STABILIZATION OF THE AXISYMMETRIC
INSTABILITY IN THE PDX TOKAMAK

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Stabilization of the Axisymmetric Instability
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ABSTRACT

Passive stabilization, associated with the mutual inductances of the field shaping coils with the plasma and with each other, can stabilize the axisymmetric magnetohydrodynamic instability in the Princeton Poloidal Divertor Experiment device that was observed in earlier numerical studies.

I. INTRODUCTION

The Princeton Poloidal Divertor Experiment¹ will be one of the first large tokamaks in which the current carrying plasma is in close proximity to poloidal field coils which carry currents comparable in magnitude to the current in the discharge. While having the desired effect of producing a magnetic divertor configuration, these coils also modify the shape of the plasma cross section and thereby affect the magnetohydrodynamic stability of the device.

It was reported previously² that typical numerically generated Princeton Poloidal Divertor Experiment "equilibrium" configurations are numerically found to be unstable to a rapidly growing axisymmetric magnetohydrodynamic instability which consists of both a vertical displacement and a complicated change of shape of the plasma cross section. It was suggested there that this instability may saturate nonlinearly due to large sheet currents induced in the surface of the plasma itself. Even if this is so, the saturated state may not be a desirable equilibrium and there is some doubt as to the effectiveness of this saturation mechanism over a resistive skin time in which the induced sheet current would dissipate.

Previous stability calculations have treated the current carrying coils as constant current sources. Here, we extend and refine the previous study by allowing the field shaping coils near the surface of the plasma to interact with each other and with the plasma as determined by their mutual inductances and by

appropriate circuit equations. This simulation of the naturally occurring passive feedback of the coils significantly improves the realism of our model of the actual physical experiment. This feedback can stabilize the axisymmetric instability in the Princeton Poloidal Divertor Experiment device.

II. PHYSICAL MODEL

The plasma is taken to be separated from the vacuum by a sharp boundary. In the plasma region, we solve the full set of nonlinear time-dependent ideal magnetohydrodynamic equations. These provide a good description of low-frequency global motions of a high temperature tokamak plasma. The plasma equations are integrated forward in time using a finite difference method³ based on magnetic flux coordinates. The finite difference equations are solved using a partially implicit inversion procedure which enables integration over the relatively long time scales associated with the axisymmetric instability. Since magnetic flux coordinates are utilized, the location of the plasma vacuum interface remains well represented.

The magnetic field in the vacuum is represented as

$$\vec{B} = \frac{1}{2\pi} \nabla\phi \times \nabla\chi + R_0 B_0 \nabla\phi \quad (1)$$

Here χ is the vacuum poloidal magnetic flux, ϕ is the symmetry angle, R_0 is the initial location of the magnetic axis, and B_0 is a spatial constant determined by the requirement that

the toroidal flux in the vacuum region remain constant. The solution for χ is obtained from

$$\chi(\underline{x}_T, t) = \chi_w + \sum_{m=1}^M I_m(t) G(\underline{x}_T, \underline{x}_m) + \frac{1}{2\pi} \oint \frac{d\ell_S}{X_S} G(\underline{x}_T, \underline{x}_S) \frac{\partial \chi}{\partial n_S}(\underline{x}_S) . \quad (2)$$

Here, $[I_m(t), \underline{x}_m ; m = 1, M]$ are the currents and the locations of the poloidal field coils, χ_w is the poloidal flux at the outer vacuum wall, and $G(\underline{x}_T, \underline{x}_S)$ is the infinite medium Green's function

$$G(\underline{x}_T, \underline{x}_S) = (X_T X_S)^{-1/2} k^{-3/2} [(2 - k^2) K(k^2) - 2E(k^2)] , \quad (3)$$

where $K(k^2)$ and $E(k^2)$ are the usual complete elliptic integrals with

$$k^2 \equiv 4X_T X_S / [(X_T + X_S)^2 + (z_T - z_S)^2] .$$

The contour of integration in Eq. (2) is the plasma vacuum interface, which is itself a function of time.

