The P-Factor and Atomic Mass Systematics: Application to Medium Mass Nuclei

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Recently it has been shown¹ that a measure of the average number of interactions of each valence nucleon with those of the other type given by the parameter $P = N_p N_n / (N_p + N_n)$ [where $N_p(N_n)$ are the number of valence protons (neutrons)] provides a general and physically meaningful explanation for the development of collectivity and the onset of deformation in medium and heavy nuclei. For example, as shown in Fig. 1, the energy ratio E_{4+1}/E_{2+1} plotted versus P exhibits remarkably similar behavior independent of mass region and, in all cases indicates a transition to deformation ($E_{4+1}/E_{2+1} \approx 3.0$) when $P \approx 4-5$. This rule provides a necessary condition for deformation that the nuclear ground state cannot be deformed for medium or heavy nuclei if the number of valence nucleons of either type is less than 4. Furthermore, this result was seen as a manifestation of the competition between the like nucleon pairing and p-n quadrupole interactions since the pairing strength, which is about 1 MeV and which supports sphericity, is approximately off-set by the quadrupole deformation-driving term when 4-5 valence p-n interactions (each with strength ≈ 200 keV) are present.

It is of interest to investigate whether the success of the P formalism can be extended to systematics of the mass surface. It has long been known that mass data reflect variation in stability which accompany changes in nuclear structure and shape. Such quantities as S_{2n} and S_{2p} , the nucleon pair separation energies, have been used to illustrate the effects of shell closures and the onset of stable ground state deformation. An example of such systematics is shown in Fig. 2 where S_{2n} , the two-neutron separation energy, is plotted for isotopic chains $Z=30-49$. The decreased binding beyond the $N=50$ shell closure and the increased binding associated with the onset of deformation around $N=60$ in the Zr region are clearly visible. While S_{2n} systematics have been extremely useful for illustrating and identifying shell closures and new regions of deformation, they are not well suited to quantitative applications such as the prediction of masses of unknown nuclides since extrapolation of non-linear systematics is involved.

In applying the P formalism to masses it is essential to separate two components which together comprise the atomic mass. The overwhelmingly dominant one is a strong, basically structure independent (macroscopic) component, that scales with A , Z or N . The second (microscopic) component contains the structure and deformation dependence and it is in this regime where the influence of the valence p-n interaction might be expected to manifest itself. In order to isolate

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Figure 1: Smoothed P plots of E_{4+1}/E_{2+1} for six mass regions.

Figure 2: Two-neutron separation energies for isotopic chains in the Zr region.

this latter component we have in each case subtracted the calculated spherical macroscopic mass from the experimental mass to arrive at a semi-empirical microscopic (SEM) mass which is presumed to be a measure of the structure dependent part of the mass. The spherical masses were taken from the model calculations of Möller and Nix² and the experimental masses from the compilation of Wapstra *et al.*³.

Figure 3 shows plots of SEM-masses versus N_n and the P-factor for the actinide region ($Z=84-104$ with $N>128$). Note that points lying higher on the plots represent lower binding energies and that the open circles in Fig. 3b are for data with $N>146$. Data are shown up to midshell with the assumption that the next shell closures occur at $Z=126$ and $N=184$. These choices of shell definition, which in the case of the actinides may be altered depending on the exact locations of the next magic numbers, are important aspects of our approach. We shall elaborate on this point when we consider the effects of subshell closures in the Zr region. At first glance there appears to be no overwhelming simplification in the mass systematics from using the P formalism. However, on closer examination of separate isotopic and isotonic sequences, the P-factor plots exhibit some interesting and useful features. Figure 4 shows such sequences for the actinide region. There are three noteworthy results: first, the SEM masses for individual isotopic or isotonic sequences fall on straight lines. Second, the isotopic sequence (e.g., the Po nuclei with $Z=84$ ($N_p=2$)) falls on the same straight line as the corresponding isotonic sequence (the $N=128$ isotones ($N_n=2$)). This fact provides a means for predicting unknown masses for nuclei far from stability in selected regions of the chart of the nuclides by interpolation rather than by traditional methods involving extrapolation⁴. Third, the slopes of these straight lines decrease smoothly as a function of increasing values of the constant N_p or N_n .

Figure 3: N_n and P plots for the actinides. In the N_n plot the solid lines connect isotopic chains. In part (b) the SEM masses for $N > 146$ are represented by open squares. The inset illustrates the upturn for $N > 146$ for two high Z sequences.

The linear dependence of SEM masses with P is striking but not yet fully understood theoretically. A simple model of SEM masses in terms of the p-n interaction would lead to isotopic and isotonic mass sequences proportional to $(A - B N_p) N_n$ and $(A' - B' N_n) N_p$, respectively, whereas, the best linearity is observed in P-factor rather than in $N_p N_n$ plots. Nevertheless, since the P-factor embodies the concept of the p-n interaction the implication that the structure part of the mass is somehow related to the p-n interaction among valence nucleons seems plausible.

