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REACTOR TECHNOLOGY

PM-1 TASK 5, SUBTASK 5.8
LOCAL BOILING HEAT TRANSFER TESTS

Single Tube Heat Transfer and Pressure Drop Tests

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FOREWORD

This report was prepared by the Nuclear Division of The Martin Company for submission to the U. S. Atomic Energy Commission. It describes experimental heat transfer studies performed under Contract No. AT(30-1)-2345.

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SUMMARY

Experimental heat transfer tests on simulated PM-1 tubular fuel elements are described. Local boiling heat transfer and pressure drop data were obtained.

Methods of data reduction and analysis are discussed. Local boiling pressure drop data were successfully correlated using an equation of the following form:

$$f/f_{iso} = a + b j + c Re$$

where j is the product of the Stanton number and the Prandtl number to the $2/3$ power, Re is the Reynolds number and a , b and c are constants for a particular pressure. The recommended design correlation for 1300 psia is:

$$f/f_{iso} = 0.080 + 222j + 0.601 (Re \times 10^{-5})$$

$$0.002 \leq j \leq 0.010$$

$$0.5 \times 10^5 \leq Re \leq 1.5 \times 10^5$$

Analysis of wall temperature data provided reasonable confirmation of the Jens and Lottes local boiling heat transfer correlation. However, formation of scale on the tube wall during the tests probably resulted in some bias in the wall superheat data.

Recommendations for continuation of the test program are given.

LIST OF SYMBOLS

Upper Case

A_f	Flow area inside the tube	ft^2
A_s	Heat transfer surface area inside the tube	ft^2
D	Inside diameter of tube	in.
G	Coolant mass velocity	$\text{lb}/\text{ft}^2\text{-hr}$
H	Manometer deflection	in.
I	Test section current	amperes
L	Overall heated length of tube	ft
M	Coolant mass flow rate	lb/hr
P	Power applied to test section	kw
Pr	Prandtl number	
Q	Coolant volumetric flow rate	gpm
Re	Reynolds number	
S	Volumetric heat generation	$\text{Btu}/\text{ft}^3\text{-hr}$
St	Stanton number	
T_m	Measured outside tube surface temperature	$^{\circ}\text{F}$
T_w	Tube inside wall temperature	$^{\circ}\text{F}$
V	Coolant velocity	ft/hr

Lower Case

c_p	Specific heat of coolant	$\text{Btu}/\text{lb-}^{\circ}\text{F}$
$\bar{c}_p$	Average coolant specific heat for a pressure drop increment	$\text{Btu}/\text{lb-}^{\circ}\text{F}$

f	Darcy friction factor (local boiling)	
f_{iso}	Isothermal Darcy friction factor	
g	Gravitational constant	ft/hr ²
h	Film coefficient of heat transfer	Btu/ft ² -hr-°F
$\bar{h}$	Film coefficient at midpoint of pressure drop increment	Btu/ft ² -hr-°F
j	Heat transfer factor; $(St)(Pr)^{2/3}$	
$\bar{k}$	Average thermal conductivity of tube wall	Btu/ft-hr-°F
p	Test section absolute pressure	psia
p_g	Test section gauge pressure	psig
q''	Heat flux to coolant	Btu/ft ² -hr
r_i	Tube inside radius	ft
r_o	Tube outside radius	ft
z	Axial distance from beginning of heated length	in.
$\bar{z}$	Axial distance to midpoint of pressure increment	in.
<u>Greek</u>		
ΔE	Test section voltage drop	volts
ΔH	Manometer deflection under zero differential pressure	in.
ΔT_F	Temperature difference between tube wall and bulk coolant	°F
ΔT_S	Wall superheat	°F
Δp	Pressure drop	lb/ft ² , psi

Δz	Length of pressure drop increment	in.
$\Delta \theta$	Overall coolant temperature rise	$^{\circ}\text{F}$
$\Delta \theta_{\text{sub}}$	Coolant subcooling	$^{\circ}\text{F}$
$\bar{\mu}$	Dynamic viscosity of coolant at midpoint of pressure drop increment	lb/ft-hr
ρ	Coolant density	lb/ft ³
ρ_{MF}	Density of manometer fluid	lb/ft ³
ρ_{MW}	Density of water (coolant) in manometer	lb/ft ³
$\bar{\rho}$	Coolant density at midpoint of pressure drop increment	lb/ft ³
$\Sigma \Delta P_s$	Summation of incremental static pressure drops	psi
θ	Coolant temperature	$^{\circ}\text{F}$
$\theta_{\text{in},1}$	Coolant temperature measured at inlet elbow	$^{\circ}\text{F}$
$\theta_{\text{in},2}$	Coolant temperature measured at inlet transition section	$^{\circ}\text{F}$
$\theta_{\text{ex},1}$	Coolant temperature measured at outlet transition section	$^{\circ}\text{F}$
$\theta_{\text{ex},2}$	Coolant temperature measured at outlet elbow	$^{\circ}\text{F}$
θ_{sat}	Coolant saturation temperature at test section absolute pressure	$^{\circ}\text{F}$
$\bar{\theta}$	Coolant temperature at midpoint of pressure drop increment	$^{\circ}\text{F}$

Subscripts

I	Incremental
T	Total
a	Acceleration
e	Elevation
f	Friction
s	Static
1, 2, ..., 15	Wall temperature measurement stations along the tube
1,2, ..., 5	Pressure drop increments
1, 2	End points of a pressure drop increment

Superscripts

'	Denotes laboratory reading (uncorrected)
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I. INTRODUCTION

In the design of a nuclear reactor core, thermal and hydraulic calculations are generally based upon correlations derived from experimental data. In the PM-1 reactor core, local boiling, i.e., nucleate boiling with the local bulk coolant temperature below the saturation temperature, is permitted during steady-state operation. Hence, in the design of this core, local boiling heat transfer, pressure drop and burnout correlations were employed. In general, these correlations were based on the results of a number of experimental studies, necessarily employing different experimental rigs and techniques, different heater element geometries, surface conditions, etc. The result is frequently a significant uncertainty in the predicted quantity above and beyond any random variation in the process itself. Furthermore, while several correlations may be available for predicting a quantity of interest, agreement among them is generally poor. Not only do the predicted values disagree, but the independent variables contained in the correlations are also different. Clearly, refined analysis under these circumstances is not justified nor, indeed, possible.

Consideration of the factors discussed above led to the decision to perform experimental heat transfer studies on simulated PM-1 fuel elements with heat fluxes and coolant conditions over a range encompassing PM-1 conditions. The objectives of these tests were to determine the validity of the correlations used in the PM-1 core design and to provide more definitive correlations, if necessary, for design of advanced local boiling cores.

This report describes a portion of the program devoted to heat transfer and pressure drop measurements on a single tube test section with coolant flow only on the inside. The local boiling pressure drop measurements are of particular interest because of the small amount of published data on this subject. Much of the data which has been reported cannot be used for design purposes because the data were obtained with simulated fuel elements which were instrumented for overall pressure drop only. Thus, in cases where the entire fuel element is not in local boiling, the length in local boiling cannot be separated from the overall pressure drop. Also, in the cases where the entire tube is in local boiling, the problem arises wherein the pertinent parameters over the entire tube are averaged and the entrance effects are neglected. The test tube used in this program was well instrumented, both for wall temperatures and incremental pressure drop measurements.

II. APPARATUS

A. HEAT TRANSFER LOOP

The Martin Heat Transfer Loop was designed to enable water heat transfer and pressure drop investigations to be conducted with simulated reactor fuel elements. The facility has also been instrumented to provide for burnout and void measurement studies. Pressures to 2000 psia and temperatures to 630° F are permitted. Flow rates up to 200 gpm are possible with typical test sections. The total capacity of the loop is approximately 50 gallons. Nuclear heating, i.e., a constant heat input system, is approximated by electrical heating of simulated fuel elements. Heating is provided by a 300-kw direct current power supply.

Figure 1 is a schematic diagram of the loop. Space is available in the loop for the installation of two test sections in parallel. The single-tube test section, STTS-3, is shown installed in the test section area. A photograph of the loop is given in Fig. 2. The instrument and control panels are shown in Fig. 3.

Referring again to Fig. 1 and starting with the circulating pump discharge, the water passes through a preheater and then to the flow control and measurement section. This section consists of three parallel lines, each with a Pottermeter which measures flow over a distinct range. Heat removal and coarse control of loop temperature are accomplished by a heat exchanger. System pressure is maintained by an electrically heated steam pressurizer. A small bypass line around the circulating pump is provided for continuous deionization of loop water during operation. This line contains an economizer, cooler and demineralizer. A positive displacement makeup pump automatically provides makeup water. Water makeup is controlled by the water level in the pressurizer. On charging the system, immersion heaters located in the storage tank are used to raise the temperature of the water so as to drive off some of the dissolved gases before pumping the water into the loop. An inert gas blanket is provided for the storage tank. The major components of the loop are described below.

1. Piping

The main circuit is constructed of three-inch, Schedule 180, stainless steel pipe. (The test section connecting lines used in this program were of stainless steel tubing and tube fittings which afforded ease of installation of test sections.)

2. Circulating Pump

The circulating pump is a Westinghouse canned motor unit, Type A-150-D1 with a design rating of 295 feet of head at 150 gpm.

Fig. 1. Schematic of Heat Transfer Loop

Fig. 2. Photograph of Heat Transfer Loop

Fig. 3. Instrument and Control Panels

3. Preheater

The preheater is composed of nine immersion-type electric heaters. These are arranged in two circuits of 21 kw and 10.5 kw, respectively. Saturable reactors, responsive to test section inlet temperature, are employed to give proportional control of the 21-kw heater. The 10.5-kw heater is provided with on-off control.

4. Heat Exchanger

The heat exchanger is a concentric straight-tube design containing six sets of tubes. The inner tubes, which contain the primary water, are of a stainless steel. The outer tubes, which contain the secondary fluid, are of carbon steel. Manual throttle valves are provided at the outlet of each of the six parallel secondary lines. The common outlet line contains an automatic valve. These are used for coarse control of loop temperature. (Fine control is provided by the preheater.)

5. Pressurizer

An electrical immersion heater of 31-kw capacity is used in the pressurizer. Coarse control of loop pressure is accomplished through on-off control by a pressure-sensitive mercury switch (Pressurtrol). Loop pressure is maintained at the control point by a more sensitive switch (Mercoide). A spray valve is provided as a safety override on the pressurizer system.

6. Deionizer

A high pressure resin bed unit is used which contains eight pounds of Amberlite XE-150 resin. The capacity of the circuit is two gpm.

7. Makeup Pump

A controlled volume, positive displacement, reciprocating pump (Milton Roy) with a full stroke capacity of 23 gal/hr is employed to charge the system and for automatic makeup.

8. Flow Control

The control of test section flow rate is provided by air-actuated valves. These may be operated manually or automatically and are driven by a flow controller mounted on the instrument panel.

