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AEC Research and
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UC-81, Reactors - Power
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**FAILURE ANALYSIS OF
DISPERSION FUEL ELEMENTS
BASED ON MATRIX CRACKING**

ALCO PRODUCTS, INC.

NUCLEAR POWER ENGINEERING DEPARTMENT
P. O. BOX 414, SCHENECTADY 1, N. Y.

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BASED ON MATRIX CRACKING**

By:
S. D. Beck

Approved by:
M. H. Dixon, Project Engineer

Issued: December 28, 1961

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New York Operations Office

ALCO PRODUCTS, INC.
Nuclear Power Engineering Department
Post Office Box 414
Schenectady 1, N. Y.

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ABSTRACT

A mathematical model has been developed for estimating the burnup at which mechanical failure occurs in dispersion fuel elements. It is postulated that failure at low temperature ($< 900^{\circ}\text{F}$) occurs as a result of brittle fracture of an elastic matrix when the intensity of the stress locally reaches the ultimate strength of the material. The contributions of fission gas pressure and thermal stress are incorporated in the stress analysis.

Because of the complexity of the stress distribution and the failure process, it was necessary to make stringent assumptions in order to have a workable model. Within the framework of the postulates, burnups to failure are predicted for several values of certain important parameters and compared with burnups to failure obtained from a plastic yield model for failure developed by previous investigation. The two models give good agreement for the special cases considered.

Of the parameters examined, relative density of the fuel particles has the greatest influence on predictions of allowable burnup. The thermal stress and volume fraction of fuel play minor roles.

ACKNOWLEDGEMENT

The author wishes to express his appreciation to J. G. Gallagher, A. S. Wilder and I. Segalman of ALCO Products Nuclear Power Engineering Department for their many helpful discussions and comments.

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1.0 SUMMARY

Matrix rupture in dispersion fuel elements is a complex process. By postulating a failure mechanism and making suitable assumptions and mathematical approximations, a mathematical model can be developed for predicting the burnups at which failure occurs.

The principal assumptions concern the mode of failure and the geometrical representation of the dispersion. The matrix is assumed to remain elastic under the influence of the stresses until the maximum tensile stress reaches the ultimate strength of the material. At this point, failure by matrix cracking occurs.

The stress distribution in the vicinity of the fuel particles is obtained by treating each fuel particle cavity as an isolated, spherical hole in an infinite medium. The medium is subjected both to uniform, uniaxial, compressive stress at infinity and to pressure in the hole. The compressive stress is the maximum, calculated, compressive, thermal stress due to the restraint of the side plates; pressure is induced by the accumulated fission gases released from the fuel particle. It is not clear how other mechanisms such as mechanical and hydraulic loads and bending moments influence the stress distribution; therefore, these mechanisms are ignored. Furthermore, an implication of the isolated cavity concept is that the effect of interactions of neighboring particles cannot be accurately determined.

The model is based on one developed by Weir,⁽¹⁾ who suggests a plastic yield criterion of failure. In many respects, the assumptions associated with the development of an elastic model are similar to those of Weir's model.

The permissible burnup is closely related to the amount of liberated fission gas, which may accumulate in the cavities until the pressure reaches that required for failure. The void volume in which the gas accumulates is the volume of the cavity reduced by the volume occupied by the fuel particle. A lower density fuel gives a greater void volume; therefore, a greater amount of gas may accumulate in the void to give any specific pressure.

Predictions of burnup are made using the results of both models for an assumed set of data for the SM-2 Core I⁽²⁾ fuel element operating in the SM-2. Burnup to failure predictions of 140 and 151 percent are obtained from the elastic and Weir models respectively. The effects of alternative values for certain parameters are considered. As expected from the nature of the assumptions, predictions are very sensitive to particle density, or more precisely, to initial void volume.

Work on this problem was performed under Item 3.6 of Task 3.0 of Contract No. AT(30-1)-2639, Program for Engineering Support and Development of Army Pressurized Water Reactor Power Plants, with the New York Operations Office of the U. S. Atomic Energy Commission.

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2.0 INTRODUCTION

One of the important considerations of fuel element design is the determination of the structural integrity of the element during reactor operation. The problem is generally quite complex. The present investigation considers one aspect of the structural integrity problem for dispersion fuel elements.

The dispersion fuel element is a composite structure. The meat is a sintered ceremet compact consisting of a collection of ceramic-like particles containing a fissile component dispersed in a metallic matrix. The fuel plate is fabricated by the picture frame and cover plate technique. The assembly is hot and cold rolled to a total reduction of 7:1. An aggregate of 16 or 18 fuel plates is assembled into a fuel element by welding to grooved stainless steel side plates. In the case of the SM-2 or Type 3 fuel element,⁽³⁾ the ceramic fuel is UO_2 and the metallic components are Type 347 stainless steel.

A continuing core development program at ALCO⁽⁴⁾ has as its goal the design of longer lived, more reliable and more efficient fuel elements. A valid description of the mechanical behavior of the elements would, therefore, be of value to the designer's overall efforts to improve fuel elements. The investigation described in this report is concerned with analytical examination of the mechanical behavior of the matrix in an attempt to predict the burnup of uranium at which the matrix will fail.

A companion report⁽⁵⁾ presents a review of the irradiation data now available on UO_2 - stainless steel dispersions and predicts the allowable burnup of SM-2 Core I fuel elements by extrapolating the existing test data. This method yields an allowable burnup of 88 percent.

The question of matrix failure during irradiation is, in essence, the determination of the stress distribution in the body and the determination of the mode of failure of the matrix structure. These questions are not readily resolved.

Stresses are induced in the matrix of the fuel element by several mechanisms. The fission products produced during the irradiation tend to occupy more volume than that occupied by the fissioned uranium. Fission gases may collect in void regions and act as sources of pressure. In addition, there is a tendency for fuel particles to swell as solid fission products and accumulate in the crystal structure. Due to the uneven heating of the fuel elements, there is a tendency for the hotter portions of the SM-2 element to expand relative to the side plates. This differential thermal expansion leads to thermal stresses. Among other causes of stress may be either mechanical and hydraulic loads or, if buckling occurs, bending moments. Because of the complexities and uncertainties associated with the nature of the loads, all contributions to the stresses will be neglected except those due to the temperature variation and the accumulated fission gases.

The stress picture is further complicated by the fact that the fuel plate is not a homogeneous medium. The fueled portion consists of a matrix of metal containing irregularly shaped, randomly distributed particles. The clad and side plates are metal. The elastic constants vary with temperature and extent of recoil damage.

The mode of failure is evidently quite complex. We may conceive of several mechanisms occurring simultaneously. Plastic flow, creep, and brittle and ductile rupture may all contribute. For example, the region of greatest recoil damage may be more brittle than the region of least recoil damage. As the temperature changes, the ductility may vary continuously or suddenly. The data necessary to characterize the failure are often lacking or unreliable.

With these complications evident, any proposed model will be, at best, a very rough approximation. However, the initial efforts involved in the investigation of a detailed model are worthwhile because they provide an indication of the effects of various parameters on fuel element behavior and they point the way to areas in which refinement of the model could be fruitful.

A literature search on the problem of dispersion fuel element failure has indicated a paucity of information relating to detailed mathematical models. There is, however, a significant amount of qualitative and experimental information and some efforts have been made to develop a quantitative approach to the problem.

