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Locating a Buried Earth Penetrator

Thurlow W. H. Caffey

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LOCATING A BURIED EARTH PENETRATOR

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ABSTRACT

The purpose of this work was to assist the recovery of a buried earth penetrator by locating the vertical projection of the penetrator upon the surface within a horizontal radius error of one metre. The penetrator will carry a small coil which is driven by an alternating current to form a magnetic dipole. Five measurements of the magnetic field vector upon the surface of the earth are shown to be sufficient for determining not only the xyz-coordinates of the dipole, but also the orientation of the dipole axis. The theory, computation process, and field tests are comprehensively described. Results of 26 field tests with the dipole at 9 different combinations of location and orientation are given. Average radial and vertical location errors are 0.27 m and -0.05 m, respectively, while the mean errors in the tilt and orientation angles of the dipole axis are 3 degrees and 8 degrees, respectively. The results are applied to the design of a locating system for a Pershing II penetrator which contains a recessed, rear-mounted coil.

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FOREWORD

Recovery of earth penetrator packages is necessary to see what structural damage may have occurred not only to the penetrator but to internal functioning parts. The location of the vertical projection of the penetrator upon the surface, or "ground zero," must be known if a vertical shafting recovery technique is to be used. If the depth of burial is also known, then offset directional drilling or mining can be used. Knowledge of the orientation of the penetrator axis will allow the use of remote grappling devices and avoid the safety hazards of a mining operation. Ground zero location also facilitates the placement of surface antennas for through-the-earth telemetry [1] and depth knowledge allows placement of buried antennas near the penetrator to reduce the battery requirement for through-the-earth telemetry. This work was undertaken to provide ground zero location with a radial error of one metre, but the method supplies orientation and depth as well.

The penetrator contains a small coil whose winding axis coincides with the roll axis of the vehicle. The coil is driven by a current whose frequency is chosen so that the EM fields in either the soil or the air are assumed to be those of a magnetic dipole in air. Prior location work, using a buried magnetic dipole or "BMD," was devoted to finding trapped miners or surveying underground quarries from the surface [2-7]. In these cases, the plane of the coil windings was always assumed to be horizontal, and only the point on the surface directly above the dipole could be found. The depth of the dipole could not be ascertained. The surface of the earth in the vicinity of the surface location was either taken to be smooth and level, or to consist of a tilted flat surface such as a smooth hillside. The present work does not make any of these assumptions. The plane of the coil windings is not assumed to be known. Both the buried depth and the surface location are found, and abrupt local changes in surface smoothness should not affect the results because of the choice of frequency.

This report describes, in order, the theory of the method and then the computation process to both of which Louis Romero, Organization 2642, made substantial contributions. The validity of the method is established by the field tests which are next described, and the report concludes with a discussion of how the method can be applied to earth penetrators.

LOCATING A BURIED EARTH PENETRATOR

I. Theory of Location Using Magnetic Fields

The magnetic field of a dipole is set forth, some geometrical relations are developed, and a method of locating a buried dipole by measurements of the fields is presented.

Magnetic Fields from a Small Coil

The components of the magnetic intensity vector due to a small coil of radius "a" centered at the origin of a cylindrical (ρ, ϕ, z)-coordinate system and lying in the $z=0$ plane are given by:

$$H_{\rho} = \frac{N\pi a^2 I}{4\pi} \left(\frac{3\rho z}{R^5} \right) \left\{ 1 - \frac{7a^2 \rho^2}{2R^4} \right\} \text{ A/m} \quad (1a)$$

$$H_z = \frac{N\pi a^2 I}{4\pi} \left(\frac{2z^2 - \rho^2}{R^5} \right) \left\{ 1 - \frac{3a^2(4\rho^2 R^2 - 7\rho^4)}{R^4(2z^2 - \rho^2)} \right\} \text{ A/m} \quad (1b)$$

where

$$R = (\rho^2 + z^2)^{1/2}, \text{ metres}$$

I = current through the coil, amperes

and

N = number of turns of wire whose radius is negligible with respect to "a".

The derivation of the formulas [8] assumes that $a/R \ll 1$; specifically, if $a/R \ll 0.01$, the second term in both braces can be omitted with a maximum error of less than 0.1 percent.

Tilted Coordinates

The observer on the surface locates the origin of Cartesian xyz-coordinates at some point on the surface (Figure 1) and makes measurements at point $P_i(x, y, z)$. Usually $z \neq 0$, but z can be zero or negative if P_i is in a depression relative to the elevation of the origin.

Figure 1. Cartesian Coordinate Frames

Beneath the earth another Cartesian frame, denoted by prime superscripts, provides the coordinate frame of the dipole. The z' -axis is the winding axis of the dipole and is generally tilted through angle α from the z -axis. The dipole axis will not, in general, lie in either the xz or yz planes so that a rotation angle, β , is defined as the angle between xz -plane and the projection of the z' -axis in the xy -plane. Now consider the cylindrical (ρ, ϕ, z) frame located at the origin of the $(xyz)'$ -frame with the z -axis coincident with the z' -axis (Figure 2). The plane of the coil windings is the $x'y'$ -plane and Eqs (1) apply.