9. Burnout Detector

This device contains a Wheatstone bridge circuit which compares the electrical resistance of the upper portion of the test section with that of the lower portion. The bridge circuit, if unbalanced due to a

Fig. 4. 300-kw d-c Power Supply

Fig. 5. Test Section Assembly

Scale: 2/1

Fig. 6. Pressure Tap Arrangement

hot spot on the tube, will actuate a trigger circuit connected to a d-c circuit breaker which then cuts out the power to the test section. The detector was used in this program as a safety device.

10. Interlocks

These are provided to shut down the loop in the event of overpressure, overtemperature, loss of flow, etc.

11. Power Supply

Direct current for the electrical heating of simulated fuel elements is supplied by a Sel-Rex selenium rectifier (Fig. 4). The system has a capacity of 300 kw and current can be supplied up to 7500 amperes at 40 volts or 3750 amperes at 80 volts. The system can be operated manually or automatically by an Inductrol regulator with d-c notching control.

B. TEST SECTION

The single-tube test section, STTS-3, was designed to simulate a PM-1 fuel element. Thus, the dimensions of STTS-3, identical to those of the PM-1 fuel tube, are:

Heated length	30 inches
Outside diameter	0.500 inches
Inside diameter	0.416 inches
Wall thickness	0.042 inches

The test section was made of Type 347 stainless steel.

The test section assembly is shown in Fig. 5. The tube assembly flanges were counterbored to a depth of $7/8$ inch to receive the ends of the tube, which were furnace-brazed into the flanges. Silver-lithium alloy was used in the brazing process. This provided good electrical connections at the tube-to-flange joints. Electrical connection plates were welded to the inner flange assemblies. Two plates, 120° apart, were provided at each flange. A counterweight was used to relieve the tube of the weight of the upper flange assemblies. Thus, during thermal expansion, the tube was restrained only by the frictional resistance of the counterweight system.

The pressure tap arrangement is shown in Fig. 6. Owing to the length of tubing required to connect to a manometer (about 20 feet), the lengths of $1/16$ and $1/8$ tubing were limited. The arrangement gave a time constant of about 12 minutes, while the time to reach a steady-state value

Fig. 7. STTS-3 Wall Thermocouples Installed

Fig. 8. Wall Thermocouple Installation

ΔP = pressure drop
 ΔE = voltage drop
 Tm = measured wall temperature
 θ = bulk coolant temperature

Fig. 9. Location of Instrumentation on Tube Assembly

Fig. 10. Test Section Installed in Loop

on a manometer after a step change in pressure drop was about 60 minutes. Two pressure tap installations may be seen on the test section pictured at the top of Fig. 8.

Chromel-alumel thermocouple wire of 0.010-inch diameter was used for the measurement of the tube outer wall temperatures. The test section with thermocouples installed is shown in Fig. 7. Figure 8 shows the thermocouple installation. At each thermocouple location, a strip of 0.005-inch thick mica tape was wrapped around the tube wall to isolate the thermocouple from the electrically heated tube. The bare thermocouple wire was then placed on the mica tape and wrapped 180 degrees around the tube. The thermocouple wire was held in place by glass tape and a steel clip. The clip served to maintain the couple in firm contact with the tube despite expansion and contraction of the tube during test operation. The thermocouple leads were covered with high temperature glass insulation.

Bulk coolant temperatures were measured with 1/16-inch diameter, sheathed thermocouples whose junctions were insulated from the sheaths. Stainless steel sheaths containing chromel-alumel wire of 0.010-inch diameter were used. Conax thermocouple glands were used in the installation of the bulk coolant thermocouples. The glands provided electrical isolation of the thermocouple sheaths from the test section.

Wires were fastened to the tube assembly flanges for the measurement of voltage drop across the test section.

The locations of pressure taps, thermocouples and voltage pickups are given in Fig. 9.

The test section, as installed in the loop and thermally insulated, is shown in Fig. 10. Electrical leads have not yet been attached.

C. INSTRUMENTATION

1. Flow Measurement

Flow rate was sensed upstream of the test section with a turbine-type (Potter) flowmeter. The Pottermeter signal was transmitted to the chart recorder of a flow controller and to a scaler. The recorder was used to make coarse measurements and to monitor the flow rate. The scaler, coupled with an automatic timer, was used for data points. Comparison with flow rates determined by weighed and timed samples showed that the Pottermeter was accurate to within 1/2% of the indicated flow.

2. System Pressure

System pressure was measured just upstream of the test section inlet with a precision Bourdon tube gage (Heise). The gage had a 3000-psi scale graduated in 2-psi increments. Accuracy of the gage was 0.1% of full scale (3 psi). System pressure was continuously recorded on a pressure recorder.

3. Differential Pressure

Pressure drops along the tube were measured with high pressure, inclined, well-type manometers (Mirriam) (see Fig. 11). The manometers were 48 inches in length, inclined at 15 degrees and provided with a -10-inch to +10-inch scale graduated in increments of 0.02 inch. Indicating fluid of 1.20 specific gravity was used in the five manometers which measured incremental pressure drops. Overall pressure drop was measured with indicating fluid of 2.95 specific gravity.

4. Heat Flux

The heat flux was computed from the electrical power applied to the test section. The potential drop across the test section was measured with a Weston recording potentiometer with a range of 0 to 80 volts and an accuracy of 0.4 volts. Potential drop was also indicated with a voltmeter. Current to the test section was measured with a Weston potentiometer equipped with a 50-mv calibrated shunt. This instrument had a range of 0 to 8000 amperes and was accurate to 40 amperes. The uncertainty in power was therefore 16 watts or an uncertainty of 200 Btu/hr-ft². This was about 0.2% of the minimum heat flux applied to the test section.

5. Temperatures

Wall and bulk coolant temperatures were measured with a Rubicon galvanometer-type potentiometer of 0.02-mv accuracy. (This is 1° F for chromel-alumel wire at 500° F.) Temperatures were also monitored with a Brown, 24-point, self-balancing precision indicator.

6. Thermocouple Calibration

The thermocouples were aged in a furnace for 170 hours at 500° F. Calibration of the couples was accomplished through comparison with a standard thermocouple. The couples and the standard couple were inserted in an aluminum block of two inches diameter and six inches in length. The block was placed in an electrically heated tubular furnace. A control thermocouple, also inserted in the block, was used to control the power to the furnace. Thermocouples were calibrated at increments of 25° F from 300° to 450° F and at 15° F increments from 450° to 750° F.

Fig. 11. Six High Pressure Inclined Well-Type Manometers Employed in the STTS-3 Program

III. TEST PARAMETERS*

A. LOCAL BOILING TEST PROGRAM

The program was designed to obtain local boiling pressure drop and heat transfer data inside a simulated PM-1 reactor fuel tube. Three pressures, 1100, 1200 and 1300 psia, were investigated at flow rates of 0.90 to 2.55 gpm, nominal inlet temperatures of 425 and 445° F and heat fluxes of 1.20×10^5 to 5.55×10^5 Btu/hr-ft². Included were the PM-1 design conditions of 1300 psia, 1.09 gpm and 445° F inlet temperature at heat fluxes of 1.60×10^5 to 2.35×10^5 Btu/hr-ft². The local boiling test parameters are given in Table 1.

A set of three special tests was also performed. The heat fluxes used in these tests corresponded to: (1) an average heat flux for the entire tube which is the highest steady-state flux expected to occur in the PM-1 core; (2) an average flux for the entire tube, corresponding to a 20% (transient) power increase over PM-1 design power; (3) an average flux for an entire tube which represents the peak possible PM-1 heat flux and includes hot spot factors as well as a 20% transient. The parameters for these tests are specified in Table 2.

An iterative technique was employed in the design of the test program to determine the test parameters for local boiling. The Jens and Lottes local boiling heat transfer correlation (Ref. 1) was equated to the Dittus-Boelter forced convection equation to give an estimated heat flux for incipient local boiling. The upper limit of heat flux was fixed to ensure that the coolant remained below saturation temperature. The test parameters were selected to give the effect of pressure, velocity and sub-cooling on local boiling heat transfer and pressure drop.

B. NONBOILING TEST RUNS

Nonboiling heat transfer and pressure drop tests were included in the program. These tests served as checks on the experimental technique through comparison of the nonboiling heat transfer film coefficients with those predicted by the Dittus-Boelter equation for forced convection and comparison of the nonboiling friction factors with those given by the Moody smooth tube curve. The nonboiling pressure drop data were also required for comparison with the local boiling pressure drop data. The various phases of the work are outlined below.

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1. Horizontal Isothermal Pressure Drop

The purpose of this test was to compare the differential head measurements with respect to the well established nonboiling frictional head loss data for smooth tubes. This work was performed over a range in Reynolds number similar to that of the local boiling tests. A horizontal disposition of the test section was employed to eliminate elevation head corrections. Thus, frictional head losses were obtained directly for isothermal flow conditions. The test parameters are shown in Table 3.

2. Vertical Isothermal Pressure Drop

This test was identified to the horizontal test except that the test section was mounted vertically, as it was in the local boiling tests. Heat loss measurements were made during this test. The test parameters are given in Table 3.

3. Forced Convection Heat Transfer

These runs served to check the experimental technique, since the results could be compared with the Dittus-Boelter equation for convective heat transfer. Force convection runs were repeated periodically during the course of the local boiling tests. Thus, any changes with time in the convective heat transfer and of the tube could be observed.

Forced convection runs were also made to permit the experimental determination of inside surface temperatures relative to the measured wall temperatures after the method of Wilson (Ref. 2). Table 4 gives the test parameters for the forced convection tests.

