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ANALYSIS OF HOMOGENEOUS U233 AND U235 CRITICAL ASSEMBLIES WITH ENDF/B-IV DATA

(AWBA DEVELOPMENT PROGRAM)

OCTOBER 1977

MASTER

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ANALYSIS OF HOMOGENEOUS U233 AND U235
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(AWBA DEVELOPMENT PROGRAM)

J. J. Ullo
J. Hardy, Jr.

CONTRACT No. EY-76-C-11-0014

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FOREWORD

Subsequent to development and successful operation of the Pressurized Water Reactor in the AEC-owned reactor plant at the Shippingport Atomic Power Station, the Atomic Energy Commission in 1965 undertook a research and development program on a breeder reactor for operation in the Shippingport Station. With fabrication of the Light Water Breeder Reactor (LWBR) nearing completion in 1976, the Energy Research and Development Administration established the Advanced Water Breeder Applications program (AWBA) to develop and disseminate technical information which would assist U.S. industry in evaluating the LWBR concept. All three of these reactor development projects have been administered by the Division of Naval Reactors with the goal of developing practical improvements in the utilization of nuclear fuel resources for generation of electrical energy using water-cooled nuclear reactors.

The Shippingport Atomic Power Station located in Shippingport, Pennsylvania was the first large-scale, central-station nuclear power plant in the United States and the first plant of such size in the world operated solely to produce electric power. This project was started in 1953 to confirm the practical application of nuclear power for large-scale electric power generation. It has provided much of the technology being used for design and operation of the commercial, central-station nuclear power plants now in use.

The objective of the Light Water Breeder Reactor project has been to develop a technology that would significantly improve the utilization of the nation's nuclear fuel resources employing the well-established water reactor technology. To achieve this objective, work has been directed toward analysis, design, component tests, and fabrication of a water-cooled, thorium oxide fuel cycle breeder reactor to install and operate at the Shippingport Station. Operation of the LWBR core in the Shippingport Station is expected to start in 1977 and be completed in about 3 to 4 years. Then the fissionable fuel inventory of the core will be measured. This effort, when completed in about 2 to 3 years after completion of LWBR core operation, is expected to confirm that breeding actually took place.

The Advanced Water Breeder Applications (AWBA) project was initiated to develop and disseminate technical information that will assist U.S. industry in evaluating the LWBR concept for commercial-scale applications. The project will explore some of the problems that would be faced by industry in adapting technology confirmed in the LWBR program. Information to be developed includes concepts for commercial-scale prebreeder cores which will produce uranium-233 for light water breeder cores while producing electric power, improvements for breeder cores based on the technology developed to fabricate and operate the Shippingport LWBR core, and other information and technology to aid in evaluating commercial-scale application of the LWBR concept.

Technical information developed under the Shippingport, LWBR, and AWBA projects has been and will continue to be published in technical memoranda, one of which is this present report.

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ABSTRACT

Thirty-two U233 and U235 homogeneous aqueous critical experiments were analyzed with ENDF/B-IV data. Calculated eigenvalues for both fuel types increased by nearly 2% over the range of hydrogen/uranium atomic ratio covered (from 2106 - 27.1). This is attributed mostly to an underprediction of fast leakage, with some contribution from the fission and capture resonance integrals of ENDF/B-IV U235. Eigenvalue sensitivities to several nuclear data changes were examined. Values of the thermal criticality parameter constraint K_2 for U233 and U235 were derived from the Gwin-Magnuson critical experiments at the zero leakage limit. $K_2 = (\bar{\eta}_U - 1) \bar{\sigma}_{aU} / \bar{\sigma}_{aH}$ relates the thermal-average eta for uranium and the uranium and hydrogen absorption cross sections.

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I. INTRODUCTION AND SUMMARY

Thirty-two U233 and U235 homogeneous aqueous critical experiments have been analyzed with ENDF/B-IV data. The critical eigenvalue (K_{eff}) and detailed reaction rates were calculated for each experiment. This study had two principal aims:

- 1) To examine the ability of these data to predict the criticality of systems covering a wide range of hydrogen/uranium atom ratios (H/U). The range of H/U was 119.4 - 2106 for U233 and 27.1 - 2052 for U235, with a correspondingly wide range of spectrum hardness and of leakage (from 1% - 50%). This work was prompted by a study done at Oak Ridge by McNeany and Jenkins (Ref. 1) to compare Hanson-Roach and ENDF/B-IV cross sections. They reported errors of several percent in calculated neutron multiplication factors for homogeneous U233 assemblies with low H/U ratios when ENDF/B-IV data were used. It was concluded in Ref. 1 that the ENDF/B-IV data file was not satisfactory for criticality calculations of such assemblies.

We have analyzed a set of eight U233 criticals (including several of those considered in the Oak Ridge study) and a corresponding set of U235 systems for comparison. In addition, eigenvalue sensitivities to certain data changes have been determined. These changes include the ENDF/B version of the oxygen cross sections, the mean energies and shapes of the fission neutron spectra, the hydrogen thermal absorption cross section, the uranium capture and fission resonance integrals, and the uranium inelastic cross sections.

- 2) To determine the implications for the thermal criticality parameters of U233 and U235 which can be derived from the 23 Gwin-Magnuson critical spheres and cylinders (Ref. 2). This is an extension of previous studies of the U235-H₂O spheres, which are ENDF/D thermal reactor benchmarks (Ref. 3), to emphasize low-leakage systems. It follows the methodology presented recently by Gwin (Ref. 4) who has emphasized the importance of minimizing the effects of leakage in evaluating these experiments.

The principal results of this study are summarized as follows:

1. Eigenvalues for critical U233 and U235 systems, calculated with ENDF/B-IV data, show an upward trend of approximately 2% with decreasing H/U. Similar results were previously reported in Ref. 5.
2. Criticality of the U233 systems agrees well with the results of similar studies at ORNL.
3. U233 and U235 systems show the same eigenvalue trend, which appears to be due mostly to an underprediction of fast leakage. The best-fit eigenvalues are:

Total Leakage:	0	50%
U233 λ :	.996	1.015
U235 λ :	.994	1.012

4. The most likely sources of leakage error appear to be the U233 and U235 fission spectra (mean energy and shape), and the oxygen scattering cross sections and angular patterns in the MeV range. For U235,

the resonance capture and fission integrals are significant for the eigenvalue trend.

5. The calculated eigenvalues of high leakage assemblies are reduced approximately 0.6% by use of ENDF/B-I oxygen, with negligible effect on large assemblies. The Version III oxygen shows an intermediate behavior.
6. Sensitivity to the fission spectrum mean energy (at ~50% leakage) is

$$\partial\lambda/\partial\bar{E}_f \approx -.15/\text{MeV}.$$

For a mean energy of $\bar{E}_f = 2.00$ MeV, the Watt shape reduces small-core eigenvalues by 0.3 - 0.4% compared to a Maxwellian. It is difficult to account entirely for the reactivity drift by credible hardening of the fission spectra for U233 and U235.

7. The small U235 cores are significantly sensitive to the capture and fission resonance integrals of U235. Approximately 0.5% of the eigenvalue trend can be attributed to the high fission integral, and the low capture integral, of ENDF/B-IV Mat. 1261. Improved resonance integrals for U235 along with a hardened Watt fission spectrum ($\bar{E} = 2.016$ MeV) can eliminate most of the eigenvalue trend for U235 systems.

8. The reduction of the hydrogen thermal absorption cross section by 0.002b (to 0.330b) has little effect on high leakage assemblies but increases the calculated eigenvalues for thermal assemblies by 0.3%.
9. There is negligible sensitivity to inelastic scattering cross sections of U233 and U235.
10. The Gwin-Magnuson criticals at the zero leakage limit determine a constraint K2 on the uranium thermal criticality parameters. $K2 = (\bar{\eta}_x - 1) \bar{\sigma}_{ax} / \bar{\sigma}_{aH}$ involves the thermal spectrum average (0 - .625 eV) eta for uranium, and the uranium and hydrogen absorption cross sections. The following values were obtained from the present analyses:

$$K2 = 2240 \pm 11 \quad (\text{U233})$$

$$K2 = 2155 \pm 11 \quad (\text{U235})$$

The presumed leakage underprediction of ENDF/B-IV does not significantly affect these results. The three U235 tubs have been omitted; their inclusion would raise K2 for U235 by 0.26%. This is considered to be the largest single question of interpretation of these experiments.

II. CALCULATIONAL METHODS

Nuclear data were from ENDF/B-IV, multigroup processed with the ~~ETOMX~~ and ~~ETOTX~~ programs (Bettis versions of ~~ETOG~~ (Ref. 6) and ~~ETOT~~ (Ref. 7), respectively). A detailed fission-source-in-H₂O weighting spectrum was used in the multigroup averaging. Thermal scattering kernels were generated with FLAN2 (a Bettis version of FLANGEII (Ref. 8)) from the ENDF/B S(α,β) files based on the Haywood kernel.

The criticals were analyzed with the Monte Carlo program RCP01, an extension of RECAP (Ref. 9), and with P7MG, an extension of P3MG (Ref. 10), a one-dimensional, multigroup P₄ program. RCP01 was used for the overall study of criticality versus H/U ratio. Cross section sensitivities were derived from P7MG, which was also used for the analysis of the Gwin-Magnuson criticals to infer constraints on the 2200 m/s parameters for U233 and U235. The agreement between RCP01 and P7MG was checked for five specific experiments and found to be very good.

RCP01 represented the geometry explicitly. It used detailed Doppler broadened resonance profiles over the ENDF/B resolved and unresolved resonance ranges (typically ~25,000 energies). In addition, there were multigroup cross sections, following the MUFT 54 group scheme, for inelastic group transfer (including n,2n multiplication). The elastic scattering angular distribution was described by a P₃ approximation in each multigroup. In the thermal energy range (0 - 0.625 eV), cross sections were described at 25 energies, including P₃ transfer matrices for hydrogen bound in H₂O. The small fraction of the fission source above 10 MeV was folded into the first MUFT group, and the hydrogen scattering cross section in this group was averaged accordingly.

