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SEISMOLOGICAL INVESTIGATION OF CRACK FORMATION IN HYDRAULIC
ROCK FRACTURING EXPERIMENTS AND IN NATURAL GEOTHERMAL
ENVIRONMENTS

PROGRESS REPORT

For Period September 1, 1981 - August 31, 1982

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Abstract

This is a progress report for period September 1, 1981 to August 31, 1982 on the project entitled "Seismological Investigation of Crack Formation in Hydraulic Rock Fracturing Experiments and in Natural Geothermal Environments."

In our last year's progress report C00-2534-7, we reported that 6 papers were published and 5 were submitted for publication during that one-year period. We have had another very productive year. We have published 6 papers during the reporting period, and another one has been accepted for publication. Two additional papers have been submitted for publication.

The most exciting accomplishments during the present reporting period are:

(1) High-quality digital data obtained in the crater and the flank of Mt. St. Helens by the deployment of 9 event-recorders in cooperation with Oregon State University and the U.S. Geological Survey. Discovery of striking differences in arrival times and amplitudes of seismic events between the crater station and flank stations.

(2) Successful interpretation of the above observations by a powerful new method of seismogram synthesis called "Gaussian Beam Method" developed by Czech seismologists.

(3) Evidence obtained at Mt. St. Helens for a close connection between the volcanic tremor and so-called "long-period events." In agreement with Koyanagi's observation on Kilauea volcano, volcanic tremor may be nothing but an

incessant occurrence of long-period events.

(4) Further development of our volcanic tremor model by including acoustic vibration in magma.

(5) Development of a high-temperature small-diameter borehole seismograph for use in search of a magma reservoir by listening to seismic events due to thermal stress.

Scope of Investigation

This research program has grown through participation in two major geothermal projects, namely the Hot Dry Rock Geothermal Energy Development Project of the Los Alamos National Laboratory and the Magma Tap Project of Sandia National Laboratories. In order to interpret data from various seismic experiments conducted at Fenton Hill, New Mexico, and Kilauea Iki, Hawaii, the theory and methods have been developed for seismic wave generation, transmission, scattering, and attenuation in a medium containing fluid-filled crack(s).

The results of interpretation are synthesized for each geothermal site, and the model parameters are updated as new experimental results are analyzed. The M.I.T. model is intended not only to define the geometrical and physical properties of the geothermal system but also to estimate the mass and energy transport through measurements of seismic signals generated by the geothermal system, such as volcanic tremors.

Models are being developed for the hot dry rock fracture system at Fenton Hill, magma lens in Kilauea Iki, deep and

shallow tremor sources under Kilauea, inside Mt. St. Helens and other volcanoes. The data needed for study are collected by a network of mobile digital seismographs of the Massachusetts Institute of Technology as well as from the U.S. Geological Survey, Los Alamos National Laboratory, Oregon State University, the University of Washington, the Centro de Investigación Científica y de Educación Superior de Ensenada in Mexico, and the University of Paris, through cooperative arrangements.

We are also developing a new borehole seismograph which can be operated at high temperature.

Summary of Results in the Present Reporting Period

We shall summarize the results obtained in the present reporting period (Sept. 1, 1981-Aug. 31, 1982) in the following. First we shall give a list of titles of papers published in this period about which a summary was already given in our last year's progress report: C00-2534-7. The list will be followed by summaries of papers published or submitted for publication which were not included in last year's progress report. Since copies of these papers have been sent to the DOE already, we shall be very brief in summarizing them. We shall then report in detail on several new on-going works supported by the present project.

(1) Interpretation of seismic data from hydraulic fracturing experiments at the Fenton Hill, New Mexico, Hot Dry Rock Geothermal Sites (K. Aki, M. Fehler, R.L. Aamodt, J.N.

Albright, R.M. Potte, C.M. Pearson, J.W. Tester), Journal of Geophysical Research, 87 (1982), 936-944.

(2) Source and scattering effects on the spectra of small local earthquakes (K. Aki), Bull. Seis. Soc. Am., 71 (1981), 1687-1700.

(3) Attenuation of short period seismic waves due to scattering (R.S. Wu), Geophysical Research Letters, 9 (1982), 9-12.

(4) Mean field attenuation and amplitude attenuation due to wave scattering (R.S. Wu), Wave Motion, 4 (1982), 305-316.

(5) Three-dimensional seismic inhomogeneities in the lithosphere and asthenosphere: Evidence for decoupling in the lithosphere and flow in the asthenosphere (K. Aki), Reviews of Geophysics and Space Physics, 20 (1982), 161-170.

(6) Using dual-well seismic measurements to infer the mechanical properties of a Hot Dry Rock Geothermal System (M. Fehler), Journal of Geophysical Research, 87 (1982), 5485-5494.

(7) Free surface displacements in the near field of a tensile crack expanding in three dimensions (B. Chouet). This paper was published in the journal of Geophysical Research, vol. 87, pages 3868-3872, 1982, with the following abstract.

The discrete wave number method is applied to the study of the elastic motions due to an arbitrary three-dimensional tensile source. The source elastic field is formulated as a superposition of plane waves propagating in discrete directions. The discretization stems from an assumption of

two-dimensional periodicity in the description of the source. Examples of the free surface motion induced in the near field of circular and rectangular dislocation sources expanding in a homogeneous half space are discussed.

(8) Ground motion near an expanding preexisting crack (B. Chouet). This paper is currently in the review process in the Journal of Volcanology and Geothermal Research.

Cooling and contraction in thick lava flows often produce tensile fractures which may grow vertically throughout the entire thickness of the flow dividing it into pillars known as columnar joints. The aim of this study is to provide some insight into the problem of the elastic radiation produced by the thermal fracturation process in solidifying basalt. Using the two-dimensional model of an expanding preexisting crack developed by Aki et al. (1977) we are applying the discrete wave number method (Bouchon, 1979; Chouet, 1981, 1982) to obtain a complete representation of the surface displacements near a buried vertical crack which suddenly expands in length by a small increment. Our object is to assess the characteristics of the ground motion resulting from an extension of the bottom or top tip of a crack embedded in a layered structure similar to that found in a typical Hawaiian lava lake. The results show the strong impulsive character of the dynamic motion near the source and demonstrate that the dominant period of the signal is quite sensitive to the direction of the rupture. The displacement is dominantly vertical near the epicenter but becomes predominantly horizontal beyond the immediate source

region. The presence of layers has a marked effect on the complexity and duration of the ground response.

