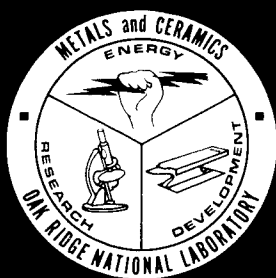


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The Thermal and Electrical Conductivity of Aluminum

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THE THERMAL AND ELECTRICAL CONDUCTIVITY OF ALUMINUM

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ABSTRACT

The thermal conductivity, electrical resistivity, and absolute Seebeck coefficient of pure aluminum were determined from 80 to 400 K by measuring all three properties of samples with resistivity ratios of 11×10^3 , 8.5×10^3 , and 9.5×10^2 using three different techniques. Measurements were made on the purest sample down to 20 K. The thermal conductivity has a broad plateau from 180 to 400 K and a possible minimum of 0.25% which is insignificant compared to the experimental errors. The same properties were measured on an aluminum alloy with a resistivity ratio of 17. Measured values of the thermal conductivity of this alloy agreed to within $\pm 1\%$ with calculated values using parameters obtained from the pure aluminum.

INTRODUCTION

In their reviews of published data on the thermal conductivity of pure aluminum, Powell et al.¹ and Powell² have shown that the room temperature conductivity is known to approximately 2%, but that the uncertainty near 200 K is appreciably larger. Some of the measurements indicate an unusually deep minimum near 200 K, while others show no minimum at all. This has led Powell² to note that there is no doubt as to the undulatory nature of the thermal conductivity, but that the data indicate the temperature at which this minimum occurs increases with sample purity.

Since the transport properties of nominally pure samples near 200 K should be determined predominantly by the strength of the electron-phonon interaction, the conjecture of Powell leads one to conclude that the thermal conductivity of Al is for some reason unusually sensitive to

impurities. This we find rather doubtful, and, suspecting that the scatter near 200 K is simply due to experimental errors, we have carried out a new investigation of the thermal conductivity, λ , and electrical resistivity, ρ , in the intermediate temperature range. We report below the results of these measurements, which yielded data from about 80 to 400 K, on four samples of different purities.

APPARATUS AND SAMPLES

We have measured the thermal conductivities, electrical resistivities, and Seebeck coefficients of four specimens which are listed in Table 1 with their residual resistivity ratios [$RRR = \rho (273.15 \text{ K}) / \rho (4.2 \text{ K})$] and electrical resistivities at 4.2 K. The three apparatus are also listed in Table 1 and their uncertainties given where NRC refers to the National Research Council of Canada and ORNL refers to the Oak Ridge National Laboratory. Apparatus NRC-1 was described by Cook et al.³ Measurements with this technique on material A were made using calibrated Cu-constantan thermocouples whereas measurements on material B used Chromel-Au (+0.07% Fe) thermocouples which were calibrated in situ against a platinum resistance thermometer. Apparatus NRC-2 and ORNL-3 were described previously by Matsumura and Laubitz⁴ and Moore et al.,⁵ respectively. There are no true thermal conductivity standards for testing the apparatus, but the uncertainty values given in Table 1 appear to be justified based on technique comparison tests reported by Laubitz and McElroy.⁶ The A1 results will be designated according to the scheme shown on the table: i.e., results on specimen A using technique No. 1 are termed A-1, etc. All ρ and λ values were corrected for thermal expansion⁷ prior to any data analysis.

RESULTS

Thermal Conductivity

Thermal conductivity results from A-1, A-2, A-3, B-1, and C-3 are shown in Fig. 1. All results have been corrected for impurity scattering

Table 1. Apparatus and materials employed in this study on aluminum

Designation of material	Apparatus and → % Uncertainty (λ ; ρ)		NRC-1 ^f (± 0.8 ; ± 0.8)	NRC-2 ^f (± 0.6 ; ± 0.4)	ORNL-3 ^f (± 1.2 ; ± 0.3)
	RRR	$\rho(4.2K)$ ($\mu\Omega$ -cm)			
A ^a	8500 ^e	2.8×10^{-4}	A-1 ^g	A-2	A-3
B ^b	11,000 ^e	2.1×10^{-4}	B-1	-	-
C ^c	950	2.5×10^{-3}	-	-	C-3
D ^d	17	0.14	-	-	D-3

^aPurchased from Cominco Ltd., Oakville, Ontario, as 69 grade Al and then annealed at NRC.

^bSame as "A" except annealed at Cominco Ltd., Oakville, Ontario.

^cCommercial purity Al.

^dNominally 99% Al with a 1.0% maximum of Fe + Si.

^eNot corrected for size effects. The diameter of A and B were 0.6 and 0.3 cm, respectively.

^fSee text for apparatus description.

^gA-1 means results obtained on material "A" using apparatus number 1, etc.

of conduction electrons using the equation

$$\lambda = \left[\frac{1}{\lambda_m} - \frac{\rho(4.2)}{L_o T} \right]^{-1} \quad (1)$$

where $\rho(4.2)$ values are given in Table 1 for each specimen, where L_o is the Sommerfeld value ($0.02443 \times 10^{-6} \text{ V}^2 \text{ K}^{-2}$) of the Lorenz function, and where λ_m is the measured value. These impurity corrections were small: the maximum correction was 0.4% at 86 K for C-3 and much less than 0.1% above 80 K for A-1, A-2, A-3, and B-1. Thus, although the corrections were somewhat uncertain, their small sizes insured that the additional uncertainty in λ was less than 0.1%. Therefore, the results in Fig. 1 for A-1, A-2, A-3, B-1, and C-3 represent the λ of pure Al as corrected for thermal expansion.