P7MG calculations were similar to the original analyses of Staub, et al. (Ref. 11). There were 57 fast groups (the MUFT 54 group scheme with three additional $\Delta u = 0.25$ groups above 10 MeV). There was one thermal group (0 - 0.625 eV) with cross sections averaged in a 25-group infinite medium calculation. The flux angular treatment was double- P_1 thermally and P_7 fast (with a simplified P_7 treatment used for spheres and cylinders). Marshak boundary conditions were used.

III. CRITICALITY VERSUS HYDROGEN/URANIUM RATIO

A. ENDF/B-IV Analysis Results

Eight U233 and nine U235 critical assemblies were analyzed in this part of the study. They are summarized in Tables 1 and 2 along with references to the original documentation.

These assemblies were spheres and cylinders containing aqueous solutions of uranyl nitrate or uranyl fluoride. They were bare, except for experiment L7, which had an essentially infinite water reflector. The range of H/U was 27.1 - 2052 for U235 and 119.4 - 2106 for U233. Leakage ranged up to about 50%.

In some cases the documentation of the experiments is limited. What are believed to be the best data describing the measurements are given in Tables 1 and 2. For a few cases the data have not been corrected for the effects of the vessel walls. Where sufficient information was available the vessels were included in the calculation. Generally, however, the effects of the vessel walls on unreflected assemblies are judged to be quite small.

The U233 results (Table 3) show generally good agreement with the Oak Ridge calculations. This suggests that, for this specific case, the use of different analysis programs and nuclear data processing procedures has little effect on the calculated results.

TABLE 1

Critical Parameters for the Uranium-233 Assemblies Used for Leakage Studies

Experiment [†]	H/U233	Critical Height (H=cm) and/or Radius (R=cm)	Critical Volume (Liters)	Critical Mass of U233 (Kg)	Comment
L1. Unreflected cylinder of aqueous $\text{UO}_2(\text{NO}_3)_2$ solu- tion (Ref. 12).	119.4	H=24.69 R=12.70	12.51	2.569	Aluminum walls, endplate, and fillpipe included in the cal- culation.
L2. Unreflected cylinder of aqueous UO_2F_2 solution (Ref. 13).	154	H=24.0 $\pm$ 0.05* R=12.75	12.257 $\pm$ 0.03*	2.02 $\pm$ 0.05*	Calculated as a bare assembly. Not reported whether data was corrected for the effects of the aluminum vessel.
L3. Unreflected sphere of aqueous UO_2F_2 solution (Ref. 13).	381	R=15.94	16.965	1.14	Same as for experiment No. 2.
L4. Unreflected sphere of aqueous $\text{UO}_2(\text{NO}_3)_2$ solu- tion (Ref. 12).	581	R=18.378	26.0	1.16	Calculated as a bare assembly.
9. Unreflected sphere of aqueous $\text{UO}_2(\text{NO}_3)_2$ solu- tion (Ref. 2).	1324	R=34.5948	173.43	3.36	Calculated as a bare assembly. Boron content 0.0912 mg/cm ³ .
5. Unreflected sphere of aqueous $\text{UO}_2(\text{NO}_3)_2$ solution (Ref. 2)	1533	R=34.5948	173.43	2.90	Calculated as a bare assembly.
11. Unreflected sphere of aqueous $\text{UO}_2(\text{NO}_3)_2$ solution (Ref. 2)	1986	R=61.0108	951.3	12.32	Calculated as a bare assembly.

(Continued)

TABLE 1 (Cont'd)

Experiment [†]	H/U233	Critical Height (H=cm) and/or Radius (R=cm)	Critical Volume (Liters)	Critical Mass of U233 (Kg)	Comment
20. Unreflected cylinder of aqueous $\text{UO}_2(\text{NO}_3)_2$ solution (Ref. 2)	2106	H=140.1572 R=77.3684	2635.68	33.04	Critical height included cor- rection for 0.53 in. bottom structure. Calculated as a tare assembly.

* Extrapolated values based on source neutron multiplication curves.

[†] I.D.'s of the Gwin-Magnuson experiments are the same as in Reference 2. Other critical experiments are denoted L1-L9.

TABLE 2

Critical Parameters for the Uranium-235 Assemblies Used for Leakage Studies

Experiment*	H/U235	Critical Height (H=cm) and/or Radius (R=cm)	Critical Volume (Liters)	Critical Mass of U235 (Kg)	Comment
L5. Unreflected cylinder of aqueous UO_2F_2 solution (Ref. 14).	27.1	H=38.9 R=12.7	19.7	16.34	Data reported to be corrected for aluminum baseplate and vessel walls. Calculated as a bare assembly.
L6. Unreflected cylinder of aqueous UO_2F_2 solution (Ref. 14).	44.3	H=35.1	17.79	9.56	Same as experiment No. 1.
L7. Reflected sphere of aqueous UO_2F_2 solution (Ref. 15).	76.1	R=11.5	6.37	2.07	Calculations included the 1.6 mm-thick aluminum vessel.
L8. Unreflected sphere of aqueous UO_2F_2 solution (Ref. 15).	1112	R=27.9	91.1	2.13	Calculations included 2.0 mm- thick aluminum vessel.
2. Unreflected sphere of aqueous $\text{UO}_2(\text{NO}_3)_2$ solu- tion (Ref. 2).	1117	R=34.5948	173.43	3.80	Calculated as a bare assembly. Boron content 0.0935 mg/cm ³ .
L9. Unreflected sphere of aqueous UO_2F_2 solution (Ref. 15).	1393	R=34.6	174.0	3.25	Calculations included 3.2 mm- thick aluminum wall.
10. Unreflected sphere of aqueous $\text{UO}_2(\text{NO}_3)_2$ solu- tion (Ref. 2).	1835	R=61.0108	951.3	13.43	Calculated as a bare assembly.
16. Unreflected cylinder of aqueous $\text{UO}_2(\text{NO}_3)_2$ solu- tion (Ref. 2).	1981	H=204.9272 R=77.3684	3853.69	54.19	Critical height included cor- rection for 0.53-in. bottom structure. Calculated as a bare assembly.

(Continued)

TABLE 2 (Cont'd)

<u>Experiment*</u>	<u>H/U235</u>	<u>Critical Height (H=cm) and/or Radius (R=cm)</u>	<u>Critical Volume (Liters)</u>	<u>Critical Mass of U235 (Kg)</u>	<u>Comment</u>
23. Unreflected cylinder of aqueous $\text{UO}_2(\text{NO}_3)_2$ solu- tion (Ref. 2).	2052	H=241.046 R=136.779	14167.35	192.82	No correction was made ex- perimentally for the effects of structure. Calculated as a base assembly.

* I.D.'s of the Gwin-Magnuson experiments are the same as in Reference 2. Other critical experiments are denoted L1-L9.

One assembly (with a thick paraffin reflector) analyzed in Ref. 1 has not been included in Table 1. However, our calculation, which was hampered by lack of a suitable thermal scattering kernel for paraffin, showed a bias of +6.5% in good agreement with the ORNL value of +7%. This is grossly out of line with results for the other critical assemblies, probably due to difficulty of interpreting the paraffin reflector.

For both U233 and U235, the ENDF/B-IV-calculated eigenvalues (Tables 3 and 4) become increasingly high relative to experiment as the H/U ratio is decreased from well thermalized systems.

Along with the decrease of H/U there is an associated increase of leakage and a hardening of the flux spectrum. This is summarized quantitatively in Table 5 where calculated leakage fractions and the ratios of epithermal to thermal fissions (δ) are listed. Corresponding to the increase in δ there is an increase in the total and fast neutron leakage fractions.

Since the trend in calculated eigenvalue errors is nearly the same for U233 and U235 systems, it is tempting to seek a common source of difficulty (e.g., oxygen scattering cross section), but this is by no means the only possibility. The following items could be significant:

- 1) oxygen scattering cross section and angular pattern (MeV range)
- 2) fission spectrum of U233 and of U235
- 3) uranium capture and fission resonance integrals, and
- 4) multigroup averaging and limited P_0 treatment in RCP01.

It seems unlikely that the trend is due to energy dependent errors of η in the thermal range (which are emphasized as the flux spectrum hardens) since both fuels show nearly the same behavior, and substantial errors would be required. Although there remain questions of interpretation about individual

TABLE 3

Results of Calculations of Some Highly Enriched U233 Critical
Experiments Using ENDF/B-IV

<u>Experiment</u>	<u>H/U233</u>	<u>Calculated λ</u>	
		<u>Present Work^a</u>	<u>Oak Ridge Calculations^{b,c}</u>
L1	119.4	1.0116 $\pm$ 0.0034	--
L2	154	1.0226 $\pm$ 0.0027	1.022 $\pm$ 0.007
L3	381	1.0110 $\pm$ 0.0035	1.007 $\pm$ 0.006
L4	581	1.0111 $\pm$ 0.0034	--
9	1324	1.0007 $\pm$ 0.0037	--
5	1533	1.0017 $\pm$ 0.0016	0.998 $\pm$ 0.004
11	1986	0.9974 $\pm$ 0.0031	0.991
20	2106	0.9956 $\pm$ 0.0008	--

^a Statistical errors are two standard deviations.

^b Statistical errors are one standard deviation.

^c These results have been updated from the results reported in
Ref. 1.

TABLE 4

Results of Calculations of Some Highly Enriched U235
Critical Experiments Using ENDF/B-IV

<u>Experiment</u>	<u>H/U235</u>	<u>Calculated λ^a</u>
L5	27.1	1.0085 ± 0.0036
L6	44.3	1.0117 ± 0.0030
L7	76.1	1.0098 ± 0.0051
L8	1112	1.0085 ± 0.0037
2	1177	1.0007 ± 0.0035
L9	1393	1.0035 ± 0.0035^b
10	1835	0.9936 ± 0.0031
16	1981	0.9939 ± 0.0008
23	2052	0.9949 ± 0.0003

^a Statistical errors are two standard deviations.

^b The corresponding result for this experiment in Ref. (1) was 1.006 ± 0.004 (1 σ).