(9) Operation of a digital seismic network on Mt. St. Helens volcano and observations of long-period seismic events that originate under the volcano (M. Fehler, B. Chouet). This paper is in press in the Geophysical Research Letters, and describes a preliminary result from our joint field-experiment on Mt. St. Helens with the Oregon State University. Since a copy of the paper has already been sent to DOE, our summary here will be brief.

During the period of May through October, 1981, we operated a nine-station digital seismic array on the flanks and within the crater of Mt. St. Helens volcano in the state of Washington. This was a joint effort undertaken by Oregon State University, Massachusetts Institute of Technology, and the United States Geological Survey. The purpose of this work was to obtain high quality digital seismic data from a dense seismic array operating near and in the crater of the volcano to facilitate the study of the near-field seismic waveforms generated by volcanic activity. Our goal is to investigate the source mechanism of volcanic tremor and seismic activity associated with magma intrusion, dome growth and steam-ash emissions occurring within the crater of Mt. St. Helens. We observe a type of seismic event that we refer to as a long-period seismic event whose occurrence does not correlate with any visual observations of changes occurring in the crater of the volcano. These events have a waveform that is quite

different from that produced by typical earthquakes. Figure 1 shows a vertical component waveform for both a local earthquake and a long-period event recorded at station BCR (Fig. 2) on September 3, 1981. The records have been artificially clipped to a maximum amplitude for the plot. The waveform of the long-period event differs from the earthquake waveform in that it is dominated by a long period component upon which a high frequency signal is superimposed. Although long period seismic events have been reported by observers studying volcanoes in other regions including Japan, New Zealand, Kamchatka, Hawaii, and Sicily, there have been few detailed studies of waveforms associated with these events. Representative examples of spectra of ground displacement corrected for instrument response (Fig. 3) are shown in Figure 4 for both long period events and earthquake. Figures 4a, 4c and 4d are spectra of ground displacement at station BCR recorded for three long period events that occurred on September 3, 1981. Figure 4c is the spectrum of the event shown at the bottom of Figure 1. Figure 4b is the spectrum of ground displacement recorded at station BCR for a small shallow earthquake which occurred on September 3, 1981, and whose waveform is shown at the top of Figure 1. The clipping which was applied when plotting Figure 1 does not affect the spectrum since the recorded trace is not clipped. Each spectrum for a long period event (Figure 4a, 4c and 4d) contains at least one dominant peak in the frequency range of 1.7 to 2.3 Hz, which has an amplitude at least one order of magnitude larger than the amplitude for frequencies

greater than 5 Hz. Interestingly, these spectra are characterized by a sharp drop off in amplitude at low frequency. They roll off roughly in proportion to ω^{-2} at high frequency. Subdominant peaks can be recognized as a stable feature between different events at least up to 10 Hz. The earthquake spectrum, on the other hand, is relatively flat between frequencies of 1 and 40 Hz. Spectral analysis of signals of long period events recorded at other locations by the digital seismic array shows that there is little variation with location in the frequency of the dominant peak in the spectrum. We do observe a decreased amplitude of the high frequency component of the waveform relative to the low frequency component at station SFT (Fig. 2) which is probably due to attenuation along the path to this distant station. The relative consistency of the spectrum at various stations means that the unique waveform of the long period events is due to a source effect rather than a path effect. The amplitude of the dominant spectral peak for the waveforms of the long period events recorded at BCR ranges between $1.1 \cdot 10^{-4}$ cms and $6.3 \cdot 10^{-4}$ cms for the events analyzed to date. Although there are slight variations in the frequencies at which the peaks in the spectra occur, this dominant frequency appears to be fairly insensitive to the amplitude of the peak. The wide variation in amplitude of the spectral peak and corresponding small variation in the frequency at which the peak occurs leads us to the conclusion that the amplitude of the seismic signal is determined by the force generating the signal rather than the size of the source region.

Figure 5 shows a plot of the waveforms recorded at station BCR, SBW and NWD. Three components of motion recorded at SBW and two components at NWD are plotted. The number below each trace is a scaling factor which was multiplied by the trace amplitude before the trace was plotted. The amplitude of the vertical component of motion recorded at BCR is approximately 15 times larger than the amplitude of the vertical component of motion recorded at SBW. The ground motion at SBW and NWD is dominantly horizontal. Polar plots of the three components of motion recorded at SBW (Fig. 6), however, show no consistent pattern of motion that can be identified as being due to a given type of wave. Figure 7 shows the filtered traces of 2 long period events obtained with 6 octave bandpass elliptic filters (see example of filter response in Fig. 8). The characteristics of the signal in the band of the dominant frequency (1-2 Hz) are remarkably consistent between different events.

We noted that long period events occurred most frequently in the time interval leading up to an eruptive episode. The lack of correlation between the occurrence of long period seismic events at Mt. St. Helens and visual changes occurring in the crater as well as the increase in the rate of occurrence of the events as an eruption approaches, leads us to conclude that the long period seismic events may be related to the excitation of some fixed cavity in or near the region where magma is stored beneath the volcano. Preliminary determinations of the locations of these events have been made using the

program Hypoinverse (Klein, 1978). For determining source locations, arrival time data from the digital seismic network was supplemented with readings from stations of the permanent University of Washington seismic network operating in the vicinity of Mt. St. Helens. We have found that these events originate in the vicinity of the crater at Mt. St. Helens at depths ranging from 0 to 5 km.