Figure 2. Dipole Location via Cylindrical Frame

It is necessary to relate the cylindrical frame to the surface frame, and the unit vector $\hat{a}_z = \hat{a}_{z'}$ can be written immediately:

$$\hat{a}_z = \hat{a}_x \cos \beta \sin \alpha + \hat{a}_y \sin \beta \sin \alpha + \hat{a}_z \cos \alpha \quad (2)$$

The total H-vector generated by the dipole can be measured on the surface as

$$\vec{H} = \hat{a}_x H_x + \hat{a}_y H_y + \hat{a}_z H_z \quad (3)$$

Because $\vec{H}$ does not have a ϕ -component, the unit vector $\hat{a}_\phi$ can be found from

$$\hat{a}_\phi = (\hat{a}_z \times \vec{H}) / |\vec{H}| \quad (4)$$

By using Eq (2) and Eq (3) in the cross-product, Eq (4) becomes

$$\begin{aligned}\hat{a}_\phi = \{ & \hat{a}_x (-H_y \cos \alpha + H_z \sin \beta \sin \alpha) + \hat{a}_y (-H_z \cos \beta \sin \alpha + H_x \cos \alpha) \\ & + \hat{a}_z (-H_x \sin \beta \sin \alpha + H_y \cos \beta \sin \alpha) \} / |\vec{H}| \quad (5)\end{aligned}$$

The set of cylindrical unit vectors is completed by using $\hat{a}_\rho = \hat{a}_\phi \times \hat{a}_z$ to obtain

$$\begin{aligned}\hat{a}_\rho = \{ & \hat{a}_x [H_x (\cos^2 \alpha + \sin^2 \beta \sin^2 \alpha) - H_y \sin \beta \cos \beta \sin^2 \alpha - H_z \cos \beta \sin \alpha \cos \alpha] \\ & + \hat{a}_y [-H_x \sin \beta \cos \beta \sin^2 \alpha + H_y (\cos \beta \sin^2 \alpha + \cos^2 \alpha) - H_z \sin \beta \sin \alpha \cos \alpha] \\ & + \hat{a}_z [-H_x \cos \beta \sin \alpha \cos \alpha - H_y \sin \beta \sin \alpha \cos \alpha + H_z \sin^2 \alpha] \} / |\vec{H}|^{-1} \quad (6)\end{aligned}$$

Locating the Dipole

Suppose measurements are made at "m" surface points denoted by subscripts 1, 2, ..., m. P_j is taken as a reference point, but this does not require that $x_j = y_j = 0$. The horizontal radius vector drawn from P_j to each of the other points P_i is given by

$$\vec{r}_i = \hat{a}_x (x_i - x_j) + \hat{a}_y (y_i - y_j) + \hat{a}_z (z_i - z_j) \quad (7)$$

and $\vec{r}_j \equiv 0$. To get from P_j to the dipole via any surface point requires traversing any of the "m" paths leading to the dipole location given by the vector $\vec{D}_i$:

$$\vec{D}_i(x, y, z) = \vec{r}_i - (\hat{a}_\rho)_i - \hat{a}_z z_i \quad i = 1, 2, \dots, m \quad (8)$$

By equating $\vec{D}_j$ to each of the other location vectors, a set of (m-1) equations can be written where each equation is of the form:

$$(\hat{a}_\rho)_j + \hat{a}_z z_j = -\vec{r}_i + (\hat{a}_\rho)_i + \hat{a}_z z_i \quad \begin{matrix} i = 1, \dots, m \\ i \neq j \end{matrix} \quad (9)$$

When each of these equations is dot-multiplied by $\hat{a}_{\phi_i}$, a typical equation takes the form:

$$(\hat{a}_{\phi_i} \cdot \hat{a}_{\rho_j}) \rho_j = -\hat{a}_{\phi_i} \cdot \vec{r}_i \quad (10)$$

since both $\hat{a}_{\phi_i} \cdot \hat{a}_z = 0$ and $\hat{a}_{\phi_i} \cdot \hat{a}_{\rho_i} = 0$.

Expanding both sides of Eq (10) gives this set of relationships:

$$K\{A_i + B_i \cos \beta \tan \alpha + C_i \sin \beta \tan \alpha\} = D_i + E_i \cos \beta \tan \alpha + F_i \sin \beta \tan \alpha \quad (11)$$

where

$$A_i = (x_j - x_i) H_{y_i} - (y_j - y_i) H_{x_i} \quad (12a)$$

$$B_i = (y_j - y_i) H_{z_i} - (z_j - z_i) H_{y_i} \quad (12b)$$

$$C_i = (z_j - z_i) H_{x_i} - (x_j - x_i) H_{z_i} \quad (12c)$$

$$D_i = H_{x_i} H_{y_j} - H_{x_j} H_{y_i} \quad (12d)$$

$$E_i = H_{y_i} H_{z_j} - H_{y_j} H_{z_i} \quad (12e)$$

$$F_i = H_{z_i} H_{x_j} - H_{z_j} H_{x_i} \quad (12f)$$

and

$$K = |\vec{H}_j|/\rho_j \quad i = 1 \dots m; i \neq j \quad (12g)$$

The method of solution, to be presented later, provides in order, β , α , and ρ_j which are needed to determine the dipole location.