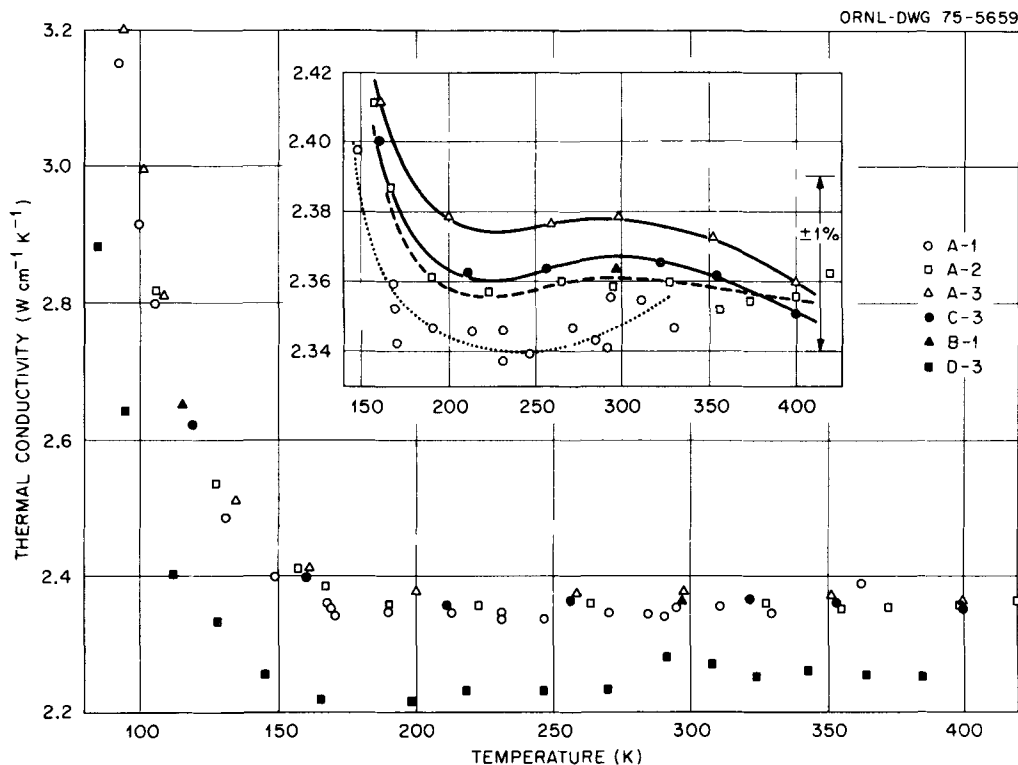


Fig. 1. Thermal conductivity from A-1, A-2, A-3, B-1, and C-3 as corrected for thermal expansion and for impurity resistance using Eq. 1. Results from D-3 were corrected only for thermal expansion.

These results show that from 80 to 160 K, λ of Al decreases rapidly with increasing temperature and then becomes nearly constant from 160 to 400 K. The latter region is shown on an expanded scale in the inset of Fig. 1; all data are within a band of $\pm 1\%$. The experimental λ results, λ_m , for D-3 are shown at the bottom of Fig. 1. These results HAVE NOT been corrected for impurity scattering because we wish to use the results in this form in a later section of this paper.

The λ data from all measurements (except D-3) need to be averaged to obtain a single curve of λ versus temperature. There are several procedures for doing this and each procedure is fraught with problems. For example, a curve could be "faired" through each data set and a value obtained from each curve at even temperature intervals. The values at each temperature could then be used to obtain a weighted mean for each temperature. The result from this procedure is usually biased by the

human who "faired" the data. This would be especially true in this case since the same curve shape would describe each data set to within experimental uncertainty.

A second possibility for combining the data would consist of fitting the λ data - by the method of linear least squares - to polynomials in T. Unfortunately, thermal conductivity is usually so complex that a two or three term polynomial is not adequate to describe experimental values.

This leads to a third method which is often used.⁸ This consists of making a plot of the deviation of each datum from the polynomial that fits the data best and then passing ("fairing") a curve through the deviations. Although this approach is very sensitive when λ varies rapidly with temperature, it gives the equivalent of a curve drawn by hand through a plot of λ versus T.

Table 2 gives results from linear least squares fits of all data to three polynomial equations which were selected to provide a good fit to the experimental data but have absolutely no theoretical significance. Data A-1, A-3, B-1, and C-3 were counted once and data A-2 were counted twice because of different measurement uncertainties. The fit to $\lambda = A + B/T^4 + CT^2$ has a variance of 0.64×10^{-3} and a maximum deviation of any datum from the line of 2.6%. The addition of the D/T term does not significantly reduce the variance or maximum deviation and the standard deviation of C becomes almost as large as C. Change of the B/T^4 term to B/T^5 effects the sign and magnitude of the other coefficients in the equation but has little effect on the variance and maximum deviation. The percentage deviations of the data about

$$\lambda = 2.4507 + 0.779 \times 10^8/T^4 - 0.1845 \times 10^{-6}T^2 - 26.27/T \quad (2)$$

are shown in Fig. 2. All data from 180 to 400 K are within $\pm 1\%$ of this equation, but the data band diverges to about $\pm 1.2\%$ at 80K. Although the experimental data were close to Eq. 2, the deviations do not appear random and each set of data appears to undulate with respect to the equation. The smooth line was drawn in the approximate middle of the experimental data from 80 to 400 K. This curve has a

Table 2. Results of λ data fits to three polynomial equations
 Data A-2 was counted twice because of greater accuracy of NRC-2.

