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A THEORETICAL STUDY OF THE TRANSIENT OPERATION AND
STABILITY OF TWO-PHASE NATURAL CIRCULATION LOOPS

by

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ABSTRACT

Natural-circulation loops have been applied in many domestic and industrial processes. Recently, new applications have been found in high-power generators, but under these conditions the operation of such loops, usually containing a mixture of vapor and liquid, has not always been predictable. In some of the proposed applications, unstable behavior could have dire effects, and for safety as well as economy, a better theoretical understanding of the loop operation is needed.

To this end, mathematical models of the time-dependent behavior of two-phase natural-circulation loops were derived from basic conservation equations and used to predict the operation and to explain the unusual instability sometimes observed. Lumped-parameter models, which essentially simulated the system as a series of cascaded mixing cells, were used. Initial solutions were obtained by using a PACE analog computer and patterning the geometry and operating conditions after those of the University of Minnesota loop from which data have been reported. The solutions predicted oscillatory behavior similar to that reported, with mean flow rates, temperatures, and qualitative trends in general agreement with experimental results. In order to obtain solutions, however, many simplifying assumptions were made, and the solutions were not completely satisfactory.

The initial results were used in formulating subsequent models of greater complexity and improved accuracy for which solutions were obtained with the UNIVAC Scientific, a large digital computer. The resulting predicted transient behavior was in close agreement with the experimental results from the University of Minnesota low-pressure loop. When the geometry and operating conditions of a 300-psia, high-pressure loop were used in the model, unstable oscillatory behavior was predicted under certain conditions, and stable behavior under others. Closed unstable regions rather than limits were predicted, and the specifications of stability in terms of a single parameter was found to be impossible. The results showed qualitative agreement with experimental results obtained at several laboratories and oscillatory frequencies of the same magnitude as those observed, but so little experimental data were available that a complete comparison was not possible. The slip ratio and the nature of its variation were found to be critical in determining the predicted loop behavior. The model finally used was versatile, reasonably fast, and quite complete.

A comparison of the causes of the oscillatory behavior at low and at high pressure indicated that they were essentially the same, and that the great difference in oscillatory frequencies that has been experimentally noted was due largely to system geometry, and partially to the nature of the oscillations themselves. The criterion for the absence of oscillations was found to be similar to one of the criteria for stability of chemical reaction systems.

INTRODUCTION

A natural-circulation loop is a closed circuit which is comprised of a heat source at one elevation, a heat sink at a higher elevation, a surge tank to accommodate changes in fluid volume and maintain constant pressure at one location within the loop, and interconnecting piping. Fluid flow is maintained by the force which arises from the difference in fluid density in the hot and cold vertical legs of the loop. Considerable engineering research and study have been devoted to the theory and operation of natural circulation systems because of their widespread domestic and industrial applications, but until recently the major portion of the research dealt with time-independent, steady-state phenomena. With the advent of nuclear reactors and other systems which generate large quantities of thermal energy and make necessary heat removal rates far greater than those conventionally encountered, controllability and burnout have become important factors, and their consideration has led to the kindling of interest in the time-dependent, transient behavior of two-phase natural-circulation systems with and without heat transfer. Only with a better understanding of the relations which exist in such systems can economic design be accomplished without extensive experimental study for each particular system.

The object of the research described herein was to create a mathematical model of a natural-circulation loop, in some portions of which two phases existed, from applications of the fundamental well-established laws of energy, mass, and momentum conservation, and of phase equilibrium, which would predict the behavior of such a loop under different operating conditions. Equations applying the fundamental concepts may be written for a differential volume element of the flow system without undue difficulty, but the solutions of the equations, which are nonlinear partial differential equations, are difficult, both because of their nonlinearities and of the empirical and sometimes uncertain nature of many of the essential elements of the problem. To obtain solutions, therefore, several approximations must be made, but if these are made properly, the important features are retained and the basic behavioral characteristics revealed. Any direct analytical solution is still probably impossible, except in trivial cases, and a reduction of the equations to some form appropriate for solution with a high-speed electronic computer must be made. In the work described, both analog and digital computers were used.

One of the most interesting of the phenomena surrounding the operation of two-phase natural-circulation loops is the occurrence under certain conditions of sustained oscillations in flow rate and associated dependent variables, even though all of the controlled independent variables and system parameters are held constant. It is with this phenomenon that the work was primarily concerned and whose prediction was sought with the use of the mathematical model(s).

In the following sections, the previous work will be briefly described, the assumptions underlying the mathematical models and the models themselves presented, comparisons between the predicted and experimental behavior presented, and the significance of the results discussed.

LITERATURE SURVEY

Experiments have been conducted and results obtained for the transient operation of many systems which embody one or more natural-circulation loops. Under an extremely wide variety of operating parameters, oscillations of the fluid temperature, pressure, and flow have been observed, even though all of the controlled variables have been held constant. The first detailed report of such oscillations was made in 1955 by Wissler, Amundson, and Isbin^(33,34) based on experiments conducted at the University of Minnesota with an electrically heated, single crossover test loop operated at atmospheric pressure. Flow rate and temperature traces exhibited nonharmonic oscillations of varying amplitudes and frequencies depending on the power input. The periods were of the order of magnitude of one minute. At low-power operation, for which no vapor was formed, and again at high-power inputs, completely stable operation was observed.

A series of experimental studies has since been made at the Argonne National Laboratory with electrically heated test loops at pressures ranging from 75 to 600 psig.⁽²¹⁾ Open loops were used in which the steam was separated and removed from the system, and makeup water added. The primary objective of the experiments was to determine the conditions accompanying the onset of the oscillations, nonrandom fluctuations of the exit steam volume fraction being the criterion used to define instability. It was found that the power level at which oscillations first occurred increased with increasing pressure, was affected by the geometry (insofar as it increased with frictional pressure drop in the downcomer and decreased with pressure drop in the heater and riser regions), and was relatively insensitive to the level of liquid above the riser section. The periods of the oscillations observed during these experiments did not show a great deal of variation and were of the order of magnitude of one second.

The General Electric Atomic Power Equipment Department has conducted experiments using an annular test section with an electrically heated core in a natural-circulation loop operated at 1000 psia.⁽²⁰⁾ In these tests the primary attention was directed toward establishing the effects of various parameters, including power input, subcooling of the fluid entering the heater, heater geometry, and system geometry on the stability of the loop and the burnout heat flux. Transient temperature, pressure drop, and flow traces were recorded; instability, as evidenced by a variation in pressure drop across the test section of greater than $\pm 10\%$, could not be correlated as a function of any single parameter, such as exit

steam volume fraction, but occurred over a large range of any single parameter with variations in the other parameters. The period of the oscillations was again found relatively insensitive to changes in operating conditions and was usually between 2 and 4 sec. Among the general observations made were that increased subcooling of the inlet water resulted in greater instability, increased frictional pressure drop in the downcomer resulted in lower flow rates and greater stability, and that an increase in the size of the electrically heated core was accompanied by a decrease in flow rate and an increase in the loop stability.

The Westinghouse Atomic Power Division has also been engaged in a program of experimental and theoretical investigation of electrically heated natural-circulation loops.⁽²⁴⁾ Their experiments have been made on loops containing single and multiple heated rectangular channels at pressures from 800 to 2000 psia. Flow and pressure drop oscillations were observed whose amplitude was found to increase at lower system pressures and heater inlet subcooling. The period of these oscillations was of the order of magnitude of one second.

Transient operation of boiling water nuclear reactors has been widespread, and considerable data have been published concerning the BORAX,^(15,30) SPERT,⁽²⁷⁾ EBWR,⁽¹⁵⁾ and VBWR⁽⁶⁾ reactors. Two approaches have been used in experiments conducted with these reactors to determine their stability. One is to measure the experimental response of reactor power, temperature, etc., to a step or ramp change in reactivity, controlled by control rod position, whereas the other is to measure the response of the reactor to an harmonically oscillating reactivity. Since the reactivity of the fuel elements is related to both temperature and vapor volume fraction in the coolant channels, however, the system is more complex than one electrically heated.

To construct a theoretical model based on ultimate considerations of the conservation equations, recourse must be made to basic references in the fields of thermal and fluid dynamics, and to references in applied mathematics. The most helpful of these were books by Lamb,⁽¹⁹⁾ Shapiro,⁽³¹⁾ Bird, Steward and Lightfoot,⁽⁸⁾ Mickley, Sherwood and Reed,⁽²⁵⁾ and Miller.⁽²⁶⁾ In the field of oscillations, a book by Andronow and Chaikin⁽³⁾ was enlightening.

Any model of a system in which two-phase flow occurs involves a description of this type of flow and the associated pressure drops due to frictional losses. No completely adequate models exist for describing either the type of flow or the pressure drop, but extensive data are available. Many experiments have been run at the Argonne National Laboratory on two-phase vertical flow systems.^(21,23) The data from such experiments were used to prepare empirical correlations of slip ratio (the ratio of vapor to liquid velocity) with pressure, vapor quality, and superficial liquid velocity.⁽²³⁾

Several experimental and theoretical studies of two-phase pressure drop have been made. A comprehensive literature survey of work in this field until 1954 was made by Isbin, Moen, and Mosher.⁽¹⁶⁾ More recently, work apropos steam-water systems has been summarized by Marchaterre.⁽²³⁾

The purpose of most theoretical analyses has been to predict the occurrence and nature of oscillations as functions of system parameters. Two distinct general approaches are evident. One has used the most general time-dependent equations describing the conservation and equilibrium relations. Under suitable restraints and approximations, solutions to these equations may be obtained and studied for an indication of the influence of system parameters on stability. The other approach has isolated a single phenomenon as being the determining factor in the occurrence of oscillations, and has sought criteria for the existence of oscillations solely on the basis of this phenomenon.

The theoretical analysis made by Wissler at the University of Minnesota used the first approach together with experimental pressure drop data and flow characteristics.⁽³³⁾ The analysis was programmed for a digital computer, but the solutions did not converge. A subsequent model due to George,⁽¹⁴⁾ utilizing the same data but making several simplifying assumptions, did converge to solutions which agreed very closely with the experimental results obtained by Wissler.

Similar in the sense that the starting points have been the general conservative equations, but different in both the methods of simplification and application, are the analyses made by Quandt of the Westinghouse Corporation.⁽²⁸⁾ These have been applied to high-pressure systems within which the pressure drop across the heater and riser have been assumed constant, and have led, via linearized stability studies, to predictions of the existence and frequency of oscillations which apparently agree with experiment.

The second approach has been used by Chilton,⁽⁹⁾ who assumes that instability will occur at the conditions accompanying a maximum in the plot of pressure drop vs mass flow rate. With this model, rather precise statements may be made concerning the conditions under which flow oscillations will occur, but recent experimental data tend to cast doubt on the original premise.⁽²⁾

A model for vertical two-phase flow kinematics based on the potential energy of the static head in the two-phase region has been designed by Beckjord.⁽²⁰⁾ This model may be used to predict the existence of flow oscillations and their periods, and although the predicted periods have agreed satisfactorily with those observed, attempts to predict thresholds at which oscillations occur have not been successful. Levy⁽²⁰⁾ has proposed a model which attributes instability to the inability of a fluid

phase to dissipate at the channel wall all of the shearing energy it receives. Its validity has not been conclusively demonstrated, and it is unable to predict oscillatory frequency or parametric dependency. A model based on the general equations of conservation is that due to Fleck and Huseby.⁽¹²⁾ The results, however, have shown "remarkable stability," and have apparently not been compared with experimental results.

In solving the system of time-dependent equations, high-speed analog and digital computers were used. Although a knowledge of the operational characteristics of such devices is becoming almost routine among chemical engineers, good references are necessary if routine pitfalls are to be avoided. For the operation of the analog computer, books by Korn and Korn,⁽¹⁸⁾ and Johnson⁽¹⁷⁾ were of particular value, and a publication by Scott⁽²⁹⁾ concerning the specific features of the ANL analog facility was useful.

DERIVATION OF EQUATIONS

The purpose of the investigation was not merely to obtain criteria for avoiding oscillations, but rather to investigate the general behavior of two-phase natural-circulation loops and to design a model which could be used to deduce new information about their operation. Therefore, a general consideration of the problem of describing mathematically the operation was made. It showed that equations ultimately based on the physical principles of conservation of mass, energy, and momentum, and on the nature and degree of equilibrium existing between the phases, must be used. Time-dependent conservation equations for natural-circulation loops have been derived by Alstad⁽¹⁾ and Wissler⁽³³⁾ for the cases of single-phase and two-phase flow, respectively. The equations are partial differential equations in time and space, and, provided terms of sufficient generality are used to express some of the less well-known characteristics inherent in any combined heat transfer and flow system, no particular difficulty arises in the derivations.

In seeking solutions, however, difficulties of considerable magnitude do arise. Since several terms in the equations are highly nonlinear, particularly for two-phase flow, a direct analytical solution is impossible, and one must resort to one of several alternatives. The system may be so simplified that the set of equations is reduced to a linear system, and an analytic solution obtained. The range of validity of such a solution is, however, limited, and such results will generally be regarded with suspicion. A second alternative is to study the stability of such a system by using perturbations about some initial steady state, and applying the methods of nonlinear mechanics in investigating their time-dependent behavior to determine whether the result indicates unstable, stable, or oscillatory behavior. This is the general approach taken by Quandt in his analyses and has led to some very interesting results. The model assumes, however, that the mathematical model used to describe the system is accurate, and that the conditions which satisfy the existence of oscillations in the system of linearized equations also satisfy the existence of oscillations in the nonlinear system. It has been shown that a linearization about the steady state and the subsequent analysis of the linear system is a valid operation which describes the physical system in the neighborhood of the steady state, provided the system is either definitely stable or unstable.⁽⁷⁾ However, if the linear system exhibits sustained oscillations, it may be shown that the nonlinear system may have any of three types of behavior, stable, unstable, or oscillatory.

A third alternative is to attempt a solution of the equations using a high-speed electronic computer. The nonlinear differential equations may, by numerical analysis, be reduced to an equivalent set of finite difference equations and the solutions obtained with a large digital computer. In order to use an analog computer, the equations must be reduced to a set of ordinary differential equations. This may be done in various ways, of which one

will be discussed in detail later. The chief restriction encountered in analog simulation is in limited facilities available for generating nonlinear functions or in solving the extremely nonlinear equations.

Since the objective was to study natural-circulation system behavior in general, it was felt that a useful approach to the problem would be to study a simplified lumped-parameter model, which might be programmed for and solved by an analog computer. This would allow one to determine the characteristics of the mathematical system and determine which parameters have the most influence on the vital transient behavior. To this end, the parameters of the system were lumped in space, and the system of partial differential equations reduced to one of ordinary differential equations. The conservation equations for a lumped system may be derived by considering a volume element of pipe, as shown in Figure 1.

Figure 1

A Volume Element of Pipe Containing a Flowing Two-phase Mixture.

It is assumed that the element contains a one-component mixture of vapor and liquid which occupy distinct cross-sectional areas A_s and A_w . If it is further assumed that the velocity of each phase is uniform throughout the area occupied by it, and that its properties of density, enthalpy, viscosity, etc., are uniform throughout the volume element, then the conservation of mass may be written as

$$(A_w \rho_w V_w + A_s \rho_s V_s)_{j,i} - (A_w \rho_w V_w + A_s \rho_s V_s)_{j,o} = \frac{d}{dt} [(A_w \rho_w + A_s \rho_s) \Delta x]_j. \quad (1)$$

The subscript j designates the volume element, and subscripts i and o refer to inlet and outlet conditions, respectively. The remaining notation is standard and fully defined below.

