PPPL-1840

PPPL-1840

DC20F

180-82
5-9-82
(H)

I-4611

①

dr. 727

MASTER

ANALYSIS OF SAWTOOTH RELAXATION OSCILLATIONS
IN TOKAMAKS

by

K. Yamazaki, K. McGuire, and M. Okabayashi

July 1982

PLASMA
PHYSICS
LABORATORY

PRINCETON UNIVERSITY
PRINCETON, NEW JERSEY

PREPARED FOR THE U.S. DEPARTMENT OF ENERGY,
UNDER CONTRACT DE-AC02-76-CNO-3073.

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

ANALYSIS OF SAWTOOTH RELAXATION OSCILLATIONS

IN TOKAMAKS

PPPL--1840

DEG2 018141

K. Yamazaki,* K. McGuire, M. Okabayashi

Plasma Physics Laboratory, Princeton University

Princeton, New Jersey 08544

Abstract

Sawtooth relaxation oscillations are analyzed using the Kadomtsev's disruption model and a thermal relaxation model. The sawtooth period is found to be very sensitive to the thermal conduction loss. Qualitative agreement between these calculations and the sawtooth period observed in several tokamaks is demonstrated.

DISCLAIMER
This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, or method disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not constitute an endorsement, recommendation, or approval by the United States Government or any agency thereof. The views and opinions of authors included herein do not necessarily state or reflect those of the United States Government or any agency thereof.

* On leave from: Institute of Plasma Physics, Nagoya University,
Nagoya 464, Japan.

1. Introduction

Internal disruptions due to $m=1$ MHD activities degrade the confinement of the internal region of tokamak plasmas abruptly, and the total energy confined in the hot plasma region is strongly affected by the period of sawtooth relaxation oscillations. Therefore, it is important to obtain scaling laws for the sawtooth period.

There are some analytical scaling laws [1-2] for sawtooth periods estimated using the $m=1$ linear-mode island width. The basic idea in these analyses is based on the fact that the relaxation period is determined by the growth of the resistive $m=1$ tearing mode. There are some detailed computer simulations to describe the relaxation process before and after internal disruptions [3-6].

In this paper, scaling laws for the sawtooth period in tokamaks are studied with a simple analysis which includes an energy/current relaxation process and Kadomtsev's heuristic reconnection treatment [7] for the $m=1$ MHD mode. An advantage of adopting Kadomtsev's model is that it is free from the linear growth rate of the mode; thus, the relaxation process can be easily included in the analysis. In this model it is assumed that the effect of the $m=1$ magnetic island is to modify the current profile and the transport process in the core of the plasma in a shorter time than the sawtooth oscillation period. Effects of neutral beam heating can be included in this analysis.

2. Model and Analysis

The sawtooth oscillation is a periodic $m=0$, $n=0$ mode caused by the growth of an internal $m=1$, $n=1$ tearing mode. A disruptive phase is separable from a relaxation phase as shown in Fig. 1. Thus it is adequate to use the Kadomtsev's disruption model [7] including magnetic flux conservation and

energy conservation for the disruptive phase, while we may use the energy and current relaxation equations for the relaxation process.

In the relaxation phase ($0 \leq t \leq t_0$), the $m=0$ evolution of current and temperature profile in the cylindrical geometry (r, θ, z) is given by

$$\frac{\partial}{\partial t} \mu = \frac{1}{r} \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\eta}{\mu_0} \frac{\partial}{\partial r} (r^2 \mu) \right), \quad (1)$$

and

$$\begin{aligned} \frac{\partial}{\partial t} T_e = & \frac{1}{nr} \frac{\partial}{\partial r} \left(nr \chi_e \frac{\partial}{\partial r} T_e \right) + \frac{2}{3} \frac{\eta j_z^2}{n} \\ & + \frac{2}{3n} (P_{Be} + P_{ie} - P_R). \end{aligned} \quad (2)$$