Experimental results from several UO_2 dispersion fuel element irradiations have been summarized in recent reports.^(5, 6, 7) Since these data are derived from a wide variety of fuel element specifications, it is difficult to separate the effects of the individual parameters. Generally speaking, the data indicate that larger particles, lower temperatures, and smaller fractional loadings of UO_2 permit higher burnups.

The effect of the UO_2 particle density is not incorporated, except for the indication that higher density particles have less tendency toward stringing during fabrication and thus give superior burnup performance. Failures that occur are characterized by matrix cracking, clad blistering or cracking and release of fission gases.

In an effort to develop an analytic approach to the failure problem, White⁽⁸⁾ considers the problem from the viewpoint of minimizing radiation damage to the matrix. This approach leads to valuable design suggestions, but does not provide any quantitative indication of the effect of the various parameters of the system on ultimate burnup.

Keller⁽⁹⁾ has devised a novel approach to the failure problem. The thesis is that failure occurs when a certain (unprescribed) amount of creep is attained. It is presumed that the burnup to failure is known for an irradiation of known

history. The approach then describes a method of predicting burnup to failure under other conditions for a fuel element of the same design. The approach gives only a qualitative indication of the effects of design parameters and irradiation conditions on ultimate burnup.

The investigation by Weir⁽¹⁾ provides the only completely quantitative approach to the problem of dispersion fuel element failures. Weir postulates an ideal situation in which each of the dispersed fuel particles is in the central cavity of one of many identical, hollow, elastic-plastic spheres which are free of tractions on the outer surface. Under the action of fission gas pressure, the spheres deform as elastic-plastic bodies until the entire volume is plastic, at which time the fuel element is said to have failed. The Weir analysis is used as a basis of this model; thus, it is discussed in some detail.

The model proposed in the present investigation contains many of the assumptions made by Weir. The most important of these is the description of the fission gas accumulation. At the start of irradiation, the fuel particle sinters immediately to perfect density, leaving a void region in which fission gases accumulate. All recoils collect in the void; no other particles do (there is no diffusion or evaporation). The rate of accumulation is constant. Decay of gas atoms or birth of them by decay of a precursor is not explicitly considered. Appendix C points out certain minor differences in the description of gas behavior used here as compared to Weir's assumptions.

The principal departure of the ALCO model from the Weir model is in the mode of failure. Where Weir has suggested a plastic yield criterion, the ALCO model considers a sudden fracture of an elastic medium. In addition, thermal stress will be incorporated into the model.

Matrix failure only is considered in this investigation. The questions of structural instability (buckling) and cladding failure are not considered.

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3.0 NOMENCLATURE

Symbols used frequently are defined below. Other symbols used are defined in the text. The subscript "i" has the value 1 or 2, referring to Xe and Kr respectively.

P	=	gas pressure in the cavity (psi)
Y	=	yield stress (psi)
F	=	volume fraction of UO_2
F'	=	effective volume fraction of UO_2
T	=	temperature ($^{\circ}\text{K}$)
a	=	initial particle radius (micron)
E	=	modulus of elasticity (psi)
R	=	real gas constant (liter-atm/mol - $^{\circ}\text{K}$)
b_i	=	real gas constant (liter/mol)
n_i	=	accumulated fission gas release (mol)
V_o	=	initial particle volume (liters)
V	=	void volume (liters)
d	=	initial relative density of fuel
r_i	=	recoil range of gas "i"
q_i	=	fraction of gas "i" in total gas produced
N	=	fission gas production rate (mol/liter-% burnup)
C	=	magnitude of maximum compressive thermal stress (psi)
σ_c	=	tensile stress due to thermal stress field (psi)
σ_p	=	tensile stress due to pressure (psi)
σ_u	=	fracture strength (psi)
ν	=	Poisson's Ratio
B	=	percent burnup of uranium atoms by fission

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4.0 THE WEIR MODEL

4.1 DESCRIPTION OF THE MODEL

The model suggested by Weir is one in which failure occurs by plastic flow. The fuel particles are assumed to be identical and uniformly dispersed in the matrix, which is a homogeneous, isotropic medium. Each particle may be thought of as being in the center of a characteristic regular polyhedron. The polyhedrons are then approximated by thick walled, hollow spheres.

Prior to irradiation, the particles are porous solids. It is known that within the first few percent burnup of uranium atoms that the UO_2 particles sinter to essentially 100 percent density. It is, therefore, assumed that at the start of irradiation the particles immediately sinter to 100 percent density leaving a void within the cavity surrounding the fuel particle. A portion of the fission-produced atoms recoil from the particle into the surrounding matrix. The recoiled, rare gas atoms are assumed to diffuse immediately into the void region, where they accumulate at a constant rate. Diffusion from the particle and radioactive decay of fission products are neglected.

The particles are approximately ellipsoidal, due to the fabrication process, with the long axis in the direction of rolling and a nearly circular cross section. Figure 1 gives an indication of particle shape. The analytic difficulties associated with ellipsoidal geometry preclude the incorporation of this shape into an elementary model. The particles and the cavities containing them are, therefore, treated in the analysis as though they were spherical. A correction is suggested by Weir to account for the increased stress in the neighborhood of an ellipsoidal cavity as compared to a spherical one. He indicates that the maximum stress is increased at the end of the long axis by a factor of $m/2a$, where m is the major axis of the ellipsoid and a is the initial radius of the corresponding spherical particle. The volume fraction is then increased by this factor to effect an approximate correction for the shape. In addition, since recoil is approximately proportional to the nominal surface area of the particle, gas release is multiplied by a factor g , given by Eq (6) below, which is the ratio of the surface area of a prolate spheroid and a sphere of the same volume.*

* Although this formula is not explicitly given by Weir, he remarks that gas release is greater from the ellipsoidal particle and that a correction is made.

Figure 1. Typical SM-2 Fuel Dispersion

The failure mechanism used by Weir is one of plastic flow. A thick-walled shell of perfectly plastic material is subjected to internal pressure and is free of external tractions. As the pressure builds up, the shell deforms elastically until the yield stress is reached at the inner surface. The plastic region grows with increasing pressure until the entire shell is plastic. This condition is defined to be the criterion of failure.

In summary, the Weir assumptions are:

1. The matrix is a perfectly plastic, homogeneous, isotropic medium.
2. The matrix is represented as a collection of identical, thick-walled, spherical shells uniformly distributed.
3. The fuel particles are perfectly dense spheres.
4. A void volume exists in the cavities as a consequence of the initial porosity of the particles.
5. Fission gas accumulates at a constant rate in the void volume due to recoil from the fuel particle.
6. Pressure from fission gas causes a plastic region to grow in the matrix.
7. Failure occurs when the characteristic, thick-walled, spherical shell becomes completely plastic.
8. Ellipsoidal shape is approximated by spherical geometry and correction factors.

4.2 ANALYSIS

Weir shows that the condition of failure occurs when*

$$P = (2/3) Y \ln (1/F') \quad (1)$$

at which time the radius of the cavity has become

$$a' = a \left\{ 1 + (Y/E) \left[(1-\nu)/F' - (2/3)(1-2\nu) \ln (1/F') \right] \right\} \quad (2)$$

In the case of spherical geometry, F' is the volume fraction of UO_2 . In the correction for the ellipsoidal shape, F' is the product of the volume fraction and $m/2a$.