The conservation of energy may be derived by considering the fluid within the j th element as the thermodynamic system. The rate at which potential, kinetic, and internal energies are carried into the system by convective flow is

$$A_w \rho_w V_w \left(e_w + \frac{V_w^2}{2g_c J} + \frac{gZ}{g_c J} \right) + A_s \rho_s V_s \left(e_s + \frac{V_s^2}{2g_c J} + \frac{gZ}{g_c J} \right)_{j,i} ,$$

whereas the rate at which it is transported away is

$$A_w \rho_w V_w \left(e_w + \frac{V_w^2}{2g_c J} + \frac{gZ}{g_c J} \right) + A_s \rho_s V_s \left(e_s + \frac{V_s^2}{2g_c J} + \frac{gZ}{g_c J} \right)_{j,o} .$$

Longitudinal diffusion at the entrance and exit is neglected as a mechanism of energy transport. It may be shown that, in addition, work is being done on the system by the fluid which enters, and by the system by that fluid which leaves. The difference between these terms is the net rate at which work is being done on the system due to the flowing fluid, and is given by

$$\left(\frac{A_w V_w p}{J} + \frac{A_s V_s p}{J} \right)_{j,i} - \left(\frac{A_w V_w p}{J} + \frac{A_s V_s p}{J} \right)_{j,o} .$$

By assumption, the velocity profile is flat, and therefore the velocity at the wall is not zero. Since shear stresses then exist between the fluid and the wall, these forces, acting through a distance equal to the velocity of the fluid at the wall per unit time, would actually do work on the system. In essence, this would be the thermal energy equivalent of the work done in overcoming shear forces at the wall. Even in cases of high frictional loss, however, the energy so contributed to the system will be a negligible fraction (0.5%) of the other terms and will be disregarded.

Heat can be transferred through the pipe wall to the flowing fluid, but the mechanism by which this transport occurs is an extremely complex subject and, in general, is beyond the scope of this investigation. The rate at which heat energy is transferred to the fluid will be designated as Q_j . The difference between the rates of energy input and output may now be equated to the rate of accumulation as

$$\begin{aligned}
& \left[A_w \rho_w V_w \left(e_w + \frac{p}{\rho_w J} + \frac{V_w^2}{2g_c J} + \frac{gZ}{g_c J} \right) + A_s \rho_s V_s \left(e_s + \frac{p}{\rho_s J} + \frac{V_s^2}{2g_c J} + \frac{gZ}{g_c J} \right) \right]_{j,i} \\
& - \left[\quad \right]_{j,0} + Q_j = \frac{d}{dt} \left[\Delta x \left(A_w \rho_w \left(e_w + \frac{V_w^2}{2g_c J} + \frac{gZ}{g_c J} \right) \right. \right. \\
& \left. \left. + A_s \rho_s \left(e_s + \frac{V_s^2}{2g_c J} + \frac{gZ}{g_c J} \right) \right) \right]_j . \quad (2)
\end{aligned}$$

It should be noted that it is the specific internal energy rather than enthalpy that can accumulate with time within the element.

The conservation of momentum is written for a control volume as the sum of the net rate of transfer of momentum to the fluid within the control volume and the net force acting on the fluid in the direction of flow equated to the rate of change of fluid momentum:⁽³¹⁾

$$\begin{aligned}
& (pA)_{j,i} - (pA)_{j,o} - \left[\Delta x (A_s \rho_s + A_w \rho_w) \frac{g}{g_c} \frac{\Delta Z}{\Delta x} j \right] - F'_{fj} \\
& + \frac{(A_s \rho_s V_s^2 + A_w \rho_w V_w^2)_{j,i}}{g_c} - \frac{(A_s \rho_s V_s^2 + A_w \rho_w V_w^2)_{j,o}}{g_c} \\
& = \frac{d}{dt} \left[\Delta x (A_w \rho_w V_w + A_s \rho_s V_s) \right]_j . \quad (3)
\end{aligned}$$

The first three terms express the net pressure, gravitational, and frictional forces, respectively, acting on the control volume of fluid within the j th element. The fourth and fifth terms represent the rate at which momentum is carried into and away from the control element of volume, respectively, by the flowing stream. The right-hand side is the time rate of change of fluid momentum within the element.

The energy equation may be put in more usable form by noting that the internal energy e , is related to the specific enthalpy h and the pressure p by

$$e = h - p/J,$$

and by defining a new function H as

$$H \equiv h + \frac{V^2}{2g_c J} + \frac{gZ}{g_c J} .$$

Substitution yields

$$\begin{aligned} & (A_w \rho_w V_w H_w + A_s \rho_s V_s H_s)_{j,i} - (A_w \rho_w V_w H_w + A_s \rho_s V_s H_s)_{j,o} + Q_j \\ &= \frac{d}{dt} \left[(A_w \rho_w H_w + A_s \rho_s H_s) \Delta x - \frac{p A \Delta x}{J} \right] \end{aligned} \quad (4)$$

At atmospheric pressure and velocities reasonably encountered in natural-circulation loops, neglect of the kinetic and potential energy terms and in the last term of the right-hand side of equation (4) containing the pressure leads to an error of less than 0.5%. At higher pressures, for which changes in density and pressure will be even smaller percentagewise, the error will be less. Since the assumption that these terms are negligible should not obscure any of the essential features, and does constitute a major simplification of the problem, it seems justified. Equation (4) then becomes

$$\begin{aligned} & (A_w \rho_w V_w h_w + A_s \rho_s V_s h_s)_{j,i} - (A_w \rho_w V_w h_w + A_s \rho_s V_s h_s)_{j,o} + Q_j \\ &= \frac{d}{dt} \left[\Delta x (A_w \rho_w h_w + A_s \rho_s h_s) \right] \end{aligned} \quad (5)$$

Equations (1), (3), and (5) are the general equations of mass, momentum, and energy conservation, respectively, for an element of volume which may or may not contain two phases and/or be adiabatic.

An entire loop can be represented by cascading a series of volume elements or stages, i.e., assuming that the output of the j th stage is the input to the $j+1$ st, and that between stages no changes occur except in total flow area. In particular, one can specify that there are no frictional losses, no delay times, and neither mass nor energy transfer between the flowing fluid and the surroundings in the (fictitious) passage between stages. Rather than representing true flow of scalar quantities, this may be thought of as a flow of information. One may then write the following equations relating the output of the j th element to the input of the $j+1$ st:

$$(A_w \rho_w V_w + A_s \rho_s V_s)_{j,o} = (A_w \rho_w V_w + A_s \rho_s V_s)_{j+1,i} \quad ;$$

$$(A_w \rho_w V_w h_w + A_s \rho_s V_s h_s)_{j,o} = (A_w \rho_w V_w h_w + A_s \rho_s V_s h_s)_{j+1,i} \quad .$$

In addition, one must relate the outlet properties to those within the stage. The simplest approach is perhaps a mixing-cell model, in which it is assumed that the contents of the entire cell have uniform properties which are identically those of the outlet. Provided the volume of the stage is properly chosen with regard to the nature of flow which occurs, this is a reasonable model. A useful guide to the proper choice of cell size may be found in papers on longitudinal diffusion of heat or mass in tubular systems.^(5,10)

For a loop consisting of $n+1$ stages designated $0, 1, \dots, n$, there will be $2(n+1)$ equations of energy and mass conservation. For all stages except the first, these will be

$$\begin{aligned} & (\rho_w V_w A_w + \rho_s V_s A_s)_{j,o} - (\rho_w V_w A_w + \rho_s V_s A_s)_{j+1,o} \\ & = \frac{d}{dt} \left[(\rho_w A_w + \rho_s A_s) \Delta x \right]_{j+1} \end{aligned} \quad (6)$$

$$\begin{aligned} & (\rho_w V_w A_w h_w + \rho_s V_s A_s h_s)_{j,o} - (\rho_w V_w A_w h_w + \rho_s V_s A_s h_s)_{j+1,o} + Q_{j+1} \\ & = \frac{d}{dt} \left[(\rho_w A_w h_w + \rho_s A_s h_s) \Delta x \right]_{j+1} , \end{aligned} \quad (7)$$

and for the "first" they will be

$$\begin{aligned} & (\rho_w V_w A_w + \rho_s V_s A_s)_{0,i} - (\rho_w V_w A_w + \rho_s V_s A_s)_{0,o} \\ & = \frac{d}{dt} \left[(\rho_w A_w + \rho_s A_s) \Delta x \right]_0 \end{aligned} \quad (8)$$

$$\begin{aligned} & (\rho_w V_w A_w h_w + \rho_s V_s A_s h_s)_{0,i} - (\rho_w V_w A_w h_w + \rho_s V_s A_s h_s)_{0,o} + Q_0 \\ & = \frac{d}{dt} \left[(\rho_w A_w h_w + \rho_s A_s h_s) \Delta x \right]_0 . \end{aligned} \quad (9)$$

If (3) is divided by A_j and the $n+1$ equations of this type summed, one obtains

$$\begin{aligned} & \sum_{j=0}^n \left[p_{j,i} - p_{j,o} - \left\{ \Delta x \left[\alpha \rho_s + (1-\alpha) \rho_w \right] \frac{\Delta z}{\Delta x} \right\}_j - \frac{F'_{fj}}{A_j} \right. \\ & \quad \left. + \frac{[\alpha \rho_s V_s^2 + (1-\alpha) \rho_w V_w^2]_{j,i}}{g_c} - \frac{[\alpha \rho_s V_s^2 + (1-\alpha) \rho_w V_w^2]_{j,o}}{g_c} \right] \\ & = \frac{d}{dt} \sum_{j=0}^n \left[\left\{ \alpha \rho_s V_s + (1-\alpha) \rho_w V_w \right\} \frac{\Delta x}{g_c} \right]_j . \end{aligned} \quad (10)$$

A reasonable boundary condition for any natural-circulation loop is that the pressure at one point is held constant by a pressurizer-surge tank. If the location is arbitrarily chosen as the outlet of the n th stage and/or the inlet to the zeroeth, then

$$P_{0,i} = P_{n,o} \quad (11)$$

In order to eliminate the pressure terms from equation (10), one must investigate the various conditions that may occur in the simulated loop. If there is no change in total cross-sectional area from the j th to the $j+1^{\text{st}}$ stage, i.e., $A_j = A_{j+1}$, then it is readily apparent from a momentum balance that one may write

$$P_{j,o} = P_{j+1,i}$$

and

$$\left(\frac{\alpha \rho_s V_s^2 + (1-\alpha) \rho_w V_w^2}{g_c} \right)_{j,o} = \left(\frac{\alpha \rho_s V_s^2 + (1-\alpha) \rho_w V_w^2}{g_c} \right)_{j+1,i} \quad (12)$$

However, if $A_j \neq A_{j+1}$, the conditions of (12) do not hold, since the change in momentum due to the change in area will effect a compensating change in the pressure. For an expansion, $A_j < A_{j+1}$; a momentum balance around the exit from j and the entrance to $j+1$ yields

$$A_j(P_{j,o} - P_{j+1,i}) + A_j(M)_{j,o} - A_{j+1}(M)_{j+1,i} = 0 \quad (13)$$

where

$$M = \frac{\alpha \rho_s V_s^2 + (1-\alpha) \rho_w V_w^2}{g_c}$$

Therefore,

$$P_{j,o} - P_{j+1,i} = \frac{A_{j+1}}{A_j} M_{j+1,i} - M_{j,o} \quad (14)$$

Similarly, for a contraction, $A_k > A_{k+1}$, one may derive the relation

$$P_{k,o} - P_{k+1,i} = M_{k+1,i} - \frac{A_k}{A_{k+1}} M_{k,o} \quad (15)$$

These equations are derived under the assumption as stated above that there are no frictional losses between stages, and, in particular, that there are no losses with the expansion or contraction that occurs. If there are many area changes in the loop or if they are located in critical positions, the losses may be taken into account by shape factors.⁽²³⁾ If the relations (11), (14), and (15) are substituted in (9), all the pressure terms of the momentum equation will be removed and one obtains

$$\begin{aligned}
& \sum_{j=0}^n \left[\left\{ \alpha \rho_s + (1-\alpha) \rho_w \right\}_j \Delta Z_j - \frac{F'_{fj}}{A_j} \right] + M_{0,i} - M_{n,o} \\
& + \sum_{\substack{k \\ A_k < A_{k+1}}} \left[\left(1 - \frac{A_{k+1}}{A_k} \right) M_{k+1,i} \right] + \sum_{\substack{m \\ A_m > A_{m+1}}} \left[\left(\frac{A_m}{A_{m+1}} - 1 \right) M_{m,o} \right] \\
& = \frac{d}{dt} \sum_{j=0}^n \left[\left\{ \alpha \rho_s V_s + (1-\alpha) \rho_w V_w \right\} \frac{\Delta x}{g_c} \right]_j .
\end{aligned}$$

In applying the general equations to an actual system, several difficulties still arise. One is the method of handling the region where the transition from one to two phases occurs. In operation, the cold leg or downcomer always contains only the liquid phase, and as the liquid phase traverses the heater and riser, boiling begins. There may not be a well-defined boundary between the single and two-phase regions because of the random nature of boiling, local boiling at the heated surface, and a velocity profile, but neither does a homogeneous fluid exist, and unless the mathematical model is to be very complex, a choice must be made of what simplification is suitable.

Two major alternatives are, first, to treat the two-phase mixture as a homogeneous fluid having characteristic mean properties of density, enthalpy, and velocity, or (2) to define a boiling boundary on one side of which a single-phase fluid exists and on the other side of which a two-phase fluid exists, the latter being additionally constrained by considerations of phase equilibrium.

