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FULL-WAVE CALCULATIONS IN FLUX COORDINATES FOR TOROIDAL GEOMETRY

PRESENTED BY

B. A. Carreras, V. E. Lynch, E. F. Jaeger, and D. B. Batchelor

Oak Ridge National Laboratory

Oak Ridge, Tennessee 37831

SHERWOOD THEORY MEETING

Gatlinburg, Tennessee

April 18-20, 1988

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ABSTRACT

A new 2-D full-wave code, HYPERION, employing a poloidal and toroidal mode expansion and including the toroidal terms arising in the wave equation has been developed. It is based on the existing modules developed for the MHD stability codes and uses as input the tokamak equilibria calculated with the RSTEQ code¹. At present the plasma response is described by the collisionally broadened cold plasma conductivity. However, the code is written in straight field line coordinates, this permits the accurate representation of $k_{||}$ and as a consequence allows the incorporation of the plasma Z functions. This code also retains the $E_{||}$ component of the electric field which will allow the study of the low density region of the plasma.

We have done detailed benchmarking of the HYPERION code in the cold plasma limit with the existing finite difference ORION² full-wave code. The agreement is very good.

¹J. A. Holmes, Y-K. M. Peng, and S. J. Lynch, J. Comput. Phys. **36**, 35 (1980).

²E. F. Jaeger, D. B. Batchelor, H. Weitzner, and J. H. Whealton, Comput. Phys. Com. **40**, 33 (1986).

FULL-WAVE CALCULATIONS IN FLUX COORDINATES FOR TOROIDAL GEOMETRY

- A new 2-D full-wave code, HYPERION, has been developed:

- 1) The code is fully toroidal and includes the effects of cross section shaping.

- 2) The code is written in straight field line coordinates, this permits the accurate representation of $k_{||}$ and as a consequence allows the incorporation of the plasma Z functions.

- 3) This code also retains the $E_{||}$ component of the electric field which allows the study of the low density region of the plasma.

- Present results:

- 1) Detailed benchmarking of the HYPERION code in the cold plasma limit with the existing finite difference ORION¹ full-wave code.

- 2) Comparison between cylindrical and toroidal calculations for TFTR parameters.

¹E. F. Jaeger, D. B. Batchelor, H. Weitzner, and J. H. Whealton, Comput. Phys. Com. **40**, 33 (1986)

EQUATIONS

In straight field line coordinates (ρ, θ, ζ) the equations solved in the HYPERION code are:

$$\begin{aligned} n\epsilon E_r &= -\omega R^2 (g^{rr} B_\theta + g^{\theta r} B_r) - i \frac{\partial E_\zeta}{\partial \rho} \\ n\epsilon E_\theta &= -\omega R^2 (g^{\theta r} B_r + g^{rr} B_\theta) - i \frac{1}{\rho} \frac{\partial E_\zeta}{\partial \theta} \\ n\epsilon B_r &= -\omega (\alpha^{rr} E_r + \alpha^{\theta r} E_\theta + \alpha^{r\zeta} E_\zeta) - i J^\theta R^2 - i \frac{\partial B_\zeta}{\partial \rho} \\ n\epsilon B_\theta &= -\omega (\alpha^{r\theta} E_r + \alpha^{\theta\theta} E_\theta + \alpha^{\theta\zeta} E_\zeta) + J^r R^2 - i \frac{1}{\rho} \frac{\partial B_\zeta}{\partial \theta} \\ 0 &= \frac{1}{\rho} \frac{\partial(\rho B_\theta)}{\partial \rho} - \frac{1}{\rho} \frac{\partial B_r}{\partial \theta} + i\omega (\epsilon \alpha^{r\zeta} E_r + \epsilon \alpha^{\theta\zeta} E_\theta + \alpha^{\zeta\zeta} E_\zeta) - J^\zeta R^2 \\ 0 &= \frac{1}{\rho} \frac{\partial E_r}{\partial \theta} - \frac{1}{\rho} \frac{\partial \rho E_\theta}{\partial \rho} + i\omega B_\zeta \end{aligned}$$

where the cold plasma conductivity tensor (α^{ij}) in this coordinate system is:

$$\begin{pmatrix} R^2 g^{rr} K_\perp & R^2 g^{r\theta} K_\perp + i K_\times \frac{F}{R_0 B} & g^{r\theta} \frac{\rho}{q} i K_\times \frac{F}{R_0 B} \\ R^2 g^{\theta r} K_\perp - i K_\times \frac{F}{R_0 B} & R^2 g^{\theta\theta} K_\perp + (K_\perp - K_\parallel) \left(\frac{F \rho \epsilon}{R B q} \right)^2 & \frac{\rho}{q} \left(i g^{\theta\theta} K_\times \frac{F}{R_0 B} + (K_\perp - K_\parallel) \left(\frac{F}{R B} \right)^2 \right) \\ -g^{r\theta} \frac{\rho}{q} i K_\times \frac{F}{R_0 B} & -\frac{\rho}{q} \left(i g^{r\theta} K_\times \frac{F}{R_0 B} (K_\perp - K_\parallel) \left(\frac{F}{R B} \right)^2 \right) & K_\perp - (K_\perp - K_\parallel) \left(\frac{F}{R B} \right)^2 \end{pmatrix}$$

with:

$$K_\perp = 1 - \sum_i \frac{\omega_{pi}^2}{\omega^2 - \Omega_i^2}, \quad K_\times = \sum_i \frac{\Omega_i}{\omega} \frac{\omega_{pi}^2}{\omega^2 - \Omega_i^2}, \quad K_\parallel = 1 - \sum_i \frac{\omega_{pi}^2}{\omega^2}$$

The lengths are normalized to the minor radius a , the times to $\tau = a/c$, the magnetic fields to the value B_0 of the toroidal field at the magnetic axis and the electric field to $E_0 = a B_0 / \tau$.