TABLE 1
Test Parameters, Local Boiling Runs

System pressure: 1100, 1200, 1300 psia

Nominal coolant inlet temperature: 425° F, 445° F

<u>Flow Rate</u> <u>(gpm)</u>	<u>Heat Flux</u> <u>(10⁵ Btu/hr-ft²)</u>
0.90	1.20
0.90	1.60
0.90	1.95
1.09	1.60
1.09	1.95
1.09	2.35

TABLE 1 (continued)

<u>Flow Rate (gpm)</u>	<u>Heat Flux (10^5 Btu/hr-ft²)</u>
1.20	1.60
1.20	1.95
1.20	2.35
1.48	1.95
1.48	2.35
1.48	3.20
1.80	2.35
1.80	3.20
1.80	3.90
2.55	3.20
2.55	3.90
2.55	5.55

TABLE 2

Test Parameters, "High" Heat Flux Runs

System pressure: 1300 psia

Flow rate: 1.09 gpm

<u>Condition (see text)</u>	<u>Inlet Temperature (°F)</u>	<u>Heat Flux (10^5 Btu/hr-ft²)</u>
(1)	362	3.06
(2)	329	3.68
(3)	283	4.54

TABLE 3

Test Parameters, Isothermal Pressure Drop

System pressure: 1300 psia

<u>Inlet Temperature (°F)</u>	<u>Flow Rate (gpm)</u>
450	0.90
425	0.90
425	1.20
450	1.20
450	1.50

TABLE 3 (continued)

<u>Inlet Temperature (°F)</u>	<u>Flow Rate (gpm)</u>
425	1.50
425	2.00
450	2.00
450	2.55
425	2.55
425	3.00
450	3.00

TABLE 4

Test Parameters, Forced Convection Heat Transfer

System pressure: 1300 psia

Coolant inlet temperature: 425° F

<u>Flow Rate (gpm)</u>	<u>Heat Flux (10⁵ Btu/hr-ft²)</u>
4.0	1.25
4.0	1.88
4.0	2.51
4.0	3.13
6.0	1.25
6.0	1.88
6.0	2.51
6.0	3.13
8.0	1.25
8.0	1.88
8.0	2.51
8.0	3.13
10.0	1.25
10.0	1.88
10.0	2.51
10.0	3.13

IV. TEST PROCEDURE*

At the start of a test run, the operating parameters of inlet temperature, inlet pressure, flow rate and heat flux were set. The system was then allowed to attain steady-state operation. This was determined by periodic recording of the instrument readings during the course of a run. Constant instrument readings over a 20-minute period were required to indicate equilibrium. It was found that thermocouple readings were steady after 30 minutes and that the manometer system required 60 minutes. About 75 minutes were required to complete a run after the establishment of a set of operating conditions. Manometer zero readings were checked at four-hour intervals.

Measurements were recorded in duplicate on data sheets. One set of data sheets was retained in the laboratory and the other set was used in the data reduction. The recorded data, pertinent to the test program, are listed in Table 5.

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TABLE 5
Recorded Data

<u>Datum</u>	<u>Symbol</u>	<u>Units</u>
Run number	-	-
Time of run	-	hour-minutes
Inlet pressure	P_g	psig
Flow rate, recording meter	Q	gpm
Pottermeter counter	-	counts
Pottermeter, counting time	-	minutes
Flow rate, Pottermeter	Q	gpm
Voltage drop across test section, recording voltmeter	ΔE	volts
Voltage drop across test section, indicating voltmeter	ΔE	volts
Current to test section	I	amperes
Power to test section	P	kilowatts
Coolant inlet temperature at inlet line	$\theta_{in.1}$	millivolts
Coolant inlet temperature at tube assembly flange	$\theta_{in.2}$	millivolts
Coolant outlet temperature at tube assembly flange	θ_{ex1}	millivolts
Coolant outlet temperature at outlet line	θ_{ex2}	millivolts
Outer wall temperature	T_m	millivolts
Ambient temperature	-	°F
Overall pressure drop	H_T	inches of indicating fluid

TABLE 5 (continued)

<u>Datum</u>	<u>Symbol</u>	<u>Units</u>
Incremental pressure drop	H	inches of indicating fluid
Manometer zero reading	ΔH	inches of indicating fluid

V. DATA REDUCTION*

The data recorded in the laboratory are shown on the sample data sheets, Tables 6 and 7, for Run 96. Only the information which appears on the last line was used**, since this is for the steady-state condition, all the other data having been recorded to indicate the attainment of equilibrium.

From the raw data, various heat transfer and pressure drop parameters were calculated. The calculated values may be seen on the sample data reduction sheets for Run 96, Tables 8 and 9.

A. HEAT TRANSFER DATA SHEET

On the heat transfer sheet (Table 8), the 15 entries in the column of measured outside wall temperatures and the first two and last two entries in the coolant temperature column were completed first, using the 19 thermocouple readings from Sheet 2 (Table 9) of the recorded data. The conversion from millivolts to degrees Fahrenheit was made, using the standard table for chromel-alumel thermocouples. A calibration correction was then made, using an equation of the form (a list of the equations for the thermocouples appears in Appendix B):

$$T_m = a + b T'_m \quad (1)$$

In Eq (1), a and b are constants determined from a least squares regression of data obtained in calibrations with a standard thermocouple. The equations were based on calibration data at temperatures of 375, 400, 425° F and temperatures over a range from 450 to 690° F in 15° F increments.

The power and volumetric flow rate were transferred from Sheet 1 of the recorded data (Table 6) to the bottom of the heat transfer page (Table 8). The absolute test section pressure was found by:

$$p = p_g + 15 \quad (2)$$

Since the flow measuring device was located between the heaters and the entrance to the test section, the proper coolant density for use in computing the mass flow rate is one determined at the inlet temperature. The

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**The exception is the data which appears in Table 6 in the columns marked monometer ΔP_x , initial and which were taken when zero differential pressure existed, so that the departure from a zero reading could be used later to correct the manometer deflection.

TABLE 6
Sample Data Sheet, Run No. 96

Test Unit: Task 5--Subtask 5.8

Test Name: Single Tube Test Section STTS-3

Time	Run Number	Wheelco Preheater Temperature	Potter Meter Count	Potter Meter Time	Potter Meter Flow (calculated)	TEE Temperature	Secondary Heat Exchanger Flow, Left-Hand Column	Secondary Heat Exchanger Flow, Right-Hand Column	Secondary Heat Exchanger Flow, (calculated)	Secondary Heat Exchanger Inlet Temperature	Secondary Heat Exchanger Outlet Temperature	Primary Heat Exchanger Outlet Temperature	Hesse Gauge	Recording Ammeter	Recording Voltmeter	Indicating Voltmeter	Manometer ΔP_1 Initial	Manometer ΔP_1	Manometer ΔP_2 Initial	Manometer ΔP_2	Manometer ΔP_3 Initial	Manometer ΔP_3	Manometer ΔP_4 Initial	Manometer ΔP_4	Manometer ΔP_5 Initial	Manometer ΔP_5	Manometer ΔP_6 Initial	Manometer ΔP_6	Power, P (calculated)
Units		°F	Count	min	gpm	°F	in.	in.	gpm	°F	°F	°F	psig	amp	Volts	Volts	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	kw
Mult Factor		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1850	All manometers bled																-0.8		0.09		0.18		0.13		0.12				
1710	All manometers cut in																												
1725	96	475	3759	3	0.926	458						390	1085	840	15.2	15.2		-2.20		-6.00		-3.09		-3.33		-3.22			12.77
1740	96	475	3758	3	0.926	458						390	1085	840	15.2	15.2		-2.20		-6.32		-3.37		-3.70		-3.48			12.77
1755	96	475	3756	3	0.925	458						390	1085	840	15.2	15.2		-2.20		-6.39		-3.42		-3.40		-3.50			12.77
1805	96	475	3755	3	0.925	458						390	1085	840	15.2	15.2		-2.20		-6.40		-3.42		-3.70		-3.48			12.77
1815	96	475	3755	3	0.925	458						390	1085	840	15.2	15.2		-2.20		-6.40		-3.42		-3.70		-3.48			12.77

Sample Data Sheet, Run No. 96

TABLE 7

Test Unit: PM-1 Task 5--Subtask 5.8--Heat Transfer
Test Name: Single Tube Test Section STTS-3

Thermocouples									
Time	Run Number	Ambient Temp (Thermometer)	°F	Units	Mult	Factor	Remarks	High Ther- mocouple Rec. (T ₁₈) °F	Read. °F
1730	86	80	9.30	9.32	80	1	Rezereoed potentiometer		
1740	86	80	9.29	9.32	80	1			
1755	86	80	9.28	9.32	80	1			
1805	86	80	9.28	9.32	80	1			
1815	86	80	9.28	9.32	80	1			
1730	80						Rezereoed potentiometer		
1740	80								
1755	80								
1805	80								
1815	80								
1730	96	80	11.30	11.46	80	1	Rezereoed potentiometer		
1740	96	80	11.27	11.46	80	1			
1755	96	80	11.28	11.46	80	1			
1805	96	80	11.28	11.47	80	1			
1815	96	80	11.28	11.46	80	1			
1730	96	80	12.84	12.86	80	1	Rezereoed potentiometer		
1740	96	80	12.85	12.86	80	1			
1755	96	80	12.85	12.86	80	1			
1805	96	80	12.85	12.86	80	1			
1815	96	80	12.85	12.86	80	1			
1730	96	80	12.87	12.87	80	1	Rezereoed potentiometer		
1740	96	80	12.87	12.87	80	1			
1755	96	80	12.87	12.87	80	1			
1805	96	80	12.87	12.87	80	1			
1815	96	80	12.87	12.87	80	1			
1730	96	80	12.89	12.89	80	1	Rezereoed potentiometer		
1740	96	80	12.89	12.89	80	1			
1755	96	80	12.89	12.89	80	1			
1805	96	80	12.89	12.89	80	1			
1815	96	80	12.89	12.89	80	1			
1730	96	80	12.90	12.90	80	1	Rezereoed potentiometer		
1740	96	80	12.90	12.90	80	1			
1755	96	80	12.90	12.90	80	1			
1805	96	80	12.90	12.90	80	1			
1815	96	80	12.90	12.90	80	1			
1730	96	80	12.87	12.87	80	1	Rezereoed potentiometer		
1740	96	80	12.87	12.87	80	1			
1755	96	80	12.87	12.87	80	1			
1805	96	80	12.87	12.87	80	1			
1815	96	80	12.87	12.87	80	1			
1730	96	80	12.88	12.88	80	1	Rezereoed potentiometer		
1740	96	80	12.88	12.88	80	1			
1755	96	80	12.88	12.88	80	1			
1805	96	80	12.88	12.88	80	1			
1815	96	80	12.88	12.88	80	1			

TABLE 8

Sample Data Reduction Sheet, Run 96, Heat Transfer

Test Name and Number: PM-1 Heat Transfer Tests--Heat Transfer Parameters

Location	Z	T _m	T _w	θ	ΔT _F	q"		h	Δθ _{sub}	ΔT _s
	Axial Distance	Outside Measured Temp	Inside Wall Temp	Coolant Temp	Film Drop	Heat Flux		Film Coefficient	Subcooling	Wall Super-heat
	in.	°F	°F	°F	°F	Btu / ft ² -hr x 10 ⁻³		Btu / ft ² -hr-°F x 10 ⁻³	°F	°F
Inl 1				439.9						
Inl 2				439.8						
1	1	582.5	558.1	443.0	115.1	160.08		1.391	113.3	1.8
2	3	589.7	565.4	449.5	115.9			1.381	106.8	9.1
3	5	596.3	572.0	456.0	116.0			1.380	100.3	15.7
4	7	596.8	572.6	462.5	110.1			1.454	93.8	16.3
5	9	597.9	573.7	469.0	104.7			1.529	87.3	17.4
6	11	600.2	576.0	475.5	100.5			1.593	80.8	19.7
7	13	596.5	572.2	482.0	90.2			1.775	74.3	15.9
8	15	599.8	575.6	488.5	87.1			1.838	67.8	19.3
9	17	598.0	573.8	495.0	78.8			2.031	61.3	17.5
10	19	597.4	573.2	501.5	71.7			2.232	54.8	16.9
11	21	597.9	573.7	507.9	65.8			2.433	48.4	17.4
12	23	596.3	572.0	514.4	57.6			2.779	41.9	15.7
13	25	592.3	568.0	520.9	47.1			3.399	35.4	11.7
14	27	595.1	570.8	527.4	43.4			3.688	28.9	14.5
15	29	595.9	571.6	533.9	37.7	160.08		4.246	22.4	15.3
Out 1				534.2						
Out 2				529.2						
Power (P) = 12.77 kw										
Volumetric flow (Q) = 0.925 gpm										
Mass velocity (G) = 4.10 x 10 ⁵ lb/ft ² -hr										
Pressure (p) = 1100 psia										
Mass flow (M) = 387 lb/hr										