Many investigators have used the homogeneous model.^(14,34) It has the advantage of enabling one to ignore any boundary, real or imagined, between the single- and two-phase regions, and simplifies the calculations of the transient behavior in this manner. However, difficulties arise in defining and using the mean properties. For example, mean properties may be defined as

$$\bar{\rho} \equiv \frac{A_s \rho_s + A_w \rho_w}{A} = \alpha \rho_s + (1-\alpha) \rho_w \quad (17)$$

$$\bar{h} \equiv \frac{\alpha \rho_s h_s + (1-\alpha) \rho_w h_w}{\bar{\rho}} \quad (18)$$

$$\bar{V} \equiv \frac{\alpha \rho_s V_s + (1-\alpha) \rho_w V_w}{\bar{\rho}} \quad (19)$$

for the mean density, specific enthalpy, and velocity, respectively, for the homogeneous fluid. The rates of convective transport of mass, energy, and momentum are then conveniently written in terms of these averaged properties, so that

$$\begin{aligned}\text{rate of mass transport} &= W = A\bar{\rho}\bar{V} \\ \text{rate of energy transport} &= A\bar{\rho}\bar{V}\bar{h} \\ \text{rate of momentum transport} &= A\bar{\rho}\bar{V}^2\end{aligned}\quad (20)$$

But on comparing these with the true rates of transport,

$$\begin{aligned}\text{true rate of mass transport} &= A_W\rho_W V_W + A_S\rho_S V_S \\ \text{true rate of energy transport} &= A_W\rho_W V_W h_W + A_S\rho_S V_S h_S \\ \text{true rate of momentum transport} &= A_W\rho_W V_W^2 + A_S\rho_S V_S^2\end{aligned}\quad (21)$$

it may be seen that only when V_S and V_W are equal, i.e., steam and water in the H_2O system flow at the same velocities, will (20) and (21) be identical. Hence, in writing the dynamic equations in terms of the mean properties as defined in (20), one must use correcting functions, themselves functions of the system parameters, or make the simplifying assumption that a "fog flow" model in which liquid and vapor move at the same velocity through the channel is adequate. Wissler used the functions ϕ_1 and ϕ_2 , defined as

$$\begin{aligned}\phi_1 &\equiv \frac{\text{true rate of momentum transport}}{A\bar{\rho}\bar{V}^2} \\ \phi_2 &\equiv \frac{\text{true rate of energy transport}}{A\bar{\rho}\bar{V}\bar{h}}\end{aligned}\quad (22)$$

as correcting functions. These would generally be functions of the liquid and vapor velocities and of the fluid thermodynamic properties, but on the basis of forced-convection steady-state data from the loop used, could be correlated as functions only of $\bar{\rho}$. The use of the average properties (20) and the correcting functions (22) in equations (6), (7), and (16) leads to the following simplified set:

$$W_{j,i} - W_{j,o} = \frac{d}{dt} (\bar{\rho}xA)_j \quad (23)$$

$$(Wh\phi_2)_{j,i} - (Wh\phi_2)_{j,o} + Q_j = \frac{d}{dt} (\bar{\rho}hAx)_j \quad (24)$$

$$\begin{aligned}
& \sum_{j=0}^n \left(-A \Delta x \bar{\rho} \frac{g \Delta Z}{g_c \Delta x} - F'_f \right)_j + \frac{(A \bar{\rho} \bar{V}^2 \phi_1)_{0,i}}{g_c} - \frac{(A \bar{\rho} \bar{V}^2 \phi_1)_{n,0}}{g_c} \\
& = \sum_{j=0}^n \frac{d}{dt} \frac{(\bar{\rho} \bar{V} \Delta x A)_j}{g_c} \quad . \quad (25)
\end{aligned}$$

An alternative to choosing a homogeneous model is to assume that there is a well-defined boundary between boiling and nonboiling regions. In a model consisting of lumped-parameter stages, this means that a particular stage either does or does not contain two phases. For one phase, h_w is one of the dependent variables and A_g is identically zero. For two phases, h_w , the specific enthalpy of the liquid, is a function of the pressure by virtue of the phase equilibrium assumed to exist, and A_g is one of the dependent variables. Thus the set of variables used in describing the processes occurring in a particular stage depends on its contents.

With use of the boiling-boundary model, calculations must continually be made to determine whether one or two phases are contained in a stage, so that the nature of the ensuing calculations can be fixed. Although more complicated in some respects, this model has a decided advantage in that the slip ratio may be used to characterize the flow. This ratio, γ , is a parameter that has been experimentally determined and correlated by several investigators, and leads to a far more generally applicable model than the use of a "corrected" homogeneous model, the corrections for which have not been determined for most systems. The use of slip ratios also permits a clearer physical model of the conditions underlying oscillatory behavior.

PROGRAMMING AND SOLUTIONS WITH AN ANALOG COMPUTER

The type of equations arising from a model in which the parameters are lumped in space are ordinary differential equations. They are highly nonlinear, but with a suitable analog computer solutions can be obtained. The degree of complexity which may be handled is limited mainly by the amount and type of equipment one has at his disposal.

A general-purpose analog computer is relatively easy to program, inexpensive to operate, and ideally suited for investigating, first, the general qualitative aspects of a model, and, second, the effect on the system behavior of varying parameters over a wide range of values. Little investigation of this sort has been reported in the literature. Wissler derived equations which could be solved on a REAC computer at the University of Minnesota, but was restricted in two ways: the average density of the two-phase fluid, regarded as homogeneous, could only vary linearly with temperature or specific enthalpy and could, therefore, exceed that of water, or even at the other extreme, become negative. Second, it was necessary to restrict boiling to a single portion of the riser. L. K. Holland of the General Electric Company has derived a model suited to analog simulation, but obtained no results, turning rather to numerical procedures.⁽¹³⁾

A preliminary investigation into the feasibility of a lumped-parameter representation of a low-pressure natural-circulation loop and into the general qualitative effects of several of the problem parameters was consequently undertaken. For even elementary models, it is necessary with a problem so complex to be able to generate nonlinear functions and to solve nonlinear equations, and, fortunately, arrangements were made to use the facilities of the analog computer laboratory of the Argonne National Laboratory at Lemont, Illinois. The fundamental computing capability consisted of two PACE computers (model EAI 131R) which could be operated separately or jointly and were well equipped with auxiliary multiplying, function generation, and recording equipment.

Model A

A model which seems to require the bare minimum of computing equipment, but still incorporates the essential features of a natural-circulation loop, is one consisting of four stages: heater, riser, cooler, and downcomer. This simple model and the results obtained are interesting since they illustrate very nicely the method of simulation on a general purpose analog, the amount of equipment necessary, and the sensitivity of the behavior of the loop to a variety of conditions.

A schematic of the general features of the model, subsequently referred to as model A, is shown in Figure 2. In deriving the equations ultimately used in the computer solution, it was assumed that the "homogeneous fluid" equations (23, 24, 25) could be applied. To simplify these

Figure 2. Schematic of Analog Model A

equations further, the following assumptions were made:

(a) A pressurizer-surge tank maintains constant pressure between the cooler and downcomer.

(b) The fluid density is constant with time and equal in all stages except in the riser.

(c) The momentum and energy transfer functions, ϕ_1 and ϕ_2 , as defined above are unity.

(d) The riser and downcomer are adiabatic. The rate of heat transfer to the fluid in the heater, Q_1 , is an arbitrary value, independent of the fluid or flow characteristics. The rate of heat removal from the fluid in the cooler, $-Q_3$, is proportional to the average enthalpy of the fluid in the cooler, $\bar{H}_3$, corresponding to an assumption of constant heat transfer coefficient and coolant temperature.

(e) The cross-sectional area of flow is constant with time and equal for all stages.

(f) The enthalpy of fluid entering the downcomer is the same as that of the fluid leaving the cooler, i.e., no time lag or mixing is caused by the surge tank.

(g) The pressure in the riser is constant.

Upon application of these assumptions to equations (23-25) and using the model of Figure 2, the following set of equations was obtained:

$$\begin{aligned}
 W_{0,i} &= W_0 = W_1 & W_0(\bar{H}_0 - \bar{H}_1) + Q_1 &= (A\Delta x \bar{\rho})_1 \frac{d\bar{H}_1}{dt} \\
 W_2 &= W_3 & W_0(\bar{H}_1 - \bar{H}_2) &= (A\Delta x \bar{\rho})_2 \frac{d\bar{H}_2}{dt} \\
 W_0 - W_2 &= (A\Delta x)_2 \frac{d\bar{\rho}_2}{dt} & W_2(\bar{H}_2 - \bar{H}_3) - Q_3 &= (A\Delta x \bar{\rho})_3 \frac{d\bar{H}_3}{dt} \\
 W_0(\bar{H}_3 - \bar{H}_0) &= (A\Delta x \bar{\rho})_0 \frac{d\bar{H}_0}{dt} \\
 \sum_{j=0}^3 [- (\bar{\rho} \Delta Z) - F_f]_j &= \frac{d}{g_c dt} \sum_{j=0}^3 (W \Delta x)_j \quad .
 \end{aligned} \tag{26}$$

With the use of a fog flow model, implicit in assumption (c) above, the average fluid density in a two-phase region is a function only of the average enthalpy, and its time derivative may be written in terms of the time derivative of the enthalpy. Thus,

$$\bar{\rho}_2 = \bar{\rho}_2(\bar{H}_2) \text{ and } \frac{d\bar{\rho}_2}{dt} = \frac{d\bar{\rho}_2(\bar{H}_2)}{d\bar{H}_2} \frac{d\bar{H}_2}{dt} \quad (27)$$

The functions $\bar{\rho}_2$ and $d\bar{\rho}_2/d\bar{H}_2$ were generated in the analog solution by two methods. Initially, a curve-follower technique was used wherein the voltage

Figure 3. Density Functions for Analog Model A

representing a nonlinear function of a single variable is generated by a combination of a curve traced on graph paper with a fine conducting wire, an oscillator, a pick-up head, and a plotting board. Later, diode function generators were used. The latter technique was found more reliable and introduced less noise into the circuit. It essentially approximates a nonlinear function by a series of ramp functions. With the PACE computer a maximum of twenty such ramps in series could be used.

The natures of $\bar{\rho}_2$ and $d\bar{\rho}_2/d\bar{H}_2$ are shown in Figure 3. It was intended to approximate the calculated density of a two-phase homogeneous mixture with unit slip ratio at one atmosphere pressure, but producing functions with such sharp changes or such severe slopes by either of the techniques described was impossible, and somewhat smoother curves were actually generated. The

smoother curves represent more realistic functions, in a sense, since they show effects similar to those due to local boiling and slip ratio greater than one.

The frictional term F_{fj} was expressed as a power function of the mass flow rate in single-phase regions by

$$F'_{fj} = \alpha_j W_j^{1.72} \quad j=0,1,3$$

where

$$\alpha_j = \frac{0.42 L_j}{2 g_c \rho_j A_j D_j \left(\frac{4}{\pi D \mu} \right)^{0.28}} \quad (28)$$

This expression was the result of using a Fanning friction model. For the riser, $j=2$, steady-state, experimental, two-phase pressure drop data from the University of Minnesota loop could be correlated to within $\pm 5\%$ by the following relation:

$$F'_{f2} = 4820(\bar{\rho}_2)^{-1.71}(W_2)^{1.79}(A\Delta x)_2 \quad (29)$$

With the substitution of (28) and (29) and the use of the density and density-enthalpy derivative functions of Figure 2, equations (26) were rearranged for simulation on the PACE computer. A schematic of the wiring diagram is shown in Figure 4. The dimensions, heat fluxes, and scaling factors necessary for reasonable solutions of time and problem frequencies are presented in Appendix A. The dimensions and power inputs conform with those used in the operation of the University of Minnesota loop with the exception of the heater volume, whose problem value is one-tenth the actual value. This exception arose because of the assumption of a constant cross-sectional area of the loop.

The procedure followed in obtaining a solution was to select a set of initial conditions and begin operation. The initial conditions were arbitrary and were varied widely from calculated steady-state values to heavily unbalanced values. An initial transient period would soon be followed by one of three conditions: a steady state, a steady oscillatory state, or divergent oscillations. The divergence was limited by the limit of ± 100 volts which, when reached, would cause an overload.

Figures 5, 6 and 7 are typical of the results obtained. These are direct copies of traces drawn by the recorder, a Variplotter, used in conjunction with the computer. All of the traces are of mass flow rate in the downcomer and are representative of the steady behavior after initial transients have been damped out.

In Figure 5, the effect of increasing friction in the riser on mass flow rate is shown. It was not possible, using the calculated friction of equation (29), to obtain well-behaved solutions, and F_{f2} of the figure is one-third of that calculated. As friction is increased, the amplitude of the oscillations increases, but the period remains relatively constant. This effect was noted in experiments at ANL, but at considerably higher pressures.

Figure 6 shows the effect of increasing the frictional losses in the downcomer and heater. The combined friction loss in the two stages is denoted F_{fL} . Its increase results in a reduced mass flow rate and increased stability in the sense that the amplitude of the oscillations is greatly diminished. This effect has been experimentally observed by nearly everyone who has done work in the field and is a well-established phenomenon.

Figure 4. Schematic for Wiring Circuit of Model A

Figure 5. Effect of Riser Friction Using Model A
 $Q = 11.8 \text{ Btu/sec}$

Figure 6. Effect of Cold Leg Friction using Model A.
 $Q = 11.8 \text{ Btu/sec}$

The curves of Figure 7 show the effect of increasing power input on flow rate. All of the stages are assumed adiabatic except the heater ($j=1$), and normal friction coefficients are in use except in the riser, where the expression of equation (29) has been reduced by a factor of three. The

Figure 7 Effect of Power Input Using Model A

result of increasing the power input was a striking reduction of the flow oscillations and a moderate increase in the mean flow rate. These results agree with the observations of Wissler.⁽³³⁾ Decreasing the power to low values resulted in unreliable behavior, not shown, in the sense that the computer results were not repeatable. This was due to slight voltage drifts within the computer. When operating with a low heat flux, no steam exists in the loop over a considerable fraction of the oscillatory period, and the sole driving force available to accelerate the fluid is that due to the difference in density between the liquid in the hot and cold legs of the loop. It is a slight difference, and as the problem was programmed was calculated by extracting the difference between two rather large voltages. A slight variation in either meant a large percentage deviation in the difference. It was not uncommon

to be presented with a negative driving force tending to reverse the direction of flow, a situation with which the model was quite unprepared to cope.

The results of model A were general, but, however general, they are interesting for at least two reasons. First, they demonstrate that a lumped-parameter model in its simplest form will produce oscillatory solutions which agree qualitatively in all, and quantitatively in some, of the characteristic features of natural-circulation loops. Second, they point out some of the improvements needed for a model which could be used with a higher degree of assurance.

Model B

A succeeding model was designed and denoted as model B, which incorporated several improvements in degree and some changes or refinements in principle over model A. The improvements in degree were:

- (1) The riser was represented by two stages rather than one.
- (2) The downcomer was represented by eleven stages rather than one.
- (3) The geometry was changed to conform more closely with that of the University of Minnesota loop.

The refinements in principle were:

- (1) Pressure variation in the riser due to changes in static head were calculated and the effect on the boiling point of the liquid taken into account.
- (2) A difference in vapor and liquid velocity was acknowledged and the effect on the mean density of the fluid considered.

Figure 8. Schematic for Analog Model B

The numerical stage designation is indicated in Figure 8. As noted above, the riser was represented by two stages. The pressure in the upper stage ($j=2,1$) was assumed constant, but that in the lower ($j=2,0$) varied with the static head above it, i.e., was a function of the fluid density in the upper riser.

The equations governing the downcomer were essentially unchanged from those of model A, but with several stages the number is increased. It was assumed that no vapor existed in the downcomer at any time, and that enthalpy (or temperature) variation was so slight that the liquid density

in each downcomer stage could be assumed constant and equal, i.e., an incompressible fluid was present, and one could write

$$W_{0,0} = W_{0,1} = \dots = W_{0,10} \quad (30)$$

The energy equations for an adiabatic downcomer are (see equation 24):

$$W_0(\bar{H}_3 - \bar{H}_{0,0}) = (A\Delta x \bar{\rho})_{0,0} \frac{d\bar{H}_{0,0}}{dt}$$

$$W_0(\bar{H}_{0,0} - \bar{H}_{0,1}) = (A\Delta x \bar{\rho})_{0,1} \frac{d\bar{H}_{0,1}}{dt}$$

$$W_0(\bar{H}_{0,9} - \bar{H}_{0,10}) = (A\Delta x \bar{\rho})_{0,10} \frac{d\bar{H}_{0,10}}{dt} \quad (31)$$

The heater equations are unchanged from those of model A. It was assumed that there is no boiling in the heated portion, although the model need not be restricted to this case. Since the geometry was patterned after that of the University of Minnesota loop in which the heater is located at the very bottom of the loop and constitutes a very small fraction of the length of the hot channel, the assumption of no net vapor formation is reasonable.