All of the quantities in Eq (8) are now known except for $\vec{H}_j$ which can be found directly from the magnetic fields, Eq (1). The magnetic fields in the cylindrical coordinates of the dipole can be related to the values measured on the surface by

$$\vec{H}_{\theta_i} = a_{\theta} \cdot \vec{H}_j \quad (13a)$$

and

$$\vec{H}_{\rho_j} = \vec{H}_j - \vec{H}_{\theta_j} \quad (13b)$$

These values also provide an alternative to Eq (6) because

$$\hat{a}_{\rho_j} = \hat{H}_{\rho_j} / |\hat{H}_{\rho_j}| \quad (13c)$$

Indicial subscripts will now be omitted and γ defined by

$$\gamma = |\hat{H}_x| / |\hat{H}_\rho| \quad (14)$$

Use is made of Eq (1) and the definition of γ to write this polynomial in x :

$$\begin{aligned} x^6 + (-3\gamma\rho)x^5 + \left(\frac{7\rho^2}{2}\right)x^4 + (-6\gamma\rho^3)x^3 + (\rho^4 - 6a^2\rho^2)x^2 \\ + \left(\frac{21a^2\gamma\rho^3 - 6\gamma\rho^5}{2}\right)x + \left(\frac{9a^2\rho^4 - \rho^6}{2}\right) = 0 \end{aligned} \quad (15)$$

If the last term in braces in Eq (1) is neglected, the x -polynomial becomes a quadratic with two solutions for x :

$$x = \frac{3\gamma\rho}{4} \{1 \pm \sqrt{1 + 8/9\gamma^2}\} \quad (16)$$

The use in Eq (8) of the results for α , β , ρ , and x together with the expressions for the cylindrical unit vectors leads to $(m-1)$ solutions for the dipole location for each value of x .

Solutions for the Angles

It was desired to have a solution method which would avoid extracting angle values as roots of inverse trigonometric quantities because it was feared that such methods would be too sensitive to the three-digit precision of field data. The method used is Gauss elimination [9], except that intermediate divisions are shown to be unnecessary. Several of the coefficients $(A \dots F)_i$ can be zero without causing division by zero to occur. The minimum value of m , which avoids the extraction of roots, is $m=5$.

It will be necessary later on to have the four sets of coefficients $(A \dots F)_i$ normalized by the largest coefficient magnitude in any of the sets. The symbols $(A \dots F)_i$ hereafter refer to normalized quantities and Eq (11) is written as shown below.

$$K\{A_1 + B_1\delta + C_1\epsilon\} = D_1 + E_1\delta + F_1\epsilon \quad (17a)$$

$$K\{A_4 + B_4\delta + C_4\epsilon\} = D_4 + E_4\delta + F_4\epsilon \quad (17d)$$

where

$$\delta = \cos\beta \tan\alpha$$

and

$$\epsilon = \sin\beta \tan\alpha$$

Dividing through each equation by its own D_i produces another set of four equations:

$$\frac{K}{D_1} \{A_1 + B_1\delta + C_1\epsilon\} = 1 + (E_1\delta + F_1\epsilon)/D_1 \quad (18a)$$

$$\frac{K}{D_4} \{A_4 + B_4\delta + C_4\epsilon\} = 1 + (E_4\delta + F_4\epsilon)/D_4 \quad (18d)$$

Subtracting the (i+1) equation from the i-th equation and making further use of the first equation by subtracting it from the fourth gives this second set of four equations:

$$K\{P_i + Q_i\delta + R_i\epsilon\} = S_i\delta + T_i\epsilon \quad i = 1, \dots, 4 \quad (19)$$

where

$$P_i = \frac{A_i D_j - A_j D_i}{D_i D_j} \quad (20a)$$

$$Q_i = \frac{B_i D_j - B_j D_i}{D_i D_j} \quad (20b)$$

$$R_i = \frac{C_i D_j - C_j D_i}{D_i D_j} \quad (20c)$$

$$S_i = \frac{E_i D_j - E_j D_i}{D_i D_j} \quad (20e)$$

and

$$T_i = \frac{F_i D_j - F_j D_i}{D_i D_j} \quad (20f)$$

where $i = 1, 2, 3$ and $j = i + 1$, except for the fourth difference where terms have the general form

$$P_4 = \frac{-(A_1 D_4 - A_4 D_1)}{D_1 D_4} \quad (20g)$$

If a conventional Gaussian reduction had been used to eliminate D_1 to provide the four relations of Eqs (19), it would have been necessary to have a set of five relations in Eqs (17). The subtraction of the first from the fourth of Eqs (18) reduces the required value of m by one.

Each of Eqs (19) is divided by its own S_i which eliminates the denominator term $D_i D_j$ so that the indicated division by $D_i D_j$ in the coefficients $(P, \dots T)_i$ can be omitted. This eliminates the possibility of dividing by zero. The first equation is not subtracted from the fourth in order to reduce the four equations to three equations of this form:

$$K(A_i + B_i \delta + C_i \epsilon) = D_i \epsilon \quad i = 1, 2, 3 \quad (21)$$

where the coefficients are now defined as:

$$A_i = \frac{P_i S_j - P_j S_i}{S_i S_j}, \quad B_i = \frac{Q_i S_j - Q_j S_i}{S_i S_j} \quad (22a, b)$$

$$C_i = \frac{R_i S_j - R_j S_i}{S_i S_j}, \quad D_i = \frac{T_i S_j - T_j S_i}{S_i S_j} \quad (22c, d)$$

Each of Eqs (21) is divided by its own D_i and subtracted as before to get two equations,

$$K\{P_i + Q_i \delta + R_i \epsilon\} = 0 \quad i = 1, 2 \quad (23)$$

where

$$P_i = \frac{A_i D_j - A_j D_i}{D_i D_j}, \quad Q_i = \frac{B_i D_j - B_j D_i}{D_i D_j} \quad (24a, b)$$

$$R_i = \frac{C_i D_j - C_j D_i}{D_i D_j} \quad (24c)$$

Hence, either $K \equiv 0$ or

$$Q_1 \delta + R_1 \epsilon = -P_1 \quad (25a)$$

$$Q_2 \delta + R_2 \epsilon = -P_2 \quad (25b)$$

The solution for the angle $\beta = \text{Arctan}(\epsilon/\delta)$ is given by

$$\beta = \text{Arctan} \left(\frac{R_1 P_2 - R_2 P_1}{Q_2 P_1 - Q_1 P_2} \right) \quad (26)$$

The use of β in Eq (25) gives a value for α and the use of these angles in Eq (10) provides ρ . The selected use of implicit multiple solutions will be discussed in the next section.