Equation (2) is inverted to give $\partial \chi / \partial n$ on the vacuum side of the interface as a functional of the location of the interface and of the currents in the external coils. This feeds back into the plasma region by way of the boundary condition that

$$P \equiv \frac{1}{2} [(\partial \chi / \partial n)^2 / (2\pi X)^2 + B_0^2 R_0^2 / X^2] + p$$

is continuous across the plasma vacuum interface. The currents in the external coils, $I_m(t)$, change in time as described in Appendix A. Each coil has a finite mutual inductance with every other coil and with the plasma.

III. EQUILIBRIUM AND CIRCUITS

The determination of stability is treated as an initial value problem. First, a plasma equilibrium, consistent with a set of prescribed fixed currents, $I_m(0)$, is found using the Princeton Equilibrium Program.⁴ The plasma is then displaced slightly (0.02 minor radii) either upward or downward. The currents are allowed to vary according to the appropriate circuit equations and the dynamics of the system are followed in time. For an unstable configuration, the initial displacement should grow exponentially; in a stable configuration it should oscillate about its initial location.

We consider the plasma equilibrium shown in Fig. 1. The equilibrium has a vertical field such that the plasma assumes a mild dee-shape, tied to the inside divertors. This has $B_0 = 25\text{kG}$, $I = 500\text{kA}$, $R_0 = 1.24\text{ m}$, plasma diameter = 0.6 m , $2.57 \leq q \leq 3.19$, $\beta_p = 0.42$, $\beta \equiv 2\langle p \rangle / B_0^2 = 0.6\%$, with p and χB_T linear functions of $\chi_B - \chi$. The plasma is supported by an external field associated with the fourteen current-carrying coils listed in Table 1. Evaluation of the localized stability criteria⁵ shows that the system is stable with respect to both ideal and resistive interchanges -- both D_I and D_R are negative, increasing monotonically from the magnetic axis to the interface. The

linear magnetohydrodynamic spectrum for this equilibrium has been evaluated using the Princeton Equilibrium Stability and Transport⁶ code. There is one unstable $n = 1$ mode [where $\xi \propto \exp(-in\phi)$] with $\Omega^2 = -0.0029$ [normalized to $\Omega_0^{-1} = \mu_0 \rho_0 (a^2/B_0^2)^{1/2}$]. This is essentially an $m = 4$ kink concentrated near the plasma vacuum boundary. There is also one unstable axisymmetric mode ($n = 0$) with $\Omega^2 = -0.0028$.

We consider here the circuit equations for the poloidal field coils shown in Fig. 2. In circuit 1, each of the top-bottom circuit pairs are connected together in parallel with all the pairs being joined together in series. In circuit 2, all of the top coils are connected in series, all of the bottom coils are connected in series, and these are joined together in parallel. Circuit 2 is most easily achieved experimentally since it requires the fewest connections. In circuit 3, the coils are assumed to have finite impedance so that the coil currents are not changed by the instability.

IV. CALCULATED RESULTS

We show in Fig. 3 the Z coordinate of the magnetic axis versus time for the three circuit configurations. The time behavior is not exactly sinusoidal or exponential since the initial perturbation is not an exact eigenmode, but is rather a superposition of eigenmodes. The Cases 2 and 3, associated with external coil circuits 2 and 3, exhibit exponential growth and are clearly unstable configurations. However, Case 1 is stable over the time interval considered. Since the three cases are

identical except for the circuitry of the coils, stability is due to passive feedback stabilization.

We illustrate one of these calculations in Figs. 4 and 5. Where we show the magnetic surfaces for Case 2 at the initial time and after 73.0 Alfvén periods. The instability is not merely a rigid displacement but is larger on the inside of the torus and significantly distorts the plasma cross section.

V. DISCUSSION

The circuitry of the external poloidal field coils can play a dominant role in the axisymmetric instability in the Princeton Poloidal Divertor Experiment tokamak. To better understand this result, we examine separately the forces acting to stabilize and to destabilize the axisymmetric instability in the two equilibrium configurations. We consider two effects: 1) external vertical field curvature, and 2) passive feedback stabilization due to the poloidal field coils.