For the two-stage riser the continuity and energy equations are:

$$W_1 - W_{2,0} = (A\Delta x)_{2,0} \frac{d\bar{\rho}_{2,0}}{dt} \quad (32)$$

$$W_{2,0} - W_{2,1} = (A\Delta x)_{2,1} \frac{d\bar{\rho}_{2,1}}{dt} \quad (33)$$

$$W_1\bar{H}_1 - W_{2,0}\bar{H}_{2,0} = (A\Delta x)_{2,0} \frac{d}{dt} (\bar{\rho}\bar{H})_{2,0} \quad (34)$$

$$W_{2,0}\bar{H}_{2,0} - W_{2,1}\bar{H}_{2,1} = (A\Delta x)_{2,1} \frac{d}{dt} (\bar{\rho}\bar{H})_{2,1} \quad (35)$$

Combining equations (32) and (34), and (33) and (35), one obtains a more tractable pair:

$$W_1(\bar{H}_1 - \bar{H}_{2,0}) = (A\Delta x)_{2,0} \bar{\rho}_{2,0} \frac{d\bar{H}_{2,0}}{dt} \quad (36)$$

$$W_{2,0}(\bar{H}_{2,0} - \bar{H}_{2,1}) = (A\Delta x)_{2,1} \bar{\rho}_{2,1} \frac{d\bar{H}_{2,1}}{dt} \quad (37)$$

The vapor quality in the jth stage may be written

$$x_j = \frac{\bar{H}_j - h_{fj}}{h_{fgj}} \quad (38)$$

The average density may be written in terms of saturation properties and vapor volume fraction as

$$\bar{\rho}_j = [\alpha \rho_s + (1-\alpha)\rho_w]_j \quad (39)$$

But a mass balance across any plane perpendicular to the direction of flow gives

$$\frac{x}{1-x} = \frac{\alpha \rho_s V_s}{(1-\alpha) \rho_w V_w} = \frac{\alpha}{(1-\alpha)} \frac{\rho_s}{\rho_w} \gamma \quad (40)$$

and since the saturation properties ρ_s , ρ_w , and h_{fg} are nearly constant with the normally small pressure changes found in any stage (see Appendix A), one may write $\bar{\rho}_j$ as a function of $(\bar{H}_j - h_{fj})$:

$$\bar{\rho}_j = \bar{\rho}_j(\bar{H}_j - h_{fj}) = \bar{\rho}_j(\bar{S}_j) \quad (41)$$

where

$$\bar{S}_j = \bar{H}_j - h_{fj} \quad .$$

Then, if the pressure in the upper stage of the riser is constant, $h_{f2,1}$ will be constant, and

$$\bar{\rho}_{2,1} = \bar{\rho}_{2,1}(\bar{H}_{2,1})$$

and

$$\frac{d}{dt} \bar{\rho}_{2,1} = \frac{d}{d\bar{H}_{2,1}} \bar{\rho}(\bar{H}_{2,1}) \frac{d}{dt} \bar{H}_{2,1} \quad (42)$$

whereas

$$\bar{\rho}_{2,0} = \bar{\rho}_{2,0}(\bar{S}_{2,0}) \quad (43)$$

but

$$\frac{d}{dt} \bar{\rho}_{2,0} = \frac{d}{d\bar{S}_{2,0}} \bar{\rho}_{2,0} \frac{d}{dt} \bar{S}_{2,0} \quad (44)$$

where

$$\frac{d}{dt} \bar{S}_{2,0} = \frac{d}{dt} \bar{H}_{2,0} - \frac{d}{dt} h_{f2,0} \quad .$$

The liquid saturation enthalpy $h_{f2,0}$ will depend very strongly on pressure, particularly at low pressure, and to evaluate the second term of the right-hand side of (44), this dependence must be known as well as the time rate of variation of the pressure.

To predict accurately the time variation of pressure at a point in a transient two-phase flow system, one must calculate the time-varying frictional, acceleration, and static head losses. Calculations showed that the variation of frictional losses with changing conditions was negligibly small compared with the change in static head losses, whereas the incorporation of the necessary equations to approximate the acceleration loss changes with time was believed to be more complex than either the facilities of the computer or the scope of the investigation warranted. Consequently, the time variation of pressure was calculated solely from the time variation of static head.

Since $p_{2,0}$ will vary linearly with the average density of the upper riser stage fluid, $\bar{\rho}_{2,1}$, and since over the range of pressure encountered $h_{f2,0}$ is approximately linear in pressure, one may write

$$h_{f2,0} = a\bar{\rho}_{2,1} + b \quad . \quad (45)$$

where a and b are constants.

Equations (35), (37), and (42) may be combined to give

$$W_{2,0} = W_{2,1} + W_{2,0}(\bar{H}_{2,0} - \bar{H}_{2,1}) \frac{d \ln \bar{\rho}_{2,1}}{d \bar{H}_{2,1}} \quad . \quad (46)$$

Equations (34) and (35) could be used in an analog circuit, but in order to extract any single variable, a divide circuit would have to be used. Such circuits were undesirable, since they introduce considerable noise into the problem. The direct use of (34) and (35), however, could be avoided by using (46) and combining equations (34), (36), (44), and (45) to give

$$W_1 = W_{2,0} + \frac{d \ln \bar{\rho}_{2,0}}{d \bar{S}_{2,0}} \left[W_1(\bar{H}_1 - \bar{H}_{2,0}) - a\bar{\rho}_{2,0} \frac{d \ln \bar{\rho}_{2,1}}{d \bar{H}_{2,1}} W_{2,0}(\bar{H}_{2,0} - \bar{H}_{2,1}) \right] , \quad (47)$$

which could be solved if adequate function generation and multiplying equipment were available.

The complete set of equations for model B is given below.

$$W_0 = W_{0,0} = W_{0,1} = \dots = W_{0,10} = W_1 \quad (a)$$

$$W_{2,0} = W_1 - \frac{d \ln \bar{\rho}_{2,0}}{d \bar{S}_{2,0}} \left[W_1 (\bar{H}_1 - \bar{H}_{2,0}) - a \bar{\rho}_{2,0} \frac{d \ln \bar{\rho}_{2,1}}{d \bar{H}_{2,1}} W_{2,0} (\bar{H}_{2,0} - \bar{H}_{2,1}) \right] \quad (b)$$

$$\sum_{j=0}^n \frac{1}{g_c} \frac{d}{dt} W_j \Delta x_j = \sum_j \left[- (A \bar{\rho} \Delta Z)_j - F_{fj} \right] \quad (c)$$

$$\frac{d \bar{H}_{0,0}}{dt} = \frac{1}{(A \Delta x \bar{\rho})_{0,0}} (\bar{H}_3 - \bar{H}_{0,0}) W_0 \quad (d) \quad (48)$$

$$\frac{d \bar{H}_{0,1}}{dt} = \frac{1}{(A \Delta x \bar{\rho})_{0,1}} (\bar{H}_{0,0} - \bar{H}_{0,1}) W_0 \quad (e)$$

$$\vdots \quad \quad \quad \vdots \quad \quad \quad \vdots$$

$$\frac{d \bar{H}_{0,10}}{dt} = \frac{1}{(A \Delta x \bar{\rho})_{0,10}} (\bar{H}_{0,9} - \bar{H}_{0,10}) W_0 \quad (n)$$

$$\frac{d \bar{H}_3}{dt} = \frac{1}{(A \Delta x \bar{\rho})_3} \left[W_{2,1} (\bar{H}_{2,1} - \bar{H}_3) + Q_3 \right] \quad (o)$$

$$\frac{d \bar{H}_1}{dt} = \frac{1}{(A \Delta x \bar{\rho})_1} \left[W_0 (\bar{H}_{0,10} - \bar{H}_1) + Q_1 \right] \quad (p)$$

$$\frac{d \bar{H}_{2,0}}{dt} = \frac{1}{(A \Delta x \bar{\rho})_{2,0}} \left[W_0 (\bar{H}_1 - \bar{H}_{2,0}) + Q_{2,0} \right] \quad (q) \quad (48)$$

$$\frac{d \bar{H}_{2,1}}{dt} = \frac{1}{(A \Delta x \bar{\rho})_{2,1}} \left[W_{2,0} (\bar{H}_{2,0} - \bar{H}_{2,1}) + Q_{2,1} \right] \quad (r)$$

The functions $\bar{\rho}_{2,0}$, $\bar{\rho}_{2,1}$, $d \ln \bar{\rho}_{2,0} / d \bar{S}_{2,0}$, and $d \ln \bar{\rho}_{2,1} / d \bar{H}_{2,1}$ were necessary in the solution and were generated as functions of $\bar{S}_{2,0}$ or $\bar{H}_{2,1}$. The slip ratio γ was employed as a parameter in the problem and assumed to be constant with respect to position and time. Its variation for different problems was not a simple matter of turning a dial, but involved changing the complete set of four functions listed above.

At one atmosphere pressure, $\bar{\rho}$ may be calculated as a function of $\bar{H}$ according to equations (39) and (40) for various values of γ . Curves for typical values are shown on Figure 26. (see p. 72). The derivative $d\ln\bar{\rho}/d\bar{H}$ as a function of $\bar{H}$ may also be determined. The pressure in the upper riser stage was assumed to be constant at one atmosphere, and therefore the functions $\bar{\rho}$ and $d\ln\bar{\rho}/d\bar{H}$ are precisely those desired for $\bar{\rho}_{2,1}$ and $d\ln\bar{\rho}_{2,1}/d\bar{H}_{2,1}$. Furthermore, if the change in properties of saturated steam is not great over the pressure range encountered in going from the upper to the lower riser, then $\bar{\rho}_{2,0}(\bar{S}_{2,0})$ and $d\ln\bar{\rho}_{2,0}(\bar{S}_{2,0})/d\bar{S}_{2,0}$ may be generated from the same curves, but with the axis shifted.

Frictional forces were calculated using the same functions described for model A. The combination of frictional terms for the two riser stages was, therefore,

$$F'_{f2} = 4820 \frac{(A\Delta x)_{2,0} W_{2,0}^{1.79}}{\bar{\rho}_{2,0}^{1.71}} + \frac{(A\Delta x)_{2,1} W_{2,1}^{1.79}}{\bar{\rho}_{2,1}^{1.71}}, \quad (49)$$

and for the single-phase regions

$$F'_{fj} = \left[\frac{0.42}{2g_c \rho AD \left(\frac{4}{\pi D \mu} \right)^{0.28}} \right]_j L_j W_j^{1.72} \quad ; \quad j=0,1,3. \quad (50)$$

A comparison of the number of analog computer elements necessary for solving the equation of (1) model A; (2) model B but retaining only a single-stage downcomer; and (3) model B with an eleven-stage downcomer is shown in Table 1.

Table 1

ANALOG EQUIPMENT NECESSARY FOR DIFFERENT MODELS

Number of Model	A	B(1)	B(11)
Integrators	5	6	16
Summing Amplifiers	33	52	72
Divide Circuits	1	2	2
Servo Multipliers	4	8	11
Nonlinear Function Generators	3	6	6

The extension to a two-stage riser leads to a considerably greater amount of equipment need, even when no increase is made in the number of downcomer stages. More pertinent than numbers is the type of equipment. Servo-multipliers and nonlinear function generators are more complex than summing amplifiers and integrators, and require both care and time in setting up and preliminary balancing. Hence a doubling in the requirement of nonlinear generating equipment from three to six required for doubling the number of riser stages is more difficult than the addition of thirty amplifiers required for going from a single- to an eleven-stage downcomer.

Equations (48) were programmed for solution by the two interlocked consoles of the PACE computer. The six nonlinear functions were generated either with diode function generators or resistance-padded Servo multiplier brushes.⁽²⁴⁾ A schematic of the circuit is included in Appendix A, as are also the dimensions used for the loop, the power inputs, scaling factors, and parametric conditions.

The results obtained were significantly improved from those of model A, particularly in the following respects:

- (1) The use of reasonable slip ratios resulted in well-behaved oscillatory solutions using the actual geometry and frictional values in contrast with the uncontrolled oscillations of model A.
- (2) The period of the oscillations was found to be strongly affected by the geometry of the loop and by the number of stages used in representing different portions. An increase in the volume of any portion caused a corresponding increase in the period. An increase in the number of downcomer stages increased the period significantly, a feature which is discussed more fully later. On the other hand, a change in slip ratio had no apparent effect on the period.
- (3) Stable behavior at both high and low heat fluxes and unstable behavior (steady oscillations) at intermediate fluxes were predicted in agreement with observed data. The average liquid densities in the hot and cold legs of the loop were chosen in such a way that the effect of temperature on liquid density would be preserved. Hence at low heat fluxes, when periods of time exist where no vapor is formed, the loop operates as a stable single-phase loop.
- (4) The amplitude of the oscillations was found to be affected by most of the adjustable parameters, but particularly by the slip ratio γ . The effect of other parameters, such as the frictional forces, loop geometry, and power input, was predictably the same as with the simpler model A, but was overshadowed by the variation with γ .

These results are illustrated in Figures 9 and 10, where several mass flow rate traces are shown. Figure 9 illustrates the gradation in behavior of the mass flow rate with an increase of power input. The heater dimensions used in the problem were less than the actual dimensions so that the trend could be better illustrated. With the larger dimensions, the oscillations were damped to the point where a comparison was difficult. The flow traces bear a surprising similarity to the experimental traces, shown in Figure 11. Figure 10 illustrates the effect of varying slip ratio on the transient flow when the actual volume is used with normal frictional terms. The amplitude is drastically attenuated as γ is increased, and the choice of flow representation would appear to be of great importance in predicting conditions under which oscillations occur. The effect of heater volume may be observed by comparing the upper right-hand curve of Figure 9 with the middle curve of Figure 10. The only difference in the conditions is the volume, and its effect is to change the period and the amplitude of the oscillations.

Figure 9. Effect of Power Input Using Model B

The use of the lumped-parameter model and its solution with an analog computer gave useful results in showing that such a model could be programmed relatively easily, and would give rapid and qualitatively agreeable solutions to the set of equations, with a good deal of inherent flexibility in varying the system parameters. It was especially useful for giving an intuitive "feel" for the relations involved in the problem.

Figure 10. Effect of Slip Ratio Ratio Using Analog B

Figure 11. Experimental Temperature and Flow Traces at Atmospheric Pressure from the Natural-circulation Loop at the University of Minnesota

However, the usefulness of such a system seemed to be limited if one were to seek more reliable and generally refined results. Three limitations in particular were:

- (1) The difficulty in varying certain key parameters, especially slip ratio. As the problem was set up, the slip was a constant, independent

of time and space. To allow for its time variation as a function of some other variable or group of variables would have meant a major revision.

(2) The limit imposed on the degree of refinement possible in the riser sections by the amount of nonlinear equipment needed. For example, to have made a three-stage representation of the riser would have required nine nonlinear function generators. Although still within the realms of possibility, this amount is certainly approaching a practical limit.

(3) The characteristics of the electronic components, e.g., voltage drift and Servo-multiplier sensitivity. As noted above, voltage drift under certain circumstances can cause major errors in the solutions. It was also found that the sensitivity of the Servo-multipliers could not be completely isolated from the sensitivity of the system of equations, and if oscillations did occur it was sometimes a moot question whether they were due to the parametric conditions of the system, or to the Servo gain setting. Perhaps additional skill in operating the computer would have minimized these objections, but it is doubtful that it would have completely eliminated them. For these reasons further results were pursued with a digital computer.

PROGRAMMING AND SOLUTIONS WITH A DIGITAL COMPUTER

By its nature, a digital computer allows very little drift in variables so long as certain numerical precautions are observed; the only limit imposed on the complexity of a problem is one of time rather than equipment; and the variation of parameters is no more difficult than with an analog computer. For these reasons and because of its availability in the University of Minnesota Numerical Analysis Center, a UNIVAC Scientific, model 1103, was used in the additional investigation.

