WORKSHOP ON HIGH-RESOLUTION, LARGE-ACCEPTANCE SPECTROMETERS

Argonne National Laboratory
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B. Zeidman, Physics Division,
Building 203
Argonne National Laboratory,
Argonne, Illinois 60439

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FOREWORD

Recent developments in accelerator technology have provided opportunities for investigations of nuclear reactions that are unprecedented in both scope and precision. In order to take advantage of these opportunities, new high-resolution and large-acceptance spectrometers are required. The purpose of the Workshop on High-Resolution, Large-Acceptance Spectrometers was to provide a means for exchange of information among those actively engaged in the design and construction of these new spectrometers. In addition, it is hoped that this workshop will stimulate ongoing interactions that will allow designers of new spectrometer systems to more effectively fulfill experimental physics needs.

The Workshop was characterized by vigorous, stimulating discussions which started with the introductory remarks and continued throughout the week. The program was basically organized into subject areas, although allowances were made for travel plans. Contributions are arranged by subject area and generally follow the order of presentation, except for written contributions from those who were unable to make a physical appearance.

The success of the Workshop resulted from the active involvement and cooperation of all participants. We thank them and Barbara Kponou, the Workshop Secretary, for their efforts. We also are indebted to the Argonne Universities Association for their support.

B. Zeidman, Editor
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>SECTION</th>
<th>CONTRIBUTION</th>
</tr>
</thead>
</table>
| I.      | High Resolution Magnetic Spectrometers  
          S. Kowalski | A |
| II.     | CALCULATIONAL TOOLS  
          High Energy Charged Particle Optics  
          Computer Programs  
          D. C. Carey | A |
|         | New Features in TRANSPORT  
          D. C. Carey | B |
|         | Ray Trace  
          S. Kowalski | C |
|         | Modifications to POISSON  
          L. H. Harwood | D |
|         | Field Calculations - Choice of Variables  
          and Methods  
          L. R. Turner | E |
|         | Examples of 3-D Field Calculations Using GFUN  
          R. J. Lari | F |
| III.    | COMPONENTS AND TECHNOLOGY  
          Superconducting Magnet Technology  
          R. P. Smith | A |
|         | Why Use Superconducting Coils in Conventional  
          Spectrograph Magnets?  
          J. A. Nolen, Jr. | B |
|         | Dipole Design Studies for the MSU 1.2 GeV/c  
          Superconducting Spectrograph  
          A. F. Zeller, J. A. Nolen, Jr., and  
          L. H. Harwood | C |
|         | Superconducting "Panofsky" Quadrupoles  
          L. H. Harwood | D |
|         | Phase Space Rotation with Solenoids and  
          Quadrupoles  
          J. A. Nolen, Jr. | E |
|         | Magnet Construction - Conventional  
          H. A. Enge | F |
|         | Homogenizing the Field in a Picture-Frame Magnet  
          P. Debenham | G |
|         | Understanding the Purcell Filter  
          P. Debenham | H |
|         | Harmonic Content of PEP Dipoles and Some Related  
          Effects and Lessons  
          J. E. Spencer | I |
<table>
<thead>
<tr>
<th>SECTION</th>
<th>CONTRIBUTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>IV. DESIGN CONSIDERATIONS: DETECTORS, BEAMS, SYSTEMS</td>
<td></td>
</tr>
<tr>
<td>Spectrometer Detectors</td>
<td>A</td>
</tr>
<tr>
<td>C. L. Morris</td>
<td></td>
</tr>
<tr>
<td>General Design Methods</td>
<td>B</td>
</tr>
<tr>
<td>H. A. Enge</td>
<td></td>
</tr>
<tr>
<td>Optimizing Energy Resolution in Accelerator-Spectrograph Systems</td>
<td>C</td>
</tr>
<tr>
<td>E. Kashy</td>
<td></td>
</tr>
<tr>
<td>Optimization of Induced Aberrations in Ion Optical Systems</td>
<td>D</td>
</tr>
<tr>
<td>L. H. Harwood, J. A. Nolen, Jr., and K. L. Brown</td>
<td></td>
</tr>
<tr>
<td>Integrated Magnet, Optics and Detector Designs for a Spectrograph</td>
<td>E</td>
</tr>
<tr>
<td>J. A. Nolen, Jr.</td>
<td></td>
</tr>
<tr>
<td>The MSU 1.6 GeV/c Beam Analysis System</td>
<td>F</td>
</tr>
<tr>
<td>J. A. Nolen, Jr.</td>
<td></td>
</tr>
<tr>
<td>The Beam Handling System at NIKHEF-K</td>
<td>G</td>
</tr>
<tr>
<td>H. de Vries</td>
<td></td>
</tr>
<tr>
<td>Storage Rings and Such</td>
<td>H</td>
</tr>
<tr>
<td>J. E. Spencer</td>
<td></td>
</tr>
<tr>
<td>Coincidence Set-Up with a High Duty-Cycle, High Energy Electron Accelerator</td>
<td>I</td>
</tr>
<tr>
<td>Ph. Leconte</td>
<td></td>
</tr>
<tr>
<td>Spectrometer Requirements for (e,e'2N) Studies</td>
<td>J</td>
</tr>
<tr>
<td>J. W. Lightbody, Jr.</td>
<td></td>
</tr>
<tr>
<td>Systems Considerations for Large Solid Angle Detection of Coincident Reaction Products From Nuclear Electroexcitation</td>
<td>K</td>
</tr>
<tr>
<td>L. S. Cardman and C. N. Papanicolas</td>
<td></td>
</tr>
<tr>
<td>SECTION</td>
<td>CONTRIBUTION</td>
</tr>
<tr>
<td>---------</td>
<td>--------------</td>
</tr>
<tr>
<td>V.</td>
<td></td>
</tr>
<tr>
<td>SPECTROMETER DESIGNS</td>
<td></td>
</tr>
<tr>
<td>LEP Spectrometer Design</td>
<td>A</td>
</tr>
<tr>
<td>R. L. Boudrie</td>
<td></td>
</tr>
<tr>
<td>The MSU 1.2 GeV/c Spectrograph</td>
<td>B</td>
</tr>
<tr>
<td>A. F. Zeller, J. A. Nolen, Jr.,</td>
<td></td>
</tr>
<tr>
<td>L. H. Harwood and E. Kashy</td>
<td></td>
</tr>
<tr>
<td>A Superconducting Multigap Spectrograph</td>
<td>C</td>
</tr>
<tr>
<td>H. A. Enge</td>
<td></td>
</tr>
<tr>
<td>A Very Large Solid Angle Spectrometer for Single Arm Electron Scattering Experiments</td>
<td>D</td>
</tr>
<tr>
<td>Ph. Leconte</td>
<td></td>
</tr>
<tr>
<td>The QDD and QDO Spectrometers at NIKHEF-K</td>
<td>E</td>
</tr>
<tr>
<td>H. de Vries</td>
<td></td>
</tr>
<tr>
<td>Design Considerations for a Coincidence Arrangement of Two Magnetic Spectrometers with Large Solid Angle and Moment Acceptances</td>
<td>F</td>
</tr>
<tr>
<td>R. Neuhausen</td>
<td></td>
</tr>
<tr>
<td>Moderate Resolution Spectrometer</td>
<td>G</td>
</tr>
<tr>
<td>R. York and R. Minehart</td>
<td></td>
</tr>
<tr>
<td>High Resolution Spectrometer</td>
<td>H</td>
</tr>
<tr>
<td>R. York and R. Minehart</td>
<td></td>
</tr>
<tr>
<td>The IUCF Double Magnetic Spectrometer System</td>
<td>I</td>
</tr>
<tr>
<td>H. Nann</td>
<td></td>
</tr>
<tr>
<td>A Design for the Large Electron Spectrometer (LES)</td>
<td>J</td>
</tr>
<tr>
<td>B. Zeidman</td>
<td></td>
</tr>
<tr>
<td>VI.</td>
<td></td>
</tr>
<tr>
<td>Program</td>
<td>A</td>
</tr>
<tr>
<td>List of Attendees</td>
<td>B</td>
</tr>
</tbody>
</table>
I. INTRODUCTION

The detailed microscopic probing of nuclear structure with energetic charged particles (electrons, protons, pions, etc.) and the study of nuclear reaction products often requires high energy resolution. This is particularly important for achieving adequate signal-to-noise in separating closely spaced resonances. In addition, large solid angles of acceptance are essential for the measurement of very small cross sections. Magnetic spectrometers remain in many cases, the only practical instruments for making such precise measurements ($\Delta p/p < 1 \times 10^{-4}$) of charged particle momenta. The usually divergent requirements of resolution and acceptance place serious constraints on the design of such systems.

During the past decade or so, important advances and improvements have been made in the design and construction of high resolution, large solid angle magnetic spectrometers. These have occurred in the areas of ion-optics, magnetics, materials, construction techniques and detectors. In this review we examine some of the important characteristics of a variety of state-of-the-art instruments currently under design, construction or operation in various nuclear laboratories. Included are devices such as the QDD, QDDD, QDQ, QQSP and various single dipole configurations as well. In addition, in some cases, use has been made of the 'energy-loss' (dispersion matching) technique. The correct matching of the beam analysis system in this case to the spectrometer optics allows the use of accelerator beams of 10-100× worse energy spread than the desired momentum resolution.

The use of superconducting technology is still relatively novel as applied to magnetic spectrometers and will not be
discussed here. Other contributions to this workshop will touch on some of these aspects as applied to the design of specific magnetic elements. It is likely that superconducting devices will play an important role in the construction of magnetic elements for a 2 GeV electron accelerator.
II. HIGH RESOLUTION SPECTROGRAPHS

Magnetic spectrometers designed for use with medium energy accelerators are in general very large instruments. Radii of curvature range from one to four meters and weights of several hundred tons are not unusual. The physics requirements for large solid angles (≥40 msr) leads to gap dimensions of some tens of cm and pole areas of several square meters. Achieving the required field homogeneity or precision over such large magnetic volumes for an extended range of operating fields is a formidable problem.

A spectrometer design is initiated by specifying a number of physics and technical criteria. These include,

i) the momentum resolution

ii) the spatial resolution, momentum range and type of particle detector to be used (this determines the momentum dispersion required and places constraints on the focal plane angle).

iii) the phase space acceptance (solid angle, source size and momentum range).

iv) the first order ion-optics in both planes (determined by the physics and the detector characteristics).

v) maximum particle momentum (sets limits on maximum radius of curvature for allowed magnetic fields).

1. OPTICS

Most of the powerful computational tools we currently use in spectrometer design have been built around a very general terminology. We will review briefly some of the more important aspects as a background to our discussion of specific instruments.

Consider a very general ion-optical system (Fig. 1) consisting of dipoles, quadrupoles, solenoids and higher order multipoles, etc. A particle passing through the point $x_{1}, y_{1}$ in the plane $z_{1}=0$, traverses the system and exits through the
point \( x_2, y_2 \) in the plane \( z_2 = 0 \). The particle directions in the system \( x, y, z \) are given by the angles \( \theta \) and \( \phi \). The reference momentum for a beam of particles is \( p_0 \) and the deviation from the central momentum for a particular particle is given as \( \delta = \Delta p/p \). The system is assumed to possess a mechanical mid-plane of symmetry (\( x-z \) plane) which for dipoles is the bend plane. On this plane the magnetic field only has a component perpendicular to the plane.

The principal objective in any ion-optical calculation is to describe the position and direction of an arbitrary ray at any point in the system, given its position and direction at some previous point. We want to find a solution for

\[
x_2 = f(x_1, \theta_1, y_1, \phi_1, \delta)
\]

and similar expressions for \( \theta_2, y_2 \) and \( \phi_2 \).

If we define a central ray for the system and restrict ourselves to small deviations from this ray, the solutions may be written as a five-fold Taylor expansion about the central trajectory as a function of the distance, \( t \), measured along the central ray.

\[
x_2(t) = \sum (x/x^\kappa \theta^\mu y^\lambda \phi^\nu z^\chi) x_1^\kappa \theta_1^\mu y_1^\lambda \phi_1^\nu z_1^\chi.
\]

\[
y_2(t) = \sum (y/x^\kappa \theta^\mu y^\lambda \phi^\nu \delta^\chi) x_1^\kappa \theta_1^\mu y_1^\lambda \phi_1^\nu \delta_1^\chi.
\]

\[
\phi_2(t) = \sum (\phi/x^\kappa \theta^\mu y^\lambda \phi^\nu \delta^\chi) x_1^\kappa \theta_1^\mu y_1^\lambda \phi_1^\nu \delta_1^\chi.
\]

The "matrix elements" in brackets are symbols for the Taylor expansion coefficients. The first part of the symbol defines the coordinate represented by the expansion, and the second part refers to the term in question. The summation extends from zero and over all positive integer values of the exponents \( \kappa, \mu, \lambda, \nu, \) and \( \chi \). A reference to first order corresponds to \( \kappa + \mu + \lambda + \nu + \chi = 1 \), while second order corresponds to \( \kappa + \mu + \lambda + \nu + \chi = 2 \), etc. The constant term \( (\kappa + \mu + \lambda + \nu + \chi = 0) \) is zero.
since the central trajectory is chosen as the reference axis. The first order Taylor coupling coefficients for \( x(t) \) and \( y(t) \) are often referred to as the "characteristic first order trajectories" with the corresponding coefficients for \( \theta(t) \) and \( \phi(t) \) being their derivatives with respect to \( t \).

As an example, the general Taylor expansion through second order for the radial and transverse displacements \((x, y)\) at \( t = f \) of any particle in terms of its initial coordinates \((x_0, \theta_0, y_0, \phi_0, \delta_0)\) becomes:

\[
\begin{align*}
x_f &= (x/x)x_0 + (x/\theta)\theta_0 + (x/\delta)\delta_0 + (x/x^2)x_0^2 + (x/x\theta)x_0\theta_0 + \\
&\quad (x/\theta^2)\theta_0^2 + (x/y^2)y_0^2 + (x/y\phi)y_0\phi_0 + (x/\phi^2)\phi_0^2 + \\
&\quad (x/x\delta)x_0\delta_0 + (x/\theta\delta)\theta_0\delta_0 + (x/\delta^2)\delta_0^2 + \\
&\quad \text{higher order terms.}
\end{align*}
\]

\[
\begin{align*}
y_f &= (y/y)y_0 + (y/\phi)\phi_0 + (y/xy)x_0y_0 + (y/\delta y)\theta_0 y_0 + \\
&\quad (y/x\phi)x_0\phi_0 + (y/\theta\phi)\theta_0\phi_0 + (y/\phi\delta)y_0\delta_0 + (y/\delta\delta)\phi_0\delta_0 + \\
&\quad \text{higher order terms.}
\end{align*}
\]

The transfer coefficients (in parenthesis) are a property of the ion optical system itself. The term \((x/x)\) for example is the magnification of the system in the \( x \)-direction and \((x/\delta)\) is the dispersion; it measures the displacement in the \( x \)-direction at the exit per unit change in \( \delta \). Because of the assumed symmetry about the median plane, terms of the form \((x/y^n\phi^m)\) are zero unless the sum \( n + m \) is even.

The detector is usually located at the position of the first order focus, i.e. where \((x/\delta) = 0\). If the target spot size is small (not true for energy-loss systems) so that we can neglect all non-linear terms in \( x_0 \) then we have a simple result useful in the discussion of many spectrometer systems.

\[
\begin{align*}
x_f &= (x/x)x_0 + (x/\delta)\delta_0 + (x/\theta^2)\theta_0^2 + (x/\theta\delta)\theta_0\delta_0 + \\
&\quad (x/\phi^2)\phi_0^2 + (x/\delta^2)\delta_0^2 + \text{higher order terms.}
\end{align*}
\]
The first order resolving power of the system is defined as the inverse of the fractional change in momentum needed to displace the first order image a distance equal to its width,

\[ R = \frac{p}{\Delta p} = \frac{x/\delta}{(x/x) \Delta x_o} = \frac{Dp}{M \Delta X_o} = 1 \int_{0}^{t} S_x(\tau) h(\tau) \, d\tau \]

\[ = \frac{(\ell-\ell_o)}{\theta_o \Delta x_o} \quad (5) \]

In the integral expression (KBI)² for \( R \), \( h(\tau) \, d\tau \) is the differential angle of bend of the central trajectory; \((\ell-\ell_o)\) is the path length difference between the outermost ray with entrance angle, \( \theta_o \), and the central trajectory.

If we examine some of the higher order terms, we note that the term \((x/\delta^2)\) and others of the form \((x/\delta^n)\) produce a non-linearity in the dispersion at the focal plane. This may not in general be desirable but is easily corrected for in practice.

The term \((x/\theta \delta)\) can be shown to be related to the focal plane angle, \( \psi \), between the z-axis and the normal to the focal surface.

\[ \tan \psi = \frac{-(x/\theta \delta)}{(\theta/\delta)(x/\delta)} \quad (6) \]

Higher order terms of the form \((x/\theta \delta^n)\) produce curvatures of the focal surface which are often correctable in actual designs.

The most important terms are the so-called aberration terms \((x/\theta^2), (x/\phi^2), (x/\theta^3), (x/\theta \phi^2)\), etc. since they lead to an actual blurring of the image formed of the object (target spot) by particles having the same momenta. If we let \( \Delta x_{ab} \) represent an appropriate measure of the increase in width of the image due to these aberrative terms, the obtainable resolving power (assuming a point object) becomes
In our discussions so far, we have mainly considered the effects of aberrative terms in the dispersive $x$-direction. These are of primary importance since they have a direct effect on the resolving power. In many cases the aberrations in the $y$-direction are also important. Modern instruments make use of both $x$ and $y$ coordinates together with angle measurements at the focal plane for purposes of software corrections both for kinematic and uncorrected aberrative effects.

2. DESIGN PROCEDURE

Given a specification of the general physics and technical requirements, in general, several acceptable first-order designs may be readily obtained. These are usually based on previous experience or variations on existing instruments. It is important to remember that since the first-order optics couples to all higher-order aberrations this choice will determine the ease with which aberrations may be minimized by introducing various multipole components into the design. For example, suppose one is faced with 4 second-order aberrations to be eliminated. This would in general require 4 variable sextupole strengths to eliminate them simultaneously. A different first order design could possibly reduce this to 3 variables. The very serious difficulty of having to reduce several aberration coefficients simultaneously is that invariably their dependence upon the multipole strength is not very "orthogonal". This has the tendency of forcing the required strengths beyond practical limits.

Another technique which was introduced into the design of practical high-resolution instruments, which overcomes some of the above difficulties, is the formation of an intermediate image usually in the $y$-plane. Since a multi-
pole located at the position of an intermediate image will not couple to the aberration coefficients in that plane, it is thus possible to achieve a higher degree of "orthogonality" in minimizing $x$ and $y$ aberrations simultaneously.

A typical operational design procedure proceeds as follows:

i) Try out a variety of first-order solutions using a code such as TRANSPORT.

ii) Calculate the second-order aberration coefficients using TRANSPORT and determine which ones must be minimized. Use the coupling coefficients of these aberrations to determine the strengths and locations of the required sextupole components. The possibility of using a non-normal focal plane angle usually helps to reduce the required sextupole strengths. If a suitable second-order solution does not exist it may be necessary to make some alterations in the first-order design.

iii) Past second-order it is essential to use ray-tracing techniques to calculate and eliminate higher order aberrations. As noted earlier, the coupling of these aberrations to the higher multipoles may be predicted and the strengths suitably adjusted. This procedure is extended to the capabilities of the RAYTRACE code or until the design objectives have been achieved.
III. SPECTROMETER EXAMPLES

We will now briefly review some specific spectrometer examples which represent different optical choices including single and double focusing, single and multi-element types and which include examples of energy-loss (dispersion matching) systems. Two recent reviews\(^3\),\(^10\) by H. A. Enge, summarize the characteristics of a number of examples of magnetic spectrographs, including broad range types, some of which are also included here. They are for the most part hardware spectrometers as opposed to software. In that case most of the important aberrations have been accounted for in the initial design. Many of the instruments use additional angle and/or position measurements in the image plane to correct for both kinematic effects and for aberration effects that could not be conveniently eliminated or which were the result of imperfect manufacturing tolerances or calculations.

The most important features of the spectrographs discussed in this section are summarized in Table 1. Most of the column parameter headings are self-explanatory. The last column is a number \(Q\), introduced by Enge\(^9\), which roughly measures the data-taking power of a spectrograph. It is defined as \(Q = \frac{\Omega}{n} \frac{\ln(p_{\text{max}}/p_{\text{min}})}{\ln 2}\)\(^8\). 

\[
Q = \frac{\Omega}{n} \frac{\ln(p_{\text{max}}/p_{\text{min}})}{\ln 2}
\]

1. Saclay - (\(\varphi\)00)\(^1\)\(^1\)

This stigmatic instrument (\(x/\vartheta=0\) and \(y/\phi=0\)) is derived from the original 'magic angle' solution. The second order field parameters of the magic angle spectrometer have been modified slightly to preserve transverse focusing over the entire momentum acceptance. To accomplish this, pole curvatures were added at the entrance and exist boundaries
along with a modified second order field index. The spectrometer is operating with a resolving power \( R \geq 10,000 \). It is used in a physics program of precision high energy elastic and inelastic electron scattering and also as an analyzer of high energy protons in \((\gamma,p)\) and \((e,e'p)\) coincidence experiments. The dipole layout is shown in Fig. 2. The system can cover the scattering angular range of \(25^\circ - 155^\circ\).

2. **Saclay - (600)**

Designed specifically to be used together with the '900' spectrometer in coincidence experiments; especially to detect electrons in \((e,e'p)\) experiments. The device (Fig. 3) is non-stigmatic with point-to-parallel optics in the transverse plane and very large momentum acceptance \((0.7 - 1.1)p_0\). The bend angle is derived from the Elbek type dipole (homogeneous field, large momentum acceptance) by \(\alpha' = \sqrt{2}\alpha\), to retain some of its properties while using a field gradient \(n=1/2\). Measured resolution, at a solid angle of 8 msr, varies from \((3.5 - 7)x 10^{-4}\) along a 2m long focal surface. Transverse image size for the same range varies from \((7 - 8.5)cm\). The spectrometer can be rotated to cover a scattering angle range of \(34^\circ - 146^\circ\).

3. **Julich - (QQDDQ)**

A magnetic spectrograph (Fig. 4) consisting of two dipoles and three quadrupoles was designed by K. Brown for use on the cyclotron facility of the Kernforschungsanlage Julich. The instrument has stigmatic focussing overall. The quadrupole doublet\((Q_1-Q_2)\) produces a horizontal image \((x/\beta=0)\) before the first dipole and a vertical image \((y/\phi=0)\) between the two dipoles. This vertical phase-space matching ensures small transverse beam dimensions in the dipoles which minimize airgaps and pole widths. The last quadrupole, \(Q_3\), produces a \(y\) image on the detector.

Higher order aberrations in the spectrometer have been corrected by curvatures on the pole boundaries of each of the dipoles. A magnetic multipole element, located at the
transverse waist is used for kinematic corrections. Inside the dipoles the magnification ($y/y_0 - 20$.) and careful attention must be paid to the vertical position and size of the primary beam on the target.

4. **MIT-(DD)**

This instrument is used at the Bates Laboratory of MIT in a program of high resolution elastic and inelastic electron scattering. It is one of the first large spectrometer systems designed specifically to be used in an energy-loss configuration. The ion-optics is point-to-point in the median plane and parallel-to-point in the transverse plane. All important higher order aberrations (for extended target) have been corrected by pole curvatures and higher order contouring at the entrance and exit surfaces of each of the dipoles. The transverse plane optics allows a position measurement at the focal surface to determine the relative scattering angle to high accuracy. This is useful in making both kinematic and software aberration corrections on an event-by-event basis. The instrument normally works with resolving powers of $R - 10,000$. A recent experiment using two position measurements for better definition of median plane angles, was performed with a resolving power of $R - 20,000$.

The magnetic design of the spectrometer includes both field clamps and double Purcell filtering. Pole cap material consists of a very high quality magnetic iron in the form of high permeability Ni-steel.

The layout of the spectrograph is shown in Fig. 5 and the first order optical properties in Fig. 6. Its important characteristics are summarized in Table 1.

5. **LAMPF-HRS(QDD)**

This is also an energy-loss design which is in use for high resolution proton scattering studies. It consists of a quadrupole lens followed by two bending magnets. The dipoles
I. A-12

have both entrance and exit edges rotated and curved in order to provide focussing and aberration corrections. First order ion-optics is point-to-point in the bend plane and parallel-to-point \((y/y=0)\) in the transverse plane. In addition a transverse image \((y/\phi=0)\) is located between the two dipoles. Position and angle measurements in both coordinates at the focal surface provide for kinematic corrections and for reduction of other aberrative terms. Instrument operates in the energy-loss mode with resolving power in excess of 20,000. The magnetic design uses field clamps at all dipole edges and an arrangement of \(H_t\) windings for correction of field inhomogeneities and aberration corrections as well.

The layout of the spectrograph is shown in Fig. 7 and the first order optics is as indicated in Fig. 8. Important characteristics are summarized in Table 1.

Of very similar design is a QDD\(^{15}\) spectrometer recently built and now in use at IKO. This instrument is also an energy-loss optimized design and will be used in a physics program of high energy elastic and inelastic electron scattering. First tests in a beam, with almost no tuneup or optimization, resulted in resolving powers in excess of 5000 at full solid angle.

As in the LAMPF design, the quadrupole provides the focussing which produces an intermediate image in the transverse plane between the dipoles. This keeps both the dipole gap dimensions small while at the same time increases the KBIntegral for improved D/M. In the IKO instrument, higher order multipole components up to dodecupole have been incorporated into the quadrupole design. The dipole entrance and exit surfaces are described by a seventh order polynomial. In addition, to provide for corrections for slight deviations in the actual performance from the design, a multipole element has been installed between the two dipoles. Since the instrument has a cross-over in the transverse plane in this region, the multipole is particularly useful for correcting
chromatic effects at the focal plane.

6. **IKO - (QDQ)**\(^{15}\)

This spectrometer will be used to detect hadrons in coincidence experiments of the type \((e,e'N)\). It has modest resolution but relatively large solid angle. Both quadrupoles have higher order multipole corrections up to dodecupoole built-in. Entrance and exit faces of the dipole are curved and additional corrections are incorporated into a small split cut as a \(V\)-shaped groove across the pole pieces in the center of the dipole. Fig. 9 shows the arrangement of the IKO QDD and QDQ spectrometers around a common pivot as they will be used for making coincidence measurements.

7. **MSU - (QQDD)**\(^{16}\)

A large spectrometer with mass energy product \(K=800\) MeV-amu is under design for the Michigan State University heavy-ion facility. It will be operated in an energy-loss mode in order to match the 0.1% accelerator energy spread to the desired experimental energy resolution of 0.01%. The typical \(VHV\) configuration (Vertical analysis/Horizontal scattering/Vertical analysis) will be used for ease in tune-up and for purposes of decoupling the scattering angle and dispersion measurements.

The spectrometer optics (Fig. 10) is quite similar to both the LAMPF and IKO designs. Separated detectors will provide the necessary position and angular information. First order focussing is provided by the entrance quadrupole doublet and dipole edge angles. Higher order corrections are achieved by pole face curvatures. Both dipoles and quadrupoles, operating at a moderately low field strength, will however, use superconducting coils for the purpose of reducing power consumption. Design resolving power is in excess of 10,000 for a solid angle of 20 msr.

8. **Groningen - (QDDD)**\(^{7}\)

This version is one of the most recent designs of a series of instruments called the Q3D's. The basic Q3D was the first instrument to use a transverse \((y\)-direction) image in the
middle of the spectrograph ($y/\phi=0$) and reimage this again at the detector. As indicated earlier, this allows for a higher degree of orthogonality in minimizing x and y aberrations simultaneously.

The QMG2 (Fig. 11 - Groningen, Berlin, Daresbury, Peking) has been modified to provide ($\theta/\beta$)=0 at the exit. This reduces the dispersion and makes it possible to extend the range. The focal plane is straight and has practically linear calibration over the full range. A central multipole is used for kinematic corrections. Dipole boundaries, field clamps and snakes\textsuperscript{18} are used to make corrections for the usual aberration coefficients.

9. **MIT-MEPS (QQSP)\textsuperscript{19}**

This spectrometer will be used at Bates to measure pion photo-production differential cross-sections to discrete nuclear states. Main requirements included a large solid angle (35msr) and short path length for measuring pion momenta in the range from 20 to 250 MeV with a resolution of 100 to 300 KeV. In combination with other spectrometers on a common pivot, it will also be used in coincidence measurements such as the quasi-free ($\gamma,n\Lambda$) process.

The layout and median plane optics is shown in Fig. 12. In the transverse plane there is an intermediate image between the dipoles and overall it has parallel-to-point optics. Higher order aberrations have been corrected by pole curvatures on the dipoles and multipole corrections built into the two entrance quadrupoles. This optimization included effects due to energy-loss mode operation as well. The instrument is designed to have a central momentum resolving power $R>2000$ and an angular resolution of <5mr.