TABLE 9
Sample Data Reduction Sheet, Run No. 96, Pressure Drop

Test Name: PM-1 Heat Transfer Test--Pressure Drop Parameters

	Z	ΔZ	H	ΔP_s	θ	ρ	ΔP_a	ΔP_e	ΔP_f	f	$\Delta \theta_{sub}$	f/f _{iso}	Re x 10 ⁻⁵	St	j
Interval Numbers	Axial Distance in.	Interval Length in.	Manometer Reading (Corrected) in.	Static Pressure Drop psi	Coolant Temperature °F	Coolant Density lb/ft ³	Accelera- tion Pres- sure Drop psi	Elevation Pressure Drop psi	Friction Pressure Drop psi	Friction Factor (Average)	Subcooling (Average) °F		Reynolds Number	Stanton Number	Heat Transfer Factor
1	0.5	5.8	-6.49		441.4	52.1									
			-6.49	0.3699			0.00192	0.3430	0.02498	0.0327	96.1	1.35	0.5067	0.00310	0.00289
2	6.3	5.8	-6.49		460.2	51.1									
3	12.1	5.8	-3.60	0.1824	479.1	50.3	0.00113	0.1672	0.01407	0.0359	67.8	1.50	0.5374	0.00387	0.00360
4	17.9	5.8	-3.83	0.1807	497.9	49.3	0.00130	0.1636	0.01580	0.0394	49.0	1.67	0.5607	0.00494	0.00462
5	23.7	5.8	-3.60	0.1824	516.7	48.2	0.00161	0.1596	0.02119	0.0516	30.2	2.21	0.5849	0.00720	0.00681
	29.5				535.5	46.9									
			$\Sigma \Delta P_s = 0.9154$ psi												
Density of water in manometer, $\rho_{MW} = 62.189$ lb/ft ³															
Densities of manometer fluid, $\rho_{MFI} = 74.844$ lb/ft ³ ; $\rho_{MFT} = 183.989$ lb/ft ³															
$\Delta P_{sT} = 1.0634$ psi															

temperature $\theta_{in,2}^t$, obtained from the coolant thermocouple in the inlet tube assembly flange (Fig. 9), was used, since it gave a better result on the test section heat balance*. The mass flow rate, then, was found from:

$$M = 8.021 \rho Q \quad (3)$$

where, in functional notation,

$$\rho = \rho(\theta_{in,2}^t, p) \quad (4)$$

The mass velocity is then

$$G = \frac{M}{A_F} \quad (5)$$

Initially, the difference between the measured outside temperature and the inside wall temperature was calculated by Wilson's method (Ref. 2); however, highly erroneous results were obtained and a different method was then employed.

The results of a number of runs with no power applied to the test section showed the magnitude of the heat loss to be within the accuracy of the instruments. The heat loss was therefore considered negligible. Also, of the four coolant temperature measurements, the pair which gave the best heat balance was at the inlet, $\theta_{in,2}$ and $\theta_{ex,1}$; however, even this combination indicated in many cases that slightly more (within 5%) heat was removed by the coolant than was applied to the test section, as determined by electrical measurements. Thus, the most accurate estimate of the heat removed by the coolant was taken to be the power applied to the test section, as determined by the measurements of current and voltage.

The diffusion equation, considering heat conduction only in the radial direction, in a tube with an insulated outer surface and a uniform internal source can be solved to give

$$T_w = T_m - \frac{S}{2\bar{k}} \left[r_o^2 \ln \left(\frac{r_o}{r_i} \right) - \frac{r_o^2 - r_i^2}{2} \right] \quad (6)$$

*The thermocouple in the inlet elbow was replaced after Run 102 and subsequently $\theta_{in,1}$ was taken as the more accurate measure of the inlet coolant temperature.

where

$$S = \frac{3413 P}{\pi(r_o^2 - r_i^2) L} \quad (7)$$

and

$$\bar{k} = 8.27 + 0.004861 \left(\frac{T_m + T_w}{2} \right) \quad (8)$$

Incorporating the values of r_o , r_i and L for the test section, Eqs (6) and (7) were combined to yield:

$$T_w = T_m - 21.1 \frac{P}{\bar{k}} \quad (9)$$

Equations (8) and (9) can usually be solved in a single iteration. The constants in Eq (8) were determined by assuming a linear relationship between the values of thermal conductivity at 212° F and 932° F for Type 347 stainless steel (Ref. 3).

To find the overall coolant temperature rise, the following equation based on the applied power was used:

$$\Delta \theta = \frac{3413 P}{Mc_p} \quad (10)$$

where c_p was taken as:

$$c_p = c_p \left(\frac{\theta_{in.2} + \theta_{ex1}}{2}, p \right) \quad (11)$$

To find the coolant temperature opposite each thermocouple along the tube, this value of $\Delta \theta$ was divided linearly and added to $\theta_{in.2}$ as follows:

$$\theta(Z) = \theta_{in.2} + \frac{Z}{L} \Delta \theta \quad (12)$$

The outlet coolant temperature might have been used as a base for these calculations, but the two inlet coolant thermocouples generally showed better agreement with each other than did the two outlet coolant thermocouples; therefore, $\theta_{in.2}$ rather than θ_{ex1} was used. The error in

using Eq (11) instead of performing an iteration using

$$c_p = c_p \left(\theta_{in. 2} + \frac{\Delta \theta}{2}, p \right) \quad (13)$$

was negligible.

The remaining columns on the heat transfer data reduction sheet (Table 8) were calculated by the following equations:

$$\Delta T_F = T_w - \theta \quad (14)$$

$$q'' = \frac{3413 P}{A_s} \quad (15)$$

$$h = \frac{q''}{\Delta T_F} \quad (16)$$

$$\Delta \theta_{sub} = \theta_{sat} - \theta \quad (17)$$

$$\Delta T_S = T_w - \theta_{sat} \quad (18)$$

B. PRESSURE DROP DATA SHEET

On the data reduction sheet for pressure drop parameters (Table 9), the corrected manometer reading for each increment of tube length was entered opposite the axial distance from the tube inlet to the beginning of the increment. The exceptions are the readings for the first two increments, which were combined owing to the failure of one of the six high pressure manometers. As mentioned above, the manometer readings were corrected using the zero readings. The equation is

$$H = H' - \Delta H \quad (19)$$

Since the scale of the inclined, well-type manometer is calibrated to account for both the angle of inclination and the small deflection of the fluid in the well, the value, H , is equivalent to the total deflection of a U-tube manometer. The static pressure drop is, thus,

$$\Delta p_s = \left(\rho_{MF} - \rho_{MW} \right) \frac{H}{1728} + \rho_{MW} \frac{\Delta z}{1728} \quad (20)$$

where the densities of the manometer fluid and the water in the manometer are taken at room temperature. The densities of the manometer fluids were found with the use of the curves of specific gravity versus temperature supplied by the manufacturer (Figs. 12 and 13). The 1.2

Fig. 12. Specific Gravity Versus Temperature

Fig. 13. Specific Gravity Versus Temperature

specific gravity fluid was used in the manometers which measured the incremental pressure drops, while the heavier fluid was used in the instrument measuring the drop over the whole length of tube. The latter measurement was corrected using Eq (19) and converted to psi with Eq (20), and it appears as the last item on the pressure drop page to serve as a check on the sum of the incremental drops, $\Sigma \Delta p_s$. However, the taps across which the total measurement was made span a larger interval than the interval between the first and last incremental taps (Fig. 9), the ratio of the intervals being 1.162. The use of this ratio, then, provides a rough comparison between $\Sigma \Delta p_s$ and $\Sigma \Delta p_{sT}$, since a linear extrapolation is not strictly correct unless the pressure gradient is uniform.

The coolant temperature opposite each incremental pressure tap was calculated using Eq (12) in the manner described for the heat transfer data reduction sheet. The coolant densities were then found at the same locations, as a function of the coolant temperature and system pressure.

In order to obtain the friction pressure drop from the measured static pressure drop, it is necessary to integrate the following differential equation (Ref. 4):

$$-\frac{dp}{\rho} = \frac{fV^2}{2gD} dg + d\left(\frac{V^2}{2g}\right) + dz \quad (21)$$

For uniform cross section and over a small increment, the result can be written:

$$\Delta p_f = \Delta p_s - \Delta p_a - \Delta p_e \quad (22)$$

where

$$\Delta p_s = p_1 - p_2 \quad (23)$$

$$\Delta p_a = \frac{G^2}{g} \left(\frac{1}{\rho_2} - \frac{1}{\rho_1} \right) \quad (24)$$

$$\Delta p_e = \bar{\rho} \Delta z \quad (25)$$

$$\Delta p_f = \frac{G^2}{2gD} \frac{\bar{f}}{\bar{\rho}} \Delta s \quad (26)$$

Thus, the acceleration and elevation drops were evaluated with Eqs (24) and (25) and, together with the static drop found with Eq (20), were substituted into Eq (22) to yield the friction drop. The Darcy friction factor was then determined by evaluating Eq (26) in the form

$$f = \frac{2g D \bar{\rho} \Delta p_f}{G^2 \Delta z} \quad (27)$$

where the bar notation on f has been dropped, though it should be kept in mind that the value represents an average for the increment.

