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Model for Ion Confinement in a Hot-Electron
Tandem Mirror Anchor

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MODEL FOR ION CONFINEMENT IN A HOT-ELECTRON TANDEM MIRROR ANCHOR

ABSTRACT

Anisotropic, hot electrons trapped in local minimum-B wells have been proposed as MHD-stabilizing anchors to an otherwise axisymmetric tandem configuration. This work describes a model for plasma confinement between the anchors and the remainder of the system and calculates the power loss implied by maintenance of this plasma.

I. INTRODUCTION

A recently proposed tandem mirror experiment by Post et al.,¹ uses hot electron plasma in minimum-B cells to anchor an otherwise axisymmetric solenoid and plug configuration. The anchor cells are to be located at the axial ends of the device, outside the peak ion-confining potential see (Fig. 1). All confined hot ions see only axisymmetric fields and are therefore essentially free from resonant transport radial loss.

This configuration bears a prima facie resemblance to a column that is stabilized by line tying to conducting walls. The important difference here is that the tying is to anchors that are themselves separated from the walls. The prohibitive power loss characteristic of normal line tying should consequently be greatly reduced. However, maintenance of sufficient plasma between the bad curvature regions and the anchors (in what is here called the "transition region") will require an end-loss current that is enhanced over that of the column itself, and this enhanced loss will increase the electron thermal contact with the walls. It is important,

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therefore, to model the required confinement in this transition region, to determine the density of plasma that can be held in the transition region, and to calculate the subsequent power drain.

The problem splits naturally into two parts, only one of which will be treated here. The first, which is not treated, deals with the amount of plasma required in the transition to effectively anchor the system to low- β interchange modes. Within anisotropic MHD, the interchange problem and the companion one of finite- β ballooning limits are being calculated for high- m by Kaiser.² The subject should also be addressed kinetically to treat more carefully the electrical contact between two regions of finite β by a region of extremely low β and to treat the non-MHD effects of the fast-drifting hot electrons. At the outset, one would expect that the required ratio of density in the transition to that in the column would be a good deal less than that required for an equivalent line tying to a conducting wall. This follows because of the important role played by the sheath that forms at the wall, which has been shown to greatly restrict the plasma electrical contact with the wall.³ The ultimate utility of the idea of using hot-electron anchors cannot be assessed until the required density in the transition region has been quantified.

The problem treated here deals with the degree of enhanced end loss that will result for a given transition region plasma density required for stability. The experiment design group has succeeded in developing a magnetic profile having very slow drift speeds, so it is plausible that end loss will dominate radial loss. This will be confirmed once characteristic temperatures, densities, etc. are established.

That the anchor cells themselves can be generated by electron cyclotron resonant heating (ECRH) of plasma to the required β -values has been demonstrated by the INTEREM experiment. In this minimum-B mirror, hot-

electron plasmas (~ 300 to 600 keV) having β -values up to 50% or above and densities in the range 10^{11} to low 10^{12} cm^{-3} were routinely generated by single or multiple microwave frequencies.⁴⁻⁶ Use of multiple-frequency heating avoided the onset of electron instabilities that occurred with a single frequency, presumably by decreasing the anisotropy. The success of these experiments bodes well for the formation of tandem mirror anchor cells.

In the following, we shall first model the ion component in INTEREM-type hot-electron plasmas. The required electrostatic confinement of ions that neutralize the hot electrons in that experiment is very similar to that required to confine ions in the transition region of the tandem mirror. The results of INTEREM could then be used as a test case for our description; however, a number of important parameters such as the ion temperature and the thermal electron density fraction were not measured.

II. ISOLATED HOT-ELECTRON MIRROR MACHINES

We avoid describing the formation of the hot-electron component by ECRH and take the density and temperature, N_H and T_H , as parameters given by experimental measurement. Ions, produced at a rate $j_i/\text{cm}^{-3} \cdot \text{s}^{-1}$ by ionization of background gas, must be confined at a sufficient density to neutralize both these hot, magnetically confined electrons and a thermal electron component that extends beyond the mirror throats. This thermal component is in turn required to neutralize the escaping ions as they flow from the region of hot electrons to the walls. Confinement of thermal electrons will require the overall system to float at a positive potential having a peak ϕ_0 , which has a relative dip $\Delta\phi$ in the region of the hot electrons that confines thermal ions.