10. **Karlsruhe - (SP) or (QQQSP)**

Of recent design is a large solid angle low energy pion spectrometer for SIN, consisting of a quadrupole triplet in front
of a split-pole (SP) spectrograph. The quadrupoles refocus
the target spot onto an object detector for the x-z plane and
to a waist in the middle of the SP for the y-z plane. This
latter condition serves to keep the gap dimensions as small as
possible. The split-pole part of the instrument is basically
a stand-alone spectrometer and the scattered pion momentum is
determined by measurements at the object and focal-plane
detectors. When used in an energy-loss mode, the object de-
tector then provides a measure of the incident pion energy.

The SP is designed to be non-stigmatic focussing. Second
order aberrations \((x/\theta^2)\) and \((x/\theta\delta)\) for a focal plane angle of
approximately 43° have been corrected with pole curvatures on
the first dipole. The usually important 3rd order coefficients,
\((x/\theta^3)\) and \((x/\theta^2\delta)\) turn out to be smaller than expected for
this type of instrument. A similar result is seen in the
"clam shell" instrument and is probably the result of not
requiring stigmatic focussing, i.e. \((y/\phi)\) at the focal plane.
Angle measurements at the focal plane will be used for making
software kinematic corrections and should improve the real
resolving power substantially. The first order ion-optics
is shown in Fig. 13 and the plan view of the instrument is
seen in Fig. 14. Table 1 summarizes some of the important
parameters.

11. LAMPF LEP-(D)

A novel new spectrometer, currently under design, is
intended for use as with low energy pions and is similar to
devices previously used for beta spectroscopy. The so-called
"clam shell" design consists of a single dipole with non-
uniform field. The pole faces are planes which are tilted
with respect to each other, producing a wedge-shaped airgap
(Fig. 15).

The spectrometer is single focussing resulting in better
than expected higher order aberrations. Second order terms
\((x/\theta^2)\) and \((x/\theta\delta)\) are corrected by entrance and exit pole face
curvatures. The focal plane angle is approximately 38°. It has a large momentum acceptance (±30%), large solid angle (~ 40msr) and is quite short. Detailed optics studies are still in progress but indications are that with proper software corrections it will be possible to obtain resolving powers in excess of 3000 in the central 15% of momentum acceptance.
IV. SUMMARY

The spectrometer systems we have reviewed form the heart of many current large research efforts in medium energy nuclear physics. They represent the state-of-the-art capability in calculation, design and manufacture of high resolution and large solid angle devices. That is not to say that improvements in the future will not be made, but rather, that they are likely to be both difficult and very expensive.

What kind of spectrometers we will build in the future depends to a large extent on the physics that is the most important and exciting to do. In the case of a (1-2) GeV cw electron accelerator, a large number of such interesting experiments have been examined in detail in the recently published report of the Workshop on Future Directions in Electromagnetic Nuclear Physics. They include single-arm scattering experiments and those in which two or more particles are detected in coincidence. These latter experiments may require two large magnetic spectrographs of novel design around a common pivot. Given the likely (as usual) tight budgetary constraints, some priorities still remain to be determined in selecting the initial instrument or instruments which are likely to lead to the greatest overall physics impact.

On the more technical side, important questions which need to be reexamined again include;

i) Hardware vs. Software
ii) Room Temperature vs. Superconducting
iii) Single - vs. Multi-Gap
iv) Resolution vs. Solid Angle

The question of hardware vs. software in spectrometer design is being asked more often now as we attempt to build larger and consequently more expensive devices. The motivation for software spectrometers seems to be the feeling that it is expensive to calculate and manufacture the complicated
curvatures needed in dipoles or the 'fancy' pole shapes in quadrupoles/multipoles. In our experience, most large manufacturing machinery today is computer controlled and the precision contouring adds only a relatively small additional cost to the total system.

Software spectrometers have a serious and often fatal drawback in that one needs very exacting information about positions and angles to obtain adequate momentum resolution. One is very sensitive to multiple scattering in vacuum windows, detector chambers and air. Unlike the possibility with π-, K- beams, etc., it is probably impossible to use detectors at the input to a spectrometer on an electron machine. There is in this case, an enormous background due to photons, low energy charged particles and neutrons.

We now see in the design of large solid angle devices that aberrations such as $(x/\theta^5)$, etc. are becoming very important. One needs a very precise angle measurement to be able to correct for such effects.

Hardware spectrometers in general have a focal plane with the best possible image. This can result in smaller and/or fewer detectors. For background and practical reasons this is very important. The necessary software corrections are reduced and the required angle measurements, if any, may be a lot easier.

Detectors do not currently appear to be a major limitation in the construction of precise instruments. As standard practise one easily obtains spatial resolutions of 0.1-0.2mm in several chamber designs. Given the radii of curvature required, for given particle momenta, even with superconducting field strengths, this spatial resolution is sufficient to achieve resolving powers of $10^5$.

One has of necessity often explicitly or implicitly traded solid angle for resolution in magnetic spectrometers. With high energy accelerators, new considerations arise. Due to the rapid variation in form factor with momentum transfer
we will be forced to accept a larger fraction of the solid angle out of the scattering plane; e.g. devices with conical acceptance. Out of the plane ($\phi - \pi/2$) measurements may also be important in some types of coincidence experiments as well. Unfortunately most of the current large acceptance devices, including those on the drawing board, have nearly equal amounts of angular acceptance in each plane.

It appears that with devices we currently see on the horizon it should be possible to achieve $\sim 40$ msr with $k \geq 10,000$ using a reasonable amount of software correction in a simple design such as the LAMPF - ("clamshell") or the Karlsruhe - (SP).
REFERENCES


### TABLE I

**SPECTROGRAPH CHARACTERISTICS**

<table>
<thead>
<tr>
<th>SPECTROGRAPH</th>
<th>MEAN RADIUS R (CM)</th>
<th>AIRGAP d (CM)</th>
<th>RANGE P&lt;sub&gt;max&lt;/sub&gt;/P&lt;sub&gt;min&lt;/sub&gt;</th>
<th>SOLID ANGLE ρ (msr)</th>
<th>DISPERSION D&lt;sub&gt;r&lt;/sub&gt; = (x/ρ)/ρ</th>
<th>MAGNIFICATION M&lt;sub&gt;x&lt;/sub&gt; M&lt;sub&gt;y&lt;/sub&gt;</th>
<th>D/M&lt;sub&gt;x&lt;/sub&gt;</th>
<th>RES.-pow&lt;sub&gt;a&lt;/sub&gt; (approx.)</th>
<th>FOCAL ψ (degrees)</th>
<th>Q (msr)</th>
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<tr>
<td>SACLAY (900)</td>
<td>180</td>
<td>12.0</td>
<td>1.10</td>
<td>5.6</td>
<td>4.0</td>
<td>1.0 1.0</td>
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<td>2.5-3.5</td>
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<td>.5 3.4</td>
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<td>38.0</td>
<td>35.8</td>
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<sup>a</sup> An approximate measure of the theoretical aberration - limited resolving power at the solid angle listed.
Figure 1. Coordinate systems used in optical beam transfer calculations.

Figure 2. SACLAY '900' Spectrometer Layout.
Figure 3. SACLAY '600' Spectrometer Layout.
Figure 4. The QQDDQ at the Kernforschungsanlage, Jülich West Germany. Note the horizontal cross-over \((x/\phi=0)\) between Q2 and D1. Not shown is a vertical image \((y/\phi=0)\) at the location of the multipole M.
Figure 5. Design details of the MIT Energy Loss Spectrometer.
Figure 6. Characteristic ion-optics for the MIT Energy Loss Spectrometer scaled to a radius of curvature
\( \rho = 1.0 \text{ m}. \)
Figure 7. Design details of the LAMPF High Resolution Spectrometer.
Figure 8. Characteristic ion-optics of the HRS scaled to a radius of curvature $\rho = 1.0$ m.
Figure 9. Schematic layout of the IKO QDD and QDQ spectrometer systems around a common pivot. Note the shielding arrangement around the focal plane detectors.
Figure 10. Proposed $K = 800$ MeV-amu spectrograph for Michigan State University.

Figure 11. The latest of the family of Q3D spectrographs designed for use in heavy-ion reaction studies at Groningen, Berlin, Daresbury and Peking.
Figure 12. a) Plan layout for Bates-MEFS (QQSP). b) Characteristic ion-optical trajectories scaled to an instrument with radius-of-curvature $\rho = 1$ meter.
Figure 13. Characteristic ion-optical trajectories for Karlsruhe (QQQSP) spectrometer.
Figure 14. Proposed Karlsruhe (QQQSP) spectrometer system for use in low energy pion reaction studies at SIN. Note the location of a horizontal image \((x/\theta = 0)\) at the object detector just in front of the split-pole.
Figure 15. Proposed "clamshell" spectrometer for low energy pion reaction studies at LAMPF.
The computer programs TRANSPORT and TURTLE are described, with special emphasis on recent developments. TRANSPORT is a general matrix evaluation and fitting program. First and second-order transfer matrix elements, including those contributing to time-of-flight differences can be evaluated. Matrix elements of both orders can be fit, separately or simultaneously. Floor coordinates of the beam line may be calculated and included in any fits. Tables of results of misalignments, including effects of bilinear terms can be produced. Fringe fields and pole face rotation angles of bending magnets may be included and also adjusted automatically during the fitting process to produce rectangular magnets. A great variety of output options is available.

TURTLE is a Monte Carlo program used to simulate beam line performance. It includes second-order terms and aperture constraints. Replacable subroutines allow an unlimited variety of input beam distributions, scattering algorithms, variables which can be histogrammed, and aperture shapes. Histograms of beam loss can also be produced. Rectangular zero-gradient bending magnets with proper circular trajectories, sagitta offsets, pole face rotation angles, and aperture constraints can be included. The effect of multiple components of quadrupoles up to 40 pole can be evaluated.
Introduction

A complete design of a beam line for transmission of charged particles involves two stages. First, one must determine certain quantifiable characteristics that the beam line must possess, and produce a design which optimizes the conformance to these characteristics. Second, one must evaluate the performance of the system produced. The latter might involve a determination of beam profiles, acceptances, and effects of magnet imperfections.

We describe here two computer programs developed to achieve the two purposes described above. A beam design, including all element spacings and magnetic fields, is produced using the program TRANSPORT. Once this design is achieved, it may be simulated using the Monte Carlo program TURTLE. The two programs use the same input data format making the transition from one to another quite simple.

Both programs are described in detail in their respective manuals and, to some extent, in the published literature. Their use is sufficiently widespread so there is no point in giving a detailed description of either. For completeness, we give a short description of each, with a greater elaboration of recent developments. Some of the more recent developments have been or will be published elsewhere. Others will appear only in this article.
TRANSPORT

TRANSPORT is a general matrix evaluation and fitting program. It can evaluate various matrices which represent the transmission of particles through a beam line, and vary the physical parameters of the beam line to fit elements of such matrices to desired values. A schematic illustration of a beam line is shown in figure 1.

TRANSPORT considers a beam line to be comprised of a set of magnetic elements placed sequentially at intervals along an assumed reference trajectory. The reference trajectory is taken to be a path of a charged particle passing through idealized magnets (no fringing fields) and having the central design momentum of the beam line. Therefore, through a bending magnet, the reference trajectory is the arc of a circle, while through all other magnetic elements it is a straight line. The input data to TRANSPORT contain the initial floor coordinates and direction of the reference trajectory, and the sequence of elements comprising the beam line. The elements include both drift spaces and magnetic elements, which are specified by their lengths, magnetic fields, and other relevant quantities. TRANSPORT can then calculate the floor coordinates of the reference trajectory at the interface between any two elements.

A local coordinate system is attached to each point on the reference trajectory. As a particle moves down the beam
line, its transverse position and direction of motion are referred to this local coordinate system. An illustration of this local coordinate system is shown in figure 2. A six component vector is used to describe a particle trajectory at a given position along the beam line, i.e.

\[
\mathbf{x} = \begin{bmatrix}
x \\
\theta \\
y \\
\phi \\
\ell \\
\delta
\end{bmatrix}
\]  \hspace{1cm} (1)

where:

- \(x\) = the horizontal displacement of the ray with respect to the reference trajectory
- \(\theta\) = the angle the ray makes is the horizontal plane with the reference trajectory
- \(y\) = the vertical displacement of the ray with respect to the reference trajectory
- \(\phi\) = the vertical angle the ray makes with the reference trajectory
- \(\ell\) = the longitudinal separation between the ray and the central trajectory
- \(\delta\) = \(\Delta p/p\) is the fractional momentum deviation of the ray from that of the reference trajectory.
The value of this vector at any location is the beam line may be determined from its initial value by means of a transfer matrix $R$, so that

$$ X(1) = R X(0) $$

(2)

where the arguments (0) and (1) indicate the initial location and the point of interest, respectively. The six by six matrix $R$ takes on a simple form if the system has midplane symmetry, where all the magnetic potentials are odd in the vertical coordinate. Then we have

$$ \begin{bmatrix} x_1 \\ y_1 \\ \phi_1 \\ l_1 \\ \delta \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} & 0 & 0 & 0 & R_{16} \\ R_{21} & R_{22} & 0 & 0 & 0 & R_{26} \\ 0 & 0 & R_{33} & R_{34} & 0 & 0 \\ 0 & 0 & R_{43} & R_{44} & 0 & 0 \\ R_{51} & R_{52} & 0 & 0 & 1 & R_{56} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ \theta_0 \\ y_0 \\ \phi_0 \\ l_0 \\ \delta \end{bmatrix} $$

(3)

In all cases, whether or not we have midplane symmetry, all entries in the fifth column, except the fifth, and all entries in the sixth row, except the sixth, will be zero. Thus nothing affects the momentum, and the longitudinal separation affects "no transverse coordinate. TRANSPORT can print the transfer matrix at any or all locations in matrix format or in a single line which contains only those elements in the first four rows which survive midplane symmetry.
The matrix formalism can be regarded as the first term in a Taylor's series and extended to second order via the equation

\[ x_i(1) = \sum_j R_{ij} x_j(0) + \sum_{jk} T_{ijk} x_j(0) x_k(0) \]  

(4)

The second-order matrix elements which contribute to the transverse coordinates have been calculated by Brown, and have previously been published. The terms which contribute to longitudinal separation have been derived by several interested parties, and have been or will be published.

In accelerator and beam transport systems, the behavior of an individual particle is often of less concern than is the behavior of a bundle of particles (the beam), of which the individual particle is a member. An extension of the matrix algebra of eq. (2) provides a convenient means for defining and manipulating this beam. TRANSPORT assumes that the beam may be correctly represented in phase space by an ellipsoid in the six-dimensional coordinate system described above. Particles in a beam are assumed to occupy the volume enclosed by the ellipsoid, each point representing a possible ray. The sum total of all phase points, the phase space volume, is commonly referred to as the "phase space" occupied by the beam. A diagram of a two-dimensional cross section of this six-dimensional ellipsoid is shown in figure 3.
The equation of the six-dimensional ellipsoid is

\[ x^T \sigma^{-1} x = 1 \]  

(5)

where \( \sigma \) is the beam or sigma matrix, some of whose elements are found in the illustration. The correlation terms \( r \) are given in terms of the off-diagonal elements of \( \sigma \) by

\[ r_{ij} = \frac{\sigma_{ij}}{\sqrt{\sigma_{ii} \sigma_{jj}}} \]  

(6)

The ellipse at one location in the beam line can be transformed into one at another location by means of the transfer matrix between the two locations, so that

\[ \sigma_{ij} = \sum_k R_{ik} R_{jk} \sigma_{kl}(0) \]  

(7)

An alternate interpretation of the sigma matrix is that it provides the second moments of a six-dimensional Gaussian distribution. The fourth moments can then be calculated and second order terms taken into account. The sigma matrix now transforms as

\[ x_i(1) = \sum_{jk} T_{ijk} \sigma_{jk}(0) \]  

(8)

\[ \sigma_{ij}(1) = \sum_k R_{ik} R_{jl} \sigma_{kl}(0) + 2 \sum_{lm} \left( \sum_k T_{ikl} \sigma_{km}(0) \right) \left( \sum_n T_{jln} \sigma_{ln}(0) \right) \]
The distribution is no longer an ellipsoid and the centroid at the final position is no longer the image of the centroid at the initial position. Nevertheless, the sigma matrix does provide an estimate of the beam dimensions and is of use in determining the magnitude of the net contribution of second-order aberrations. The second order transformation of the sigma matrix has been described elsewhere.

A number of the physical parameters describing the magnets or their locations or orientations may be varied by the program. A list of elements which are physical or have parameters which may be varied is given in table 1. Almost all items require no explanation. The single possible exception is a bending magnet, whose configuration can sometimes be quite complicated.

A bend magnet element specifies a sector bend magnet where the field boundaries are infinitely sharp and form a plane perpendicular to the reference trajectory at the input and output faces of the magnet. A field boundary making an angle with the perpendicular plane is specified as a separate element which precedes or follows the magnet. The rotation of the field boundary acts as a quadrupole component which can affect the first-order transfer matrix. A quadratic variation of the central field of a bend magnet, or a curvature of its entrance or exit field boundaries can contribute a sextupole component which will affect the second-order transfer
matrix. A diagram of a general bending magnet is shown in figure 4.

The values of the parameters to be varied will be found which will satisfy any user-imposed constraints. A variety of constraints is available and a list is given in table II. Any assortment of constraints can be fit simultaneously by TRANSPORT, providing the configuration is physically possible. First- and second-order constraints may be mixed and all parameters describing magnetic elements or their intended location or orientation may be varied in either a first- or second-order run. The only exceptions are that parameters which affect only the second-order characteristics of a beam may not be varied and second-order constraints may not be imposed in a first-order run. Parameters directly describing the beam ellipse and misalignment parameters may be varied only in first order.

The misalignment tolerances of the magnets in a beam line can also be determined by TRANSPORT. The complete theory of magnet alignment tolerances has been given elsewhere and will be described briefly here. A picture of a misaligned bending magnet is shown in figure 5.

The most immediate effect of the misalignment of a magnetic element is a displacement of the reference trajectory. This would add a term to equation (2) which was not dependent on the values of the initial coordinates of the trajectory.
However, many possible misalignments, such as the rotation of a quadrupole about its axis would not be included in such a term. We therefore add a second term which is bilinear in the extent of the misalignment and arrive at

\[ X(l) = RX(0) + Fm + GX(0)m \]  

(9)

where \( m \) is a vector of misalignment parameters given by

\[
\begin{bmatrix}
\delta x \\
\theta_x \\
\delta y \\
\theta_y \\
\delta z \\
\theta_z \\
\end{bmatrix}
\]

(10)

The six components indicate displacements and rotations with respect to the three axes of the reference coordinate system at the entrance face of the magnet.

In TRANSPORT a number of possible elements or portions of the beam line may be misaligned. An individual element or section of a beam line can be misaligned, and misalignments can be nested. Also TRANSPORT can be instructed to misalign all quadrupoles and/or bending magnets by a given amount.

The effects of misalignments are shown in the beam matrix. The misalignments may be of two types. A known misalignment of a magnet will produce a displacement of the
beam centroid. The new beam centroid and sigma matrix are given by

\[ \bar{X} = Fm \]

\[ \sigma(1) = R\sigma(0)R^T + G\sigma(0)mR^T + R\sigma(\theta)m^T g^T + G\sigma(0)m^T g^T \] (11)

An uncertainty in position will not affect the beam centroid, but will produce an altered beam ellipse given by

\[ \sigma(1) = R\sigma(0)R^T + F<mm^T>G^T + G\sigma(0)<mm^T>G^T \] (12)

The matrix \( <mm^T> \) represents an ellipsoid of uncertainty in the six-dimensional space of misalignment parameters. If the misalignments are uncorrelated this ellipsoid will be upright. If the initial dimensions of the beam ellipse are zero, then the beam matrix will represent the envelope of possible locations of the reference trajectory.

The results of the misalignments may be represented in either the beam matrix or in a special misalignment table. If the beam matrix is used, the results of the misalignment of all magnets in all coordinates will be lumped together, to give an aggregate result. The misalignment parameters may then be fit via constraints on the beam matrix. The misalignment table consists of altered facsimiles of the beam matrix, reproduced a number of times. If the misalignment table is used, then the results of misaligning each magnet in each coordinate can be shown individually.
**TURTLE**

TURTLE is a Monte Carlo program used to simulate beam line performance. It can produce histograms and scatter plots showing beam profiles and any distribution or correlation of any physical quantity. It also includes second-order and many higher-order terms and aperture constraints.

The input deck is the same as that used for TRANSPORT. Three changes are required to make it into a deck for running TURTLE. First, the computer must be instructed to run TURTLE instead of TRANSPORT. Second, the TRANSPORT indicator card indicating whether the problem is new or a continuation of an old problem is changed to the number of rays to be run through the system. Third, the histogram requests must be inserted.

The transformations through quadrupoles, sextupoles, and solenoids are done individually for each momentum. The geometric effects on trajectories are limited to second order. For a general bending magnet, a second-order transformation in both chromatic and geometric effects is used. The pole face rotations with their accompanying fringe fields, and the body of the bending magnet are all included in a single second-order transformation.

If the field gradient of a magnet is zero, then the trajectory of a particle through a magnet will be the arc of a circle. A rectangular magnet with a sample circular trajectory is shown in figure 6. If \( \alpha \) is the bend angle of
a sector bending magnet, then the output coordinates expressed in terms of the input coordinates are

\[
\sin \theta_1 = \sin \theta_0 \cos a - \frac{\sin^2 \theta_0 \sin a}{1 + \cos \theta_0} - x_0 \frac{\sin a}{\rho} + \frac{\delta}{1 + \delta} \sin a
\]

\[
x_1 = x_0 \left[ \cos a + \frac{\sin a (\sin(\theta + a) + \sin \theta_1)}{\cos(\theta_0 + a) + \cos \theta_1} \right] + L \frac{\sin a}{\alpha} \left( \frac{\sin \theta_0 + \sin \theta_1}{\cos(\theta_0 + a) + \cos \theta_1} \right) \left[ 1 + \frac{\sin a}{1 + \cos a} \frac{\sin \theta_1 - \sin \theta_0}{\cos \theta_1 + \cos \theta_0} \right]
\]

\[
y_1 = y_0 + y_0' \frac{\cos \theta_0}{\cos \frac{1}{2}(\theta_0 + a + \theta_1)} - \frac{1}{2} \frac{(a + \theta_0 - \theta_1)}{\sin \frac{1}{2}(a + \theta_0 - \theta_1)} \left[ x_0 \sin a + L \frac{\sin a}{\alpha} \right]
\]

\[
y_1' = y_0' \frac{\cos \theta_0}{\cos \theta_1}
\]

The quantity L is the length of the bend magnet, \( \rho \) the radius of curvature of the individual trajectory, and \( \theta_0 \) the inverse tangent of \( x_0' \). For a rectangular zero-gradient bending magnet, the effect of the pole face rotations is included via a second-order transfer matrix. Also for a rectangular bending magnet, the proper sagitta offset and corresponding aperture limitations can be calculated automatically.

The effect of non-linearities in quadripole fields may also be calculated automatically. The magnitude of the error field is represented by a multipole expansion.
The mathematical procedure for calculating the effect of the error field has been described elsewhere. Briefly, the trajectory is transformed to the longitudinal center of the quadrupole by an ordinary transfer matrix. At that point it is perturbed by an integral of the multipole field, where the strengths of the components are determined from the magnitude of the ray at the longitudinal midpoint. Then it is transformed to the end of the quadrupole. Multipoles up to the 40-pole may be included.

Replacable subroutines allow a variety of input phase space distributions, scattering distributions, slit shapes, and histogrammable variables. The normal phase space is rectangular in \( x \) vs. \( x' \) or \( y \) vs. \( y' \), but circular in \( x \) vs. \( y \) or \( x' \) vs. \( y' \). It is also uniform in momentum. Alternate distributions representing particle production models have been used.

Scattering may be introduced at any point in the beam line. The default distribution is the same as for the input phase space. The subroutine producing the scattering may be replaced by one giving a Gaussian or other desired distribution.

Slits of any size and shape can also be inserted anywhere.
in the beam line. The program allows a slit in any of the trajectory coordinates. A replacable subroutine allows any other slit that can be formulated mathematically. This feature is useful in defining irregularly shaped apertures of magnets.

Histograms and scatter plots of trajectory coordinates may also be produced. A histogram of the longitudinal position of beam loss is also available. Replacable subroutines allow calculation of any quantity which is related to the trajectory coordinates for inclusion in histograms or scatter plots. Examples might be kinematic variables associated with a particular reaction, the angle of a cone of Cerenkov light, or a reconstruction of an initial trajectory coordinate from chamber readings. The scatter plot may contain any two variables, which need not occur at the same location in the beam line.

Histograms and scatter plots may be flagged. A trajectory coordinate will not be entered into the histogram unless the ray remains in the beam when the flag is encountered. This feature is especially useful for determining the effective acceptance of a beam line.
REFERENCES


<table>
<thead>
<tr>
<th>Element</th>
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</tr>
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<tr>
<td>Initial beam ellipse</td>
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<tr>
<td>Drift space</td>
<td>Length</td>
</tr>
<tr>
<td>Bend magnet</td>
<td>Length, field, field gradient</td>
</tr>
<tr>
<td>Quadrupole</td>
<td>Length, pole tip field</td>
</tr>
<tr>
<td>Initial beam centroid shift</td>
<td>All dimensions</td>
</tr>
<tr>
<td>Alignment tolerance</td>
<td>All dimensions</td>
</tr>
<tr>
<td>Accelerator</td>
<td></td>
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<tr>
<td>Arbitrary Matrix</td>
<td>All elements</td>
</tr>
<tr>
<td>Initial floor coordinates</td>
<td>All positions and angles</td>
</tr>
<tr>
<td>BM sextupole components</td>
<td>Strength</td>
</tr>
<tr>
<td>Sextupole</td>
<td>Field</td>
</tr>
<tr>
<td>Solenoid</td>
<td>Length, field</td>
</tr>
<tr>
<td>Coordinate rotation</td>
<td>Angle of rotation</td>
</tr>
</tbody>
</table>

Table I. Physical and Variable TRANSPORT Elements
Zero'th Order
Total system length
Floor coordinates -- three positions and three angles

First Order
Transfer matrix element -- $R_{ij}$
Beam size in any coordinate
Beam matrix element -- $\sigma_{ij}$
Beam correlation matrix element -- $r_{ij} = \frac{\sigma_{ij}}{\sqrt{\sigma_{ii}\sigma_{jj}}}$
First moments of the beam -- centroid
Phase advance -- Trace of R matrix in one plane

Second Order
Transfer matrix element -- $T_{ijk}$
Second order contributions to beam size in any coordinate

Table II. Fitting Constraints
Figure Captions

Fig. 1: A charged particle beam line, with the reference trajectory shown.

Fig. 2: The local coordinate system for determining the local coordinates of a particle trajectory.

Fig. 3: A two-dimensional beam phase ellipse.

Fig. 4: Field boundaries for a general bending magnet.

Fig. 5: Perfectly aligned and misaligned bending magnets.

Fig. 6: A zero-gradient rectangular bending magnet. The reference trajectory and another sample particle trajectory are shown.
\[ \sigma_{\text{max}} = \sqrt{\sigma_{22}} \]
\[ \sigma_{\text{int}} = \sqrt{\sigma_{22} (1 - r_{12}^2)} \]
\[ \text{slope} = \frac{1}{r_{12}} \sqrt{\frac{\sigma_{22}}{\sigma_{11}}} \]
\[ \sqrt{\sigma_{11}} = x_{\text{max}} \]
\[ \sqrt{\sigma_{11} (1 - r_{12}^2)} = x_{\text{int}} \]

**Figure 3.**

\[ x_1 \]
\[ Z_1 \]
\[ x_2 \]
\[ Z_2 \]
\[ \beta_1 \]
\[ \beta_2 \]
\[ R_1 \]
\[ R_2 \]
\[ \rho = \frac{1}{h} \]

**Figure 4.**
REFERENCE TRAJECTORY IN
REFERENCE TRAJECTORY OUT

PERFECTLY ALIGNED

MISALIGNED

REFERENCE TRAJECTORY IN
REFERENCE TRAJECTORY OUT

Figure 5.

REFERENCE TRAJECTORY
PARTICLE TRAJECTORY

Figure 6.
New Features in TRANSPORT

David C. Carey

September 7, 1981

The computer program TRANSPORT, used for designing charged particle beam transport systems, has been modified to include a number of new features. Among them are the possibility of using accelerator notation to specify the beam matrix, expanded fitting capabilities including the ability to constrain algebraic combinations of matrix elements, and more flexible means of specifying individual or groups of beam line elements.

This document is to be used as a supplement to the TRANSPORT manual. It is organized by type code, as is the manual. The contents of this report will eventually be written into a revised TRANSPORT manual. In a few instances, the material of the manual has been rewritten, but in most cases it has simply been expanded.
 INPUT BEAM: Type code 1.0

The beam matrix may also be specified in terms of accelerator notation using the parameters $\beta$, $\alpha$, and $\eta$. A 13. 7. ; element placed before the beam element will indicate that the beam matrix input and output and beam matrix constraints are all expressed in accelerator notation. The meaning of the accelerator parameters $\beta$ and $\alpha$ is shown in the illustration below. The parameters $\beta$ and $\alpha$ may be defined for each transverse plane. The parameter $\eta$ is used to characterize the behavior of off-momentum trajectories. The use of this option is not compatible with either misalignments (type code 8.) or second order (type code 17.).