Function	A (S.D.)*	B (S.D.)	C (S.D.)	D (S.D.)	Variance	Max % Deviation
$\lambda = A + \frac{B}{T^4} + CT^2$	2.286 (0.007)	0.6883×10^8 (0.719×10^6)	0.533×10^{-6} (0.76×10^{-7})	0	0.64×10^{-3}	2.6
$\lambda = A + \frac{B}{T^4} + CT^2 + \frac{D}{T}$	2.4507 (0.029)	0.779×10^8 (0.169×10^7)	-0.1845×10^{-6} (0.14×10^{-6})	-26.27 (4.58)	0.43×10^{-3}	2.4
$\lambda = A + \frac{B}{T^5} + CT^2 + \frac{D}{T}$	2.241 (0.029)	0.5303×10^{10} (0.131×10^9)	0.50×10^{-6} (0.147×10^{-6})	17.47 (4.2)	0.55×10^{-3}	2.0

*The standard deviation of each parameter is shown in parenthesis beneath the parameter.

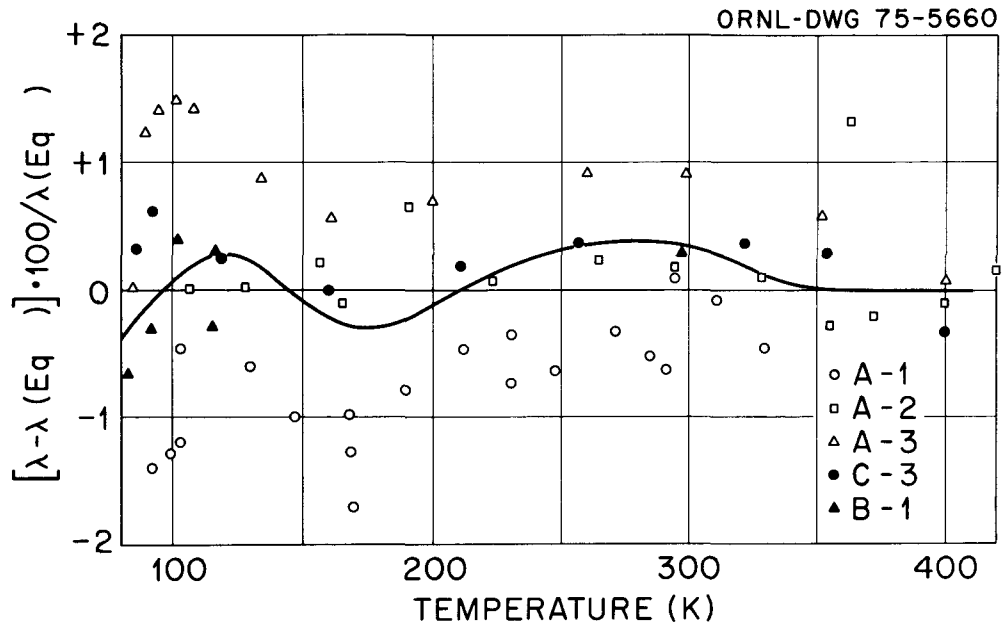


Fig. 2. Deviations of λ results from Eq. 2. The λ values from A-1, A-2, A-3, B-1, and C-3 were corrected for thermal expansion and for impurity resistance. The solid line represents $\bar{\lambda}$ which is the average value for pure Al from these results.

maximum deviation of +0.4% from Eq. 2 near 280 K. Smoothed values of λ at even temperature intervals were obtained from this smooth curve. We believe that values obtained in this manner represent the thermal conductivity of pure aluminum as corrected for thermal expansion and small concentrations of impurities. These values are given in Table 3 for the temperature range 80 to 400 K.

The first approach mentioned above was also employed and this yielded smoothed results which were within $\pm 0.2\%$ of the values listed in Table 3. Data B-1 was the only set of data that extended below 80 K and the $\bar{\lambda}$ values in Table 3 below 80 K were obtained by correcting this set for impurity scattering and smoothing the results.

Electrical Resistivity

The electrical resistivities of these Al specimens vary by 10^5 over the temperature range covered by this study, and this makes a sensitive comparison of data extremely difficult. Therefore, we have

Table 3. Thermal conductivity and electrical resistivity of high purity aluminum from this study and calculated values of λ_{ℓ} and $L(T)$

T (K)	$\bar{\lambda}$ (W cm ⁻¹ K ⁻¹)	$\bar{\rho}$ ($\mu\Omega$ cm)	^a λ_{ℓ} (W cm ⁻¹ K ⁻¹)	λ_e (W cm ⁻¹ K ⁻¹)	$L(T) \times 10^8$ (V ² K ⁻²)
20	147.	0.0008	0.073	147.	0.588
30	48.7	0.0043	0.137	48.6	0.696
40	21.2	0.0179	0.183	21.0	0.941
50	10.9	0.0472	0.202	10.7	1.010
60	7.05	0.0953	0.203	6.85	1.088
70	5.03	0.1618	0.195	4.84	1.118
80	4.007	0.2439	0.182	3.825	1.166
90	3.370	0.3377	0.169	3.201	1.201
100	2.968	0.4401	0.157	2.810	1.237
120	2.612	0.6601	0.135	2.476	1.362
140	2.465	0.8899	0.118	2.347	1.492
160	2.396	1.123	0.104	2.292	1.609
180	2.367	1.356	0.093	2.274	1.713
200	2.358	1.589	0.084	2.274	1.807
220	2.358	1.820	0.077	2.281	1.887
240	2.359	2.049	0.070	2.289	1.954
260	2.362	2.278	0.065	2.296	2.012
280	2.364	2.506	0.060	2.304	2.062
300	2.364	2.733	0.056	2.308	2.103
320	2.362	2.961	0.053	2.309	2.137
340	2.360	3.189	0.050	2.310	2.167
360	2.358	3.416	0.047	2.311	2.193
380	2.359	3.645	0.045	2.314	2.220
400	2.359	3.875	0.042	2.316	2.244