The anticipated qualitative density and potential profiles are shown in Fig. 2(a). The hot electrons N_H are confined between the mirrors; the ion density n_i peaks with the hot electrons, but is non-zero outside the mirrors; the thermal electrons n_e neutralize the ions outside and penetrate with reduced density into the midplane. In all that follows, we shall neglect profile effects and replace these profiles by equivalent square profiles.

The ionization current is given by

$$j_i = n_0 \left[n_e \langle \sigma v \rangle_e + N_H \langle \sigma v \rangle_H \right] , \quad (1)$$

where n_0 is the density of neutral atoms which we take to be given by $n_0 = 3.2 \times 10^{16} p_0(\text{Torr}) \text{ cm}^{-3}$, where p_0 is the base pressure in Torr. The ionization rates for atomic and molecular hydrogen, obtained from a product of cross section⁷ and velocity, are shown in Fig. 3. At the densities and dimensions of interest, the plasma is transparent even to room-temperature neutrals.

For typical parameters, $n_0 \approx 10^{11} \text{ cm}^{-3}$, $T_H \approx 300 \text{ keV}$, $N_H \approx 2 \times 10^{11} \text{ cm}^{-3}$, and $j_i \approx 2 \times 10^{14} \text{ cm}^{-3} \cdot \text{s}^{-1}$, there is required a cold-ion confinement parameter $\pi \tau \approx 2 \times 10^8 \text{ cm}^{-3} \text{ s}$. Electrostatic confinement in the potential dip $\Delta\phi$ is important in confining cold ions with this $\pi \tau$ value.

In modelling this confinement, we shall use the result that in the intermediate regime between long and short mean free paths, the confinement time can be represented by the sum of the two limiting times⁸

$$\tau_i = \left(RL \left(\frac{\pi m_i}{2 T_i} \right)^{1/2} + g(r) \tau_0 \frac{\Delta\phi}{T_i} \right) \exp \frac{\Delta\phi}{T_i} , \quad (2)$$

where R is the mirror ratio, $g(R) = 2 \div 3$, and τ_0 is the 90° scatter time. Here L is an effective plasma length. In isolated mirrors having loss out both ends, such as INTEREM, L is the plasma length. In mirror regions plugged at one end by a high potential so that all of the loss is out one end, such as the anchor of a tandem mirror, L is twice the overall length. For typical parameters, the two terms in Eq. (2) are comparable, although usually the first dominates.

We do not attempt to model the ion temperature. Ionization of the molecular hydrogen by few eV electrons, through intermediate singly charged states, results in a source energy distribution having typically 2 eV average energy.⁷ Here, we shall treat T_i as a parameter, expected to be in the range of 2 eV in INTEREM where the electron temperature was about 10 to 15 eV.

Because the entire potential is positive with respect to ground by an amount larger than $\Delta\phi$, the thermal electrons will be Boltzmann distributed between the midplane, where their density is n_e , and the mirror throat, where their density is n_m :

$$\Delta\phi = T_e \ln \left(\frac{n_m}{n_e} \right) \quad (3)$$

The ion density at the mirror throat is critical; only by n_m exceeding the thermal electron density at the midplane can $\Delta\phi$ be positive. n_m is itself related to the ionization rate by

$$n_m = j_i R L \left(\frac{\pi m_i}{2 T_i} \right)^{1/2} \quad (4)$$

where we have assumed the ion distribution in the throat to be Maxwellian. This is quite a good assumption considering the near-collisional character of the confinement.

Writing

$$j_i = \frac{n_i}{\tau_i} \quad , \quad (5)$$

where $n_i = N_H + n_e$ is the midplane ion density, and using Eqs. (3) through (5) gives

$$\exp \Delta \psi \left(\frac{1}{T_e} + \frac{1}{T_i} \right) = \frac{n_e + N_H}{n_e} \frac{\tau_f}{\tau_i} \quad , \quad (6)$$

where $\tau_f = RL(\pi m_i / 2 T_i)^{1/2}$, and

$$j_i = \frac{n_e}{\tau_f} \left(\frac{n_e + N_H}{n_e} \frac{\tau_f}{\tau_i} \right)^{\frac{T_i}{T_i + T_e}} \quad . \quad (7)$$