Here the inverse safety factor μ ($\equiv RB_\theta/rB_z$) and the electron temperature T_e are functions of both time t and radius r . Equation (1) including plasma resistivity η shows the current relaxation, and Eq. (2) is the temperature relaxation with the assumption of constant plasma density n , where P_{Be} , P_R , and P_{ie} denote beam heating power, radiation power loss, and power transferred from ions to electrons, respectively. By assuming small perturbations, such as

$$\mu = 1 + \hat{\mu}, \quad |\hat{\mu}| \ll 1$$

and

$$T_e = T_{e0} (1 + \hat{T}_e), \quad |\hat{T}_e| \ll 1,$$

and by introducing characteristic times related to the singular radius $r = r_s$,
skin time,

$$\tau_{\text{skin}} = \mu_0 r_s^2 / \eta_0 ,$$

ohmic heating time,

$$\tau_{\text{OH}} = 3 n T_{\text{eo}} / (2 \eta_0 j_{\text{zo}}^2) ,$$

beam heating time,

$$\tau_B = 3 n T_{\text{eo}} / (2 P_{\text{Bo}}) ,$$

thermal conduction time,

$$\tau_E = r_s^2 / \chi_e ,$$

and radiation cooling time,

$$\tau_R = 3 n T_{\text{eo}} / (2 P_{\text{Ro}}) ,$$

we obtain the dimensionless linearized equation with a normalized distance
($x = r/r_s$) and a normalized time ($\lambda = t/\tau_{\text{skin}}$),

$$\frac{\partial}{\partial \lambda} \hat{\mu} \approx - \frac{3s}{x} \frac{\partial}{\partial x} \hat{T}_e + \frac{1}{x} \frac{\partial}{\partial x} \left(\frac{1}{x} \frac{\partial}{\partial x} x^2 \hat{\mu} \right) , \quad (3)$$

and

$$\frac{\partial}{\partial \lambda} \tilde{T}_e = \frac{\tau_{\text{skin}}}{\tau_E} \frac{1}{x} \frac{\partial}{\partial x} \left(x \frac{\partial}{\partial x} \tilde{T}_e \right) + \frac{2\tau_{\text{skin}}}{\tau_{\text{OH}}} \frac{\partial}{\partial x} (x^2 \tilde{\mu}) - \frac{\tau_{\text{skin}}}{\tau_*} \tilde{T}_e + \frac{\tau_{\text{skin}}}{\tau_Q}, \quad (4)$$

where

$$\frac{1}{\tau_*} \equiv -\frac{s_\eta}{\tau_{\text{OH}}} - \frac{s_B}{\tau_B} - \frac{s_{ie}}{\tau_{ie}} + \frac{s_R}{\tau_R},$$

and

$$\frac{1}{\tau_Q} \equiv \frac{1}{\tau_{\text{OH}}} + \frac{1}{\tau_B} + \frac{1}{\tau_{ie}} - \frac{1}{\tau_R}.$$

We assumed the following temperature dependence given by exponents s ,

$$\eta \propto T_e^{s_\eta},$$

$$P_R \propto T_e^{s_R},$$

$$P_B \propto T_e^{s_B},$$

and

$$P_{ie} \propto T_e^{s_{ie}}.$$

In the case of normal resistivity, $s_\eta \sim -3/2$, while for anomalous resistivity in the center or outside the hot core, we can use $s_\eta > -3/2$ or $s_\eta < -3/2$, respectively. The bremsstrahlung dominant radiation case leads to $s_R = 1/2$; on the other hand, the impurity dominant radiation gives rise to $s_R \lesssim 0$. For beam heating power contributing to electrons, we can write $s_B \sim -3/2$, and, for electron ion power transfer in high electron temperature and high ion temperature cases, we may use $s_{ie} \sim -1/2$ and $s_{ie} \sim -3/2$, respectively.