The subcooling for each increment was found from

$$\Delta \theta_{\text{sub}} = \theta_{\text{sat}} - \bar{\theta} \quad (28)$$

and the Reynolds number was determined by:

$$\text{Re} = \frac{G D}{\bar{\mu}} \quad (29)$$

where

$$\bar{\mu} = \mu(\bar{\theta}, p) \quad (30)$$

The isothermal friction factor was determined with an empirical equation found by means of a least squares fit of data obtained in the preliminary vertical isothermal runs:

$$f_{\text{iso}} = \frac{0.0374}{(\text{Re} \times 10^{-4})^{0.267}} \quad (31)$$

The Stanton number is defined as:

$$\text{St} = \frac{\bar{h}}{G \bar{c}_p} \quad (32)$$

where

$$\bar{c}_p = c_p(\bar{\theta}, p) \quad (33)$$

and the film coefficient,

$$\bar{h} = h(\bar{z}) \quad (34)$$

was found by interpolation on the heat transfer data reduction sheet. Finally, the heat transfer factor is defined as

$$j = (\text{St}) (\text{Pr})^{2/3} \quad (35)$$

where the Prandtl number,

$$\text{Pr} = \text{Pr}(\theta, p) = \frac{\bar{c}_p \bar{\mu}}{\bar{k}} \quad (36)$$

Fig. 14. Temperature Versus Distance from Inlet--Run 25 (nonboiling)

Fig. 15. Temperature Versus Distance from Inlet--Run 59 (nonboiling)

Fig. 16. Temperature Versus Distance from Inlet--Run 79 (nonboiling)

Fig. 17. Temperature Versus Distance from Inlet--Run 133 (nonboiling)

Fig. 18. Temperature Versus Distance from Inlet--Run 147 (nonboiling)

Fig. 19. Temperature Versus Distance from Inlet--Run 60 (nonboiling)

Fig. 20. Temperature Versus Distance from Inlet--Run 80 (nonboiling)

Power = 10.11 kw
Pressure = 1300 psia
Flow = 6.02 gpm

Fig. 21. Temperature Versus Distance from Inlet--Run 132 (nonboiling)

Fig. 22. Temperature Versus Distance from Inlet--Run 146 (nonboiling)

VI. DATA ANALYSIS*

A. FORCED CONVECTION HEAT TRANSFER

To check the reproducibility of the data, nonboiling runs at the same conditions of heat flux, pressure and flow rate were made at regular intervals throughout the duration of the program. Graphs of temperature versus axial distance for these runs are shown in Figs. 14 through 18 for 4 gpm and Figs. 19 through 22 for 6 gpm. The actual date of the run appears at the top of each chart. Since the change in film coefficient due to changes in the thermal properties of the coolant between inlet and outlet is negligible, the difference between the inside wall temperature and bulk coolant temperature (film drop) should be independent of distance from the inlet. The dashed line is drawn assuming the film drop at the inlet to be the true value. The difference between the inside wall temperature and the dashed line at any axial location is then a measure of the thickness of boiler scale at the location.

There are several things worth noting in this set of curves. A buildup of scale is indicated over the first 11 inches of the test section, after which the thickness remains constant. This buildup was also found in a visual inspection of the tube with a boroscope, at the end of the test program. Secondly, it can be seen that the scale thickness increases with time-- the greatest increase occurring between Runs 59 and 60, and 79' and 80.** The scale thickness, prior and subsequent to this series of runs is, for the most part, constant. Finally, the high degree of consistency in the shapes of the wall temperature curves lends credulity to the above conclusions.

The scale effect is also demonstrated in the comparison of the results of the same nonboiling runs with the Dittus-Boelter equation in Figs. 23 through 27. Since the inside surface temperature was calculated without accounting for the temperature difference across the scale, it is higher than it should be. Therefore the apparent film drop is too high and the calculated film coefficient and Nusselt number are too low. Thus, on each chart there is a cluster of points below the Dittus-Boelter line.

The several points which consistently appear above the line are actually from measurements near the beginning of the heated section, although there is no indication of axial distance on the figures. That these points are higher than the cluster of points is another indication of the increase in scale thickness with length over the first portion of the

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**The prime beside a run number indicates a repetition of a run of the same number.

Fig. 23. Nusselt Number Versus Reynolds Number for Nonboiling Runs

Fig. 24. Nusselt Number Versus Reynolds Number for Nonboiling Runs

Fig. 25. Nusselt Number Versus Reynolds Number for Nonboiling Runs

Fig. 26. Nusselt Number Versus Reynolds Number for Nonboiling Runs

tube. That these points are above the Dittus-Boelter line could be due in part to the entrance effect. Before fully developed turbulent flow is attained, the laminar sublayer is not fully formed and there is more turbulence at the tube wall. Using the equation of Hausen (Ref. 16), a Nusselt number which is at most 10% greater than Dittus-Boelter predicts can be calculated for this effect at the first thermocouple location. Several more percent may be added for axial heat conduction to the inlet header, but the remainder of the departure from Dittus-Boelter must be attributed to the $\pm 25\%$ accuracy limits associated with that equation.

B. LOCAL BOILING HEAT TRANSFER

In analyzing the local boiling heat transfer data, the inside wall temperature was plotted against distance from the tube inlet from data of 10 runs chosen at random from the total of 96 local boiling runs (Figs. 28 through 37). These graphs indicated that the wall temperature does, as has been found by previous investigators, remain constant over the length of tube in local boiling. Therefore, in plotting wall superheat versus heat flux, only one value for each run was used, where this value was the arithmetic mean of all readings in local boiling. Rather than plot wall temperature versus distance along the tube for each run, the point of incipient local boiling was determined by inspection of the tabulated data. All values of wall superheat after this point were used to compute the mean.

The plots of wall superheat versus heat flux (Figs. 38 through 40) fall slightly above the Jens and Lottes equation. At first glance, this difference might be attributed to the scale; however, re-examination of the nonboiling Figs. 16 through 18 and 20 through 22 indicates the temperature difference across the scale to be from 20 to 30° F at a heat flux as low as 1.25×10^5 Btu/ft²-hr (10 kw). If this difference were used to correct the wall superheats, the conclusion would be that the scale-water interface temperature was below saturation and, consequently, that local boiling did not occur. Since there is no doubt that the test section was operating in the local boiling region, it must be concluded that the effect of the scale was smaller in local boiling (the temperature difference across it was smaller) than in nonboiling.

The reason for this is not known with any certainty. The nature of the scale, which was soft, flaky and quite porous, leads to the consideration of one possibility. That is, that the porosity of the scale did not greatly inhibit the mechanism of local boiling but did increase the effective thickness of the laminar sublayer during nonboiling heat transfer.

The mean deviations for the wall superheat data relative to the wall superheats predicted by the Jens and Lottes equation are listed in Table 10. It is seen that the mean deviations varied from 32 to 56%. The Jens

Fig. 28. Inside Wall Temperature Versus Axial Distance--Run 41

Fig. 29. Inside Wall Temperature Versus Axial Distance--Run 53A

Fig. 30. Inside Wall Temperature Versus Axial Distance--Run 65

Fig. 31. Inside Wall Temperature Versus Axial Distance--Run 77

Fig. 32. Inside Wall Temperature Versus Axial Distance--Run 85

Fig. 33. Inside Wall Temperature Versus Axial Distance--Run 94

Fig. 34. Inside Wall Temperature Versus Axial Distance--Run 109

Fig. 35. Inside Wall Temperature Versus Distance--Run 119

Fig. 36. Inside Wall Temperature Versus Axial Distance--Run 130

Fig. 37. Inside Wall Temperature Versus Axial--Run 143

Fig. 38. Wall Superheat Versus Heat Flux--1300 psia

Fig. 39. Wall Superheat Versus Heat Flux--1100 psia

Fig. 40. Wall Superheat Versus Heat Flux--1200 psia

and Lottes equation, however, gives wall superheats to an accuracy of 14° F; and in the range of superheats covered in this program, this amounts to 100% or greater.

TABLE 10
Mean Deviation of Wall Superheat Data Relative
to Jens and Lottes Equation

<u>Figure</u>	<u>Pressure (psia)</u>	<u>Mean Deviation (%)</u>
38	1300	48.0
39	1100	40.1
40	1200	32.5
41	1300, Runs 41 through 58	55.7
42	1300, Runs 61 through 78	38.9

In an attempt to explain the greater amount of scatter in the 1300-psia runs (Fig. 38), which were made before the 1100- and 1200-psia runs (Figs. 39 and 40), the data for 1300 psia were divided into two groups: (1) before scaling, Runs 41 through 58' (Group I); and (2) during or after scaling, Runs 61 through 78 (Group II). It can be seen that the amount of scatter in the Group I data (Fig. 41) is less than that in the Group II data (Fig. 42).

C. LOCAL BOILING PRESSURE DROP

In spite of the scale and its uncertain effect on the mechanism of local boiling heat transfer, considerable success was achieved in correlating local boiling friction factor in terms of Reynolds number and heat transfer factor in an equation of the form

$$f/f_{iso} = a + bj + cRe \quad (37)$$

The constants a, b, and c were determined by a least squares regression performed on the data for each pressure (the statistical analysis is discussed in Appendix C). In Figs. 43 through 45, f/f_{iso} is plotted against Reynolds number, with the heat transfer factor as a parameter. In order to draw in the fitted lines, the average heat transfer factor for each group was substituted into the empirical equation. The information presented on each chart includes the empirical equation as well as a table of the j interval, number of points, average j and the symbol for each group.

Fig. 41. Wall Superheat Versus Heat Flux--1300 psia

Fig. 42. Wall Superheat Versus Heat Flux--1300 psia

Fig. 43. f/f_{iso} Versus Reynolds Number for 1300 psia

Fig. 44. f/f_{iso} Versus Reynolds Number for 1100 psia

Fig. 45. f/f_{iso} Versus Reynolds Number for 1200 psia

Only in the 1300-psia runs did nonboiling exist over a length of tube substantial enough to result in $f/f_{iso} = 1$. It should be recalled that the first pressure drop increment was a double one, 11.6 inches long, and local boiling over the latter part could yield an $f/f_{iso} > 1$ despite non-boiling over the lower portion of the increment. By inspection of the plotted points for 1300 psia (Fig. 43), it was decided to consider these points with $Re < 0.65 \times 10^5$ and belonging to the lowest j group ($j < 0.003$) separately from all other points at 1300 psia.

The f_{iso} (Eq (31)) was determined from vertical, zero power runs made within the same period, in regard to scale buildup on the test section, as the Group I local boiling runs (Runs 41 through 58'). Therefore, the ratio, f/f_{iso} , should essentially remove the scale effect from the friction factors of that group. Data of this group are plotted separately in Fig. 46 and are slightly lower than those of Fig. 43. The 95% confidence limits for the Group I equation are shown in Fig. 47. (For the equation of these limits, see Appendix C.) The limits are within 6% of the fitted equation for the investigated ranges of the independent variables. A comparison of the actual values of f/f_{iso} with the values calculated from the equation is made in Fig. 48.

Data of J. B. Reynolds* (Ref. 5) were also correlated successfully in the form of Eq (37) and appear in Fig. 49. The objective of this effort was to determine whether the form of Eq (37) was applicable to other local boiling pressure drop data. The data for each of four pressures were first correlated separately in an attempt to find a dependency of the constants a , b , and c on pressure, but the spread on pressure was apparently insufficient to show an effect, and the data were then grouped for a range of 45 to 100 psia.