A Mathematical Quality Test

Equations (17) can be put into a form with which the quality of numerical results can be tested. Each of Eqs (17) is divided by its right-hand side and each such result is set equal to the next result in turn. This eliminates K , and multiplication followed by collection of terms gives three equations of this form:

$$(GA + GB + GC + GE + GF)_i = 0, \quad i = 1, 2, 3 \quad (27)$$

where

$$GA_i = \{A_i D_j - A_j D_i\} \cos^2 \alpha \quad (28a)$$

$$GB_i = \{C_i F_j - C_j F_i\} \sin^2 \beta \sin^2 \alpha \quad (28b)$$

$$GC_i = \{B_i F_j + C_i E_j - B_j F_i - C_j E_i\} \sin \beta \cos \beta \sin^2 \alpha \quad (28c)$$

$$GD_i = \{A_i F_j + C_i D_j - A_j F_i - C_j D_i\} \sin \beta \sin \alpha \cos \alpha \quad (28d)$$

$$GE_i = \{B_i E_j - B_j E_i\} \cos^2 \beta \sin^2 \alpha \quad (28e)$$

$$GF_i = \{A_i E_j + B_i D_j - A_j E_i - B_j D_i\} \cos \beta \sin \alpha \cos \alpha, \quad j = i + 1 \quad (28f)$$

When unlimited accuracy applies,

$$\sum_{i=1}^3 (GA + GB + GC + GD + GE + GF)_i = 0 \quad (29)$$

so that the amount by which the sum is nonzero is an indication of the quality of the results for (β, α) .

The next section explains in some detail how the computations, based on this theory, are done.

II. Computation Process

The location theory provides the dipole coordinates in a direct computation process when the input data have five-digit precision with an uncertainty of only one least-significant digit. Actual input data have a precision of only three digits, and their accuracy is further degraded by noise. It is necessary both to take advantage of the redundancy implicit in the theory and to add a search algorithm. Redundancy is used in both the analytic and search modes, but it is only used where experience has shown it to be necessary. This section will outline the computer program in a general way to point out where unused computational paths are available and to introduce terms for understanding the field experiments.

Tracing the Use of Surface Points

In the previous section, an analytic solution based on the use of five surface points was outlined in which one point was arbitrarily selected as a reference. If only five points are used in a field test, there is no margin for error in measuring the three components of the field at each point. Rather than require virtual perfection, it was decided to use six surface points and make six sets of five-point computations (Figure 3). A "trace" number is generated to identify each set of five points and to identify which of those five points is chosen as a reference. The trace is also used to limit the total number of cases which might be otherwise considered in the search mode.

Computing Angle β

Once the reference point, P_j , is selected the four sets of coefficients, $(A, \dots, F)_i$, are computed in normalized form. This brings the process to Eqs (17) where there are three choices of right hand coefficients by which to divide, and there is no reason for a preferred choice. If the differences are not distinguished by algebraic sign, there are 15 sets of four differences that can be formed from Eqs (18). Rather than use all 15 sets, the computation uses one of those three sets in which each index subscript appears the same number of times to provide equal weighting. The set of coefficients, $(P, \dots, T)_i$, are now found from Eqs (20), except that the denominator is set to unity because subsequent division would remove it. There are 20 ways to form a set of three equations from Eqs (19), and all 20 sets are used. The possibility of dividing by T_i in addition to S_i is foregone, and the coefficients, $(A, \dots, D)_i$, are recomputed from Eqs (22) with the denominator set to unity. All three ways of forming Eqs (23) are used, and Eqs (26) return β values in only the first and fourth quadrants. Because the tangent function is symmetric about zero with a period of Π , fourth quadrant values have Π added to them so that all retained values of β lie in the range $0 < \beta < \Pi$. It is possible, with field data, that all of the angles β , will differ appreciably from each other. Rather than submit up to 180 values to subsequent processing, only those values are retained which agree with another value to within an absolute difference of one degree. This may still leave 180 values if the field data are of perfect quality, so β values which are redundant to within an absolute difference of 0.01 degrees are retained only as a single mean-value. Of course, 180 values can still remain if they are all separated by an amount greater than 0.01 degrees and less than 1 degree, but such a case has not been observed.

Figure 3. Computation Flow Diagram

Computing Angle α

The process of eliminating nonpaired and redundant values of β also eliminates the possibility of direct substitution into Eq (25). For each β -value, Eq (11) becomes

$$K\{A_1 + S_1 \tan \alpha\} = D_1 + T_1 \tan \alpha \quad (30a)$$

$$K\{A_4 + S_4 \tan \alpha\} = D_4 + T_4 \tan \alpha \quad (30d)$$

where

$$S_i = B_i \cos \beta + C_i \sin \beta \quad (30e)$$

and

$$T_i = E_i \cos \beta + F_i \sin \beta \quad (30f)$$

in terms of Eq (12).