LIST OF FIGURES

- Fig. 1 Vertical component records obtained at station BCR within the crater of Mt. St. Helens. The event at the top of the figure is a shallow earthquake and the event at the bottom is a long-period event. The arrows indicate the absolute time. Numbers at the lower right of each trace are factors by which the trace was multiplied before plotting. The traces were artificially clipped to fit the size of the plot.
- Fig. 2 Map of the Mt. St. Helens area showing the disposition of the seismic network (open circles).
- Fig. 3 Amplitude-frequency response of the seismic system obtained at the maximum amplification.
- Fig. 4 Frequency spectra of the vertical components of four events recorded at station BCR. Events a, c and d are long-period events. Event b is the earthquake depicted at the top of Fig. 1 and event c is the long-period event shown at the bottom of that figure.
- Fig. 5 Records obtained at 3 stations for the long-period event shown in Fig. 1. The figure shows the three

components of motion at station SBW, two components at station NWD, and the vertical component at station BCR. The arrow indicates the absolute time and the numbers at the lower right of each trace are factors by which the trace was multiplied prior to plotting.

Fig. 6 a) original and filtered (band 1 to 3Hz) traces of the three components of motion of a long-period event recorded at station SBW. The vertical, east-west, and north-south components are shown from top to bottom. b), c), d), e) Polar plots of the ground motion produced by this event. Shown from top to bottom are the motion obtained in the three orthogonal planes defined by each component pair, i.e., vertical versus east-west, vertical versus north-south, and north-south versus east-west, respectively. The plots are obtained from the filtered traces shown in Fig. 6a and are separated into individual time slices of two seconds starting at the beginning of the record and ending 20 seconds later.

Fig. 7 Filtered traces of two long-period events obtained in the 6 octave bands of 0.5 to 1Hz, 1 to 2Hz, 2 to 4Hz, 4 to 8Hz, 8 to 16Hz, and 16 to 32Hz.

Fig. 8 Typical response of the elliptic bandpass filter used in the analysis of the long-period events. The filter is designed with a rejection of 58.707dB at frequencies 10% beyond the bandwidth and has a 0.1dB passband ripple.

0 1 2 3 4 5
TIME (SEC)

BCR Z

↑ 246 5 7 18

0.0103

BCR Z

↑ 246 7 24 45

0.0660

SBW UD 3 | 246 4 58 39 5.499808

| BANDWIDTH 1.0 TO 3.0 HZ

SBW EW 3 | 246 4 58 39 20.783920

| BANDWIDTH 1.0 TO 3.0 HZ

SBW NS 3 | 246 4 58 39 20.016098

| BANDWIDTH 1.0 TO 3.0 HZ

0 5 10
TIME (SEC)

0 - 2 SEC

2 - 4 SEC

4 - 6 SEC

6 - 8 SEC

8 - 10 SEC

10 - 12 SEC

12 - 14 SEC

14 - 16 SEC

e

16 - 18 SEC

18 - 20 SEC

0.077480

BANDWIDTH 0.5 TO 1.0 HZ

0.197698

BANDWIDTH 0.5 TO 1.0 HZ

9.071245

BANDWIDTH 1.0 TO 2.0 HZ

12.010989

BANDWIDTH 1.0 TO 2.0 HZ

6.742924

BANDWIDTH 2.0 TO 4.0 HZ

9.585993

BANDWIDTH 2.0 TO 4.0 HZ

5.525514

BANDWIDTH 4.0 TO 8.0 HZ

6.430227

BANDWIDTH 4.0 TO 8.0 HZ

8.830456

BANDWIDTH 8.0 TO 16.0 HZ

6.746917

BANDWIDTH 8.0 TO 16.0 HZ

13.634722

BANDWIDTH 16.0 TO 32.0 HZ

13.991732

BANDWIDTH 16.0 TO 32.0 HZ

BCR UD 44 | 246 10 22 5

BCR UD 47 | 246 10 36 42

0 5 10
TIME (SEC)

(10) Interpretation of seismograms of high-frequency events of Mt. St. Helens by the method of "Gaussian Beams" (K. Aki).

Many of the events recorded in early September, 1981 by our event recorders at Mt. St. Helens looked like typical earthquakes. To distinguish them from the long-period events discussed earlier they are called "high-frequency events." Examples of seismograms of high-frequency events are shown in Fig. 9 for the event of 249 Julian date, 8h 56m 36s and in Fig 10 for the event of 246 day, 5h 41m 50s. In each figure, the amplitude is scaled by a factor shown at the left of the seismogram. For example, a signal at EDM with amplitude equal to that at SBW has an absolute value 0.25 times that of the latter, as shown in Fig. 9. Fig. 10 shows that the absolute amplitude at BCR is an order of magnitude greater than at SBW.

As a first step, the station site effect was examined using coda waves, using the method developed by Aki (1969), Aki and Chouet (1975) and Tsujiura (1978). According to these studies, the later part of seismograms of a given local earthquake must have the same amplitude spectra at a given time at all stations, except for the station site effect.

Applying the above principle, we find that stations BCR (crater), SRW and WRW have nearly the same site amplification effect, but they are somewhat (about 50%) greater than those at SBW and NWD. On the other hand, station EDM seems to have a serious coupling problem. The site amplification factor at EDM is about 1/20 that of SBW at 1Hz and 1/10 at 5Hz. Station SFT showed a very low high-frequency response, and the site

amplification factor is $1/20$ that of SBW at 2Hz. Because of these poor site conditions, the records obtained at EDM and SFT were not very useful.

Knowing the station site effect from coda waves, we now examine the arrival time and amplitude of direct waves. Two most significant results are that (1) they arrive at the crater station (BCR) earlier by 0.4 to 0.7s than at the flank stations (NWD and SBW), and that (2) the amplitude is about 10 times greater at the crater station than at the flank stations. Since there were only small differences in site effect among these stations, we must conclude that the observed large difference in amplitude is due to the propagation path effect. Also consistently observed for all events are that (3) the arrival times at NWD and SBW are always coincident, and (4) that the arrival time at SRW is delayed by 0.1 to 0.4s relative to NWD or SBW. The above observations suggest that the seismic source is located symmetric with respect to NWD and SBW, but shifted to north relative to the point of equal distance from the three stations (slightly east of BCR). This is consistent with the earthquake epicenters determined by the University of Washington.

In order to explain the two most significant observations, that is, the 0.4-0.7s earlier arrival and an order of magnitude greater amplitude at the crater station than the flank stations, we applied a new synthetic-seismogram method developed recently in Czechoslovakia, namely the Gaussian beam method of Cerveny, Psencik and Popov.