The results of Figs. 43 to 49 are summarized in Table 11 together with the multiple correlation coefficient (r) and the number of points used in each fit (n). Although there is no consistent trend of the constants with pressure within the low pressure group of equations and within the high pressure group of equations, the difference between the two sets is great enough to indicate a dependency on pressure. This dependency is also reflected by the f/f_{iso} values for $j = 0.003$ and $Re = 10^5$ in the last column of the table.

*A contemporary investigator not to be confused with Osborne Reynolds, after whom the Reynolds number was named.

$$f/f_{iso} = 0.080 + 222j + 0.601 Re \times 10^{-5}$$

Fig. 46. f/f_{iso} Versus Reynolds Number for 1300 psia--Runs 41 to 58

Fig. 47. 95% Confidence Limits for f/f_{iso} Versus Reynolds Number at 1300 psia-- Runs 41 to 58

Fig. 48. Actual f/f_{iso} Versus Calculated f/f_{iso} 1300 psia--Runs 41 to 58

Fig. 49. f/f_{iso} Versus Reynolds Number for Pressures of 45 to 100 psia

TABLE 11
Summary of f/f_{iso} Correlations in the Form of Equation (37)

	p (psia)	a	b	c ($\times 10^{-5}$)	r	n	$\left(\frac{f}{f_{iso}}\right)$	$\bar{J}$	$\overline{Re}$ ($\times 10^{-5}$)	$\frac{f}{f_{iso}}$ (0.003, 10^5)
STTS-3	1100	0.189	176	1.35	0.95	119	1.87	0.00380	0.753	2.07
	1200	0.402	151	1.14	0.92	116	1.76	0.00341	0.741	2.00
	1300	0.032	224	0.702	0.94	109	1.41	0.00331	0.901	1.41
	1300 (Group I)	0.080	222	0.601	0.92	61	1.40	0.00352	0.900	1.35
J. B. Reynolds	45	-3.10	1340	1.75	0.90	13	2.41	0.00258	1.17	2.67
	60	-2.23	1120	1.65	0.86	15	2.72	0.00255	1.27	2.78
	75	-3.99	1730	1.74	0.96	15	2.51	0.00240	1.34	2.94
	100	-1.11	1130	0.62	0.85	13	2.60	0.00251	1.40	2.90
	45 to 100	-2.54	1326	1.37	0.87	56	2.56	0.00251	1.30	2.77

Perhaps the most significant values in Table 11 are the multiple correlation coefficients (r). The very high r 's indicate that the linearity of Eq (37) is not just a convenient form for performing the regression, but that the relationship of the three parameters is truly linear. The linear form was further verified by a χ^2 test that showed no significant difference between the distribution of the associated standardized variable and the normal distribution. This linear relationship between the heat transfer parameter, j , and the momentum transfer parameter, f/f_{iso} , has theoretical basis in the analogy drawn by Osborne Reynolds between the two exchange mechanisms (Ref. 11). Use of the analogy in the local boiling region was suggested in a paper by Sabersky and Mulligan (Ref. 6). It is usually presented with the Stanton number as a linear function of the friction factor and sometimes includes a dependency on the Prandtl number (Ref. 7). A further dependency of the relationship on Reynolds number has also been presented in connection with a fluidized bed (Ref. 8).

In Fig. 50, the equation for the Group I data at 1300 psia has been extrapolated to the nonboiling region by substituting $f/f_{iso} = 1$. The agreement with the Dittus-Boelter equation is good over a limited Reynolds number range in spite of the extrapolation and in spite of the rearrangement of the fitted equation. Since the regression done was f/f_{iso} on j and Re , no form other than that of Eq (37) is correct using the coefficients a , b , and c ; however, the correlation coefficient is 0.92 and if it were 1.00 the regression coefficients would be the same regardless of the arrangement of the equation.

The Group I data were plotted in Fig. 51 for comparison with the Westinghouse equation (Ref. 9). The data were higher than that equation predicts and did not correlate well with subcooling.

Better correlation was obtained using Rohde's parameter $(q''/q_0'' - 1)^*$. A comparison with Rohde's correlation (Ref. 10) of the Buchberg data (Ref. 12) is made in Fig. 52. Here, too, the STTS-3 data are slightly higher, although the Buchberg pressure drops were measured over the entire length of the test section and might be too low as a consequence of including a portion in nonboiling. The result of a least squares fit of the Group I data is shown in the figure. The correlation coefficient for that equation is 0.79.

*Where q_0 is the nonboiling heat flux which the same film drop and the Dittus-Boelter coefficient would yield.

Fig. 50. Heat Transfer Factor Versus Reynolds Number

Fig. 51. f/f_{iso} Versus $\Delta\theta_{sub}$ at 1300 psia--Runs 41 to 58

Fig. 52. (f/f_{iso}) Versus $(\frac{q''}{q_0''} - 1)$ for 1300 psia--Runs 41 to 58

VII. CONCLUSIONS AND RECOMMENDATIONS

The recommended design correlation for local boiling friction factor at 1300 psia is

$$\frac{f}{f_{iso}} = 0.080 + 222 j + 0.601 (Re \times 10^{-5}) \quad (38)$$

$$0.002 \leq j \leq 0.010$$

$$0.5 \times 10^5 \leq Re \leq 1.5 \times 10^5$$

The local boiling friction factor and isothermal friction factor data which were used in the above correlation were obtained during the same period of time. It is felt, therefore, that the influence of scale deposition has been minimized in the correlation. The correlation was obtained by a least squares regression which gave a correlation coefficient of 0.92.

As the discussion below will indicate, the existence of scale introduced uncertainty into the determination of the wall superheat. On the other hand, the temperature difference between the wall-water interface and the coolant is much greater than the wall superheat, hence less in error. That this temperature difference was used to determine the film coefficient which, in turn, was used in the parameter, j , of the friction factor correlation probably accounts for the fact that the correlation showed no effect of the scale.

The recommended friction factor correlation (Eq. 38) has been incorporated in the BITE (Boiling in Tubular Elements) code. This IBM 7090 program is used for thermal and hydraulic analysis on cores with tubular fuel elements; hence, the results of the experimental study are available for all future design effort.

An accurate local boiling heat transfer correlation is also required by the reactor designer. For tubular fuel elements, the heat flux is not an independent variable (as is the case for rod-type elements). The relative amounts of heat which pass across the inner and outer walls are calculated, using both the diffusion equation and the heat transfer conditions at these boundaries.

In the design of the PM-1 core, the division of heat flow to the inner and outer surfaces of those fuel elements in local boiling was calculated using the Jens and Lottes equation to specify the tube boundary conditions. It was found that approximately one-half of the heat generated within the element was transferred to the coolant at each surface

of the tube. The ratio of coolant flow rate inside and outside of the fuel elements was then selected to result in equal outlet coolant temperatures.

The local boiling heat transfer data obtained in this test program confirmed the Jens and Lottes correlation reasonably well, considering the increase in wall superheat which would be expected, due to the presence of scale. The mean deviations of the wall superheat data for the various pressures, as compared to the Jens and Lottes equation, ranged from 32% at 1200 psia to 56% at 1300 psia. The Jens and Lottes equation, however, gives wall superheats to within 14° F and, in the range of wall superheats observed in the program, this would amount to a deviation of approximately 100%. Thus, the experimental data fall within the uncertainty associated with the Jens and Lottes equation.

It was observed that scale deposition has a much smaller effect on local boiling heat transfer than on forced convection heat transfer. Nevertheless, it is felt that additional heat transfer data should be obtained before a local boiling heat transfer correlation can be recommended.

It is recommended that the work on local boiling heat transfer and pressure drop inside tubes be continued. The work is applicable to tubular fuel elements, and the single-tube test section can be well instrumented to yield high quality data. The following recommendations are specified:

- (1) That a second program be conducted with the single-tube test section, the parameters to cover a wider range than those of the initial program. Consideration is to be given, in the program design, to the accuracy of pressure drop measurements. Thus, velocities may be selected which give readily measured pressure drops.
- (2) That the program be preceded by tests designed to check out a water treatment procedure, which ensured the absence or minimization of scale deposition.
- (3) That refined techniques for wall temperature measurements be employed. These may include use of a guard heater to eliminate the need for corrections due to heat losses and/or use of three-wire thermocouples welded directly to the tube wall.

VIII. REFERENCES

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APPENDIX A

A. WATER TREATMENT

It has been observed that the deposition of corrosion products occurs preferentially on heat transfer surfaces in pressurized water loops. This scale deposition can give rise to both erroneous pressure drop measurements and false heat transfer information. Thus, the water treatment employed in this program was chosen with the objective of minimizing scale formation on the heat transfer surface. Present data indicate that fouling is caused by the combined condition of low pH (5 to 7) and the presence of large quantities of oxygen. This condition had been satisfied in a prior program (STTS-2), despite the fact that demineralized and deaerated water was charged into the loop. The water pH was neutral, that is, 7, but a continuous source of oxygen was provided by the dissociation of water at the heat transfer surface. This dissociation is encountered in both nuclear reactors and pressurized water loops employing direct current resistance heating due to the ionizing potential present at the heat transfer surface. A hard oxide film was developed on the heat transfer surface despite the fact that the water purity was maintained at a resistivity of approximately 800,000 ohms.

During tests on STTS-3, a second approach, based on the findings of the prior program and the mechanism described above, was employed. This method required that the pH be maintained between 9.5 and 10.5. A pH of 9.5 was generally maintained by using 75 ppm of 67% disodium phosphate and 33% trisodium phosphate. Deaerated and demineralized (1×10^6 ohm-cm) water was again charged into the loop and the phosphate was added. However, the results were still not satisfactory, and the bulk of the data obtained in this program was realized with a test section coated with a soft, flaky scale several mils thick. The scale thickness increased with distance along the length of the test section, with the maximum thickness occurring at the outlet. Emission spectroscopy revealed the light brown scale to be heavy in phosphorous, with the principal metal component being magnesium. Moderate quantities of iron and copper were also found, while calcium and zinc occurred to a minor degree.

Since both of the approaches enumerated above failed to give results satisfactory for experimental heat transfer and pressure drop studies, a third method will be employed in future tests. This method consists of controlling the pH of the working fluid by means of a mixed bed resin, Amberlite XE-170. This resin is composed of a mixture of a strongly acidic cation exchange resin, Amberlite XE-169 and a strongly basic anion exchange resin, Amberlite XE-78. An ammonium-to-hydroxide ratio of 1:1 is obtained with the effluent product being dilute NH_4OH .

Control of pH will be obtained by adjusting the flow of water through the demineralizer bed. This water treatment will be checked out in preliminary forced convection runs before actual data runs are undertaken.