* Logarithms abbreviated in $\ln$ are base e ; abbreviated $\log$ are base 10.

The pressure exerted by the accumulated fission gas is

$$P = 14.7 RT \left[n_1 / (V - n_1 b_1) + n_2 / (V - n_2 b_2) \right] \quad (3)$$

The void volume in which the gas is confined is the volume of the cavity less the volume occupied by the fuel particle.

$$V = V_0 \left[(a'/a)^3 - d \right] \quad (4)$$

The quantity of gas released by recoil is

$$n_i = (q_i V_0 N d g / 4) \left[1 - (1 - r_i / a d)^{1/3} \right]^3 B \quad (5)$$

where i is 1 or 2 corresponding to Xe or Kr respectively, and where (10)

$$g = \left[c^2 + \frac{m^2 c}{\sqrt{m^2 - c^2}} \cos^{-1} \frac{c}{m} \right] / 8 a^2. \quad (6)$$

The diameter of the ellipsoid is c .

Defining for simplification in solving the above equations:

$$K = \frac{8Y \ln(1/F')}{3 (14.7) RNT \text{ gd}} = \frac{4 P}{(14.7) RNT \text{ gd}} \quad (7)$$

$$k_i = q_i \left[1 - (1 - r_i / a d)^{1/3} \right]^3 \quad (8)$$

$$t_i = N \text{ gd } b_i / 4 \quad (9)$$

$$v = V/V_0 = (a'/a)^3 - d \quad (10)$$

$$\alpha = (K t_1 t_2 + t_1 + t_2) k_1 k_2 \quad (11)$$

$$\beta = K (t_1 k_1 + t_2 k_2) + k_1 + k_2 \quad (12)$$

Equating the right hand sides of Eqs. (1) and (3), and substituting the pertinent expressions gives, after some algebraic manipulation, the predicted burnup at failure as the smaller root of

$$\alpha B^2 - \beta v B + v^2 K = 0 \quad (13)$$

That is

$$B = (v/2\alpha) (\beta - \sqrt{\beta^2 - 4\alpha K}) \quad (14)$$

4.3 NUMERICAL EXAMPLE

As an illustration of Weir's model we calculate the burnup to failure for the SM-2 fuel element. The assumed data are estimates and may not be completely accurate.

R	=	0.08207 liter-atm./mole - °K
b ₁	=	0.0511 liter/mole
b ₂	=	0.0398 liter/mole
N	=	0.098 mole/liter - % burnup
q ₁	=	0.865
q ₂	=	0.135
r ₁	=	7.2 microns
r ₂	=	9.2 microns
T	=	605°K
Y	=	6.7 (10) ⁴ psi
Y/E	=	2.7 (10) ⁻³
ν	=	0.27
d	=	0.92
a	=	46.6 micron
m	=	167 micron
c	=	69.7 micron
F	=	0.22

The data on the gas constants, fission gas production, proportions of Xe and Kr, yield strength, and Y/E are taken from Weir's report; β is taken from Reference 2; F, a, m, and c were estimated from Figure 1; and d was obtained from a private conversation with A. S. Wilder. T and r_1 are obtained in Appendices B and C. The following quantities are calculated:

$$\begin{aligned}
 F' &= 0.394 \\
 g &= 1.119 \\
 P &= 4.16 (10)^4 \text{ psi} \\
 K &= 2.26 (10)^3 \\
 k_1 &= 0.350 \\
 k_2 &= 0.0666 \\
 t_1 &= 1.289 (10)^{-3} \\
 t_2 &= 1.004 (10)^{-3} \\
 v &= 0.0927 \\
 \alpha &= 1.217 (10)^{-4} \\
 \beta &= 1.588
 \end{aligned}$$

For this case, the predicted burnup to failure is 151 percent.

Several alternate cases were considered. Each of these incorporated the modification of one of the parameters. The results are tabulated below. The given parameter is the one modified in each case.

Standard Case		B	=	151%
Density	d = 0.928	B	=	137%
	d = 0.96	B	=	83%
Volume Fraction	F = 0.242	B	=	144%
	F = 0.33	B	=	119%
	F = 0.44	B	=	77%
Yield Stress	Y = $6.03 (10)^4$ psi	B	=	144%
	Y = $3.35 (10)^4$ psi	B	=	110%

The variation of density d corresponds to reductions of 10 and 50 percent in the initial void volume. The modified values of Y represent 10 and 50 percent reductions. The values of F represent 10, 50 and 100 percent increases over the nominal value.

Note that the initial void volume significantly affects the predicted burnup. Since only a small variation in fuel density corresponds to a much larger change in void volume, the predicted burnup is particularly sensitive to density. This effect has been pointed out by Weir.

4.4 REMARKS ON WEIR'S MODEL

Certain implications of the Weir analysis should be pointed out. The concept of each particle residing in a distinct sphere is an attempt to account for the volume fraction of fuel. The assumption partially incorporates the interactions between particles in that a domain of influence of each particle is prescribed. The stress-free boundary condition tends to lead to a conservative (low) estimate of burnup, since one would expect a compressive, normal stress on a plane midway between adjacent particles. Thus the idealized sphere is free from constraints of external pressure and deforms more readily.

The rate of gas accumulation is assumed constant. Although the assumption of a linear accumulation rate is clearly inaccurate, the approximate nature of the analysis does not justify the complications imposed by a more sophisticated (and not necessarily more accurate) assumption.

There are several other considerations which are neglected in the Weir model. Temperature variation across the width of the fuel plates and loads other than gas pressure are ignored. Variations of material properties are also neglected. As in the case with other approximations associated with the model, their incorporation would lead to additional complications and to refinements of uncertain validity.

The question remains as to whether the matrix would actually fail by plastic yielding at the low temperatures considered. The mechanism of failure is complex and probably includes brittle fracture, creep, and plastic flow, coupled with annealing out of stresses and defects. Thus, we can not discard plastic flow as a mechanism of failure, nor can we rely on the assumption that it is the principal mode. In the next section, we shall consider an alternative model, similar to Weir's, but assuming brittle fracture of an elastic medium.

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5.0 THE ELASTIC MODEL

5.1 DESCRIPTION OF THE MODEL

As an alternative model, we shall consider one in which the matrix behaves elastically under the combined action of the pressure in the cavities and the thermal stress until such time as the ultimate or fracture stress of the matrix material is reached. An approximation to the stress distribution in the neighborhood of each cavity is obtained by treating each cavity as a spherical hole in an infinite medium and subject to a uniform uniaxial compressive stress at infinity and to pressure in the cavity. The pressure is due only to accumulated fission gases. The assumptions concerning gas behavior are precisely the same as those used in the Weir model.

The compressive stress at infinity is found from the thermal stress analysis. The thermal stress field is calculated as though the medium were a homogeneous, isotropic plate subject to temperature variation only across the width. The material properties are taken to be those of the matrix material. This part of the analysis neglects the effect of the fuel particles which act as inclusions in the medium. Because the plate is thin in comparison to its other characteristic dimensions we make the usual assumption of the theory of elasticity that the stress normal to the plane of the plate is zero. The thermal stresses, therefore, are normal to a cross section of the plate and vary across the width. The greatest compressive stress occurs at the hottest part of the plate. The details of the analysis are discussed in Appendix B.

