 + \frac{\alpha}{\text{UL}} \left( \frac{u_{i-1/2,j}^n - u_{i+1/2,j}^n}{2} \right) \right| \right\} \]

\[- \frac{\Delta t}{r_{i+1/2} \sin \theta_j (\theta_{i+1/2} - \theta_{j-1/2})} \left\{ \sin \theta_j, \frac{V_T \cdot \text{UT} + \alpha (V_T)}{2} \right\} - \sin \theta_{j-1/2} \left| V_T \cdot \text{UB} + \alpha \text{VB} \right| \]

\[ \frac{(u_{i+1/2,j}^n - u_{i+1/2,j}^{n+1})}{2} \left\{ \frac{\Delta t}{r_{i+1/2}^2 \sin \theta_j} \left| V_T \cdot \text{UT} + \alpha (V_T) \right| - \frac{\Delta t}{r_{i+1/2}^2 \sin \theta_j} \left| V_T \cdot \text{UB} + \alpha \text{VB} \right| \right\} \]

\[ - \frac{(u_{i+1/2,j}^n - u_{i+1/2,j}^{n+1})}{2} \left\{ \frac{\Delta t}{r_{i+1/2}^2 \sin \theta_j} \left| V_T \cdot \text{UT} + \alpha (V_T) \right| - \frac{\Delta t}{r_{i+1/2}^2 \sin \theta_j} \left| V_T \cdot \text{UB} + \alpha \text{VB} \right| \right\} \]

\[ \frac{(u_{i+1/2,j+1}^n - u_{i+1/2,j-1}^n)}{\theta_j - \theta_{j-1}} \left\{ \frac{1}{\theta_j - \theta_{j-1}} - \frac{\Delta t}{r_{i+1/2}^2 \sin \theta_j} \left| V_T \cdot \text{UT} + \alpha (V_T) \right| - \frac{\Delta t}{r_{i+1/2}^2 \sin \theta_j} \left| V_T \cdot \text{UB} + \alpha \text{VB} \right| \right\} \]

\[ - \frac{(r_{i+1/2,j}^n - r_{i+1/2,j}^{n+1})}{\rho_j - \rho_{j-1}} \left\{ \frac{1}{\rho_j - \rho_{j-1}} - \frac{\Delta t}{\rho_{j+1}^n - \rho_j^n} \left| \rho_{j+1}^n - \rho_j^n \right| \right\} \]

Fig. 4 Finite-difference form of the radial momentum equation.
\[
\tilde{v}_{i+1/2}^n = \frac{\Delta t}{r_i^2 (r_{i+1/2}^2 - r_{i-1/2}^2)} \left[ r_{i+1/2}^2 \left[ UR \cdot VR + a |UL| \left( \frac{v_{i+1/2}^n - v_{i+1, i+1/2}^n}{2} \right) \right] - r_{i-1/2}^2 \left[ UL \cdot VL + a |UL| \right] \right]
\]

\[
\left( \frac{v_{i-1,i+1/2}^n - v_{i,i+1/2}^n}{2} \right) \right] - \frac{\Delta t}{r_i \sin \theta_{j+1/2} (\theta_{j+1} - \theta_j)} \left[ \sin \theta_{j+1} \left[ VT^2 + a |VT| \left( \frac{v_{i+1/2}^n - v_{i,j+1/2}^n}{2} \right) \right] \right]
\]

\[
- \sin \theta \left[ V_B^2 + a |V_B| \left( \frac{v_{i-1,i+1/2}^n - v_{i,i+1/2}^n}{2} \right) \right] - \frac{\Delta t}{r_i} v_{i+1/2}^n u_{i+1/2}^n + \frac{\Delta t}{r_i} \left( \frac{r_{i+1} v_{i+1,i+1/2}^n - v_{i,i+1/2}^n}{r_{i+1} - r_i} \right)
\]

\[
\frac{v_{i+1, i+1/2}^n}{r_i (r_{i+1/2}^2 - r_{i-1/2}^2)} \left( \frac{v_{i+1/2}^n - v_{i-1/2}^n}{r_i - r_{i-1}} \right) \right] - \frac{\Delta t}{r_i} \left( \frac{v_{i+1, i+1/2}^n - v_{i,i+1/2}^n}{\theta_{j+1} - \theta_j} \right)
\]

\[
\frac{v_{i+1, i+1/2}^n}{r_i (r_{i+1/2}^2 - r_{i-1/2}^2)} \left( \frac{v_{i+1, i+1/2}^n - v_{i,i+1/2}^n}{\theta_{j+1} - \theta_j} \right) \right] - \frac{\Delta t}{r_i} \left( \frac{v_{i+1, i+1/2}^n - v_{i,i+1/2}^n}{\theta_{j+1} - \theta_j} \right)
\]