In the disruptive phase ($t_0 \leq t \leq t_1$), we will treat the magnetic flux conservation and energy conservation similar to that in Refs. [5-7]. In the following, let us analyze a case with parabolic current profile within the singular surface,

$$\hat{\mu}(\lambda, x) = U_0(\lambda) + U_1(\lambda) x^2. \quad (5)$$

With the hypothesis that the magnetic flux and the total energy are conserved within the mixing zone of radius $\sqrt{2} r_s$, we obtain

$$\begin{aligned} U_0(x=0) &= 0, \\ U_1(x=0) &= -\Delta_j/4, \\ U_0(\lambda=\lambda_1) &= \Delta_j, \\ U_1(\lambda=\lambda_0) &= -\Delta_j, \end{aligned} \quad (6)$$

and

$$\begin{aligned} &\int_0^{\sqrt{2}} [\hat{T}_e(\lambda_0, x) - \hat{T}_e(0, x)] x dx \\ &= \frac{1}{\beta_{\theta s}} \int_0^{\sqrt{2}} [\hat{\mu}(0, x)^2 - \hat{\mu}(\lambda_0, x)^2] x^3 dx \\ &= \frac{5}{35} \frac{\Delta_j^2}{\beta_{\theta s}} \sim 0, \end{aligned} \quad (7)$$

where Δ_j denotes a current perturbation in the plasma center and

$$\beta_{\theta s} = \frac{n T_{eo}}{B_0^2 (r_s)^2 / 2 \mu_0}.$$

Substituting Eq. (5) into Eqs. (3) and (4) and using boundary conditions (6) and (7), we obtain the temperature perturbation

$$\hat{T}_e(\lambda, x) = A_2(\lambda) x^4 + A_1(\lambda) x^2 + A_0(\lambda),$$

where coefficients $A_j(\lambda)$ are given in the Appendix A. The functions $U_j(\lambda)$ are also given there as functions of several characteristic times and sawtooth time ($\lambda_{\text{saw}} \sim \lambda_0$).

Assuming six characteristic times and changing the sawtooth period, we can obtain the time and space variations of the perturbed electron temperature which can fit some experimental results.

3. Results of the Calculation and Comparisons with Experiments

First, we plotted experimental sawtooth period normalized by skin time in many tokamaks as a function of normalized ohmic heating time in Fig. 2. From this figure we can obtain a rough empirical scaling law for the sawtooth periods,

$$\tau_{\text{saw}} = (0.1 - 0.2) \tau_{\text{OH}}^{1/2} \tau_{\text{skin}}^{1/2}. \quad (8)$$

This simple scaling can be obtained analytically as shown in Appendix B using Eqs. (3), (5), (6), and

$$\begin{aligned} \frac{d}{dt} \hat{T}_e &\sim \frac{1}{\tau_{\text{OH}}} \text{ at } r = 0, \\ \hat{T}_e &\sim 0 \quad \text{at } r = r_s, \end{aligned} \quad (9)$$

instead of the detailed transport equation [Eq.(4)]. Here it is assumed that

$$\tau_{\text{saw}} \lesssim \tau_{\text{OH}} \lesssim \tau_{\text{skin}},$$

which is satisfied in most tokamaks.

In order to determine the sawtooth period, we took the criteria that central temperature should increase, outside temperature should decrease and temperature on the singular surface should be held constant. Figure 3 shows the effect of thermal transport on the sawtooth period. Here, energy confinement time τ_E defined by the singular surface radius is smaller than the global confinement time τ_E^G

$$\tau_E \sim K \left(\frac{r_s}{a} \right)^2 \tau_E^G,$$

and

$$\tau_E^G \sim \tau_{\text{OH}},$$

where $K \sim 4-6$ depending on the profile. Therefore,

$$\tau_E \sim (0.2 - 1.0) \tau_{\text{OH}}$$

is acceptable in an ohmic heating experiment. For fixed singular surface confinement time τ_E , we obtained two sawtooth regions: one for $\tau_{\text{skin}} > \tau_{\text{OH}}$ and the other for $\tau_{\text{skin}} \lesssim \tau_{\text{OH}}$ area. The former domain is not shown in Fig. 3 and is characterized by the fact that the amplitude of the current peaking is as large as the amplitude of temperature peaking, as discussed ("thermal

instability") in Ref. [8]. The latter domain shows sawtooth oscillation related to actual experiments. Calculated time-variations of the electron temperature are shown in Fig. 4 for the experimental data corresponding to PDX.

In the model, thermal conduction loss reduces the sawtooth period for a fixed ohmic heating time τ_{OH} , because the current peaking is essentially derived from rapid thermal transport. The amplitude of sawtooth oscillations decrease as thermal conduction increases for the fixed ohmic heating rate.