Before leaving the actinides a closer look at Fig. 3 is warranted. Note that the open squares on the global P-plot (Fig. 3b) show a different feature which is illustrated in more detail in the inset. Starting from $N=146$ (the leftmost open square) the data level off relative to the descending line defined at lower P-values. Then as N increases further, an up-bend occurs. A simple explanation for this behavior is possible. There is some evidence in the actinide region for proton and neutron shells or subshells at $Z=114$ and $N=152$. If these are assumed valid instead of $Z=126$ and $N=184$ then points for $N \geq 140$ are plotted at incorrect P values in Fig. 3 and should be omitted from Fig. 4 since they fall in a different quadrant of p-n space. Another explanation is also possible in terms of the Fermion Dynamical Symmetry Model. We will not go into this explanation here but instead refer the reader to Refs. 4 and 5 for details.

We have extended our investigation of P-factor mass systematics to other regions of medium and heavy nuclei. In particular, a discussion of the rare earth region can be found in a forthcoming publication⁴. A feature of this region that complicates the analysis is the presence of a strong subshell closure at $Z=64$. A strict application of the P-formalism in this case dictates that the number of valence protons be counted as particles or as holes (whichever is appropriate) relative to $Z=64$ for $N \leq 90$. It is well known that for $N \leq 90$ the traditional shell definitions ($Z=50-82$, $N=82-126$) are operable. Thus the P-formalism as currently

Figure 4: P factor plots for selected N_p and N_n values for actinide nuclei. Dots represent isotopic sequences and crosses are for isotonic chains. Panel (f) summarizes the slopes (per unit P) of the fitted straight lines as a function of N_p or N_n .

prescribed requires an abrupt transition in the number of valence protons for elements with $58 < Z < 72$ when $N=90$. Recent studies of experimental g-factors and $B(E2)$ values of 2^+_1 states in this region⁶ have shown that one should more appropriately use N_p^{eff} values which reflect a smoothed transition in shell definition in the region of the Z=64 subshell. It is not surprising, therefore, to find that the presence of subshells causes some difficulties for any scheme (including the P-factor method) which relies solely on counting valence nucleons. This will become more evident as we examine mass systematics in the Zr region.

Figure 5 shows isotopic and isotonic SEM masses as a function of P for part of the "Zr region" which for our purposes we designate as either the mass space confined within the Z=28-50, N=50-82 shells or within the Z=38-50, N=50-82 region depending on whether one ignores or invokes the shell closure at Z=38. Systematics for the former are shown to the left (panels a-e) in Fig. 5 and those which presume a strong subshell closure at Z=38 are shown to the right (panels f-h). In the former case, we see that certain general features which were seen in the actinide region are again present here, namely, that the isotopic and isotonic data fall on more-or-less straight lines when plotted versus P and that the slope decreases as the number of valence nucleons of either types increases. A closer look at Fig. 5, however, reveals two discrepant features. In each case the isotonic data (crosses) depart from linearity by "rolling over" for higher values of P. In addition, as N_1 increases there is a separation of the isotopic and isotonic systematics which require two, nearly parallel curves, instead of one, to represent the data. If we

Figure 5: P factor plots for selected N_p and N_n values in the Zr region. Dots represent isotopic data and crosses are for isotonic chains. Panels (a-e) display data for p-hole, n-particle nuclei in the region defined by the shells $Z=28-50$, $N=50-82$ while panels (f-h) assume a proton shell closure at $Z=38$. Panels (i-j) display data for the p-particle, n-particle quadrant defined by the $Z=38-50$, $N=50-82$ shells.

instead incorporate a $Z=38$ subshell in valence particle counting we see (right hand side of Fig. 5) that the systematics are much improved for $N_1 = 2, 4$ and 6 (the $N_1 = 8$ and 10 cases no longer fall in the neutron-particle, proton-hole mass quadrant when the $Z=38$ subshell is invoked) and a very linear systematics in P

results, again suggesting that the p - n interaction, as manifest in the P -factor², seems to describe to a remarkable extent the structure-dependent part of the mass. One must be cautious, however, because this agreement has actually been achieved by shifting the offending isotonic data to another mass quadrant (proton-particle, neutron-particle) rather than by bringing them onto a common line consistent with the other points. Examples of these data are shown in panels i-j. Neither the isotopic nor the shifted isotonic data display convincing linearity in the particle-particle quadrant nor is there any congruence. The best that can be said is that the isotonic data are more linear in P when the $Z=38$ subshell is invoked.

In summary, the SEM masses display remarkable linearity when plotted vs P in the neutron-particle, proton-hole space when a strong $Z=38$ subshell is invoked (a similar statement would apply were $Z=40$ assumed as the subshell closure), and the decrease in slope with increasing N_1 is consistent with the expectation of binding effects between valence protons and neutrons. To the extent that deviations from linearity occur, we may be observing effects arising from the simplicity of our assumptions about the abrupt termination of shells and subshells.

In conclusion, we have applied the P formalism to atomic mass systematics for medium and heavy nuclei. The P -factor linearizes the structure dependent part of the nuclear mass in those regions which are free from subshell effects indicating that the attractive quadrupole p - n force plays an important role in determining the binding of valence nucleons. Where marked non-linearities occur, the P -factor provides a means for recognizing subshell closures and/or other structural features not embodied in the simple assumptions of abrupt shell or subshell changes. These are thought to be regions where the monopole part of the p - n interaction is highly orbit dependent and alters the underlying single particle structure as a function of A , N or Z . Finally, in those regions where the systematics are smooth and subshells are absent, the P -factor provides a means for predicting masses of some nuclei far-from-stability by interpolation rather than by extrapolation.

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