Fig. 5 Finite-difference form of the azimuthal momentum equation.
\begin{equation}
\frac{T_{ij}^{n+1} - T_{ij}^n}{\Delta t} = \frac{\Delta t}{r_i^2 (r_{i+1/2} - r_{i-1/2})} \left\{ \left[ \text{UR} \cdot TR + \sigma |ULR| \left( \frac{T_{ij}^n - T_{ij+1}^n}{2} \right) \right] - r_{i-1/2} \left[ \text{UL} \cdot TL + \sigma |UL| (T_{i-1,j}^n - T_{ij}^n) \right] \right\} \\
- \frac{\Delta t}{r_i \sin \theta_i (\theta_{i+1/2} - \theta_{i-1/2})} \left\{ \sin \theta_i \left[ VT \cdot TT + \sigma |VT| \left( \frac{T_{ij}^n - T_{ij+1}^n}{2} \right) \right] \right\} - \sin \theta_{i-1/2} \left\{ VB \cdot TB + \sigma |VB| \right\} \\
\left( \frac{T_{ij}^n - T_{ij}^0}{2} \right) + \frac{\Delta t}{r_i^2 (r_{i+1/2} - r_{i-1/2})} \left\{ \left( T_{ij+1}^n - T_{ij}^n \right) \frac{r_{i+1/2}^2 - r_{i+1/2,j}}{r_{i+1} - r_i} - \frac{r_{i-1/2}^2 - r_{i-1/2,j}}{r_{i-1} - r_{i-1}} \right\} \\
+ \frac{\Delta t}{r_i^2 \sin \theta_i (\theta_{i+1/2} - \theta_{i-1/2})} \left\{ \left( T_{ij+1}^n - T_{ij}^n \right) \frac{\sin \theta_{i+1} a_{i,j+1/2} - \sin \theta_i a_{i,j+1/2}}{\theta_{i+1} - \theta_i} - \frac{\sin \theta_{i-1} a_{i,j-1/2} - \sin \theta_i a_{i,j-1/2}}{\theta_{i-1} - \theta_i} \right\} 
\end{equation}

Fig. 6 Finite-difference form of the energy equation.
Continuity Equation

\[
\frac{1}{r^2_i} \left[ \frac{r^2_{i+1/2} u^n_{i+1/2,j} - r^2_{i-1/2} u^n_{i-1/2,j}}{r_{i+1/2} - r_{i-1/2}} \right] = \frac{1}{r_i \sin \theta_j} \left[ \frac{\sin \theta_{j+1/2} v^n_{i,j+1/2} - \sin \theta_{j-1/2} v^n_{i,j-1/2}}{\theta_{j+1/2} - \theta_{j-1/2}} \right] = 0
\]

NUMERICAL SOLUTION PROCEDURE

The numerical solution of these finite-difference equations proceeds in three steps. The first step is the explicit solution of equations 5 and 6 for \( \tilde{u}^n_{i+1/2,j} \) and \( \tilde{v}^n_{i,j+1/2} \). The tilde denotes that these are tentative velocities which are not complete because they do not in general satisfy conservation of mass. In the second step of the solution, equation 8 is imposed and pressures and velocities are simultaneously iterated upon until mass conservation is achieved. An equivalent technique was introduced by Chorin (11) and has been applied by others (12, 13). Pressure-velocity iterations are performed in such a way as to preserve the vorticity of the original tentative velocity field. Iterations are continued until the maximum cell divergence drops below a specified limit. At this point the solution for velocity and pressure at the new time step is achieved. In the third step, equation 7 is solved explicitly for temperature using the velocity solution from step two. Details of this solution technique are discussed in Ref. 9.
HEAT TRANSFER EQUATIONS