The method of Gaussian beam has been described in a lecture note given by Professor Cervený at Utrecht in April-May 1981. The method is applicable to 3-D inhomogeneous media with irregular interfaces. First, we trace a ray in such a medium, and transform the wave equation into so-called "ray-centered coordinate." Then, by applying the parabolic approximation, we find an asymptotic solution in the neighbourhood of the ray. The solution can be obtained by solving a pair of linear differential equations called "dynamic ray tracing system." Introducing an imaginary part into the solution of the dynamic ray tracing system, we obtain a "Gaussian beam" representing a wave field concentrated near the ray. The basic idea of the Gaussian beam method is to construct wave field from a given wave-source by a superposition of Gaussian beams. This idea has a strong appeal to one's intuition, although it has to be tested with problems for which solutions are known.

A computer package has been kindly supplied to us by Dr. Psencik. It contains routines for kinematic and dynamic ray tracing in 2-D media. A graduate student at M.I.T., R. Nowack, made a small modification of the program for synthesizing the Gaussian beam using formulas given in Cervený's lecture note.

We have tested this program for a line source in a homogeneous medium, and obtained a solution which agrees well with the asymptotic approximation to the exact solution as expected. So far, we have not been able to test the effect of irregular interface including the topography. We shall, therefore, model the interior of Mt. St. Helens by a

continuously inhomogeneous medium, and neglect the effect of topography.

Fig. 11 shows a preliminary model of 2-D P velocity structure under Mt. St. Helens which we are tentatively adopting. This is based on evidence obtained by various workers on Mt. St. Helens (Malone, 1982) as well as other similar types of volcanoes such as St. Augustine in Alaska (Kienle et al., 1979) and Showa-shinzan in Japan (Hayakawa, 1957). In all these cases, the internal structure of a volcano is depicted as a high-velocity magma body with P velocity about 4km/s covered by a low velocity layer.

At first, we thought that the observed high amplitude and early arrival may be explained by locating the earthquake foci at the bottom of the hypothesized high-velocity column. Fig. 12 shows the ray-tracing result for a focus about 3.5km below the summit. We found that the high-velocity column tends to diverge the rays away from the summit. The Gaussian-beam seismograms at stations indicated by triangles in Fig. 11 are shown in Figs. 13, 14 and 15, respectively, for signals with frequencies of 5, 10 and 20Hz. We find that amplitudes at flank stations (2km away from the summit corresponding to SBW, NWD and SRW) are comparable to those at the crater station, and the arrival time differences are within 0.25s.

If we move the focal depth upward, however, we find a drastically changed pattern of seismic signals. Fig. 16 shows the result of ray-tracing for a focal depth of about 1km from the summit. In this case, the ray paths going directly upward

are dense and short, while those toward the flank of volcano are sparse and take a detour. The resultant seismograms for 5, 10 and 20Hz (Figs. 17, 18, and 19, respectively) show an order of magnitude greater amplitude at the crater station than at the flank stations located 2km away, and about 0.5 sec earlier arrival.

Thus, the P-velocity structure depicted in Fig. 11 and the focal depths less than 1km from the summit can explain our most significant observations.

A similar observation was made for St. Augustine volcano by Lalla and Kienle (1982), who discovered a correlation between the amplitude ratio of summit to flank station and the focal depth. For focal depths shallower than about 2km, the summit amplitude was greater by an order of magnitude than the flank amplitude, and for deeper depths, the opposite was observed.

In any case, the sensitivity of the pattern of seismic signal to the focal depth is remarkable and is due to the strongly heterogeneous velocity structure under a volcano.

The result also suggests a promising way of accurately locating volcanic earthquakes using the distribution of seismic amplitude as well as arrival time. The accurate location of volcanic earthquakes is a crucial requirement for reliable prediction of a volcanic eruption.

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- Fig. 9 Seismograms of a high-frequency event which occurred on the 249th Julian day, 8h 56m 36s. (See Fig. 2 for station location.)
- Fig. 10 Seismograms of a high-frequency event which occurred on the 246th Julian day, 5h 41m 50s. (See Fig. 2 for station location.)
- Fig. 11 A 2-D model of the interior of Mt. St. Helens. The hand-written numbers show P velocity in km/s. The triangles show stations at which synthetic seismograms are computed.
- Fig. 12 Ray paths from a source at a depth of 3.5km below the summit for the velocity model shown in Fig. 11.
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- Fig. 17 Synthetic seismograms for 5Hz.
- Fig. 18 Synthetic seismograms for 10Hz.
- Fig. 19 Synthetic seismograms for 20Hz.

Fig. 9

CITY DISTRIBUTION
INES CONSTRUCTED FROM 1.00000 TO 6.00000 WITH INCREMENT 0.50000

Fig. 12

Fig. 14₃₆

Fig. 15

Fig. 16

SEC

2

1

0

0

1

2

3

4

5

6

7

8

DISTANCE IN KM

MT ST HFI FNS

Fig. 17³⁹

Fig. 18

Fig. 19

(11) A source model for the long period events observed at Mt. St. Helens volcano (B. Chouet).

As discussed above, the well-defined character of the spectral peak observed for the long period events suggests that their origin may lay in the excitation of a fixed cavity under the active crater. Although the manner in which it was generated is not known, we assume that this excitation results from the existence of free oscillatory fluid motion within the volcanic system. Our object is to study the natural frequencies of vibration and associated damping of the fluid oscillation in a plausible volcanic system such as shown in Figure 20. A large reservoir of fluid is connected to a long thin vertical pipe of uniform section through a small orifice. For a slender tube having a diameter much smaller than the wavelengths of interest, sound pressure and axial particle displacement are taken to be functions of one coordinate and time only. Elastic yielding of the wall does result in some radial motion, but radial pressure gradients accompanying this motion are too small to modify the piston-like motion in the axial direction. Motion in the axial direction is caused by the pressure gradient along the axis. The usual approach (e.g. Wylie and Streeter, 1978) is to write the equations of motion and continuity for the axial coordinates. The equation of continuity yield terms accounting for the fluid compressibility, the radial change in conduit size, and the elongation of the conduit; these relations are then developed into expressions involving only pressure. With the assumption of low frequencies, or slowly changing pressure gradients, the radial motion is assumed to be in equilibrium with the pressure existing at a given time. The same assumption of low frequencies also ensures that the pressure is essentially uniform over distances of several pipe diameters in the axial direction. With this limiting case in mind, one might expect the relation between pressure and radial expansion of the wall to be adequately described