To estimate the effect of vertical field curvature, we approximate the plasma by a filament current source displaced upward a distance δZ from the symmetry plane at radial location x_p and with strength I_p . The radial field at the plasma due to the external coils is given by

$$\delta B_r = \frac{\partial B_r}{\partial Z} \delta Z = \frac{1}{2\pi x_p} \frac{\partial^2 \chi}{\partial Z^2} \delta Z = \frac{1}{2\pi} \sum_{m=1}^M \frac{I_m}{p} \frac{\partial^2 G_m}{\partial Z^2} \delta Z$$

This corresponds to an energy change

$$\delta W_k = \pi \rho_0 \gamma_k^2 = \frac{\mu_0 I_p^2}{2\pi X_p} \sum_{m=1}^M I_m \frac{\partial^2 G}{\partial Z^2} (\underline{x}_p, \underline{x}_m) .$$

This will be stabilizing at some points on the plasma midplane and destabilizing at other points.

Again, approximating the plasma by a filament current source at location $\underline{x}_p$, we can estimate δW_F , the energy change due to the passive feedback of the coils. If the plasma is displaced upward from $(\underline{x}_p, 0)$ to $(\underline{x}_p, \delta Z)$, each coil will see a flux change

$$\delta \chi_{i'j'l'k'} = I_p \frac{\partial G}{\partial Z} (\underline{x}_p, \underline{x}_{i'j'l'k'}) \delta Z .$$

This will induce currents in the coils

$$\delta I_{ijl} = \sum_{i'j'l'k'} C_{ijl, i'j'l'} n_{i'j'l'k'} \delta \chi_{i'j'l'k'} .$$

where $C \equiv B^{-1}A$, A and B being defined in Appendix A.

These induced currents will produce a field acting back on the plasma producing a force opposing the original motion with energy change

$$\delta W_F = \pi \rho_0 \gamma_F^2 = \frac{\mu_0}{2\pi X_p} I_p^2 \sum_{ijlk} \sum_{i'j'l'k'} n_{ijlk} n_{i'j'l'k'} C_{ijl; i'j'l'}$$

$$\frac{\partial G}{\partial Z} (\underline{x}_p, \underline{x}_{i'j'l'k'}) \frac{\partial G}{\partial Z} (\underline{x}_{ijlk}, \underline{x}_p)$$

the magnitude of this term depends on the coil circuit, but it will be stabilizing for every value of X_p .

In Fig. 6 we plot these terms as a function of X_p across the midplane for coil circuits 1 and 2. Several things are to be noted. First, we see that coil circuit 1 is significantly more stabilizing than is coil circuit 2. The reasons for this are physically clear. The dominant feedback effect comes from the two pair of oppositely directed coils labeled 1,2,3,4 in Fig. 1. If the plasma is displaced slightly upward, then the maximum restoring force would come from a current $+\delta I$, $-\delta I$, $+\delta I$, $-\delta I$ induced in coils 1,2,3,4, respectively. This pattern is allowed by the circuit configuration 1 but not by circuit 2.

The second thing to note from Fig. 6 is that the total $\delta W = \delta W_K + W_F$ considered as a function of X_p , is positive on the inside of the plasma cross section while going negative toward the center. The actual δW is some kind of a weighted average over X_p of that plotted in these figures. Since the quantity to be averaged passes through zero in all cases, we cannot determine stability from this simple estimate but we can see that any instability should distort the inside of the plasma more than the outside.

Saturation due to skin currents induced in the plasma does not appear effective in Princeton Poloidal Divertor Experiment. This saturation has been observed numerically³ in simple cylindrical geometries with widely separated divertor coils and there has been speculation that it may serve to restore equilibrium to a

linearly unstable toroidal divertor configuration. This saturation has not been observed in the geometry considered here.

Since the equilibrium configuration relies on induced currents in the coils for stability, it should slowly go unstable over the time it takes for the induced currents in the coils to resistively decay away. This time, the L/R time of the coils, is typically on the order of 100 msec while the unstabilized instability grows over a time more like 10 μ sec. Thus, passive stabilization should be useful for slowing down an instability so that dynamic stabilization is possible.