For simplicity we shall assume $T_i \ll T_e$. This is fairly good for INTEREM and very good for the tandem mirror. Then, from Eq. (1)

$$T_i (\text{eV}) = \left(\frac{n_0 RL \sqrt{\mu}}{7.8 \times 10^5} \right)^2 \left(\left(\frac{\sigma v}{n_e} \right)_e + \frac{N_H}{n_e} \left(\frac{\sigma v}{n_e} \right)_H \right)^2 \quad , \quad (8)$$

where μ is the atomic mass number. This expression relates T_i , $n_0 RL$, and N_H/n_e . If it cannot be satisfied, e.g., by choosing too high an n_0 for given T_i , the system cannot get rid of ions as quickly as they are ionized. Physically, the density will peak up, causing a fall in potential to the mirror throat so that ions are accelerated out on the time scale of the ion acoustic speed. In using Eq. (2) for the ion lifetime, we have assumed that there is no such acceleration--the transit rate is at the ion thermal velocity. By displacing thermal electrons, the hot electrons in

conjunction with the ion density at the mirror throat allow the potential to stay flat and in fact to have a slight rise $\Delta\phi$.

For INTEREM, we use $T_e = 15$ eV (inferred from a minimum energy of end-loss ions of 40 eV⁵), $T_i = 2$ eV (not measured and guessed from typical ionization energies⁷ of molecular hydrogen through the H_2^+ channel for $T_e = 15$ eV), $T_H = 300$ keV as typical, and $RL = 50$ cm. Then for molecular hydrogen

$$(\sigma v)_e \approx 1.5 \times 10^{-8} \text{ cm}^3 \cdot \text{s}^{-1} ,$$

and

$$(\sigma v)_H \approx 8.5 \times 10^{-9} \text{ cm}^3 \cdot \text{s}^{-1} .$$

Equation (8) becomes

$$n_0 \left(1 + .57 \frac{n_H}{n_e} \right) = 1.0 \times 10^{12} \text{ cm}^{-3} .$$

At a neutral pressure of 10^{-6} Torr, this gives a thermal electron fraction of only 2% at the midplane. Because $\tau_i \approx 2\tau_f$, using Eq. (6) this gives $\Delta\phi/T_i \approx 3.2$.

At a volume of $V = 11$ l, the power carried out by the thermal electrons,

$$\text{Power} = j_i (\phi_e + T_e) V ,$$

with $\phi_e \approx 4 T_e$ is only about 9 W, 1% to 10% of the injected ECRH power.

Equation (8) followed from three basic points: (i) that the ionization rate is given by Eq. (1); (ii) that n_e was very close to the ion density in the mirror throat; and (iii) that the throat density was related to the ionization rate by Eq. (4). For INTEREM parameters, they lead to the

thermal electron fractions given above. Such low values must be consistent with microstability of the compound distribution and with the availability of a source, or feed, for the hot electrons. An important conclusion is that n_e can be determined experimentally by measuring n_m , so that theoretically predicted warm-electron fractions can be measured.

The assumed value for T_i was important in obtaining the low value of n_e/N_H for INTEREM. For example, were the ion temperature closer to room temperature, the calculated n_e/N_H ratio would be 10 times higher. One mechanism for such a reduction in T_i is through charge-exchange cooling. We have already assumed ionization $H_2 \rightarrow H_2^+$, followed by a rapid dissociation into $H + H^+$ or $2H^+$ with $T_i \approx 2$ eV. Further charge exchange on cold atomic hydrogen would result in a cold end product. The ultimate temperature would lie somewhere between the two extremes. Also, were the experiment to run on atomic as opposed to molecular hydrogen, all daughters would be cold, and dramatic increases in n_H n_0/n_e should occur.

III. TANDEM CONFIGURATION

The results of the preceding section can be carried over directly to the tandem mirror anchor. From the midplane of the anchor outwards [(rightward in Fig. (2b)], the profiles are the same. Inward, toward the solenoid, there are ions trapped between the small rise $\Delta\phi$ and the large potential of the main plug [(to the left in Fig. (2b)]. As before, for $\Delta\phi$ to be positive, the thermal electron density in the transition region cannot exceed n_m , the ion density in the outer mirror throat. Again as before, the thermal electron density must be nearly constant from the outer mirror to the anchor midplane and the transition region. The conclusion is that

n_m sets the density in the transition region, and the formulation developed in Sec. II can be carried over directly.