TABLE 5

Calculated Leakage Fractions and Fission Ratios Using ENDF/B-IV

<u>Experiment</u>	<u>H/U</u>	<u>Total Leakage Fraction (L)</u>	<u>Leakage to 5.53 keV (L_{F1})</u>	<u>$\ln(1-L_{F1})$</u>	<u>Leakage to 0.625 eV (L_{F2})</u>	<u>$\ln(1-L_{F2})$</u>	<u>δ^a</u>
<u>U233 Experiments</u>							
L1	119.4	0.5149	0.3944	-0.5015	0.4912	-0.6757	0.4493
L2	154	0.5081	0.3855	-0.4869	0.4799	-0.6537	0.3466
L3	381	0.4617	0.3316	-0.4029	0.4164	-0.5385	0.1526
L4	581	0.4081	0.2823	-0.3317	0.3562	-0.4404	0.1094
9	1324	0.1742	0.1109	-0.1175	0.1413	-0.1529	0.0635
5	1533	0.1776	0.1104	-0.1170	0.1414	-0.1524	0.0548
11	1986	0.0683	0.0402	-0.0421	0.0526	-0.0540	0.0463
20	2106	0.0402	0.0238	-0.0241	0.0305	-0.0310	0.0461
<u>U235 Experiments</u>							
L5	27.1	0.4380	0.3562	-0.4404	0.4312	-0.5642	0.8175
L6	44.3	0.4501	0.3566	-0.4410	0.4384	-0.5771	0.4722
L7	76.1	0.4661	0.4977	-0.6886	0.5534	-0.8061	0.1962
L8	1112	0.2333	0.1490	-0.1613	0.1921	-0.2133	0.0251
2	1177	0.1678	0.1076	-0.1138	0.1383	-0.1488	0.0255
L9	1393	0.1703	0.1055	-0.1115	0.1365	-0.1468	0.0214
10	1835	0.0663	0.0402	-0.0410	0.0517	-0.0531	0.0179
16	1981	0.0329	0.0198	-0.0200	0.0256	-0.0259	0.0169
23	2052	0.0141	0.0084	-0.0084	0.0109	-0.0118	0.0165

^aRatio of epithermal (>0.625 eV) to thermal (<0.625 eV) fissions.

experiments, this is judged not to be significant for the overall pattern.

Furthermore, calculational approximations (including data processing) are thought to be relatively unimportant. This is supported by the good agreement obtained with the Oak Ridge Analysis (Ref. 1) and also by a comparison of RCP01-calculated leakage with results obtained by H. H. Henryson II at the Argonne National Laboratory with the VIM Monte Carlo Program (Ref. 16). VIM employed a strictly pointwise representation of the data and a detailed elastic scattering angular treatment (20 equal $\cos\theta$ intervals). The comparison was made for a small cylinder of U^{235} - H_2O in vacuum, with a spatially flat fission source. ENDF/B-IV data were used. Results for total leakage (10 MeV - 1.12 eV) were:

$$\begin{array}{rcl} \text{RCP01:} & 0.5285 & + 0.0006 \\ \text{VIM:} & 0.5267 & \pm 0.0007 \\ \hline \text{RCP01-VIM:} & 0.0018 & \pm 0.0009 \end{array}$$

Agreement is very good within statistics (one standard deviation) and, if anything, RCP01 produces slightly more leakage than VIM.

That the criticality bias is not attributable to spectrum hardening alone can perhaps be inferred from the Gwin-Magnuson U^{233} criticals. (The considerably larger scatter of λ for the corresponding U^{235} systems obscures the argument in that case). Figure 1 shows λ for these systems plotted against δ^{23} , the ratio of epithermal-to-thermal U^{233} fissions. The argument is simply this: If the bias drift in Figure 1 for the unpoisoned cores is attributed to the spectrum hardening which accompanies increased leakage, then additional hardening obtained solely by boron poisoning should continue the trend, but this does not occur. Although the range of hardening is quite limited, this argument at least suggests that the more likely source of difficulty is leakage itself (compare also Figure 13). For U^{235} , however, there is some sensitivity to the capture and fission resonance integrals (see the following section).

Eigenvalues of Gwin-Magnuson U233 Criticals as a
Function of Epithermal/Thermal U233 Fission Ratio

FIGURE 1

The effect of a leakage underprediction on criticality (items 1-4 above) can be characterized by means of the critical equation,

$$\lambda = K_{\infty} \frac{K(B, E_c)}{1 + B^2 L_T^2}, \quad (1)$$

where

λ is the effective neutron multiplication factor (eigenvalue)

B is the buckling

L_T is the thermal diffusion length

$K(B, E_c)$ is the nonleakage probability during slowing down to energy E_c

K_{∞} is the infinite medium multiplication factor.

Since $K(B, E_c)$ for an infinite system ($B=0$) is unity, the nonleakage probability for small B^2 can be obtained if $K(B, E_c)$ is expanded as follows:

$$K(B, E_c) = K(0, E_c) + \frac{1}{2} B^2 \frac{\partial^2 K(B, E_c)}{\partial B^2} \Big|_{B=0} + \dots \quad (2)$$

$$= K(0, E_c) \left(1 + \frac{1}{2} \frac{B^2}{K(0, E_c)} \frac{\partial^2 K(B, E_c)}{\partial B^2} \Big|_{B=0} + \dots \right) \quad (3)$$

$$\approx (1 - B^2 L^2(E_c)) \approx e^{-B^2 L^2(E_c)} \quad (4)$$

where

$$L^2(E_c) = - \frac{1}{2} \frac{\partial^2 K(B, E_c)}{\partial B^2} \Big|_{B=0} \quad (5)$$

$L(E_c)$ is the slowing down length from fission neutron energies to energy E_c . Equation (1) can now be written in the "Fermi-age form,"

$$\lambda = \frac{K_{\infty} e^{-B^2 L^2(E_c)}}{1 + B^2 L_T^2} \quad (6)$$

It is possible starting from Eqn. (6) to deduce a simple logarithmic relation between the calculated reactivity bias $\delta\lambda$ and the fast neutron leakage fraction L_F if an error is introduced into the square of the slowing down length. Differentiation of Eqn. (6) with respect to $L^2(E_c)$ yields

$$\frac{\delta\lambda}{\lambda} = \frac{\delta L^2(E_c)}{L^2(E_c)} \ln(1 - L_F(E_c)), \quad (7)$$

where $L_F(E_c)$ is the calculated leakage of all neutrons with energies above a cutoff energy, E_c . If the relative error in the square of the slowing down length is assumed to be nearly constant for the assemblies studied in this work, then a linear relation between $\delta\lambda$ and $\ln(1 - L_F(E_c))$ is suggested by Eq. (7):

$$\delta\lambda = a + b \ln(1 - L_F(E_c)), \quad (8)$$

where $b = \frac{\delta L^2(E_c)}{L^2(E_c)}$ is the relative error in the square of the slowing down length to the cutoff energy E_c . The constant a is included since it is possible that all the calculated results could deviate from unity by a

practically constant value due to error in the assumed number of neutrons produced per fission.

With the calculated leakage fractions (Table 5) to cutoff energies of $E_c = 5.53$ keV (L_{F1}) and $E_c = 0.625$ eV (L_{F2}), weighted least-squares analyses were made using Eqn. (8) (Figs. 2-5). The results for the fitted constants a and b are summarized in Table 6. Within statistics the trend in calculated eigenvalues is independent of fuel type. Since Eqn. (6) and Eqn. (8) are valid only for large thermal assemblies with small fast neutron leakage fractions, for those fits including all the assemblies of a given fuel type the interpretation of the parameter b may be questionable. When fits using Eqn. (8) are made including only the thermal assemblies, the values obtained for b are again similar for both fuels and more indicative of the relative error in the square of the slowing down length. The fit of the calculated U235 results using Eqn. (8) is improved (Table 6) if experiment No. 18 ($H/U = 1112$) is excluded. Why the calculation of this experiment is out of line with the others could not be determined, but the origin of the discrepancy is thought to be experimental.

B. Sensitivities of Calculated Eigenvalues to Nuclear Data

Sensitivities of the calculated eigenvalues to several nuclear data changes were determined for selected U233 and U235 critical assemblies with a wide range of leakage. These changes include the ENDF/B version of the oxygen cross sections, the hydrogen thermal absorption cross section, the uranium capture and fission resonance integrals, the uranium inelastic cross sections, and representations of the fission neutron spectra.

TABLE 6

Results of Studies Using Eq. (8)

Correlation	U-235 Assemblies	
	a [†]	b [†]
$\delta K_{eff} = a+b \ln(1-L_{F1})$	-0.007 ± 0.002	-0.037 ± 0.017
$\delta K_{eff} = a+b \ln(1-L_{F1})^*$	-0.007 ± 0.002	-0.036 ± 0.013
$\delta K_{eff} = a+b \ln(1-L_{F2})$	-0.007 ± 0.002	-0.030 ± 0.012
$\delta K_{eff} = a+b \ln(1-L_{F2})^*$	-0.007 ± 0.002	-0.029 ± 0.010
$\delta K_{eff} = a+b \ln(1-L_{F1})^{**}$	-0.007 ± 0.002	-0.074 ± 0.037
	U-233 Assemblies	
	a [†]	b [†]
$\delta K_{eff} = a+b \ln(1-L_{F1})$	-0.005 ± 0.002	-0.048 ± 0.011
$\delta K_{eff} = a+b \ln(1-L_{F2})$	-0.005 ± 0.002	-0.035 ± 0.009
$\delta K_{eff} = a+b \ln(1-L_{F1})^{***}$	-0.006 ± 0.001	-0.064 ± 0.008

* Did not include experiment L8 (H/U = 1112).

** Included only the five thermal assemblies (H/U = 1177 - 2052).

*** Included only the four thermal assemblies (H/U = 1324 - 2106).

[†] All uncertainties are one standard deviation.