$${}^a\lambda_{\ell} = [T/17 + 5000/T^2]^{-1}$$

$${}^bL(T) = (\bar{\lambda} - \lambda_{\ell}) \times \bar{\rho}/T$$

compared our ρ results to the Bloch-Gruneisen equation⁹ which is of limited theoretical value but describes the temperature dependence of ρ for many metals. Figure 3 compares our ρ data and some literature values to this equation which can be written

$$\rho(T) = \frac{C}{\theta_R(T)} \left(\frac{T}{\theta_R(T)} \right)^5 \int_0^{\frac{\theta_R(T)}{T}} z^5 dz / \left[(e^z - 1)(1 - e^{-z}) \right] \quad (3)$$

where $\theta_R(T)$, the Debye temperature at T can be expressed as

$$\theta_R(T) = \theta_R(\theta) \exp \left[\int_{\theta}^T (-3 \alpha \gamma dT) \right] \quad (4)$$

The latter equation corrects the Bloch-Gruneisen equation for lattice dilation which causes the Debye temperature to vary. In this equation we assumed the Gruneisen constant, γ , equal to 2.14¹⁰ and the value of α was mentioned previously.⁷ We assumed a constant C, and both C and $\theta_R(T)$ were determined by fitting Eq. (3) to the recent ρ data of Seth and Woods between 30 and 310 K.¹¹ This procedure, which was described for sodium by Cook et al.,³ yielded a $\theta_R(\theta)$ value of 383.5 K. All data shown in Fig. 3 as deviations from Eq. (3) were corrected for thermal expansion effects on the form factors⁷ and for impurity resistance by subtracting the proper $\rho(4.2)$ from the ρ of each specimen.

The deviations of results about the equation are not random and we cannot take values from the equation to represent our best values for high purity aluminum. Therefore, the "best values," which we denote as $\bar{\rho}$, were obtained in the manner described for $\bar{\lambda}$ by passing a smooth curve approximately midway between the high and low results for the pure specimens. Values from this smooth curve at even temperature intervals are given in Table 3. The curve in Fig. 3 that represents our best ρ values

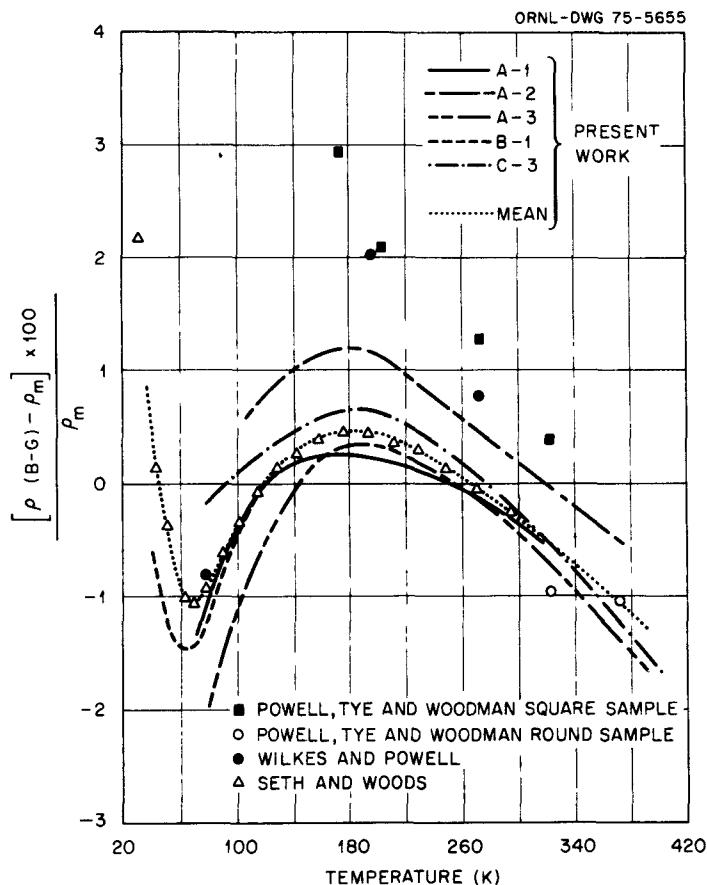


Fig. 3. Electrical resistivity of Al as deviations from Eq. (3). The dotted line represents ρ which is the average value for pure Al from these results.

is in good agreement with data by Seth and Woods¹¹ around 80 K, and we have merely extended the curve through their ρ results at lower temperatures.

Seebeck Coefficient

Each of the various experiments also yielded data on the Seebeck coefficient of Al relative to one of the thermocouple legs. In separate experiments S of these various thermocouple legs was determined versus Pb or Pt, and S of these standard materials was then subtracted (Christian et al.¹² Moore and Graves¹³). Since all results agreed within the experimental scatter of $\pm 0.05 \mu\text{v/K}$ at 300 K, and $\pm 0.15 \mu\text{v/K}$ at 100 K,

with the published $S(\text{Al})$ values of Gripshover et al.¹⁴ and Huebener,¹⁵ we have omitted a plot of our data.