The local heat transfer coefficient at the inner sphere wall is

\[ h_w = -k \frac{dT}{dr} \left( \frac{1}{T_w - T_m} \right) \tag{9} \]

where \( T_w \) is the local temperature at the inner sphere and \( T_m \) is the mean or mixing-cup temperature calculated from the expression

\[ T_m = \frac{\int_{R_1}^{R_2} V T \, dA}{\int_{R_1}^{R_2} V \, dA} \tag{10} \]

Using equation 9, an expression for local Nusselt number can be written as

\[ N_u = \frac{\frac{dT}{dr} (R_2 - R_1)}{(T_w - T_m)} \tag{11} \]

In addition to local properties, it is of interest to compute the overall or bulk heat transfer properties based on the total amount of heat transferred. The bulk heat transfer from the inner sphere can be calculated from the expression

\[ \bar{Q} = mc (T_{out} - T_{in}) \tag{12} \]
By combining the above equation with the definition of bulk heat transfer coefficient

\[ Q = \dot{H} A (T_w - T_B) \]

(13)

an equation for the bulk heat transfer coefficient \( \dot{H} \) can be found that does not contain the total heat transfer \( Q \). This expression is then used to calculate the bulk Nusselt number

\[ \dot{Nu}_g = \frac{\dot{m} c (T_{out} - T_{in})}{A (T_w - T_B)} \left( \frac{R_2 - R_1}{\rho k} \right) \]

(14)

where \( T_B \) is the bulk fluid temperature calculated as the arithmetic mean of the inlet and outlet temperatures.

The average velocity varies with angle due to area changes in the spherical annulus. The velocity used to calculate the Reynolds number is the average velocity at \( \Theta = 90^\circ \) from the expression

\[ \bar{V} = \frac{\dot{m}}{\rho \pi (R_2^2 - R_1^2)} \]

(15)
VI. COMPUTATIONAL RESULTS

In this section, results of numerical computations are presented. All calculations are for a 0°C isothermal inner sphere and an insulated outer sphere. No-slip velocity conditions are applied at sphere walls and the incoming fluid temperature is 50°C. Properties of water at the inlet temperature were used for all calculations. In each calculation the transient solution was carried out in time until the maximum time rate of change of temperature at any point was less than 0.05°C/sec.

The last points to reach steady state were always near the outer wall and beyond θ = 90°. Temperatures in the main area of interest (near the inner sphere) reached steady state much sooner than anywhere else.

Unless otherwise indicated, all calculations are made on the variable mesh shown in Fig. 7. There are 16 radial cells, 84 azimuthal cells, 4 cells across the inlet, and 3 cells across the outlet. A geometrically stretched mesh is used to improve resolution near the inner sphere and in the inlet region. The radial stretching factor is 1.07 and the azimuthal stretching factor is 1.007. An inner sphere radius of 13.97 cm and an outer sphere radius of 16.83 cm with inlet and outlet tubes of 3.81 cm diameter were chosen to match a geometry studied in Refs. (3, 4, 5). The radius ratio for this geometry is $R_o/R_i = 1.2$.

Velocity Profiles

Azimuthal velocity profiles at various angles are plotted in Figs. 8a and 8b for $Re_g = 22.0$. The inlet flow velocity is 1.86 cm/sec, the mass flow rate is 21.20 g/sec, and the annulus gap is 2.86 cm.

Flow enters the annulus, impinges on the inner sphere and jets off, forming a boundary jet along the inner sphere and a reverse flow region near the outer sphere. The boundary jet thickens and decelerates due to
Fig. 7 Computational mesh.
Fig. 8 Azimuthal velocity profiles at various angles.
the increasing flow area as the equator (Θ = 90°) is approached. The action of viscosity and an adverse pressure gradient consume the wall jet momentum and cause separation at Θ = 48.7 degrees. The presence of separation is indicated by an inflection in the velocity profile at Θ = 50 degrees shown in Fig. 8a. The flow continues to decelerate and the velocity profiles flatten out beyond the separation point until the equator is reached. Past there, the flow accelerates due to the decreasing area and velocity profiles resembling fully-developed parabolic flow appear, as seen in Fig. 8b. The flow continues to accelerate until the effect of the outlet tube appears as seen in the profile at Θ = 165 degrees.