The radiation has no significant effects on the sawtooth period as far as medium radiation losses ($\tau_R < \tau_{OH}$) are concerned. Most of the experimental data in tokamaks with low-Z material limiters show hollow profiles of radiated power which do not contribute to the energy balance in the plasma core.

Figure 5 shows an example of the effects of neutral beam heating on the sawtooth period. In the actual experiment, both the ohmic heating time and the skin time increase due to the rise of electron temperature and the decrease of the loop voltage. The radius of the $q=1$ surface increases which leads to an increase of τ_E , if the thermal conduction coefficient χ_e does not change. Thus, all characteristic parameters change; however, the ratio of the measured sawtooth time (τ_{saw}) to the singular surface skin time (τ_{skin}) remains constant or slightly decreases due to neutral beam heating. A detailed comparison of the analysis to the PDX experiment with neutral beam heating is being carried out.

4. Discussions

We obtained qualitative agreement between the analysis and many experiments using Kadomtsev's parabolic current sawtooth model. The sawtooth period is found to be sensitive to the thermal conduction loss. The neutral

beam heating may change basic characteristic times such as skin time, ohmic heating time, and overall heating time and make the sawtooth period longer. By changing the boundary condition of Eq. (6), we can study sawtooth oscillations in hollow current profiles as discussed in Ref. [6].

In order to improve the present model, we should include higher order expansion of the equations in Appendix A to take into account the effects of the outside edge of the disruptive area. Changing the zeroth order temperature profile from uniform to parabolic would also be helpful.

ACKNOWLEDGMENT

This work was supported by the U.S. Department of Energy, Contract Number DE-AC02-76-CHO-3073.

REFERENCES

- [1] a) G.L. Jahns, M. Soler, B.V. Waddell, J.D. Callen, and H.R. Hicks, Nucl. Fusion 18 (1978) 609.
 b) B.V. Waddell, G.L. Jahns, J.D. Callen and H.R. Hicks, Nucl. Fusion 18 (1978) 735.
- [2] K. McGuire and D.C. Robinson, Nucl. Fusion 19 (1979) 505.
- [3] A. Sykes and J.A. Wesson, Phys. Rev. Lett. 37 (1976) 140.
- [4] B.V. Waddell, M.N. Rosenbluth, D.A. Monticello, and R.B. White, Nucl. Fusion 16 (1976) 528.
- [5] Yu. N. Dnestrovskii, S.E. Lysenko, and R. Smith, Fizika Plasmy 3 (1977) 18 [Sov. J. Plasma Phys. 3 (1977) 9].
- [6] V.V. Parail and G.V. Pereverzev, Fizika Plazmy 6 (1980) 27 [Sov. J. Plasma Phys. 6 (1980) 14].
- [7] B.B. Kadomtsev, Fizika Plazmy 1 (1975) 710 [Sov. J. Plasma Phys. 1 (1975) 389].
- [8] H.P. Furth, M.N. Rosenbluth, P.H. Rutherford, and W. Stodiek, Phys. Fluids 13 (1970) 3020.

APPENDIX A

According to Kadomtsev [7], we use the helical flux ψ^* defined by

$$\frac{\partial}{\partial r} \psi^* = B_\theta - \frac{r}{R} B_z \quad (A-1)$$

like

$$\psi^* = \sum_{j=0}^N k_j(\lambda) x^{2j}, \quad (A-2)$$

where the initial and final values of the coefficients, $k_j(\lambda = 0)$ and $k_j(\lambda = \lambda_{\text{saw}})$, are related to each other by the flux conserving condition during the internal disruption. We can choose an appropriate N value to describe the time evolution of the helical flux. Using Eq. (A-2), we can obtain

$$\hat{\mu} = \sum_{j=0}^N \frac{1}{2} U_j(\lambda) x^{2j}, \quad (A-3)$$

where

$$U_j(\lambda) = \frac{j+1}{2\psi_s^*} k_{j+1}(\lambda),$$

and the boundary values, $U_j(0)$ and $U_j(\lambda_{\text{saw}})$ are known.