The set of equations for which the digital computer was programmed was different from that used with either of the models solved by the analog computer. The reasons are perhaps obvious, but first was the fact that the two types of computer are fundamentally different in technique and capability, second that the objectives were new and different, and third that the analog models were less flexible than desired.

The objectives of approximating and solving the system of equations numerically with a digital computer were:

- (1) to determine whether quantitative agreement could be achieved through the use of a more precise instrument;
- (2) to compare the behavior of such a system at low and at high pressure, and determine if fundamentally different causes were responsible for oscillations in the two cases; and
- (3) to allow the slip ratio more flexible and realistic behavior.

It was felt that the lumped-parameter model had been justified as a realistic model by the results obtained from the analog computer, since it was not unduly complex, its limitations were unobscured, and it had given results qualitatively in agreement with experiment. Therefore, instead of reverting to the original partial differential equations, the general lumped-parameter concept was retained. The "homogeneous fluid" model was rejected, however, in favor of a separated-flow model in which the boundary between boiling and nonboiling regions could be defined and used to determine the particular set of equations that would apply in a specific location. Pressure variations in the riser stages, and the resulting changes in liquid and vapor saturation properties due to changes in gravitational pressure drop, were calculated. At high pressures, the saturation properties vary only slowly with pressure, so that these calculations are not important, but near atmospheric pressure the properties change rapidly and are important.

If equations (6), (7), (8), (9), and (16), general conservation equations of mass, energy, and momentum, are written in terms of slip ratio γ and vapor volume fraction α the following are obtained:

$$V_{w0,i}[(1-\alpha)\rho_w + \alpha\gamma\rho_s]_{0,i} - V_{w0}[(1-\alpha)\rho_w + \alpha\gamma\rho_s]_0 = \frac{d}{dt} [\{(1-\alpha)\rho_w + \alpha\gamma\rho_s\}\Delta x]_0 \quad (51a)$$

$$V_{wj-1}[(1-\alpha)\rho_w + \alpha\gamma\rho_s]_{j-1} - \frac{A_j}{A_{j-1}} V_{wj} [(1-\alpha)\rho_w + \alpha\gamma\rho_s]_j = \frac{d}{dt} [\{(1-\alpha)\rho_w + \alpha\gamma\rho_s\}\Delta x]_j \quad (51b)$$

$$V_{w0,i}[(1-\alpha)\rho_w h_w + \alpha\gamma\rho_s h_s]_{0,i} - V_{w0}[(1-\alpha)\rho_w h_w + \alpha\gamma\rho_s h_s]_0 + \frac{Q_0}{A_0} = \frac{d}{dt} [\{(1-\alpha)\rho_w h_w + \alpha\gamma\rho_s h_s\}\Delta x]_0 \quad (52a)$$

$$V_{wj-1}[(1-\alpha)\rho_w h_w + \alpha\gamma\rho_s h_s]_{j-1} - \frac{A_j}{A_{j-1}} V_{wj} [(1-\alpha)\rho_w h_w + \alpha\gamma\rho_s h_s]_j + \frac{Q_j}{A_j} = \frac{d}{dt} [\{(1-\alpha)\rho_w h_w + \alpha\gamma\rho_s h_s\}\Delta x]_j \quad (52b)$$

$$\begin{aligned} \sum_{j=0}^n [\{(\alpha\rho_s + (1-\alpha)\rho_w)\Delta Z\}_j - F_{ij}] + M_{0,i} - M_{n,o} + \sum_k \left[\left(1 - \frac{A_{k+1}}{A_k}\right) M_{k+1,i} \right] \\ + \sum_m \left[\left(\frac{A_m}{A_{m+1}} - 1\right) M_{m,o} \right] = \frac{d}{dt} \sum_{j=0}^n \left[\left\{ (1-\alpha)\rho_w + \alpha\gamma\rho_s \right\} \frac{V_w \Delta x}{g_c} \right]_j \end{aligned} \quad (53)$$

Equations (51) to (53) are general in the sense that they apply to stages which contain either liquid or a two-phase mixture. The specific form taken by the expanded right-hand sides, however, depends on whether one or two phases occur. For the cases in which only the liquid phase is present, i.e., $\alpha_j = 0$, and where two phases occur, $\alpha_j \neq 0$, the right-hand sides of (51) and (52) and the j th term of (53) are given in Table 2.

Table 2

RIGHT-HAND SIDES OF EQUATIONS (51) to (53) FOR LIQUID ONLY
AND FOR LIQUID-VAPOR MIXTURES

Equation No.	Term	Liquid Only $\alpha_j = 0$	Liquid and Vapor $\alpha_j \neq 0$
51	$\frac{d}{dt} [\{(1-\alpha)\rho_w + \alpha\gamma\rho_s\}\Delta x]_j =$	0	$[(\rho_s - \rho_w)\Delta x]_j \frac{d\alpha_j}{dt}$
52	$\frac{d}{dt} [\{(1-\alpha)\rho_w h_w + \alpha\gamma\rho_s h_s\}\Delta x]_j =$	$(\rho_w \Delta x)_j \frac{dh_{wj}}{dt}$	$[(\rho_s h_s - \rho_w h_w)\Delta x]_j \frac{d\alpha_j}{dt}$
53	$\frac{d}{dt} \left[\left\{ (1-\alpha)\rho_w + \alpha\gamma\rho_s \right\} \frac{V_w \Delta x}{g_c} \right]_j =$	$\frac{(\rho_w \Delta x)_j}{g_c} \frac{dV_{wj}}{dt}$	$\frac{\Delta x_j}{g_c} \left[\left\{ (1-\alpha)\rho_w + \alpha\gamma\rho_s \right\} \frac{dV_{wj}}{dt} + V_{wj} (\gamma\rho_s - \rho_w)_j \frac{d\alpha_j}{dt} \right]$

For all cases, $d\rho_w/dt$ is assumed equal to zero. For $\alpha_j = 0$, the liquid specific enthalpy h_w is a variable whose value depends on the solution of the energy equation, and hence on the flow, inlet enthalpy, and heat transfer rates. However, for $\alpha_j \neq 0$, the saturation properties ρ_s , h_s , ρ_w , and h_w are determined by the pressure, since phase equilibrium is assumed. The magnitudes of terms containing their time derivatives will, therefore, depend on the rate at which the pressure changes. Although this is difficult to assess accurately, it can be approximated, and such an approximation shows conclusively that these time-derivative terms are of far smaller magnitude than the terms which contain the time derivative of α_j and V_w .

A preliminary investigation of the possibility of obtaining a numerical solution to the set of equations (51), (52), and (53) indicated that to achieve reasonable solution times, high-speed access storage capacity would not be adequate unless further simplifications were made. In order to comply with this requirement, while still retaining the essential two-phase features of the system, the temperature feedback through the cooler and downcomer to the heater entrance was neglected, i.e., a feed of constant inlet temperature to the downcomer was assumed. Experiments at the Argonne National Laboratory⁽²¹⁾ and at the Westinghouse Electric Corporation⁽²⁴⁾ have demonstrated that oscillations do occur in systems where the heater inlet fluid temperature is held constant, so that temperature feedback is not the sole cause of oscillatory behavior. However, it is possible that if the downcomer temperature varies considerably with time, the period and amplitude of any oscillations will probably be affected, especially at low pressure.

The simplification eliminates the need for solving the energy equations in the cooler and downcomer regions. Since the fluid in these regions is entirely liquid, the solution to the transient energy equations is straightforward, and if adequate storage capacity were available could be incorporated without difficulty.

The final form of equations (51) and (53) and the criteria for deciding the proper form of the right-hand sides are then

$$\frac{A_{j-1}}{A_j} V_{wj-1} [(1-\alpha)\rho_w + \alpha\gamma\rho_s]_{j-1} - V_{wj} [(1-\alpha)\rho_w + \alpha\gamma\rho_s]_j$$

$$= \begin{cases} 0, & \text{for } \alpha_j \leq 0 \text{ and } h_{wj} \leq h_{fj} \\ (\rho_s - \rho_w)_j \Delta x_j \frac{d\alpha_j}{dt}, & \text{for } \alpha_j > 0 \text{ or } h_{wj} > h_{fj} \end{cases} \quad (54a)$$

$$(54b)$$

$$\frac{A_{j-1}}{A_j} V_{wj-1} [(1-\alpha)\rho_w h_w + \alpha\gamma\rho_s h_s]_{j-1} - V_{wj} [(1-\alpha)\rho_w h_w + \alpha\gamma\rho_s h_s]_j$$

$$+ \frac{Q_j}{A_j} = \begin{cases} (\Delta x \rho)_j \frac{dh_{wj}}{dt}, & \text{for } \alpha_j \leq 0 \text{ and } h_{wj} \leq h_{fj} \\ (\rho_s h_s - \rho_w h_w)_j \Delta x_j \frac{d\alpha_j}{dt}, & \text{for } \alpha_j > 0 \text{ or } h_{wj} > h_{fj} \end{cases} \quad (55a)$$

$$(55b)$$

$$\sum_{j=0}^n \{ [\alpha\rho_s + (1-\alpha)\rho_w]_j \Delta Z_j - F_{fj} \} + M_{0,i} - M_{n,o}$$

$$+ \sum_{\substack{k \\ A_k < A_{k+1}}} \left[\left(1 - \frac{A_{k+1}}{A_k} \right) M_{k+1,i} \right] + \sum_{\substack{m \\ A_m > A_{m+1}}} \left[\left(\frac{A_m}{A_{m+1}} - 1 \right) M_{m,o} \right]$$

$$= \frac{d}{dt} \sum_{j=0}^n \left[\left\{ (1-\alpha)\rho_w + \alpha\gamma\rho_s \right\}_j \frac{V_{wj} \Delta x_j}{g_c} \right] \quad (56)$$

The general method of solution is outlined in the following steps:

- (1) Knowing h_{wj} and α_j for all j at time t , the quantities ρ_{wj} , ρ_{sj} , h_{fj} , and h_{sj} were calculated from equilibrium data, based on steam tables, for all j .
- (2) For $j = 1$, the first heater stage, α_j and h_{wj} were observed.
- (3) If $\alpha_j \leq 0$ and $h_{wj} \leq h_{fj}$,
 - (a) $\dot{\alpha}_j$ was set equal to zero.*
 - (b) Knowing V_{wj-1} , equation (54a) was used to calculate V_{wj} .
 - (c) Equation (55a) was used to calculate $\dot{h}_{wj}$.

*A dotted quantity represents its time derivative.

- (4) If $\alpha_j > 0$ or if $h_{wj} > h_{fj}$,
 - (a) h_{wj} was equated with h_{fj} , and $\dot{h}_{wj}$ was set equal to zero.
 - (b) Equations (54b) and (55b) were solved simultaneously for V_{wj} .
 - (c) Equation (55b) was used to calculate $\dot{\alpha}_j$.
- (5) Steps 2, 3, and 4 were repeated for all values of j up to but not including $j = n$, the cooler.
- (6) The velocity of liquid leaving the cooler, V_{wn} , was calculated by means of equation (54a).
- (7) Equation (56) was used to calculate $\dot{V}_{w0}$. The values of V_{wj} ($j \neq 0$) were necessary for this calculation, and for the first iteration in time $\dot{V}_{wj}$ from the previous time interval was used.
- (8) Knowing $\dot{\alpha}_j$ or $\dot{h}_{wj}$ ($j = 1, 2, \dots, n-1$), $\dot{V}_{w0}$, and Δt , the values of h_j , α_j , and V_{w0} at time $t + \Delta t$ were calculated, i.e., Euler's method was used as a predictor.
- (9) The trapezoidal rule was used as a corrector in reiterating steps 1 through 8 until successive iterations produced no further change. The value of Δt and the number of iterations was determined by trial.

The program was essentially one of applying a forward differencing technique to a system of partial differential equations, and was quite similar to that used by George.⁽¹⁴⁾ The system of equations and the functions retained differed considerably, however. It was difficult to establish rigorously that the numerical procedure was stable to roundoff error and did not give spurious oscillations, although hand calculations gave an initial estimate. However, to test the reliability of the computed results, a variety of initial conditions was used under the same operating conditions. Repetition of the same pattern of steady behavior was taken as proof of the stability of the numerical technique.

The program was applied by using the University of Minnesota loop geometry and power inputs in the same range as reported by Wissler.⁽³³⁾ Stage designation is indicated in Figure 12. The number of stages was limited only by the high-speed access storage available in the UNIVAC Scientific and the economics of purchasing computer time.

Figure 12. Schematic for the Digital Computer Model.

The pressure at which the loop was operated was approximately one atmosphere, since its surge tank was exposed to the ambient room conditions. At such low pressures, and the corresponding vapor qualities encountered, the art of accurately predicting the steam-water slip ratio is not well developed, since it is difficult to obtain accurate data in this range, even for steady-state operation. However, it is possible to bracket the value to be used within reasonable limits, and this was done. The average value of γ chosen was held constant with time and assumed not to vary with position. This was for convenience and not a restriction of the model, since it was capable of calculating γ as a function of time and/or space, given a suitable functional relation.

Frictional losses were calculated as a function of the actual liquid velocity alone, using a relation of the form

$$F_{fj} = f_j V_{wj}^2,$$

where f_j was a constant whose value depended on the mean liquid velocity. Again, the model could have incorporated a more complex function, but the high-speed storage limitation of the computer and the uncertainty of even more complex models led to the use of this expression.

To start the calculations, initial conditions of velocity, enthalpy, and vapor volume fraction were chosen and designated in the program. These were in no way intended to approximate a steady state of the system and were arbitrarily chosen. As with a real operation, initial transients soon damped out and were followed by either a steady state or an oscillatory condition. Figures 13 and 14 are plots of digital output data for two runs made under identical operating conditions, but with different initial conditions. The four curves traced on Figure 13 represent vapor volume fraction from the exit riser, specific enthalpy of the heater outlet, specific enthalpy of the middle riser section, and the liquid velocity in the cold leg. The specific enthalpies h_{w1} and h_{w3} may be compared directly with temperatures T_1 and T_2 , respectively, of the experimental results shown on Figure 11. The exit vapor volume fraction α_5 was plotted in order to give insight into the mechanism underlying low-pressure flow oscillations. Conditions under which the loop is operating are indicated on each figure.

Figure 13

Transient Velocity,
 Enthalpy, and Void
 Fraction; $p = 1 \text{ Atm}$,
 $Q = 12.5 \text{ Btu/sec}$.

Figure 14
 Time Variations in Velocity,
 $p = 1 \text{ Atm}$, $Q = 12.5 \text{ Btu/sec}$.

A comparison of Figure 13 and Figure 14 shows that after the initial transient has decayed, identical steady-state flow oscillations are observed for each case, effectively eliminating the possibility of cyclic behavior arising solely from the numerical treatment of the differential equations.

Figures 15 through 17 give results obtained at different levels of power input corresponding to various experimental runs already reported and shown above. The shapes of the flow and enthalpy (or temperature) curves are remarkably similar to those found experimentally, and the phase relations between variables, as well as the trend with increasing power input, show very good agreement. In these respects the model seems very adequate. Furthermore, the mean mass flow rates and oscillatory amplitudes agree to within the experimental error.

Figure 15. Transient Velocity, Enthalpy, and Void Fraction; $p = 1$ Atm, $Q = 5.84$ Btu/sec

Figure 16. Transient Velocity, Enthalpy, and Void Fraction; $p = 1$ Atm, $Q = 9.44$ Btu/sec

The largest and most significant deviation from the observed results is in the period of the oscillations. Those obtained using this mathematical model are shorter than those observed experimentally by a factor of nearly two. This deviation will be discussed more fully below. The change in characteristics with heat input is in good agreement, and, as found experimentally, the loop is stable at both high and low heat fluxes.