The overall flow pattern can be seen in the velocity vectors and profiles shown in Fig. 9. The profiles and vectors are plotted in relation to their location in the annulus allowing easier visualization of the overall flow development. Separation and reattachment points along the inner sphere are marked indicating velocity reversal points. The profiles shown in Figs. 8 and 9 portray steady flow patterns typical of calculations at Reynolds numbers considered in this study.

Temperature Profiles

Radial temperature profiles at various angles are shown in Figs. 10a and 10b. Temperature profiles change rapidly from inlet to separation as indicated by the curves at Θ = 15, 30 and 45 degrees. After separation, the change is more gradual with no more than 5°C change in temperature occurring anywhere beyond 90 degrees. Temperature gradients are large near the inner sphere, particularly near the inlet and decrease as the angle increases and the boundary layer thickens. Large velocities and heat transfer rates near the inlet resulting from the jetting action of
Fig. 9 Velocity profiles and vectors for $Re_g = 22$. 
Fig. 10 Temperature profiles at various angles for $Re_g = 22$. 
the flow make the inlet region a very important part of the spherical shell heat exchanger. For this example, with $Re_g = 22$, the flow separated at 48.7 degrees and the region between inlet and separation accounted for nearly half of the total heat transfer with less than one-fifth of the total area.

Flow Separation Point

In all of the calculations in which separation occurred, the flow separated from the inner sphere. The separation angle varied with Reynolds number as shown in Fig. 11. For $Re_g < 22$ the flow did not separate. The location of separation in the calculations was identified by zero velocity gradient at the wall followed by a flow reversal near the inner sphere.

In general, for a boundary layer to separate there must be a positive pressure gradient. Fig. 12 is a plot of non-dimensional pressure versus angle at the inner sphere wall for several Reynolds numbers. The location of separation is indicated on each curve. In all cases, pressure is maximum at the stagnation point and decreases rapidly until about 20 degrees where the gradient becomes positive and remains so until separation occurs. In the case of $Re_g = 440$, the pressure gradient becomes negative a second time before becoming positive again just prior to separation. This effect may be due to the area decrease that occurs beyond 90 degrees. Downstream of separation, the pressure remains nearly constant until reaching a point where the outlet causes a slight dip. For the unseparated flow at $Re_g = 4.4$, the pressure gradient is negative everywhere except for a small region of positive gradient from about 18 to 30 degrees. In this case the momentum of the wall jet was able to overcome the "pressure hill" and separation did not occur.
Fig. 11 Computed separation angle versus gap Reynolds number, $Re_0 = 1.2$.

Fig. 12 Non-dimensional pressure versus angle at inner sphere wall for various Reynolds numbers.
The presence of an outer sphere imposes an influence on the pressure gradient and, therefore, on the location of separation. The fluid viscosity and the curvature of the inner sphere also play important roles in the location of separation. For constant sphere size and inlet flow rate, the separation angle decreases as the fluid viscosity is increased. Thus, the viscosity plays a major role in opposing the momentum of the wall jet. Separation angles for varying fluid viscosity and constant flow rate were calculated and were found to agree with the angles calculated for the same Reynolds numbers at constant viscosity and varying flow rate. These calculations covered a range of Reynolds numbers from 4.4 to 440 and demonstrate that the location of separation is dependent only on the balance between viscous and inertia forces.

The effects of separation and Reynolds number on average temperature are shown in Fig. 13. The mean temperature was calculated from equation 11. Separation causes dips in the temperature curves which become less and less pronounced as the Reynolds number increases. The flow did not separate at \( \text{Re}_g = 4.4 \), and therefore the profile of mean temperature does not have the characteristic dip as seen in the other curves. The sudden decrease in mean temperature near 180 degrees is due to cold fluid near the inner sphere turning toward the outlet tube.

In Fig. 14, the Nusselt number profile is affected by the combination of wall jet separation and reattachment. The effect becomes more and more pronounced as Reynolds number is increased and for \( \text{Re}_g = 110 \) and above, there is a double hump in the curve immediately following the initial dip at the separation angle. This effect is a result of the recirculation eddy formed near the inner sphere downstream of separation. Maximum heat transfer occurs just beyond the stagnation
Fig. 13 Effect of separation and Reynolds number on mean temperature.