by the application of static elasticity to the same geometry. This approach was taken by Halliwell (1963) to study the water hammer in tunnels embedded into a homogeneous, isotropic, elastic full space. A derivation of the radial displacement due to a pressure on the interior of a thick-walled tube can be found in many works (e.g., Timoshenko, 1951). Neglecting the effect of gravity, the equation of motion yields

$$\frac{1}{\rho} p_x + \frac{4}{\rho D} \tau + vv_x + vt = 0 \quad (1)$$

where ρ is the fluid density, D is the pipe diameter, τ is the wall shear stress, p is the pressure, v is the velocity of the fluid, and the subscripts x and t denote partial derivatives with respect to the axial coordinate and time, respectively. The equation of continuity gives

$$v_x + \frac{1}{\rho a^2} (vp_x + pt) = 0 \quad (2)$$

where a is the acoustic velocity in the fluid given by

$$a = \left(\frac{\frac{K}{\rho}}{1 + (1+\nu) \frac{2K}{E}} \right)^{1/2} \quad (3)$$

in which K represents the bulk modulus of the fluid, E is Young's modulus, and ν is Poisson's ratio for the wall material.

The continuity and momentum equations form a pair of quasi-linear hyperbolic partial differential equations in terms of two dependent variables, velocity and pressure, and two independent variables, distance along pipe and time. These equations are usually converted into four ordinary differential equations by the method of characteristics, then solved using first-order finite-difference approximations after introducing appropriate boundary conditions. To simplify the analysis of the system depicted in Figure 9 we shall neglect the convective terms vv_x and vp_x as well as the effect of viscous drag. Then equations (1) and (2) become

$$p_x + \rho v_t = 0 \quad (4)$$

$$v_x + \frac{1}{\rho a} p_t = 0 \quad (5)$$

Taking partial derivatives we obtain

$$p_{xx} + \rho v_{xt} = 0 \quad (6)$$

$$v_{xt} + \frac{1}{\rho a} p_{tt} = 0 \quad (7)$$

yielding the one-dimensional wave equations

$$p_{xx} = \frac{1}{a^2} p_{tt} \quad (8)$$

$$v_{xx} = \frac{1}{a^2} v_{tt} \quad (9)$$

with the solutions

$$p = (A_1 e^{\gamma x} + A_2 e^{-\gamma x}) e^{st} = p(x) e^{st} \quad (10)$$

$$v = -\frac{\gamma}{\rho s} (A_1 e^{\gamma x} - A_2 e^{-\gamma x}) e^{st} = v(x) e^{st} \quad (11)$$

where A_1 and A_2 are constants and s represents the complex frequency with real and imaginary parts σ and ω , respectively

$$s = \sigma + i\omega \quad (12)$$

in which σ denotes the damping of the fluid oscillation and ω is the circular frequency.

The constant γ is the complex wave number defined as

$$\gamma = \frac{s}{a} \quad (13)$$

Let us now consider the boundary conditions of the system depicted in Fig. 20. At the top of the tube we assume an elastic interface. To describe the boundary condition at this interface we consider the static solution obtained for a penny-shaped crack under pressure, i.e. (Sheddon and Lowengrub, 1969),

$$u(r) = \frac{\lambda+2\mu}{\mu(\lambda+\mu)} \frac{1}{\pi} \Delta p \sqrt{R^2 - r^2} \quad (14)$$

where $u(r)$ denotes the transverse displacement of the crack face as a function of the radius r , R is the radius of the crack, Δp is the applied pressure within the crack, and λ and μ are the Lamé constants. Assuming $\lambda=\mu$ we obtain the average displacement $\bar{u}$ as

$$\bar{u} = \frac{R}{\pi\mu} \Delta p \quad (15)$$

and applying this boundary condition to our problem gives the relation

$$p_t = kv \quad (16)$$

where $k = \pi\mu/R$, R being taken here as the radius of the tube. Putting (16) into (10) and (11) yields, at $x=0$

$$A_2 = \frac{\frac{\gamma k}{\rho s} + s}{\frac{\gamma k}{\rho s} - s} A_1 = CA_1 \quad (17)$$

Following Wylie and Streeter (1978) we define the hydraulic impedance $Z(x)$ as the ratio of the complex head $H(x)$ to the complex volume flow rate $Q(x)$ at a particular point x in the system

$$Z(x) = \frac{H(x)}{Q(x)} \quad (18)$$

From Fig. 20, using the subscript o to refer to the position of the orifice and the subscripts ℓ and r to denote the positions immediately upstream and downstream of the orifice, respectively, we obtain the head loss across the orifice as

$$H_\ell - H_r = Z_o Q_o \quad (19)$$

where Z_o is the impedance of the orifice and Q_o is the flow rate at the orifice. The continuity requires

$$Q_o = Q_\ell = Q_r \quad (20)$$

and since $H = p/(\rho g)$, where g is the acceleration due to gravity, and $Q = Av$, where A is the cross-sectional area of the flow, we can use (19) and (20) to obtain the relations

$$p_r = p_\ell - \rho g Z_0 A_\ell v_\ell \quad (21)$$

$$v_r = \frac{A_\ell}{A_r} v_\ell \quad (22)$$

from which we derive the boundary condition at the downstream end of the tube

$$Z_r = \frac{H_r}{Q_r} = \frac{1}{\rho g A_\ell} \frac{p_\ell - \rho g A_\ell Z_0 v_\ell}{v_\ell} \quad (23)$$

where Z_r denotes the impedance of the reservoir. Defining the characteristic impedance of the fluid in the pipe by $Z_c = a/(gA_\ell)$ and using (10), (11), (17) and (23), we get

$$-\frac{Z_r}{Z_0} = \frac{\left(\frac{Z_c}{Z_0} + 1\right) + \left(\frac{Z_c}{Z_0} - 1\right)Ce^{-2\gamma\ell}}{1 - Ce^{-2\gamma\ell}} \quad (24)$$

For a fixed orifice the relationship between oscillatory flow and head is given by Wylie and Streeter (1978) as

$$Z_0 = \frac{2\bar{H}}{\bar{Q}} \quad (25)$$

where $\bar{H}$ is the head drop across the orifice for the mean flow discharge $\bar{Q}$.