For analysis of the local stress distribution in the neighborhood of a cavity, finite dimensions of the plate are disregarded. The cavity is then treated as a single, spherical hole in an infinite medium. This implies that cavities are assumed remotely dispersed so that interactions may be neglected, and that cavity dimensions are small compared to the thickness of the plate.

Appendix A treats the details of the local stress analysis. The compressive boundary condition gives rise to a tensile stress at the pole of the spherical cavity (see Fig. 2). To this stress, the hoop stress resulting from pressure is added in accordance with the principle of superposition.

The mathematical treatment of the elastic model makes no provision for the volume fraction of UO_2 in the plate. To get some indication of how the predicted burnup is affected by this volume fraction, it shall be assumed that the calculated local stress is increased by the correction factor $1/(1-F)$ to compensate for the reduced cross-sectional area.

A second correction factor is applied to the calculated stresses to compensate for the ellipsoidal shape of the particles and cavities. In accordance with the cor-

Figure 2. Tensile Stress in the Neighborhood of a Cavity with a Uniaxial Compressive Field at Infinity

rection suggested by Weir, it is assumed that the stresses are increased by the factor $m/2a$. The additional gas release due to the increased area will be treated as in the analysis of the Weir model.

The matrix is assumed to behave as a linear-elastic material until the maximum tensile stress reaches the ultimate stress. This state is defined to be the condition of failure. The behavior of the material after the failure criterion has been achieved is not considered here, although this consideration has significance with respect to the failure process.

5.2 ANALYSIS

The contribution σ_c due to the compression at infinity to the maximum tensile stress in the vicinity of a cavity is given by σ_θ or σ_ϕ , Eqs. (A-8), evaluated at $\theta = 0$ and $r = a$. We find

$$\sigma_c = \frac{1.5 (1 + 5\nu) C}{7 - 5\nu} \quad (15)$$

The contribution σ_p due to the pressure is given by σ_θ or σ_ϕ Eqs. (A-13), evaluated at $r = a$.

$$\sigma_p = \frac{P}{2} \quad (16)$$

The maximum tensile stress σ is given by

$$\sigma = \sigma_p + \sigma_c = \frac{P}{2} + \frac{1.5 (1 + 5\nu) C}{7 - 5\nu} \quad (17)$$

The stress given by Eq. (17) is exact for the single, spherical cavity in an infinite medium with the prescribed loading. We now modify this expression in accordance with the assumed correction factors to compensate for the infinite volume fraction of fuel and the ellipsoidal shape. We have then

$$\sigma = \left[\frac{1}{1 - F} \right] \left[\frac{m}{2a} \right] \left[\frac{P}{2} + \frac{1.5 (1 + 5\nu) C}{7 - 5\nu} \right] \quad (18)$$

Failure occurs when σ becomes σ_u . We make this substitution in Eq. (18) and solve for the failure pressure

$$P = \frac{4 a (1 - F) \sigma_u}{m} - \frac{3 (1 + 5\nu) C}{7 - 5\nu} \quad (19)$$

The pressure exerted by the accumulated fission gases is given by Eq. (3). At failure then, the right hand members of Eqs. (3) and (19) are equated.

The volume V in which the fission gas collects is the total volume of the cavity less the volume occupied by the sintered fuel particles. Eq. (A-18) gives the volume of the cavity. The void volume V is therefore given by

$$V = V_0 \left[1 + \frac{P(1+\nu) - 3C(1-\nu)}{2E} - d \right] \quad (20)$$

The calculation of predicted burnup is now reduced to the solution of Eq. (3) for B , where P is given by Eq. (19), n_1 and n_2 by Eq. (5) and V by Eq. (20). We define

$$K = \frac{4P}{(14.7) RNTgd} \quad (21)$$

and

$$v = 1 - d + (1/2 E) \left[P(1+\nu) - 3C(1-\nu) \right] \quad (22)$$

Eqs. (8), (9), (11) and (12) are used to define the quantities k_1 , t_1 , α and β , respectively. B , then, is given by Eq. (14) as in the case of the Weir model. However, some of the symbols in that equation have different numerical values as a consequence of the different development associated with the elastic model.

5.3 NUMERICAL EXAMPLE

The calculation of burnup to failure is illustrated for the same conditions as in the example (SM-2 Core I) used for Weir's model. In addition to the data listed there, we assume

$$\begin{aligned} E &= 2.5 (10)^7 \text{ psi} \\ \sigma_u &= 6.9 (10)^4 \text{ psi} \end{aligned}$$

The value of E is from Reference (2). That for σ_u is typical of the values given by the manufacturers for Type 347 stainless steel. Since irradiation increases the ultimate strength, the use of the value for unirradiated steel will be conservative. The maximum compressive thermal stress from the solid plate analysis is used as the compressive stress at infinity. This value is found in Appendix B to be

$$C = 6190 \text{ psi, compression}$$

The values of k_1 , k_2 , t_1 , t_2 and g are the same as for the Weir example. The other calculated values are

$$\begin{aligned}
 P &= 5.24 (10)^4 \text{ psi} \\
 K &= 2.84 (10)^3 \\
 v &= 0.0811 \\
 \alpha &= 1.393 (10)^{-4} \\
 \beta &= 1.891
 \end{aligned}$$

The predicted burnup for this case is

$$B = 140\%$$

We shall treat this example as the standard. We examine several alternate cases, each of which involves the modification of one parameter. The tabulation below lists these alternates with corresponding value of predicted burnup.

Standard Case		B = 140%
Density	d = 0.928	B = 125%
	d = 0.96	B = 69%
Volume fraction	F = 0.242	B = 139%
	F = 0.33	B = 133%
	F = 0.44	B = 125%
Ultimate strength	$\sigma_u = 6.21 (10)^4 \text{ psi}$	B = 135%
	$\sigma_u = 3.45 (10)^4 \text{ psi}$	B = 105%
Thermal stress	C = 6.809 (10) ³ psi	B = 139%
	C = 9.285 (10) ³ psi	B = 137%
	C = 1.238 (10) ⁴ psi	B = 134%

We notice that a change in initial void space has the most significant effect on the prediction. At the assumed level of the parameters, the contributions of the thermal stress and the volume fraction of fuel are seen to be minor. The ultimate strength has intermediate significance.

5.4 REMARKS ON THE ELASTIC MODEL

The approximate stress distribution to the vicinity of the cavity neglects the fact that there are many particles in the matrix and that the matrix is finite.

These particles vary in size and shape and are not uniformly spaced. Although engineering approximations are employed to account for the effects of volume fraction of UO_2 in the matrix and the ellipsoidal shape of the particles, the approximations are crude and do not necessarily give accurate insight into the effects of the geometrical considerations. Furthermore, the varying thermal stress is taken as constant for the local stress analysis. The error caused by these assumptions may be moderate if the particles are small compared to the characteristic dimensions of the plate and to the distance between particles. If the particle spacing is less than the particle size, there may be greatly increased stresses due to interactions. A more sophisticated analysis might consider two particles and incorporate the concept of spherical dipolar coordinates as developed by Sternberg and Sadowsky⁽¹¹⁾. The complexities of the resulting analysis, however, are severe and not justified in the present approximate model. For a specific case discussed in their paper, uniform hydrostatic tension at infinity - cavity spacing equal to the cavity diameter, they found only a 5 percent increase in maximum tensile stress compared to the single cavity case. The empirical correction factor for the ellipsoidal geometry is also a source of error. An exact analysis for the case of an ellipsoidal cavity in an infinite medium has also been developed by Sadowsky and Sternberg.⁽¹²⁾ This analysis is also extremely complicated and has not been incorporated here. For the development of a more exact model, however, the use of this method should be considered.

