MHD KINK-DRIVEN INSTABILITIES IN NET-CURRENT-FREE STELLARATORS

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ABSTRACT

The Pfirsch-Schlüter current, which is induced in a toroidal device to keep the plasma current divergence-free, is shown to drive a free-boundary instability in a model of a net-current-free ATF-1 stellarator if $\langle \beta \rangle = 2.60$.

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I. INTRODUCTION

The stellarator approach to fusion has been the subject of renewed interest [1]. This stellarator revival was based on the experimental observation of improved confinement when operated with no net toroidal current, the theoretical indication that transport could be improved over what had previously been predicted, and the recognition that steady-state operation in a modular system is possible. A prediction that stable operation with high $\beta \equiv 8\pi p/B^2$ could be achieved [2,3] has contributed significantly to the attractiveness of the approach. This possibility arises from the argument that the Pfirsch-Schlüter currents, which are induced in the plasma to keep the current divergence-free, shift the magnetic axis outward and create an average magnetic well. This well, coupled with the relatively short connection lengths between regions of favorable and unfavorable field line curvature, provides stability with respect to localized magnetohydrodynamic (MHD) instabilities associated with both the Mercier criterion and ballooning-mode formalisms. The short-wavelength assumption used in obtaining these criteria eliminates forces associated with force-free current from consideration. It has been expected [2,3] that, since no net toroidal current flows on the magnetic surfaces in a stellarator to drive kink modes, stability considerations should impose no limitations on the value of β that can be achieved. Since the magnitude of the Pfirsch-Schlüter currents can be large, it is, however, possible they may affect the stability properties. Until now, no stability analysis which properly incorporates them has been made. It is the purpose of this paper to show that they may indeed impose a stability limitation on an interesting stellarator configuration.

In order to study this problem, we use modifications [4,5] of the PEST equilibrium and stability codes [6,7] which incorporate the stellarator

expansion [8], a large-aspect-ratio model in which the effects of the helical field modulation are treated analytically so that the stellarator problem is reduced to a two-dimensional one. This expansion assumes an ordering

$$(B_{\delta}/B_0)^2 \sim B_K/B_0 \sim B_{\beta}/B_0 \sim B_{\sigma}/B_0 \sim a/R \ll 1, \quad (1)$$

where B_0 is a uniform (to lowest order) toroidal field, B_{δ} is a multipolar field with periodicity over a length of the order of the plasma radius a ; B_K contains the correction to B_0 associated with toroidal curvature, B_{β} is a toroidal field associated with the plasma pressure, and $B_{\sigma}(r, \theta)$ is the field created by toroidal axisymmetric currents inside and outside the plasma. In this model [4,8], the lowest-order magnetic surfaces are given by

$$\Psi^{(0)} = \frac{2\pi R}{B_0} \left(B_{\delta r} \int B_{\delta \theta} d\phi \right) + 2\pi R A, \quad (2)$$

with the stream function A satisfying

$$\nabla^2 A = -B_0^2 R p'(\Psi^{(0)}) Q + G(\Psi^{(0)}) \quad (3)$$

Here

$$Q \equiv \frac{\langle B_{\delta}^2 \rangle}{B_0^2} + \frac{2\pi}{R} \cos \theta \quad (4)$$

measures the effective curvature of a magnetic field line and the brackets $\langle \rangle$ denote an average over the multipolar field period. The boundary conditions on A in this generalization of the Grad-Shafranov equation are provided by the externally imposed axisymmetric field far from the plasma [4,8].

The Lagrangian associated with a displacement $\xi^{(0)}$ from an equilibrium reduces to [5,9]

$$\begin{aligned} \mathcal{L} = & \frac{1}{2} \omega^2 \int_p d\tau \rho |\xi_{\perp}^{(0)}|^2 - \frac{1}{2} \int_p d\tau (|Q_{\perp}|^2 + 4\pi J_{\perp} \times \xi_{\perp}^{(0)*} \cdot Q_{\perp} \\ & + 4\pi \xi_{\perp}^{(0)} \cdot \nabla p \xi_{\perp}^{(0)*} \cdot \nabla \eta) - \frac{1}{2} \int_v d\tau |\delta B|^2, \end{aligned} \quad (5)$$

with

$$\xi_{\perp}^{(0)} = \nabla \phi \times \nabla \eta, \quad (6)$$

$$Q_{\perp} = \nabla \phi \times \nabla \left(\frac{B}{R} \frac{\partial}{\partial \phi} + \nabla \phi \times \nabla \Psi^{(0)} \cdot \nabla \right) \eta. \quad (7)$$

Only the components of $\xi^{(0)}$ which are divergence-free and do not vary with ϕ over the period of the multipolar field survive the extremization leading to Eq. (5). As mentioned earlier, the assumption of localization associated with rapid radial variation or high toroidal mode number n , used in obtaining the Mercier and ballooning mode criteria, eliminates the $J_{\perp} \times \xi_{\perp}^{(0)*} \cdot Q_{\perp}$ term in Eq. (5). To our knowledge, all previous stability analyses which treated this kink term have considered systems with a nonzero net current.