There are 12 ways to form equation pairs in Eqs (30) and solve for $\tan \alpha$ by eliminating K , but a solution to a quadratic in $\tan \alpha$ is not desired. Instead, the β -solution process will be employed again by dividing each of Eq (27) by its own D_i and subtracting to provide six differences. The solutions which begin by dividing by T_i are omitted. Fifteen sets of two differences can be formed, and each member of a typical pair of differences is of the form:

$$K\{P_i + Q_i \tan \alpha\} = R_i \tan \alpha \quad (31a)$$

where

$$P_i = \frac{A_i D_j - A_j D_i}{D_i D_j}, \quad Q_i = \frac{S_i D_j - S_j D_i}{D_i D_j}, \quad (31b, c)$$

and

$$R_i = \frac{T_i D_j - T_j D_i}{D_i D_j} \quad (31d)$$

Each member equation is divided by its own R_i and subtraction gives

$$P_1 R_2 - P_2 R_1 + (Q_1 R_2 - Q_2 R_1) \tan \alpha = 0 \quad (32a)$$

so that

$$\alpha = \text{Arctan} \left(\frac{P_2 R_1 - P_1 R_2}{Q_1 R_2 - Q_2 R_1} \right) \quad 0 \leq \alpha \leq \pi \quad (32b)$$

Again, all values of α which do not form a pair to within one degree are eliminated and those which are redundant to within 0.01 degree are retained as a single mean-value. Each (β, α) -pair is subjected to the sum-to-zero test, Eq (29), and pairs whose sum exceeds 0.001 are dropped.

Computing Spatial Coordinates

It is now possible to compute $\hat{a}_{\phi_i}$ from Eq (5) and $\hat{a}_{\rho_j}$ from Eq (6) for each pair of angles (β, α) . The results are used to solve Eq (10) in which both subscripts are allowed full range:

$$\rho_j = \frac{-\hat{a}_{\phi_i} \cdot \hat{r}_i}{\hat{a}_{\phi_i} \cdot \hat{a}_{\rho_j}} \quad \text{metres} \quad \begin{matrix} i = 1, \dots, 6 \\ j = 1, \dots, 6 \\ i \neq j \end{matrix} \quad (33)$$

There are 15 values of ρ_j computed for each (β, α) -pair and surface reference position P_j . It is assumed that the condition $a < R$ applies, so that two values of ρ_j are computed from Eq (16) for each value of ρ_j .

The dipole coordinates $(x, y, z)_d$ with respect to the surface origin of coordinates are obtained from Eqs (8):

$$\begin{aligned} x_d &= \hat{a}_x \cdot \hat{D}_i - x_j \\ y_d &= \hat{a}_y \cdot \hat{D}_i - y_j \\ z_d &= \hat{a}_z \cdot \hat{D}_i - z_j \end{aligned} \quad i = 1, \dots, 5; i \neq j \quad (34)$$

Some practical conditions which can be used to eliminate some dipole locations are:

1. The dipole must be underground, that is, z_d must be negative.
2. The dipole depth must exceed an estimated minimum value. This minimum can be a negative infinitesimal if desired.
3. The dipole depth must not exceed a maximum value which is estimated from knowledge of the soil structure, the impact angle, and the impact velocity. This maximum can simply be set to a large negative number if knowledge of the soil structure and terminal conditions is poor.
4. The radial distance to ground zero must not exceed some maximum value.

Usually, several candidate locations still remain and a way to select the most valid location is needed.

Validity Testing and Ranking

The set of measured H-components is the means by which the validity of a dipole location is established. The measured H-values are normalized by the largest value and stored. Equation (1) is used to compute, for each candidate dipole location, the Cartesian H-components at each surface point. The set of such values is normalized by the maximum of the set and is compared with the set of normalized measured values, using the rms difference over all 15 H-values as a criterion. Those locations whose "V-test" exceeds 20% are eliminated because experience has shown that they will not be useful solutions. The remaining locations are ranked in ascending order of V-test values, and the location with the smallest value is taken as the dipole location. All locations whose V-tests are within one percent of the minimum V-test are retained as candidates for the search schemes.

Searching for the Improved Solution

A straightforward scheme, using trial values of (β, α) near the analytic (β, α) -pairs, was chosen for simplicity. Trial values of β are assumed over the range of $\pm 10\%$ of the analytic- β in increments of 0.5%. Trial values of α are likewise assumed over a range of $\pm 10\%$ of the analytic- α in increments of 0.5% for each trial value of β . Because the analytic solution for both β and α is based on the tangent function, both the search ranges and increments are divided by the tangent of the analytic β and α , respectively, when the tangents exceed unity. This reduces the search range, but still leaves 1600 (β, α) -trial pairs for each analytic solution. It is necessary both to reduce the number of solutions for which a search will be made and to reduce the number of search (β, α) -pairs.

Study of the analytic solutions shows that (β, α) -pairs are clustered by trace number, and the number of unique search possibilities is the same as the number of unique traces. Solutions with both the same trace number and (β, α) -pair, but slightly differing values of $(x, y, z)_d$, are reduced to one (β, α) -pair for search purposes. In this way, the maximum number of analytic (β, α) -pairs upon which a search will be made is limited to 36 which would occur only if all the data were perfect. At the other extreme, if there is only one trace number, the data are minimally acceptable. Therefore, the number of traces can serve as a basis for reducing the number of search (β, α) -pairs. The number of search (β, α) -pairs considered for each analytic (β, α) is reduced by increasing the search increment according to the number of traces while keeping the search range fixed. If there are more than 6 unique traces, the angle increments become 1%; and if there are more than 12 traces, the increments are increased to 2%. The break point of 6 trace numbers corresponds to the mean of the trace number distribution computed for the field tests. The break point of 12 trace numbers corresponds to the sum of the mean and one standard deviation. Hence, the relationship between the number of search (β, α) -pairs and the number of traces is shown below:

<u>Number of Unique Traces</u>	<u>Number of (β, α) Search Pairs/Trace</u>
1 - 6	1600
7 - 12	400
13 - 36	100

It can be shown that both the zero-sum check and the $(x, y, z)_d$ are the same whether (β, α) or the complimentary pair $(\pi + \beta, \pi - \alpha)$ is used. A search is made for each complimentary analytic (β, α) -pair, because the increments differ from those of the original (β, α) -pair and the search involves the testing of a different set of locations. For each (β, α) pair, whether original or complimentary, the zero-elimination is used as in the analytic mode. The dipole locations are computed, tested against physical constraints, and ranked in ascending order of V-test values as before. The process is halted in progress if any V-test is less than 1.1% so that, in the event of very good data, the search is not prolonged. The final location values are the means of all those locations whose V-test values are within 0.1% of the minimum V-test value.

The computer program requires 57000₈ core locations in a CDC-6600 computer. The program, with its selected multiple computation paths in the analytic mode and particular choice of search limits and increments, was developed partly as a result of the field tests which are described in the next section. The tests involve a variety of locations, tilt angles, and apparent measurement accuracies so that the computation process is regarded as well-tested.

III. Field Tests

The conduct of the field tests is described in this section, and the difference between computed and measured radial offsets, depths, and orientation angles is discussed. The tests were a sequence of development efforts rather than confirmations of developed field technique and computation method. All of the results have been uniformly processed by the most recent computer program using appropriate minimum and maximum depth limits. Progressive developments in equipment, experimental technique, and operating frequency were made during the tests, and the effect of changes are discussed.

Choice of Frequency

The operating frequency had to be chosen so that the fields from the buried magnetic dipole (BMD) are given by Eq (1) to within some small error. The fields from a tilted BMD can be resolved into a combination of the fields from two horizontal dipoles, respectively aligned with the x-axis and y-axis, together with a dipole along the z-axis. Each surface field component is made up of contributions from these three dipoles, but for the sake of simplicity the H_z -field from only the vertical dipole was used to choose the operating frequency.

The H_z -field at the surface of the earth from a vertical BMD in a homogeneous, conducting soil is given by [2]:

$$H_z = \frac{N\pi a^2 I}{2\pi} \int_0^\infty \frac{\lambda^3 e^{-uZ}}{\lambda + u} J_0(\lambda r) d\lambda \quad (35)$$

where

r = the radial offset of the surface point, metres

λ = the separation constant of Bessels equation of order zero which results from solving the wave equation with azimuthal symmetry, metres⁻¹

$u = (\lambda^2 + \gamma^2)^{1/2}$, $\gamma^2 = j\mu w(\sigma - j\epsilon_r \epsilon_0)$, metres⁻¹

σ = soil conductivity, siemens/metre

ϵ_r = relative dielectric constant of the soil

ϵ_0 = permittivity of free space, farads/metre

μ = soil permeability, henries/metre

$w = 2\pi$ times the operating frequency, f ; radians/second

J_0 = Bessel's function of zero order.

The other quantities have been previously defined. The error in using Eq (1a) in place of Eq (35) can now be examined for choices of r , z , σ , ϵ , μ , and frequency. Assuming that the soil has the permeability of air, as is often the case, and has worst-case values of $\sigma = 0.2$ S/m and $\epsilon_r = 1000$, Figure 4 shows the error in H_z as a function of depth with both (r/z) and frequency as parameters. The initial operating frequency of 40 Hz was chosen because it preserved a useful ratio of $(r/z) = 0.5$ at an error limit of one percent and depth of 50 metres.

The Buried Magnetic Dipole

The BMD package (Figure 5) consists of an aluminum cylinder 127 mm OD by 630 mm long which supports a small, tiltable coil on the rear. The cylinder contains batteries, an oscillator, and a power amplifier to drive the coil. The package was lowered down a vertical shaft located at Edgewood, New Mexico. Rotation of the package in a horizontal plane was prevented by foam rings which were glued to the cylinder and pressed against the shaft wall. However, rotation of the package during descent still occurred. Two lights mounted on the rear were used to estimate angle β once the package was at depth. The shaft was cased with 150-mm PVC pipe, plugged to a finished depth of 52.3 metres, and was kept dry. The local soil conductivity is modeled as three horizontal layers having conductivities of 24, 165, and 46 mS/m with interfaces at depths of 9.1 and 49.4 metres [1].

Figure 4. Error in Hz vs Depth with Frequency and Normalized Offset as Parameters

Figure 5. Buried Dipole Package

Surface Apparatus and Layout

Two large coils were mounted on dielectric tripods equipped with leveling screws (Figure 6). Each leveled platform includes a compass so that the winding axis of the coil can be set to a particular magnetic azimuth when the coil axis is tilted from the vertical. The coil diameter is 0.61 metres, and the total turns-area product is 100 square metres. Each coil is switch-selected into one common amplifier and filter located about 50 metres away where the induced voltage is read on a DVM and noted (Figure 7). The use of only one measurement system assures a higher accuracy, since voltages must be compared, than would the use of two systems.

Figure 6. Tripod-Mounted Surface Coil

Figure 7. Surface Layout and Block Diagram

One coil is kept at P_1 for use as a phase reference while the other coil is moved successively to the other points. A total of six points is used rather than the minimum of five to provide redundancy, and six groups of five points each are analyzed for each test. The geometric figure outlined by connecting the successive positions of the moving coil is a crude pentagon (Figure 7) whose radius is chosen to be about half the estimated maximum depth. A crude pentagon is used so that sample points will be widely spaced around the reference point. The BMD can be inside or outside of the layout boundary.