Figure 17. Transient Velocity, Enthalpy, and Void Fraction; $p = 1 \text{ Atm}$, $Q = 12.70 \text{ Btu/sec}$

Considerable insight into the nature of the oscillatory behavior at low pressures can be obtained by observing the simultaneous variations in vapor volume fraction, temperature, and velocity. At low heat-input rates, oscillations will not occur as long as the flowrate supported by the difference in liquid density is great enough to prevent formation of vapor in the riser. The loop then operates as a typically stable, single-phase natural-circulation loop. An increase in heat input or an analogous increase in feed temperature which permits vapor to be formed will result in oscillations having the form of those shown in Figure 15. The inception of such oscillations is due to the very large difference between the specific volume

of water and steam at low pressure, which corresponds to a tremendous change in the average density of the riser fluid with the formation of even a small amount of vapor. This change causes a sudden increase in the gravitational driving force acting to accelerate the fluid flow, and the increase in velocity is large enough to expel all vapor and return the loop to its original single-phase condition. The amount of vapor formation necessary for this phenomenon to occur is very slight. For example, the maximum vapor volume fraction of Figure 15, $\alpha_5 = 0.20$, corresponds to a weight fraction of steam less than 0.0009 at atmospheric pressure and a slip ratio of five.

The oscillations become more pronounced as the heat input is increased, but then subside if it is increased even further. It appears that the oscillations cease even with maximum flow, because vapor formation within the riser is still possible, and hence no sudden change in driving force can occur. Figure 18 illustrates the computed results for an inlet feed very near the saturation temperature corresponding to the pressure in the upper riser. With any heat transferred to this fluid, vapor will be formed, and it will be impossible to operate for any period of time in the single-phase region. The plot of velocity vs. time illustrates how strongly damped the transient behavior is, and how stable such operating conditions are.

Figure 18. Time Variations in Velocity,
 $Q = 10$ Btu/sec, Feed at
 Boiling Point

The interpretation of the mechanism of oscillation reveals why the computed periods are somewhat shorter than those found experimentally. From a general consideration of the processes which occur, one can see that the period will be determined by (1) the amount of time needed to expel

the vapor which has formed, and (2) the time necessary for the temperature of the liquid to again reach the boiling point. The latter term will depend largely on the time required for an element of fluid in the heater, heated to a temperature which corresponds to the boiling point in the upper portion of the riser, to be propagated along the length of the riser. To account accurately for this propagation time, a mixing-cell model will be adequate only if the length of each cell is determined by a consideration of the longitudinal diffusion coefficient. A precise estimate may be made by referring to work done in the field of longitudinal diffusion of scalar properties in tubular chemical reactors,^(5,10) but an order-of-magnitude approximation is that the length of each cell be equal to its diameter. The five-stage riser used in this model has a length-to-diameter ratio of 50 for each cell, and hence will result in a far faster rate of propagation.

HIGH-PRESSURE OPERATION

Natural circulation loops containing two-phase steam-water mixtures have been operated at pressures ranging from 150 to 1600 psia. Under certain operating conditions, oscillatory behavior has also been observed at these high pressures. The oscillations have been similar to those found at lower pressures in the sense that they occur when all controlled variables, such as power input, pressure, or inlet subcooling, are held constant, but have had amplitudes and periods quite different from the low-pressure oscillations. Because of these marked differences, some investigators have felt that the causes were different in the two cases.

Published experimental data in the high-pressure field are quite limited, and in cases where oscillations are reported, the complete circumstances surrounding their occurrence are seldom known. This is primarily due to the fact that oscillatory behavior was an incidental occurrence rather than the primary objective of the experimental investigations.

Some of the general conclusions that have been drawn from experimental results of high-pressure natural-circulation loop operation at the Argonne National Laboratory, Westinghouse Electric Company, and General Electric Company are:

- (1) The power densities at which oscillations occur increase with pressure.
- (2) An increase in downcomer or cold leg resistance stabilizes, or tends to stabilize, the system.
- (3) Oscillatory frequencies at high pressure are markedly higher than at low pressure.
- (4) Oscillations at high pressures do not occur until after the peak circulation rate is reached. The peak circulation rate itself is apparently independent of the stability.
- (5) Criteria for the existence of oscillations cannot be formulated in terms of a single parameter.
- (6) Increased subcooling decreases the stability of the system.

Since the digital computer model used at low pressures was general and not restricted to a particular system or set of operating conditions, it was applied at a higher pressure. The geometric configuration conformed closely to that of the Argonne National Laboratory 600-psia loop,⁽²¹⁾ although some simplification was made in order to conserve computing time.

The specific geometry is given in Appendix B. Essentially, the simulated loop consisted of a rectangular heater and riser, $\frac{1}{4} \times 2 \times 89$ in., with an unrestricted downcomer and crossover piping. For the purpose of calculating frictional losses, the diameter of the downcomer and crossovers was assumed to be 2 in. The actual loop at Argonne included a steam-water separator and an additional upper crossover, not included in the model. A pressure of 300 psia was arbitrarily chosen as high enough to be typical of high-pressure runs in general.

The computer simulation was similar to that used at atmospheric pressure. The heater and riser were represented by a five-stage mixing-cell model, of which three comprised the heater. The initial conditions were arbitrarily chosen, and the time increments by which the problem solution was advanced were decreased until shorter increments resulted in no change in the results.

Initial runs were made under conditions of constant heat input, inlet subcooling, and slip ratio. Under all conditions chosen, however, the loop exhibited stability, initial transients damping out rapidly, with sustained oscillations never being supported. Power density, subcooling, and the slip ratio were varied from run to run, but results were always similar to those shown in Figure 19, a plot of the damping of an initial transient and the relatively rapid attainment of a steady state. The flow rate and vapor volume fraction so calculated were very reasonable, and agreed with what experimental data were available to within 10%, lending limited confidence to the method of calculating frictional losses and driving forces.

Figure 19

Transient Velocity and Void
Fraction, $Q = 70 \text{ kw/l}$,
 $p = 300 \text{ psia}$, $\gamma = 2.0$

The power inputs were expressed in units of kilowatts per liter for purposes of comparison with experimental data. The degree of subcooling of the liquid entering the heater is given as $h_{f5} - h_0$, the difference between the saturated liquid enthalpy in the top riser stage, and the incoming feed liquid enthalpy.

Several investigators have reported a variation of slip ratio with superficial velocity.^(16,23) Since the model was so formulated that variations of γ with time could be easily accommodated, experimental data were used to determine the functional relation between γ and V_{w6} , the exit velocity. The reported correlations were based on steady-state runs, and, as far as is known, no slip ratios have been experimentally determined in transient systems. Hence the application of these steady-state data to a transient system is based on expedience rather than rigor.

Based on an interpolation of the data of Lottes *et al.*,⁽²³⁾ and using an average quality and velocity at which the loop would be expected to operate, the slip ratio was approximated as a linear function of the superficial outlet cooler velocity:

$$\gamma = a - bV_{w6} \quad (57)$$

where a , b are constants.

With the same operating and initial conditions as used in the run shown in Figure 19, but using (57) for γ , with " a " and " b " equal to 3.3 and 0.30, respectively, instead of $\gamma = 2.0$, the initial transient was much less strongly damped, although a steady state was ultimately reached, as the results plotted in Figure 20 indicate.

Figure 20

Transient Velocity and Void Fraction; $p = 300 \text{ psia}$
 $Q = 70 \text{ kw/l}, \gamma = 3.3 - 0.3 V_{w6}$

A comparison of Figures 19 and 20 reveals, however, the decided effect that the inclusion of a variable slip ratio has on the calculated behavior at high pressure.

It has been noted above that, at one atmosphere, both experimental and predicted results were extremely stable at high power inputs, or conversely, at inlet feed temperatures near the boiling point. The reason for this stability was the fact that the entire heater and riser were filled with vapor, resulting in a maximum driving force, and regardless of how rapidly the fluid was accelerated in the loop, it was impossible to accelerate it fast enough to reduce the void fraction materially. Analogous behavior might be expected at higher pressures, and experimental results have indeed indicated a stabilizing influence as the inlet feed subcooling is decreased. Figure 21 shows the predicted results of a run in which the subcooling is only 5 Btu/lb. Here the exit vapor volume fraction has levelled off at 0.67, above the value at which oscillations have occurred at this pressure. A further decrease in the subcooling (not shown) resulted in even more strongly damped oscillations. It may be noted also that the mean velocity is now somewhat less than in Figure 20, where the subcooling was greater and the exit void fraction less, indicating that the point of maximum flow has been exceeded.

Figure 21

Transient Velocity and Void Fraction; $p = 300 \text{ psia}$
 $Q = 70 \text{ kw/l}$, $\gamma = 2.2 - 0.3 V_{w0}$

Between the regions of stable flow illustrated in Figures 20 and 21 very interesting phenomena occurred, for a step change of inlet feed temperature resulted in oscillations with an amplitude so large that the computer was unable to continue the calculations because of overloading registers. Figure 22 illustrates the results obtained when the subcooling was changed to 10 Btu/lb from two initially stable conditions, one with a

higher and the other with a lower subcooling. In both cases the transient oscillations diverged until some of the computer registers were overloaded. Changes in scaling were made, but these did not materially alter the results.

Figure 22

Divergent Oscillations at SC = 10;
 $p = 300 \text{ psia}$, $Q = 70 \text{ kw/l}$.

It was felt that the magnitude of the step change might be so great that the divergence was due to a strong initial transient, and subsequently smaller step changes approaching the same operating condition of 10 Btu/lb subcooling were made. Starting with stable operation with feed at 5-Btu/lb subcooling, the inlet temperature was changed by steps to 7.5 and later to 9.0-Btu/lb subcooling. At 7.5 Btu/lb the behavior was stable and all variables settled down to steady, nonoscillatory values, but at 9.0 Btu/lb steady oscillations were observed. The transient behavior of two of the velocities and the exit vapor volume fraction as a result of the step change to 9.0-Btu/lb subcooling are shown in Figure 23. The period of the steady oscillations was approximately one second, in agreement with experimental values.

From this steady oscillatory condition at a subcooling of 9.0 Btu/lb, a step change of one unit to 10 Btu/lb was made. Again, as shown in Figure 24, the oscillations diverged and ultimately caused a computer overload. Results shown on Figures 23 and 24 were achieved by slowly increasing the subcooling of the feed by small increments, but similar results were obtained when the subcooling was decreased from 20 to 10 Btu/lb by small increments. At 20 and at 15 Btu/lb subcooling, the behavior was ultimately stable, but at 12 Btu/lb steady oscillations occurred, and again with the feed subcooled 10 Btu/lb, the results of Figure 24 were obtained.

Figure 23

Steady Oscillations at
SC = 9, $p = 300$ psia

Figure 24

Transient Velocity for a Step
Change of Subcooling from
9 to 10.

From these computed results at both high and low pressures a number of observations may be made:

(1) The power density (power per unit heater volume) at which oscillations occur has increased by nearly two orders of magnitude in going from one atmosphere to 300 psia. The lowest rate of heat input at which oscillations were predicted at one atmosphere was 6 Btu/sec, corresponding to a power density of 1.37 kw/liter. At 15 Btu/sec, the low-pressure operations were completely stable. However, at 300 psia the oscillations occurred at 70 kw/liter. The trend of increasing stability with pressure has been observed with all experimental test loops.

(2) The specification of power density alone, or of any other single variable, is not sufficient for determining the unstable region. The instability is a complex function of power input, system geometry, inlet subcooling, and perhaps other variables. This feature is illustrated by the computed results which show that a region supporting sustained oscillations may be reached by varying either the subcooling or the power input. A similar conclusion, based on experimental results, has been made by Levy and Beckjord.⁽²⁰⁾

(3) The region of instability appears to be a closed region, outside of which stable operation is possible. At both one atmosphere and at 300 psia, oscillatory behavior occurred at an intermediate range of operating conditions with steady behavior at extremes. There are, however, limitations imposed on the model used in the computer studies that may invalidate any practical application. An example of such a limitation is the possibility of burnout at high power inputs, or at lower inputs if oscillations alter the mechanism of heat transfer. In the model, an arbitrary heat source was assumed to exist without regard to any mechanism.

(4) The frequency of oscillations at high pressure is approximately 30 times higher than that of the low-pressure oscillations, in agreement with experimental results. The absolute agreement of predicted and experimental frequencies is difficult to determine and not of vital concern, since no attempt was made to simulate exactly an existing physical apparatus. However, one may observe that at high pressure the frequency is approximately the natural frequency of the system, and the oscillations themselves are approximately harmonic. The natural frequency may be obtained by deriving equations in a LaGrangian system of coordinates, i.e., coordinates which move with the mean velocity of the fluid, and considering natural oscillations about this mean. Hence, the geometry, the mass of fluid within the loop, and the gradient of driving force and resisting force all play an important role in determining the frequency. At low pressure, the oscillations are not harmonic, but resemble relaxation oscillations. Here, again, the oscillatory period will depend on parameters of the system, but the actual and natural frequencies may be completely different. The actual frequencies are strong functions of the transport times within the loop.

(5) The mechanism of two-phase vapor-liquid fluid flow and its transient behavior must be known accurately in order to predict the conditions under which instabilities occur, especially at high pressure. From the computed results one can see that solutions are very sensitive to the specification of slip ratio. Although the modification of the problem to incorporate a variation of slip ratio with superficial velocity seems reasonable in the light of experimental results, it has not been established that the relations valid for steady flow are also valid for continuously time-varying flow.

(6) The mechanism by which oscillations are initiated and sustained at high pressure appears to be essentially the same as that at low pressure despite the fact that in one case the oscillations appear to be relaxation oscillations and in the other case nearly harmonic. Differences do exist in the conditions surrounding the oscillatory states, but in each case the cause appears identical. Graphically one can obtain insight into both the causes and some of the general features of oscillations by plotting the steady-state solutions of equations (54) to (56) as functions of the steady rate of power input. This is most conveniently done by plotting the gravitational driving

Figure 25. Calculated Steady State Behavior at High and Low Pressure.

300 psia; at a subcooling of 10 Btu/lb, at which value no steady solution could be obtained, the difference in gradients is greatest.

Table 3

CALCULATED STEADY-STATE BEHAVIOR AT HIGH AND AT LOW PRESSURE

A. P = 300 psia, Q = 70 kw/liter				B. P = 1 atm, SC = 20.0 Btu/lb			
Steady-state Point	Subcooling (Btu/lb)	Velocity (ft/sec)	$\frac{d}{dW} (DF - RF)_s$	Steady-state Point	Q (Btu/sec)	Velocity (ft/sec)	$\frac{d}{dW} (DF - RF)_s$
a'	20.0	4.37	32	a	3.0	0.68	30
b'	15.0	4.18	55	b	5.84	1.16	847
c'	10.0	3.94	96	c	9.44	1.82	668
d'	7.5	3.83	75	d	12.70	2.31	410
e'	5.0	3.68	70	e	15.00	3.03	233

(7) At high pressures, it has been experimentally noted that the mass flow rate in a natural circulation loop will, with an increase in power input, pass through a maximum.^(2,20,24) When oscillations have been observed in such systems, the conditions have usually been beyond those surrounding the maximum flow.^(2,20) At 300 psia, the computer model also shows this type of behavior.