If accelerator notation is to be used, there are eight entries to be made on the beam card.

1 - The type code 1.0 (specifies a beam entry follows).
2 - $\beta_x$
3 - $\alpha_x$
4 - $\beta_y$
5 - $\alpha_y$
6 - 0.
7 - 0.
8 - The momentum of the central trajectory [$p(0)$] (GeV/c).

The units for $\beta$ are the quotient of the transverse length units and the transverse angle units. Conventionally $\beta$ is expressed in mm/mr or meters. Following this convention requires a change in the transverse distance unit to mm using the 15. type code. The six components of the $\eta$ vector are specified
using the centroid shift (type code 7.). If no type code 7 element is present, the fractional momentum deviation of $\eta$ from the beam design momentum is one unit in fractional momentum spread. Normally this is in percent $\Delta p/p$, but can also be changed via a 15. type code entry. All other coordinates of $\eta$ are then taken to equal zero. A beam correlation entry (type code 12.) is not used with accelerator notation since $\alpha_x$ and $\alpha_y$ express the degree of correlation between position and angle in the two transverse planes.

The beam matrix, whenever printed, will be expressed in accelerator notation. If the beam matrix is printed after every physical element, by the insertion of a 13. 3. ; element, a heading will be printed to identify the numbers in the beam matrix. The beam matrix will be printed in a single line. The quantities printed are

$$\psi_x \quad \psi_y \quad \beta_x \quad \beta_y \quad \alpha_x \quad \alpha_y \quad \eta_x \quad \eta_y \quad \eta'_x \quad \eta'_y$$

The parameters $\psi_x$ and $\psi_y$ are the phase advances in the horizontal and vertical planes. The phase advance is given by

$$\psi = \int \frac{ds}{\beta}$$

where $s$ is the distance along the reference trajectory. The units for $\psi$ are degrees. If the beam matrix is printed only after selected elements via a 13. 1. ; type code entry, no heading is printed.
The area of the ellipse is given by:

$$A = \pi (\det \sigma)^{\frac{1}{2}} = \pi x_{\text{max}} \theta_{\text{max}} = \pi x_{\text{int}} \theta_{\text{int}}$$

The equation of the ellipse is:

$$\gamma x^2 + 2a x \theta + \beta \theta^2 = \varepsilon$$

where

$$\sigma = \begin{bmatrix} \sigma_{11} & \sigma_{21} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{bmatrix}$$

and

$$\beta \gamma - \alpha^2 = 1, \quad r_{21} = r_{12} = \frac{-\alpha}{\sqrt{1+\alpha^2}} = \frac{-\alpha}{\sqrt{\beta \gamma}}$$
FRINGING FIELDS and POLE-FACE ROTATIONS for bending magnets
Type code 2.0

If the bending magnets to be specified are rectangular as seen from the top, then the sum of the input and output pole face rotation angles will equal the bend angle. Several options exist for automatically setting the pole face rotation angles in terms of the bend angle. Such options are given by entries of 13. 40. ; through 13. 43. ; inclusive. Even when one of these options is selected, the 2. element must be included in the data before and after the type code 4. entry if fringing-field effects are to be calculated. Details of the various options are found in the section describing type code 13.
WEDGE BENDING MAGNET: Type code 4.0

Alternatively, a wedge bending magnet may be specified in terms of its length and bend angle. The four parameters to be specified are then

1 - Type code 4.0 (specifying a wedge bending magnet)
2 - The (effective) length $L$ of the central trajectory in meters.
3 - The bend angle $\alpha$ in degrees.
4 - The field gradient (n-value).

The quantities $L$, $\alpha$, and $n$ may be varied. The units for bend angle may be changed via a 15. 7. element preceding the beam card.

If a 13. 48. ; entry is inserted in the data, all subsequent bend magnet entries are taken to be expressed in terms of $L$, $\alpha$, and $n$. At a later point one can revert to the normal description in terms of $L$, $B$, and $n$ by the insertion of a 13. 47. ; element.
QUADRUPOLE: Type code 5.0

A device which is focusing in both transverse planes or defocusing in both transverse planes may also be represented via a 5. type code. The description is the same as for an ordinary quadrupole, except that the sign of B now applies to both planes. A 13. 98. ; element inserted before a 5. type code will cause it to be taken as such a device. A subsequent 13. 97. ; entry causes the 5. type code to be taken as representing a quadrupole.
SHIFT IN THE BEAM CENTROID: Type code 7.0

If accelerator notation is selected for the beam matrix via a 13. 7. ; element, then the 7. type code is used to represent the η function. The coordinates specified are those of the initial η function. The η function is conventionally taken to represent a trajectory whose momentum differs by one unit from the design momentum of the beam line. In this case the δ parameter on the shift element would be 1.0.
VARY CODES and FITTING CONSTRAINTS: Type code 10.0

First-order vary codes

In a first-order run, the additional quantities marked v may be varied.

BEAM (rotated ellipse). 12.vvvvvvvvvv - All elements of the correlation matrix may be varied.

Second-order vary codes

In a second-order run the following parameters may be varied:

ROTAT............ 2.v - The pole face angle of a bending magnet may be varied.

DRIFT............ 3.v - The drift length may be varied.

BEND............. 4.vvv - The length, the field, and/or the n-value may be varied. Alternatively, the length, the bend angle, and/or the n-value may be varied.

QUAD.............. 5.vv0 - The length may be varied; the field may be; the aperture may not be.

MATRIX............ 14.vvvvvv0 - Any of the first order matrix elements may be varied.

INITIAL COORDINATES.... 16.0v - Any of the three initial position floor coordinates or two angle coordinates may be varied.

\( \varepsilon(1) \)............ 16.0v 1. - The normalized quadratic term (sextupole component) in the midplane expansion for the field of a bending magnet may be varied.

1/R1............. 16.0v 12. - The pole face curvature of a bending magnet entrance may be varied.
1/R2......... 16.0v 13. - The pole face curvature of a bending magnet exit may be varied.
SEXTUPOLE..... 18.0v - The field strength may be varied.
SOLENOID....... 19.vv - The length and/or field may be varied.
BEAM ROTATION.. 20.v - The angle of rotation may be varied.

Variation of any parameter which may also be varied in first order should be done with caution as it may affect the first-order properties of the beam line. But, inversely coupled drift spaces straddling a sextupole will, for example, show only second-order effects.

The special parameter cards (type code 16.0) once introduces apply to all subsequent magnets in a beam line until another type code 16.0 specifying the same parameter is encountered. Thus, if such a parameter is varied, the variation will apply simultaneously to all subsequent magnets to which it pertains. The variation will persist until the parameter or vary code attached to the parameter is changed by the introduction of another type code 16.0 element specifying the same parameter.

The first-order parameters of a pole face rotation, bend, or solenoid may not be fit to satisfy a second-order constraint. They are variable in a second-order run only to permit first- and second-order constraints to be imposed simultaneously.

Possible fitting constraints

Any elements of the first-order transfer matrix, the floor coordinates, and the system length may now also be fit in a second-order run.
Beam matrix fitting constraints using accelerator notation

The correspondence between the notation for a beam constraint and the convention for beam parameter input is the same as when accelerator notation is not used. There are five parameters to be specified.

1 - Type code 10.
2 - Code digit (i).
3 - Code digit (j).
4 - Desired value of parameter to be fit.
5 - Desired accuracy of fit (standard deviation).

If i = j, the constraint applies to the i'th physical parameter on the beam (type code 1.) element. For example, if \( \beta_x \) is to be constrained, then i = j = 1. If i = 0, the constraint applies to the j'th physical parameter on the shift (type code 7.) element. Thus, to fit \( \eta_x \), one would set j equal to 1.

Some typical accelerator notation beam constraints are as follows:

<table>
<thead>
<tr>
<th>Desired optical condition</th>
<th>Typical fitting constraint</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_x = .5 )</td>
<td>10. 1. 1. .5 0.001 'F1' ;</td>
</tr>
<tr>
<td>( \alpha_x = 0 )</td>
<td>10. 2. 2. 0. 0.001 'F2' ;</td>
</tr>
<tr>
<td>( \beta_y = .3 )</td>
<td>10. 3. 3. .3 0.001 'F3' ;</td>
</tr>
<tr>
<td>( \alpha_y = -.2 )</td>
<td>10. 4. 4. -.2 0.001 'F4' ;</td>
</tr>
<tr>
<td>( \eta_x = .5 )</td>
<td>10. 0. 1. .5 0.001 'F5' ;</td>
</tr>
<tr>
<td>( \eta_y = -.2 )</td>
<td>10. 0. 4. -.2 0.001 'F6' ;</td>
</tr>
</tbody>
</table>
Algebraic combination of matrix elements constraint

Any quantity which can be constrained may also be used to form an algebraic expression which may itself be constrained. The algebraic combinations are formed using the 22. and 23. type code elements. The results are placed in numbered registers. To constrain the contents of a numbered register, five parameters are required.

1 - Type code 10.
2 - Code digit 9.
3 - Register number (j).
4 - Desired value of register contents.
5 - Desired accuracy of fit.

Complete descriptions on the formation of algebraic combinations of matrix elements and their placement in storage registers are given in the sections on type codes 22. and 23.

The following example shows the use of the 22, 23, and 10 type code elements. It shows a fit of \( R_{11} + R_{22} \) to a value of \( .2 \).

\[
\begin{align*}
22. & -1. 1. 1. ; \\
22. & -2. 2. 2. ; \\
23. & 1. 2. 1. 1. ; \\
10. & 9. 1. .2 0.001 ; 
\end{align*}
\]
INPUT-OUTPUT options: Type code 13.

Several new options for input-output control have been implemented. The format for type code 13 is:

1 - Type code 13.0
2 - Code number

The new options with their code numbers are as follows;

Accelerator notation for beam matrix 7.

Both input and output for the beam matrix are in accelerator notation. The parameters $\beta, \alpha, \text{ and } \eta,$ of accelerator theory are used. Constraints on the beam are also taken to be in terms of $\beta, \alpha, \text{ and } \eta.$ The forms of input and output for the beam matrix are explained under type codes 1 and 7. The form of constraints on the beam are explained under type code 10. To specify that accelerator notation is to be used in a run, the 13. 7. ; element must be before the beam element (type code 1,).

Positions of waists 14.

The longitudinal positions and transverse dimensions of (real or virtual) waists in both transverse planes are printed at the point of insertion of this element. The waist location and characteristics are computed from the dimensions of the current location assuming infinitely long drift spaces both upstream and downstream.

Inverse transfer matrix print control -4.

The inverse of the current transformation matrix $R_1$ will
be printed by this code. If the program is computing a second-order matrix, the inverse of the second-order transformation matrix will be included in the print out.

**Refer transfer matrix to original coordinate system 13.**

If the transverse coordinates are rotated through an angle via a 20. type code entry, the rotation is taken only as specifying the orientation of the rotated magnet. The transfer matrix, whenever printed, is given in the unrotated coordinate system. The 13. 13. ; element must precede any 20. type code elements to be used. When using a 13. 13. ; element, all coordinate rotations must be (positive or negative) multiples of 90°.

**Shift in reference trajectory 9.**

At the location of this element, the reference trajectory is shifted to line up with the first-order image of the original beam centroid. Thus, the reference trajectory of the beam line, as followed by the program will not be continuous. Quadrupoles will also show dispersive effects. All the fitting options connected with this element operate as expected. Thus the original beam centroid displacement parameters can be varied to fit values of floor coordinates specified after the shift.

**Precise values of varied parameters 16.**

The values of any varied parameters will be printed in F18.10 format. This option is useful primarily for investigating the mathematical characteristics of a solution.
Pole face rotation angle specification 40., 41., 42., 43.

The pole face rotation angle normally specified with type code 2 may alternatively be calculated from the bend angle of the associated bend magnet. The pole face rotation element must still be present. The options described here merely cause the value of $\beta$ (the pole face rotation angle) to be filled in automatically by the program.

The element specifying the means of determination of the pole face rotation angle must precede the bending magnet specification, including pole face rotation angle elements, to which it applies. It will remain in effect until the option is respecified via another 13. type code element. Some caution must be taken in the use of these options. If a bend magnet is segmented, then the pole face rotation angle will be calculated from the bend angle of the adjacent segment.

(13. 41. ;) Both input and output pole face rotation angles are equal to half the bend angle.

(13. 42. ;) The entry pole face rotation angle will be zero. The exit pole face rotation angle will equal the bend angle.

(13. 43. ;) The entry pole face rotation angle will equal the bend angle. The exit pole face rotation angle will be zero.

(13. 40. ;) The normal option is restored. The pole face rotation angles will be read from the data.
Bend magnet input specifications 47., 48.

(13. 48. ;) Bending magnets encountered subsequent to the insertion of this element are to be specified by the length, bend angle, and n value. Thus a bend magnet element will take the form

4. L α n ;

(13. 47. ;) The normal option for specification of a bending magnet is restored by this element. A bend magnet element now takes the form

4. L B n ;

Lithium or plasma lens 97., 98.

(13. 98. ;) All quadrupoles subsequent to this element will be taken as having the same effect in both transverse planes. Thus, if the pole tip field is positive, both planes will be focusing. If the pole tip field is negative, both planes will be defocusing.

(13. 97. ;) The normal option is restored for a quadrupole element. It is now focusing in one plane and defocusing in the other.
SPECIAL INPUT PARAMETERS: Type code 16.0

Code digit for new special parameter

2. $K_0$ - An integral related to the transverse displacement of the reference trajectory arising from passage through the fringe field of a bending magnet. If the (16. 5. g/2 ;) element has been inserted, the program assumes a default value of $K_0 = 0.5$. Insertion of the 16. 2. $K_0$ element supplants this default value. The transverse displacement is given by $\Delta x = g^2 K_0 \rho$. The actual value of $K_0$ is given by a double integral

$$K_0 = \frac{1}{g^2 B_0 \cos^2 \beta} \int_{-s_1}^{s_1} ds \int_{-s_1}^{s_1} (B_0 - B(s)) ds$$

where $s$ is measured perpendicular to the pole face. The parameter $\beta$ is the pole face rotation angle and $B_0$ is the field interior to the magnet. The value of $s_1$ is chosen so as to be well within the interior of the magnet.
COORDINATE ROTATION: Type code 20.0

Normally, the transformation matrix is expressed in the rotated system. If a 13. 13. element is included in the data before the beam (type code 1.) element, the transformation matrix will be expressed in the original (unrotated) coordinate system. In this latter case, all coordinate rotations must be (positive or negative) multiples of $90^\circ$. 
ALGEBRAIC COMBINATIONS OF MATRIX ELEMENTS -
DEFINING REGISTER CONTENTS: Type code 22.

Any of the quantities which can be constrained can also be used to form algebraic combinations which in turn may be constrained. A set of ten storage registers is available for formation of such algebraic combinations. The 22. type code element is used for storage into these registers. Four parameters are required.

1 - Type code 22. (indicating a storage operation).
2 - Code digit i.
3 - Code digit j.
4 - Register number

The indices i and j have the same meaning as for the fitting constraint (10.) type code. Thus, for example, if one wished to place the $R_{34}$ matrix element into register 1, one would use the following element

22. -3. 4. 1. ;

Numerical constants may also be placed in registers for use in forming algebraic expressions. If i is equal to 100, then the third parameter is taken to be the numerical constant. For example, the element

22. 100. 3.14159 7. ;

causes the number 3.14159 to be placed in register 7.

The contents of a register may be constrained by a fitting constraint element (type code 10.). Details are given in the section describing fitting constraints.
ALGEBRAIC COMBINATIONS OF MATRIX ELEMENTS -
FORMING COMBINATIONS: Type code 23.

Algebraic combinations of elements in storage registers may be formed and themselves placed in storage registers. The combinations formed may be constrained or used to form further algebraic combinations.

Five parameters are required:
1 - Type code 23. (signifying an algebraic combination)
2 - First input storage register
3 - Second input storage register
4 - Algebraic operation to be performed
5 - Output storage register

The algebraic operations are referred to by number according to the following list
1 - addition (+)
2 - subtraction (-)
3 - multiplication (×)
4 - division (÷)
5 - square root (√)

If the square root is selected, only the first input register is used. A dummy index should be inserted for the second input register.

As an example of the use of the algebraic combination element we illustrate the process of taking the quotient of the contents of registers 2 and 7 and placing the result in register 2

23. 2. 7. 4. 2. ;
DEFINED SECTION: Type code 24.

A system may contain a section which is repeated at some later point. The repeated section may not occur immediately after the original section, so that the repeat element (type code 9.) is not appropriate. It may also be useful to repeat a section, but with the elements listed in the reverse order.

The 24. type code element allows definition of the section to be repeated. It also indicates the locations at which the section is to be repeated and whether the repetition is to be forwards or backwards.

Three parameters are required:
1 - Type code 24.
2 - Code digit.
3 - Section name. The section name is a maximum of four characters long and is enclosed in single quotes.

The meaning of the code digit is as follows:
1 - Indicates the beginning of the section to be defined.
2 - Indicates the end of the section to be defined.
3 - The section named is to be repeated at the present location in the forwards direction.
4 - The section named is to be repeated at the present location with the elements in reverse order.

A number of rules apply to the use of the defined section element. Each defined section must have both its beginning and its end indicated, and the beginning must precede the end. The
A definition of a section must completely precede its use. A given name can be used only once to define a section, although it can be used many times to indicate a repeat of a section. A defined section may not refer to itself, either explicitly or implicitly. The number of defined sections is limited to 10. The total z rotation (type code 20.) within a defined section must sum to zero. Finally, defined sections must nest properly with the repeat code (type code 9.). Defined sections, however, need not nest properly with each other.

Example of a defined section

24. 1. 'DOUB' ; ———— Begin defined section
5. 10. 5. 2. ;
3. 8. ;
5. 10. -5. 2. ;
24. 2. 'DOUB' ; ———— End defined section
    }
24. 3. 'DOUB' ; ———— Repeat defined section
    }
24. 4. 'DOUB' ; ———— Repeat defined section in reverse order
During the past decade, a very general RAYTRACE code has been developed at M.I.T. for following the trajectories of charged particles through ion-optical systems. The motion of a particle carrying charge \( Q \) is governed by the Lorentz equation,

\[
F = Q[\mathbf{E} + \mathbf{v} \times \mathbf{B}],
\]

where \( \mathbf{E} \) is the electric field and \( \mathbf{B} \) is the magnetic field. In a rectangular \( (x,y,z) \) coordinate system the equations of motion along each of the axes may be written as

\[
\begin{align*}
mx &= Q(E_x + v_yB_z - v_zB_y) \\
my &= Q(E_y + v_zB_x - v_xB_z) \\
 mz &= Q(E_z + v_xB_y - v_yB_x).
\end{align*}
\]

These three particle differential equations of motion are solved by means of a step-by-step numerical integration with time as the independent variable. Accuracy is limited only by the uncertainties in our knowledge of the electric and magnetic fields. Current versions of the code may be used to calculate trajectories through an arbitrary arrangement of elements including dipoles, quadrupoles, general multipoles, solenoids, velocity selectors (\( \mathbf{E} \times \mathbf{B} \) fields) drifts and thin lenses. The modular nature of the code allows easy access for addition of new devices or modification of the 'field' routines to correspond to specific needs. Versions of the code exist for calculating particle orbits in
II. C-2

microtron and/or recirculator type accelerator systems as well.

In each of the magnetic elements, the particle typically moves through three distinct regions: the entrance fringing field, the 'uniform' field and the exit fringing field. In Figure 1 we show the layout for a dipole deflecting magnet. The different coordinate systems are related by a geometrical ray ABCD which is a straight line from A to B, a circular arc from B to C, and a straight line from C to D. All calculations are made with reference to the four rectangular coordinate systems with origins at A, B, C and D.

In the case of the dipole, the effective edge corresponding to the pole boundary is described by the expression

$$z = -R \frac{8}{n=2} S_n \left(\frac{x}{R}\right)^n$$

where $R$ is the layout radius and $S_2, S_3, S_4, \text{etc.}$ are parameters corresponding to the strength of second- (sextupole), third- (octupole), fourth- (decupole), etc. order multipole corrections applied to the shape of the pole boundary (Fig. 1).

The "uniform field" region assumes either that $B_y = B_o$, a constant in the case of uniform field dipoles or more generally,

$$B_y = B_o [1 - n \left(\frac{\Delta x}{R}\right) + \beta \left(\frac{\Delta x}{R}\right)^2 + \gamma \left(\frac{\Delta x}{R}\right)^3 + \delta \left(\frac{\Delta x}{R}\right)^4]$$

for the case of the standard non-uniform field description.

In the median plane of the deflecting magnet the fringing field is described by the equation,

$$B_y = B_o / (1 + e^S)$$

where $S$ is the polynomial

$$S = C_0 + C_1 s + C_2 s^2 + C_3 s^3 + C_4 s^4 + C_5 s^5,$$

and $s$ is approximately the distance measured along the normal to the pole boundary in units of gap widths, $D$. A typical fringe field shape is shown in Figure 2.
The coefficients $C_0$,...,$C_5$ which completely describe the shape of the fringing field in the median plane are usually obtained in one of two ways. Where possible the best way is a fit to measurements on existing similar poleface, coil and/or clamp configurations. Often this is not possible and in that case one should calculate a fringe field shape using one of several good magnetostatic codes which are currently available.

Equation (5) describes the fringing field on the median plane only; i.e. for $y = 0$. Off the median plane, the field components $B_x$, $B_y$ and $B_z$ at point $(x,y,z)$ are computed by means of a Taylor expansion in $y$ off the midplane. The required derivatives are evaluated numerically from the values of $B_y$ ($y=0$) in a thirteen point grid (Fig. 3) centered at $(x,o,z)$. This expansion is carried to fourth order.

The treatment of the fringing fields in the other magnetic elements is similar to that described for the dipole case. In the case of quadrupoles, for example, the gradient rather than the field itself is described by a formula similar to Eq.(5).

The usual procedure in evaluating the performance of a magnetic spectrometer using the RAYTRACE code is to study the intercepts in the image plane of sets of standard rays. These rays are chosen to represent the phase space area of the beam. The intercepts of the rays with the image plane, may also be used to calculate the Taylor expansion coefficients (matrix elements) of the instrument. A spectrometer designed for a "point" source target requires 14 such rays (Fig. 4), for each momentum studied, to obtain all these coefficients through fifth order. To obtain all the coefficients (non point source) in second order and the important ones in third- and fourth orders requires 46 rays to be traced through the system.

The total width of the image as evaluated from the ray intercepts with the image plane (surface defined by intersection of rays with the same momentum) is used as a measure of the performance of the instrument. Chromatic effects are studied by repeating the calculations for several spaced momenta.
Improvements in this performance are made by making adjustments to the multipole strength coefficients $S_2, S_3, S_4$, etc. at dipole boundaries or equivalently by adjusting pure multipole strengths which may be present. The effect on an aberration coefficient due to a change in multipole strength of the same order may be predicted in advance. This has been shown formally by K. L. Brown and is a very useful technique for rapid convergence of the process of making final corrections in the design of a precision ion-optical instrument.

The M.I.T. version of RAYTRACE in its present form contains no automatic fitting procedures. Some versions exist which include some automated features and others incorporate a Monte-Carlo study of hundreds of trajectories at the image plane to more exactly determine the performance of the system. Use has also been made of experimentally determined field maps.

Recent modifications to RAYTRACE at M.I.T. have included explicit calculation of path-length matrix elements together with the addition of graphics output. The graphic features are very important from a designers aspect as it provides for rapid interaction with the necessary practical layout of poles, coils, vacuum chambers, etc.
REFERENCES

1. S. Kowalski and H. A. Enge, unpublished


Figure 1. Dipole deflecting magnet layout as used in the code RAYTRACE.
Figure 2. Typical fringe field shape in the midplane of a dipole deflecting magnet. The distance, \( Z/D \), in units of gap widths \( D \) is measured relative to the effective edge.
Figure 3. Grid pattern centered at a field point (x,0,z) used in evaluating the field components $B_x$, $B_y$ and $B_z$ off the median plane.

Figure 4. Ray bundle for evaluating point source aberration coefficients and spectrometer performance.
Modifications to POISSON

L.H. Harwood
Cyclotron Laboratory, East Lansing, MI 48824

At MSU we have used the POISSON(1) family of programs extensively for magnetic field calculations. In our presently super-saturated computer situation, reducing the run time for the program is imperative. Thus, I have made a series of modifications to POISSON to speed up convergence. Two of the modifications aim at having the "first guess" solution as close as possible to the final solution. The other two aim at increasing the convergence rate. In this discussion, a working knowledge of POISSON is assumed. The amount of new code and expected time saving for each modification is discussed.

The first modification is quite simple. POISSON allows the result of a previous calculation to be used as the start for a subsequent case if the latter differs from the former by only an overall scaling of the current densities involved. This is accomplished with a scaling factor XJFACT. The full potential of this feature is not utilized by POISSON. By adding a single line of code, a time savings of 5-90% can be achieved. Quite simply, originally when the program changed the current densities, it did not scale the vector potential array. In an infinite permeability or an iron-free case, the vector potential scales linearly with current density. If this scaling is included then the overrelaxation needs only to account for saturation effects. At very low or very
high fields, this can mean a 90% time savings. At intermediate fields, the savings might only be 5%. However, the simple, one-line addition should always save some time.

A slightly more involved modification (about one-half page of coding) allows the results of a previous calculation to be used with a different lattice. I reasoned that if only small changes were made in the lattice (eg. change a pole shim detail) then only small changes would occur in the vector potential array. It is now possible to read an old solution in as the "first guess" for a different lattice. Savings can be from 5% to 90% depending upon how large the scale of the change.

As a companion to the last modification, another option was added. When the change in the lattice is small, its effect should be fairly localized. Thus, it seems somewhat wasteful to do the overrelaxation for the entire lattice. To take advantage of the localization, a new option lets the user have a portion of the problem "overrelaxed" more frequently than the bulk of the problem. This option can save 5% to 50% of the time used without the option.

The final modification is rather subtle. POISSON always accesses the points in the same order, "lower left" to "upper right". This leads to a nonuniform convergence rate, faster at the "lower left" and slower at the "upper right". By reversing the order of the points each cycle, a time savings of about 5% is achieved with the addition of only a few lines of code.
New options and modifications are continually being added to the MSU version of POISSON. Those interested in the above changes or any others should contact the author.

References

1) R.F. Holsinger, private communication.
Magnetostatic calculations can involve (in order of increasing complexity) conductors only, material with constant or infinite permeability, or material with variable permeability. We consider here only the most general case, calculations involving ferritic material with variable permeability.

Variables suitable for magnetostatic calculations are the magnetic field, the magnetic vector potential, and the magnetic scalar potential. For two-dimensional calculations the potentials, which each have only one component, have advantages over the field, which has two components. Because it is a single-valued variable, the vector potential is perhaps the best variable for two-dimensional calculations.

In three dimensions, both the field and the vector potential have three components; the scalar potential, with only one component, provides a much smaller system of equations to be solved. However the scalar potential is not single-valued. To circumvent this problem, a calculation with two scalar potentials can be performed. The scalar potential whose source is the conductors can be calculated directly by the Biot-Savart law, and the scalar potential whose source is the magnetized material is single valued. However in some situations, the fields from the two potentials nearly cancel; and the numerical accuracy is lost. The 3-D magnetostatic program TOSCA employs a single total scalar potential; the program GFUN uses the magnetic field as its variable.

There are three methods which may be used in formulating a magnetostatic computation: Finite Difference Method (FDM), Finite Element Method (FEM), and Integral Equation Method. There are important differences between the FDM and FEM, but here we shall consider them together. In FDM and FEM, the value of the field or potential at each mesh point is determined from the values at neighboring points. Hence the resulting system of equations has a sparse matrix, which expedites solution of the equations. Boundary conditions must be imposed explicitly, and the entire domain of the problem must be included.
in the mesh. Advantages of the FDM and the FEM are their sparse matrices and the extensive theory that has been worked out in other contexts (e.g. stress analysis) but can be applied to magnetostatics.

In the integral formulation, the field at each element is a function of the field at every other element; hence the system of equations has a dense matrix. However only the iron region has to be included in the mesh, and the number of variables can be much smaller. Advantages of the integral method are its smaller number of variables and its avoidance of problems with boundary conditions, particularly boundary conditions at infinity.

There is no clear-cut choice as to which method to use. Programs employing the FDM or FEM, such as TOSCA, are clearly better for geometries with thin iron plates, small gaps in the iron, or complex iron shapes. Programs employing the integral method, such as GFUN, are clearly better for problems with simple iron geometry and complex conductor geometry or for finding the field far from the iron. For most applications, either may be used.
Magnetostatic Problems

- Conductor Only

- Constant \( \mu \) (or \( \infty \mu \))

- Variable \( \mu \) (iron)

We consider the most general case, variable \( \mu \)

Larry Turner
ANL
## 2D or 3D Geometry

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>2D</th>
<th>3D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Vector Potential</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Scalar Potential</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

- Not Single-Valued

Two Scalar Potentials 1 1

- Can lead to cancellations
II. E-5

METHODS

- Finite - Difference:
  Used since early 1960's.