The thermal stress distribution in the plate is based on the assumption that the temperature varies only across the width of the plate. With this assumption, variation of temperature across the thickness and thermal stresses which this variation generates are neglected. In the case of the SM-2 Core I fuel element operating in the SM-2⁽²⁾, the maximum compressive stress has been calculated to be of the same order of magnitude as the compressive stress due to the variation across the width. The stress field is biaxial and could be represented as compressive stresses

$$\sigma_x = \sigma_y = -C = \text{constant} \quad (23)$$

at infinity. It may be shown that this boundary condition leads to a maximum tensile stress in the vicinity of the spherical cavity of

$$\sigma = \frac{3C}{(7-5\nu)} \quad (24)$$

which is less than that due to an equivalent uniaxial field as long as $\nu \geq 0.2$ [see Eq. (15)]. We have neglected this effect, since its contribution to the prediction of allowable burnup is apparently small and its incorporation would add additional complexity (without compensating additional accuracy) to the model. Consider as a worst case the event in which the maximum biaxial thermal stress and the maximum uniaxial stress occur at the same point. The tension due to

the thermal stress would be increased by a factor of probably not more than two. For the given numerical example, doubling the thermal stress reduces, the predicted allowable burnup by only about 4 percent. To more accurately describe the stresses, a two-dimensional thermal stress analysis should be considered instead of the sum of two one-dimensional problems. Further refinement of the theory may be justified if it were found that, for certain important combinations of parameters, the prediction of allowable burnup is sensitive to the assumed value of the thermal stress.

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6.0 DISCUSSION AND CONCLUSIONS

Two models are now available for the prediction of uranium atom burnup which may be sustained in dispersion fuel elements before failure occurs. Both have relied on several severe assumptions and engineering approximations. The justification for these simplifications of a complex process is twofold. It is necessary to idealize the mechanism to obtain a model which is manageable analytically. Secondly, the many uncertainties in the properties and behavior of the system may leave the results of a more sophisticated representation still inaccurate.

The fuel element designer must be aware that the results of these analyses are to be used with caution. At best, they give an indication of the quantitative effect of the various parameters. They do not give a reliable prediction of the precise burnup at which failure will occur.

In each of the models, the most significant parameter examined is initial fuel density. This condition comes about as a consequence of the assumption that the volume in which the fission gas may accumulate is predominantly the residual void volume left by the sintering of the fuel particle. However, if the assumed immediate sintering does not indeed occur, the low density fuel particle may permit significant diffusion of fission gases, especially at high temperatures, more than offsetting the beneficial effect of low density. An additional enigma comes about because the fabricator strives for high density particles in order to reduce the deformations and stringing that accompany the rolling process. This final difficulty might be overcome if pressure bonding, with its associated negligible deformations, could be employed as the fabrication method.

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7.0 RECOMMENDATIONS FOR FUTURE RESEARCH

It is evident from a search of the literature that very little analytic work has been done on the problem of dispersion fuel element failure. Greater analytic effort is indicated to supplement the larger experimental efforts in the field.

The analytic formulation of the model for the failure problem can be improved if more rigorous treatment of certain questions could be established. These include effects of the following considerations:

1. Interactions and non-uniform spacing of particles.
2. Ellipsoidal shape of particles.
3. Two-dimensional thermal stress distribution.
4. Finite diffusion rate and radioactive decay of fission gases.
5. Solid fission products.
6. Non-homogeneity of mechanical properties.

The effects of neglecting the first and second of these considerations may be severe. In an improved analysis, an empirical method could be used to describe the worst (i. e., closest) spacing of fuel particles which occurs in a fuel plate. The technique for the analysis of a system containing two spherical cavities⁽¹¹⁾ could then give a stress concentration factor for these closest particles. The stress concentration due to the ellipsoidal cavity shape may be determined separately.⁽¹²⁾ The worst stress may then be more accurately approximated by the product of the stress concentration factors at a critical point on the cavity surface. The use of high speed computers may make the incorporation of these considerations feasible. The uniform field for these analyses should be determined from a two-dimensional thermal stress analysis.

The fourth consideration introduces time as a parameter in the analysis. As accumulation of fission gas may be treated independently of the stress analysis, the basic formulation of the problem might be recast to determine allowable fission gas accumulation or pressure as the criterion of failure. The final determination of burnup would then be obtained by incorporating burnup and finite diffusion rates and radioactive decay in the calculation of the critical accumulation. Fission product release from spherical particles, with decay and production accounted for, has been investigated⁽¹³⁾; relatively simple formulas for accumulation of fission gas released from spherical particles are available.

The incorporation of the fifth and sixth consideration may be impractical because of the uncertainties associated with their effects. For the greatest accuracy, however, these should be considered. Certain experimental effort is needed to obtain more knowledge about the failure process. The concept of instantaneous sintering of the UO_2 particles to 100 percent density with consequential initial voids may require modification. The question of whether the failure mechanism is plastic or elastic remains unresolved. Clad failure, which has not been considered in this investigation, may be important. The mechanisms may vary in importance for one design operated at different temperature levels and irradiation intensities. These fundamental questions may need to be settled experimentally or at least by further evaluation of previously performed experiments.

Finally, much more information is needed about the data used in the calculations. Values for properties of irradiated material at elevated temperature are especially difficult to find. In many cases these data can be obtained only by experiment. In others, a more thorough search of present knowledge may turn up reliable data.

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APPENDIX A - STRESSES AND DEFLECTIONS IN THE VICINITY OF CAVITY

The stress distribution in the matrix in the vicinity of a fuel particle is approximated by the following mathematical model. Let the matrix be represented by an infinite medium subjected to a uniform uniaxial compressive stress $\sigma_z = -c$ ($\sigma_x = \sigma_y = \tau_{xy} = \tau_{yz} = \tau_{zx} = 0$) at infinity. The fuel particle is contained in a spherical cavity at uniform pressure P . The matrix is a homogeneous, isotropic elastic medium, with no body forces.

It is convenient to describe a spherical coordinate system (Figure A-1) whose origin is at the center of the cavity and which is related to the cartesian coordinates by

$$\left. \begin{aligned} x &= r \sin \theta \cos \phi \\ y &= r \sin \theta \sin \phi \\ z &= r \cos \theta \end{aligned} \right\} \quad (A-1)$$

The line $\theta = 0$ and π coincides with the positive and negative branches of the z axis and is the axis of symmetry for the described problem.