APPENDIX B

Thermocouple Calibration Correction Equations

θ_{in_1}	=	$-3.83 + 1.00092 (\theta_{in_1})$	
θ_{in_1}	=	$-5.55 + 1.01020 (\theta_{in_1})$	(used for Runs 101A and up)*
θ_{in_2}	=	$-4.07 + 0.99735 (\theta_{in_2})$	
T_{m_1}	=	$-7.29 + 1.00482 (T_{m_1})$	
T_{m_2}	=	$-6.81 + 1.00426 (T_{m_2})$	
T_{m_3}	=	$-7.01 + 1.00471 (T_{m_3})$	
T_{m_4}	=	$-2.92 + 0.99875 (T_{m_4})$	
T_{m_5}	=	$-3.20 + 0.99928 (T_{m_5})$	
T_{m_6}	=	$-1.78 + 0.99668 (T_{m_6})$	
T_{m_7}	=	$-2.43 + 0.99823 (T_{m_7})$	
T_{m_8}	=	$-6.91 + 1.00541 (T_{m_8})$	
T_{m_9}	=	$-7.38 + 1.00653 (T_{m_9})$	
$T_{m_{10}}$	=	$-3.07 + 0.99827 (T_{m_{10}})$	
$T_{m_{11}}$	=	$-6.69 + 1.00435 (T_{m_{11}})$	
$T_{m_{12}}$	=	$-5.25 + 1.00173 (T_{m_{12}})$	
$T_{m_{13}}$	=	$-6.05 + 1.00288 (T_{m_{13}})$	
$T_{m_{14}}$	=	$-4.15 + 1.00038 (T_{m_{14}})$	
$T_{m_{15}}$	=	$-5.32 + 1.00255 (T_{m_{15}})$	
θ_{ex_1}	=	$-3.43 + 0.99685 (\theta_{ex_1})$	

*The thermocouple in the inlet elbow was changed after Run 102.

$$\begin{aligned}\theta_{ex_2} &= -2.54 + 1.00045 (\theta_{ex_2}) \\ \theta_{ex_2} &= -3.51 + 1.00609 (\theta_{ex_2}) \text{ (used for Runs 134 and up)**}\end{aligned}$$

****The thermocouple in the outlet elbow was changed after Run 133.**

APPENDIX C

Statistical Analysis

As mentioned in the text, the independent variables, j and Re , were suggested by the paper of Sabersky and Mulligan (Ref. 6) and previous work in the field of fluidization (Ref. 8). After plotting f/f_{iso} against the parameters and noting the linearity, it was decided to perform a regression analysis with an equation of the form*

$$(Y - \bar{y}) = b(x_1 - \bar{x}_1) + c(x_2 - \bar{x}_2) \quad (1)$$

which can be rearranged

$$Y = a + bx_1 + cx_2 \quad (2)$$

thus defining

$$a = \bar{y} - b\bar{x}_1 - c\bar{x}_2 \quad (3)$$

In such an analysis, the quantity, $\sum_{i=1}^n (y_i - Y_i)^2$, is minimized, thus minimizing the deviation, $y_i - Y_i$, which is the vertical distance between the experimental point (x_{1i}, x_{2i}, y_i) and the corresponding point (x_{1i}, x_{2i}, Y_i) on the empirical regression plane. Having minimized the sum of the squares of the deviations by the method from the calculus, the estimates of the regression coefficients, b and c , can be seen to

be dependent upon $\sum_{i=1}^n x_{1i}$, $\sum_{i=1}^n x_{2i}$, $\sum_{i=1}^n y_i$, $\sum_{i=1}^n x_{1i}^2$, $\sum_{i=1}^n x_{2i}^2$, $\sum_{i=1}^n x_{1i}x_{2i}$,

$\sum_{i=1}^n x_{1i}y_i$ and $\sum_{i=1}^n x_{2i}y_i$. Several techniques for calculating the coefficients are available. The one used was derived and begins by calculating the following:

$$\alpha = n \sum_{i=1}^n x_{2i}^2 - \left(\sum_{i=1}^n x_{2i} \right)^2 \quad (4)$$

* See the end of this Appendix for list of symbols used.

$$\beta = n \sum_{i=1}^n x_{1i} y_i - \sum_{i=1}^n x_{1i} \sum_{i=1}^n y_i \quad (5)$$

$$\gamma = n \sum_{i=1}^n x_{1i} x_{2i} - \sum_{i=1}^n x_{1i} \sum_{i=1}^n x_{2i} \quad (6)$$

$$\delta = n \sum_{i=1}^n x_{2i} y_i - \sum_{i=1}^n x_{2i} \sum_{i=1}^n y_i \quad (7)$$

$$\epsilon = n \sum_{i=1}^n x_{1i}^2 - \left(\sum_{i=1}^n x_{1i} \right)^2 \quad (8)$$

The equations for the coefficients are then:

$$b = \frac{\alpha \beta - \gamma \delta}{\alpha \epsilon - \gamma^2} \quad (9)$$

$$c = \frac{\delta \epsilon - \beta \gamma}{\alpha \epsilon - \gamma^2} \quad (10)$$

The formula for the estimate of the multiple correlation coefficient is given by Cramer (Ref. 18) and can be rewritten, in terms of the above quantities,

$$r = \left[\frac{\alpha \beta^2 + \delta^2 \epsilon - 2 \beta \gamma \delta}{\rho (\alpha \epsilon - \gamma^2)} \right]^{1/2} \quad (11)$$

where

$$\rho = n \sum_{i=1}^n y_i^2 - \left(\sum_{i=1}^n y_i \right)^2 \quad (12)$$

In order to compute the confidence interval to be used when predicting with the empirically derived equation, the method given by Hald was employed (Ref. 17). The corrected sample variance (square of the standard deviation), which is also called the residual variance (Ref. 18), is first computed according to

$$s^2 = \frac{1}{n-3} \sum_{i=1}^n (y_i - Y_1)^2 \quad (15)$$

where the "correction" refers to the reduction of the number of degrees of freedom from n to $(n-3)$ because three additional equations (constraints) connected with the three constants in the equation for Y (Eq 2) must be satisfied. A second variance consisting of variances of the three constants is then computed by

$$s_y^2 = s^2 \left\{ \frac{1}{n} + \frac{1}{D} \left[(x_1 - \bar{x}_1)^2 \sum_{i=1}^n (x_{2i} - \bar{x}_2)^2 \right. \right. \quad (14)$$

$$- 2(x_1 - \bar{x}_1)(x_2 - \bar{x}_2) \sum_{i=1}^n (x_{1i} - \bar{x}_1)(x_{2i} - \bar{x}_2)$$

$$\left. \left. + (x_2 - \bar{x}_2)^2 \sum_{i=1}^n (x_{1i} - \bar{x}_1)^2 \right] \right\}$$

where

$$D = \sum_{i=1}^n (x_{1i} - \bar{x}_1)^2 \sum_{i=1}^n (x_{2i} - \bar{x}_2)^2 - \left[\sum_{i=1}^n (x_{1i} - \bar{x}_1)(x_{2i} - \bar{x}_2) \right]^2 \quad (15)$$

The standardized variable, $\frac{Y - \eta}{s_y}$, which may now be formed, is t -distributed, with $(n-3)$ degrees of freedom and we have

$$P(Y - t_p s_y < \eta < Y + t_p s_y) = 1 - \frac{P}{100} \quad (16)$$

Equation 16 states that we have the probability $(100 - p)\%$ that the two random variables $Y \pm t_p s_y$ will take values including between them the fixed, but unknown, true value of the parameter, η , which is the mean of the dependent variable population. We thus have a confidence interval specified by Eq (16), which provides us with a rule for estimating the parameter, η , with a constant risk of error equal to $p\%$. The value of p may be arbitrarily chosen, and the corresponding value of t_p is tabulated as a function of p and the number of degrees of freedom (Ref. 18).

It should be noted that the variance s_y^2 , in Eq (14), is a function of the independent variables, x_1 and x_2 , since all arithmetic means and other summations are numbers calculated from the data. Thus the confidence limits, $Y \pm t_p s_y$, are also functions of x_1 and x_2 . Since s_y^2 is a minimum for $x_1 = \bar{x}_1$ and $x_2 = \bar{x}_2$, it follows that we may have the most confidence in predicting with Eq (2) when the values of the independent variable coincide with the means of the experimental data used to determine the predicting equation.

Although the multiple correlation coefficient provides a good check on the assumed linear form, perfect dependency being indicated by a value of 1.0, a further check of the function type can be made by comparing the distribution of the standardized variable, $\frac{y - Y}{s}$, with a normal distribution with mean equal to zero and standard deviation equal to one (Ref. 17). To test the hypothesis that $\frac{y - Y}{s}$ is normal, the chi-squared (χ^2) test may be used (Ref. 18). After computing all the $\frac{y_i - Y_i}{s}$, these values should be grouped into u groups so that there are at least ten values in each group ($f_j \geq 10$) (Ref. 18). The limits which define each group are then used to compute the probability element (expected group frequency) of that group from the tabulation of the normal distribution (Ref. 18) as follows: If the class limits of the j^{th} group are l and m , determine $\Phi(l)$ and $\Phi(m)$ from the table. The probability element for that group is, then

$$n\pi_j = n[\Phi(m) - \Phi(l)] \quad (17)$$

Next calculate

$$\chi^2 = \sum_{j=1}^u \frac{(f_j - n\pi_j)^2}{n\pi_j} \quad (18)$$

It can be seen that in χ^2 we have a direct measure of the departure of the actual distribution, represented by the actual group frequencies, f_j , from the theoretical (in this case the normal) distribution, represented by the theoretical group frequencies, $n\pi_j$. Thus, the smaller χ^2 , the closer the actual distribution resembles the theoretical.

Since there are q unknown parameters estimated from the data (in our case $q = 3$) and an additional linear constraint, the number of degrees of freedom to be used with the χ^2 of Eq (18) is $u - q - 1$.

What is tabulated for the χ^2 distribution (Ref. 18) is χ_p^2 as a function of the number of degrees of freedom, and p , where we have

$$P(\chi^2 > \chi_p^2) = \frac{P}{100} \quad (19)$$

and reference to the table shows χ_p^2 to be directly proportional to the number of degrees of freedom and inversely proportional to p . Thus, for 3 degrees of freedom for $p = 90\%$ (the 90% level of significance), $\chi_p^2 = 0.584$ while for $p = 5\%$ (the 5% level of significance), $\chi_p^2 = 7.815$. We are free to test our value of χ^2 as calculated from Eq (18) on any level we choose. If we have 3 degrees of freedom and we decide to test on the 90% level, we have a large probability (90%) that our χ^2 will be greater than 0.584; so that if χ^2 turns out to be less, we have an excellent test result (the hypothesis that our experimental distribution resembles a normal distribution is acceptable). A less stringent test may be made on the 5% level where $p(\chi^2 > 7.815) = 0.05$, but this test is generally considered adequate for checking the hypothesis.

The above analytical methods are demonstrated in the tables which follow using the data from Runs 41 through 58.