- Finite - Elements:
  Methods adapted from Stress Analysis.

- Integrated Equation Methods:
  Relate each element to all other elements.
II. E-6

COMPARISON OF METHODS

ADVANTAGES OF F.D. AND F.E.

- Sparse Matrix
- Theory Has Been Worked Out Extensively

ADVANTAGES OF INTEGRAL METHODS

- Fewer Elements:
  Only Iron Region Divided Into Elements
- No Problem with Boundary Conditions at $\infty$
WHICH METHOD TO USE?

- **Finite - Element or Finite - Difference**
  Clearly Better with Small Gaps or a Complex Iron Geometry.

- **Integral Method Clearly Better with Simple Iron Geometry and Complex Conductor Geometry**
  or in Finding the Field Far From the Iron.

- For Most Applications, Either May Be Used.
I will describe here some magnets that were calculated using GFUN. The first figure shows the computing system at ANL. A four-step procedure is used to calculate magnets using GFUN. First, the interactive TSO system is used to draw the geometry of the magnet on a Tektronix 4012 graphic display unit. When the geometry is correct, it is stored on a disc file that is shared by the batch computers 3033 and 195. A file of JCL and data can be created on TSO and submitted to the batch computers to calculate the magnetization of the steel tetrahedron elements. The results of this ~1 hour batch job are stored on disc. In the same job, or in a separate one, the fields can be calculated at desired points and stored on a "shared" disc. The fourth step is to plot these fields interactively in TSO.

The second figure shows the 12-foot bubble chamber magnet with S.C. coils and the next figure shows it split up into tetrahedrons. Results of the field calculations compared with measurements are in agreement to ±0.8% over the volume of interest. It is noted that this magnet is ideal for a GFUN calculation since the field points are far from the steel elements.

Figure 5 shows the iron arrangement and field uniformity in a high field "C" magnet with a circular pole tip.

A 40-inch gap "H" window magnet was calculated and is shown in the next two figures. As expected the smaller errors of ±0.5% are far from the steel, and errors of ±2.0% occur closer to the steel.
The next figure shows a S.C. coil with four bars and one brick of steel. The DRAW commands necessary to establish this geometry are also listed and the results shown in the next figure. After the magnetizations of the steel elements are calculated the fields may be calculated from the current plus the steel, or by setting the current density to a very small value, the nonuniformity of the field due to the steel can be calculated and plotted as shown.

Two coil shapes from the menu of coils are shown in the next figure. One is a solenoid the other is a constant perimeter end coil that produces a dipole field. Graphical output of the dipole and solenoid field are shown below.

A dipole "C" magnet with solenoid coil and triangular pole tip is shown in the next figure. A beam of particles comes in from the left and is deflected downward. This is path 1. A second beam comes from the bottom and is deflected to the right. This is path 2. The fields calculated along these paths are shown as the solid curves of the next figure. By changing the arrangement of tetrahedrons in the pole tip, a second set of curves (shown dashed) was obtained. This is clearly not acceptable for tracking particles and is due to the small air gap of the magnet and the relatively large tetrahedron size.

To solve this problem, I calculate the edge field using TRIM and created a stand-alone program called TRACK from the TRACK/BEAM subroutine of GFUN as described in the next figure. The parameters of the TRACK command are given in the next figure. The last two figures show how to input data and some output results.

The limitations of the code are the 200 steel element limit and the time it takes to calculate the magnetization.
Current Configuration

IBM 3033 with Wylib3r, ASP, Express & Batch Processing; second IBM 3033 with VM/370 supporting multiple virtual systems; 370/195 with Batch Processing.
Fig. 2
Overall View of the ANL 12' BC showing the location of the chamber, expansion system, coil assembly and cryostat with respect to the iron yoke and particle beam access window.
1. Draw symm = -3, mate = 0, sw = 0, aw = 0, H3 = 0, H1 = 76.2, R1 = 97.5
   R2 = 12.24, f = 274.905

2. Draw symm = 3, mate = 3, shape = -7, x1 = -2.015, x3 = 2.015

3. y1 = 26.395, y2 = 32.025, x1 = -17.75, x5 = 0, N2 = 2

4. Draw shape = -7, y1 = 0, x5 = 60, N2 = 6

5. Draw shape = -7, x1 = -2.25, x3 = 2.25, y1 = 42.20, y2 = 46.70

6. x1 = -40, x5 = 0, N2 = 4

7. Draw shape = -7, x1 = 0, x5 = 60, N2 = 6

8. Draw shape = -7, x1 = -12.7, x3 = 12.7, y1 = 0, y2 = 36.83

9. z1 = -26.67, z5 = -17.75, N3 = 2, NY = 2
II. F-11

Field due to coil + steel.

Set $j = 0.01$ and get field due to steel only.
II. F-15

TRACK-A Program to Track Charged Particles
Through A Magnetic Field and Plot the Path

Robert J. Lari
December 20, 1978
Accelerator Research Facilities
Argonne National Laboratory

Introduction

The TRACK/BEAM commands of GPWU 3-D are useful for tracking a charged particle through a magnetic field and plotting the path. It was felt the subroutines associated with these two commands would form a useful stand-alone program where the user supplied the magnetic field instead of the GPWU 3-D calculated field. Hence, measured fields could also be used. The user supplied fields are stored on disk as a field map or as the edge field of a uniform field magnet.

This report is intended as a user guide for the two programs TRACKEDG and TRACKMAP which are members of the cataloged partitioned data set B06643.TRACK.LOAD. A file FT19F001 must contain the field values. A second file FT16F001 is needed to store the plot information. The graphic subroutines have been written for use on the TEKTRONIX 4012 graphic display console in the TSO mode.

TRACKEDG

This program assumes a triangular pole where the field along every normal to the 3 edges is the same. The geometry is specified
TABLE 2 - TRACK COMMAND PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Default</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>1.0</td>
<td>Particle momentum GeV/c (do not use if energy is specified)</td>
</tr>
<tr>
<td>CHARGE</td>
<td>+1.0</td>
<td>Particle charge</td>
</tr>
<tr>
<td>MASS</td>
<td>938.28</td>
<td>Particle rest mass MeV</td>
</tr>
<tr>
<td>EW</td>
<td>0.0</td>
<td>Electric field in principal direction</td>
</tr>
<tr>
<td>ENERGY</td>
<td></td>
<td>Initial energy eV, then $P = \sqrt{2Em}$ energy x mass Not required if momentum supplied.</td>
</tr>
<tr>
<td>BEAM</td>
<td></td>
<td>Defines beam or ray initial direction. BEAM-X, implies U=Y, V=Z BEAM-Y, implies U=Z, V=X BEAM-Z, implies U=X, V=Y</td>
</tr>
<tr>
<td>U</td>
<td>0.0</td>
<td>Initial position</td>
</tr>
<tr>
<td>UANGLE</td>
<td>0.0</td>
<td>Initial angle degrees</td>
</tr>
<tr>
<td>V</td>
<td>0.0</td>
<td>Initial position</td>
</tr>
<tr>
<td>VANGLE</td>
<td>0.0</td>
<td>Initial angle degrees</td>
</tr>
<tr>
<td>W1</td>
<td>0.0</td>
<td>Initial value of beam or ray variable</td>
</tr>
<tr>
<td>W2</td>
<td>100.0</td>
<td>Final value of beam or ray variable</td>
</tr>
<tr>
<td>DW</td>
<td>0.5</td>
<td>Initial increment</td>
</tr>
<tr>
<td>START</td>
<td></td>
<td>STAR=YES for a new set of tracks</td>
</tr>
<tr>
<td>PRINT</td>
<td>YES</td>
<td>Coordinates displayed</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Type C for continue Type S for stop</td>
</tr>
<tr>
<td>EMITTANCE</td>
<td></td>
<td>Beam emittance rad.cm.</td>
</tr>
<tr>
<td>NTKS</td>
<td>1</td>
<td>Number of rays (cannot set this parameter)</td>
</tr>
<tr>
<td>EB1</td>
<td>0.001</td>
<td>Truncation error bound</td>
</tr>
<tr>
<td>EB2</td>
<td>0.00001</td>
<td>Truncation error bound</td>
</tr>
<tr>
<td>IPRI</td>
<td>0.0</td>
<td>If IPRI=1, $x, y, z, E_x, E_y, E_z$ are printed on unit 6 If IPRI=2, $x, y, z, E_x, E_y, E_z$ are printed on unit 1</td>
</tr>
</tbody>
</table>

Particle Mass

Electron → 0.511 MeV
Proton → 938.28 MeV
Deuteron → 1875.6 MeV

\[ P = \sqrt{2m_0 E} \]

| P → GeV/c |
| m → MeV |
| E0 → Volts |
| x → radians |
| B → g/cm² |
| l → inches |
**INPUT DATA CASES:**

1. \( N_{\text{MAX}}, K_{\text{MAX}} \) (213 Rows)
2. \( B_{\text{ZERO}} \) (90, 3 "")
3. \( x_1, y_1, x_2, y_2, x_3, y_3, x_3 \) (9F82 Format)

**WHERE:**

- \( N_{\text{MAX}} \) is the number of field values in the edge field table.
- \( K_{\text{MAX}} \) is the number of edge field values at different \( B \).
- \( B_{\text{ZERO}} \) is the value of the uniform field region in the control.
- \( x_1, y_1, x_2, y_2, x_3 \) are the effective edge intercept and angle, units are cm and degrees.

**Figure 1**
SUPERCONDUCTING MAGNET TECHNOLOGY

Richard P. Smith
ACCELERATOR RESEARCH FACILITIES

PRESENTED AT

WORKSHOP ON HIGH RESOLUTION AND LARGE ACCEPTANCE SPECTROMETERS
ARGONNE NATIONAL LABORATORY

SEPTEMBER 8, 1981
WHY USE SUPERCONDUCTING MAGNETS?

- HIGH FIELD: \( B > 2 \) Tesla
- LARGE FIELD VOLUME: \( V > 1 \) m\(^3\)
- COSTLY IRON YOKE OPTIONAL
- HIGH FIELD QUALITY: \( \Delta B / B < 10^{-4} \)
- LOW OPERATING COST: Megawatts → Kilowatts
- HIGH FIELD STABILITY: \( \Delta B / \Delta T < 10^{-7} \) T/sec
- SIMPLIFIED SUPPORTING FACILITIES

SOME SUPERCONDUCTING MAGNET DESIGN ALTERNATIVES

- CHOICE OF CONDUCTOR STABILITY
- REFRIGERATION TECHNIQUE
- OPERATING TEMPERATURE
- CONDUCTOR DESIGN
- USE OF IRON
BASIC PHYSICS OF SUPERCONDUCTORS

FOR MAGNET TECHNOLOGY

- $\text{NbTi}$: GOOD $T_C$ vs $B$, FABRICABLE
- $\text{Nb}_3\text{Sn}$: HIGH $T_C$ vs $B$, DIFFICULT TO FABRICATE
- $\text{VA}_3\text{Ga}, \text{Nb}_3\text{Ge}$; TERNARIES: UNDER DEVELOPMENT
III. A-4

CONDUCTOR STABILITY

- INTRINSIC STABILITY
- CRYOSTATIC STABILITY
- CONDITIONAL STABILITY

INTRINSIC STABILITY - Multifilamentary superconductor in a low-resistivity normal metal matrix

For Type II superconductor, magnetization current screen an external field up to about \( B \leq 0.2 \) Tesla. For higher fields, field penetrates superconductor a distance \( D \):

\[
D = \frac{1}{\mu_0} \frac{B}{J_C} \quad \text{(LINEAR BEAN MODEL)}
\]

A conductor is adiabatically stable if for an increase of field \( \Delta B \) below the critical field the conductor remains superconducting:

\[
\left[ + \Delta B + + \Delta U + + \Delta T + - \Delta J_C \right]
\]

This process converges before \( J_C \) reaches zero if the size of each \( \Delta T \) decreases.

This condition leads to a restriction on the size of the conductor:

\[
D^2 < \frac{3}{J_C^2 \mu_0} (C_p \delta) \left(-J_C/\frac{DJ_C}{DT}\right)
\]

For NbTi, \( C_p \delta \) \( 1.1 \times 10^{-3} \text{Ws/cm}^3\text{K} \), \( J_C \sim 3 \times 10^5 \text{A/cm}^2 \), \(-J_C/\frac{DJ_C}{DT}\) \(~5\text{K}\), hence \( D < 350 \text{ microns} \).
III. A-5

The conductor is dynamically stable if the fluxjump heating diffuses away through the normal metal faster than the magnetic field (hence current) can move into the normal metal due to the magnetic diffusivity of the normal metal. This leads to a more restricted condition on the conductor size:

\[
D < \frac{6K}{\pi^2} \frac{1 - \lambda}{\lambda} \frac{1}{\rho J_c^2} \left( - \frac{1}{J_c} \frac{DJ_c}{DT} \right)^{-1}
\]

\( K = \text{thermal conductivity of matrix} \)
\( \rho = \text{electrical resistivity of matrix} \)
\( \lambda = \text{ratio of superconductor to matrix} \)

Metallurgy shows \( \lambda \) can exceed 1.0, but for \( \lambda = .5 \), we get \( D < 25 \text{ microns} \).

Note a kindness of Mother Nature: The process of creating very small filaments helps generate a defect-rich structure in the superconductor due to the cold work. These defects help increase the "pinning strength" \( \alpha \), which prevents the flux from moving into Type II superconductors. And in the Kim-Anderson model this is the \( \alpha \) which explains the observed behavior

\[
J_c \sim \frac{\alpha}{B}
\]

Thus the metallurgy used to create the small filaments can also increase the \( J_c \) performance of the alloy.
It is important to note that for intrinsic stability there is no additional thermal stability against inelastic mechanical disturbances, ohmic heating, etc. In principle, any effect which deposits only a few millijoules per cubic centimeter of conductor will raise the local temperature above the critical temperature and the conductor will be driven normal.

Cryostatic Stability - Augment an intrinsically stable conductor with extra normal metal which can safely carry the current while the superconductor recovers from the perturbation that drove it normal; provide cooling to dissipate this ohmic heating during the recovery process.

Measure pool-cooling heat transfer of liquid helium:

\[ \dot{Q} \quad (\text{W/cm}^2) \]

\[ \log \Delta T \quad (\text{K}) \]

Cryostability Condition

\[ I^2R < \dot{Q} \times \text{(cooled surface)} \]
REFRIGERATION TECHNIQUES

Pool Boiling
- Easy to control conductor temperature
- Easy to provide refrigeration
- Large thermal reserve always available
- Heat flux characteristics to liquid known

Super Critical Forced Flow
- Reduced helium volume
- Heat transfer more straightforward
- Expanded range of operating temperatures possible

- Probably not possible to achieve classical "warm end" cryostability
- Cryogenic system complicated
- Limited excess refrigeration available
III. A-8

Superfluid He II

Reduced temperature improves $J_C$ of superconductor

Non-classical heat transfer probably much better than pool boiling.

Quenches less likely to lead to loss of helium

* * * * *

Cryogenic system more complicated

Limited experience available to date.

Refrigeration Costs:

Current Leads: 1.5 liquid liters/hr per kiloamp

Cold Mass Support: $5 \times 10^{-2}$ watts per ton supported achievable for large systems

Resistive Joints: $R \sim 10^{-9}$ ohms

Apparent thermal conduction from multilayer insulation

(80 K - 4 K): $5 \times 10^{-2}$ watts/meter$^2$

Transfer line losses: $\leq .1$ watt per meter

Helium heat of Vaporization: 1 watt vaporized 1.4 $\ell$/hr

(20.4 J/g)

Helium bas enthalpy (4 K - 300 K): 1542 J/gm (1 atm)
III. A-9

OPERATING TEMPERATURE

* Correlations for NbTi (kA/cm²)

\[ J_c = J_c(4.2 \text{ K}) + \Delta J(T) \]

\[ J_c(4.2 \text{ K}) = 137 + [22 + 10 (4 - B)] (4 - B) \]
\[ = 5 + 22 (10 - B) \]
\[ B < 4T \]
\[ = 5 + 22 (10 - B) \]
\[ B > 4T \]

\[ \Delta J(T) = (4.2 - T) (54 - 2.33B) + 62 - 4B \]
\[ T < 3K \]
\[ = (4.2 - T) (54 - 2.33B) + (62 - 4B) (4.2 - T) \]
\[ 1.2 \quad 3 < T < 4.2 \]
\[ = (4.2 - 7) (54 - 2.33B) \]
\[ T > 4.2 \]

* Correlation for NbTiTa

\[ J_c = 468 - 25B - (30 + 1.5B) T \]
\[ 1.8 < T < 4 \]
\[ B < 12T \]

* Correlation for Nb₃Sn

\[ J_c = [1 - (.032 + .017B) (T - 4.2)] \phi \]

\[ \phi = 30.34 - .8161 B + 1000e^{-.378B} \]
\[ B < 13T \]
\[ = 27.1 - 3.01 (B - 13) \]
\[ B > 13T \]

One can at present design for \( J_c = 1.5 - 2.0 \times 10^5 \text{ A/cm}^2 \) at 4.2 K for NbTi at 5.0 T for example, and from the above correlations, design for about twice this at 1.8 K and 5 T.
III. A-10

Note that at 4.2 K and $B \approx 9 - 10 \, T$ NbTi no longer carries interesting currents. At 9 T, Nb$_3$Sn can carry 3 - 4 times the current of NbTi, and will carry as much current at 15 T as NbTi will at 9 T.
HIGH FIELD APPLICATIONS

High Performance Small Bore Dipoles and Quadrupoles

Assume stored energy < Few Megajoules
OK to Use Intrinsic Stability Only

Assume Fields > 2 T
OK to Omit Iron Except for Shielding

Example: Design a 5 T dipole magnet, 1 meter long and a 10 cm cold bore:

Field at \( w^* \) due to a current at \( z^* \) is given by

\[
\frac{dB}{dz} = \frac{\mu_0 Jr \, dr \, d\theta}{2\pi (w^* - z^*)} \quad \quad z^* = re^{-i\theta}
\]

For a "round" dipole we can place 2 current filaments symmetrically and integrate over annuli from \( R_1 \) to \( R_2 \)

Maxwell's equations ensure analyticity so we can expand the function and integrate term-by-term
III. A-12

The lowest order term is the dipole field. Note for the choice \( \phi = \pi / 3 \) the \( k = 1, 4, \ldots \) terms are zero.

The higher order fields in fact sum to only a few parts in one thousand of the dipole field at a radius of \( 2/3 \) of the winding radius.

For higher field precision a second layer, or additional blocks of current in the first layer of varying current density can be used to achieve field purities of a few parts in \( 10^{-4} \). With several layers or blocks one can reach \( \sim 10^{-5} \).

An alternate design approach is based on the observation that overlapping circles of uniform current, displaced a distance \( d \), or overlapping ellipses as well, generates a pure dipole field:

\[
\begin{align*}
B &= \frac{\mu_0 J d}{2} \\
B &= \frac{\mu_0 J bd}{(a+b)}
\end{align*}
\]

A \( \cos \theta \) current distribution on a cylinder also generates a pure dipole field.
III. A-13

In practice a finite number of discrete blocks of current are used to approximate these winding shapes.

For our example, we use the simple-block winding with $\phi = \pi/3$:

$$B = \frac{\mu_0 J}{\pi} (R_2^2 - R_1^2) \frac{\sqrt{3}}{2}$$

How do we find $J$??

Assuming the peak field in the winding does not exceed 5.5 T and the operating temperature never exceeds 4.5 K we find that for NbTi, $J_c = 1 \times 10^5 \text{ A/cm}^2$. It is possible to fabricate the multifilamentary conductor with as little as 2:1, Cu: NbTi so averaged over the conductor (Cu + NbTi), $J_c = 3.3 \times 10^4 \text{ A/cm}^2$. Insulation between turns, and perhaps some helium in the windings would give a packing factor $\lambda$ of say, 0.8, so that $\lambda J = 2.6 \times 10^4 \text{ A/cm}^2$. One prudently operates the magnet at somewhat less than the critical current, so the actual operating density might be $2.1 \times 10^4 \text{ A/cm}^2$

Finally, 

$$R_2 = \frac{2\pi B}{\sqrt{3} \mu_0 J} + R_1 = 0.069 + 0.05 = 0.119 \text{ m}$$

We note that for the shape chosen, the next highest nonzero multipole is $B_2 = 18$ pole, and at $r = 4 \text{ cm} = 0.8$ of the winding radius,

$$\frac{B_2}{B_0} = 0.0018$$

Remember of course that there are in fact nonzero "non-allowed" multipoles that stem from construction asymmetries.
III. A-14

Now for the Cost:

The ampere turns are \( NI = JA = J \times \frac{2\pi}{3} \times \frac{1}{2} (R_2^2 - R_1^2) \)

\[ = 2.6 \times 10^6 \text{ AMPERE TURNS} \]

Finished NbTi conductor can cost almost $5 per kilo-amp meter for 5 T service, so for our 1 meter long magnet, the conductor will cost

Conductor Cost = 2 x 1 x $5 x 2.6 \times 10^3

\[ = \$26 \text{ K} \]

To support the conductor against the Lorentz forces, we can get an insight into the task by noting that the 5 T field in the bore tube exerts a magnetic pressure of

\[ P = \frac{1}{2} \mu_0 B^2 = 9.95 \times 10^6 \text{ psi} = 1450 \text{ psi} \]

The stress patterns are analyzed in detail by calculating the 2D forces on one quadrant

Strong clamping is needed to contain the burst forces and the compressive loads toward the median plane must not crush the insulation on the conductor. In fact the Lorentz stresses must be carefully equilibrated everywhere to avoid conductor motion which will surely cause quenching. There are axial hoop stresses in the conductor caused by the ends of the magnet. These 3D forces are somewhat more difficult to analyze.
If an iron yoke is desired it can be placed outside the coil, perhaps far enough away to avoid saturation effects. Then the image currents in the iron all linearly to the field of the current in a helpful way. Generally it is found that the cost and complexity of the iron more than out weighs the cost of extra conductor needed to get the same total field.

The choice of operating current is yet to be made. A con current cheapens the cryogenic costs (and power supply costs), but engenders risk due to coil burnout during a quench. In fact when examining quench safety one finds:

\[ T \propto I \]

Example - A single choice of current won't optimize both temperature and voltage safety during a quench. Remember that the inductance of the coil goes as the square of the number of turns which in turn goes inversely as the operating current.

For a magnet whose coils are not directly cooled by helium the peak adiabatic temperature rise \( T \), during a quench is given by

\[
\int_{0}^{\infty} J^2 dt = \int_{0}^{T} \frac{c_\delta}{\rho} d\theta
\]

One usually desires the quench to propagate very rapidly so as much as possible of the magnet's mass can participate in absorbing the stored energy, rendering \( T \) as small as possible. Clearly if the entire coil is not driven normal, \( T \) can become dangerously large.
One can extract energy from the magnet with the aid of an external voltage, but the onset of the quench must be detected in order to trigger the protection system.

Example: High performance quadrupoles can be built following similar procedures. For a magnet that has a field purity of a few parts in 1000 at $\approx 2/3$ of the winding radius, a single block design is sufficient. In this case, the lowest order term is

$$B_0 = \frac{2i\mu_0 J}{\pi} \ln \left( \frac{R_2}{R_1} \right) \sin 2\phi$$

$$G = \frac{2B}{\omega_0} = \frac{2\mu_0 J}{\pi} \ln \left( \frac{R_2}{R_1} \right) \sin 2\phi$$

For the choice $\phi = \pi/6$, the first higher multipole is zero, and the field purity is as good as the dipole.

The ampere turns is readily calculated and expression for the field within the windings themselves evaluated to find the peak field, in order to design the conductor. One integrates the field expressions to find the stored energy,

$$U = \frac{1}{2\mu_0} \int B^2 \, dv$$

so that the inductance is known.

Large Analyzers and Bending Elements.

When the stored energy of the magnet exceeds a few megajoules, the magnet can be designed to be cryostable if pool cooling is chosen. At present the cryostable magnet is the only system for which design principles exist that when followed can essentially guarantee that the magnet will not quench. The same cannot be said of the various schemes for forced flow cooling.
III. A-17

The cyrostability idea begins with the simple concept presented above: Complete local recovery from an incident which drives the conductor completely normal.

Let us consider a large magnet, with iron, producing a field of 2 T in a gap 10 cm wide. Let the pole faces be 1 m x 1 m.

An estimate of the amount of current required is obtained by noting that in the iron, $H \approx 0$.

$$\int H \, dl = I$$

$$I = \frac{B \cdot l}{\mu_0} = \frac{2 \times 0.1}{4\pi \times 10^{-7}} = 1.6 \times 10^5$$

Assuming it is possible to wind the coils with a square conductor having $f = 50\%$ of the forces exposed to helium, and choosing the conservative recovery heat flux of $\dot{Q} = 0.15 \text{ W/cm}^2$, we have

$$I^2 R \approx \dot{Q} \rho$$

$$I^2 A^2 f L = \dot{Q} f 4 \omega L$$

$$I^2 \rho \omega = \dot{Q} f 4 \omega$$

At 2 T, the copper resistivity is

$$\rho = \frac{1}{150} \left(1.7 \times 10^{-6}\right) + 2 \times 4 \times 10^{-9} \quad \Omega \text{cm}$$

$$= 1.9 \times 10^{-8} \quad \Omega \text{cm}$$
CHOOSING A CONDUCTOR 1 CM SQUARE, \(J = 3973 \text{ A/cm}^2\). Since the conductor area is 1 cm\(^2\), the operating unit is 4000 amperes. Of course, not all the conductor area is copper. We find that at 2 T, the NbTi conductor will require \(4000/4 \times 10^5 = .01 \text{ cm}^2\) total area. We might make a small NbTi multiwire cable, and solder it into a groove in the copper bar.

With turn-to-turn insulation, suitably designed to support the Lorentz loads yet permit 50% face cooling, the packing fraction might be \(\sim 0.8\), hence each coil pack becomes

\[
A = \frac{1}{2} \frac{NI}{J} = \frac{1}{2} \frac{1.6 \times 10^5}{4000 \text{ A/cm}^2} = 20 \text{ cm}^2
\]

about 4.5 cm on a side. There are 20 turns in each coil. The coil winding is enclosed in a \(\approx 50\) psi stainless steel pressure vessel casing, then supported from the vacuum vessel with stiff struts against the coil-iron forces. Within the vacuum space is superinsulation and a sheet metal shield cooled to 80 K with LN\(_2\). Depending on how the cryogenic supply neck is designed, and the cold-mass support struts, the overall coil is quite compact.

For a 1 m \(\times\) 1 m coil, the superconduction conductor would cost about

\[
\frac{\$}{5} \times \frac{2T}{5T} \times \frac{4m}{4m} \times \frac{1.6 \times 10^5}{4 \text{ kA/turn}} = \$1.3 \times 10^3
\]

There is only about 100 kg of copper in the conductor, so in fact the conductor cost is likely to be the cheapest element. For such a magnet the iron will be the most costly element.
The refrigeration costs for this magnet should not exceed 25 kW. A conventional magnet of this size is in the MW range for power consumption.

\[ \frac{W_c}{Q} \text{Carnot} = \frac{T_o - T}{T} \]

### Reversible Power Requirements

<table>
<thead>
<tr>
<th>Fluid</th>
<th>T (K)</th>
<th>Refrigeration (W/W)</th>
<th>Liquefaction (W hr/liter)</th>
<th>Evaporating Liquid Refrigeration (W/W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Helium</td>
<td>4.2</td>
<td>70.4</td>
<td>236</td>
<td>326</td>
</tr>
<tr>
<td>Hydrogen</td>
<td>20.4</td>
<td>13.7</td>
<td>278</td>
<td>31.7</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>77.4</td>
<td>2.88</td>
<td>173</td>
<td>3.87</td>
</tr>
</tbody>
</table>

*Obtained by dividing the ideal liquefaction power requirement by the heat of vaporization of the fluid.

---

**Figure 1.** Efficiency of low temperature refrigerators and liquefiers as a function of refrigeration capacity.

**Figure 4.** Cost of low temperature refrigerators and liquefiers as a function of installed input power.
USEFUL REFERENCES

A PRIMARY REFERENCE FOR NEARLY EVERYTHING CONCERNING SUPERCONDUCTING MAGNETS IS:


MORE RECENT LITERATURE, PARTICULARLY USEFUL FOR DETAILS:


3. "Advances in Cryogenic Engineering" Annual Volume. These are the proceedings of the International Cryogenic Engineering Conferences.

4. The monthly periodical "Cryogenics".
Why use superconducting coils in conventional spectrograph magnets?

J.A. Nolen, Jr.
National Superconducting Cyclotron Lab
Michigan State University
East Lansing, MI 48824

ABSTRACT

This report presents an analysis of the costs of large spectrograph dipoles with conventional copper coils compared with ones utilizing superconducting coils. For large gap magnets (~ 15 cm), the superconducting version is significantly less expensive to build as well as to operate. The copper-coil dipole would also be about 50% more massive than the superconducting version.