The voltage induced in a coil when its winding axis is vertical is V_z and is proportional to H_z . The location of P_1 requires first, that V_{z1} have a signal-to-noise ratio of such quality that the DVM changes at most by a single unit in the third digit. The SNR of V_{z1} must therefore be at least 40 dB, and if it is not, a new location for P_1 is tried. When the SNR for V_{z1} is satisfied, the coil is tilted to a horizontal position and an attempt is made to locate two magnetic bearings, 90 degrees apart, for which the induced voltages vary no more than one unit from the mean. When these horizontal

criteria are met, the voltages become V_{x1} and V_{y1} , respectively. The moving coil is placed some distance out on the x-axis, as explained earlier, and P_2 is thus established. Successive points are stepped off around the perimeter using, if desired, the compass at P_1 to estimate the angular separation. If at any point there should be a null, the point is adjusted in radius until at least 30-dB readings are obtained. During the 45 minutes or so required for the voltage measurements, the transmitter current may decrease. This decrease is observed by measuring V_{z1} before and after the three measurements at each of the other points. These observations also establish whether each voltage is in phase or out of phase with respect to V_{z1} . Measured values are corrected within the program for decreases in V_{z1} before other computations take place.

While the voltages are being measured, a surveyor's transit is positioned to the side and used to measure the elevation of the coil center at each perimeter point with respect to the elevation of the center of the reference coil. The metre stick attached to the side of the coil frames is used for this purpose. The elevation of the center of the reference coil is also measured with respect to the earth's surface. Each point is marked with a stake and after the voltage measurements are completed the layout is surveyed, taking P_1 as the origin.

The entire measurement process, including both voltage measurement and survey, can be completed in one hour when using only two coils and three men. Most of the time is spent in moving the perimeter coil, and the use of an additional coil and man could reduce the time to one-half hour.

Field Test Results

Measurements were made after the voltage data were complete to determine the actual location and orientation of the BMD. These are shown in Table I together with transmitter data. The uncertainty is ± 2 cm in each of (x, y, z) , $\pm 8^\circ$ in β , and $\pm 0.5^\circ$ in α ; there is no β -angle for Test 23 because of the zero tilt. The large uncertainty in β represents one-half the spread of observations made from the top of the pipe. This is mostly due to parallax which could not be easily reduced due to the interference of the suspension rope. The error in each location parameter for each test is shown in Figure 8. The radial error is well within the one metre objective, but in four of the last six tests the depth error increased noticeably with respect to the previous tests. The rotation angle errors are all within the uncertainty to which β could be estimated, and the tilt angle errors are all on the positive side with but one exception. The twenty-six tests comprise five test groups, and a discussion of each group will give a better understanding of the test results.

Figure 8. Location Errors

TABLE I
BMD Location, Orientation, and Electrical Data

Test No.	Date 1976	Location, metres			Angles, Degrees		Transmitter Data		
		x	y	z	β	α	Amps, rms	Amp-m ²	Watts
1-7	24 Aug	3.74	-4.09	-10.0	132	45	0.96	0.95	10.1
8-13	14 Sep	7.14	-3.26	-10.0	150	40	1.04	1.04	11.9
14-20	15 Sep	7.14	-3.26	-10.0	170	45	1.04	1.04	11.9
21	7 Oct	10.17	-5.87	-15.0	135	40	0.11	0.38	0.11
22	7 Oct	-4.02	-11.03	-8.54	50	24	0.21	0.72	0.41
23	13 Dec	8.50	2.32	-15.0	NA	0	0.13	0.45	0.15
24	14 Dec	6.58	7.74	-18.8	10	20			
25	14 Dec	11.40	0.10	-20.0	158	65			
26	13 Dec	5.72	6.89	-18.0	40	90	0.13	0.45	0.15

The first three test groups were based on a six-point search algorithm in existence at the time. A seven-point layout and hexagon perimeter were used for each of these groups. The subsequent development of a five-point search algorithm permits the data to be analyzed as seven tests of six points each. Tests 1 through 7 was the first test group and both the BMD location and surface points were preplanned (Figure 9). When the package rotated upon descent, an unplanned, near-null in V_z occurred at Point 4. The number of traces was minimal for this test group except for Test 4 where Point 4 was omitted and four unique traces were computed. The location errors in Test 4 were comparable to those of the other tests in this group which shows that the computations are insensitive to a null in one component. There are two reasons why this is so. One reason is the use of redundancy so that combinations of points which do not include the null are considered. Another reason is that angle β usually cannot be found for that combination of points which includes the null because the error in the null measurement is apt to be large.

The second group consists of Tests 8 through 13 and was a second, seven-point preplanned test. Winds up to 2.7 m/s made this test take 90 minutes, the most time needed for any test. The wind reduces the signal-to-noise ratio in two ways. The coil orientations are disturbed which makes the signal amplitude vary, and the coils are moved within the earth's magnetic field which is greater than the maximum field component established by the BMD by a factor of 10^{11} . A very slight disturbance of the coil in the earth's field is sufficient to generate large amounts of flux noise in the receiver. The data appeared satisfactory in the field, but the combination of points 2 through 7 did not return an answer to the analytic routine. The following day a special effort was made to arrive early at the test site. The winds were dead calm, and the location of one point was adjusted to increase voltage levels. The third group, Tests 14 through 20, provided the smallest radial offset errors and V-test values of any test group. The receiver bandwidth used in all tests so far was 1 Hz with a noise figure of 6.5 dB.