The mathematical formulation of this problem is to determine the set of stresses which satisfies the equations of equilibrium and the boundary conditions. Axial symmetry implies that

$$\tau_{\phi r} = \tau_{\phi \theta} = 0 \quad (A-2)$$

and that derivatives with respect to ϕ are zero. The equations of equilibrium in spherical coordinates for an axially symmetric stress distribution are ^(14a.)

$$\begin{aligned} \frac{\partial \sigma_r}{\partial r} + \frac{1}{r} \frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{1}{r} (2\sigma_r - \sigma_\theta - \sigma_\phi + \tau_{r\theta} \cot \theta) &= 0 \\ \frac{\partial \tau_{r\theta}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_\theta}{\partial \theta} + \frac{1}{r} \left[(\sigma_\theta - \sigma_\phi) \cot \theta - 3\tau_{r\theta} \right] &= 0 \end{aligned} \quad (A-3)$$

Figure A-1. Spherical and Cartesian Coordinates

The boundary conditions are

$$\left. \begin{aligned} \sigma_r &= -C \cos^2 \theta \\ \sigma_\theta &= -C \sin^2 \theta \\ \tau_{r\theta} &= C \sin \theta \cos \theta \\ \sigma_\phi &= 0 \end{aligned} \right\} \quad \text{at } r = \infty \quad (\text{A-4})$$

and

$$\left. \begin{aligned} \sigma_r &= -P \\ \tau_{r\theta} &= 0 \end{aligned} \right\} \quad \text{at } r = a \quad (\text{A-5})$$

The principle of superposition enables reduction of the problem to two simpler problems, whose solutions when added together give the solution to the complete problem. Each portion must satisfy the equilibrium conditions (A-3). The first part is that of uniaxial compression, with stress-free surface in the cavity. This requires the boundary conditions to be Equations (A-4 at infinity, and

$$\sigma_r = \tau_{r\theta} = 0, \quad \text{at } r = a \quad (\text{A-6})$$

The second problem is for uniform pressure on the cavity surface and stress-free at infinity. The boundary conditions for the internal pressure problem are Equations (A-5 at $r = a$, and

$$\sigma_r = \sigma_\theta = \sigma_\phi = \tau_{r\theta} = 0, \quad \text{at } r = \infty \quad (\text{A-7})$$

For each of the problems, the unique solution is that one which satisfies the equilibrium conditions (A-3) and the appropriate boundary conditions, and which may be derived from an admissible displacement field.

The infinite medium with uniaxial compression at infinity is discussed by Timoshenko and Goodier.⁽¹⁵⁾ They give their results in terms of a superposition of several stress distributions in mixed spherical and cylindrical coordinates. However, we are able to transform their results to spherical coordinates by a somewhat lengthy calculation which we will not describe here. It is sufficient to present the resulting stress field. It is readily verified that the field satisfies

the equilibrium conditions (A-3) and the boundary conditions (A-4, A-6). The compatible deflection field will be obtained below. The stress field thus obtained will be the unique solution to the prescribed problem. The stress field is:

$$\begin{aligned}
 \sigma_r &= -\frac{C}{7-5\nu} \left\{ (7-5\nu) \cos^2 \theta + [6-5(5-\nu) \cos^2 \theta] \left(\frac{a}{r}\right)^3 + 6(3 \cos^2 \theta - 1) \left(\frac{a}{r}\right)^5 \right\} \\
 \sigma_\theta &= -\frac{C}{2(7-5\nu)} \left\{ 2(7-5\nu) \sin^2 \theta + [3(3-5\nu) - 5(1-2\nu) \sin^2 \theta] \left(\frac{a}{r}\right)^3 + 3(7 \sin^2 \theta - 4) \left(\frac{a}{r}\right)^5 \right\} \\
 \sigma_\phi &= -\frac{3C}{2(7-5\nu)} \left\{ [5(1-2\nu) \cos^2 \theta - (2-5\nu)] \left(\frac{a}{r}\right)^3 + (1-5 \cos^2 \theta) \left(\frac{a}{r}\right)^5 \right\} \\
 \tau_{r\theta} &= \frac{C \sin \theta \cos \theta}{7-5\nu} \left\{ 7-5\nu + 5(1+\nu) \left(\frac{a}{r}\right)^3 - 12 \left(\frac{a}{r}\right)^5 \right\} \\
 \tau_{\phi r} &= \tau_{\theta \phi} = 0
 \end{aligned} \tag{A-8}$$

The strains are obtained with the stress-strain relationships

$$\left. \begin{aligned}
 \epsilon_\alpha &= \frac{1}{E} \left[(1+\nu) \sigma_\alpha - \nu \Theta \right] \\
 \epsilon_{\alpha\beta} &= \frac{1+\nu}{E} \sigma_{\alpha\beta} \quad (\alpha \neq \beta)
 \end{aligned} \right\} \tag{A-9}$$

where

$$\Theta = \sigma_r + \sigma_\theta + \sigma_\phi$$

and the subscripts α and β may have any of the values r , θ or ϕ . We find the following strain field:

$$\epsilon_r = -\frac{C}{E(7-5\nu)} \left\{ (7-5\nu) \left[(1+\nu) \cos^2 \theta - \nu \right] + (1+\nu) \left[6-5\nu-5(5-4\nu) \cos^2 \theta \right] \left(\frac{a}{r} \right)^3 + 6(1+\nu) (3 \cos^2 \theta - 1) \left(\frac{a}{r} \right)^5 \right\}$$

$$\epsilon_\theta = -\frac{C}{2E(7-5\nu)} \left\{ 2(7-5\nu) \left[1-(1+\nu) \cos^2 \theta \right] + (1+\nu) \left[4-15\nu+5(1+4\nu) \cos^2 \theta \right] \left(\frac{a}{r} \right)^3 + 3(1+\nu) (3-7 \cos^2 \theta) \left(\frac{a}{r} \right)^5 \right\} \quad (A-10)$$

$$\epsilon_\phi = -\frac{C}{2E(7-5\nu)} \left\{ -2\nu(7-5\nu) + (1+\nu) (5\nu-6+15 \cos^2 \theta) \left(\frac{a}{r} \right)^3 + 3(1+\nu) (1-5 \cos^2 \theta) \left(\frac{a}{r} \right)^5 \right\}$$

$$\epsilon_{r\theta} = \frac{C(1+\nu) \sin \theta \cos \theta}{E(7-5\nu)} \left\{ (7-5\nu) + 5(1+\nu) \left(\frac{a}{r} \right)^3 - 12 \left(\frac{a}{r} \right)^5 \right\}$$

$$\epsilon_{\theta r} = \epsilon_{\theta\phi} = 0$$

The strains are related to the displacements by (14b).

$$\frac{\partial u}{\partial r} = \epsilon_r$$

$$\frac{v \cot \theta}{r} + \frac{u}{r} = \epsilon_\theta \quad (A-11)$$

$$\frac{1}{r} \frac{\partial v}{\partial \theta} + \frac{u}{r} = \epsilon_\theta$$

$$\frac{1}{2} \left[r \frac{\partial}{\partial r} \left(\frac{v}{r} \right) + \frac{1}{r} \frac{\partial u}{\partial \theta} \right] = \epsilon_{r\theta}$$

where u and v are respectively the radial displacement and the displacement in the θ direction. Displacements in the ϕ direction vanish identically because of radial symmetry.