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APPENDIX

We consider the general circuit configuration as shown in Fig. 7. A coil is specified by the four indices $(ijkl)$. Every coil has a mutual inductance with every other coil $M_{ijkl;i'j'l'k'}$. Each coil has a two-dimensional position vector $\underline{x}_{ijkl}$ and has n_{ijkl} turns, each with current I_{ijl} , resistance r_{ijkl} and voltage v_{ijkl} . The circuit equations can be written as

$$\sum_{i'j'l'k'} M_{i'j'l'k';ijkl} \frac{d}{dt} I_{i'j'l'} + n_{ijkl} \frac{d}{dt} K_{ijkl} + |n_{ijkl}| r_{ijkl} I_{ijl} = n_{ijkl} v_{ijkl}$$

where

$$M_{i'j'l'k';ijkl} \equiv - n_{ijkl} n_{i'j'l'k'} G(\underline{x}_{ijkl}, \underline{x}_{i'j'l'k'})$$

is the mutual inductance between two coils, and

$$K_{ijkl} = - \frac{1}{(2\pi)^2} \oint \frac{d\ell_s}{x_s} \frac{\partial \chi}{\partial n}(\underline{x}_s) G(\underline{x}_{ijkl}, \underline{x}_s)$$

is the poloidal flux at the coil $ijkl$ due to the plasma current, the line integral being over the plasma vacuum interface. If $i'j'l'k' = ijlk$, we calculate M from an asymptotic formula⁷ for the self inductance of a circular coil of square cross section.

Application of Kirchhoff's laws to the circuit equations yields the system of equations

$$\frac{dI_{ij\ell}}{dt} = \sum_{i''j''\ell''} B_{i''j''\ell'';ij\ell}^{-1} \left[\sum_{i',j',\ell'} A_{i''j''\ell'';i'j'\ell'} \left(\frac{d}{dt} K_{i'j'\ell'} + R_{i'j'\ell'} I_{i'j'\ell'} \right) + \frac{V_{i''}}{Z_{i''} T_{i''j''}} \right]$$

where

$$B_{ij\ell, i''j''\ell''} \equiv \delta_{ii''} \delta_{jj''} \delta_{\ell\ell''} L_{i''j''\ell''} - \sum_{i',j',\ell'} A_{ij\ell; i'j'\ell'}$$

$$(M_{i'j'\ell'; i''j''\ell''} - \delta_{i'i''} \delta_{j'j''} \delta_{\ell'\ell''} L_{i'j'\ell'}) ,$$

$$A_{ij\ell; i'j'\ell'} \equiv -\delta_{ii'} \delta_{jj'} \delta_{\ell\ell'} + \frac{\delta_{ii'} \delta_{jj'}}{T_{ij} L_{i'j'\ell'}} - \frac{\delta_{ii'}}{T_{ij} Z_i T_{i'j'} L_{i'j'\ell'}} ,$$

$$Z_i \equiv \sum_j \frac{1}{T_{ij}} ,$$

$$K_{ij\ell} \equiv \sum_k K_{ij\ell k} n_{ij\ell k} ,$$

$$T_{ij} \equiv \sum_{\ell} \frac{1}{L_{ij\ell}} ,$$

$$M_{ij\ell; i'j'\ell'} \equiv \sum_{kk'} M_{ij\ell k; i'j'\ell' k'} ,$$

$$V_i = \sum_{jk} n_{ij\ell k} v_{ij\ell k} ,$$

$$L_{ij\ell} \equiv \sum_k L_{ij\ell k} ,$$

$$R_{ij\ell} \equiv \sum_k n_{ij\ell k} r_{ij\ell k} .$$

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TABLE 1

<u>Coil Pairs</u>	<u>Number of Turns</u>	<u>Initial Currents (kA)</u>
1,2	+8	140.0
3,4	-8	-140.0
5,6	+8	140.0
7,8	-3	- 52.5
9,10	-8	-140.0
11,12	+8	140.0
13,14	+6	105.0

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Fig. 1. PDX equilibrium showing the locations of the poloidal divertor coils.

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 Fig. 2. Poloidal divertor coils are connected in one of three possible circuit configurations. Circuit 3 keeps the currents fixed at their initial values.

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Fig. 3. The Z-coordinate of the magnetic axis vs time for the three circuit configurations of Fig. 2.

Fig. 4. Magnetic surfaces for Case 2 at time $t = 0$. Dashed surfaces lie in vacuum. 772475

Fig. 5. Magnetic surfaces for Case 2 at time $t = 73$. 772477

Fig. 6. δW vs X_p . δW_3 is due to external field curvature, δW_1 and δW_2 are the total $\delta W = \delta W_3 + \delta W_F$ for circuits 1 and 2.

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Fig. 7. General circuit configuration. A coil is specified by the four indices $(ijlk)$.

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