Test Number	Surface Points Used in Test	
1	All	Except P1
2		P2
3		P3
4		P4
5		P5
6		P6
7	All	Except P7

Figure 9. Layout for Tests 1 through 7

The fourth group is distinguished by many changes. Only a six-point layout was beginning with these tests, the operating frequency was increased to 390 Hz, and the bandwidth was increased by about the same factor to 11 Hz. This change in frequency and bandwidth should have increased signal-to-thermal noise ratio by $(390/40)^2/11$ or 8.6, and the current, dipole moment, and power in the buried transmitter was accordingly reduced (Table I). The noise figure of the receiver remained at 6.5 dB. One of the surface coils was cast in epoxy to add mass to reduce wind noise, while the second coil was left unchanged. A side-by-side comparison in a 2.6 m/s wind showed that the noise from the potted coil was 6-dB less than that from the unpotted coil. The first of these two tests was planned only in depth and tilt angle; the second test conditions were not chosen beforehand in any way. The location errors are satisfactory and appear to justify the many changes.

All tests so far had been conducted at tilt angles of 50 degrees or less at depths of no more than 15 metres. The fifth group consists of four tests of at least 15 metres depth, with tilt angles of 0, 20, 65, and 90 degrees. The acceptable location errors confirm that the location method can handle the full range of tilt angles.

Computation Times

Analytic and search times for each test are given in Figure 10. The time for analytic computation increases beginning at Test 14 because of improvement in data quality by either the avoidance of nulls, calm wind conditions, or mass loading of the coils. There is a wide variation in search times; but again, longer search times were required beginning at Test 14 as the data improved. The increase in search times for Test 15 by comparison to Test 14 is of special interest because they are both in the third test group and both gave a radial error of 19 cm. The search in Tests 14, 16, and 17 was terminated when a V-test value of 1.1% was found. In both Tests 15 and 18, the minimum V-test value was 1.2% which was just above the search-stop value of 1.1%. The search scheme had to process 767711 and 238434 possible locations, respectively, with a large increase in time.

Figure 10. Computation Times .

The field tests have shown that it is possible to locate a BMD within acceptable error limits from the surface of the earth. The next section establishes the requirements for finding a deeply buried earth penetrator.

IV. Application to Earth Penetrators

A system design which applies the previous work to the location of a Pershing II penetrator with a recessed antenna is described. The design is based on a maximum depth of 50 metres in a soil with a conductivity of 0.2 s/m and a relative dielectric constant of 1000 or less. It is assumed that the reference surface coil can be placed, by preliminary measurements of the field, to within a radial offset of 25 metres from ground zero. If it is further assumed that the recessed coil in the penetrator provides a dipole field structure, then Figure 4 applies at an operating frequency of 40 Hz and the following design can be made.

Surface Equipment

A signal-to-noise ratio of 40 db is required for V_{z1} ; the thermal noise of the 40-Hz system, terminated with a resistance equal to that of the surface coil and including 50 metres of cable, has been measured as 2.1 nV. If a filter with a 0.01-Hz bandwidth is used, the required value of V_{z1} becomes 21 nV. There are three ways to realize the $Q = 4000$ filter [10, 11, 12], and the center frequencies of each filter type can be adjusted to allow for both transmitter offset and drift. The very narrow bandwidth used to reduce thermal noise also increases the time required for each measurement. Typically, a settling time of 4 inverse filter bandwidths or 400 seconds will be required for each measurement and at least 18 measurements must be made. At least 2 hours would be required to gather the data if measurements are made only in succession, but the use of a larger field crew and multiple surface coils would permit the measurement of several points simultaneously. It will also be necessary to add mass to the surface coils to keep wind-induced noise below the thermal noise level. It is planned to base the mass increase on a wind speed of 5 m/s.

Penetrator Transmitter

The recessed transmitter coil presently used in the Pershing II penetrator has a total turns area product of 1.28 metres². Using this area in Eq (1b) and the relationship

$$V_{z1} = 21\text{nV} = \frac{\mu\omega(\text{Surface Coil Area})(\text{Xmtr Coil Area})I}{4\pi R^5} (2\gamma^2 - \rho^2) \quad (36)$$

the required current is found to be 0.82 amperes when $2\rho = \gamma = 50$ metres. The inductive reactance of the coil at 40 Hz is less than the dc coil resistance of 12 ohms, so the present Class-D driver cannot be used. However, another type of driver using the present supply of +28 V can provide the required current. The power dissipated in the coil is 8.1 watts and the power dissipated in the driver will be about one-half the coil power; therefore, the total current drawn will be at least 0.44 amperes.

Downlink Control

The large current requirement and the uncertainty in the period of time between penetrator impact and the arrival of the field party each make it necessary to control the penetrator transmitter from the surface. Wireless control of a buried transmitter to depths of 50 metres has been demonstrated, [1] and some of the equipment is still on hand. The operating mode will be to turn on and turn off the penetrator transmitter as desired, rather than turning on the transmitter and allowing it to operate under control of a timer. The optimum frequency for the downlink is 4100 Hz [13] which is close enough to the present downlink frequency of 3990 Hz that present equipment can be adapted easily.

Conclusion

It appears that the method used to find a BMD can be used to find a buried penetrator if it can be verified that the recessed coil provides a dipole field structure. The two major problems of the narrow bandpass filter on the surface and the power requirement in the penetrator will require development.

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