$\delta\lambda$ vs $\ln(1 - L_{F1})$ for the U233
Assemblies Using ENDF/B-IV

FIGURE 2

$\delta\lambda$ vs $\ln(1 - L_{F1})$ for the U235
Assemblies Using ENDF/B-IV

FIGURE 3

$\delta\lambda$ vs $\ln(1 - L_{F2})$ for the U233
Assemblies Using ENDF/B-IV

FIGURE 4

$\delta\lambda$ vs $\ln(1-L_{F2})$ for the U235
Assemblies Using ENDF/B-IV

FIGURE 5

A reference eigenvalue was computed for each assembly with P7MG and ENDF/B-IV data. Then for each change in data a $\delta\lambda$ was determined relative to this reference value. The sensitivities of λ to the various parameters are summarized in Tables 7 and 8. The following points are noteworthy:

- 1) Sensitivities to the ENDF/B version of oxygen are significant for the high-leakage assemblies and negligible for the thermal assemblies. The successive versions of oxygen (ENDF/B-I to ENDF/B-IV) show an increase in backscattering. For each oxygen change the $\delta\lambda$ versus epithermal (> 0.625 eV) neutron leakage fraction is plotted for the U233 assemblies (Fig. 6) and the U235 assemblies (Fig. 7). The approximate linear variation for each case can be used to estimate $\delta\lambda$ for the same changes in the oxygen cross sections for other assemblies having intermediate epithermal neutron leakage fractions (Table 5).
- 2) The fission neutron spectrum significantly affects fast neutron leakage. An estimate of the spectrum hardening increase necessary to eliminate the eigenvalue error was made as follows. Using a Maxwellian fission spectrum at two mean energies ($\bar{E} = 2.0$ MeV and 2.1 MeV) one finds that for the high leakage assemblies (Tables 7A and 7B).

$$\left(\frac{\delta\lambda}{\delta\bar{E}} \right)_{U233} \approx - \frac{0.0170}{0.10} = -0.170 \text{ MeV}^{-1} \quad (9)$$

$$\left(\frac{\delta\lambda}{\delta\bar{E}} \right)_{U235} \approx - \frac{0.014}{0.10} = -0.140 \text{ MeV}^{-1}. \quad (10)$$

TABLE 7A

Sensitivities of U233 Critical Assemblies to Changes in Nuclear Data

<u>Data Component</u>	<u>Change in Data Component</u>	<u>Change in Eigenvalue ($\delta\lambda$)</u>			
		<u>H/U233-154(L2)^a</u>	<u>381(L3)</u>	<u>1533(5)</u>	<u>1986(11)</u>
<u>Oxygen Cross Sections</u>	ENDF/B-IV $\rightarrow$ ENDF/B-III	-0.0041	-0.0031	-0.0008	-0.0001
	ENDF/B-IV $\rightarrow$ ENDF/B-I	-0.0062	-0.0044	-0.0012	-0.0002
<u>Fission Spectrum^b</u>	Maxwellian spectrum $\left\{ \begin{array}{l} \bar{E}=2.0 \text{ MeV} \\ T=1.333 \text{ MeV} \end{array} \right.$	+0.0036	+0.0029	+0.0009	+0.0003
	Maxwellian spectrum $\left\{ \begin{array}{l} \bar{E}=2.1 \text{ MeV} \\ T=1.40 \text{ MeV} \end{array} \right.$	-0.0134	-0.0109	-0.0049	-0.0024
	Watt spectrum ($\bar{E} = 2.0 \text{ MeV}$)	-0.0001	+0.0001	+0.0005	+0.0004
<u>Hydrogen Cross Sections</u>	Reduced thermal absorption cross section from 0.332b to 0.330b	+0.0004	+0.0009	+0.0026	+0.0030
<u>Uranium-233 Cross Sections</u>	Reduced the inelastic cross section by 20%.	-0.0004	-0.0001	0.0000	0.0000
	Inelastic cross sections reduced by 20% and the elastic cross section increased by an equivalent amount to maintain the same total cross section.	-0.0003	-0.0001	0.0000	0.0000

^aExperiment ID's correspond to those in Table 1.

^bAll $\delta\lambda$ are relative to the ENDF/B-IV fission neutron spectrum for U233.

TABLE 7B

Sensitivities of U235 Critical Assemblies to Changes in Nuclear Data

Data Component	Change in Data Component	Change in Eigenvalue ($\delta\lambda$)				
		H/U235-27.1(L5) ^a	44.3(L6)	1112(L8)	1177(2)	1835(10)
<u>Oxygen Cross Sections</u>	ENDF/B-IV $\rightarrow$ ENDF/B-III	-0.0033	-0.0034	-0.0013	-0.0008	-0.0001
	ENDF/B-IV $\rightarrow$ ENDF/B-I	-0.0049	-0.0050	-0.0018	-0.0012	-0.0002
<u>Fission Spectrum</u> ^b	ENDF/B-III U235 fission neutron spectrum	+0.0049	+0.0051	+0.0027	+0.0020	+0.0009
	Maxwellian spectrum { $\bar{E}=2.0$ MeV	-0.0022	-0.0023	-0.0012	-0.0009	-0.0004
	{ $T=1.333$ MeV					
	Maxwellian spectrum { $\bar{E}=2.1$ MeV	-0.0157	-0.0165	-0.0087	-0.0067	-0.0031
	{ $T=1.40$ MeV					
	Watt spectrum ($\bar{E}=2.0$ MeV)	-0.0053	-0.0054	-0.0020	-0.0015	-0.0004
<u>Hydrogen Cross Sections</u>	Reduced thermal absorption cross section from 0.332b to 0.330b	+0.0001	+0.0001	+0.0021	+0.0020	+0.0028
<u>Uranium-235 Cross Sections</u>	Reduced the inelastic cross section by 20%	-0.0016	-0.0011	0.0000	0.0000	0.0000
	Inelastic cross section reduced by 20% and the elastic cross section increased by an equivalent amount	-0.0009	-0.0006	0.0000	0.0000	0.0000

^a Experiment ID's correspond to those in Table 2.

^b All $\delta\lambda$ are relative to the ENDF/B-IV fission neutron spectrum for U235.

$\delta\lambda$ vs Epithermal Neutron Leakage Fraction L_F (> 0.625 eV) for Various Oxygen Changes

FIGURE 6

$\delta\lambda$ vs Epithermal Neutron Leakage Fraction L_F (> 0.625 eV) for Various Oxygen Changes

FIGURE 7

Since the change of calculated eigenvalue from zero to 50% total leakage is +0.019 for U233 assemblies, Eq. (9) implies that the mean energy of the U233 fission neutron spectrum would have to be increased by ~0.1 MeV, and a similar conclusion holds for the U235 assemblies. Such hardening of the fission spectra is inconsistent with the available data, and it is concluded that changes in the fission spectrum mean energy alone will not compensate for the increasing error in the calculated eigenvalues with fast neutron leakage fraction.

Sensitivity to the shape of the fission spectrum was also studied. The two most commonly used analytical representations of the experimental fission spectrum data are the Maxwellian,

$$N(E) = \frac{2}{\sqrt{\pi} T} \sqrt{\frac{E}{T}} e^{-E/T} \quad (11)$$

and the Watt spectrum,

$$N(E) = C e^{-AE} \sinh (BE)^{\frac{1}{2}}. \quad (12)$$

In Eqn. (11) the temperature T is varied to fit the experimental data, and in Eqn. (12) C is a normalization constant, and A and B are varied to fit the data. The effect of the slight difference in shape between the Watt spectrum and the Maxwellian was determined for a mean fission neutron energy of 2.0 MeV. For the high leakage U233 assembly ($H/U = 154$) use of the Watt spectrum reduced the calculated reactivity by 0.0037 relative to that computed with the Maxwellian spectrum. For the thermal assemblies the difference

was negligible. A similar result was obtained for the high leakage U235 assemblies (the Watt spectrum lowered λ by 0.0031 relative to a Maxwellian for the high leakage assemblies). These and other results are plotted in Figures 8 and 9.

Reasonable increases (~ 0.02 MeV) in the mean energies of the fission spectra can result in eigenvalue reductions of $\sim 0.3\%$ in the high leakage assemblies, with negligible effect at low leakage. In addition, for a 2.0 MeV mean energy the shape of the Watt spectrum leads to $\sim 0.3\%$ lower eigenvalues than a Maxwellian for the high-leakage criticals. Consequently, realistic variations in the energy distribution of the fission spectra are approximately equal to the mean energy in importance to calculated eigenvalues of high-leakage assemblies. For U235 ($\bar{E} = 1.985$ MeV) such spectrum changes are not inconsistent with recent measurements (Ref. 17) which favor a harder spectrum of the Watt shape, especially at high energies. Increasing the mean energy of the ENDF/B-IV U233 fission spectrum ($\bar{E} = 2.012$ MeV) by 0.02 MeV seems less credible (Ref. 18).

- 3) Sensitivity to the hydrogen thermal absorption cross section was studied. Specifically, the 2200 m/sec cross section (σ_{aH}) was reduced from 0.332b to 0.330b. This change increases the reactivity of the thermal assemblies by 0.3% and leaves the high leakage assemblies relatively unchanged. Other studies (Ref. 4) have suggested that σ_{aH} might be reduced to 0.330b, and such a change would reduce the value of $\frac{\partial \lambda}{\partial L_F}$ (L_F = epithermal neutron leakage fraction) which results from calculations of homogeneous aqueous critical assemblies using ENDF/B-IV.

Sensitivities to Fission Spectra Vs Epithermal Leakage Fraction
 $L_F (> 0.625 \text{ eV})$

FIGURE 2

Sensitivities to Fission Spectra Vs Epithermal Leakage Fraction
 L_F ($> 0.625 \text{ eV}$)

FIGURE 9

- 4) The uranium capture and fission resonance integrals were varied as shown in Table 8. In each case this was achieved by a uniform percentage change of the smooth multigroup cross sections. There are significant sensitivities for the U235 systems but not for U233. The relative importance of these sensitivities can be seen from the resonance integrals shown in Table 9. The ENDF/B-IV U235 fission integral is too large and the capture integral is low. This can account for at least 0.5% of the eigenvalue difference between small and large cores.
- 5) The inelastic cross sections for both fuel types were reduced by 20%. The calculated $\delta\lambda$'s (Tables 7A and 7B) were negligible. In fact, it is reasonable that the reduction of the inelastic scattering should be added to the elastic cross section so as to keep the total cross section unchanged. This was done, and the resulting $\delta\lambda$'s were again negligible. It is concluded that reasonable perturbations of the inelastic cross sections of both fuels do not affect the trend in the calculated eigenvalues seen in this work.