Supported in part by the NSF Grant # PHY 80-17605 and DOE Grant # DEAC-02-80ER-10579
The k-800 (1.2 GeV/c) spectrograph to be constructed at the National Superconducting Cyclotron Lab is described in a separate paper at this conference. This is a large solid angle (20 msr) high resolution spectrograph which requires two large gap (15 cm) dipoles with good field uniformity. The maximum field required in these dipoles is 15 kG. The purpose of this paper is to compare the cost of construction and operation of these dipoles with superconducting coils with those associated with conventional copper coils.

For the purposes of this comparison consider the H-frame magnet illustrated schematically in figure 1. This shows the results of a POISSON calculation for a preliminary dipole design using a superconducting coil. This dipole has a 15 cm gap and a useful field width of 70 cm as required by the spectrograph. It has two small superconducting coils (the blackened region), one above and one below the median plane, which carry 130,000 amp-turns each to produce a 15 kG field. The top and side pieces are fairly thin (40 cm and 37 cm, respectively) and run at 21-22 kG when the central field is 15 kG. The 15 cm gap would require only 100,000 amp-turns if the top and side pieces were much thicker. The resulting dipoles are relatively lightweight, less than 70 tons each for a central path length of 3.5 m. A similar design optimized for a conventional copper coil would be about 50% more massive because the top and side pieces would be thickened to reduce the power requirements.
We will first present an estimate of the costs associated with construction and operation of superconducting coils and then compare them with estimates for conventional copper coils.

A summary of the total costs of the superconducting coils for two dipoles is given in Table 1. Each dipole has two coils with 130,000 amp turns per coil and each coil has a circumference of 33 ft. for a total conductor requirement of $17 \times 10^6$ amp-ft. Since the peak field in the conductor region is very low (< 2T.) and the space available for the coil is adequate, there are no special problems associated with the conductor. The conductor cost estimate based on standard commercially available NbTi wire is about $10,000. The cost of power supplies depends on the final choice of operating current and turn-on time requirements, but 10V at 1000 a or 10kW per dipole leads to a conservative total estimate of $10,000 for power supplies.

Quite often the least certain costs are those listed under items I.3 and I.4 in Table 1, the construction and assembly of the cryogenic components. The estimate of $100,000 and $25,000, respectively, are scaled from recent cost experience associated with similar components of the k=500 and k=800 superconducting cyclotrons at MSU. If the superconducting coils are flat, as in an H-frame magnet, and do not have negative curvatures, then many components and construction methods will, in fact, be very similar to those for the cyclotrons. Hence, in this case, these estimates are expected to be reliable.
III. B-4

Item 1.5, the refrigerator, may or may not be a real cost, depending on the laboratory. At MSU the large refrigerator system under construction for the two superconducting cyclotrons will have adequate capacity to service superconducting experimental apparatus when not in the cool-down mode for one of the cyclotrons. The estimate of $100,000 given in the table is for the case where the spectrograph requirements represent a sizeable fraction of a laboratory's liquid helium usage.

The operational costs for the superconducting dipoles are mainly power to run the refrigerator and maintenance of the refrigerator, and possibly some maintenance of the cryogenic components. Both costs for the refrigerator depend on the size of the system and its related efficiency. A conservative estimate of 20 kW of electricity at a cost of $0.05 per kW-hr would be $8,000 per year or $80,000 over a ten year period. With an estimated $2,000 per year for a share of the refrigerator maintenance and miscellaneous costs, the estimated total operational costs for 10 years is $100,000.

A version of these dipoles could be built with conventional copper coils by just filling the cryostat space (18" tall x 7" wide) with copper. However, due to the saturated iron and high current density the peak power requirement for two such dipoles would be over 500 kW, resulting in an estimated power cost over a 10 year period of over $500,000 (assuming a 20% duty factor). Hence, to optimize the conventional coil dipoles,
such that the total of initial and operational costs over 10 years is a minimum, requires (a) thicker top and side steel pieces, and (b) larger copper coil area.

The estimates of the costs of approximately optimized conventional coil dipoles are listed in Table 2. As in the estimates for the superconducting coil version, these costs do not include the cost of the steel for the base magnets or other miscellaneous costs such as support structures and vacuum components. Only costs directly associated with the coil and incremental costs of steel are included. The extra steel indicated under item 1.3 is mostly required to reduce the extra power dissipation caused by saturation of the top and side pieces, i.e. to reduce the ampere turns per dipole from 260,000 to 200,000. A small component of extra steel is also required in order to increase the space available for the coil from 18" x 7" to 18" x 11.5". This reduces the average current density in the coil to about 1000 amp/in², or about 1700 amp/in² in the copper with a packing factor of 60%.

The copper coils used in this design are relatively large (15 tons for the two dipoles), but this is required for an optimized design. Reducing the coil mass increases the power supply cost and the cost of electricity. If the copper coils cost more than the $10.00 per pound indicated, then the balance between coil size and electrical costs would shift, but the total costs would obviously be larger. The average cost of electricity for the 10 year period, taken here to be $0.05/kW-hr,
may be different at different laboratories. The estimated duty factor of 20% comes from a combination of usage time for the spectrograph and the relative power at which it is used.

In Table 3, the total costs of both systems are compared. The superconducting version of these dipoles wins by about a factor of 2 in both initial cost and operational cost. The savings are substantial, several hundred thousand dollars, in this example.

This comparison has been done for one specific dipole design, but generalization to other cases is fairly easy. The costs of the copper coil system scale very rapidly with the gap of the magnet, whereas the costs of the superconducting coil scale very slowly with the magnet gap. Hence, for larger gap magnets, e.g. 20 to 30 cm or more, there is absolutely no question that superconducting coils are much more cost effective than conventional coils.

On the other hand, scaling the present estimates down to smaller gaps, e.g. 5 to 10 cm, the cost advantages of the superconducting coils for this style dipole tend to go away. For such smaller dipoles, however, other magnet designs may be more cost effective. We have done preliminary studies of 5 cm window-frame dipoles with superconducting saddle coils. By using cold steel in these designs very compact systems can be built. In any case, before concluding that superconducting coils are not cost effective for small dipoles, various options must be considered.
In conclusion, it is clear that any new large solid angle, high resolving power spectrograph for momenta above 1 GeV/c superconducting coils can permit substantial savings in both initial cost and operational cost.
### Table 1  Superconducting Coil Costs for Two Dipoles

#### I. Initial Costs

1. Conductor (NbTi) 17x10^6 amp-ft  \[ $10,000.00 \]
2. Power supplies 2x10 kW  \[ $10,000.00 \]
3. Cryogenic components  \[ $100,000.00 \]
   - Bobbin
   - Coil supports
   - LN\textsubscript{2} Shield
   - Cryostat container
   - Leads and diagnostics
   - Flexible 4.2k connections
   - Cryo lines to refrigerator
4. Assembly 7 man-months \[ $25,000.00 \]
   - Coil winding 560 turns total @ 1000 amp.
   - Cryostat
5. Refrigerator 20 watts at 4.2k \[ $100,000.00 \]
   (fraction of a larger system)

**Subtotal** \[ $245,000.00 \]

#### II. Operational costs (10 years)

1. Power to run refrigerator 20 kW \[ $80,000.00 \]
2. Refrigerator and Cryogenic maintenance \[ $20,000.00 \]

**Subtotal** \[ $100,000.00 \]

**Total** \[ $345,000.00 \]
Table 2  Conventional Coil Costs

I. **Initial Costs**

1. Copper coils 15 tons @ $10.00 per pound $300,000.00  
2. Power supplies 2 x 100 kW 100,000.00  
3. Extra Steel (50%) 70 tons 200,000.00  

Subtotal $600,000.00

II. **Operational Costs** (10 years)

1. Electricity* 200 kW @ $0.05 / kW-hr 200,000.00  
   x 20% duty factor  

Subtotal $200,000.00  

Total 800,000.00

* The cost of cooling water has been neglected here, but in some laboratories this could be a significant percentage of the cost of electrical power.
Table 3 Comparison of the costs estimated for superconducting and copper spectrograph coils.

<table>
<thead>
<tr>
<th></th>
<th>Copper</th>
<th>Superconductor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Cost</td>
<td>$600,000.00</td>
<td>$245,000.00*</td>
</tr>
<tr>
<td>Operational Cost</td>
<td>$200,000.00</td>
<td>$100,000.00</td>
</tr>
<tr>
<td>Total</td>
<td>$800,000.00</td>
<td>$345,000.00*</td>
</tr>
</tbody>
</table>

* Reduce by $100,000 if adequate refrigeration capacity already exists at the facility.
Figure 1. A field plot from a POISSON calculation of a dipole with a superconducting coil. One quarter of the dipole cross section is shown. The superconductor is in the blackened region.
Dipole Design Studies for the MSU 1.2 GeV/c Superconducting Spectrograph*  

A.F. Zeller, J.A. Nolen, Jr., and L.H. Harwood  

Cyclotron Laboratory  

Michigan State University  

East Lansing, MI 48824  

ABSTRACT  

This report summarizes the results of a series of calculations for a large gap (6") "window-frame" dipole with superconducting coils. These studies led to a design which has a large region of good field, low mass, and large dynamic range [5-16 kG]. It's disadvantage is relatively complicated superconducting saddle coils.  

* Research supported by NSF PHY 80-17605 and  

DOE DEAC-02-80-ER-10579
Once the decision to use superconducting coils in a low field dipole is made, the task becomes one of optimizing the coil design to produce the required field shape using the minimum amount of iron. The resulting magnet should have the required operating range, smallest cost for iron and should not have excessive force on the coil which would increase the complexity of the support structure.

Magnet design calculations were performed with the code POISSON* using a permability curve, shown in fig. 1 as "Lukens". It is modeled after the curve given by Lukens Steel Company as "Lukens H P Steel", with a carbon content <.08%. The curve labeled 1015 is that given for commercial 1015 grade steel. Both curves use the same parameters for $B^{>20kG}$; ie, those used in calculations for the MSU Superconducting Cyclotron magnets.

The present design studies are for window-frame type magnets with room temperature steel and superconducting coils. Since the cryostat is in the gap of the magnet the coil does not fill the gap vertically. Calculations were mostly done in Cartesian coordinates, using a quarter of the magnet, but after a reasonable design was found, cylindrical coordinate calculations were done to optimize the yoke on the inner and outer sides. For the design in fig. 2. calculations produced the gradient and the field distributions shown in fig. 3. In this figure, and in the following ones, the nominal central field is 15 kG unless otherwise specified. As can be seen from the shape of the gradient, a symmetric sextupole component has been introduced.

into the field. This component is included in the ray tracing calculations and, therefore, the choice of curvatures at the dipole entrances and exits.

Generally the width of good field does not depend on the relative width of the inner and outer yokes, as long as sufficient area is available to maintain the central field at the required level. However, the position of the center of the magnet, where the sextupole component's gradient changes sign, does depend on the widths. It was found by cylindrical coordinate calculations that the removal of 2 cm from the outer yoke (initially the same width as the inner) and adding 4 cm to the inner resulted in the magnetic center coinciding with the physical center.

For a given pole width and gap, the thicknesses of the top and side yokes are quickly determined by systematically reducing them until significant saturation begins to occur. Figures 4 and 5 show calculations for a base design (see Fig. 2) having a 125 cm pole width and a 15 cm gap. Figure 4 shows six of the designs that were considered, with the criterion for a good design being the greatest width of field with a gradient $\leq 1 G/cm$. The numbers on the curves correspond to the coil regions shown on the side of the figure. It is interesting to observe that essentially the same window-frame design can have characteristics of either a window-frame (a positive gradient approaching the coil) or an H-frame (a negative gradient near the coil). Design 1 is a conventional window frame except
that the coil does not fill the entire gap; it displays the
typical, positive gradient. Designs 2 and 3 are similar to
1 except that a 1 cm pole tip has been added, and they display
an increasing gradient until ~ 35 cm from the center where
the field suddenly begins to fall off. The addition of a thin
pole tip partially compensates for the natural saturation of
the window-frame magnet and the concentrated coil. The gradient
is strongly negative until close to the coil where it increases
very rapidly, reaching a peak field at the conductor of 20
kG.

Changes in the size of the coil do not produce as striking
effects as the introduction of a pole or a slot. Designs 3
and 4 differ only in the height of the coil (coil 4 is, however,
2.5 cm closer to the center than coil 3), but the shape of
the gradient is essentially unchanged, though the region of
good field is decreased.

The introduction of a slot produces a larger region of
good field than such a design without a pole or slot, eg, 5
vs 1. Clearly design 6 has too much slot and pole, but illustrates
that field shapes can be dramatically changed by use of slots
and/or a small pole.

Figure 5 shows a more detailed study of the effects of
slots. The curve labelled "no slot" shows the base shape and
position used in the other calculation, labelled 1-4 and shown
at the bottom of the figure. For a given position of the slot
there is a maximum in good field for a particular height of
the slot. However, as illustrated by curves 2 and 3, the maximum
is relatively broad for a variation of several centimeters. The position of the slot is much more critical, both to the region of good field and the sign of the gradient. By moving the slot from point 1, the gradient changes from one characteristic of an H-frame back to one of a window-frame without a slot. There is seen to exist an intermediate position which shows the character of a modified window frame, i.e. a rising gradient before a small fall off when the gradient changes sign before rising sharply. Moving the position of the slot from 40 to 45 cm from the center increases the width of good field from 29 to 38 cm.

Because the optics calculations for the spectrograph require a good field of ~35 cm (half width) for the largest solid angle/energy bite that will pass through the quadrupoles, a smaller magnet than the one shown in the preceding figures should be acceptable. This smaller magnet, the final design, is about half the mass of the previous example and is shown in cross section in figure 6b. Fig. 6a shows an enlarged view of the coil region and the calculated gradient for fields of 5-16.4kG and for 15kG using infinite permeability for the iron. Note that the 5kG solution is better than for infinite permeability, due to the saturation of the pole. A more detailed study of the coil and pole design is summarized in figures 7 and 8.

For a fixed coil height (the coil width is not a critical parameter and is essentially fixed by current density and cryostat considerations) variations of a few tenths of a millimeter
in pole tip thickness produce significant changes in the shape of the gradients, as in fig. 7. Calculations are for central fields of 15 kG, except those with "(5)" which are for central fields of 5kG. The labels are pole thicknesses in centimeters. We see that each 1/2 mm (from .4 to .5cm) in pole tip thickness produces 1 cm additional good field at 15kG. However, the field must also be good at low field, hence producing a broad range device. Since the 5 mm pole calculation has too much negative gradient at 5kG to meet this requirement the 4.5 mm thickness is the optimum choice.

Instead of fixing the coil size and finding the pole thickness which maximizes the good field, it is alternatively possible to choose a pole thickness and vary the coil height, as shown in figure 8. The curves are labelled with the coil height, in cm, for a 5 mm pole thickness and at a central field of 15kG. We see that the sign of the gradient can be changed by simple changes in the coil height.

Other types of designs, like those shown for the earlier, bigger magnet, do not produce sufficient regions of good field. Figure 9 shows the gradient for an optimized design (5 cm tall, coil and 4.5 mm thick pole). Also shown are the calculations for the same magnet without a pole (curve #2) and with a slot (curve #3), neither of which produces as good a solution as the curve labelled 1. Difficulties with finding a satisfactory location and height for a slot arises, in part, because the thickness of the top yoke was constrained to minimize the mass
of steel.

It is apparent from figures 7 and 8 that many combinations of coil height and pole thicknesses are available which produce adequate regions of good field at 15 kG. Other criteria are also used to decide on an appropriate combination: 1) It must have a good field over a wide range of field strengths (e.g., Figure 6), it must minimize the amount of iron needed, and hence the cost (the final design used only ~ 1/2 the iron required for the magnet shown in figure 2, approximately 50 tons per dipole), and 3) it must be reasonably insensitive to small changes in yoke thickness. Criterion 3 produces a flexibility in the choice of machining facilities. A uniform reduction of the top yoke thickness from 40 to 39 cm results in a decrease in the region of good field of less than .5 cm for the 4.5 mm pole -5 cm coil design. Additionally, coil size also dictates the magnitude of the forces on the coil at full field. A large coil generally has a lower peak field, and hence, less force on it than a smaller one. However, practical limitations on cryostat design limit the coil height to ~ 5 cm in the 7.5 cm half-gap.

The unpredictability of permeability curves for steel require caution in the design since the permeabilities may not be as large as used in the calculations. To test the sensitivity to the permeability curves, the curve labelled 1015 in figure 1 was used to calculate the gradients shown in figure 10. The curves are labelled with the central field values and the solid and dashed curves, are respectively, those with the $\mu_{\text{max}} = 2200$
III. C-8

(peak permability for Luckens HP steel) and \( \mu_{\text{max}} = 1500 \) (peak for commercial 1015 steel). The 1015 steel hopefully represents an extreme case since the anticipated specification is for a carbon content of \( \leq 0.08\% \), i.e. 1006 steel. It is seen, though, that even the 1015 steel reduces the region of good field by only 1 cm, although the gradient has increased.

The magnet design presented here is essentially a "window-frame" type and, consequently, involves saddle coils for particle entrance and exit. The feasibility and cost of winding such complicated superconducting are currently being investigated. Calculations are also currently being carried out for an "H-frame" type dipole with flat superconducting coils above and below the median plane. Preliminary results show that such magnets are at least 50% more massive for comparable volumes of good field, but the coil geometry is considerably simpler and the forces on the coil are greatly reduced.

Note Added in Proof:

The "H-frame" dipole studies have continued since the conference. It now appears that an H-frame magnet with flat superconducting coils will be our choice for this spectrograph. Reduced forces on the coil as well as the simpler coil geometry strongly influenced this choice.
Fig. 1. Permeability curves used in relaxation calculation.

Fig. 2. Cross sectional view of the top half of preliminary magnet design.

Fig. 3. Gradient and field plots, for the magnet in figure 2.

Fig. 4. Miscellaneous design calculations for the magnet in figure 2.

Fig. 5. The effects of vertical slots for the magnet in figure 2.

Fig. 6. a) An enlarged cross section of the coil region and gradients for several fields in an optimized design. 
            b) A cross sectional view of this magnet.

Fig. 7. The effects of changes in the pole thickness.

Fig. 8. The effects of changes in coil height.

Fig. 9. The effects of changes for the magnet shown in figure 6.

Fig. 10. The effects of using the different permeability curves shown in figure 1.
PERMEABILITY CURVES USED IN RELAXATION CALCULATIONS

PERMEABILITY ($\mu \times 100$) vs. B (kG)

LUKENS

1015
MISCELLANEOUS DESIGN CALCULATIONS

![Graph showing dB/dr vs r (cm) with various curves and labels from 1 to 6.](image-url)
5 cm COIL
VARIABLE POLE THICKNESS

$\frac{dB}{dr}$ (G/cm)
r (cm)

-3 -5

0.4
0.4(5)

0.45
0.45(5)

0.5
0.5(5)

POLE
THICKNESS
0.5 cm POLE THICKNESS
VARIABLE COIL HEIGHT

\[ \frac{dB}{dr} \text{ (G/cm)} \]

\[ r \text{ (cm)} \]
DIFFERENT STEEL

- $\mu_{\text{max}} = 2200$
- $\mu_{\text{max}} = 1500$

$\frac{\text{dB}}{\text{dr}} \text{ (G/cm)}$

$r \text{ (cm)}$

$\mu$
Superconducting "Panofsky" Quadrupoles

Leigh H. Harwood
Cyclotron Laboratory,
East Lansing, MI 48824

A design for a rectangular aperture quadrupole magnet without pole-tips was introduced by Hand and Panofsky in 1959 (1). This design was quite radical but simple to construct. Few magnets of this design were ever built because of the large power needed. With the advent of superconducting coils there has been a renewed interest in them. The mathematical basis, field characteristics, and present and future construction of these magnets will be described below.

Mathematical basis: The P.Q. design is shown in fig 1. That a perfect quadrupole field can be produced with so simple a design is initially quite surprising. However, the P.Q. is a member of a family of magnet designs called "current sheet magnets" by Ikegami (2). These magnets are based on a cavity in a shell of infinite permeability iron which is lined with an infinitesimal current sheet. If $\phi$ is the magnetostatic potential for the problem and $j$ is the linear current density, then (2) $j = -\frac{2\phi}{2s}$ where $\frac{2\phi}{2s}$ is the derivative of the potential tangent to the surface. For a 2N-pole field, $\phi = A_{2N} r^N \sin(N\theta)$. Thus, $j = -NA_{2N} r^{N-1} \sin(\alpha + (N-1)\theta)$ where $\alpha$ is the angle between the tangent to the surface and the x-axis.

Consider a rectangular shell. If we substitute $N=2$ (quadrupole) and $\alpha = \frac{\pi}{2}$ (a vertical surface), we get

$$j = NA_{2N} r \cos(\theta) = \beta x$$

where $\beta$ is a constant. For this
vertical surface \( x = a = \text{constant} \); therefore \( j = \text{constant} \) and we get the P.Q. configuration. If on the other hand we have a cylindrical shell, then \( r = \text{constant} \) and \( \alpha = \theta + \frac{\pi}{2} \). Thus, \( j = C_{2N} \cos (N\theta) \) where \( C_{2N} = \text{constant} \). This is the familiar "\( \cos N\theta \)" multipole design. The interesting fact is that the two "competing" designs for superconducting quadrupoles are mathematically similar.

The "current sheet magnet" concept can be applied to any multipole field and any arbitrary iron shape. This will be addressed later in this discussion.

Advantages and disadvantages of the P.Q.

As stated above, the primary problem with the P.Q. is power consumption. This is due to an increased in needed ampere-turns. For a given field gradient \( B' \) (gauss/inch), a conventional quadrupole needs \( 8 \ B'R^2 \) ampere-turns (1) for an aperture with a radius of \( R \) inches. A P.Q. needs \( 4B'(LW + \frac{a}{2}) \) ampere-turns (1) for the same gradient where \( L \) and \( W \) are the full-aperture dimensions (in inches) and \( a \) is the total cross-sectional area of windings. If we set \( L = W = 2R \), i.e., a square aperture with radius \( R \), and assume the coil area is negligible, then a P.Q. requires \( 16 \ B'R^2 \) ampere-turns which is twice that for a conventional magnet. If the same current is used in both types of magnet then the P.Q. needs twice as many turns, thereby doubling the coil's resistance and doubling its power loss. Of course holding the number of turns constant and doubling the current would require four times the power for the P.Q. as required for a conventional magnet with all other
parameters equal. With superconducting coils, power losses become unimportant.

The higher current densities available with superconducting cable also decrease the "wasted" aperture. Without power losses as a problem and the small coil size, it is possible to conveniently increase the ampere-turns to get large "pole-tip" fields (eg. 20 kG) which yield large gradients (eg 10 kG/in). Table 1 lists the harmonic analysis coefficients calculated with POISSON for a P.Q. This field is quite good. Since a conventional magnet relies on the iron to shape the field, field quality for a pole-tip magnet would suffer noticeably if it were pushed to the 20 kG region.

Mechanically, the P.Q. has advantages and disadvantages. A major advantage is the rectangular cross-section of all components. These components are considerably easier to produce accurately than the sophisticated surfaces (circular or, preferably, hyperbolic) needed for high-quality "pole-tip" magnets. This accuracy cannot be wasted, however. Since the field shape is dictated quite sensitively by the current distribution, careful manufacturing and positioning of the coils is essential. Likewise are the accurate machining and assembly of the iron return path important. The sensitivity of the field quality to the construction will be discussed below.

An additional disadvantage/advantage of the superconducting P.Q. lies in the proximity of the coils and iron which necessitates
using "cold" (i.e., 4.5°K) iron. This greatly increases the cool-down time because the large iron mass must be cooled. The iron does act as a large heat sink in case of small-scale quenches and helps stop their propagation.

A rather subtle advantage to the P.Q. over pole-tip magnets is possible due to the usual construction of a P.Q. Instead of making a side of a P.Q. from a single coil, it is usually broken into at least four parts. The return path for each part is on a fall 90° from the first face. This is illustrated in fig 1. By energizing the coils on a given face differently, it is possible to introduce sizeable octupole or dodecapole components into the field. If regions 1 and 3 are one coil and 2 and 4 another, then unbalancing the two currents produces an octupole field of good quality. Table 2 lists the harmonic components of the field for this case. (Please note that the current in coil 1-3 is the negative of the one in 2-4. Thus these calculations show only the field due to the imbalance. The main quadrupole field would be produced by adding the same current to 1-3 and 2-4.) If one ties 1 to 4 and 2 to 3, a dodecapole field is produced as shown in table 3 (The quadrupole component would simply add or subtract from the main quadrupole field.) Thus one can fairly simply use the P.Q. as a multipole magnet with tuneable components.

Built or planned magnets - The number of documented P.Q. magnets has been small. The Stanford magnet (1) was the first. Brookhaven built a superconducting P.Q. in the late
1960's (3) but did not pursue it. P.Q. magnets were also built and used at KEK in Japan (4,5). A P.Q. was also used at the ZGS at Argonne National Lab (6).

The most recent magnet to be built was for a beam-line at Saturne II (7). This magnet was quite well documented and its builder's experiences prove to be informative. They are summarized below. Required field quality was specified as an integral (longitudinal) gradient homogeneity of $3 \times 10^{-3}$. They found that a shift of a coil by 0.1 mm would produce this level of effect; thus placement of the coils was shown to be critical. The magnet showed dramatic gradient homogeneity changes as the excitation was changed. The conclusion was that these were saturation effects due to small air gaps left in the iron yoke during assembly. Measurements showed that the magnet had integral nonhomogeneities of typically $10^{-2}$ as it was initially assembled. Careful reassembly improved the homogeneity to $\leq 5 \times 10^{-3}$. By turning on a small adjustment current in one of the four coils on each corner, the homogeneity improved to $\leq 10^{-3}$.

The Saturn II magnet paid careful attention to the fringe field effects. The coils were wound so as to minimize the contribution of the ends of the coil to the useful aperture field. Iron shims were also used to help the fringing fields. The final positions of these shims were determined at room temperature by running a small current through the coil, doing field maps and repositioning the shims. The builders point out that the room temperature measurements were sufficient to accurately position the shims for higher fields.

NSCL plans to use many superconducting magnets. The
two main cyclotron magnets being the first. Plans call for all beam line magnets to be superconducting; all new experimental apparatus is to use superconducting magnets as well. The choice of superconducting magnets had several reasons. For the beam-line magnets, operating cost made superconducting quadrupoles imperative. The large spectrograph (see the report by A. Zeller at this workshop) will use two superconducting P.Q. magnets. The first one is high field (2T "pole-tip" field) magnet and the second must accommodate a short, wide beam (approximately 2:1 aspect ratio). This is the situation for which the P.Q. was first intended (1). As described earlier, the P.Q. design allows us to conveniently introduce variable multipole components into the quadrupoles for the spectrograph and thereby make the focusing for the spectrograph more flexible.

Conclusion "Panofsky" quadrupoles have come into their own with the availability of superconducting coils. Their simple design and high field gradients make them advantageous for many application. Any laboratory which is considering new quadrupoles should seriously consider their use, especially if they already have superconducting technology in the lab.
References

3) W. Sampson, Brookhaven National Laboratory, a private communication.
6) R. Pardo, Argonne National Laboratory, private communication.
### Table 1

**Harmonic Component of a "Panofsky" Quadrupole**

<table>
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<tr>
<th>N (2)</th>
<th>$B_n/B_2$ (3)</th>
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<tr>
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<tr>
<td>10</td>
<td>$-6 \times 10^{-4}$</td>
</tr>
<tr>
<td>14</td>
<td>$-1 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

1) NSCL S800 Q1, 4 in. radius  
2) $B(R) = \sum_{N} B_N (R/3.5 \text{ in.})$  
3) $B_2 = 1.75 \times 10^4 \text{ G}$

### Table 2

**Octupole Perturbation to a "Panofsky" Quadrupole**

<table>
<thead>
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<th>N (1)</th>
<th>$B_N/B_4$</th>
</tr>
</thead>
<tbody>
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<td>$-3 \times 10^{-5}$</td>
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<tr>
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<tr>
<td>6</td>
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<td>10</td>
<td>$-1 \times 10^{-3}$</td>
</tr>
<tr>
<td>12</td>
<td>$-6 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

1) $B(R) = \sum_{N} B_N (R/4 \text{ in.})^{N-1}$
### Table 3

Dodecapole Perturbation to a "Panofsky" Quadrupole

<table>
<thead>
<tr>
<th>(N^{(1)})</th>
<th>(B_N/B_2)</th>
</tr>
</thead>
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<td>(-3\times10^{-2})</td>
</tr>
<tr>
<td>12</td>
<td>(-3\times10^{-4})</td>
</tr>
</tbody>
</table>

1) \(B(R) = \sum_{N} B_N \left(\frac{R}{4 \text{ in.}}\right)^{N-1}\)

---

**Figure Caption**

1) Illustration of a quarter of a square aperature Panofsky quadrupole.
Phase Space Rotation with Solenoids and Quadrupoles*

J.A. Nolen, Jr.
National Superconducting Cyclotron Laboratory
Michigan State University
East Lansing, MI 48824

ABSTRACT

In this report a standard five-quadrupole phase-space rotation system is discussed and compared with a possible alternative - two superconducting solenoids which accomplish the same job in a different way. In some laboratories the solenoid system may be advantageous.