It has been demonstrated [10] that equilibrium calculations made with this model are in much better agreement with results from three-dimensional studies than one has any right to expect. Therefore, it may be worthwhile to use the code to investigate the equilibrium and stability properties of existing and proposed stellarators. We have applied it to a preliminary version of the ATF-1 configuration which will be constructed at the Oak Ridge National Laboratory [11,12]. Previous studies of fixed boundary modes in this

configuration [13] with a parabolic pressure profile led to estimates of stability if $\langle \beta \rangle \lesssim 7.5\%$. Similarly, no unstable free-boundary modes were found [5] at $\langle \beta \rangle = 2.5\%$. Here we describe free-boundary instabilities for a slightly higher pressure, $\langle \beta \rangle = 2.6\%$, and show that the Pfirsch-Schlüter current is largely responsible for these modes.

II. ATF-1 EQUILIBRIUM AND STABILITY CALCULATIONS

The ATF-1 stellarator is an $\ell = 2$ torsatron with twelve field periods and an aspect ratio $R/a = 7$. It has been optimized to possess a vacuum-field magnetic well and a rotational transform ι increasing from 0.3 at the magnetic axis to 0.9 at the plasma edge. We report here on studies using the fields of a preliminary conceptual design [12]. As in most earlier studies we assume a pressure distribution

$$p = p(0) \left[1 - \left(\frac{\psi^{(0)} - \psi_{\min}^{(0)}}{\psi_{\lim}^{(0)} - \psi_{\min}^{(0)}} \right)^2 \right] \quad (8)$$

We fix the vertical height of the outermost zeroth-order magnetic surface $\psi^{(0)}$, but allow the shape of this surface to adjust to satisfy the externally imposed boundary conditions.

The lowest-order surfaces for a configuration with $\beta(0) = 7.5\%$, $\langle \beta \rangle = 2.6\%$, are shown in Fig. 1. The coordinates ϕ , $\psi^{(0)}$, and $\theta \equiv 2\pi B_0 \oint [\psi^{(0)}]^{1/2} ds / |\nabla \psi^{(0)}|$, with ds an element of arc length along a constant ϕ cut, are chosen to make the magnetic field lines straight [9] and are used in the stability analysis. The real flux surfaces have an $\ell = 2$ helical distortion superimposed on them. The most important equilibrium properties $\oint [\psi^{(0)}]$ and $V'[\oint [\psi^{(0)}]]$, with $\oint$ the toroidal flux, are shown in Fig. 2. We have not included the part of V' which is associated with the diamagnetic current since

it does not enter the stability analysis. The most striking effect of the plasma pressure is the increase of the rotational transform near the magnetic axis from its vacuum-field value of 0.3. This introduces two " $q = 2$ " (or $\lambda = 0.5$) surfaces into the plasma, the outer one occurring in a region of positive $v''(\phi)$. In the earlier calculations [5] with $\langle \beta \rangle = 2.5\%$ where no unstable modes were found, both $\lambda = 0.5$ surfaces occurred in regions of favorable average field-line curvature. The large change in the shape of $\lambda[\psi^{(0)}]$ from its vacuum-field profile because of the Pfirsch-Schlüter current associated with plasma pressure makes suspect any equilibrium or stability calculations based on vacuum magnetic fields.

We treat modes with

$$\xi(\psi^{(0)}, \theta, \phi) = \nabla\phi \times \nabla \sum_m \eta_m(\psi^{(0)}) \exp[i(m\theta - n\phi)]. \quad (9)$$

The different values of n decouple. The displacement vector that extremizes the Lagrangian of Eq. (5) for an $n = 1$ mode is shown in Fig. 3 with its associated stream function given in Fig. 4. The length of the arrow denotes the relative magnitude of the displacement. Of particular interest are the facts that the mode is strongly localized to the outside of the torus where the local curvature is unfavorable, that the displacement of the magnetic axis is kink-like and large, and that the perturbation (particularly the $m = 0, 1$, and 2 components of the stream function) is large at the plasma surface.

Higher- n modes are also unstable. The eigenfunction for $n = 3$ is shown in Figs. 5 and 6. As might have been expected, the mode has considerably more structure than that with $n = 1$. It shows even more ballooning-like character and very little kink structure. As occurs in all MHD studies, the different m components adjust themselves to be relatively large at the magnetic surfaces

where they can resonate.

Ballooning modes are found in this model by going to a large n limit inside the ordering $n < R/a$. Using a modified form of the standard axisymmetric ballooning mode code which incorporates the stellarator expansion model, we find that MHD ballooning modes are stable for this equilibrium.