Substituting Eq. (A-3) into Eq. (3),

$$\hat{T}_e = \sum_{j=0}^N A_j(\lambda) x^{2j} \quad (A-4)$$

where

$$\tau_N(\lambda) = -\frac{1}{6(N+1)} \frac{d}{d\lambda} U_N(\lambda)$$

and

$$A_j(\lambda) = -\frac{1}{6(j+1)} \left[\frac{d}{d\lambda} U_j(\lambda) - 4(j+1)/(j+2) U_{j+1}(\lambda) \right] \\ (j = 1, 2, \dots, N-1).$$

The temperature balance equation, Eq. (4), leads to

$$\frac{d}{d\lambda} A_N + \frac{A_N}{\lambda_*} = 0$$

$$\frac{d}{d\lambda} A_j + \frac{A_j}{\lambda_*} = \frac{4}{\lambda_E} (j+1)^2 A_{j+1} + 4(j+1) U_j / \lambda_{OH} \\ (j = 1, 2, \dots, N-1)$$

and

$$\frac{d}{d\lambda} A_0 + \frac{A_0}{\lambda_*} = 4 \frac{A_1}{\lambda_*} + \frac{4U_0}{\lambda_{OH}} + \frac{1}{\lambda_Q}.$$

Finally, using energy conservation during the internal disruption, Eq. (7), we can get the solution.

For example, the explicit solutions for $N = 4$ case is as follows:

$$\mu \equiv u(\lambda) + v(\lambda) x^2 + w(\lambda) x^4,$$

$$\tilde{T}_e \equiv a(\lambda) x^6 + b(\lambda) x^4 + c(\lambda) x^2 + d(\lambda).$$

where

$$w(\lambda) = w_1 + w_2 \exp(-\lambda/\lambda_*) ,$$

$$v(\lambda) = v_1 + v_2 \lambda + (v_3 + v_4 \lambda) \exp(-\lambda/\lambda_*) ,$$

$$u(\lambda) = u_1 + u_2 \lambda + u_3 \lambda^2 + (u_4 + u_5 \lambda + u_6 \lambda^2) \exp(-\lambda/\lambda_*) ,$$

and

$$a(\lambda) = a_1 \exp(-\lambda/\lambda_*) ,$$

$$b(\lambda) = (b_1 + b_2 \lambda) \exp(-\lambda/\lambda_*) + b_3 ,$$

$$c(\lambda) = (c_1 + c_2 \lambda + c_3 \lambda^2) \exp(-\lambda/\lambda_*) + c_4 \lambda + c_5 ,$$

$$d(\lambda) = (d_1 + d_2 \lambda + d_3 \lambda^2 + d_4 \lambda^3) \exp(-\lambda/\lambda_*) + d_5 \lambda^2 + d_6 \lambda + d_7 .$$

Here time-independent coefficients are given by

$$w_2 = [w(0) - w(\lambda_{\text{saw}})] / [1 - \exp(-\lambda_{\text{saw}}/\lambda_*)]$$

$$w_1 = w(0) - w_2$$

$$a_1 = w_2 / 18 \lambda_*$$

$$b_2 = 36 a_1 / \lambda_E + 6 w_2 / \lambda_{\text{OH}}$$

$$b_3 = 6 \lambda_* w_1 / \lambda_{\text{OH}}$$

$$v_2 = 24 w_1 - 12 b_3$$

$$v_4 = 12 b_2 \lambda_*$$

$$v_3 = [v(0) - v(\lambda_{\text{saw}}) + v_2 \lambda_{\text{saw}} + v_4 \lambda_{\text{saw}} \exp(-\lambda_{\text{saw}}/\lambda_*)] / [1 - \exp(-\lambda_{\text{saw}}/\lambda_*)]$$