The strain field, and thus the stress field, is compatible if a displacement field exists from which the strains can be derived. The displacements are found by solving the first two Equations (A-11). The last two equations are satisfied by the resulting solutions. We find

$$u = -\frac{Cr}{2E(7-5\nu)} \left\{ 2(7-5\nu) \left[(1+\nu) \cos^2 \theta - \nu \right] \right. \\ \left. - (1+\nu) \left[6-5\nu - 5(5-4\nu) \cos^2 \theta \right] \left(\frac{a}{r} \right)^3 - 3(1+\nu)(3 \cos^2 \theta - 1) \left(\frac{a}{r} \right)^5 \right\} \\ v = \frac{Cr(1+\nu) \sin \theta \cos \theta}{E(7-5\nu)} \left\{ (7-5\nu) + 5(1-2\nu) \left(\frac{a}{r} \right)^3 + 3 \left(\frac{a}{r} \right)^5 \right\} \quad (A-12)$$

Since the four equations are satisfied by the two functions u and v , we have the unique solution to the first stress problem.

The solution to the second problem (pressure in the hole) is well known^(14c). The stresses are given by

$$\sigma_r = -\frac{Pa^3}{r^3} \\ \sigma_\theta = \sigma_\phi = \frac{Pa^3}{2r^3} \quad (A-13)$$

and the radial displacement is

$$u = \frac{P(1+\nu)a^3}{2Er^2} \quad (A-14)$$

The remaining stresses and displacements are zero.

The stresses and deflections in the combined problem are the sums of the stresses and deflections for the two separate problems.

We next consider the volume of the deformed cavity. If ρ is the local radius of a cavity which is both axially symmetric and symmetric with respect to the plane $\theta = \pi/2$, its volume H is given in spherical coordinates by

$$H = \frac{4\pi}{3} \int_0^{\pi/2} \rho^3 \sin \theta d\theta \quad (A-15)$$

For the case we are considering ρ is the sum of the initial radius a and the deflections at $r = a$ due to the internal pressure and the uniaxial field.

$$\rho = a \left\{ 1 - \frac{3C(1-\nu)}{2E(7-5\nu)} \left[10(1+\nu) \cos^2 \theta - (1+5\nu) \right] + \frac{P(1+\nu)}{2E} \right\} \quad (A-16)$$

Since the final two terms are small compared to unity, we may approximate

$$\rho^3 = a^3 \left\{ 1 - \frac{9C(1-\nu)}{2E(7-5\nu)} \left[10(1+\nu) \cos^2 \theta - (1+5\nu) \right] + \frac{3P(1+\nu)}{2E} \right\} \quad (A-17)$$

Substituting Equation (A-17) into (A-15) and integrating gives

$$H = \frac{4\pi a^3}{3} \left\{ 1 - \frac{3C(1-\nu)}{2E} + \frac{P(1+\nu)}{2E} \right\} \quad (A-18)$$

The volume of the cavity before deformation was simply

$$V_0 = \frac{4\pi a^3}{3} \quad (A-19)$$

Although the volume of the cavity has not changed appreciably, it is noted that the initial cavity was in part filled with fuel material. Thus, the available void volume is more significantly changed by the change in the volume of the cavity.

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APPENDIX B. THERMAL STRESS DISTRIBUTION IN THE PLATE

The temperature and stress distribution in the plate will be represented by the approximation suggested in APAE No. 69. (2) Because of an error in the integration there, the solution and certain simplifications shall be outlined here. Furthermore, we shall use the data presented in that report to generate numerical values.

The assumed temperature variation is that across the width of the plate. This temperature distribution leads to stresses normal to a cross section across the width of the plate. Although the temperature does vary across the thickness and in the axial directions, these variations will be neglected, since the additional complications imposed are not justified by the relatively small contribution of the thermal stress to the postulated failure mechanism.

The proposed heat equations are based on a fin analysis. The plate is divided into three regions as shown in Figure B-1; region 1 is the fuel bearing portion of the plate in which heat is produced; region 2 is a non-heat-producing conductor; region 3 is at the bulk temperature of the water.

Figure B-1. Quarter Cross-Section of Fuel Element Component

The fuel element consists of a set of identical parallel thin plates connected by side plates into a composite structure. The component described in the analysis is a quarter plate and its accompanying side plate which comprise a unit of symmetry.

Symbols:

U_i = temperature excess above bulk coolant temperature

K = thermal conductivity

x = coordinate of position in the plate

$Q \cosh \beta x$ = empirical heat generation function

σ = stress normal to cross section

ϵ = strain normal to cross section

α = thermal expansion coefficient

E = modulus of elasticity

W_i, H_i = dimensions of fuel element component

h = film coefficient for heat loss to the coolant

M^2 = h/KH_1

The heat equations are written

$$\frac{d^2 U_1}{dx^2} - M^2 U_1 + \frac{Q}{K} \cosh \beta x = 0$$

(B-1)

$$\frac{d^2 U_2}{dx^2} - M^2 U_2 = 0$$

The boundary conditions are

$$\begin{aligned}
 \frac{dU_1}{dx} &= 0 & \text{at } x &= 0 \\
 \left. \begin{aligned} U_1 &= U_2 \\ \frac{dU_1}{dx} &= \frac{dU_2}{dx} \end{aligned} \right\} & \text{at } x &= W_1 \\
 U_2 &= 0 & \text{at } x &= W_2
 \end{aligned} \tag{B-2}$$

We find as the solution to Eqs (B-1)

$$\begin{aligned}
 U_1 &= A_1 \cosh Mx + B_1 \sinh Mx + S \cosh \beta x \\
 U_2 &= A_2 \cosh Mx + B_2 \sinh Mx
 \end{aligned} \tag{B-3}$$

where

$$\begin{aligned}
 S &= \frac{Q}{K(M^2 - \beta^2)} \\
 A_1 &= S \left[\cosh \beta W_1 (\tanh MW_2 \sinh MW_1 - \cosh MW_1) \right. \\
 &\quad \left. - (\beta/M \sinh \beta W_1 (\tanh MW_2 \cosh MW_1 - \sinh MW_1)) \right] \\
 B_1 &= 0 \\
 A_2 &= S \tanh MW_2 (\sinh MW_1 \cosh \beta W_1 - (\beta/M) \cosh MW_1 \sinh \beta W_1) \\
 B_2 &= S (\sinh MW_1 \cosh \beta W_1 - (\beta/M) \cosh MW_1 \sinh \beta W_1)
 \end{aligned} \tag{B-4}$$

Note that MW_1 and MW_2 in general, and that Mx in region 2 are large. Furthermore, the coefficient A_1 is so small that in region 1 the factor $\cosh Mx$ is significant only for Mx large. Thus, the following simplifications may be used:

$$\begin{aligned}
 \tanh MW_2 &= 1 - 2e^{-2MW_2} + 2e^{-4MW_2} - \dots \\
 \cosh Mx &= \frac{e^{Mx}}{2} + \frac{e^{-Mx}}{2} \approx \frac{e^{Mx}}{2} \\
 \sinh Mx &= \frac{e^{Mx} - e^{-Mx}}{2} \approx \frac{e^{Mx}}{2} \\
 \tanh MW_2 \cosh Mx - \sinh Mx &\approx e^{-Mx} - e^{-M(2W_2 - x)} \\
 \tanh MW_2 \sinh Mx - \cosh Mx &\approx -e^{-Mx} - e^{-M(2W_2 - x)}
 \end{aligned} \tag{B-5}$$