NUMERICAL APPROACH

- The new 2-D full-wave code, HYPERION, employs a poloidal and toroidal mode expansion and includes the toroidal terms arising in the wave equation. It is based on existing modules developed for the MHD stability codes and uses as input the tokamak equilibria calculated with the RSTEQ code

- To remove the singularities: 1) the mass m_j has been replaced by $m_j(1+iv/\omega)$, and 2) the equations have been multiplied by the function:

$$\Gamma = \prod_j \left\{ \left[(\omega^2 - v^2) \left(\frac{R}{R_0} \right)^2 - \Omega_{0j}^2 f(\rho, \theta) \right]^2 + 4 \omega^2 v^2 \left(\frac{R}{R_0} \right)^4 \right\}$$

where

$$f(\rho, \theta) = \frac{F}{R_0 B_0} \left(1 + \frac{\varepsilon^2 \rho^2}{q^2} g^{\rho\rho} \right) \equiv \left(\frac{RB}{R_0 B_0} \right)^2$$

The input equilibrium parameters, $R(\rho, \theta)$, $Z(\rho, \theta)$, and $g^{ij}(\rho, \theta)$ are given as a Fourier series expansion. In this way, the (α^j) once multiplied by Γ are directly expressed through convolutions of the equilibrium parameters and no Fourier transform are required.

- The code uses finite differences in the "radial" direction. A two grid method is used, with the fields E_ρ, B_θ , and B_ζ defined on the half grid points and the fields B_ρ, E_θ , and E_ζ on the integer grid points. In this way the boundary conditions are only relevant for the latter.

NUMERICAL TESTS

- To benchmark the HYPERION code detailed comparisons have been made with the results of ORION code in the cylindrical limit.
- As an illustration, we present a case for TFTR parameters similar to one of the cases studied by Smithie et al (Nuclear Fusion **27**,1319,1987)

$$B=4 \text{ T}$$

$$n(0)= 3.3 \times 10^{13} \text{ cm}^{-3}$$

$$R_0= 3 \text{ m}$$

$$a_{\text{wall}}= 1 \text{ m}$$

$$a_{\text{plasma}}= 0.99 \text{ m}$$

$$a_{\text{antenna}}= 0.95 \text{ m}$$

$$\eta = \frac{n(\text{H})}{n(\text{D})} = 0.05$$

$$\frac{v}{\omega} = 0.01$$

$$f=55 \text{ MHz}$$

- The antenna is a half loop antenna with poloidal width $\pi/2$, and toroidal length of 10 cm.

HYPERION

ORION

HYPERION

ORION

HYPERION

ORION

CONVERGENCE STUDIES

- The convergence with poloidal mode number m is surprisingly good. For the same equilibrium parameters and antenna configuration the toroidal case has a broader m spectrum than the corresponding cylindrical limit

- However, few modes are required to give a converged value for the plasma loading resistance

COMPARISON BETWEEN CYLINDRICAL AND TOROIDAL RESULTS

- To compare cylindrical and toroidal results we have considered a toroidal plasma equilibrium with the same q -profile as the cylindrical case and low beta. The antenna dimensions are taken to be the same in both cases as well as the total current in the antenna.
- For the case of low field antenna, the plasma loading resistance n -spectrum is broader and with higher values for the toroidal case

- Note that in the cylindrical case $k_z = n/R_0$ and in the toroidal case $k_z = n/R$ with $R > R_0$ in the low field.

TFTR PARAMETERS HALF LOOP ANTENNA

low field antenna

$n=1$

$|E_+|$

cylindrical

toroidal

plasma loading resistance

9.30 Ohm

10.2 Ohm

TFTR PARAMETERS HALF LOOP ANTENNA

high field antenna

$n=1$ $|E_+|$

cylindrical

toroidal

plasma loading resistance

8.84 Ohm

5.19 Ohm

COMPARISON BETWEEN CYLINDRICAL AND TOROIDAL RESULTS

- For a high field antenna the plasma loading resistance n-spectrum is less broad and with lower values for the toroidal case. Note that for a high field antenna and a given toroidal mode number n the cylindrical k_z is lower than the effective toroidal k_z .
- Using the cold plasma local dispersion relation for $n=40$ with $k_z=n/R$ and the parameters considered here

Cylindrical high field antenna $n=40$

Toroidal high field antenna $n=40$

POLOIDAL MODE NUMBER SPECTRUM

- The poloidal mode number (m) spectrum of the E-field in toroidal geometry is characterized by a peak at low m values ($m < 7$) plus a broad background of larger m modes

- The poloidal localization of the antenna seems to mostly affect the width of the central peak

FLEXIBILITY OF THE HYPERION CODE

- The equations in the HYPERION code are expressed in flux coordinates, thus the cross section shaping effects are automatically included.
- HYPERION uses as input the equilibria from the RSTEQ code. These are the same equilibria used for MHD stability calculations. As an example here is shown the $|E_+|$ for an equilibrium with CIT parameters

FUTURE WORK

- In the next few months, we expect to move in the following directions:
 - 1) Improve the efficiency of the code, reducing the storage requirements.
 - 2) Include warm plasma effects with the proper representation of $k_{||}$.
 - 3) Evaluate the possibility of generalizing this approach to 3-D configurations.