DISCUSSION OF RESULTS

Thermal Conductivity of High-Purity Aluminum

The values of $\bar{\lambda}$ from Table 3 are shown in Fig. 4 with a solid line drawn through these values and a $\pm 1\%$ band about the line. Results from others are also shown for comparison. The present results are within $\pm 1.5\%$ of recommended values of Powell, Ho, and Liley,¹ and, thus, the present results would suggest only small modifications in the 1966 recommended values. The data of Wilkes and Powell¹⁶ are within 0.5% of the present results from 120 to 400 K whereas the data of Flynn¹⁷ differ by less than 1.5% .^{*} In both cases these small differences are within experimental uncertainty. The results of Powell, Tye, and Woodman¹⁸ on one sample from 320 to 400 K are higher than the present results by about 1.5% ; but, below 280 K, their results on a second sample deviate and are 5% low at 200 K. We tend to discard these results as spurious. We also discard results obtained by one of us¹⁹ on an Al specimen with a RRR of 520, which were discussed at the Sixth Thermal Conductivity Conference. After that conference, we found two small errors and one large error in the technique. The latter was due to loosening of the thermal clamp to the heat sink caused by creep of the very soft specimen. These older data are spurious and will be ignored.

Figure 4 shows a distinct plateau in $\bar{\lambda}$ from about 200 to 400 K but the present data do not show a "minimum" within normal experimental scatter. The low value of 2.358 near 230 K and the high value of 2.364 near 290 K differ by only 0.25% whereas normal experimental scatter with each technique is about $\pm 0.3\%$.⁶

The polynomial equations described in Table 2 describe the λ data well and this provides another way of examining this question. Figure 5 shows $d\lambda/dT$ from each polynomial as a function of T and all three curves

*The data from Flynn differ somewhat from those quoted by Powell, Ho, and Liley¹ and by Powell.²

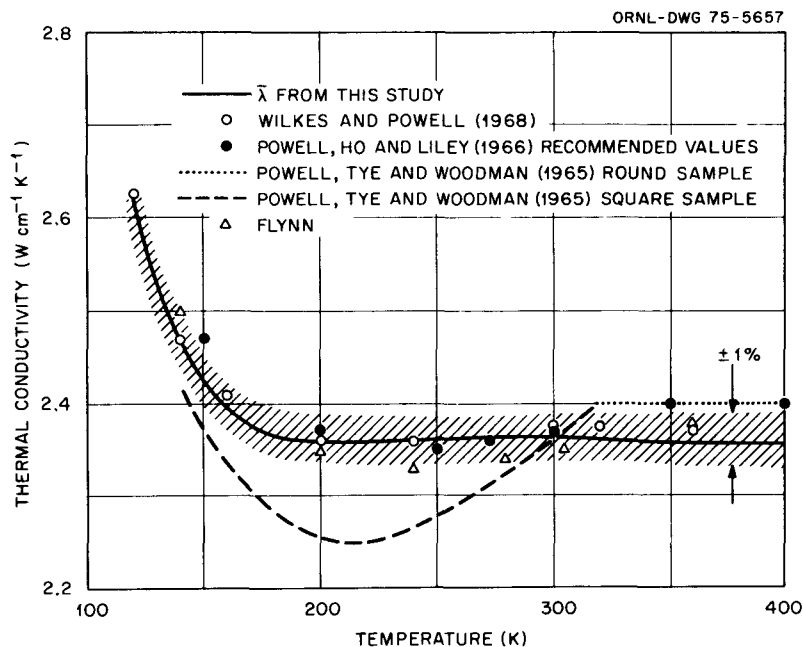


Fig. 4. Comparison of $\bar{\lambda}$ to results from others. The hashed band represents the maximum spread in the present data.

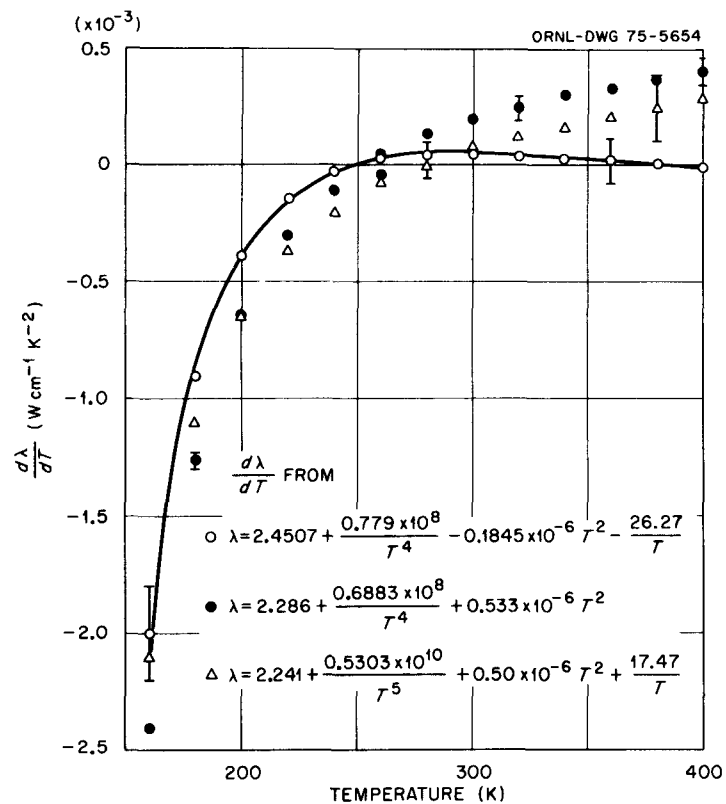


Fig. 5. $d\lambda/dT$ as calculated using the polynomial equations described in Table 2. The bands represent the standard deviation of $d\lambda/dT$ as determined from the standard deviations of the parameters from each least squares fit.

pass from negative to positive with increasing temperature. This sign change in $d\lambda/dT$ from negative to positive would indicate a minimum in λ . However, the standard deviations of the calculated values of $d\lambda/dT^*$ from the best fit (the second equation) are large enough to encompass the horizontal line representing $d\lambda/dT = 0$. Therefore, it is impossible to state that a minimum exists in the total thermal conductivity and we shall now examine the electronic component, λ_e .