$$v_1 = v(0) - v_3$$

$$b_1 = (v_3/\lambda_* + 24w_2 - 12b_2\lambda_*) / 12$$

$$c_2 = 16 b_1/\lambda_E + v_3/\lambda_{OH}$$

$$c_3 = 8 b_2/\lambda_E + 2 v_4/\lambda_{OH}$$

$$c_4 = 4\lambda_* v_2/\lambda_{OH}$$

$$c_5 = (16 b_3/\lambda_E + 4 v_1/\lambda_{OH} - c_4) \lambda_*$$

$$u_2 = 8 v_1 - 6 c_5$$

$$u_3 = 4 v_2 - 3 c_4$$

$$u_5 = -\lambda_* (8v_4 - 6c_2) + 12\lambda_*^2 c_3$$

$$u_6 = 6\lambda_* c_3$$

$$u_4 = [u(0) - u(\lambda_{saw}) + u_2 \lambda_{saw} + u_3 \lambda_{saw}^2 + (u_5 \lambda_{saw} + u_6 \lambda_{saw}^2) \exp(-\lambda_{saw}/\lambda)] / [1 - \exp(-\lambda_{saw}/\lambda_*)]$$

$$u_1 = u(0) - u_4$$

$$c_1 = [8u_3 + \lambda_* (8u_4 - 6 c_2) - (2\lambda_*^2 c_3 + u_4/\lambda_*)] / 6$$

$$d_2 = 4 c_1/\lambda_E + 2 u_4/\lambda_{OH}$$

$$d_3 = 2 c_2/\lambda_E + u_5/\lambda_{OH}$$

$$d_4 = (4 c_3/\lambda_E + 2 u_6/\lambda_{OH}) / 3$$

$$d_5 = 2 u_3 \lambda_*/\lambda_H$$

$$d_6 = (4 c_4/\lambda_E + 2 u_2/\lambda_{OH} - 2 d_5) \lambda_*$$

$$d_7 = (4 c_5/\lambda_E + 2 u_1/\lambda_{OH} + 1/\lambda_Q - d_6) \lambda_*$$

The value d_1 can be determined by Eq. (7).

APPENDIX B

Neglecting the last term in Eq. (3) and using Eqs. (5), (6), and (9), we obtain simple forms for the inverse safety factor perturbation and the temperature perturbation:

$$\hat{\mu} = \frac{\Delta_j}{8} \left[\left(3 \frac{t}{\tau_{\text{saw}}} + 5 \frac{t^2}{\tau_{\text{saw}}^2} \right) - 2 \left(1 + 3 \frac{t}{\tau_{\text{saw}}} \right) x^2 \right], \quad (\text{B-1})$$

$$\hat{T}_e = (1 - x^2) \left(\frac{t}{\tau_{\text{OH}}} - \frac{\Delta_j \tau_{\text{skin}}}{16 \tau_{\text{saw}}} x^2 \right), \quad (\text{B-2})$$

and

$$\frac{1}{\tau_{\text{OH}}} = \frac{5}{24} \Delta_j \frac{\tau_{\text{skin}}}{\tau_{\text{saw}}^2}.$$

These equations satisfy the energy conservation equation, Eq. (7).

Thus, we obtain a simple scaling law for the sawtooth period:

$$\tau_{\text{saw}} = C \tau_{\text{OH}}^{1/2} \tau_{\text{skin}}^{1/2},$$

$$C \equiv \sqrt{\frac{5}{24} \Delta_j}.$$

FIGURE CAPTIONS

Fig. 1. Sawtooth oscillation model with relaxation phase I and disruptive phase II.

Fig. 2 Empirical scaling for sawtooth oscillations. Sawtooth period and ohmic heating time are normalized by skin time. Broken line shows

$$\tau_{\text{saw}} \sim (0.1 - 0.2) \tau_{\text{OH}}^{1/2} \tau_{\text{skin}}.$$

Fig. 3 Effects of thermal conduction loss on the sawtooth period in ohmic heating plasmas.

Fig. 4 Time variation of calculated temperature as a function of radius using PDX parameter ($\tau_E \sim \tau_{\text{OH}} \sim 0.1 \tau_{\text{skin}}, \tau_{\text{saw}} \sim 0.05 \tau_{\text{skin}}$).

Fig. 5 Effects of neutral beam heating on the period of sawtooth oscillations.

81X1079

18

Fig. 1

Fig. 2

Fig. 3

81X1080

$r=0$ $r=0.5 r_s$ $r=r_s$ $r=\sqrt{2} r_s$

Fig. 4

Fig. 5