Fig. 14 Effect of separation and Reynolds number on local Nusselt number.
point and then decreases rapidly until separation occurs. The shape of the Nusselt number profile in the stagnation region is similar to that of a round jet impinging on a flat plate (14).

Temperature contours for various Reynolds numbers over the range studied are shown in Fig. 15. The contours were calculated for a constant fluid viscosity with varying flow rates and represent 1.5°C increments. The effect of separation and thermal boundary layer growth can be seen.

Effect of Mesh

To determine the effect of mesh on the solution, runs were made for the 16 x 84 mesh shown in Fig. 7 with various degrees of geometrical stretching. Mesh stretching was increased to give finer and finer resolution of the boundary layer near the inner sphere. Radial stretching factors of 1.03, 1.05, and 1.07 were applied for \( \text{Re}_g = 22 \). With 16 radial cells, these stretching factors resulted in the following mesh increments near the inner sphere:

<table>
<thead>
<tr>
<th>GFR</th>
<th>( \Delta R^1 \text{-cm} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>.179</td>
</tr>
<tr>
<td>1.03</td>
<td>.142</td>
</tr>
<tr>
<td>1.05</td>
<td>.121</td>
</tr>
<tr>
<td>1.07</td>
<td>.103</td>
</tr>
</tbody>
</table>
Fig. 15 Steady temperature contours at 1.5°C increments for various Reynolds numbers.
The maximum effect on the velocity solution occurred near separation. Calculated velocities differed by at most 11.1% for the different meshes and in most instances the differences were indistinguishable when plotted. The bulk Nusselt number increased with increased stretching and showed a maximum change of 5.7%. The calculated separation angle increased asymptotically as the first cell became smaller. The separation angle increased by 3.1% while the first cell increment decreased by 42.5%.

Calculations on a 16 x 84 mesh and a 22 x 84 mesh with GFR = 1.07 were performed to determine the effect of overall radial mesh resolution. The calculated bulk Nusselt number differed by 1.9% for the two meshes and the calculated separation angle differed by 2.9%. Based on these comparisons, the 16 x 84 mesh with GFR = 1.07 was chosen as the baseline mesh for all calculations.

Heat Transfer

In each calculation, local and average heat transfer coefficients were computed. The local heat transfer coefficient in the form of the Nusselt number has been presented in Fig. 14. Fig. 16a shows calculated bulk Nusselt number versus gap Reynolds number. Below \( \text{Re}_g = 22 \), the flow did not separate and the effect appears as a change in slope of the curve. The experimental correlation of Rundell (4) is also plotted for comparison.

Fig. 16b is a plot of Nusselt number versus Reynolds number based on inner sphere diameter. Experimental correlations from Rundell (4) and Newton (8) are plotted. Astill's (7) computed correlation is also plotted along with the computed results from this study. The results of Astill and Newton are for air and those of Rundell and the present study are for water.
Fig. 16 Comparison of computed and experimental bulk Nusselt number versus Reynolds number based on (a) gap, (b) inner sphere diameter.
Turbulence in the experiments is responsible for the discrepancy between the results computed in this study and the experimental results of Rundell. The presence of swirling vorticities and irregular flow patterns have been reported in every spherical annulus experimental investigation in the literature (3, 4, 5, 8). Bozeman et al (5) reported swirling vortex motion in the entrance region for Re as low as 200. Both Ward (3) and Rundell reported a high degree of irregularity for all flow rates studied. Turbulence has the effect of enhancing heat transfer and from a computational point of view, increasing the effective viscosity of the fluid. Rundell reported a flow rate independent separation point located between 45 and 50 degrees downstream of the inlet. The fact that flow rate did not effect the separation angle suggests the balance between viscous and inertia forces is not the determining factor but that the effects of turbulence are dominating the flow. In addition, it is interesting to note that the minimum calculated separation angle agrees with the constant angle observed by Rundell.