*Supported by the National Science Foundation-Grant No. PHY 78-22696 and Department of Energy Grant No. DOE-DEAC-02-80-ER-10579.
Several laboratories which have high resolution energy-loss spectrograph systems currently use a five-quadrupole phase space rotation system, e.g. MIT-Bates and LAMPF in the United States. In these particular cases the phase space rotation is between the beam dispersion system and the spectrograph. The x and y phase spaces must be interchanged because the dispersion of the beam is in the horizontal plane whereas the spectrograph dispersion is in the vertical plane.

Five quadrupoles are the minimum which can be used to interchange x and y phase space and also be optically equivalent to a drift of the same length as the quadrupole system. To mix x and y phase spaces the quadrupoles are oriented with their x and y axes at 45° to those of the laboratory. A solution can be obtained with a symmetric quadrupole system of the form: 

\[ A_y d_1 B_x d_2 C_y d_2 B_x d_1 A_y, \]

where \( A_y, B_x, \) and \( C_y \) represent quadrupole strength while \( d_1 \) and \( d_2 \) are drifts. Sample TRANSPORT input and output for a 1.6 GeV/c system are given in table I. The beam envelopes for this calculation are shown in figure 1. Initially the beam was large in x with a small \( \theta \) divergence and small in y with a large \( \phi \) divergence. It is easy to see from figure 1 that the quadrupoles have interchanged these phase spaces. This is also displayed by the final transfer matrix given in table I. It is equivalent to the negative unit matrix times a drift of length 7 meters except that the final x and \( \theta \) coordinates are determined by the initial y and \( \phi \), and vice versa. In the TRANSPORT input of table I the +45° and -45° rotations before and after each quadrupole are only used so that the beam envelope coordinates
and transform matrices through the system are given relative to the laboratory x and y axes.

The constraint that the "grad drift-twister" illustrated above be optically equivalent to a drift is convenient for tuning purposes. The beam line focussing elements can first be tuned with the drift-twister turned off. Then energizing the drift-twister should interchange x and y coordinates at the images, but not change the focussing. This has the additional advantage that the phase space rotation is optional, i.e. it can be turned on or off without affecting the imaging, otherwise.

The disadvantages of the "drift-twister" system for some applications is that it is inherently a rather long system, 6 m from the entrance of the first quad to the exit of the fifth in this example at 1.6 GeV/c. The length can be reduced only by using shorter quads with larger gradients (for fixed beam rigidity). (The present solution is a scaled up version of a solution used by Karl Brown at lower momentum). This system seems particularly long when one realizes that all focussing elements must be outside this 6 m drift region. If the beam phase space is not very small such a long drift may lead to quite large beam envelopes in the system. In the present example, with a beam full width of 1 mm and full divergence of 5 mr, the envelope is nearly 4 cm wide by the end of the drift.
An alternative to the five quad drift-twister is a two solenoid "unit matrix twister". Solenoids have been used as focussing elements for many years, especially in low energy electron optics where room temperature magnets are adequate. Superconducting solenoids have been used in higher energy applications and as for the muon channel at SIN, but to my knowledge have not been used to replace quadrupoles on accelerator beam lines at momenta above 100 MeV/c. This is generally true because for normal apertures quadrupoles are stronger focussing elements than solenoids:

\[
\frac{1}{f_{QD}} \approx \frac{(B/a)^2 L_Q^2 L_d}{[Bp]^2} \quad m^{-1}
\]

whereas

\[
\frac{1}{f_S} \approx \frac{B^2 L}{4 [Bp]^2} \quad m^{-1}
\]

where \( f_{QD} \) and \( f_S \) are the focal length of a quadrupole doublet and a solenoid, respectively, \( (B/a) \) is the quadrupole gradient in Tesla per meter, \( L_Q, L_d, \) and \( L \) are the lengths in meters, of the quadrupole singlets, their separation, and the solenoid and \([Bp]\) is the magnetic rigidity of the charged particle in Tesla-meters. A quadrupole doublet with pole tip field \( B = 1 \) T and aperture radius \( a = 0.05 \) m, \( L_Q = 0.4 \) m, \( L_d = 0.2 \) m, and \([Bp] = 5 \) T-m, the focal length is:

\[
f_{QD} \sim 2. \text{ m}
\]

For a solenoid of length \( L = 1 \) m to have the same focal length at the same rigidity requires \( B \sim 7 \) T.
If, however, the quadrupoles were either shorter in length or larger in aperture the comparison would be more favorable for the solenoid. This is because the quadrupole doublet focal length scales as $L^{-3}$ and $a^2$, whereas the solenoid focal length scales as $L^{-1}$ and is independent of aperture radius.

The feature of solenoids which makes them potentially useful for phase space rotation is that the charged particles spiral around the central axis as they are focussed. There is a rotation of transverse coordinates by an amount:

$$\theta = \frac{\pi}{2} \frac{BL}{\rho} [B\rho]$$

The direction of rotation depends on the sign of the magnetic field, B, while the focussing depends only on the magnitude of the field. Hence, a symmetric two solenoid system can be used to perform point-to-point imaging as well as phase space rotation. If the currents in the solenoids are of the same sign phase space is rotated, as in figure 2, whereas if the currents are opposite, phase space is not rotated, as in figure 3. In either case, with this symmetric geometry, the overall transform is the negative unit matrix, i.e. point-to-point and parallel-to-parallel with unit magnification.

For a numerical example of the sizes of the required solenoids consider a phase space rotation angle $\theta = \pi/2$ at a beam momentum of 1.6 Gev/c, which is a rigidity of 5.3 T-m. From the equations above for phase space rotation we have for $\theta = \pi/2$: 
III. E-6

\[ BL = \pi [B \rho] = 16.8 \text{ T-m} \]

total for both solenoids, or \( BL = 8.4 \text{ T-m per solenoid.} \) For NbTi superconductor a reasonably economical upper limit for the operating field is 7T. Therefore, \( L = 1.2 \text{ m per solenoid,} \) and in the focal length equation \( B^2 L = 58.9 \text{ T}^2 \text{-m,} \) or

\[ \frac{1}{f_s} \sim \frac{B^2 L}{4[B \rho]^2} \sim 0.52 \text{ m}^{-1}. \]

This gives an overall system length \( l \sim 4f \sim 8m. \) The TRANSPORT calculations shown in figures 2 and 3 give the more accurate total length of 8.4 m with two 1.2 m long solenoids operating at 7T fields for \( p/q = 1.6 \text{ GeV/c.} \)

The solenoid unit matrix-twister (figure 2) has three potential advantages over the quad drift-twisters. (figure 1). Firstly, the quadrupole system may actually require more floor space (length) by the time the focussing elements are added to the "drift" section illustrated in figure 1. Secondly, the beam envelope is kept much smaller in the solenoid system because it is a focussing system as opposed to a drift. And thirdly, the solenoid system employs only two identical elements (running at equal currents) whereas the quadrupole system requires a minimum of 7 or possibly 9 quadrupoles to do the equivalent combination of focussing, plus phase space rotation. Hence, tuning and adjustment problems are potentially more troublesome with the quadrupole system.
A cost comparison of the two options for MSU is in progress. In our case the two superconducting solenoids would replace nine superconducting Panofsky quadrupoles. In the high field solenoids most of the cost goes into superconductor, whereas, in the quadrupoles it is dominated by the cryostats and coil winding.

As higher field superconductors become practical and more economical solenoids as focusing elements and phase space rotators will tend to become even more attractive alternatives to quadrupoles.
Table I. The input parameters for a TRANSPORT calculation of a five quadrupole drift-twister. The resulting transfer matrix and beam envelope are also given.

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*TRANSFORM 1*

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1. Beam envelopes calculated by the program TRANSPORT for a five quadrupole drift-twister. The input parameters and transfer matrix for this calculation are given in Table I.
2. Beam envelopes from a TRANSPORT calculation of a two solenoid unit matrix twister. The solenoids are each 1.2 m long with central field strengths of 7.7 T. At a beam momentum of 1.6 GeV/c this system produces a 90° phase space rotation plus focusing as a negative unit matrix, i.e. point-to-point and parallel-to-parallel.
3. Beam envelopes calculated for the same two solenoid system as in figure two, except with the current reversed in the second solenoid. In this case the net phase space rotation is zero, but the focussing is still the negative unit matrix. Note that the beam envelopes in figures 2 and 3 remain significantly smaller than in figure 1.
Magnet Construction - Conventional
Harald A. Enge

1. General procedure in spectrograph design.

First- and second-order design are first done with TRANS-PORT and later checked and adjusted with RAYTRACE. The adjustments are generally about 0.5% in quad strength and <1 degree on shim angles. Second-order adjustment (sextupoles or radii of boundary) generally less than 5 percent. Method for higher order corrections described in next talk. Next step: Lay-out drawing to check if drift distances are adequate, etc., i.e., is there room for coils, yoke, etc.?

2. Some design details.

Fig. 1a. Pole profile where EFB lines up with pole root. This simplifies the work of the magnet engineer. Fig. 1b - same with field clamp, Fig. 1c - same with adjustable insert, and Fig. 2 with flexible "snake." The clamp, insert, and snake can all be used to provide vernier adjustments of the shape of the EFB, either in the design stage or after initial testing. In the latter case the clamps or the inserts have to be remachined, the snake adjusted by a series of bolts.

Figs. 1 and 2 show homogenizer gaps (or Purcell filters) which serve two functions. The first is to attenuate the effects of the reluctance in the yoke - in particular for magnets with large pole areas. The second is to mechanically decouple the
pole pieces from the yoke. In a horizontal spectrograph the lower pole piece is supported on the bottom yoke by nonmagnetic spacers and the upper pole piece is supported on the lower by spacers. There is no spacer between the upper pole piece and the top yoke. Deflection of the yokes by magnetic forces will then have a minimum effect on the homogeneity of the field in the main gap. The homogenizer gap will be distorted, but this is much less serious than a nonuniformity of the main gap. It is not recommended to use homogenizer gaps for magnet designed for nonuniform fields.

3. **Dynamic adjustments in dipoles.**
   a) Surface windings. For small adjustments.
   b) Halbach's HT windings. Imbedded in poles from the back side, but relatively close to the pole surface to set up gradients in the tangential direction.

4. **Multipoles with dynamic, independent adjustments.**
   Used in Q3D series of spectrographs. Different execution on Osaka QDDQ spectrograph. Fig. 3 shows the Osaka version. It is a superposition of Panofsky multipoles.

5. **Combining quadrupoles and higher-order multipole.**
   The subroutine POLES in RAYTRACE allows for a superposition of quadrupole, sextupole, etc., fields. The strength is given as the field, $B_Q$ for instance, at the pole tip of the element of aperture radius $R$ if it were acting alone. If the higher order multipoles are weak compared to the quadrupole, the element can be
Magnet Construction - Conventional
Harald A. Enge

designed basically as a quadrupole with distorted poles. Fig. 4 shows a somewhat extreme example where also the amperturns on the two sets of poles are different. At nominal aperture R=10 cm the multipole field components are $B_Q = 8.40 \text{ kG}$, $B_H = -3.4 \text{ kG}$, $B_O = 1.56 \text{ kG}$, and $B_D = 0.60 \text{ kG}$. Kowalski will talk more about these techniques.
Fig. 2

MEDIAN PLANE

BY

MEDIAN PLANE

Pole

YOKE

COIL

SNAKE

30° 75°

30°

0.1%

1%

10%

2 Gaps

EFB 1

EFB 2

B_y/B_0

0

0.2

0.4

0.6

0.8

1.0

Fig. 2
Fig. 4
HOMOGENIZING THE FIELD IN A PICTURE-FRAME MAGNET

P. Debenham
National Bureau of Standards
Washington, D.C. 20234

Traditionally, most magnetic spectrometers have been H-magnets, i.e., magnets with H-shaped air gaps. An advantage of this design is the presence of the pole pieces which separate the two sides of the H. These may be designed to homogenize the field in the air gap. A disadvantage of the H-magnet is that the pole corners produce a region of nonuniform field. Consequently, the poles must be wider than the beam by several gap heights, and the spectrometer must be larger than desired. The picture-frame magnet design, as shown in Figure 1, avoids the size problem by virtue of having no pole corners and can result in a more economical magnet. It has not been used extensively in spectrometers because of its very simplicity - it lacks the design flexibility required to produce a very homogeneous field. The present work is a study of how to modify such a magnet for homogeneous field.

Two-dimensional magnetic field calculations have been performed using the computer program TRIM for the magnet shown in Figure 1. For expediency, rectangular rather than cylindrical symmetry was assumed for the calculation. The calculated magnetic field on the midplane of this magnet is shown in Figure 2 for a range of field strengths typical of a spectrometer. The curve labelled $\mu = \infty$ in Figure 2 is the result of a calculation for which the iron is assumed to have infinite permeability. This calculation, which ideally would yield a uniform field, yields
field inhomogeneities of $1 \times 10^{-5} B_0$. This gives a measure of the error in the calculation. It is important to perform a $\mu = \infty$ calculation at the start of any new magnetostatic problem in order to select a grid which will provide an acceptable level of error. For the remaining curves in Figure 2 the iron was assumed to have the magnetization curve of an annealed, low-carbon steel such as AISI 1010. The non-uniform component of the field is a sextupole with an amplitude of $4.5 \times 10^{-4} B_0$ at $x = 35$ cm, the largest $x$-value at which the calculation is reliable, almost independent of $B_0$. In a spectrometer magnet, a pure sextupole component of this magnitude can be handled in several ways. One possibility is to include the effect of the sextupole component in software focal plane corrections. A second possibility is to homogenize the field using pole face correction coils of the printed-circuit-board variety.\textsuperscript{2} Since the error field shape is independent of field strength, a single, low-power coil with appropriate sextupole windings would suffice.

Another way to homogenize the field is to change the magnet design. In a picture-frame magnet there is a non-zero angle, $\theta$, between the field lines in the pole iron and the normal to the pole surface (see Figure 3). It can be shown\textsuperscript{3} that the sextupole field is proportional to $\tan \theta$. In a spectrometer magnet, which must typically operate over a range of field values and with the iron approaching saturation, it is desirable to homogenize the field (i.e. set $\theta = 0$ across the entire aperture) without resorting to shims in the gap. A magnet design which meets this requirement was developed for the end magnets of the NBS-LANL racetrack microtron.\textsuperscript{4} It can be adapted to a picture-frame magnet as illustrated in Figure 4. The design includes two homogenizing air gaps, or Purcell filters,\textsuperscript{5} of width $a$ and the same length as the aperture ($2w$),...
parallel to the main gap. The parallel homogenizing gaps are connected to the main gap by inclined homogenizing gaps of width b above and below the coils. Dimension b is chosen to equalize the magnetic vector potential in the main and homogenizing gaps at x = w, i.e., at the end of the aperture. Since the vector potential is necessarily the same on the symmetry axis at x = 0, the potential will then be equal in the main and homogenizing gaps at all x, and any field line will intersect both gaps at the same x. This sets $\phi = 0$. An approximate expression for the required value of b can be derived for $\mu = \mu_0$ using the orthogonal analog model.\(^3\) In this derivation the homogenizing gap and main half-gap g are treated as two parallel conducting paths, as discussed in reference 6, and the resistance of the inclined gap is adjusted to equalize the voltage in the two paths at x = w. The result is

$$b = \frac{2ag(h + \delta)}{w(g - a)}$$

For finite $\mu$, a value of b slightly different from this expression is required to achieve $\phi = 0$ in a TRIM calculation with a realistic magnetization curve for the iron. The pole thickness, t, is chosen to be approximately half the coil width, c. To minimize coil power, a is chosen as small as possible consistent with the considerations given in reference 5 and with an expression for the field non-uniformity that is caused by the expected error, $\Delta a$, in the homogenizing gap height:

$$\frac{\Delta B}{B} = \frac{x^2 \Delta a}{6G uta}$$
III. G-4

The effect of the homogenizing gaps on the magnetic flux pattern at 1.3 Tesla may be seen in Figure 5. This magnet has the same aperture and external dimensions as that shown in Figure 1. The angle of the field lines in the iron above the homogenizing gap is similar to that in Figure 3, but the field lines are almost perpendicular to the main gap surface. The only deviation from perpendicularity occurs near the edge of the aperture where the field lines in the pole are curved. The curvature, which is due to the nonlinear magnetization curve of iron, is more pronounced at 1.6 Tesla, as shown in Figure 6.

The midplane magnetic field calculated for the filtered picture-frame magnet (see Figure 7) is uniform to within the accuracy of the calculation ($\pm 2 \times 10^{-5}$) over 88% of the aperture and over a magnetic field range of 0.5 Tesla to 1.4 Tesla. Above 1.4 Tesla the effects of nonlinear magnetization in the pole iron begin to appear. The non-uniform field component is $1 \times 10^{-4} B_o$ at 1.5 Tesla, and by 1.6 Tesla it has increased to $4 \times 10^{-4} B_o$, the same as in the picture-frame magnet. Nevertheless, for fields up to 1.4 Tesla the Purcell filters produce a field uniformity of $\pm 2 \times 10^{-5}$ or better.

In a picture-frame magnet, the magnetic forces distort the yoke pieces and produce a non-uniform air gap with a non-uniform magnetic field. The filtered design provides a way to alleviate this effect, as pointed out in reference 5. If the pole pieces are mechanically isolated from the yokes, distortion in the yoke is not transmitted to the poles. There is no net magnetic force on the poles in the region above and below the aperture since the field strength is equal on both sides of the poles. The only residual forces which can deform the poles
are then both relatively small: a net vertical magnetic force in the coil region, which can be supported with gap spacers, and the gravitational forces.

In summary, we have seen that a picture-frame spectrometer magnet can produce fields in the range 0.5 Tesla to 1.6 Tesla having a sextupole component of magnitude $5 \times 10^{-4} \, T$. The sextupole component can be compensated either with software focal plane corrections or with low-power, pole-face correction coils. We have also seen that by introducing homogenizing gaps into the picture-frame design we can improve the field uniformity in the range 0.5 Tesla to 1.4 Tesla to $\pm 2 \times 10^{-5} \, T$ without using correction coils. An additional advantage of the latter design is the possibility of reducing magnetic distortions of the gap.
III. G-6

References

Figure Captions

1. Picture frame magnet with appropriate dimensions for a 1 GeV/c spectrometer.

2. Calculated midplane field for the magnet of Figure 1. Relative field variation is plotted versus distance from the center of the gap.

3. Calculated flux distribution in a quadrant of the magnet of Figure 1. The field strength in the air gap is 1.3 Tesla.


5. Calculated flux distribution in a magnet of the type shown in Figure 4 at 1.3 Tesla.

6. Flux distribution corresponding to Figure 4 at 1.6 Tesla.

7. Calculated midplane field for a magnet of the type shown in Figure 4.
\[ B(x) / B(0) \times 10^4 \]

Legend:
- 0.5 Tesla
- 1.0 Tesla
- 1.4 Tesla
- 1.5 Tesla

Shaded area represents the magnetic field distribution.
UNDERSTANDING THE PURCELL FILTER

P. Debenham
National Bureau of Standards
Washington, D.C. 20234

A Purcell filter (or homogenizing gap) is an extra air gap which separates the pole piece of a magnet from the yoke in order to make the magnetic field in the main air gap more uniform. Enge has explained how Purcell filters work in terms of magnetic circuits\(^1\) and has used them with success in a variety of magnets. On the other hand, statements have been made at this workshop about magnets which have Purcell filters but turn out not to have as good field uniformity as expected. I would like to show that the effectiveness of Purcell filters is not automatic but depends on the pole-end and coil geometry.

A quarter section of an H-magnet with a Purcell filter is shown in Figure 1a. The Purcell filter has height \(a\), and the main gap half-height is \(b\). The field in the main gap is uniform when the field lines in the pole iron are perpendicular to the gap surface. Since field lines are lines of constant magnetic potential, \(A_z\), the preceding condition for uniform field is met if \(A_z\) has the same value in both gaps for every value of \(x\). \(A_z\) is constant on the symmetry axis at \(x = 0\). If in addition the value of \(A_z\) is the same in the two gaps for any other value of \(x\), which we can call \(x_u\), in the uniform gap range, then the condition holds for all \(x\)-values in the uniform gap range. The problem of homogenizing the field is thus reduced to achieving the same value of the vector potential in the two gaps for some \(x\)-value, \(x_u\).

An enlightening guide for solving this problem is the Orthogonal Analog Model, or OAM. I am indebted to Klaus Halbach for pointing out
the usefulness of this method. The basis for the OAM, as explained in reference 2, is the fact that the magnetic and electric Maxwell's equations have the same form. The voltage, $V$, in the electric equations takes the role of $A_z$ in the magnetic equations. In the OAM, the steel in the magnetic problem is replaced with an insulator, and the air is replaced with a conducting sheet. The conducting sheet can be cut to a shape that produces the desired voltage pattern. In the present case, however, we will use the OAM only as a thinking tool to gain insight into the solution of the magnet design problem.

The analog for the H-magnet is shown in Figure 1b. Current is injected into the conducting sheet in the coil region and flows through two current paths (corresponding to the two air gaps) to a ground at the $y$-axis. In order for the voltage to be the same on both current paths at $x_1$, the ratio of resistances in the two paths between $x_1$ and the coil at $x_2$ must be the same as the ratio of resistances in the two paths between 0 and $x_1$. That is,

$$\frac{R_a(x_1,x_2)}{R_b(x_1,x_2)} = \frac{R_a(0,x_1)}{R_b(0,x_1)} = \frac{b}{a}.$$

Returning to the magnetic problem, we have found the condition for a uniform magnetic field. The coil position and the profile of the pole end which faces the coil must be designed to achieve the above resistance ratio in the analog model. If they are not, the main gap field will not be uniform despite the Purcell filter. This accounts for the observation that Purcell filters do not always homogenize the field.
III. H-3

References


Figure Captions

1a. Quarter section of a two-dimensional H-magnet with a Purcell filter. The magnet is symmetric about the x- and y-axes.

1b. Orthogonal Analog Model of the magnet in Figure 1a.
Figure 1a.

Yoke
$\mu = \infty$

Pole
$\mu = \infty$

Insulator
$\varepsilon = \infty$

Conductor $\varepsilon = 1$

Figure 1b.

Insulator
$\varepsilon = \infty$

Insulator
$\varepsilon = \infty$

Conductor $\varepsilon = 1$
The harmonic content of magnets such as the standard PEP bend is (among other things, a function of excitation current, the way the current is set and even the magnetization history. For instance, harmonic strengths generally vary not only with the magnitude of the current but the direction and rate at which the current is approached and set. The field distribution resulting from different procedures can vary markedly depending on both the mechanical and magnetic design and the degree to which eddy current effects are emphasized. Variations among magnets of the same design result from variations in the iron (e.g. its chemistry, lamination fabrication and treatment procedures etc.) as well as overall magnet fabrication procedures (e.g. from unobtainable tolerances on important assembly steps). Because the field distribution may also depend on the previous history of a magnet, all PEP dipoles were subjected to what are called "magnetization" and "standardization" cycles before measurement - the latter depending on the former and intended to set the initial conditions of the magnet to a reproducible standard\(^{(1)}\). The primary goal of the magnetic measurements was then to determine the dipole strength as a function of current for each magnet based on a practical setting algorithm. The main constraints on the algorithm were reproducibility of the integrated field\(\int B_y \, dz\), speed, power and reduction of higher harmonics. Quadrupole and sextupole strengths were also measured on about one-half of the magnets at one current \((I = 1250A/\mathcal{E} = 17.43\text{ GeV})\). This note presents the data and discusses it from the viewpoint of subsequent measurements with stored beams. The most important (and obvious) conclusion is that failure to distribute laminations according to heat number and/or strike number results in "magnetic personalities" among the magnets which are quite difficult to deal with afterwards.

\(^{(1)}\) Because of the comparatively low field desired in these magnets \((B \ll B_{\text{sat}})\), to keep the radiated power down, it is not possible to significantly saturate them. This is not a problem so long as the magnetizing current is never exceeded \((I_{\text{mag}} = 2000A)\).
Bead Magnet Description

Figure 1 shows a schematic of the bottom half of a PEP bend, sliced through the median plane relative to the central design trajectory. Each magnet is formed from 16 gauge (1.59 mm thick), annealed and decarburized steel laminations (carbon content < 0.01%). The magnet has a rectangular pole with a nominal gap height of \( G = 70 \text{ mm} \) and a mechanical design gradient across the gap of \( \frac{dG}{dx} = +0.53 \pm 0.05 \text{ mr} \) — intended to compensate the longer flux path through the lamination iron with decreasing values of \( x \).

This is illustrated in Fig. 2 which shows the upper half of the magnet cross section, sliced through the \( x-y \) plane, as generated\(^{(2)}\) with POISSON. Since the magnet is rectangular and long in the \( z \)-direction (\( L/G > 76 \)), such 2-dimensional calculations should provide a good representation for a magnet whose steel conforms to the assumed permeability table except for residual field effects related to the coercive force \( (H_c) \) of the steel\(^{(3)}\), eddy current effects associated with setting the magnet and mechanical distortions such as those associated with the mutual attraction of the poles or from the support members. Such distortion effects can easily nulify the predictions of the magnetic design calculations. While H-type magnets would have been preferable on the above points as well as their lower power and capital costs, the C-magnet allowed better vacuum chamber access and therefore schedule flexibility.

A total of 194 of these magnets were mechanically and magnetically measured prior to installation. Based on the measurements, the 192 magnets installed in the ring were divided into 3-classes numbering 120, 48 and 24 depending on whether their trim windings were to be excited and whether they were also near the vertical steering magnets in cells 4 and 8.

\(^{(2)}\) R. Early and G. Fischer.

\(^{(3)}\) These become increasingly important with decreasing field strengths and are difficult to control. We will attempt to use the code PANDIRA to study them.
Bend Measurement Techniques

The magnetic induction integral, $\int(B_y(x,y,z)_{\text{coil}} \, dz)$, was measured along lines parallel to the mechanical centerline of the magnet (shown as the straight dashed line in Fig. 1 and $x = y = 0$ in Fig. 2) using a rotating, long, pancake-coil (10-turns) having a rotation rate of $\sim 2$Hz and a wire radius of 2.352 cm about the rotation axis. Because three magnets were measured simultaneously, the different coils were cross-calibrated against one another in the same magnet and field as well as against a point-by-point integral obtained using a Hall-probe and NMR on a calibrated lead-screw. An ideal measurement coil of this form is sensitive to only the symmetry-allowed harmonics of a bend e.g. the dipole, sextupole and successive even $n$ ($2n + 2$ number of physical poles) harmonics. For a sufficiently long coil in a field having no variation of $B_y(x,y,z)$ with $x$ (the direction parallel to the rectangular pole edge) and symmetry about the median plane ($y = 0$), it follows that the coil measures only

$$\int B_y(x,y,z)dz = \int B_y(0,0,z)dz.$$ (1)

Thus, assuming the magnet is magnetically well designed and doesn't distort pathologically, one expects to measure only the integral of the vertical dipole component along the rotation axis of the coil whenever it is near the mechanical centerline. The accuracy of measurements for any one magnet and the rms variation over all magnets depends on many factors, however, none were found to be $>1:30,000$ for the final measuring system used to measure all magnets. The best test was to remeasure bend #15 after 10 months of measuring more than 140 additional magnets (up to bend #156) with essentially the same results over the excitation range from 5.0 - 25.6 GeV of $(\Delta BL/BL)< 1:10,000$ for all measurements.

(4) The coils were straight with lengths $L = 6219$ mm and $(L-L)/2G > 6$. The coil centerline had a range of travel of $\leq \pm 4.5$ cm about the mechanical centerline of the magnet. $\Delta B/B < 0.2\%$ over this range.
Dipole Effects

The primary purpose of the PEP bend measurements was to determine the dipole integral of each magnet on a fixed mesh of currents\(^{(5)}\) which would allow setting the ring to any energy for injection or ramping a stored beam up or down in energy. Errors in the dipole integral for individual magnets are understood as deviations from the mean value. Representative results are given in Table I and shown in Fig. 3. Whereas the relative current stability of the power supply improved with increasing currents the field integral spread between magnets increases as the permeability begins to decrease above 1000A. An attempt was made to assign magnets into Class 1 in such a way as to minimize its rms spread since these magnets would not have supplies for exciting their backleg trims.