Use of these formulas enables us to write the simplification of the solution to Eqs. (B-1).

$$\left. \begin{aligned}
 U_1 &= S (\cosh \beta x - D_1 e^{Mx}) \\
 U_2 &= S D_2 e^{MW_1} \left(e^{-Mx} - e^{-M(2W_2 - x)} \right)
 \end{aligned} \right\} \tag{B-6}$$

where

$$\left. \begin{aligned}
 D_1 &= (1/2) \left[e^{-MW_1} + e^{-M(2W_2 - W_1)} \cosh \beta W_1 \right. \\
 &\quad \left. + (\beta/M) \left(e^{-MW_1} - e^{-M(2W_2 - W_1)} \right) \sinh \beta W_1 \right] \\
 D_2 &= (1/2) \left[\cosh \beta W_1 - (\beta/M) \sinh \beta W_1 \right]
 \end{aligned} \right\} \tag{B-7}$$

The maximum temperature in the plate is particular interest. This occurs in region 1 at the value x^* that satisfies

$$\frac{dU_1}{dx} = 0 \tag{B-8}$$

This implies

$$M D_1 e^{Mx^*} = \beta \sinh \beta x^* \quad (B-9)$$

Taking $\log_{10}$ of both sides gives

$$x^* = \frac{2.30259}{M} \log \left[\left(\frac{\beta}{MD_1} \right) \sinh \beta x^* \right] \quad (B-10)$$

We may find x^* by an iteration process which rapidly converges to the appropriate value.

We wish next to calculate the thermal stress induced by the temperature distribution Eqs. (B-6). We assume plane strain. Then

$$\epsilon = \frac{\sigma}{E} + \alpha U \quad (B-11)$$

is a constant. The stress is then

$$\sigma = E (\epsilon - \alpha U) \quad (B-12)$$

The resultant force on any cross section is zero. Therefore, if E and α are assumed constant

$$\int_0^{W_1} H_1 (\epsilon - \alpha U_1) dx + \int_{W_1}^{W_2} H_1 (\epsilon - \alpha U_2) dx + \int_{W_2}^{W_3} H_3 \epsilon = 0 \quad (B-13)$$

Therefore

$$\frac{\epsilon}{\alpha} = \frac{S}{W_2 + (W_3 - W_2) H_3/H_1} \left[\int_0^{W_1} U_1 dx + \int_{W_1}^{W_2} U_2 dx \right] \quad (B-14)$$

$$\begin{aligned} \frac{\epsilon}{\alpha} = \frac{S}{W_2 (W_3 - W_2) H_3/H_1} & \left\{ \frac{\sinh \beta W_1}{\beta} \left[1 - \frac{\beta^2}{M^2} \left(1 - e^{-M(W_2 - W_1)} \right) \right. \right. \\ & \left. \left. - \frac{e^{-MW_1} - e^{-M(2W_2 - W_1)}}{2} \right] - \frac{\cosh \beta W_1}{M} \left[e^{-M(W_2 - W_1)} \right. \right. \\ & \left. \left. - \frac{e^{-MW_1} + e^{-M(2W_2 - W_1)}}{2} \right] \right\} \quad (B-15) \end{aligned}$$

In many cases, it is adequate to write

$$\frac{\epsilon}{\alpha} = \frac{S}{W_2 + (W_3 - W_2)H_3/H_1} \frac{\sinh \beta W_1}{\beta} \quad (B-16)$$

The maximum magnitude of stress in region 1 occurs where the temperature is a maximum.

$$\sigma = E\alpha \left[\epsilon/\alpha - U_1(x^*) \right] \quad (B-17)$$

where ϵ/α is obtained from Eq. (B-14) and $U_1(x^*)$ from Eq. (B-6) and the solution to Eq. (B-9).

The data used in the calculations are those given in APAE No. 69 (2).

W_1	=	1.335 in.
W_2	=	1.395 in.
$W_3 - W_2$	=	$4.000 (10)^{-2}$ in.
H_1	=	$2.000 (10)^{-2}$ in.
H_3	=	$8.100 (10)^{-2}$ in.
K	=	11.00 Btu/hr-ft ⁰ F
Q	=	$3.310 (10)^8$ Btu/hr-ft ³
M	=	48.00 in. ⁻¹
β	=	0.5000 in. ⁻¹
E	=	$26 (10)^6$ lbs/in ²
α	=	$9.7 (10)^{-6}$ °F ⁻¹

We calculate the following parameters:

$$S = 90.71^{\circ}\text{F}$$

$$\cosh \beta W_1 = 1.2312$$

$$\sinh \beta W_1 = 0.7182$$

$$e^{-MW_1} = 1.4805 (10)^{-28}$$

$$e^{-M(2W_2 - W_1)} = 4.665 (10)^{-31}$$

$$e^{-M(W_2 - W_1)} = 5.613 (10)^{-2}$$

$$D_1 = 9.198 (10)^{-29}$$

$$x^* = 1.2412$$

$$e^{Mx^*} = 7.486 (10)^{25}$$

$$\cosh \beta x^* = 1.1988$$

$$U^* = 108.12^{\circ}\text{F}$$

$$\epsilon/\alpha = 83.59^{\circ}\text{F}$$

$$C = \sigma_{\max} = 6.19 (10)^3 \text{ psi, compression}$$

The bulk temperature in the coolant is assumed to be 520°F . Thus, the maximum calculated plate temperature is 628°F , or approximately 605°K .

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APPENDIX C. FISSION GAS ACCUMULATION

The release of fission gases is assumed to be essentially as described by Weir. However, certain of these parameters are modified. Specifically, for the number of moles of gas generated by 1% burnup of the uranium atoms in one liter of UO_2 , it is found that

$$N = \frac{.24 \left[\frac{\text{atoms gas}}{\text{atom U bu}} \right] 2.46 (10)^{25} \left[\frac{\text{atoms U}}{\text{liter } UO_2} \right]}{100 \left[\frac{\%}{\%} \right] 6.025 (10)^{23} \left[\frac{\text{atoms}}{\text{mol}} \right]}$$

$$N = 9.8 (10)^{-2} \frac{\text{moles}}{\text{liter } UO_2 - \% \text{ burnup}}$$

Weir used the figure $N = 9.7 (10)^{-2}$

For the recoil ranges in UO_2 data given in the Reactor Handbook (16) is extrapolated. For U_3O_8 , the range of light fragments is given as 10.0 mg/cm^2 . Assume the range is the same in UO_2 and convert to range in microns as follows:

$$r_2 = \frac{10.0 \left[\frac{\text{mg}}{\text{cm}^2} \right] (10)^4 \left[\frac{\text{micron}}{\text{cm}} \right]}{10.90 \left[\frac{\text{gm}}{\text{cm}^3} \right] 10^3 \left[\frac{\text{mg}}{\text{gm}} \right]}$$

$$r_2 = 9.2 \text{ microns}$$

Further, it is assumed that the ranges of light and heavy fragments in UO_2 are in the same ratio as in air. Thus

$$r_1 = 9.2 \left(\frac{1.8}{2.3} \right) = 7.2 \text{ microns}$$

These values are used for both the Weir and the elastic models instead of 6.21 and 4.86 micron as used by Weir.