The total thermal conductivity of a metal is generally written as

$$\lambda = \lambda_e + \lambda_l \quad (5)$$

where λ_l is the lattice conduction component. The electronic component is related to the electrical resistivity with

$$\lambda_e = \frac{L(T)T}{\rho} \quad (6)$$

where $L(T)$ is an appropriate Lorenz function. To determine λ_e from the experimental data, the lattice component has been assumed to be

$$\lambda_l = \left[\frac{T}{17} + \frac{5000}{T^2} \right]^{-1} \quad (7)$$

At the higher temperatures this λ_l reduces to $17/T$, which is one-half the value of the Leibfried-Schloemann equation for λ_l limited by phonon-phonon scattering. The low temperature limit of $\lambda_l = T^2/5000$ was obtained from 20.3 of Klemens²⁰ for λ_l determined by electron-phonon scattering. These estimates are supported by the experiments of Powell et al.²¹ and Taubert et al.²² but not by the measurements of Sirota et al.²³ Sirota

*The standard deviation of $d\lambda/dT$ is equal to the square root of the variance which is

$$\text{Variance } \frac{d\lambda}{dT} = \left(\frac{3 \times 0.169 \times 10^7}{T^5} \right)^2 + (2 \times 0.14 \times 10^{-6} \times T)^2 + \left(\frac{4.58}{T^2} \right)^2$$

et al.²³ found values for λ_ℓ from a total λ measured in a magnetic field of 50 koe that are an order of magnitude larger than our assumed λ_ℓ . We believe their λ_ℓ is inaccurate since 50 koe is not adequate to quench the electronic component of $\bar{\lambda}$ completely at the high temperature end of their measurement range. Further, their large λ_ℓ value at 50 K in zero field indicates the presence of other experimental difficulties. Results by Klaffky et al.²⁴ which became available to us only recently, would indicate about $11 \times 10^3/T^2$ for the low temperature term.

The $\bar{\lambda}$ and $\bar{\rho}$ values of Table 3 were used to calculate λ_e and $L(T)$ for pure Al, and the results of these calculations are also shown in Table 3. Although there is only a plateau in $\bar{\lambda}$ to within experimental uncertainty, there is a minimum of 1.8% in λ_e based on λ_e values of $2.316 \text{ W cm}^{-1}\text{K}^{-1}$ and $2.274 \text{ W cm}^{-1}\text{K}^{-1}$ at 400 and 190 K, respectively. We do not wish to attach much importance to this results, however, since the presence or absence of a minimum simply indicates the relative strengths of the vertical and horizontal electron-phonon scattering processes.

Thermal Conductivity of D-3

The $L(T)$ values for pure Al listed in Table 3 permit calculation of the thermal conductivity of sample D-3 for comparison to the experimental results. Calculations were made using

$$\lambda(D-3) = \left[\frac{\rho_0}{L_0 T} + \frac{\bar{\rho}}{L(T)T} \right]^{-1} + \left[\frac{T}{17} + \frac{5000}{T^2} \right]^{-1} \quad (8)$$

for two cases. The first assumed that $\rho_0 = \rho(4.2 \text{ K})$ and the second assumed that $\rho_0 = \rho(D-3) - \bar{\rho}$. Both calculated values are within $\pm 1\%$ of the experimental results above 110 K, and both values diverge from experimental results on D-3 by about 4% at 80 K. This disagreement at the lower end of the measurement range may be due to a difference in λ_ℓ for the pure and impure materials. Since the two values agree, we have calculated λ for the impurity resistances shown in Fig. 6 with the assumption that $\rho_0 = \rho(4.2 \text{ K})$.

As the impurity resistance increases, the calculated curves show deeper minima with the temperatures of the minima occurring at lower temperatures. This continues until the low temperature peak has been eliminated. Thus, the depth and location of any minimum in Al alloys should behave as Powell² described when Matthiesen's rule is approximately valid. This effect, however, cannot be observed on a series of purer samples where RRR of all samples are greater than 100, such as those examined by Powell,² because the λ difference near 200 K between the most pure and most impure specimens would only be about 1% which is not observable with present state-of-the-art measurements. Since λ_e dominates the thermal conductivity of Al, Fig. 6 is similar to Fig. IX 9 of Wilson.²⁵