TABLE 8

Sensitivities of Eigenvalues to Uranium
Capture and Fission Resonance Integrals

	Exp.:	Change in Eigenvalue					
		<u>L1</u>	<u>L2</u>	<u>L3</u>	<u>L4</u>	<u>5</u>	<u>20</u>
<u>U233</u>							
Smooth capture integral increased 6.0 b		-.0021	-.0017	-.0007	-.0005	-.0002	-.0001
Smooth fission integral increased 13.0 b		+.0020	+.0016	+.0009	+.0008	+.0005	+.0005
<u>U235</u>							
	Exp.:	<u>L5</u>	<u>L6</u>	<u>L7</u>	<u>L8</u>		<u>21</u>
Smooth capture integral increased 6.0 b		-.0077	-.0053	-.0028	-.0003		-.0002
Smooth fission integral increased 10.0 b		+.0080	+.0053	+.0027	+.0005		+.0004

TABLE 9

Uranium Resonance Capture and Fission Integrals

		Resonance Integral Above 0.625 eV (barns)	
		<u>Fission</u>	<u>Capture</u>
U235	ENDF/B-IV (Mat. 1261)	268	136
U235	BNL-325 (3 rd ed.) (Ref. 19)	259 $\pm$ 5	142 $\pm$ 6
U233	ENDF/B-IV (Mat. 1260)	733	132
U233	BNL-325 (3 rd ed.)	735 $\pm$ 13	138 $\pm$ 6

IV. THERMAL PARAMETER CONSTRAINTS DERIVED FROM THE GWIN-MAGNUSON CRITICALS

In this section the thermal criticality parameter constraint implied by the Gwin-Magnuson critical experiments is evaluated. Some details of the calculations are given in Part A, followed by calculated eigenvalues and reaction rates (Part B). Finally (Part C) the form of the constraint is derived and values are obtained for U233 and U235.

A. P7MG Calculations

The 23 Gwin-Magnuson U233 and U235 critical spheres and cylinders were analyzed with P7MG and ENDF/B-IV data. As a check, five experiments were also analyzed with RCP01. These systems comprise spheres of 27.24-in. and 48.04-in. diameter, cylinders of 60.92-in. diameter and three U235 tubs of 107.7-in. diameter. Three of the U235 spheres and four U233 spheres contained boron poison. Leakage ranged from 1% to 17%.

The present analysis is similar to previous studies by Staub et al. (Ref. 11) and by Steen (Ref. 18). The P7MG program performed a one-dimensional multigroup P_L calculation. In both axial and radial traverses a mesh of ~0.1 cm was used for the outer 1-2 cm, 0.25 cm for 4 cm, and 1 cm for the remainder. In the largest criticals, a 2 cm mesh was used in the central portion. Halving the mesh in selected problems produced completely insignificant changes of λ . Use of a P_1 flux angular approximation hardly affected results: for example, leakage changed by +0.0005 (out of 0.03034) and λ changed by -0.0005 for experiment 21. This does not seem surprising in view of the small leakage in these cores and resultant near isotropy of the flux.

The cylinders were analyzed in two steps:

(1) A radial traverse, including the thin container wall. (In this calculation, axial leakage was represented with a single transverse buckling B_z^2 obtained from the measured critical solution height and an extrapolation distance of 2.5 cm). Radial leakage was edited at the solution boundary in four few-groups: 21 MeV - 821 keV - 5530 eV - 0.625 eV - 0 eV. In each few-group, a buckling was obtained which, when used in an infinite medium calculation, would reproduce the observed leakage. That is

$$\left(\frac{\int J \cdot dS}{\int \Sigma_A \phi dV} \right)_{\text{Radial Traverse}} \stackrel{\text{set}}{=} \left(\frac{DB_r^2 \phi}{\Sigma_A \phi} \right)_{\infty \text{ Medium}} \quad (13)$$

Assuming that the few-group Σ_A and D remain the same (a good approximation because leakage is small)

$$B_r^2 = \frac{\int J \cdot dS}{D \int \phi dV} = \frac{2\pi R J}{D \phi} \quad (14)$$

(2) An axial traverse with few-group transverse B_r^2 values obtained from the radial calculation. It was verified that the resulting four-group radial leakages agreed adequately with those calculated explicitly in (1).

This procedure for a typical cylinder (Experiment 16) gave results shown in Table 10. (Although an exact match of leakage could easily be obtained by adjusting the B_r^2 values, this was judged to be not worth doing.) The large differences among these few-group bucklings (more than a factor of two) are noteworthy.

TABLE 10

Calculated Few Group Radial Leakages and Inferred Bucklings
(Experiment 16)

<u>Group</u>	<u>Radial Traverse</u>			<u>Resulting $D B_r^2 \varphi$</u> (Axial Calculation)
	<u>$D\phi$</u>	<u>$2\pi RJ$</u>	<u>Inferred $B_r^2 \times 10^3 \text{ (cm}^{-2}\text{)}$</u>	
1	14.1787	0.00938	0.662	0.00942
2	6.9112	0.00590	0.853	0.00591
3	3.8152	0.00414	1.086	0.00415
4	<u>3.8766</u>	<u>0.00624</u>	<u>1.610</u>	<u>0.00624</u>
TOTAL	28.7817	0.02566	0.892	0.02572

NOTE: The inferred $B_r^2 = \frac{2\pi RJ}{D\phi}$.

The single net B_r^2 derived by this method to match the calculated total radial leakage is $0.892 \times 10^{-3} \text{ cm}^{-2}$. This differs slightly from the B^2 of the simply-buckled flux shape which is established over the central portion of the core. Figure 10 shows $J_o(B_r)$, with $B_r^2 = 0.892 \times 10^{-3} \text{ cm}^{-2}$, in comparison with the individual few-group shapes.

Table 11 shows the single B_r^2 value inferred from the total radial leakage calculated for each core. Agreement with experiment is good for the 60.92-in. diameter cylinders, but the calculation is 2.4% high for the 107.7-in. diameter tubs.

The single B_z^2 necessary to reproduce the calculated total axial leakage was inferred in a similar manner. Values are shown in Table 12 along with the corresponding total bucklings. A comparison of calculated and measured total bucklings is shown in Figure 11. The calculation lies about halfway between the measured fast and thermal B^2 values except for experiments 21 and 23 which are significantly out of line. Table 13 shows bucklings inferred from the calculated leakages for the spheres.

Figure 12 shows calculated few-group axial flux shapes for Experiment 16, along with the simply-buckled shape which matches the total leakage ($B_z^2 = 0.2224 \times 10^{-3} \text{ cm}^{-2}$). This buckling is about 0.6% smaller than that of the simply-buckled flux shape established over the central portion of the core. (Although relatively small, this affects the comparison between measured and calculated bucklings shown in Figure 11). In addition, individual few-group leakages obtained with this single B_z^2 differ substantially from those actually calculated, as shown in Table 14A (such behavior is mentioned in Ref. 2, p. 368).

Few Group Radial Flux Shapes Near the Boundary (Exp. 16)

FIGURE 10

TABLE 11

Comparison of Calculated and Measured
Radial Bucklings for Cylinders

<u>Experiment</u>		$B_r^2 \times 10^3 \text{ (cm}^{-2}\text{)}$	
		<u>Calculated</u>	<u>Measured (Ref. 2)</u>
U235	12	0.878	0.883 0.890
(60.92-in. diam.)	13	0.881	
	14	0.886	
	15	0.890	
	16	0.892	
U233	17	0.882	
(60.92-in. diam.)	18	0.885	
	19	0.889	
	20	<u>0.893</u>	
Average		0.886	
U235	21	0.294	0.288
(107.7-in. diam.)	22	0.295	
	23	<u>0.296</u>	
Average		0.295	

TABLE 12

Comparison of Calculated and Measured Total Bucklings for
Cylinders and Tubs

<u>Exp.</u>	$B^2 \text{ (} \times 10^3 \text{ cm}^{-2} \text{)}$			<u>Measured (Ref. 2)</u>	
	<u>Calculated</u>				
	<u>Axial</u>	<u>Radial</u>	<u>Total</u>	<u>Thermal</u>	<u>Fast</u>
12	3.633	0.878	4.511	4.67 $\pm$.11	4.46
13	3.366	0.881	4.247	4.31 $\pm$.10	4.17
14	1.548	0.886	2.434	2.47 $\pm$.05	2.42
15	0.7806	0.890	1.671	1.67 $\pm$.03	1.66
16	0.2224	0.892	1.114	1.11 $\pm$.02	1.10
17	3.102	0.882	3.984	4.09 $\pm$.10	3.93
18	2.257	0.885	3.142	3.20 $\pm$.07	3.11
19	1.380	0.889	2.269	2.29 $\pm$.05	2.25
20	0.4653	0.893	1.358	1.35 $\pm$.03	1.35
21	1.062	0.294	1.356	1.31 $\pm$.02	
22	0.6333	0.295	0.928	0.93 $\pm$.01	
23	0.1624	0.296	0.458	0.44 $\pm$.01

Ratio of Measured to Calculated Buckling
for Gwin-Magnuson Cylinders

FIGURE 11

TABLE 13

Bucklings Inferred from Calculated Leakages for Spheres

<u>Experiment</u>	<u>$B^2 \times 10^3 \text{ (cm}^{-2}\text{)}$</u>
1	6.860
2	6.842
3	6.828
4	6.823
5	6.872
6	6.867
7	6.862
8	6.857
9	6.853
10	2.402
11	2.406

Few Group Axial Flux Shapes Near the
Boundary (Experiment 16)
FIGURE 12

TABLE 14A

Comparison of Calculated Few Group Axial
Leakages at the Core Boundary (Experiment 16)

<u>Group</u>	<u>D_g</u>	<u>Inferred B_z² × 10³ (cm⁻²)</u>	<u>J</u>	<u>D_g × < B_z² ></u>
1	14.2271	0.1631	0.00232	0.00316
2	6.9317	0.2192	0.00152	0.00154
3	3.8181	0.2907	0.00111	0.00085
4	<u>3.8764</u>	<u>0.3792</u>	<u>0.00147</u>	<u>0.00086</u>
Total	28.8533	0.2224	0.00642	0.00642

*Note $\langle B_z^2 \rangle = 0.2224 \times 10^{-3} \text{ cm}^{-2}$.

The inferred $B_z^2 = \frac{J}{D_g}$.

At a distance of 9.46 cm inside the core boundary, these differences are much reduced, as shown in Table 14B. Now, the individual B_z^2 values which match the few-group leakages are nearly the same, and the B_z^2 inferred from the total leakage is close to that of the single simply-buckled flux shape ($B_z^2 = .2237 \times 10^{-3} \text{ cm}^{-2}$) established over the interior of the core (Figure 12).