These observations lead to the general conclusion regarding the results obtained from a digital computer that qualitatively the predicted behavior agrees with the observed experimental behavior in all respects, whereas quantitatively the frequency and amplitude characteristics of oscillations are within order-of-magnitude agreement; other characteristics, such as mean flow rate and conditions surrounding the oscillations, are in closer agreement, so far as any comparison is possible.

GENERAL DISCUSSION AND SUMMARY

The purpose of this investigation was to create a mathematical model of a two-phase natural-circulation loop and with it examine the behavior of such a system. Previous theoretical investigations have dealt primarily with determining the conditions under which a system would be unstable to small perturbations about a steady state rather than with the prediction of the general transient phenomena. One exception has been the work of Wissler and George, but in this case, the system described is of limited application.

In achieving the goal sought, an evolutionary process has been used wherein initial models, solved with an analog computer, gave insight into some of the significant areas in which improvement was needed, and led to more accurate and efficient models. The final model, which was programmed for solution with a digital computer, is believed to satisfy the objectives and to prove useful in both predicting and explaining the operation of two-phase natural-circulation loops.

The validity of any hypothesized model can only be tested by comparing its predictions with the results of the actual process it is intended to describe. Comparisons of this sort at high and low pressures and in systems of widely differing geometry have shown general qualitative agreement, and quantitative agreement where such comparisons were not precluded by a scarcity of data, in every significant respect except for the oscillatory frequency at low pressure.

Since few data pertaining directly to instability phenomena were available for comparison, the model was kept general and its geometry simple. It may be applied to far more complex systems and could include such refinements as use of empirical or semi-empirical frictional pressure-drop correlations, spatial as well as temporal variation of slip ratio and power input, and more complex geometric configurations. With the continual increase in speed and storage capacity of computers, procedures presently found unwieldy may become practical. The model is believed to be both complete and versatile, taking account, as it does, of the effects on energy, mass, and momentum transport of the slip ratio, and being applicable to systems operating over a wide range of pressures.

The results show conclusively that high-pressure oscillations due to hydraulic phenomena alone are predicted in certain regions of operating variables, and that these phenomena are not essentially different from those at low pressure. The mechanism causing oscillations is clearly related to the difference between the gradients of driving force and resisting force, and, in fact, when the difference exceeds a certain limit, oscillations occur. This criterion is similar to the criterion for stability in a stirred tank chemical reaction system,⁽⁴⁾ in which a similar condition may be shown to exist (Appendix C).

In order to obtain solutions with the digital computer in reasonable lengths of time, the effect of thermal feedback through the cold leg (down-comer) has been neglected, and an inlet feed of constant temperature to the heater has been assumed. The assumption should not seriously affect any of the results at high pressure, but at low pressure the effect may be quite pronounced. It is not hard to imagine a situation in which the temperature feedback would strongly abet or annul the oscillatory tendency. Including the feedback in a digital solution poses no particularly difficult problems except one of storage, since the cold leg contains a single liquid phase at all times and the problem is merely a transient one-dimensional heat convection problem, for which solutions of suitable accuracy are straightforward.

In applying the model, the method of lumping parameters is of more than routine interest. Because of the complexity of the equations involved in simultaneous two-phase flow and heat transfer, however, only the most elementary lumping has been undertaken, and the resulting model has been essentially a mixing-cell model. One would like to combine lumped parameters with pure delays in such a way that the true distributed-parameter system is more accurately portrayed, but attempts to do this have not been successful. Of especial difficulty has been the problem of simulating continuously variable time delays within the framework of the analog facilities and within the high-speed storage capacity of UNIVAC Scientific, and combining these delays with the simultaneous transport of energy, mass, and momentum.

CONCLUSIONS

Based on the investigation which was carried out concerning the nature of two-phase natural-circulation loops, it has been concluded that:

(1) Models of a two-phase natural-circulation loop, based on a concept of cascaded mixing cells, have shown versatility, speed, and reliability in predicting the high- and low-pressure operations of such loops. The models employed with an analog computer appear to be restricted to rather general investigations, but those used in conjunction with the digital computer appear capable of a high degree of refinement if the necessary computer capacity and time are available.

(2) Initiation of hydraulic oscillations at both high and low pressures is caused by a large difference between the gradients of driving force and resisting force with flow rate.

(3) Notable differences between the predicted and observed periods of low-pressure oscillations can be expected, since transport times are not accurately represented with mixing cells unless their dimensions are considerably smaller than the ones used in this problem.

(4) Negative driving forces, which sometimes result from small variations in voltage, lack of flexibility in varying some of the problem parameters, and the requirement of an inordinately large amount of equipment are disadvantages of an analog model of this problem. However, such a model is flexible in many respects, particularly in varying most parameters, and is an excellent means of gaining insight into the general behavior of the mathematical system.

(5) Evidence is strong that oscillations occur within a closed region of operating conditions, and not simply beyond a certain boundary.

RECOMMENDATIONS

Many possibilities for further research exist in this field. As projects that would aid in better understanding the complex phenomena, it is recommended that:

(1) Specific systems at high pressure should be represented by the computer model and the results compared with those obtained experimentally. Recent studies have been in progress which are directly concerned with the question of oscillations, and data should be available for such comparisons.

(2) Operation of two-phase natural-circulation loops with feed containing both vapor and liquid should be investigated. In this region, stability seems assured, while the only possible disadvantage is one of burnout.

(3) The effect of the magnitudes of perturbations about an initial steady state should be investigated. In many respects, natural-circulation loops behave like a pendulum clock, and in developing a theory of the clock, Andronow and Chaikin⁽³⁾ have shown that initial displacements from equilibrium must exceed some minimum in order for oscillations to be sustained.

(4) A homogeneous-fluid model should be developed to incorporate the variation in slip ratio more easily. This type of model is more easily programmed, especially with an analog computer, than a separated-flow model.

(5) New methods of simulation which would combine a system of lumped parameters and pure delays should be tried and tested so that the time behavior may be more neatly and accurately obtained.

NOTATION

Primary English Letter Symbols

a	constant
A	total cross sectional flow area, sq ft
b	constant
d	differential operator
D	diameter, ft
e	internal energy, Btu/lb
f	Fanning friction factor
f	constant, defined in text
F	force per unit area $\text{lb}_f/\text{sq ft}$
F'	total force, lb_f
g_c	conversion factor, $\text{lb}_m\text{-ft}/\text{lb}_f\text{-sec}^2$
$\bar{h}$, $\bar{H}$	mean specific enthalpy, Btu/lb
h	specific enthalpy, Btu/lb
H	function, defined in text
J	conversion factor, $\text{ft-lb}_f/\text{Btu}$
M	function, defined in text
L	length, ft
L	equivalent length, ft
p	pressure, $\text{lb}_f/\text{sq ft}$
Q	rate of heat transfer, Btu/sec
S	function, defined in text
t	time, sec
V	velocity, ft/sec
$\bar{V}$	mean velocity, defined in text, ft/sec
W	mass flow rate, lb/sec
x	weight fraction vapor, dimensionless
x	incremental distance, ft
Z, z	elevation, ft

Primary Greek Letter Symbols

α	vapor volume fraction, dimensionless
Δ	finite increment or difference
γ	slip ratio, (vapor velocity)/(liquid velocity), dimensionless
ρ	density, lb/cu ft
$\bar{\rho}$	mean density, defined in text, lb/cu ft
ϕ	function, defined in text, dimensionless
μ	viscosity, lb _m ft/sec

Subscripts

f	friction
f	saturated liquid
fg	liquid to vapor transformation
i	inlet
j, k, m	general element designation
o	outlet
s	vapor, always saturated
w	liquid

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Appendix A

ANALOG COMPUTER MODELS

Analog Model A

In the analog solutions to equations (26), the geometric constants of Table 4 were used.

Table 4

GEOMETRY OF ANALOG MODEL A, WITH
 $A_j = 0.00415 \text{ ft}^2$ and $D_j = 0.0727 \text{ ft}$

j	$\Delta x_j(\text{ft})$	$L_j(\text{ft})$	$\Delta Z_j(\text{ft})$
0	28.0	30.0	-24.0
1	4.0	5.0	4.0
2	20.0	-	20.0
3	4.0	5.0	0.0

The values for L_j included corrections for expansion, contraction, and bend losses, and hence are somewhat larger than the corresponding Δx_j .

The liquid density in each of stages $j = 0, 1$, and 3 was assumed constant and equal:

$$\rho_0 = \rho_1 = \rho_3 = 60.0 \text{ lb/cu ft.}$$

The viscosity of water, which appears in equation (28) and must be used in calculating the frictional losses, was assumed to be that of saturated water at 200°F:

$$\mu = 64 \times 10^{-7} \text{ lb}_m/\text{ft-sec.}$$

The heat removed from the fluid at the cooler was assumed to be proportional to the difference in enthalpy between the cooled and the cooling fluids;

$$Q_3 = K(\bar{H}_3 - \bar{H}_{CW})$$

From data presented by Wissler at 11.8 Btu/sec, K was calculated to be 0.421 (Btu/sec)/(Btu/lb) and was assumed to be constant over the entire range of heat inputs. The enthalpy of the cooling water, $\bar{H}_{CW}$, was taken as 130 Btu/lb. This was subsequently used as the datum of enthalpy from which all values of $\bar{H}$ were calculated.

In solving equations (26), new variables were defined, and in this way scaling factors introduced so that machine voltages would not exceed their limiting values or be so small that they would lose all significance. The machine variables were defined as

$$\begin{aligned}\tau &= t/\alpha & V' &= V/\epsilon \\ W' &= W/\beta & H' &= H/\gamma \\ \rho' &= \rho/\delta\end{aligned}$$

where α , β , δ , ϵ , and γ are scaling factors. The values of these constants were

$$\begin{aligned}\alpha &= 1.0 & \delta &= 1.0 \\ \beta &= 0.05 & \epsilon &= 0.05 \\ \gamma &= 5.0\end{aligned}$$

Using these scaling factors and the geometry specified above, potentiometer settings for Model A were calculated. These settings are listed in Table 5. Potentiometer numbers refer to those of Figure 4. The settings of potentiometers 11 through 16 represent a typical set of initial conditions.

Table 5

POTENTIOMETER SETTINGS FOR ANALOG MODEL A

Potentiometer Number	Setting (corrected for loading)
18	0.9960
19	0.0237
21	0.0621
22	0.6128
24	0.3320
25	0.7786
26	0.0952
29	0.0502
30	0.4231
32	0.0718
34	0.5015
41	0.3160
57	0.2012
58	0.7500
Initial conditions:	
11	0.1854 (x + 100V)
12	0.1003 (x - 100V)
13	0.0560 (x + 100V)
15	0.0560 (x - 100V)
16	0.1003 (x + 100V)

The riser fluid density was generated as a function of the average enthalpy by a diode function generator as described on page 24. The effects of varying many of the problem parameters, e.g., power input, frictional coefficients, stage volume, and cooler heat transfer coefficient, could be easily determined by changing the potentiometer settings and observing the new results. Results were recorded in the form of pen or stylus traces on a Variplotter and a four-channel Sanborn recorder.

Analog Model B

In the analog solutions to equations (48), the geometric constants of Table 6 were used. The model did not allow for the formation of vapor in the heater, but two phases could exist in either or both of the riser stages. The density of the liquid in the downcomer stages, the heater, and the cooler was assumed to be constant, but not equal for all stages. A difference was retained to reflect the difference in mean temperature. Similarly, the liquid density in the riser stages was less than in the downcomer. The following values were used:

$$\rho_0 = \rho_3 = 60.0 \text{ lb/cu ft}; \quad \rho_1 = \rho_{w2} = 59.7 \text{ lb/cu ft}.$$

The viscosity μ and the cooler heat transfer coefficient were the same as for Model A, given above.

Table 6

GEOMETRY OF ANALOG MODEL B

j	$A_j(\text{ft})^2$	$x_j(\text{ft})$	$D_j(\text{ft})$	$L_j(\text{ft})$	$Z_j(\text{ft})$
0,0	0.00415	2.54	0.0727	30.0	-2.54
0,1	0.00415	2.54	0.0727		-2.54
.	.	.	.		.
.	.	.	.		.
.	.	.	.		.
0,8	0.00415	2.54	0.0727	5.0	-2.54
0,9	0.00415	2.54	0.0727		-1.10
0,10	0.00415	2.54	0.0727		0
1	0.0575 (0.00415)	4.00	0.2705 (0.0727)		+4.00
2,0	0.00415	10.0	0.0727	-	+10.0
2,1	0.00415	10.0	0.0727	-	+10.0
3	0.00415	4.0	0.0727	-	0

The two-phase density function was very important in determining the behavior of the loop. To provide better-defined and more realistic functions than those used in Model A, a slip ratio was taken into account by using equations (39) and (40), and the thermodynamic properties of saturated steam and water. The calculated values of $\bar{\rho}$ as a function of the mean enthalpy for different values of γ are listed in Table 7. Figure 26 is a plot of some of the curves derived.

Table 7

MEAN TWO-PHASE DENSITY OF STEAM-WATER
MIXTURES AT 1 ATM PRESSURE CALCULATED AS
A FUNCTION OF MEAN ENTHALPY AND SLIP RATIO

$\bar{h}$ (Btu/lb)	$\bar{\rho}$ (lb/cu ft)					
	($\gamma=1$)	($\gamma=2$)	($\gamma=5$)	($\gamma=10$)	($\gamma=20$)	($\gamma=50$)
180.0	59.81	59.81	59.81	59.81	59.81	59.81
181.0	23.59	33.81	45.77	51.87	55.55	58.03
182.0	14.26	23.05	36.50	45.35	51.57	56.23
183.0	10.26	17.49	30.35	40.27	48.16	54.52
184.0	7.99	14.09	25.98	36.20	45.11	52.94
185.0	6.55	11.75	22.69	32.93	42.48	51.39
186.0	5.54	10.14	20.18	30.16	40.09	49.95
188.0	4.22	7.93	16.48	25.80	36.03	47.32
190.0	3.45	6.49	13.91	22.51	32.74	44.93
195.0	2.37	4.46	9.96	17.07	26.58	39.85
200.0	1.77	3.38	7.81	13.73	23.53	35.79

Figure 26

Effect of Slip Ratio on Average Two-phase Density (1 atmosphere).

The amount by which the average fluid enthalpy exceeds the liquid saturation enthalpy, $\bar{h} - h_f$, and the enthalpy of vaporization, h_{fg} , determines the vapor quality, and if h_{fg} does not change significantly over the pressures encountered, the quality will be a function only of $\bar{h} - h_f$, which has been called $\bar{S}$ in the text. Hence, if the ratio of saturated liquid specific volume to saturated vapor specific volume does not suffer large changes over the pressure range of interest, the average density as calculated above may be expressed accurately as a function of $\bar{S}$ even though pressure changes do occur. To check this assumption, the average density was calculated by this method at pressures of 14.7, 16.0 and 18.0 psia and with a slip ratio γ of one, to determine the change in the plot of $\bar{\rho}$ vs $\bar{S}$ with pressure. As shown in Figure 27, the variation is very slight.

Figure 27. Effect of Pressure on Mean Density for Model B.

The riser densities and density functions necessary for the solution were therefore calculated as functions of $\bar{S}$. In the upper riser, $\bar{S}$ was simply the difference between the mean enthalpy and the constant enthalpy of saturated liquid at one atmosphere. In the lower riser, the saturated liquid enthalpy h_f varied with pressure, so in order to obtain $\bar{S}$ it was necessary to first calculate h_f and then obtain the difference between it and the average fluid enthalpy $\bar{h}$.