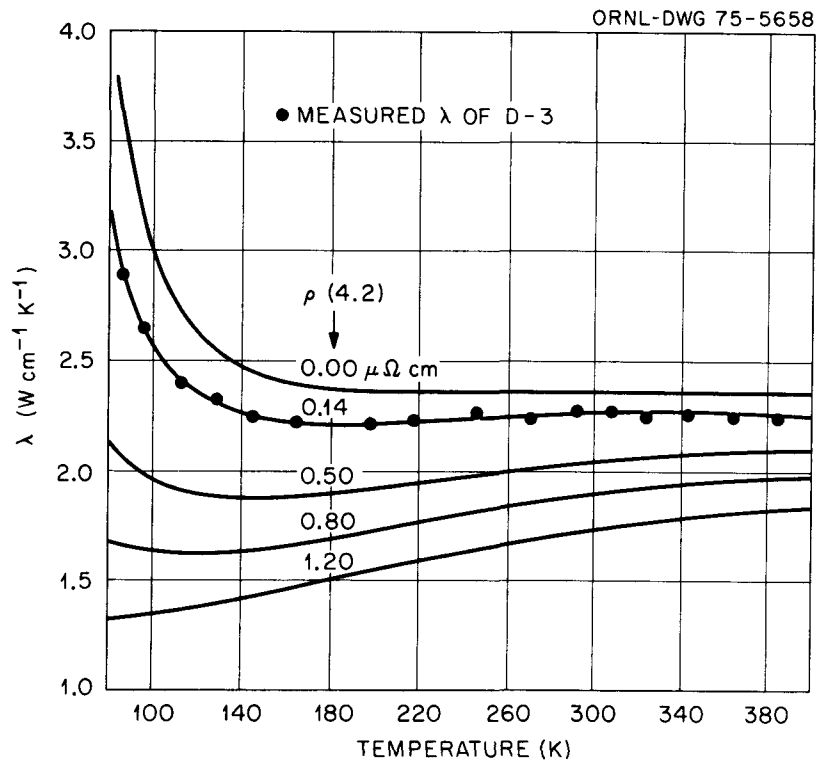


Fig. 6. The thermal conductivity of D-3 compared to $\bar{\lambda}$ and to a calculated curve for the λ of D-3. Calculated curves for more impure (hypothetical) specimens are also shown.

The Lorenz function, $L(T)$, was calculated using Eq. (6) and assuming that λ_ℓ was given by Eq. (7) (these $L(T)$ values are in Table 3), and then assuming that λ_ℓ was equal to zero. These two functions were normalized to L_0 and are shown in Fig. 7. The decrease of these functions with decreasing temperature is of course due to the increasingly inelastic nature of electron-phonon scattering; had we not corrected for impurity scattering these curves would have returned to unity at very low temperatures. A slight hump is visible in both curves near 80 K which is approximately one-fifth of the specific heat Debye temperature. Similar bumps may be found in the temperature range $0.14 - 0.20\theta_D$ in the alkali and noble metals. In each case the hump is found at those temperatures where the strengths of elastic and inelastic electron-phonon scattering are approximately equal.

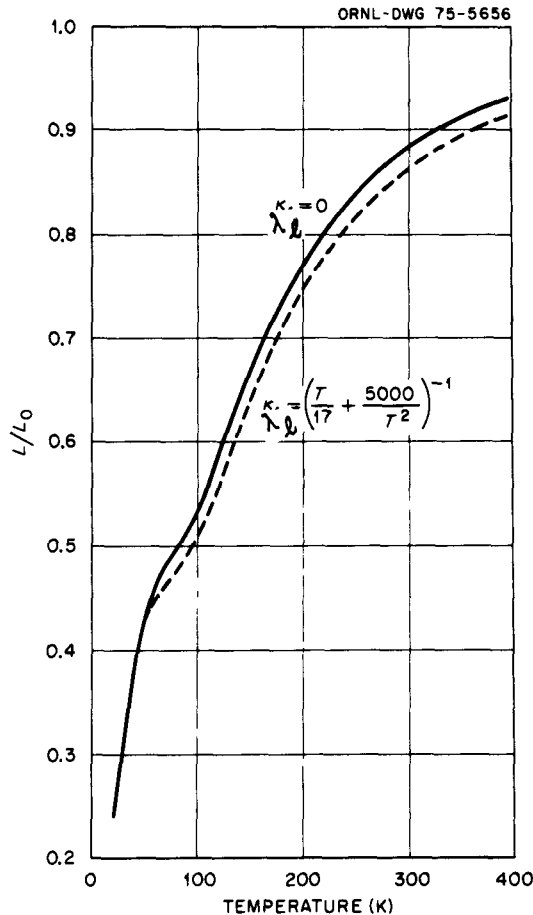


Fig. 7. The ratio of $L(T)$ to L_0 with the assumptions that $\lambda_\ell = 0$ and that λ_ℓ is given by Eq. 7.

The Low Temperature Thermal Resistivity

According to simple theory, the low temperature thermal resistivity, $W = \lambda^{-1}$, should vary quadratically with temperature. Generally, two methods are used to compare experimental results to this theory. Most often, the data are plotted against temperature in such a way that to a first approximation the effects of electron-phonon scattering and electron-impurity scattering may be separated (see e.g. Seeberg and Olsen²⁶). Alternately (see e.g. Van Baarle et al.,²⁷ Kos²⁸) it is recognized that only W_v , i.e. that part of the thermal resistivity due to "vertical" or small-angle electron phonon scattering, should vary quadratically with temperature.

We have followed the latter method by plotting $W_v = 1/(\bar{\lambda} - \lambda_l) - \bar{\rho}/L_o T$, where λ_l is the lattice conductivity, and $W_H = \rho/L_o T$ is the thermal resistivity due to "horizontal" large-scale scattering of electrons (we ignore interference between vertical and horizontal processes). Our results are shown in Fig. 8. The dashed lines give W_H , equal to $\rho/L_o T$ at low temperatures.* We see the familiar hump in W_v centered near one-quarter the Debye temperature, but at low temperatures W_v does not vary with T^2 as simple theory would predict: a change in slope near 10 K is clearly visible. This result disagrees with the conclusion reached by Seeberg and Olsen: namely that the thermal resistivity of Al varies quadratically with temperature, and the constant of proportionality varies with sample purity.