B. Calculated Results

Tables 15 and 16 show criticality results and major reaction rates of interest. The experimental eigenvalues quoted for the spheres are the "corrected values" from Ref. 11, which incorporate corrections for several small effects. For the cylinders, only the room return correction is applicable, and this has been obtained by the method of Ref. 11.

Figures 13 and 14 show $\lambda_{\text{calc}} - \lambda_{\text{exp}}$ versus leakage along with the best-fit line obtained from the ENDF/B-IV analysis of small cores. The Gwin-Magnuson criticals show a leakage dependence of λ_{calc} and this appears to be consistent with that inferred from analysis of the small cores.

As a check, the P7MG results are compared with RCP Monte Carlo calculations which were done for experiments 1, 5, 16, 20, and 23. Table 17 shows a comparison of the P7MG and RCP eigenvalues, and calculated total leakages are compared in Table 18. The P7MG eigenvalues are consistently higher than those from RCP by about 0.11%. This is attributed to differences between the two calculational methods, mostly in leakage rates (Table 18), as well as possible slight lack of eigenfunction convergence of the RCP results for the very large Experiment #23. The leakage results are consistent with a fixed leakage difference of about 0.0007 between RCP and P7MG.

TABLE 14B

Comparison of Calculated Few Group Axial
Leakages at a Plane 9.46 cm from the Core Boundary (Exp. 16)

<u>Group</u>	<u>D²</u>	<u>Inferred B_z² × 10³ (cm⁻²)</u>	<u>J</u>	<u>D² × < B_z² ></u>
1	14.0126	0.2176	0.00305	0.00314
2	6.8302	0.2254	0.00154	0.00153
3	3.7621	0.2285	0.00086	0.00084
4	<u>3.8181</u>	<u>0.2383</u>	<u>0.00091</u>	<u>0.00085</u>
	28.4230	0.2237	0.00636	0.00636

Note $\langle B_z^2 \rangle = 0.2237 \times 10^{-3} \text{ cm}^{-2}$.

The inferred $B_z^2 = \frac{J}{D^2}$.

TABLE 15

Calculated Reaction Rates for U235 Criticals

<u>Exp.</u>	<u>$\lambda_{exp.}$</u>	<u>$\lambda_{calc.}$</u>	<u>Ther. $\nu F_{iss.}$</u>	<u>Epi. $\nu F_{iss.}$</u>	<u>Ther. A</u>	<u>Ther. A_x</u>	<u>Leakage</u>
1	1.00026	0.99831	0.97708	0.02123	0.80460	0.47271	0.17285
2*	0.99975	0.99600	0.97324	0.02477	0.80421	0.47090	0.16992
3*	0.99994	0.99494	0.96682	0.02811	0.80316	0.46786	0.16777
4*	0.99924	0.99637	0.96652	0.02985	0.80244	0.46773	0.16688
10	1.00031	0.99567	0.97821	0.01746	0.81212	0.47320	0.06702
12	0.99970	0.99647	0.97931	0.01915	0.85866	0.47375	0.11968
13		0.99761	0.97870	0.03891	0.86580	0.47345	0.11262
14		0.99802	0.98043	0.01759	0.91117	0.47428	0.06788
15		0.99797	0.98089	0.01708	0.93178	0.47449	0.04748
16		0.99480	0.97818	0.01662	0.94732	0.47317	0.03214
21		0.99338	0.97664	0.01675	0.94049	0.47243	0.03880
22		0.99383	0.97734	0.01649	0.95252	0.47277	0.02686
23		0.99574	0.97948	0.01626	0.95604	0.47380	0.01341

* Boron loaded

Ther A_x is the thermal absorption rate in fissile uranium.

TABLE 16

Calculated Reaction Rates for U233 Criticals

<u>Exp.</u>	<u>$\lambda_{exp.}$</u>	<u>$\lambda_{calc.}$</u>	<u>Ther. νFiss.</u>	<u>Epi. νFiss.</u>	<u>Ther. A</u>	<u>Ther. A_x</u>	<u>Leakage</u>
5	0.99949	1.00319	0.94957	0.05362	0.78882	0.41443	0.17688
6*	1.00009	1.00370	0.94786	0.05584	0.78846	0.41370	0.17592
7*	1.00015	1.00350	0.94564	0.05786	0.78807	0.41274	0.17510
8*	0.99930	1.00346	0.94360	0.05985	0.78764	0.41184	0.17434
9*	0.99942	1.00286	0.94107	0.06179	0.78718	0.41075	0.17364
11	0.99944	0.99814	0.95257	0.04556	0.90020	0.41576	0.06855
17	0.99970	0.99899	0.95087	0.04811	0.85839	0.41493	0.10938
18	↓	1.00023	0.95332	0.04691	0.88021	0.41600	0.08800
19		0.99909	0.95360	0.04549	0.90386	0.41612	0.06490
20		0.99669	0.95269	0.04400	0.92957	0.41572	0.03975

* Boron loaded.

Ther A_x is the thermal absorption rate in fissile uranium.

Calculated Eigenvalue Versus Total Leakage Fraction
for U233 Criticals

FIGURE 13

Calculated Eigenvalue Versus Total Leakage Fraction
for U235 Criticals

FIGURE 14

TABLE 17

Comparison of RCP01 and P7MG Eigenvalue Results

<u>Experiment</u>		<u>RCP01 ($\pm 2\sigma$)</u>	<u>P7MG</u>	<u>P7MG/RCP01 ($\pm 2\sigma$)</u>
U235	1	0.9969 $\pm$.0015	0.9983	1.0014 $\pm$.0015
	16	0.9939 $\pm$.0008	0.9948	1.0009 $\pm$.0008
	23	0.9949 $\pm$.0003	0.9957	1.0008 $\pm$.0003
U233	5	1.0017 $\pm$.0016	1.0032	1.0015 $\pm$.0016
	20	0.9956 $\pm$.0008	0.9967	1.0011 $\pm$.0008
				<u>1.0011 Average</u>

TABLE 18

Comparison of RCP01 and P7MG Calculated Leakages

<u>Experiment</u>		<u>RCP01 ($\pm \sigma$)</u>	<u>P7MG</u>	<u>RCP01-P7MG($\pm \sigma$)</u>
U235	1	0.1738 $\pm$.0006	0.1729	0.0009 $\pm$.0006
	16	0.0329 $\pm$.0003	0.0321	0.0008 $\pm$.0003
	23	0.0141 $\pm$.0001	0.0134	0.0007 $\pm$.0001
U233	5	0.1776 $\pm$.0006	0.1769	0.0007 $\pm$.0006
	20	0.0402 $\pm$.0003	0.0398	0.0004 $\pm$.0003

NOTE: Statistical uncertainties of RCP01 leakage rates do not include eigen-function convergence effects.

C. Thermal Criticality Parameter Constraint

The microscopic criticality balance (which expresses the constraint on fissile thermal parameters) was obtained as follows from edits of the calculated whole-core macroscopic reaction rates (see Tables 15 and 16).

In the actual core, which may not be exactly critical,

$$\lambda_{\text{exp}} = \frac{\text{Thermal } \nu\text{Fiss} + \text{Epi } \nu\text{Fiss}}{\text{Source}} \quad (15)$$

$$\text{Source} = \text{Thermal A} + \text{Epi A} + \text{Leakage} = 1.0 \quad (16)$$

Reaction rates are for the whole core and are normalized to one neutron born of fission. In the present calculations "Thermal" means 0 - 0.625 eV, "Epi" means $E > 0.625$ eV, A is absorption, Fiss is fission, and νFiss is neutron production. Leakage refers to all energies including thermal.

Thermal absorption and neutron production can be separated out by casting Eqn. (15) in the form

$$\lambda_{\text{exp}} = \frac{\text{Thermal } \nu\text{Fiss}}{\text{Thermal A}} \text{FP} \quad (17)$$

where

$$\text{FP} = \left(1 + \frac{\text{Epi } \nu\text{Fiss}}{\text{Thermal } \nu\text{Fiss}} \right) (\text{Thermal A}) \quad (18)$$

$$= \left(1 + \frac{\text{Epi } \nu\text{Fiss}}{\text{Thermal } \nu\text{Fiss}} \right) (1 - \text{Epi A} - \text{Leakage}). \quad (19)$$

Multiplication by $\frac{\sigma_{\text{aH}} / \langle \sigma_{\text{aH}} \rangle}{\phi_{\text{Th}} V N_{\text{x}}}$ converts the thermal absorption and fission rates to microscopic cross sections (denoted $\bar{\sigma}$). $\sigma_{\text{aH}} / \langle \sigma_{\text{aH}} \rangle$ is the ratio of the 2200 m/s absorption cross section for hydrogen to the spectrum-averaged value for the range 0 - 0.625 eV. $\phi_{\text{Th}} V N_{\text{x}}$ is the volume-energy integrated

thermal flux times the fissile uranium atomic number density.