Scaling factors were the same as for Model A. Potentiometer settings were calculated in the same manner. The circuit schematic was rather complex and will not be reproduced here. For Model B, using an eleven-stage downcomer, the number of components necessary is given in Table 8.

Table 8

ANALOG COMPONENTS NECESSARY FOR ANALOG MODEL B

Type	Quantity
Integrating Amplifiers	16
Summing Amplifiers	72
Servomultipliers	13
High-gain Amplifiers	2
Nonlinear Function Generators	6
Coefficient Potentiometers	33
Initial Condition Potentiometers	16

Appendix B

DIGITAL COMPUTER MODELS

Low-pressure Calculations

The designation of stages for the low-pressure model has been given in Figure 12. The dimensions of these stages used in the numerical solution to equations (51) to (53) are given in Table 9.

Table 9

DIMENSIONS AND SHAPE FACTORS FOR THE LOW-PRESSURE
DIGITAL COMPUTER MODEL, WITH $A_j = 0.00415 \text{ ft}^2$

j	$\Delta x_j(\text{ft})$	$\Delta Z_j(\text{ft})$	$L_j(\text{ft})$	$f'_j (\text{lb}_m/\text{ft}^3)$
0	24.0	-20.0	29.35	241.42
1	4.0	4.0	5.00	1.04
2	4.0	4.0	5.00	41.127
3	4.0	4.0	5.00	41.127
4	4.0	4.0	5.00	41.127
5	4.0	4.0	5.00	41.127
6	4.0	0	5.00	41.127

The dimensions are essentially those of the University of Minnesota loop. As noted in the main body of the report, frictional losses were based on a Fanning friction model using only the liquid-phase velocity. The frictional term of equation (53) was calculated by means of the expression

$$F_{fj} = f_j L_j \rho_{wj} V_j^2 / 2 D_j g_c \quad ,$$

where

L_j = an effective length, ft

ρ_{wj} = density of liquid, lb_m/ft^3

f_j = Fanning friction factor, dimensionless

D_j = cylindrical pipe inner diameter, ft.

The density ρ_{wj} in the friction term was assumed to remain constant at $59.8 \text{ lb}/\text{cu ft}$. The Fanning friction factor, based on the mean Reynolds number of the liquid, was taken as 0.02 in all stages except the heater, for which it was assumed to be 0.03.

The term $f_j^!$ denotes a collection of constants:

$$f_j^! = f_j L_j \rho_{wj} / 2D_j \quad .$$

The physical and thermodynamic constants used in the solution were:

$$p_5 = 16.5 \text{ psia}$$

$$h_{fj} = 0.1032 p_j^! + 186.125$$

$$\rho_{sj} = 8.303 \times 10^{-5} p_j^! + 0.04163$$

$$h_{sj} = 0.03747 p_j^! + 1152.4$$

$$\rho_{wj} = -0.02645 h_{wj} + 64.6004$$

where

$$p_j^! = \sum_{k=j+1}^5 [\rho_w(1-\alpha) + \rho_s \alpha]_k \quad , \quad (j = 1, 2, 3, 4)$$

The pressure in the top riser stage, p_5 , was greater than atmospheric by an amount equal to the pressure drop sustained in the cooler, estimated for the mean flow rate through the cooler. The actual pressure has little effect on the general characteristics of operation, but does affect the heat input-subcooling conditions at which sustained oscillations will commence. All relations above are based on steam table values, and are within one per cent accuracy over the pressure range encountered in any of the problem conditions.

In solving the equations numerically, the time increments by which the solution was advanced depended on the enthalpy at the top of the riser. The reason for this was that, when no boiling occurred within the loop, the behavior was quiet and without rapid transients, whereas when the loop was at the point of incipient boiling, or actually contained vapor, severe transients might well occur, and the time intervals needed to be shorter. The value of the increments was chosen by trial, and was continually reduced until any further reduction resulted in no significant change in the problem solution. The actual time increments used and the criterion for their selection were:

$$\Delta t = \frac{1}{16} \text{ sec} \quad , \quad h_{w5} > 186.0 \text{ Btu/lb};$$

$$\Delta t = \frac{1}{4} \text{ sec} \quad , \quad h_{w5} \leq 186.0 \text{ Btu/lb}.$$

The saturation enthalpy for $j = 5$, ($p = 16.5 \text{ psia}$), was

$$h_{f5} = 186.125 \text{ Btu/lb}.$$

Using the trapezoidal rule, five corrective iterations were made within each time increment. The computer was so programmed that V_{wj} , h_{wj} , and α_j , for all values of j , could be punched on paper tape at every tenth time increment, and V_{w0} and α_5 could be printed by the typewriter at the operator's discretion. The actual machine time required for the advance of one complete increment in time was 5.1 sec. The complete program occupied 1002 words of electrostatic core storage in the UNIVAC Scientific. It is on file at the University of Minnesota, Department of Chemical Engineering.

In solving the equations, some of the momentum terms of the left hand side of equation (53) remain, since $A_0 = A_2 \neq A_1$. The terms which remain for this particular geometry are.

$$\left(1 - \frac{A_1}{A_0}\right)M_{1,1} + \left(\frac{A_1}{A_2} - 1\right)M_{1,0}$$

However, since boiling never occurs in the heater ($j=1$), and the change in liquid density with temperature is so slight that it may be ignored,

$$M_{1,i} = \frac{(\rho_w V_w^2)_{1,i}}{g_c} \quad \text{and} \quad M_{1,o} = \frac{(\rho_w V_w^2)_{1,o}}{g_c}$$

but for an incompressible fluid

$$V_{wj,i} = V_{wj,o}$$

and hence

$$M_{1,1} = M_{1,0} = M_1$$

Then since $A_0 = A_2$, the sum of the momentum terms is equal to zero.

High-pressure Calculations

The dimensions used in simulating the high-pressure, 300-psia loop are given in Table 10.

Table 10

DIMENSIONS AND SHAPE FACTORS FOR THE HIGH-PRESSURE DIGITAL COMPUTER MODEL

j	$A_j(\text{ft})^2$	$\Delta x_j(\text{ft})$	$\Delta Z_j(\text{ft})$	$f'_j (\text{lb}_m/\text{ft}^3)$
0	0.020833	9.5	-7.5	26.586
1	0.0034722	1.5	1.5	16.281
2	0.0034722	1.5	1.5	16.281
3	0.0034722	1.5	1.5	16.281
4	0.0034722	1.5	1.5	16.281
5	0.0034722	1.5	1.5	16.281
6	0.020833	2.0	0	5.597

Stages 1, 2, and 3 constituted the heater, 4 and 5 the riser, and 6 and 1 the crossover and downcomer portions of the loops. The actual loop consisted of a single $\frac{1}{4} \times 2 \times 60$ -in. heated channel and a $\frac{1}{4} \times 2 \times 29$ -in. riser with an "unrestricted" downcomer, so that the computer model is a reasonable facsimile.

Using the definition of p_j^i given on page 76, the saturation properties were calculated from the following expressions:

$$h_{fj} = 0.003418 p_j^i + 393.84 \text{ (Btu/lb}_m\text{)}$$

$$\rho_{sj} = 2.207 \times 10^{-5} p_j^i + 0.6480 \text{ (lb}_m\text{/cu ft)}$$

$$h_{sj} = 2.083 \times 10^{-4} p_j^i + 1202.8 \text{ (Btu/lb}_m\text{)}$$

The variation of liquid density was calculated as a function of enthalpy as

$$\rho_{wj} = 67.940 - 0.03812 h_{wj} \text{ (lb}_m\text{/cu ft)}.$$

Frictional losses were calculated by means of the relation

$$F_{fj} = \frac{f_j V_{wj}^2}{g_c} = \left(\frac{fL}{2D} \right)_j \frac{V_{wj}^2}{g_c}$$

In stages where no boiling occurred ($j = 0, 6$), the average Reynolds number was approximately 1.3×10^5 . Based on this, the average Moody friction factor for smooth pipes was 0.017. In the smaller heater and riser stages ($j = 1, 2, \dots, 5$), an average liquid linear velocity of 5 ft/sec was assumed. This gave a mean Reynolds number of 3.7×10^5 , which, for a rectangular channel, would give a friction factor f of 0.015. The averaged values were assumed constant and used throughout the 300-psia calculations.

The method of solving the system of equations and obtaining solutions was essentially the same as that used for the runs at atmospheric pressure. Since the characteristic oscillations observed at 300 psia were nearly harmonic, there were not the alternating periods of activity and relaxation observed at the low pressure. Hence, the time increments for the solution were fixed. The magnitude of these increments was again determined by reducing it until any further reduction did not change the results. The most commonly used increment was 0.025 sec. Heater inlet enthalpy h_{w0} and power input could be arbitrarily fixed. The rate of power input was constant with time and unaffected by changes in fluid flow or temperature. The time required for the computer to advance the solution by one time increment was unchanged from the low-pressure solution, 5.1 sec. The program occupied 1014 words of electrostatic core storage.

Appendix C

CHEMICAL REACTOR STABILITY

Stability in chemical reaction systems has been thoroughly discussed by Amundson and Aris.⁽⁴⁾ It is of interest to compare the conditions necessary for stability in such systems with those in a two-phase natural-circulation loop.

If a chemical reaction system consists of a single continuous agitated vessel of volume V , inlet and outlet volumetric flow rates q , and a reaction expressed as

$$\sum_{j=1}^{n-1} A_j X_j - X = 0 \quad (C1)$$

where,

n = number of chemical species

X_j = species designation

A_j = stoichiometric coefficients

undesigned X = species that would be first exhausted if the reaction were not limited by equilibrium,

then mass and heat balances for the reactor give

$$V \frac{dx}{dt} = q(x_o - x) - VR \quad (C2)$$

$$Vc\rho \frac{dT}{dt} = qc\rho(T_o - T) - VU^* - (-\Delta H)VR \quad (C3)$$

where

x, x_o = concentration (moles per unit volume) and influent concentration

t = time (sec)

R = reaction rate = $-\left(\frac{dx}{dt}\right)_r$, where the subscript r refers to reaction alone

C = specific heat of reacting mixture

ρ = density of reacting mixture

T_0 , T = inlet temperature of reactants, temperature of mixture

U^* = heat per unit volume per unit time removed from the reactor by the cooling system

$(-\Delta H)$ = heat evolved per mole of X formed.

Making the following substitutions:

$\tau = qt/V$ = reduced time

$\xi = x/x_0$ = reduced concentration

$\eta = c\rho T/(-\Delta H)x_0$ = reduced temperature

$P = VR/qx_0$ = reduced reaction rate

$U = VU^*/qx_0(-\Delta H)$ = reduced heat removal rate,

the mass and heat balances become

$$\dot{\xi} = 1 - \xi - P(\xi, \eta) = \bar{P}(\xi, \eta) \quad (C4)$$

$$\dot{\eta} = (\eta_0 - \eta) - P(\xi, \eta) - U(\eta) - \bar{Q}(\xi, \eta) \quad (C5)$$

The steady-state values ξ_s and η_s , obtained by setting the left-hand sides of these equations equal to zero, may be combined to give

$$\eta_s - \eta_0 + U(\eta_s) = -P[\xi_s(\eta_s), \eta_s] \quad (C6)$$

Writing equation (C6) in terms of the original variables,

$$\frac{c\rho}{(-\Delta H)x_0} (T - T_0) + \frac{VU^*}{q(-\Delta H)x_0} = -\frac{VR}{qx_0}$$

or

$$[qc\rho(T - T_0) + VU^*]_s = -[VR(-\Delta H)]_s \quad (C7)$$

it may be seen that the left-hand side of (C7) is the rate at which heat is removed from the reacting system and the right-hand side the rate of heat generation.

If equations (C4) and (C5) are written in linear form describing small departures from a steady state and if the stability of these departures are determined, Aris and Amundson have shown that the conditions necessary and sufficient for stability are

$$-(\bar{P}_\xi)_s - (\bar{Q}_\eta)_s > 0 \quad (C8)$$

and

$$(\bar{P}_\xi)_s (\bar{Q}_\eta)_s - (\bar{P}_\eta)_s (\bar{Q}_\xi)_s > 0 \quad (C9)$$

where

$$(\bar{P}_\xi)_s \text{ is } \left(\frac{\partial \bar{P}}{\partial \xi} \right) \text{ evaluated at } \xi_s, \eta_s, \text{ etc.}$$

Expanding (C9) in a more complete form by means of equations (C4) and (C5), one obtains

$$[1 + (P_\xi)_s] [1 + (U_\eta)_s] > - (P_\eta)_s \quad (C10)$$

Using equations (C6) and (C7), one can obtain the derivatives with respect to temperature of the left- and right-hand sides of (C7):

$$\frac{d}{dT} (\text{heat removal rate})_s = \frac{d}{dT} [qc\rho(T - T_o) + VU^*]_s = qc\rho(1 + U_\eta)_s \quad (C11)$$

$$\frac{d}{dT} (\text{heat generation rate})_s = \frac{d}{dT} [-VR(-\Delta H)]_s = qc\rho \left(\frac{-P_\eta}{1 + P_\xi} \right)_s \quad (C12)$$

Therefore, the necessary condition for stability of (C9) is simply that

$$\frac{d}{dT} (\text{heat removal rate})_s > \frac{d}{dT} (\text{heat generation rate})_s \quad (C13)$$

This has been shown graphically by the authors to be clearly necessary for stable operation.

The significance of (C8) is less obvious, however. If it is expanded by using (C4) and (C5), the condition may be written as

$$(1 + P_\xi) + (1 + U_\eta) > -P_\eta \quad (C14)$$

By rearranging and adding the quantity $\left(1 + 2U_\eta + \frac{P_\eta}{1 + P_\xi}\right)$ to each side of the inequality (C14), one obtains

$$P_\xi + P_\eta + 2U_\eta + 3 + \frac{P_\eta}{1 + P_\xi} > (1 + U_\eta) - \left(\frac{-P_\eta}{1 + P_\xi} \right) \quad (C15)$$

The right-hand side of (C15) is

$$\frac{1}{qc\rho} \left[\frac{d}{dT} (\text{heat removal rate})_s - \frac{d}{dT} (\text{heat generation rate})_s \right]$$

or the difference between the gradients of the two quantities. The left-hand side will normally be a positive quantity, since each of the separate terms is positive:

$$(P_{\xi})_s = \frac{V}{q} \frac{\partial R}{\partial x} > 0$$

since x is a reactant and the reaction rate will normally be increased with its concentration.

$$(P_{\eta})_s = \frac{V(-\Delta H)}{qc\rho} \frac{\partial R}{\partial T} > 0,$$

for an exothermic reaction where the rate increases with temperature.

$$(U_{\eta})_s = \frac{V}{qc\rho} \frac{\partial U^*}{\partial T} > 0,$$

for normal conditions where coolant flow and temperature remain relatively constant.

Thus, the physical significance of the stability criterion expressed by equation (C8) is that the difference between gradients of the heat removal rate and the heat generation rate at the steady-state operating point must be less than some positive quantity

$$qc\rho \left[\frac{V}{q} \left(\frac{\partial R}{\partial x} \right) + \frac{V}{qc\rho} (-\Delta H) \frac{\partial R}{\partial T} + \frac{2V}{qc\rho} \frac{\partial U^*}{\partial T} + 3 + \frac{P_{\eta}}{1 + P_{\xi}} \right]$$

whose magnitude increases with system volume and heat capacity. This criterion is quite similar to the criterion deduced by observation from the computed and experimental results of two-phase natural-circulation loops.

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