Assuming no significant saturation or deformation of the steel with excitation current, one would expect the distribution of Fig. 3 to follow the distribution of mechanical gap heights, measured at zero excitation, so long as the steel was uniform and the lengths were the same because

\[
\int B_y(x, o, z) \, dz = \left(\frac{\text{Eff}}{G}\right) \left[-\frac{4\pi}{10^3}\right] \Delta I + \int_{\text{iron}(x)} \left(\frac{H_c - B_0/\mu}{\text{dl}}\right)\]

\(\text{(2)}\)

where units are Gaussian (e.g. Gauss and Oersted) except for lengths and current which are MKS. The integral in the iron follows a flux tube whose shape depends on the \(x\)-value in the median plane as seen from Fig. 2. \(H_c\) varies throughout the magnet and depends on both the steel and the distribution of maximum field seen by the iron\(^{(6)}\) i.e. on the magnetization current, \(I_{\text{mag}}\).

\(\text{(5)}\) The dipole integral for each magnet was measured following standardization to \(I = 1250A\), proceeding downwards to 50A in 150A steps and then upwards to 2000A with the same step size at a rate of 5 A/s. Consequently, the ring ramping rate is \(\leq 5\) A/s or \(\leq 70\) MeV/s. A rate in excess of this will result in an apparently low energy i.e. more efficient steel.

\(\text{(6)}\) One wants magnet steel to have a low coercive force (e.g. \(|H_c| < 1\) Oe) since this reduces the area of the hysteresis loop, magnetic ageing effects and variations between magnets.
At zero excitation, measurement of the residual field integral determines the coercive term,

\[ \frac{\text{Leff}}{\langle G \rangle} M_c = \frac{\text{Leff}}{\langle G \rangle} \int_{x=0}^{H_c} H_c \, dx = -85.0 \, \text{G-m} , \tag{3} \]

ignoring variations between magnets\(^{(7)}\). Thus, at low inductions \((B_I < \mu)\), the dipole strength is dominated by the current driving term, followed by the coercive force which contributes 1.6% at 350A (or 5 GeV).

Figure 4 shows the predicted relative strengths of Fig. 3 based solely on the measured gap heights. The mechanical length of the magnet was the most easily controlled variable in Eq. 2 and is considered constant since an error of one lam only leads to an error \(\Delta k_D \approx 10 \, \mu\text{rad}\). While there is a rough correlation with the data of Fig. 3, it is clear\(^{(8)}\) that quality control over production of large numbers of such magnets based on Eq. 2 requires some distribution or "shuffling" of laminations — the extent and parameters of the distribution depending on the range and quality of fields required. This is so important and incontrovertible that it will be called Golden Rule #1.

To complete this section, a brief tabulation of some possible corrections and other effects that might influence normal ring operation is given insofar as they relate to dipole effects. These include such things as fringing field effects, earth's field, proximity to other magnets and various hysteresis effects.

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\(^{(7)}\) The variations alone are larger than the earth's unshielded field because of variations between heats of steel which were \(0.88 \leq |H_c| \leq 1.53\) Oe and difficulty in hand-mixing the available heats between magnets.

\(^{(8)}\) Significant anomalies such as the Class 2 magnet #087 were traceable to a longer length than its neighbors. Another typical example is Class 2 magnet #001 which is formed from only one heat of steel with an \(H_c \approx -1.02\) Oe whereas the reference magnet was formed from 3 heats having a range \(1.03 \leq |H_c| \leq 1.23\) Oe. The area of all hysteresis loops and residual fields \((B_r)\) of the reference were greater than for #001 whereas the dipole integral per ampere of excitation was less for currents \(100 \leq I \leq 1900\) A. The "high" field \((I \geq I_{mag})\) behavior can be attributed to the differing chemistry and treatment of the heats leading to a relative inversion of the high field permeabilities and/or "anomalous" enhancement of \(H_c\).
1) Conversion of the long-coil measurements, taken along the dashed line in Fig. 1, to the design orbit shown as solid is given by the multiplicative constant:

\[ C_1 = \frac{L_{\text{Design}}}{L_{\text{measure}}} = \frac{(\phi/2)}{\sin(\phi/2)} \approx 1 + \frac{1}{6} \left( \frac{\phi}{2} \right)^2 = 1.0000443. \]  

This correction to all measurements was not made by the magnet group.

2) The special vertical steering magnets (cells 4 and 8) are in the coil-dominated region of their adjacent bends so they act to shield the beam trajectory from a positive vertical field. The effect is essentially independent of current so that all Class 3 magnets were corrected by the multiplicative constant:

\[ C_2 = 1.0000477 \pm 0.000018. \]  

Fig. 5 of PEP-Note-345 implies proximity to other magnets is not a problem.

3) The magnets tend to shield the earth's field from the measurement coils except near the ends so no correction was made for this although the mean value measured by the three coils was found to be:

\[ \langle B_{\text{earth}} \rangle_{\text{coil}} = 0.554 \pm 0.089 \text{ G}. \]  

The earth's field complicates measurement of any zero excitation offset but since field integrals as low as 0.1 ± 0.1 G-m were measured for virgin magnets, no correction appeared necessary.

4) The effect of an extended fringing field as compared to the equivalent sharp-cutoff approximation shown in Fig. 1 is to displace the real orbit parallel to the assumed one in the field free region if they coincide in the constant field region. There is no change in strength \( k_0 \) but only

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(9) This was encouraging considering the different locations of the coils in the measurement building since it verified that the accurate cross-calibration of the coils had been maintained over the course of the measurements (July 1978 to Sept. 1979).

an alignment error of

$$\frac{\Delta x}{G} \leq (G/\rho) / \cos^2(\phi/2) = 0.000423,$$

so this is irrelevant for a pure dipole field and is also negligible compared to the alignment errors and variations between the magnetic and mechanical center lines in these magnets ($\Delta x/G > 0.02$).

5) Trim current hysteresis is shown in Fig. 5 when the magnet has $I = 1250A$ i.e. the same as for Fig. 3. The 19-turn (10 gauge) winding can produce up to $0.033 \text{ T-m} (\sim 61 \text{ G})$ which is more than sufficient to correct its own magnet's error but this can also lead to as much as $0.0034 \text{ T-m} (0.18\%)$ ambiguity which is a significant error compared to either the spreads in Fig. 3 or the injection sensitivity observed for some configurations.

6) Hysteresis effects associated with exciting the primary windings are shown in Fig. 6. Considering only the dipole effect ($x \approx 0$), it is clear that the strength varies with the rate and direction of setting as well as the number of cycles. The potential for unrepeatability from such effects leads to Golden Rule #2 even when Golden Rule #1 has been followed:

To achieve optimal repeatability and uniformity, magnets should undergo a precondition or magnetization cycle which should not be achievable by any means during normal in situ operation. All fields should be able to be set based on the same standardization algorithm (rate and cycle procedure) used for the magnetic measurements.

Although this is most relevant for magnets which can't be "fully" saturated, it also applies to solid core magnets (e.g. spectrometer magnets) because these are generally of multi-piece construction. Although it is obvious, it is stated as a golden rule because it seems to be "followed in the breach rather than the observance". Furthermore, since one expects to see greater use of combined function magnets (11) such effects will become of increasing interest in magnet design (3,12) until eventually one will be able to calculate the optimal procedure.

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(11) One generally deals with combined function magnets anyway, except that the higher harmonics are supposed to be weak enough to ignore.

(12) The implications for quadrupole magnets will be discussed in another note on the PEP quads similar to this one.
rather than fashion it empirically. Figure 7 shows a typical set of magnetic measurements and how the magnetization cycle influences the results as a function of current.

**Quadrupole Effects**

Looking at Fig. 1 again, it is clear that the rectangular pole contour of the magnet leads to so-called "shim" angles (α, β) at entrance and exit of the magnet with α = β = -φ/2 which are equivalent to horizontally defocusing quadrupoles of strength

$$k_Q^0 = -\frac{\tan(\phi/2)}{\rho} = -9.9 \times 10^{-5} \text{ m}^{-1} . \quad (8)$$

These are constant in strength and since they are an optical effect, resulting from the geometry of the magnets, rather than magnetic, they are not relevant here except to give an idea of what could be an important magnitude for the distributed quadrupole component of the magnet. Of course, one is interested in this for its effect on the tunes of the machine.

Returning to Fig. 6, one finds a good puzzle in that the more they study it, the more confused they're likely to become. Both the sign and magnitude of the quadrupole strength may depend on whether the current is set from above or below as well as the rate of approach. For instance, the quadrupole components associated with the top and bottom curves (open circles) are nearly equal in magnitude but opposite in sign. Furthermore, the brownout data show that the magnitude depends on the final current. Finally, when the quadrupole component is unfolded, one is left with what can be called a transverse "umbrella" in the lower curves and a "dish" in the upper one — effects which are certainly related to the quadrupole variations as well as the previously mentioned dipole effects.(13)

(13) Dipole variations associated with eddy currents were observed by Cobb and Harris, Int'l Symp. on Magnet Tech. (Stanford) pg. 823 (1965).
The strength of harmonics in such a magnet can be written (14), using Eq. 1, as

$$\int B_y(x,0,z)dz = (B_0) \sum_n k_n x^n ,$$  \hspace{1cm} (9)

where we have already used $k_0 = k_D$ and $k_4 = k_Q$ in the discussion above.

The distributed quadrupole strength can be written as

$$k_Q = \left( \frac{\partial k_D}{\partial x} \right)_{x=y=0} ,$$  \hspace{1cm} (10)

regardless of its equivalent optical strength which may differ. As indicated in the discussion of Eq. 1, the sensitivity of the quadrupole measurement depends only on the step size from the centerline, which was chosen large enough to encompass the full beam spread ($4\sigma_{x_{tot}}$) over its trajectory through the magnet (Fig. 1) but small enough to ignore harmonics higher than three. Using Eq. 2 one then has

$$k_Q = - \kappa_D \left( \frac{\partial \langle G \rangle}{\partial x} \right)_{x=0} + \frac{k_D}{B_D \langle G \rangle} \left[ \frac{\partial \mathcal{H}_C}{\partial x} - \frac{\partial \mathcal{B}_M}{\partial x} \right]_{x=0} ,$$  \hspace{1cm} (11)

where $B_D$ is the effective, sharp-cutoff induction field.

With good steel, design and construction, the first term can be assumed zero, otherwise, the design constraint for a separated function, "dipole" magnet is

$$\kappa = \frac{1}{B_D} \left[ \frac{\partial \mathcal{H}_C}{\partial x} - \frac{\partial \mathcal{B}_M}{\partial x} \right] ,$$  \hspace{1cm} (12)

assuming no mechanical gradient increase with excitation (15). Thus, one desires cancellation between coercive and permeability terms somewhere toward the middle of the operating range.

(14) K. L. Brown, SLAC Report No. 75.

(15) The low fields required to reduce radiative energy loss simplify this problem but don't eliminate it because of the increasing importance of coercive contributions compared to the current driving terms.
III. I-10

Figure 8 shows the distribution of mechanical gradients, measured at zero excitation, which have a mean and rms:

$$\left( \frac{\partial \langle G \rangle}{\partial x} \right) = 0.355 \pm 0.167 \text{ m}. \quad (13)$$

Similarly, the mean and rms of the distribution of average gap heights, $\langle G \rangle$, shown in Fig. 4 is:

$$\langle G \rangle = 70.122 \pm 0.040 \text{ mm.} \quad (14)$$

with Eq. 11 defined as $\overline{k}_Q = \overline{k}_Q^G + \overline{k}_Q^c + \overline{k}_Q^u$, one has:

$$\overline{k}_Q^G \geq (1.65 \pm 0.77) \times 10^{-4} \text{ m}^{-1}, \quad (15)$$

which is considerably larger than the optical, edge contribution, Eq. 7.

The values of $\overline{k}_Q^c$ can be estimated using Fig. 2 and Eqn's 3 and 11 as:

$$\frac{\partial \mathcal{H}}{\partial x} \approx \frac{\mathcal{H}}{\mathcal{H}_o} \frac{dI}{dx} = \langle H_c \rangle \frac{dI}{dx} \approx 1.65 \text{ Oe} \quad (16)$$

i.e.

$$\overline{k}_Q^c \approx - 3.86 \times 10^{-3}/E(\text{GeV}) \text{ m}^{-1} \quad (17)$$

Similarly, the value of $\overline{k}_Q^u$ can be estimated, but with a little more care. For instance, some of the curves in Fig. 6 result from purposely inducing eddy current effects such as discussed by Halbach\(^{(16)}\). While it is difficult to totally escape such effects, the standardization and setting procedure is supposed to minimize them so that we can approximate $\overline{k}_Q^u$ by

$$\overline{k}_Q^u = - \frac{k_B}{\langle G \rangle \cos \psi} \langle 1/\mu \rangle \frac{dI}{dx} \approx 0.79 \langle 1/\mu \rangle /\cos \psi \quad (18)$$

assuming no variation of $\mu$ with $x$. Using the measured, low field permeability, $\mu \approx 3000$, and assuming it doesn't change appreciably with current gives

$$\overline{k}_Q^u \geq + 3.04 \times 10^{-4} \text{ m}^{-1} \quad (19)$$

\(^{(16)}\) K. Halbach, NIM 107, (1973) 529.
Based on these assumptions, one expects the residual field integral or coercive term to exceed the permeability contribution below 12 GeV.

Figure 9 shows the measured quadrupole strengths for \( I = 1250 \) A (or \( E = 17.43 \) GeV). At this current, the prediction, based on Eqs. 15, 17 and 19, is

\[
\overline{k}_Q \gtrsim (-0.82 \pm 0.77 \times 10^{-4}) \text{ m}^{-1}, \tag{20}
\]

i.e. \( \overline{k}_Q \) should not be more negative, so long as there is no gap closure at this excitation. The mean and rms of the measurements \(^{(17)}\) are

\[
\langle k_Q \rangle \approx (-0.42 \pm 0.44) \times 10^{-4} \text{ m}^{-1}. \tag{21}
\]

Adjusting Eq. 19 for the lower permeability at 1250A can account for the difference \(^{(18)}\) between prediction and measurement. Below 1250A, one expects \( \overline{k}^u_Q \) to decrease somewhat and \( \overline{k}^c_Q \) to also decrease (i.e. become more negative) so that \( \langle k_Q \rangle \) should become more negative (assuming no gap opening). This is not inconsistent with Fig. 6 since the so-called "brownout" data was actually taken at the same constant rate of change of current i.e. the magnet was not truly standardized. Since the 800A data is comparable to 1250A data at the same rate, one might reason that it is possible to use eddy currents to compensate such a decrease of \( \langle k_Q \rangle \) with energy.

The model for this (ignoring the laminar structure) can be written down by considering the induced eddy currents in the backleg perpendicular to the line A-B in Fig. 2:

\[
 J_z = -\frac{\sigma}{c} \langle \vec{B} \rangle x. \tag{22}
\]

---

\(^{(17)}\) Because measurements were obtained on only half the magnets, the determination of the mean is based on a weighted average of two groups. One group was composed of magnets from \#030 - 140 since, according to G. Fischer, a restrike operation began after \#029 which included a mechanical gradient of 0.53 mr for the first time. From Fig. 8, it appears that this gradient was roughly obtained until after magnet \#140 when "scrap" laminations had to be included which were made prior to \#030.

\(^{(18)}\) The calculation is admittedly crude. It was intended primarily to show how different effects contribute and thereby justify the PANDIRA calculations.
This eddy current translates into a relative variation of $\mu$ across line A-B which in turn translates into an induced variation of the gradient across the gap which can be positive or negative depending on the sign of $\hat{n}$ i.e. whether one approaches from above or below.

We complete this section by summarizing some conclusions based solely on considering the quadrupole strength of such magnets:

1) There is a definite correlation between the mechanical gradient and quadrupole strength. In fact, the simple prediction of $k_Q$ for individual magnets as well as $\bar{k}_Q$ is consistent, within the errors on $\bar{k}_Q$, with the measurement of $\langle k_Q \rangle$. The same can be said of $k_D$ and $\bar{k}_D$.

2) A much smaller variation of $\bar{k}_Q$ would have resulted from distributing the laminations serially according to magnet number as they are produced i.e. by following G. R. #1.

3) Comparison of the data in Fig. 6 obtained by ramping up at a constant rate of 100 A/s with that obtained by standardizing, i.e. following an exponential upward approach, shows the former is totally eddy-current dominated. This demonstrates how easy it is to change the character of the magnets and the optics of the ring by inconsistent setting or standardization procedures i.e. by ignoring G. R. #2.

4) From the equations above, one wants $H_c$ to have a small magnitude, one wants a high permeability, a small angle, $\psi$, and as high an operating field, $B_D$, as is compatible with a small variation of $\mu$ over the intended operating range. This last point also favors a small $\psi$ which can be obtained by either making a higher pole segment or a more symmetric design e.g. an H-magnet.

5) When one considers that a mechanical gap closure of 1 mil at the pole-edge causes a change of $\Delta k^C_Q = 1.2 \times 10^{-4}$ m$^{-1}$ and, the additional fact, that the magnet, through lack of symmetry about the y-z plane, is more sensitive to eddy currents, one has Golden Rule #3:

Don't make magnets whose mechanical symmetry doesn't conform to the desired magnetic symmetry or alternatively insure that the calculations comprehend the real magnet i.e. its steel and construction.
6) From Fig. 1 and the fact that $\langle k_Q \rangle \neq 0$, it follows that the effective dipole strength is modified because the central ray spends more time at positive than negative values of $x$ i.e. using

$$\delta k_D = \langle k_Q \rangle \langle \delta x \rangle \quad (23)$$

where

$$\langle \delta x \rangle = \rho \left[ \frac{\sin(\phi/2)}{(\phi/2)} - \frac{1}{2} \left(1 + \cos(\phi/2)\right) \right] = 3.67 \, \text{mm}, \quad (24)$$

this is equivalent to effectively less bending i.e. we would need a fractionally higher current by the amount:

$$-\left( \frac{\delta k_D}{k_D/E} \right) = 4.27 \times 10^{-6} \left( \frac{E(\text{GeV})}{17.43} \right). \quad (25)$$

This is equivalent to only 0.052 MeV at 14.5 GeV.

7) From Fig. 1, it is also clear that the equivalent, optical lens strength will be much reduced from the measured value $\langle k_Q \rangle$ because of the difference between the design orbit in the magnet and the magnetic measurement path. Thus, there is also a correction here equivalent to item (1) on page 6 for the dipole component but rather than increasing, this decreases $\langle k_Q^0 \rangle$.

While it is reasonable (i.e. at the few percent level) to approximate Eq. 8 by a thin lens at entrance and exit, it isn't reasonable to consider the distributed component this way until a more detailed calculation \(^{(19)}\) is done for the effective optical strength. However, in the spirit of Eqn's 23-25, we can get an idea of the effect by assuming a circular path which is unper-turbed by the focusing action (which is assumed weak). The result is:

$$\langle k_Q^o \rangle = \langle k_Q \rangle \left[ 1 - \frac{\sin(\phi/2)}{(\phi/2)} \right] / (1 - \cos(\phi/2)) \quad (26)$$

$$\langle k_Q^o \rangle = \langle k_Q \rangle \langle \delta x \rangle / \Delta = 0.333 \, \langle k_Q \rangle.$$  

where $\Delta$, as shown in Fig. 1, was made to be $(\rho/2)(1 - \cos \phi/2)$. \(^{(19)}\)

\[^{(19)}\] A full ray tracing calculation will be done to get the effective transfer matrix. In first order, the magnets should be partitioned into an odd number of lens, $n \geq 3$, e.g. triplets with the entrance and exit strengths modified from the value given in Eq. 8.
Together with Eq. 23, this result defines the optimal horizontal alignment of such magnets in the ring i.e. $\langle \delta x \rangle \approx 0$ or $\Delta \approx 7.33$ mm which minimizes dipole, quadrupole and to some degree, the higher order errors as well. Conversely, the horizontal alignment will influence any comparison of the expectations from magnetic measurements with storage ring measurements. Such comparisons will be discussed in the last section.

Sextupole Effects

The existence of "shim" angles ($\alpha, \beta = -\phi / 2$) generally imply the equivalent of defocusing sextupoles ($k_s < 0$) at entrance and exit of strength $^{(14,20)}$:

$$k_s^o = - \frac{1}{2 \rho R} \sec^3 (\phi / 2) \approx - 0.0032 \text{ m}^{-2},$$

(27)

where $R_\alpha = R_\beta = R > 0$ is the radius of the isoinduction lines at entrance and exit. If the magnet pole is wide enough ($W / G \gg 1$) and there is no significant saturation over the operating range, then a rectangular pole contour will have $R = \infty$ — even at the ends where the third dimension enters. Because this would be an unacustomed (if not unnecessary) extravagance one expects a small, negative sextupole strength at entrance and exit (because $W / G \approx 1.5$) which becomes increasingly negative with energy. One also expects the distributed sextupole strength to become increasingly negative with excitation.

Clearly, the upper curve in Fig. 6 has the "wrong" sign i.e. it doesn't decrease as one goes away from center but increases. Where we understood the induced gradient from eddy currents to result from the lack of symmetry in the backleg return yoke of the C-magnet, the induced sextupole can be understood from the symmetric arrangement of the coils around the pole such that the induced eddy currents approach zero near the magnet centerline and reverse when $B$ changes sign. This is essentially the "dish" effect that Halbach studied in Ref. (16).

Comparing to the lower curves in Fig. 6, obtained by standardizing, we see that the sextupole strength is significantly reduced and also has the expected sign ($k_S < 0$).

Figure 10 shows the distribution of sextupole strengths which were measured in the same way and at the same time the quadrupole data was obtained. One sees the same correlations with the magnet fabrication procedures as for the quadrupole data\(^{(17)}\) but with one significant difference — there is an apparent increase with time (or lamination number). Table II summarizes the results based on the previous partition of the data\(^{(17)}\). There are several possible explanations e.g. significant die wear at the "sharp" corners of the pole edge which became increasingly rounded — resulting in less saturation at 1250A. Unfortunately, there are no measurements at other excitations. The mean and rms are:

$$<k_S> = (+3.38 \pm 10.9) \times 10^{-4} \text{ m}^{-2},$$  \hspace{1cm} (28)

where the rms is taken simply as the weighted average of the three groups.

The same kind of analysis as done for $k_Q$ will not be done for $k_S$ except to point out one result i.e. imposition of a mechanical gradient (to optimize $k_Q$) can modify $k_S$\(^{(20)}\) and all higher orders. While this is not the explanation of Fig. 10, it provides further support for G.R. #3. The large spread in the data of Fig. 10 supports G.R. #1 and, as just discussed, the sextupole effects displayed in Fig. 6 support G.R. #2. The reason an analytic treatment of $k_S$ is not done is because there are too many terms and too little information i.e. a set of measurements similar to those of Fig. 7 should have been obtained as a function of both

---

\(^{(20)}\) Imposition of a gradient (after magnet #029) can be expected to increase $k_S$ as observed in Fig. 10. Comparing Fig's. 9 and 10 implies that the imposed gradient was roughly twice as large as it should have been — at least at this excitation.
III. I-16

current and position (x) before and after the magnetization step was done. This brings us to another important point which will be called G.R. #4:

It is preferable to do a good, complete 2-D magnetic design calculation than a poor 3-D one i.e. at least one full-scale prototype should be built which tests the proposed fabrication procedure’s ability to produce a magnet which verifies the 2-D calculations and allows us to measure and modify 3-D effects such as end contributions give.

This has a lot to say about how one designs magnets and is applicable to systems with only a few components such as spectrometers. For instance, a corollary is that one should not plan on shimming the end fields to correct errors observed in the central field (or vice-versa) because that should not be necessary! This also has something to say about how one designs optical systems e.g. short, separated-function magnets should be avoided — especially since optics considerations generally prefer combined function systems anyway (11, 21).

Discussion

The guaranteed operating range of the PEP bend magnets is from 200-1850A i.e. from 2.8 — 25.6 GeV with a standardizing current of 1250A. From Tables I and II, the relative multipole strengths measured at 1250A are:

\[
\langle k_D \rangle_{\text{class 1}} = 0.032625 \pm 0.000018,
\]

\[
\langle G \rangle \cdot \langle k_Q \rangle / \langle k_D \rangle_1 = -0.000090 \pm 0.000094,
\]

\[
\langle G \rangle^2 \cdot \langle k_S \rangle / \langle k_D \rangle_1 = 0.000051 \pm 0.000162,
\]

(29) The argument here is not whether combined-function elements are preferred but rather on how to realize them. Golden Rules #3 and #4 certainly relate to this problem which will be discussed elsewhere (Collider Memo). When fabrication mistakes occur, such magnets are natural candidates to also serve as general purpose correction magnets e.g. by adding pole-face windings for various multipoles similar to the way magnets were selected to have their backleg dipole trims excited. In this way, one might require only about 2v magnet (N \gg 2v).
with higher multipoles undetermined but convergent. One is interested in such results because the bend magnets are the largest class of magnets in the ring ($N = 192$).

An rms error in the bending strength of only 18 $\mu$rad translates into a closed orbit error at any location, $s$, of:

$$\langle \chi_{c.o.}^2(s) \rangle \approx \frac{\sqrt{\langle \beta_x \beta(s) \rangle / 8}}{|\sin \pi \sigma_s|} \sqrt{N} \sigma_{k_D} = 0.43 \sqrt{\beta(s)}$$

where we have used the ring values given in Table III for configuration #31. Even a small rms error can therefore lead to significant orbit distortions.

The question of the absolute calibration of the beam energy will be discussed in another PTM (22).

The tune shifts expected from a quadrupole strength such as given by Eq. 29 and modified by Eq. 26 will be

$$\Delta v_x \leq \frac{N}{4\pi} \langle k_Q \delta \beta_x \rangle \langle \beta_x \rangle = -0.0031$$

$$\Delta v_y \geq -\Delta v_x \langle \beta_y \rangle / \langle \beta_x \rangle = +0.0054$$

i.e. $v_x$ decreases and $v_y$ increases compared to a model which includes Eq. 8 but not Eq. 21. The inequalities should be valid below 17.4 GeV i.e. one expects $\Delta v_y - \Delta v_x (\geq 0.0085)$ to increase with decreasing energy. Comparisons with ring measurements are complicated by the fact that the most sensitive quadrupoles in the ring are in series with the bends. Fortunately, this appears to be a small effect (23).

(22) An energy error will be reflected everywhere e.g. tunes, acceptance, damping partition, etc. The higher multipoles were shown to influence this but the magnitudes don't appear significant so long as the orbit distortions are controlled.

(23) The model that has been formulated can be tested on the reference magnet.
The tune spread, which is a measure of the stopband widths, can also be calculated from Eq. 29 as:

\[
\langle \Delta v_x^2 \rangle^{\frac{1}{2}} = \frac{k_s}{4\pi} \langle \beta_x \rangle \sigma_{kQ} = 0.0007
\]  

(32)

\[
\langle \Delta v_y^2 \rangle^{\frac{1}{2}} = 0.0012.
\]

Similarly, using Courant and Snyder's formulae again\(^{(24)}\), one can write the corresponding beta mismatch as

\[
\frac{\langle \Delta \beta_x^2 \rangle^{\frac{1}{2}}}{\beta_x(s)} \approx \sqrt{2} \pi \frac{\langle \Delta v_x^2 \rangle^{\frac{1}{2}}}{|\sin 2\pi v_x|} = 0.0032
\]

(33)

\[
\frac{\langle \Delta \beta_y^2 \rangle^{\frac{1}{2}}}{\beta_y(s)} \approx 0.0053.
\]

One-half percent or so would seem quite acceptable. If the damping partition numbers are \(J_x = 1 - \mathcal{J}\) etc., then the change due to a distributed quadrupole or energy error (\(\delta = \delta E/E\)) will be:

\[
\Delta \mathcal{J} \approx \left[ \frac{n}{k_D} \right] \left[ 2 \langle k_Q \rangle - \langle k_D \rangle / \rho \right] \delta + 2 \langle k_Q \rangle
\]

(34)

i.e. \(\Delta \mathcal{J} \ll 1\) for the ring acceptance. Furthermore, imposing an energy perturbation which makes \(\Delta \mathcal{J} = 0\) can cancel any tendency to antidamp from such a quadrupole contribution — a result compatible with simply shifting the beam in the dipoles as given by Eq. 24 (within the approximations used here).

\(^{(24)}\) Courant and Snyder, Annals of Physics 3 (1958) 1 or Sands, SLAC Rpt. No. 121.
Finally, although the sextupole content in Eq. 29 is of higher-order, it can influence the tunes through residual orbit distortions such as Eq. 30 i.e.

$$\Delta \nu_{x,y} \approx \frac{N}{4\pi} \left| \langle x^6 \rangle \langle y_{c.o.} \rangle \langle \delta x_{bend}, \delta y_{bend} \rangle \right|$$

which shows that orbit distortions of the magnitude of Eq. 30 can produce significantly large tune shifts (\(\Delta \nu \approx 0.1\)). For an otherwise perfect ring, a measurement of the tunes together with Eqn's. 31 and 35 would allow determination of the distributed multipole components or knowing them, one can infer how much of the observed discrepancy with the ring model comes from this source. From the results above and the new mini-beta insertions, one expects to see a much smaller discrepancy between the model and the measurements.