Equation (25) as well as the relation for Z_c above show that Z_0 and Z_c are real numbers and so the expressions between the parentheses in (24) are real numbers too. Replacing C from (17), γ from (13) and s from (12) into (24), we obtain from the real and imaginary parts of this expression, the relations

$$-I_r = \frac{2Z_c e^{-2\ell\sigma/a} \left(2\omega c \cos\left(\frac{2\omega\ell}{a}\right) + (\omega^2 + \sigma^2 - c^2) \sin\left(\frac{2\omega\ell}{a}\right) \right)}{(c - \sigma)^2 + \omega^2 + e^{-4\ell\sigma/a} ((\sigma + c)^2 + \omega^2) + 2e^{-2\ell\sigma/a} ((\omega^2 + \sigma^2 - c^2) \cos\left(\frac{2\omega\ell}{a}\right) - 2\omega c \sin\left(\frac{2\omega\ell}{a}\right))} \quad (26)$$

- $R_r =$

$$\frac{(Z_c + Z_0)((c - \sigma)^2 + \omega^2) - 2Z_0 e^{-2\ell\sigma/a} ((c^2 - \sigma^2 - \omega^2) \cos(\frac{2\omega\ell}{a}) + 2\omega c \sin(\frac{2\omega\ell}{a})) - (Z_c - Z_0) e^{-4\ell\sigma/a} ((c + \sigma)^2 + \omega^2)}{(c - \sigma)^2 + \omega^2 + e^{-4\ell\sigma/a} ((c + \sigma)^2 + \omega^2) + 2e^{-2\ell\sigma/a} ((\omega^2 + \sigma^2 - c^2) \cos(\frac{2\omega\ell}{a}) - 2\omega c \sin(\frac{2\omega\ell}{a}))} \quad (27)$$

where $I_r = \text{Im}(Z_r)$, $R_r = \text{Re}(Z_r)$, and $c = k/(\rho a) = \pi\mu/(\rho a R)$. When Z_r is known, equations (26) and (27) can be solved to obtain the natural frequencies and damping of the fluid system. The value of σ is a property of the entire system and is independent of position and time. It is also independent of the amplitude of oscillation. It is, however, a function of frequency since the dissipative character of the system is dependent upon the relative amplitude of oscillation at the various resistive elements in the system.

Let us first consider the simple case where the pressure in the reservoir vanishes. Since Z_r is zero in that case, we obtain from (26)

$$\tan\left(\frac{2\omega\ell}{a}\right) = \frac{2\omega c}{c^2 - \omega^2 - \sigma^2} \quad (28)$$

the solution of which is represented graphically in Figure 21. The modes ω_n are indicated by the open circles; they are non-linearly related to each other. Note that we have drawn only one value of the function which appears at the right hand side of equation (28), while in fact we should have drawn a different asymptote for each mode since σ is a function of frequency. As we shall see below, however, the actual values of σ_n are very much smaller than the value of c for the lower modes. Thus $(c^2 - \sigma^2)^{1/2} \sim c$ in our case so that the modes depend essentially on the value of this constant. In a similar way, we obtain from (27)

$$\frac{Z_0}{Z_c} = \frac{(c - \sigma)^2 + \omega^2 - e^{-4\ell\sigma/a} [(c + \sigma)^2 + \omega^2]}{(c - \sigma)^2 + \omega^2 + e^{-4\ell\sigma/a} ((c + \sigma)^2 + \omega^2) + 2e^{-2\ell\sigma/a} ((\omega^2 + \sigma^2 - c^2) \cos(\frac{2\omega\ell}{a}) - 2\omega c \sin(\frac{2\omega\ell}{a}))} \quad (29)$$

Taking $\mu \sim 10^8 - 10^{11}$ dyne/cm², $\rho \sim 3$ g/cm³, $a \sim 10^4 - 10^5$ cm/s and $R \sim 10^2 - 10^3$ cm as reasonable values for the volcanic system we obtain $c \sim 10^5$ s⁻¹. On the other hand, the frequency of the dominant peak of the observed long period events is about 2 Hz, so that the corresponding ω is of the order of 10 s⁻¹. From Fig. 1 we can estimate σ to range between 10^{-1} and 10^{-2} s⁻¹. Since $\sigma \ll c$ and $\sigma \ll \omega$ in the present situation, we can simplify (29) to

$$-\frac{Z_0}{Z_c} = \frac{\omega^2 + c^2 - e^{-4\ell\sigma/a}(\omega^2 + c^2)}{\omega^2 + c^2 + e^{-4\ell\sigma/a}(\omega^2 + c^2) + 2e^{-2\ell\sigma/a}((\omega^2 - c^2)\cos(\frac{2\omega\ell}{a}) - 2\omega c \sin(\frac{2\omega\ell}{a}))} \quad (30)$$

the solution of which is

$$\sigma = -\frac{a}{2\ell} \ln \left\{ \frac{Z_0((\omega^2 - c^2)\cos(\frac{2\omega\ell}{a}) - 2\omega c \sin(\frac{2\omega\ell}{a})) \pm \sqrt{Z_c^2(\omega^2 + c^2)^2 - 2\omega^2 c^2 Z_0^2 [2 + \sin^2(\frac{2\omega\ell}{a})]}}{(Z_c - Z_0)(\omega^2 + c^2)} \right\} \quad (31)$$

In the limiting case where both ω and σ are much less than c and c is very large, we obtain from (28)

$$\tan\left(\frac{2\omega\ell}{a}\right) = 0 \quad (32)$$

yielding $\omega = \frac{n\pi}{2} \frac{a}{\ell}; \quad n = 0, 1, 2, \dots \quad (33)$

and the fundamental mode has the wavelength $\lambda = 4\ell$ as expected for a tube opened at one end. In that case equation (31) gives the result

$$\sigma = -\frac{a}{2\ell} \ln \left(\frac{-Z_0(-1)^n \pm Z_c}{Z_c - Z_0} \right) \quad (34)$$

Inasmuch as the parameters Z_c and Z_0 are positive real numbers, and the bracketed quantity must always be positive, two free oscillation cases exist.