Note that F and P are not separately defined in Eqn. (18). The most convenient definition seems to be the following. Let

$$p \equiv 1 - E_{pi} A \quad (20)$$

$$\epsilon \equiv 1 + \frac{E_{pi} \nu_{Fiss}}{\text{Thermal } \nu_{Fiss}} \quad (21)$$

$$P \equiv 1 - \text{Leakage} \quad (22)$$

Then from Eqn. (18), for small leakage

$$FP \approx \epsilon(1 - E_{pi} A) (1 - \text{Leakage}) \quad (23)$$

$$\approx \epsilon p P$$

$$\text{with } F = \epsilon p.$$

Further separation of the fissile uranium and all other thermal absorption leads to the form used by Gwin (Ref. 4):

$$\bar{\sigma}_{ax} + \frac{H}{X} \bar{\sigma}_{aH} = \nu \bar{\sigma}_{fx} \frac{FP}{\lambda_{exp}} = \bar{\eta}_x \bar{\sigma}_{ax} \frac{FP}{\lambda_{exp}}. \quad (24)$$

In this expression H/X is the effective hydrogen/fissile uranium atomic ratio, that is

$$\frac{H}{X} \bar{\sigma}_{aH} = \frac{N_H}{N_X} \bar{\sigma}_{aH} + \sum_{i \neq X, H} \frac{N_i}{N_X} \bar{\sigma}_{ai} \quad (25)$$

Neglecting for convenience the small deviations of λ_{exp} from 1.0, Eqn. (24) can be expressed as a constraint on the microscopic parameters, denoted K2:

$$K2 = (\bar{\eta}_x - 1) \left(\frac{\bar{\sigma}_{ax}}{\bar{\sigma}_{aH}} \right) = \frac{H}{X} + \frac{\bar{\sigma}_f}{\bar{\sigma}_{aH}} (1 - FP). \quad (26)$$

Gwin determines K2 as the value of H/X at FP = 1.0, obtained by a least-squares fit to H/X vs FP for many cores:

$$\frac{H}{X} = \left(\bar{\eta}_x \frac{\bar{\sigma}_{ax}}{\bar{\sigma}_{aH}} \right) FP - \left(\frac{\bar{\sigma}_{ax}}{\bar{\sigma}_{aH}} \right) \quad (27)$$

At FP = 1.0, the small epithermal multiplication just compensates for all leakage and epithermal absorption. In principle, the slope and intercept determine $\bar{\eta}_x$ and $\bar{\sigma}_{ax}/\bar{\sigma}_{aH}$ separately, but the sensitivity to errors in determining FP is significant since values of FP range only from 0.8 - 1.0, and the intercept at FP=0 is relatively distant.

Since, for fixed H/X (i.e. for a particular core), an error in FP is strongly reflected in the estimate of K2

$$\left(\frac{dK2}{dFP} \right)_{H/X} = -\bar{\eta}_x \frac{\bar{\sigma}_{ax}}{\bar{\sigma}_{aH}}, \quad (28)$$

it seems useful to display K2 estimates vs FP. Any drift in K2—equivalent to a drift in λ —is evidence of wrong determination of FP and hence of the slope $\bar{\eta}_x (\bar{\sigma}_{ax}/\bar{\sigma}_{aH})$. At the very least, there should be no visible drift of K2 in an analysis which seeks to extract useful values of slope and intercept. On the other hand, determination of K2 at the FP = 1.0 limit is relatively free from these uncertainties.

Note that in any particular calculation (as in Tables 15 and 16), the analogue of Eqn. (15) is:

$$\lambda_{\text{calc}} = \frac{\text{Thermal } \sqrt{f_{\text{iss}}}}{\text{Thermal } A} \text{ FP} \quad (29)$$

and

$$\text{Thermal } A = \frac{\text{Thermal } \sqrt{f_{\text{iss}}}}{\lambda'} \frac{\text{FP}}{\lambda_{\text{exp}}} \quad (30)$$

where

$$\lambda' = \frac{\lambda_{\text{calc}}}{\lambda_{\text{exp}}}.$$

This leads to the analogue of Eqn. (26)

$$K2 = \left(\frac{\pi_x}{\lambda'} - 1 \right) \left(\frac{\bar{\sigma}_{ax}}{\bar{\sigma}_{aH}} \right) = \frac{H}{X} + \frac{\sqrt{\sigma} f_x}{\lambda' \bar{\sigma}_{aH}} \left(1 - \frac{\text{FP}}{\lambda_{\text{exp}}} \right) \quad (31)$$

and K2 can be obtained from the calculated quantities on either side of the equation.

The calculated "g-factors" shown in Table 19 are by definition $\bar{\sigma} / \sigma$ (.0253 eV). These are significantly smaller than the usual thermal Maxwellian g-factors*, and in fact show a slight variation over the range of H/X spanned by these criticals. Values of g_a and g_f at the limit FP = 1.0, suitable for use in applying the constraint to the 2200 m/s normalization, are also shown in Table 19. Note that thermal spectrum averages in these calculations are over the flux spectrum from 0-0.625 eV.

* At the pre-Version IV thermal data task force meeting, only the eigenvalues from the calculations were used. Thermal Maxwellian g-factors were used to derive the constraint K1 ($=K2 \times \bar{\sigma}_{aH}$), and presumably these same g-factors were used in applying the constraint. In any case, this seems to be the reason for the high K1 values about which Gwin comments (Ref. 4).

TABLE 19

Additional Calculated Results[†]

Experiment	g_a	g_f	H/X	FP	K2	C'
U235						
1	.97196	.96921	1404.1	.8221	2140.8	736.7
2*	.97093	.96808	1414.0	.8247	2139.3	725.3
3*	.96996	.96699	1430.3	.8265	2149.4	719.1
4*	.96949	.96646	1427.4	.8272	2142.2	714.8
10	.97286	.97022	1856.7	.9284	2154.3	297.6
12	.97249	.96981	1625.7	.8754	2141.6	515.9
13	.97252	.96981	1658.2	.8825	2145.3	487.1
14	.97284	.97020	1843.9	.9275	2144.4	300.5
15	.97296	.97033	1929.3	.9480	2144.9	215.6
16	.97306	.97044	2006.2	.9634	2158.4	152.2
21	.97302	.97040	1983.5	.9566	2164.3	180.8
22	.97309	.97047	2031.7	.9686	2162.5	130.8
23	.97316	.97055	2080.2	.9821	2154.8	74.6
#	.97326	.97065		1.0		
U233						
5	1.00072	.99771	1554.9	.8334	2210.0	655.1
6*	1.00079	.99775	1559.3	.8349	2208.0	648.7
7*	1.00085	.99780	1565.4	.8363	2208.7	643.3
8*	1.00092	.99784	1570.9	.8376	2209.2	638.3
9*	1.00098	.99788	1577.7	.8389	2211.5	633.8
11	1.00052	.99758	2005.0	.9433	2229.2	224.2
17	1.00058	.99762	1839.2	.9018	2226.8	387.6
18	1.00055	.99759	1920.3	.9235	2221.9	301.6
19	1.00052	.99757	2017.0	.9470	2226.3	209.3
20	1.00049	.99755	2127.0	.9725	2235.8	108.8
#	1.00045	.99752		1.0		

* Boron loaded.

[†] See text for definitions of g_a , g_f , H/X, FP and K2.

$$C' = \frac{\nu \bar{\sigma}_{fx}}{\lambda' \bar{\sigma}_{aH}}$$

The values of g_a and g_f at FP = 1.0 have been obtained by extrapolation.

K2 values, calculated for each core from Eqn. (31), are shown in Table 19 and in Figures 15 and 16. Table 20 shows the limiting values of K2 at FP = 1.0 obtained from a least squares fit for various assumptions. For comparison, the same quantity obtained from a fit to Eqn. (27) is also shown. Differences between these two estimates are small.

The uncertainties shown for K2 estimates of Table 20 are standard deviations obtained from the least squares fit. They reflect only the scatter of K2 values (in Figure 15 for example) which is attributed to experimental fissile loading uncertainties. It is noteworthy that eigenvalues of the U233 criticals exhibit much less scatter than do those of the U235 criticals.

For U235 systems there is a significant difference in K2 (at FP = 1.0) depending on whether or not the three tubs (Experiments 21, 22, 23) are included in the fit: including the tubs raises K2 by 0.26%. Whether or not the boron loaded cores are included has little effect.

The nonconstancy of the K2 estimates with FP is due to the underprediction of leakage. To assess the effect of this leakage error on the limiting value of K2, a second series of calculations was done with fast leakage enhanced artificially by hardening the fission spectrum (a Maxwellian with $\bar{E} = 2.1$ Mev was used). These results are shown in Figures 15 and 17 and in Table 20. The calculations with enhanced leakage produce only small differences in the estimate of K2 at FP = 1.0. It should be noted that the device used (hardening the fission spectrum) produces a small residual eigenvalue difference in addition to the main effect, the increase of leakage.

TABLE 20

Estimates of K2 at FP = 1.0

	<u>Experiments</u>	<u>From Eqn. 27</u>	<u>From K2 vs FP</u>	<u>From K2 vs FP (enhanced leakage)</u>
U235	All	2158.6 $\pm$ 3.1	2159.5 $\pm$ 3.3	2160.9 $\pm$ 3.5
	Omit 2, 3, 4	2160.1 $\pm$ 3.7	2161.0 $\pm$ 4.0	2162.6 $\pm$ 4.3
	Omit 21, 22, 23	2153.4 $\pm$ 3.8	2154.4 $\pm$ 4.5	2155.4 $\pm$ 4.8
U233	All	2237.4 $\pm$ 2.2	2238.9 $\pm$ 2.5	2242.3 $\pm$ 2.3
	Omit 6, 7, 8, 9	2236.9 $\pm$ 3.8	2238.1 $\pm$ 4.3	2241.7 $\pm$ 4.3

Note: Uncertainties are standard deviations obtained from the least squares fit.

$$K2 = (\bar{\eta}_x - 1) \left(\frac{\bar{\sigma}_{ax}}{\bar{\sigma}_{aH}} \right)$$

See Eqn. 19 for definition of FP.

Experiments 2, 3, 4 and 6, 7, 8, 9 are boron loaded.

Experiments 21, 22, 23 are the large tubs.

Thermal Criticality Parameter K2 Versus FP, Calculated
with ENDF/B-IV and with Augmented Leakage for
U233 Criticals

FIGURE 15

Thermal Criticality Parameter K_2 Versus FP Calculated
with ENDF/B-IV for U235 Criticals

FIGURE 16

Thermal Criticality Parameter K_2 Versus FP Calculated
with Augmented Leakage for U235 Criticals

FIGURE 17

The best K2 values to be derived from this analysis are taken to be:

$$U233 \qquad 2240 \pm 11 \text{ (0.5\% } \sigma \text{)}$$

$$U235 \qquad 2155 \pm 11 \text{ (0.5\% } \sigma \text{)}$$

These are somewhat higher than values derived by Gwin (Ref. 4), which are shown in Table 21.

An overall uncertainty of ± 11 units (0.5%) has been assigned to K2 based on consideration of the items summarized in Table 22. A detailed discussion of these factors is given in Appendix I.

The U235 tubs have been omitted from the fit and the uncertainty chosen large enough to include them. Although no specific reasons can be cited for this choice, it is felt that the tubs are slightly less well understood than the spheres and cylinders (for example, no experimental support-structure correction was applied) and they were measured in a different facility. Also, consistency with the U233 results may be improved by this choice, since there were no U233 tubs.