In Fig. 16a, the calculated bulk Nusselt number increases more rapidly with Reynolds number than does Rundell's correlation. This is because the calculated separation angle increases with flow rate and hence, the effective area of the wall jet also increases. Thus, both effective heat transfer area and rate are increasing with Reynolds number. This double effect can be seen in the plots of local Nusselt number shown in Fig. 14.
VII. SUMMARY AND CONCLUSIONS

In this paper, results of numerical calculations of laminar incompressible fluid flow and heat transfer in a spherical annulus have been presented. The full transient axisymmetric Navier-Stokes equations were solved by an explicit time marching solution technique. Steady solutions were obtained by allowing the transient solution to achieve steady state. Pressure, temperature, velocity and heat transfer coefficients were presented for an isothermal inner sphere and an insulated outer sphere with radius $r_i$, $R_O = 1.2$. Gap Reynolds numbers from 4.4 to 440 were investigated.

Calculated velocity profiles show the incoming flow jetting off the inner sphere and forming a wall jet with a lower velocity reverse flow near the outer sphere. The wall jet separates due to an adverse pressure gradient and at higher Reynolds numbers a small recirculation eddy is formed downstream near the inner sphere. The velocity takes on a parabolic profile downstream of separation.

The impingement and wall jet regions account for the majority of heat transfer. At $Re_g = 22$ the wall jet separated at 48.7 degrees and the region between inlet and separation accounted for nearly half the total heat transfer with less than one-fifth of the total area. The angle of wall jet separation was found to be a function of Reynolds number, indicating the laminar separation angle is dependent upon the balance between viscous and inertia forces.

Bulk Nusselt number, when compared to an experimental correlation from the literature was found to predict lower heat transfer rates that increased more rapidly with Reynolds number than experiments. Calculated separation angles and wall jet heat transfer rates increase with
increasing Reynolds number and cause bulk heat transfer rates to increase more rapidly than experimental rates (a fixed separation angle has been reported from experiments). Turbulence and swirling vortices in the inlet region were probably responsible for the higher experimental heat transfer rates and the fixed separation angle. The laminar calculations in this study did not predict vortex shedding in the inlet region for any of the flow rates studied. Below $Re_g = 22$ the calculated wall jet does not separate and the Nusselt number versus Reynolds number curve has a lower slope. The calculated angle at which separation first occurred (48.7 degrees), is in agreement with the fixed separation angle reported in the literature (45 - 50 degrees).

Further calculations at higher Reynolds numbers will require a different solution technique than used here. A transient solution technique is expensive to use when steady state results are required. Long thermal time constants require the solution to be carried on for a prohibitive amount of time making each calculation expensive in terms of computing costs. For this reason, other radius combinations and Reynolds numbers were not considered in this study. In addition, the results of this study indicate a turbulence model is required even at relatively low Reynolds numbers. The most critical areas for turbulence modeling are the impingement and wall jet regions. Turbulence in these types of flows has been studied by others and the results could be applied to develop a model for spherical annulus flow (15, 16).

Experiments are planned which will extend the results in the literature to lower Reynolds numbers and provide information on separation location and inner sphere heat flux distribution.
   J. Franklin Institute, Sept. 1963, p. 197.

2. R. B. Bird, W. E. Stewart and E. N. Lightfoot, Transport Phenomena,

3. E. G. Ward, Flow Through the Annulus Formed Between Concentric
   Spheres, Ph.D. Thesis, University of Houston, Houston, Texas
   (1966).

4. H. A. Rundell, E. G. Ward and J. E. Cox, "Forced Convection in
   Concentric-Sphere Heat Exchangers," J. Heat Trans., Transactions

5. J. D. Bozeman and C. Dalton, "Flow in the Entrance Region of a
   Concentric Sphere Heat Exchanger," J. Heat Trans., Transactions

6. J. R. Brown, Natural Convection Heat Transfer Between Concentric
   Spheres, Ph.D. Thesis, University of Texas, Austin, Texas (1967).

7. K. N. Astill, "An Analysis of Laminar Forced Convection Between
   Concentric Spheres," J. Heat Trans., Transactions ASME, Ser. C,

8. R. L. Newton, An Experimental Investigation of Forced Convection
   Between Concentric Spheres, Masters Thesis, Tufts University,

9. D. B. Tuft, "Calculation of Laminar Incompressible Fluid Flow and
   Heat Transfer During Spherical Annulus Filling," Lawrence
   Livermore Laboratory Report UCID-18168, April 1979.


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