In order to get some understanding of what drives the low- n instabilities, we have carried through the extremization of Eq. (5), first turning off the kink terms arising from the toroidal current $J_{\parallel}$ in this equation, and then removing those associated with the curvature $\kappa = -(1/2) \nabla Q$. The resulting eigenvalues $\rho \omega^2$ are given in Table I. Clearly the kink term provides the dominant energy source for these modes, although the curvature has some effect.

It is useful to investigate the effect of a perfectly conducting wall outside the plasma on these modes. To do this we introduce a conducting surface, shaped like the zeroth-order plasma boundary. As can be seen in Fig. 7, all the low- n modes become stabilized if the wall is relatively close to the plasma, $r_{\text{wall}}/r_{\text{plasma}} \lesssim 1.5$. This is somewhat closer than has been planned in ATF-1, but the calculations are encouraging. Since the instabilities are strongly dependent on the exact shape of the rotational transform, it should be expected that relatively minor changes in the magnitude or shape of an externally imposed vertical field would have considerable effect. (Here we have adjusted the size of a uniform vertical field to keep the centroid of the boundary surface fixed as we increase $\langle \beta \rangle$.)

III. DISCUSSION

This study of an ATF-1 preliminary design suggests that the earlier estimates of stable operation at $\langle \beta \rangle = 7.5\%$ are optimistic. On the other

hand, the instability described here is very sensitive to the properties of the equilibrium, especially the shape of the rotational transform profile. Since this function can be fine-tuned by the application of relatively small vertical fields, significantly higher stable values of $\langle\beta\rangle$ than the one discussed here should be observed. In particular, the ATF-1 final design has no instability for $\langle\beta\rangle$ up to 2.7%, the highest value for which we have obtained good convergence with our equilibrium code. Even if the critical value of $\langle\beta\rangle$ is substantially less than 2.5% in the ATF-1 device, the possibility of studying the effect of MHD instability on containment at respectable but easily attainable plasma conditions should enhance the usefulness of the facility.

The important lesson to be learned from this work is that local criteria do not tell the whole stability story. This calculation provides a good illustration of a case where the energy source is primarily the Pfirsch-Schlüter current. It illustrates the necessity of considering the effect of the plasma in stellarator configurations, rather than of relying on the properties of the vacuum magnetic field. Nevertheless, the β -values discussed here are high enough to be useful for an experimental device. Since the stability properties depend strongly on those of the equilibrium, it should be possible to raise the β -limit significantly. This could be done, for example, by adjusting the magnitude and shape of small axisymmetric trimming fields. Recent calculations at Oak Ridge [14] have shown that, with the presently designed ATF-1 hardware, the rotational transform can be kept monotonic in ψ as the pressure is increased by modifying the currents in the external poloidal coils. An alternate approach would be to modify the design to minimize the effects of the Pfirsch-Schlüter current [15].

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TABLE I.

Unnormalized growth rates $\rho\omega^2$ of instabilities associated with the equilibrium of Fig. 1. The first column contains all the terms in Eq. (5). In the second column the kink term is omitted and in the third the interchange term is omitted.

	ALL TERMS	NO KINK TERM	NO INTERCHANGE TERM
$n = 1$	-0.028	-0.003	-0.019
$n = 2$	-0.027	>0	-0.004
$n = 3$	-0.017	>0	>0

FIGURES

FIG. 1. Lowest-order magnetic surfaces for the ATF-1 device with a parabolic pressure distribution and $\langle \beta \rangle = 2.6\%$. The poloidal coordinate is chosen to contain equal toroidal flux so that the magnetic field lines are straight. A higher-order multipolar distortion modifies these axisymmetric average surfaces.

FIG. 2. Equilibrium properties, $\chi(\Psi^{(0)})$ and $V'(\phi)$, for the ATF-1 configuration with $\langle \beta \rangle = 2.6\%$. The fields associated with the Pfirsch-Schlüter current have increased the rotational transform at the magnetic axis from its vacuum-field value of 0.3 so that two $\chi = 0.5$ surfaces occur in the plasma. At this value of $\langle \beta \rangle$ the outer of these lies in a region of unfavorable average magnetic field curvature.

FIG. 3. Displacement vector $\xi^{(0)}$ for an $n = 1$ instability in the configuration of Fig. 1.

FIG. 4. Stream function η associated with the displacement vector of Fig. 3.

FIG. 5. Displacement vector $\xi^{(0)}$ for an $n = 3$ instability in the configuration of Fig. 1.

FIG. 6. Stream function η associated with the displacement vector of Fig. 5.

FIG. 7. Effect of a perfectly conducting outer wall on the instabilities associated with the ATF-1 equilibrium of Fig. 1.

#83T0480

Fig. 1

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Fig. 2

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Fig. 3

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Fig. 4

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Fig. 5

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Fig. 6

Fig. 7

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