Since for all practical purposes the derived value of K2 is independent of the value taken for σ_{aH} (see Appendix II) and $K1 = K2 \times \bar{\sigma}_{aH}$, two major alternatives in the least squares adjustment are:

- (1) Let σ_{aH} be adjustable, using the constraints

$$\left(\bar{n}_x - 1 \right) \frac{\bar{\sigma}_{ax}}{\bar{\sigma}_{aH}} = K2 \pm \delta K2 \qquad (32)$$

TABLE 21

Values of K2 at FP = 1.0 Derived by Gwin (Ref. 4) with
Measured Bucklings

	<u>U233</u>	<u>U235</u>
Fast B ²	2237	2150
Thermal B ²	<u>2231</u>	<u>2145</u>
AVERAGE	2234	2148

TABLE 22

Factors Considered in Assigning Uncertainty to K2

	Units of K2	
	<u>U235</u>	<u>U233</u>
Experimental fissile loading (H/X)	<u>+ 5</u>	<u>+ 3</u>
Experimental corrections (λ_{exp})	4	4
Epithermal absorptions (F)	2	4
Leakage estimate (P)	1	3
Effect of including tubs	+6	--

and $\sigma_{aH} = .332 \pm .002$ (0.6%)

plus calculated "g-factors" (Table 19).

(2) Let σ_{aH} be held fixed, using the constraint

$$(\bar{\eta}_x - 1) \bar{\sigma}_{ax} = K1 \pm \delta K1 \quad (33)$$

where

$$K1 = K2 \times \bar{\sigma}_{aH}$$

plus calculated "g-factors" (Table 19).

The uncertainty on the fissile parameter constraint $K1 \equiv (\bar{\eta}_x - 1) \bar{\sigma}_{ax}$ is dominated by the uncertainty of σ_{aH} (.0253 eV). That is

$$\left(\frac{\delta K1}{K1} \right)^2 = \left(\frac{\delta K2}{K2} \right)^2 + \left(\frac{\delta \sigma_{aH}}{\sigma_{aH}} \right)^2 \quad (34)$$

A 0.5% uncertainty on K2 and 0.6% on σ_{aH} , implies 0.8% uncertainty on K1*. It should be noted that the value and uncertainty adopted for σ_{aH} are somewhat "conventional" and might well be re-examined.

* In a previous analysis of the U235 spheres (done in connection with ENDF/B-IV preparation), an uncertainty on λ_{calc} of 0.4% was chosen. This implies an uncertainty of ~0.8% on K1.

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APPENDIX IUncertainty of K2 Estimate

The components of uncertainty in the estimate of K2 can be seen from Eqn. 31:

$$K2 = \frac{H}{X} + c' \left(1 - \frac{FP}{\lambda_{exp}} \right) \quad (I-1)$$

where

$$c' \equiv \frac{\sqrt{\sigma} f_x}{\lambda' \sigma_{aH}} \approx 4000 \quad (\text{for both U233 and U235})$$

Now,

$$\delta(K2) = \delta(H/\lambda) + (\delta c') \left(1 - \frac{FP}{\lambda_{exp}} \right) + c' \delta \left(1 - \frac{FP}{\lambda_{exp}} \right) \quad (I-2)$$

Near $\frac{FP}{\lambda_{exp}} = 1.0$,

$$\delta(K2) = \delta(H/X) + c' \delta \left(1 - \frac{FP}{\lambda_{exp}} \right) \quad (I-3)$$

$$= \delta(H/X) - c' \delta \left(\frac{FP}{\lambda_{exp}} \right) \frac{FP}{\lambda_{exp}} = 1.0$$

The term $\delta(H/X)$ is dominated by the experimental loading uncertainty (from the least squares fit, Table 20). The calculation itself contributes negligible uncertainty here, and no account is taken of any possible systematic error in loading determination.

The significant calculational uncertainties enter through

$$\left[\delta \left(\frac{FP}{\lambda_{exp}} \right) \right] \frac{FP}{\lambda_{exp}} = 1.0$$

- (1) An uncertainty of 0.1% is assigned to λ_{exp} to account for experimental effects other than fissile inventory. Corrections (Ref. 2,11) to λ_{exp} accounting for such things as room return and support structure are relatively small. The room return correction (Ref. 11) is approximately -.0003 for all cores, but this could be doubled under different assumptions (Ref. 18). The net correction (Ref. 11) for all effects (for the spheres) is approximately -.001.
- (2) Uncertainty of F is estimated as follows. From Eqn. (23):

$$F \approx \left(1 + \frac{Epi \ vFiss}{Ther \ vFiss} \right) (1 - Epi A) \quad (I-4)$$

$$F-1 \approx Epi \ vFiss - Epi A$$

$$\approx (\nu-1) Epi Fiss - Epi C_x - Epi A_H - Epi A_O$$

where the terms represent fission, uranium capture, hydrogen absorption and oxygen absorption respectively. Other epithermal absorption is negligible.

$$\begin{aligned} \delta F &= \delta(F-1) \approx (\nu-1) \delta(Epi Fiss) + (Epi Fiss) \delta(\nu-1) \\ &\quad - \delta(Epi C_x) - \delta(Epi A_H) - \delta(Epi A_O). \end{aligned} \quad (I-5)$$

Table AI-1 shows the isotopic epithermal absorption rates for representative experiments (20 and 23). Aside from uranium, only hydrogen and oxygen are important. At this dilution, epithermal reaction rate uncertainties should be dominated by uncertainties of the absorption and fission ("resonance") integrals. Evaluation of the terms in Eqn. (I-5) is summarized in Table I-2. Depending on correlations, the net uncertainty of F is less than 0.1% and likely to be .05% for U235. For U233 these uncertainties are doubled due to the significantly larger epithermal fission integral. Values of .05% and 0.1% respectively are chosen. With $C' = 4000$, these contribute 2 and 4 units to the uncertainty of K_2 .

(3) Errors stemming from the leakage calculation (via P) are relatively small at $FP = 1.0$. The enhanced fast leakage calculation (Table 20) increased the K_2 estimate by about 1.0 unit for U235 and 3 units for U233.

TABLE I-1Epithermal Absorption Rates

	<u>Exp. 23</u>	<u>Exp. 20</u>
U235Fiss	.00667	--
Cap	.00321	--
U233Fiss	--	.01756
Cap	--	.00310
H	.00670	.00652
O	.00317	.00314
U234	.00017	.00022
U236	.00006	--
U238	.00038	.00007
N	<u>.00023</u>	<u>.00014</u>
TOTAL	.02059	.03075

TABLE I-2Factors Contributing to Uncertainty of F

<u>Term (Eqn. I-5)</u>	<u>Assigned Uncertainty</u>	<u>Contribution to δF</u>	
		<u>Exp. 23</u>	<u>Exp. 20</u>
$(\nu-1) \delta(\text{Epi Fiss})$	U235 1.8%	.00017	--
	U233 1.7%	--	.00045
$(\text{Epi Fiss}) \delta (\nu-1)$	2% on ν	.00033	--
		--	.00087
$\delta(\text{Epi } C_x)$	U235 4.2%	.00013	--
	U233 4.3%	--	.00013
$\delta(\text{Epi } A_H)$	1%	.00007	.00007
$\delta(\text{Epi } A_O)$	5%	<u>.00016</u>	<u>.00016</u>
Total (uncorrelated)		.00043	.00100
(additive)		.00086	.00168

NOTE: Uranium capture and fission integral uncertainties are taken from Ref. 19.

II-1

APPENDIX II

Sensitivity of K2 to $\sigma_{aH}(.0253 \text{ eV})$

For practical purposes, the estimate of K2 at FP = 1.0 is independent of the choice of $\sigma_{aH}(.0253 \text{ eV})$. This was verified by repeating the entire analysis with $\sigma_{aH}(.0253 \text{ eV}) = 0.330 \text{ b}$ instead of 0.332 b. Results are shown in Table II-1.

TABLE II-1

Sensitivity of K2 (FP = 1.0) to Choice of $\sigma_{aH}(.0253 \text{ eV})$

<u>$\sigma_{aH}(.0253 \text{ eV})$</u>	<u>K2</u>	
	<u>U233</u>	<u>U235</u>
0.332 b	2236.1	2159.1
0.330 b	2237.0	2159.7

Details of the sensitivity can be seen from the expression (Ref. 4):

$$K1 = \frac{H}{X} \bar{\sigma}_{aH} + \frac{\bar{\omega}_{fx}}{\lambda'} (1-FP) = \frac{H}{X} \bar{\sigma}_{aH} + c$$

and from Table II-2 which shows the component parts of K1 for two extreme cases (Experiments 1 and 23).

H/X and $\bar{\omega}_{fx}$ are essentially unchanged and c shows relatively small change, stemming from small changes of λ' and $1-FP$ which nearly cancel. For large cores, where c is relatively unimportant, the 0.6% reduction of $\bar{\sigma}_{aH}$ causes nearly a 0.6% reduction of K1 and no change of $K2 = \frac{K1}{\bar{\sigma}_{aH}}$. For the leakier cores, where c is ~25% of K1, there is a 0.45% reduction of K1 and a 0.15% increase of K2.

Since K_2 is independent of σ_{aH} for the large cores and slightly dependent for the smaller cores, one might in principle use consistency of the K_2 estimate over all cores to determine a best σ_{aH} . Unfortunately, the small variation is overwhelmed by uncertainty of the leakage calculation and by experimental scatter.

TABLE II-2Sensitivity of the Constraints to $\sigma_{aH}(.0253 \text{ eV})$

	Experiment 1			Experiment 23		
			% Change			% Change
$\sigma_{aH}(b)$	.332	.330	-.6	.332	.330	-.6
λ'	.99829	1.00063	+.23	.99592	.99889	+.30
H/X	1404.1	1404.3		2080.2	2080.4	
$\bar{\omega}_{fx}$	1372.4	1372.4		1374.3	1374.3	
1-FP	.17794	.17803		.01775	.01780	
C	244.6	244.2		24.5	24.5	
K2	2140.9	2144.2	+.15	2154.0	2154.6	+.03
K1	710.8	707.6	-.45	715.1	711.0	-.57

NOTE: These values are from preliminary calculations.