If $Z_0 > Z_c$:

$$\sigma = -\frac{a}{2\ell} \ln \left(\frac{Z_0 + Z_c}{Z_0 - Z_c} \right)$$

$$\omega = \frac{n\pi}{2} \frac{a}{\ell} \quad n = 0, 2, 4, \dots \quad (35)$$

If $Z_0 < Z_c$:

$$\sigma = -\frac{a}{2\ell} \ln \left(\frac{Z_c + Z_0}{Z_c - Z_0} \right)$$

$$\omega = \frac{n\pi}{2} \frac{a}{\ell} \quad n = 1, 3, 5, \dots \quad (36)$$

In the first case, equation (35), the free vibration frequencies are the even harmonics which means the orifice provides a response similar to a dead end. The pipe responds as if shut at both ends in that case. In the second case, equation (36), the odd harmonics exist which indicate a reflective condition at the orifice similar to a reservoir. The pipe behaves as a singly open-ended tube in that case. As Z_c and Z_0 get closer and closer together, the value of σ becomes a larger negative number. This infers a condition in which free oscillation is impossible. We note that another solution is possible in the case of the even harmonics and which corresponds to the plus sign in (34). In that case $\sigma = 0$ independently of the values attributed to Z_0 and Z_c . The same result is obtained in (35) when $Z_0 \gg Z_c$ or in (36) when $Z_0 \ll Z_c$. The system then has no damping.

Let us now consider the case where the impedance of the reservoir is non zero. We assume that flow from the pipe is compressing the reservoir fluid. The oscillatory variation of the volume of fluid in the reservoir due to the discharge is $\Delta V_r = V e^{i\omega t}$ and the flow rate associated with this variation of volume is $Q_r = i\omega V e^{i\omega t}$. Using the bulk modulus of fluid, K , we have

$$p_r = -K \frac{\Delta V_r}{V_r} \quad (37)$$

and the equivalent head in the reservoir is

$$H_r = -\frac{K}{\rho g} \frac{\Delta V_r}{V_r} \quad (38)$$

The reservoir impedance is then

$$Z_r = \frac{H_r}{Q_r} = \frac{iK}{\rho g \omega V_r} \quad (39)$$

which shows the head to be leading the discharge by $\pi/2$. For V_r constant the reservoir impedance is a pure imaginary number and we have $I_r = \text{Im}(Z_r) = K/(\rho g \omega V_r)$, $R_r = \text{Re}(Z_r) = 0$. Note that for $\omega \neq 0$, Z_r approaches zero when V_r becomes very large and we obtain the solution to ω_n and σ discussed earlier. Putting (39) into (26) we get

$$\frac{K(c^2 - \omega^2 - \sigma^2) - 2\rho g Z_c V_r c \omega^2 + (\rho g Z_c V_r \omega(c^2 - \omega^2 - \sigma^2) + 2Kc\omega) \tan(\frac{2\omega \ell}{a})}{K((c^2 + \omega^2 + \sigma^2) \cosh(\frac{2\ell\sigma}{a}) - 2c\sigma \sinh(\frac{2\ell\sigma}{a}))} = \frac{1}{\cos(\frac{2\omega \ell}{a})} \quad (40)$$

which we rewrite as

$$f_1(\omega) + f_2(\omega) \tan(\frac{2\omega \ell}{a}) = \frac{f_3(\omega)}{\cos(\frac{2\omega \ell}{a})} \quad (41)$$

with

$$\begin{aligned} f_1(\omega) &= K(c^2 - \sigma^2) - \omega^2(K + 2\rho g Z_c V_r c) \\ f_2(\omega) &= \omega(\rho g Z_c V_r (c^2 - \sigma^2 - \omega^2) + 2Kc) \\ f_3(\omega) &= K((c^2 + \sigma^2 + \omega^2) \cosh(\frac{2\ell\sigma}{a}) - 2c\sigma \sinh(\frac{2\ell\sigma}{a})) \end{aligned} \quad (42)$$

The functions $f_1(\omega)$, $f_2(\omega)$ and $f_3(\omega)$ are depicted in Figure 22. The solution of (41), shown by the intersections of the solid curves in Figure 23, indicates that the system has modes at the frequencies

$$\omega_{2n+1} = \frac{\pi}{4}(1+2n)\frac{a}{\ell} \quad n = 0, 1, 2, \dots \quad (43)$$

Depending on the values of the parameters, additional modes are possible.

These modes are indicated schematically by the open circles in Fig. 23. Note again that we have implicitly assumed that $\sigma \ll c$ in drawing these solutions.

In practice, since σ is a function of frequency, a different set of functions f_1 , f_2 and f_3 exists for each mode. The damping of the system is again given by equation (31). For the modes corresponding to equation (43) we obtain

$$\sigma = -\frac{a}{2\ell} \ln \left\{ \frac{-2Z_0\omega c(-1)^n \pm \sqrt{Z_c^2(\omega^2+c^2)^2 - 6Z_0^2\omega^2c^2}}{(Z_c-Z_0)(\omega^2+c^2)} \right\} \quad (44)$$

The limiting case when c is very large is given by

$$\sigma = -\frac{a}{2\ell} \ln \left(\frac{\pm Z_c}{Z_c-Z_0} \right) \quad (45)$$

so that we have the two cases

$$Z_c > Z_0 : \quad \sigma = -\frac{a}{2\ell} \ln \left(\frac{Z_c}{Z_c-Z_0} \right) \quad (46)$$

$$Z_c < Z_0 : \quad \sigma = -\frac{a}{2\ell} \ln \left(\frac{Z_c}{Z_0-Z_c} \right) \quad (47)$$