TABLE C-1
Pressure Drop Parameters
1300-psia Runs 41 to 58

$\frac{y}{(f/f_{iso})}$	x_1 (j)	$x_2 \times 10^{-5}$ ($Re \times 10^{-5}$)	Y	$\frac{y - Y}{s}$
0.843	0.00215	1.007	1.162	-2.157
0.896	0.00223	0.977	1.162	-1.798
0.914	0.00210	0.6760	0.952	-0.257
0.951	0.00230	0.6652	0.990	-0.264
0.957	0.00314	0.5027	1.079	-0.825
0.975	0.00250	0.6943	1.052	-0.521
0.978	0.00245	1.065	1.264	-1.934
0.983	0.00235	0.6960	1.020	-0.250
1.093	0.00241	1.046	1.244	-1.021

TABLE C-1 (continued)

y (f/f_{iso})	x_1 (j)	$x_2 \times 10^{-5}$ ($Re \times 10^{-5}$)	Y	$\frac{y-Y}{s}$
1.100	0.00265	1.107	1.334	-1.582
1.104	0.00293	0.7219	1.164	-0.406
1.112	0.00312	0.4876	1.066	0.311
1.129	0.00308	0.6614	1.161	-0.216
1.129	0.00278	0.7186	1.129	0
1.134	0.00302	0.6214	1.124	0.068
1.149	0.00286	0.6550	1.108	0.277
1.172	0.00348	0.5943	1.210	-0.257
1.197	0.00225	0.8047	1.063	0.906
1.204	0.00275	1.079	1.339	-0.913
1.207	0.00365	0.5441	1.217	-0.068
1.214	0.00227	0.8262	1.080	0.906
1.229	0.00250	0.8543	1.148	0.548
1.232	0.00292	0.8220	1.222	0.068
1.241	0.00237	0.7999	1.087	1.041
1.245	0.00251	0.8353	1.139	0.717
1.255	0.00351	0.7563	1.314	-0.399
1.255	0.00268	0.7700	1.138	0.791
1.255	0.00317	1.155	1.478	-1.508
1.259	0.00304	1.076	1.402	-0.967
1.271	0.00319	0.6865	1.201	0.473
1.281	0.00367	0.6515	12.86	-0.034
1.333	0.00291	0.8700	1.249	0.568
1.336	0.00416	0.5150	1.313	0.156
1.342	0.00291	0.8843	1.251	0.575
1.384	0.00400	0.7198	1.400	-0.108
1.385	0.00347	1.132	1.531	-0.987
1.408	0.00362	0.9077	1.429	-0.142
1.451	0.00234	1.398	1.440	0.074
1.468	0.00405	0.6247	1.354	0.771
1.489	0.00561	0.5511	1.657	-1.136
1.540	0.00376	0.8667	1.436	0.703
1.555	0.00234	1.445	1.468	0.588
1.557	0.00379	0.5258	1.237	2.164
1.564	0.00269	1.530	1.597	-0.223
1.582	0.00220	1.481	1.458	0.838
1.598	0.00197	1.438	1.382	1.460
1.600	0.00386	1.200	1.658	-0.392
1.617	0.00526	0.6598	1.644	-0.182
1.676	0.00336	1.4662	1.707	-0.210
1.731	0.00436	1.187	1.761	-0.203
1.753	0.00481	0.9203	1.701	0.352

TABLE C-1 (continued)

$\frac{y}{(f/f_{iso})}$	x_1 (j)	$x_2 \times 10^{-5}$ (Re $\times 10^{-5}$)	Y	$\frac{y - Y}{s}$
1.757	0.00479	0.6818	1.553	1.379
1.777	0.00384	1.4826	1.824	-0.318
1.797	0.00429	1.5406	1.958	-1.088
1.891	0.00305	1.566	1.698	1.305
1.897	0.00544	0.5478	1.617	1.893
1.980	0.00936	0.5729	2.502	-3.529
2.007	0.00531	0.7566	1.714	1.981
2.366	0.00582	1.245	2.120	1.663
2.382	0.00788	0.6960	2.248	0.906
2.401	0.00716	0.9604	2.247	1.041

TABLE C-2

Computation of Regression Equation
and
Multiple Correlation Coefficient

$$\sum_{i=1}^n x_{1_i} = 0.21440$$

$$\sum_{i=1}^n y_i^2 = 127.98892$$

$$\sum_{i=1}^n x_{2_i} = 54.9293 \times 10^5$$

$$\sum_{i=1}^n x_{1_i} x_{2_i} = 0.187211 \times 10^5$$

$$\sum_{i=1}^n y_i = 85.5880$$

$$\sum_{i=1}^n x_{1_i} y_i = 0.324870$$

$$\sum_{i=1}^n x_{1_i}^2 = 8.77639 \times 10^{-4}$$

$$\sum_{i=1}^n x_{2_i} y_i = 79.227480 \times 10^5$$

$$\sum_{i=1}^n x_{2_i}^2 = 55.22904 \times 10^{10}$$

TABLE C-2 (continued)

n = 61 data points

$$\bar{x}_1 = \frac{1}{n} \sum_{i=1}^n x_{1_i} = 3.5148 \times 10^{-3}; \quad \bar{x}_2 = \frac{1}{n} \sum_{i=1}^n x_{2_i} = 0.900480 \times 10^5;$$

$$\bar{y} = \frac{1}{n} \sum_{i=1}^n y_i = 1.403081$$

$$\alpha = 351.743442 \times 10^{10} \text{ (Eq 4)}$$

$$\beta = 1.463579 \text{ (Eq 5)}$$

$$\gamma = -0.359169 \times 10^5 \text{ (Eq 6)}$$

$$\delta = 131.587352 \times 10^5 \text{ (Eq 7)}$$

$$\epsilon = 75.514654 \times 10^{-4} \text{ (Eq 8)}$$

$$\rho = 482.018376 \text{ (Eq 12)}$$

$$b = 222.4 \text{ (Eq 9)}$$

$$c = 0.6012 \times 10^{-5} \text{ (Eq 10)}$$

$$a = 0.080 \text{ (Eq 3)}$$

$$\therefore Y = 0.080 + 222 x_1 + 0.601 x_2 \times 10^{-5} \text{ (Eq 2)}$$

$$\text{and } r = 0.916 \text{ (Eq 11)}$$

TABLE C-3

Computation of 95% Confidence Interval

$$s^2 = 0.02188 \text{ (Eq 13)}$$

$$s' = 0.1479$$

$$D = 6.79192 \times 10^6 \text{ (Eq 15)}$$

TABLE C-3 (continued)

$$\begin{aligned}
 s_y^2 &= 3.5869 \times 10^{-4} + 185.761 (x_1 - 0.003515)^2 \\
 &\quad + 3.9882 \times 10^{-13} (x_2 - 0.90048 \times 10^5)^2 \\
 &\quad + 3.7936 \times 10^{-6} (x_1 - 0.003515) (x_2 - 0.90048 \times 10^5) \text{ (Eq 14)}
 \end{aligned}$$

Number of degrees of freedom = $n - 3 = 58$

If we desire a 95% confidence interval, we choose $p = 5$. From the tabulated t - distribution (Ref. 18), interpolating for 58 degrees of freedom at the 5% level, we find $t_p = 2.001$. Using the s_y calculated above and the derived equation for Y , we thus have

$$P(Y - 2.001 s_y < \eta < Y + 2.001 s_y) = 1 - \frac{5}{100} = 0.95 \text{ (Eq 16)}$$

TABLE C-4

 χ^2 Test

Standardized Class Limits	ξ_j	ξ_j^2	f_j^*	$n\pi_j$	$\frac{(f_j - n\pi_j)^2}{n\pi_j}$
-3.529, -1.25	-2.0847	4.3460	6	6.4492	0.031
-1.25, -0.75	-0.9910	0.9821	7	7.3795	0.020
-0.75, -0.251	-0.3518	0.1238	8	10.6212	0.647
-0.25, 0.25	-0.0847	0.0072	15	12.0274	0.735
0.25, 0.75	0.5112	0.2613	10	10.6448	0.039
0.75, 1.25	0.9000	0.8100	8	7.3795	0.052
1.25, 2.164	1.6921	2.8632	7	5.5271	0.392
				$n = 61$	$\chi^2 = 1.916$

$$\bar{\xi} = \frac{1}{n} \sum_{j=1}^u f_j \xi_j = 0.0103$$

$$s = \left(\frac{1}{n} \sum_{j=1}^u f_j \xi_j^2 \right)^{1/2} = 1.017$$

$u = 7$ groups

$q = 3$ (the parameters estimated from the sample data are a , b and c)

$\therefore$ number of degrees of freedom = $u - q - 1 = 3$

* It can be seen that all $f_j \geq 10$. A less stringent requirement is given by Hald (Ref. 17), viz., $n\pi_j > 5$, and this requirement is satisfied.

From the χ^2 table (Ref. 18), it can be seen that for 3 degrees of freedom, $\chi^2 = 1.916$ falls between the 70 and 50% levels. Since this is less than the 5% value of χ^2_p , there is no significant difference between the tested distribution and a normal distribution with mean zero and standard deviation of one. It is therefore concluded that the assumed linear form of the regression equation is correct.

LIST OF SYMBOLS FOR APPENDIX C

Upper case

- D determinant defined by Eq (15)
- P the probability of, e.g., $P(\mu = 2)$ is read, "the probability that $\mu = 2$."
- Y the value of the dependent variable (f/f_{iso}) on the regression plane, i.e., calculated by Eq (2).

Lower case

- a estimate of the constant in the regression Eq (2)
- b estimate of the regression coefficient of the first independent variable (see Eqs 1 and 2)
- c estimate of the regression coefficient of the second independent variable (see Eqs 1 and 2)
- f frequency of occurrence
- l lower limit of the j^{th} group (see Eq 17)
- m upper limit of the j^{th} group (see Eq 17)
- n number of data points
- p probability, defined variously in Eq (16) or (19)
- q number of parameters estimated from data of the sample population

r	the estimate of the multiple correlation coefficient
s	the estimate of the standard deviation
s_Y	standard deviation defined by Eq (14)
t	the independent variable of the Student's or t distribution
u	number of groups for the χ^2 test
x	independent variable (j or Re) of the data
y	dependent variable (f/f_{iso}) of the data

Greek

α	constant defined by Eq (4)
β	constant defined by Eq (5)
γ	constant defined by Eq (6)
δ	constant defined by Eq (7)
ϵ	constant defined by Eq (8)
ρ	constant defined by Eq (12)
η	the unknown mean value of the dependent variable population
ξ	the group mean of the standardized variable $(y - Y)/s$
π	the probability element of the normal distribution (see Eq 17)
$\sum$	summation of
Φ	the normal distribution function
χ^2	independent variable of the χ^2 distribution

Subscripts

1	first independent variable (j)
2	second independent variable (Re)

- i index running from 1 to n
- j index running from 1 to u
- p denotes a value corresponding to the p% level

Superscripts

bar arithmetic mean