Because in the tandem configuration all ions ionized in the transition region are lost through the outer mirror, the density n_m is correspondingly higher, and the effective length in Eq. (8) should be twice the actual length. In more detail, the length of each region is to be weighted with its mirror ratio due to the proper $\int d\ell/B$ origin of the term. For the proposal, the anchor and transition are each about 100 cm, leading to a value $RL = 2 \times (2 \times 100 + 100) = 600$ cm. We will neglect the added ion density of $\approx 3 \times 10^9 \text{ cm}^{-3}$ resulting for solenoid and plug end-loss.

Assuming the proposal-derived value for the electron temperature in the solenoid, 400 eV, and that these electrons predominate in the anchor, we obtain a thermal electron ionization rate $(\sigma v)_e = 5.5 \times 10^{-8} \text{ cm}^3 \cdot \text{s}^{-1}$. Assuming the same molecular ion scenerio on which 2 eV in INTEREM was based, the higher T_e suggests $T_i \approx 8$ eV in the tandem.⁷ For these parameters, Eq. (8) becomes

$$n_0 \left(1 + 0.16 \frac{N_H}{n_e} \right) = 6.7 \times 10^{10} \text{ cm}^{-3} \quad ,$$

or

$$p_0(T) \left(1 + 0.16 \frac{N_H}{n_e} \right) = 1.86 \times 10^{-6} \text{ Torr} \quad ,$$

where $p_0(T)$ is the neutral pressure. Operation with $p_0 \approx 1 \div 2 \times 10^{-6}$ Torr should produce $n_e \approx N_H$, permitting densities in the transition region comparable to the hot-electron density.

For the case $n_e = N_H$, we calculate the implied power drain on the thermal electrons as

$$\text{Power} = j_f(\phi_e + T_e)V$$

For the volume, we take $V = 1/2 LR \pi r_m^2 = 65$ litres per end, recalling that $LR = 600$ cm was defined as twice the actual product and taking $r_m = 8$ cm. ϕ_e is 1 ÷ 2 times T_e ; and from Eq. (7), for $n_e = N_H = 2 \times 10^{11} \text{ cm}^{-3}$, we obtain

$$j_f = \frac{n_e}{\tau_f} \approx 7 \times 10^{14} \text{ cm}^{-3} \cdot \text{s}^{-1},$$

or about 18 kW for the two transition/anchor regions.

This power comes from the thermal electrons. It is meaningfully compared with the total power flow out of the central cell at 12 A and $\phi_e + T_e + \phi_c + T_c = 7.5 T_e + 4.5 T_c = 4.8$ keV per ion/electron pair, or a power of 58 kW. Maintenance of the hot-electron anchors, therefore, increases by about 30% the power necessary to supply to the thermal electrons in order to maintain the assumed $T_e = T_c = 400$ eV. Note that, compared to INTEREM, the increase in power is due to the larger current/volume, larger volume, and the higher energy per electron.

IV. SUMMARY

In summary, we have shown that hot-electron mirror machines remain space charge neutralized by electrostatically confining a low-energy ion component. Because on being lost these ions can only flow to the wall at the sound speed, the thermal electrons neutralizing them must be confined in a distributed Langmuir sheath, causing the overall potential to be

positive. Therefore, end lost ions will be observed at the wall to have a minimum energy of a few T_e . Importantly, the overall potential will not be negative with respect to ground as might be supposed by comparison with hot-ion mirror machines.

Most of our formulae would continue to hold under reactor conditions. Because the hot electrons would continue to have about the same energy, and the same hot electron β -values would be required, N_H would scale with B^2 . The thermal ion temperatures would remain in the few eV range. Given the electron temperature and the geometry factors, one can immediately calculate the required power.

Acknowledgement

The author wishes to thank J. R. Hiskes for information on the molecular hydrogen ionization channels and the expected typical ion energies.

Figures

- Fig. 1 The magnet set, and field, density, and potential profiles for one end of the proposed tandem experiment.
- Fig. 2 Qualitative profiles of N_H , n_e , n_i , and for (a) INTEREM and (b) a hot-electron tandem mirror anchor.
- Fig. 3 Calculated $\langle\sigma v\rangle$ products for electron ionization of atomic and molecular hydrogen. Solid curves are calculated from experimentally determined cross sections; dashed curves, from theoretical.

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ANCHOR RECIRCULATOR

BARRIER-PLUG

CENTRAL CELL

MAGNETIC FIELD

ANCHOR

PLUG

CENTRAL CELL

DENSITY

POTENTIAL

Fig. 1

Fig. 2a

Fig. 2b

Fig. 3