CONCLUSIONS

Three apparatus which represent the state-of-the-art for thermal conductivity determinations have been employed to measure λ , ρ , and S of four aluminum materials with RRR between 17 and 11,000. After small

*We are unable to compute W_v with certainty for the less pure samples examined by Seeberg and Olsen, since for such samples W_v is a small difference between the much larger $1/(\bar{\lambda} - \lambda_l)$ and λ_o/T .

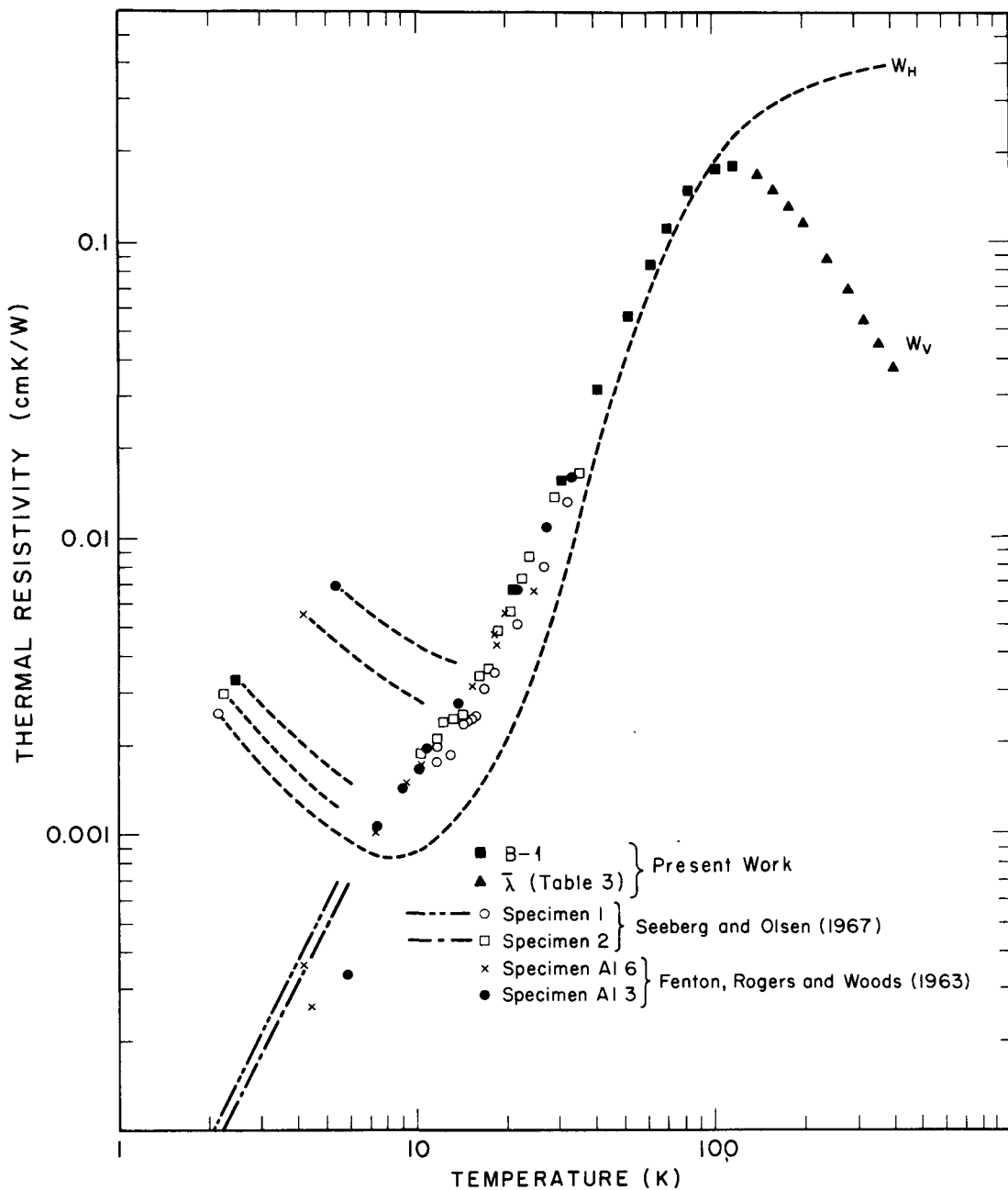


Fig. 8. Thermal resistivity of Al due to horizontal ($W_H = \rho/LT$) and vertical ($W_V = 1/(\lambda - \lambda_0) - W_H$) electron-phonon scattering. Individual points represent W_V whereas the dashed lines represent W_H . The points on the ends of the dashed lines identify the specimen on which the calculation of W_H was based. The straight-dashed lines in the lower left of the figure gives mean values of W_V determined by Seeberg and Olsen.

corrections were applied to the λ results from the three most pure specimens, results from the three agreed to within $\pm 1.0\%$ from 160 to 400 K. To within experimental uncertainty there was not a minimum in λ but there was a minimum in the electronic component of λ when a reasonable assumption was made for the lattice component of λ .

All λ and ρ results on the three most pure specimens were combined to generate average values for pure Al. These average values were then used to determine the Lorenz function for pure Al. Using this Lorenz function, calculated values of λ for an impure Al were within 1% of experimental values.

The present results showed that the thermal resistance due to vertical electron processes does not vary with T^2 as simple theory predicts.

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