In the general case, given by equation (44), the square root must be a real number. This provides the condition

$$\frac{Z_c}{Z_0} > \sqrt{6} \frac{\omega_{2n+1}^2 c^2}{\omega_{2n+1}^2 + c^2} \quad (48)$$

which, in the limit of the equal sign, yields the two cases

$$Z_c > Z_0 : \quad \sigma = -\frac{a}{2\ell} \ln \left(\frac{2Z_0 Z_c}{\sqrt{6}(Z_c-Z_0)} \right) \quad (49)$$

$$\omega_{2n+1} = \frac{\pi}{4} (2n+1) \frac{a}{\ell} \quad n = 1, 3, 5, \dots$$

$$Z_0 > Z_c : \quad \sigma = -\frac{a}{2\ell} \ln \left(\frac{2Z_0 Z_c}{\sqrt{6}(Z_0-Z_c)} \right) \quad (50)$$

$$\omega_{2n+1} = \frac{\pi}{4} (2n+1) \frac{a}{\ell} \quad n = 0, 2, 4, \dots$$

A direct analytical solution of equations (26) and (27) is not possible. To solve these equations an iterative method must be used. According to Wylie and Streeter (1978) Newton's method has been found to be quite effective in seeking the roots of such a system. The application of Newton's method requires a reasonable estimate of s near each complex root. This can be accomplished by performing a frequency scan of equation (24) with an assumed value of σ . An example of such a scan is shown in Figure 24, which depicts the modulus of the impedance function

$$Z_r - \frac{(Z_c + Z_0)(c - s) + (Z_c - Z_0)(c + s)e^{-2\ell s/a}}{-(c - s) + (c + s)e^{-2\ell s/a}} = 0 \quad (51)$$

The calculation is done assuming $\sigma = -0.4s^{-1}$, a value suggested by examination of the data of Figure 7 in the bandwidth of 1 to 2 Hz. The parameters relevant to this example are $K = \mu = 10^9$ dyne/cm², $E = 2.5 \cdot 10^9$ dyne/cm², $\nu = 0.25$, $\rho = 2g/cm^3$, $a = 1.581 \cdot 10^4$ cm/s, $R = 10^2$ cm², $g = 9.81 \cdot 10^2$ cm/s², $V_r = 10^8$ cm³, $\ell = 10^4$ cm, $Z_0 = 10^{-2}$ s/cm², $Z_c = 5.1 \cdot 10^{-4}$ s/cm², and $c = 9.934 \cdot 10^2$ s⁻¹. The function has a first minimum at the frequency $f = 0.47$ Hz. The correct value of s for this mode is then obtained by applying Newton's method to equation (51) using $s = 2\pi \cdot 0.47 - i0.4$ as a starting value.

From equations (10) and (11) we obtain at $x = 0$

$$\begin{aligned} p(x=0) &= p_0 = A_1 + A_2 \\ v(x=0) &= v_0 = -\frac{\gamma}{\rho s} (A_1 - A_2) \end{aligned} \quad (52)$$

from which we derive

$$\begin{aligned} A_1 &= \frac{1}{2} (p_0 - \frac{\rho s}{\gamma} v_0) \\ A_2 &= \frac{1}{2} (p_0 + \frac{\rho s}{\gamma} v_0) \end{aligned} \quad (53)$$

Replacing these values into (10) and (11) yields the equations

$$p(x) = p_0 \cosh \gamma x - \frac{\rho S}{\gamma} v_0 \sinh \gamma x \quad (54)$$

$$v(x) = -\frac{\gamma}{\rho S} p_0 \sinh \gamma x + v_0 \cosh \gamma x \quad (55)$$

which can be used to calculate the mode shape. Since p_0 and v_0 are related through equation (16), a specification of p_0 , or $p(x)$, or $v(x)$ at a particular position, for example at the orifice, enables us to solve for the remaining variables relative to the selected value.

LIST OF FIGURES

- Fig. 20 Source model used for the study of the long-period events (see text for details).
- Fig. 21 Natural frequencies of the system shown in Fig. 20 when the reservoir pressure vanishes.
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- Fig. 23 Natural frequencies of the system shown in Fig. 20 when the reservoir is a liquid accumulator.
- Fig. 24 Impedance function according to equation (51).

— $\tan\left(\frac{2\omega l}{a}\right)$

--- $\frac{2\omega c}{c^2 - v^2 - \omega^2}$

- A slope $\rho g Z_c V_r (c^2 - v^2) + 2Kc$
- B slope $-2[\rho g Z_c V_r (c^2 - v^2) + 2Kc]$
- 1 $[K(c^2 - v^2) / (2\rho g Z_c V_r c + K)]^{1/2}$
- 2 $\{[\rho g Z_c V_r (c^2 - v^2) + 2Kc] / (3\rho g Z_c V_r)\}^{1/2}$
- 3 $\{[\rho g Z_c V_r (c^2 - v^2) + 2Kc] / (\rho g Z_c V_r)\}^{1/2}$
- 4 $K(c^2 - v^2)$
- 5 $K[(c^2 + v^2) \cosh(\frac{2\ell v}{a}) - 2cv \sinh(\frac{2\ell v}{a})]$
- 6 $\{[\rho g Z_c V_r (c^2 - v^2) + 2Kc] / (3\rho g Z_c V_r)\}^{1/2} \cdot 2[\rho g Z_c V_r (c^2 - v^2) + 2Kc] / 3$

Fig. 24

$K = 10^9 \text{ dyne/cm}^2$, $\mu = 10^9 \text{ dyne/cm}^2$, $E = 2.5 \cdot 10^9 \text{ dyne/cm}^2$
 $\rho = 2 \text{ g/cm}^3$, $a = 1.581 \cdot 10^4 \text{ cm/s}$, $R = 10^2 \text{ cm}$, $g = 9.81 \cdot 10^2 \text{ cm/s}^2$
 $V_r = 10^8 \text{ cm}^3$, $l = 10^4 \text{ cm}$, $Z_0 = 10^{-2} \text{ s/cm}^2$, $Z_c = 5.13 \cdot 10^{-4} \text{ s/cm}^2$
 $c = 2.934 \cdot 10^2 \text{ s}^{-1}$.

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