Superconducting and permanent magnets notwithstanding, reports of the demise of the "iron age" of magnets are greatly exaggerated so long as people follow the Golden Rule i.e. *Do what they know they should.*

**Acknowledgements**

It is important to thank all members of the PEP Magnet Group but especially Marty Anderson, Bob Bell and Gerry Fischer who had the unenviable task of making these magnets. I would also like to thank the SLAC Magnetic Measurements group who were always available and built a superbly sturdy measurement system that resisted all attempts of man and nature to destroy it. Zorlab Vassillian was every bit as dependable and is the individual most responsible for the quality of the data presented here together with a great many Stanford students who helped with the measurements. I must also thank John Rees who got me to automate the data "sooner than later" since this proved to be extremely helpful in assigning the magnets into classes. Finally, I thank Margie Barnes for her help and patience with this report. To end it, I paraphrase a remark made by Newton: I do not know how it appears to others but I seem to have been like a truffling hog who has rooted out some interesting trifles which I hope will help others.
### Table I

<table>
<thead>
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<th>Current [A]</th>
<th>Class</th>
<th>#</th>
<th>$\langle f_{bdz} \rangle$ ± RMS[T-m]</th>
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### Table II

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### Table III

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<th>β̂_y</th>
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<td>14.65 ± 2.52 m</td>
<td>25.26 ± 3.33 m</td>
<td>0.89 ± 0.31 m</td>
</tr>
</tbody>
</table>
Fig. 1: Schematic cross-sectional view through the median-plane of a standard PEP bend magnet (C-type) which was designed to have an effective magnetic length, \((1/B_y) \int B_y(s) ds\), along the circular design orbit between effective field boundaries, of 5.400 m. \(\phi\) is the average bend angle per magnet in degrees i.e. the effective dipole strength. The maximum beam spreads expected in the dispersion plane in these magnets are \(\sigma_{x_{\text{tot}}} < 4\) mm.
Fig. 2: POISSON plot of the upper half of the PEP standard bend magnet showing the coordinate convention (consistent with Helm, PEP PTM-207). z is the third coordinate of a right-handed system which coincides with the mechanical centerline of the magnet. The induction is negative and the dipole strength, $k_D$, is positive. A linearly increasing negative field for positive x-values corresponds to a horizontally focusing quadrupole ($k_Q > 0$) and a quadratically increasing negative field corresponds to a horizontally focusing sextupole ($k_S > 0$). The angle here is $\psi \approx 30^\circ$. 
Fig. 3: Ratio of $\int B_y dz$ to the simultaneously measured value for the reference bend for each class defined as (1) trims not excitable (total = 120), (2) trims excitable (48) and (3) trims excitable and near a vertical steering magnet (24). The average rms error on this data is typically less than or comparable to the size of the print symbol.
Fig. 4: Ratio of the measured, average gap opening of the reference magnet to all others. Error bars represent rms uncertainty in this ratio from the rms spreads in the mean gap opening.
Fig. 6: Effects of current ramping rate and direction of approach on absolute field level and distribution in PEP bend magnet #001. The lowest curve was obtained by dumping the magnet from 1250A following a complete standardization cycle and then running up to 1250A at a constant rate of 100A/S. The highest curve resulted by then running the current up to 2000A and back down to 1250A at the same constant rate. All data is at 1250A except for that in boxes (□) which was at brownout current (800A).
Fig. 7: Field integrals measured before and after subjecting a virgin magnet to the precondition or magnetization cycle.
Fig. 8: Distribution of average, mechanical gradient measured across the gap, transversely to the longitudinal centerline for each magnet. The average error, resulting from the rms of the measurement data, is ± 0.31 mr.
Fig. 9: Sampled distribution of quadrupole strengths measured in the PEP bend magnets following standardization at 1250A.
Fig. 10: Sampled distribution of sextupole strengths measured in the PEP bend magnets following standardization at 1250A.
ABSTRACT

Comparisons of different techniques for position measurements in spectrometer focal plane detector systems are made. Capabilities and limits of gas proportional detectors are discussed.

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I. Introduction

Although the detectors used to readout positions from a large spectrometer system are a small fraction of the total spectrometer system, their performance can have a large impact on the total cost. The better the position resolution the less dispersion that is needed for a given momentum (or energy) resolution, and consequently the smaller the magnets necessary. Conversely, given a total size for a spectrometer system, its design should be optimized to make use of the position resolution available with current state of the art technology.

Most existing medium energy spectrometer systems use some version of the gaseous proportional chambers, either multiwire proportional chambers or drift chambers, to provide position information. First I will try to summarize the properties and capabilities of gas proportional chambers, and then I will describe the chambers used in some of the existing spectrometer systems, and finally I will present some recent ideas and developments.

II. Gas Proportional Chambers (General Properties)

A gas proportional chamber consists of an anode wire surrounded by some gas and cathode planes. The passage of a charged particle through the chamber creates a track of electron-ion pairs. The electrons drift toward the anode wire. If the field strength around the anode wire is sufficiently large an avalanche develops, resulting in an amplification of the original electron pulse by as much as a factor of $10^7$.

The quantities that can be measured are the time and pulse height of the anode pulses, as well as the distribution of the induced cathode pulses. Position information is commonly obtained for such a chamber by one or more of several methods. The first and crudest method is that used in a simple multiwire-proportional chamber (MWPC).\textsuperscript{1} Such a chamber consists of a plane of anode wires. For particle trajectories at normal incidence pulse formation only occurs on a single anode wire. Determining which anode wire fired on the passage of a charged particle through the chamber determines its position with an accuracy given by the anode wire separation (the minimum practical separation is about 1mm). A second method involves
determining the particle coordinate along the anode wire. Two methods to accomplish this are to use the induced cathode pulses on a delay line and measure the cathode distribution directly or to use a resistive anode wire and use a R-C or charge division type of readout. The third method is to measure the time it takes for an electron track to drift to the anode wire. Such a measurement provides the shortest distance from the particle track to the anode wire, thus improving on the position resolution obtainable for a given wire spacing over a simple MWPC. Several methods exist for determining on which side of the anode wire the track occurred.

The ultimate position resolution obtainable from such a detector is determined by the dynamics of the ion track formation and by the subsequent diffusion of the electrons in the time between the formation of the ion track and its detection. Since in 1 cm of Argon at atmospheric pressure (the typical thickness of a chamber) one out of 20 minimum ionizing particles will eject an electron whose range is greater than 100 \( \mu m \), one can expect the ultimate position resolution from a gas proportional counter to be \( \approx 100 \mu m/\sqrt{20} \) or approximately 20 \( \mu m \). Another contribution to the resolution is diffusion. In the direction parallel to the electric field, diffusion of the electrons in a drift chamber of lengths of 1 cm and with an Ar-Isobutane gas mixture contributes \( \approx 50 \mu m \) to the chamber resolution. It is interesting to note that the diffusion in the direction perpendicular to the electric field is typically 2 to 5 times smaller than in the direction parallel to the electric field, and so the expected resolution is better in chambers which measure position along the direction of the anode wire.

Although induced cathode pulse readouts are expected to give better resolutions than drift chambers, such resolutions have only been recently observed, and then only for small chamber areas. In fig. 1 we show results obtained by Charpak, et al. using a small detector. The position for each event was found by independently reading out and digitizing the induced pulse heights on several cathode strips, and then computing the centroid of the induced charge. The best resolution obtained is \( \approx 70 \mu m \) FWHM, about a factor of 2 better than can be obtained with a drift chamber. Similar results
have been obtained using a delay-line readout chamber with an active area of 5x5 cm$^2$, fig. 2.$^{11}$

The counting rate of a MWPC is limited by space charge effects. This limits the rate in a typical chamber to $\approx 10^5$/sec/cm of anode wire.$^{9}$ In spectrometer applications, where events tend to be localized on a chamber, such a rate can be exceeded at reasonably low singles rates. Another consideration which is sometimes important is the thickness of a chamber and consequent multiple scattering and energy straggling of the incident particles.

III. Spectrometer Systems

Although as mentioned in the previous section, one can obtain the best position resolution from MWPC's by finding position along the anode wire (either directly, using a delay line, or with resistive anode wires), the best resolution of $< 100 \mu$m has been obtained only recently. In chambers of reasonable length, $= 60$ cm, position resolution has been dominated by contribution from the methods used to read out the cathode information. The accuracy available from charge division$^{12}$ or delay-line readouts$^{13,14,15}$ has been limited to $\approx 500 \mu$m (FWHM). Because of the complexity of the readout systems, chambers which directly digitize the induced charges on cathode pickup strips, a method which is not expected to degrade in long chambers, have not been implemented for large spectrometer systems. In fact, chambers in the focal planes of the highest resolution medium energy spectrometers (HRS, Bates, EPICS) have used drift time information to interpolate between anode wires and consequently provide position information with resolution $\approx 150 \mu$m (FWHM).

A. Delay-line Readout Drift Chambers (DRDC)

The chambers used to readout the HRS and EPICS spectrometer focal planes, more thoroughly described elsewhere,$^{16}$ are delay-line readout drift chambers. These systems use chambers with dimensions of up to $30 \times 90$ cm$^2$. Four pairs of chamber planes, two orthogonal sets of four planes each, are used for each trajectory measurement. The planes are of an alternating gradient design$^{10}$ with 8 mm wire spacings. Alternate field defining cathode wires are bussed together, providing two cathode outputs per plane, and the anode wires are coupled
directly into a fast (2.5 ns/cm) delay line which provides two anode outputs per plane. The anode planes, which are run at high positive voltage, are separated by grounded foil cathode planes.

An assembly view of such a chamber is presented in fig. 3. The O-ring seals between each plane provide a gas seal that is sufficiently tight to allow the chamber assembly to be used in a vacuum. All connections to the chamber are made through the front cover plate using either BNC connectors for the cathode signals or SHV connectors for the anode signals. Entrance and exit windows are constructed of mylar. This chamber assembly, consisting of eight wire planes used in the front of the EPICS spectrometer and operated in a vacuum with 51 µm mylar entrance and exit windows, presents 30 mg/cm² of mass to the incident beam.

The anode signals are capacitively blocked and terminated into 100 Ω at the input to the Lecroy Systems Corp. LD604 amplifier-discriminator located approximately 1 meter from the chamber. The resultant NIM level fast logic pulses are transported to the counting house through up to 200 m of 50 Ω coaxial cable, and are then regenerated into a NIM leading edge discriminator and used as stops for CAMAC multichannel TDC's. The start signals for the TDC's are obtained from the trigger coincidence logic of the spectrometer system, which can include information from the chambers. Coarse position information is provided by the time difference between pulses from the ends of the delay lines whereas finer information is obtained from the time sum (drift time information).

The typical position resolution obtained from these chambers is 125 µm (FWHM) and is independent of the chamber size. This resolution is not compromised by counting rates of up to 10⁶/sec.

B. Vertical Drift Chamber

A novel MWPC drift chamber called the vertical drift chamber (VDC) has been described by Bertozzi, et al., and used to read out the MIT energy loss spectrometer at Bates. This device uses a single wire plane, rotated 45° with respect to the central ray leaving the spectrometer and lies approximately in the focal surface of the spectrometer. A detailed cross section view of this chamber is shown in fig. 4. Each of the active anode wires are alternately coupled
IV. A-6

through an amplifier and discriminator into one of three delay lines. This readout scheme is shown in fig. 5. As in the chamber described above, the difference in pulse arrival times from the ends of the delay lines is used to decode the wire number, while the sum is used to provide the drift time.

As can easily be seen, with these chambers one plane provides both position and angle information. The accuracy of the reconstructed track position is 120 µm (FWHM) and the measured angle resolution is 1.7 mr.

IV. Recent Developments
A. Use of Induced Cathode Pulses

An alternative method of making the Left-Right decision recently applied to individual wire readout chambers is to use the current pulses induced on the cathode field defining wires. Using the present DRDC, Left-Right information was obtained by measuring the difference in the induced currents on the cathode wires on either side of the active anode wire. To minimize the electronics needed for these measurements, every other cathode wire in a plane is bussed together, providing two cathode outputs per plane. The difference of these two signals is generated using a voltage-sensitive differential amplifier. For odd (even) wires, a positive (negative) signal is observed for particles passing to one side of the anode wire, and a negative (positive) signal is observed for particles passing to the other side. The integration time constant of the amplifier, determined by the 80 pf capacitance of the chamber and a 500 Ω terminating resistor, is about 40 ns. The analog output from such a circuit, integrated for 100 ns, is shown in fig. 6 as a function of position. One can see a clear separation between the L-R signals. Initial tests of this methods showed an erroneous left-right decision was made approximately 1% of the time.

This method reduces the number of chamber planes required to make a position measurement to one, and consequently results in a considerable simplification of the spectrometer system.
B. Reduction of Multiple Scattering

The present limit of the resolution of the EPICS spectrometer is determined by the accuracy of the angle measurements made in the front chambers (a schematic view of the spectrometer system is shown in fig. 7). The measured FWHM of this angle determination is 4 mrad, about what is expected from multiple scattering in the chamber gas. A reduction of a factor of \( \approx 2 \) can be made in this multiple scattering by using a chamber gas mixture based on Neon rather than Argon. Initial tests of such a mixture showed it to be feasible, although some trouble was encountered with obtaining an efficiency greater than 98%/chamber plane. We hope to solve this problem with more experimentation.

A further reduction in the thickness and multiple scattering in these chambers can be obtained by replacing the existing windows with preformed Beryllium windows. Windows of 0.001" thickness can be made by sputtering Beryllium onto an aluminum mandrell. Forming the windows to a spherical shape reduces the stress, and allows thinner windows to be used over a given area.

V. Conclusion

Most present spectrometer readout systems use drift chambers to provide the needed positive resolution. With such chambers, it is possible to measure positions with accuracies of \( \approx 120 \) \( \mu \)m, and angles with accuracies limited by multiple scattering. The most cost effective spectrometer designs are likely to be ones which include the possibility of software corrections which can be made with such readout systems in the design phase.
REFERENCES


FIGURE CAPTIONS

Fig. 1: Position resolution obtained by Charpak et al.\textsuperscript{3} for X-rays in a chamber with independently readout cathode strips.

Fig. 2: Position resolution obtained by Thompson et al.\textsuperscript{11} for X-rays in a short delay-line readout chamber.

Fig. 3: Assembly view of an EPICS front chamber.\textsuperscript{16}

Fig. 4: Schematic drawing of a VDC\textsuperscript{17}.

Fig. 5: Schematic drawing of the readout scheme for a VDC\textsuperscript{17}.

Fig. 6: A scatter plot showing the difference in the induced cathode pulses for events on different sides of the anode wires. Anode wires are located every 8 mm from the center of the plot.

Fig. 7: Plan view of the EPICS spectrometer as modified for a recent double charge exchange experiment. Chambers are located in both the front chamber box, as well as at the rear of the spectrometer.
Figure 1
Figure 5

BLOCK DIAGRAM: DELAY LINE READOUT
Figure 6

L-R DIFFERENCE (ARBITRARY UNITS)
IV. A-17

Figure 7
General Design Methods
Harald A. Enge

First, in response to a question from the Chairman about how well does a well-designed spectrograph conform to the calculations: The fields used in RAYTRACE do satisfy Laplace's equation and can therefore be produced with appropriate shapes of poles. So it is up to the magnet engineer. (LeVine was questioned about how well the Q3D spectrographs conformed to calculations, e.i., 10^{-4} resolution at full solid angle, and responded that the Brookhaven Q3D does not meet this but, for instance, the Saclay instrument does.)

Fig. 1 shows the trend in spectrograph design towards data-collection power. The ordinate is a product of the momentum range and the solid angle of the instruments shown. It is unlikely that the trend will continue up the same slope. Compare Fig. 2 which is self-explanatory.

I want to raise the question of if there is any advantage in use of quadrupoles in large solid-angle spectrographs. My contention is that the quads gain little or nothing for you in solid angle. For instance, the LASL "Clamshell" (Fig. 3) is designed for 40 msr without use of quads and the Blomqvist QQDD is designed for approximately the same momentum range and solid angle. However, there is a difference in the resolving power. A quadrupole helps in two ways to facilitate hardware corrections. First, multipole corrections can be included in the element itself,
second, the quad changes the aspect ratio of the particle bundle and this makes it easier to make semi-orthogonal corrections for median-plane rays and off-median-plane rays. This is utilized in the LASL-HRS instrument, the Q3D spectrographs (Fig. 4) and others.

In the previous talk I mentioned briefly the procedure of designing a spectrograph. We always start with TRANSPORT to determine first- and second-order parameters and then follow up with RAYTRACE for higher accuracy and for higher order corrections. A preliminary layout drawing has to be made at an early state in the design.

Assuming that the central ray for the central momentum in RAYTRACE passes through the origin of the coordinate systems at the entrance and exit boundaries of dipoles and through the center of multipoles, there should in principle be no first and second order changes in the aberration coefficients resulting from a third-order correction. There usually are higher-order changes, however. Since RAYTRACE truncates the series of coefficients at 5th order, the sixth and higher order changes do reflect back to the lower orders, artificially. In principle it should be possible to make corrections for higher and higher orders, one order at a time, without having to go back to the lower order. In practice, some recorrections are generally necessary.

RAYTRACE prints out third order terms such as \(x/\theta^3\), \(x/\theta^2\phi\), \(x/\theta^2\phi^2\), \(x/\theta^2\delta\), \(x/\theta^2\delta^2\) and others. Similar terms are given for
fourth and fifth orders. How can they be reduced without going through a lengthy procedure of trial and error? Fig. 5 indicates a third-order change on the exit boundary of a dipole. The ray shown suffers a change in direction $\Delta \nu$ in the $x$-$z$ plane and, if it is an off median-plane ray, a change $\Delta \phi$ in the $y$-direction. These changes in direction will cause a change in position on the detector that can be calculated by using first-order transfer matrices from where the change is being made to the detector. Since these matrices are known from TRANSPORT it is possible to work out a table of correction coefficients for any order of correction when the first-order layout has been made. So the procedure is to pre-calculate the necessary changes of, for instance, third-order boundaries and/or octupoles at selected locations and then run RAYTRACE to check the results. Next come fourth-order corrections and so forth. This procedure works very well for spectrographs with small momentum range, not so well for the Split-Pole, for instance.

Aside from calculational difficulties, the Split-Pole is a difficult instrument to correct for third orders. The original version has a large negative $x/\theta^3$. It has turned out to be hard to reduce this term substantially without increasing $x/\theta \phi^2$, $x/\theta^2\delta$ and $x/\theta \delta^2$. We discovered by accident, however, that by decreasing the object distance and not requiring $y/\theta=0$ at the detector, the third-order aberration terms could be substantially reduced. Fig. 6 shows this new version of the Split-Pole. It is a
20 msr instrument designed for SIN to be used for low-energy pions. The $y/\phi$ term and $y$ aberrations produce $\Delta y \approx 8$ cm on the detector. The gap is $D = 10$ cm.
Figure 2: A graph showing the increase in energy (eV) from 1930 to 2000. Key events include the Cosmotron, AGS, and NAL. The Earth is depicted with a magnetic field of 4 Tesla.
Fig. 3  Claw Shell Spectrometer. Scale 1/10
Optimizing Energy Resolution in
Accelerator-Spectrograph Systems*

E. Kashy

National Superconducting Cyclotron Laboratory
Michigan State University
East Lansing, MI 48824

This paper describes the optimization of energy resolution in magnetic spectrograph systems and at the same time compares two significantly different systems, the HHH and VHV systems. In the first, HHH, we have in the same plane, the incident beam dispersion, the nuclear reaction or scattering, and the spectrograph analysis, the plane being commonly the Horizontal plane. In the second scheme, VHV, which is the case for the Los Alamos HRS\(^1\) (High Resolution Spectrograph), the incident beam dispersion and the analysis of the reaction products by the spectrograph are in one plane (vertical), while the reaction plane is an orthogonal plane, (horizontal). This latter scheme, VHV, has been selected for the high resolution 1.2 GeV/c spectrograph\(^2\) to be used in the study of heavy-ion reactions at the National Superconducting Cyclotron Laboratory (NSCL) which is being constructed on the campus of Michigan State University.

We begin by referring to some empirical methods for tuning the accelerator-beam transport-spectrograph system.

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These methods were developed at MSU by Blosser et al$^{3,4}$, and by Nolen, Miller and Kashy$^{5,6}$. Adaptation and further development of these techniques have been successfully carried out at Orsay$^7$ with the MP Tandem and at Julich with their cyclotron and Big Karl magnetic spectrograph. The $0^\circ$ tuning method$^6$ has recently been used at Los Alamos with the HRS with excellent results.

In this paper we will briefly review the physics of matching, and for that we have the review article by Hendrie$^8$. We then describe the tuning scheme with the magnetic spectrograph at $0^\circ$ with respect to the incident beam, and as we shall see, a number of the spectrograph aberrations can be measured in this way, while at the same time leading to critical adjustments of the elements of the beam line and of the beam energy analysis system. As will be clear from the discussion, the spectrograph works extremely well as an analyser of the beam quality since this large solid angle instrument is used in a task where it acts essentially as a "pin-hole" camera. We shall also discuss some of the results obtained with the HHH and VHV systems.

**Physics of Matching**

Figure 1 shows a beam on target for the case where dispersion and scattering are in the same plane (HHH). To understand the characteristics of the beam so that following the nuclear reaction, narrow peaks corresponding to individual nuclear states are obtained on the focal plane of the spectrograph, we start from the focal plane for a reaction with
no kinematic energy spread (i.e., $K$, the kinematic factor defined as $\frac{1}{p} \frac{dp}{d\theta}$ is zero). We follow the central ray back to the target. We then repeat the process with rays of different momenta, and their position on target gives the required energy dispersion on target. The two orbits shown, one (slashed line) has the momentum $p^+ = p_0 (1+\delta)$ and the other (dotted line) has $p^- = p_0 (1-\delta)$, where $\delta = \frac{\Delta p}{p_0}$. Note that the rays are displaced in both position and angle, in a manner dictated by the properties of the spectrograph.

The situation where kinematic energy spread is important, i.e. $K \neq 0$ is shown in the lower part of Fig. 1. The new focal plane is displaced farther from the magnet as the kinematic correction is done by focal plane motion. As before, we follow orbits back to the target and now have to achieve both the energy/position and angle/energy correlation on target. The calculations of the settings of the beam line elements to achieve these conditions using the code TRANSPORT$^9$ is described Reich et al.$^{10}$ and Roussel et al.$^7$.

An example of the calculation of a matching condition is shown in Fig. 2, for the $x-\theta-\delta$ correlation for the beam at the target position of the Bacchus spectrograph at IPN, Orsay. The upper part of Fig. 2 shows a section of the phase space ellipsoid for the MP tandem 60 MeV $^{16}$O beam, as it appears at the entrance slits of the analyzing magnet. The observed $\delta$ for this tandem beam is in large part due to the stripping foil which results in an energy spread of about 20 keV. The lower part of the figure show the
correlation required where the reaction is elastic scattering with kinematic factor $K = 0.1$. This value of $K$ corresponds to the case of scattering of $^{20}\text{Ne}$ from $^{40}\text{Ca}$ at a laboratory angle of $12^\circ$. As is seen the beam is not focussed on target and there is little one could observe on a scintillator at the target position to ascertain that the proper condition is achieved. (As pointed out by D. Hendrie in the discussion, it is in principle possible to have a scintillator that moves back and forth to various locations ahead or past the target, and thus have a beam focus int he scattering plane at the remote location. This may not be a practical solution, however, as the motion may exceed the space available.)

Note that the complexity evident in fig. 2 arises from the need to combine kinematic and dispersion compensation, since in the HHH case the scattering is in the same plane as the dispersion. Some of the advantages of decoupling the two effects as is done with the HRS at Los Alamos have been discussed by Zeidman. For electron scattering from nuclei the kinematic effects are indeed much smaller and the advantages of decoupling the scattering and dispersion less important. For the study of two body reactions with heavy ions, large kinematic effects are present and must be compensated. This would require "retuning" the beam line as the kinematic factor $K$ varies, if scattering and dispersion are in the same plane (HHH). This variation would occur when changing targets, or when changing the
scattering angle for the same target, and has many dis-
advantages.

The VHV system chosen for the large spectrograph at
NSCL decouples these important effects and should result
in a much simpler and reliable method for consistently achieving
the highest resolution.

2. Tuning with Spectrograph at 0°

The apparatus used in the focal plane for viewing the
beam for tuning the system when the spectrograph is placed
at or near 0° is shown in Fig. 3.6 The scintillating screens
are made of a thin layer of MgO "smoke" collected on a thin
supporting foil of aluminium as described by Nolen11. Two
small wires act as fiducial marks for distance calibration.1
In this configuration the spectrometer acts as a powerful
diagnostic instrument in evaluating the ultimate resolution
potential of the system from the accelerator to the spectro-
graph. Since the solid angle of the beam entering the spectro-
graph is in general relatively small, the latter's aberrations
have therefore little effect on the resolution. Figure 4
is a schematic of the tuning procedure at zero degrees.
The goal is the narrowest "line" in the spectrograph focal
plane. The aperture sketched can be inserted in the beam
path at the focal plane of the energy analysing system.
An accurate focus can then be achieved at the target position
by looking at a scintillating screen placed there, taking
into account the signs and magnitudes of the calculated
magnifications in both horizontal and vertical planes.
The complex nature of the aperture (it resembles the Hebrew letter "h") helps considerably in the focusing. The performance of the analyser is then tested and, depending upon the adjustment provisions made in the design, can be optimized. In spite of the large number of parameters, the immediate nature of the information visually collected behind the spectrograph allows thorough tests of the various components and at the same time imparts an appreciation of the sensitivity of the resolution on their settings. This includes the effects of higher-order aberration-correcting elements such as sextupole magnets. The tuning procedure followed has to be adapted to the particular experimental layout and is considerably helped by careful precalculations of optimum conditions. Once a narrow line is obtained in the focal plane, \( \frac{x}{\theta^2} \) and \( \frac{x}{\theta^3} \) aberrations of the spectrograph can be calculated from the simple measurement of the line's displacement in the focal plane as a function of the spectrometer angle. The displacement can be observed as the spectrometer is rotated slightly to the left and right about its zero degree position. Figure 5 shows results for MSU's split pole spectrograph\(^{12,6}\). Finally, beam emittance measurements can and have been made for both the MSU Cyclotron\(^{13}\) and the IPN Tandem\(^{14}\). The knowledge of the beam emittance is essential in planning high resolution experiments. These may require that the emittance be reduced, using slits, at a cost of reduced beam current.
**Conclusion**

The excellent resolution of the spectrum by Nann and Nolen\(^1\) shown in Fig. 6 testifies to the success of the on-line tuning method used by Blosser et al.\(^3\) An example obtained with 0° tuning for a small cross section reaction, (i.e. \(\text{<} |\mu\text{b/sr}\)) for the \((^{3}\text{He}, ^{6}\text{He})\) reaction\(^16\) is shown in Fig. 7. There the weakness of the reaction yield requires that all setting be arrived at independently of the reaction studied. The spectrum in Fig. 7 is indeed an excellent result, where the resolution is dominated by target thickness effects.

In setting up for optimum resolution, the quadrupole settings for combined dispersion and kinematic matching can be calculated, but achieving the correct field gradients is in general a non-trivial task due to uncertainties in field calibration data. One knows from the results of the TRANSPORT calculation, the setting required for the various beam line elements, but achieving these setting in actual beam lines is often a difficult task. A method for overcoming this problem which has been used at Orsay\(^7\) was to

a) calculate the focus on target condition for \(K=0\),

b) adjust the quadrupole currents to values as close as possible to the calculation consistent with maintaining a sharp focus and the desired values of the magnification in both transverse directions,

c) then change the currents in the quadrupoles by the relative amounts shown in the calculation for the \(K=0\) and for the desired \(K\neq 0\) value.
This method does work well as shown in Fig. 8. In the VHV scheme at the Los Alamos HRS, a resolution of 20 keV at 500 MeV was achieved recently after 0° tuning of the spectrograph. So while both HHH and VHV can be used for the highest resolution, the advantages of VHV for heavy ions dictated that choice at NSCL.

Acknowledgements

Several helpful discussions with P. Roussel (IPN) and J.A. Nolen, Jr. (MSU) are gratefully acknowledged.
IV. C-9

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17. J.A. Nolen, Jr. and A. Thiessen, private communication.
Fig. 1 Three particle orbits chosen to demonstrate matching requirement in a spectrophotograph. Dispersion and scattering are in the same plane, and kinematic correction is made via focal plane motion.

Fig. 2 Example of a beam correlation required on target for dispersion and kinematic compensation using the Bacchus spectrometer.
Fig. 3 Viewing apparatus positioned in the spectrograph focal plane. Light from the lens traverses a plastic window before entering the television camera lens. The two vertical wires in the insert, which are 25 μm in diameter and are separated by 2 mm, serve to calibrate the width and relative position of the beam image. (Ref. 6).

Fig. 4 Block diagram of tuning method with spectrometer at 0°.
IV. C-13

![Graph]

Fig. 3 Image position as a function of spectrograph angle relative to the beam. These data determine the horizontal aberration coefficients \(y/y'\) and \(y/y''\). Three focal plane positions are shown. These data also locate precisely the zero-order focal plane position and the best “zero-degree” entrance angle for the spectrograph. (Ref. 6).

![Graph]

Fig. 6 Spectrum of protons inelastically scattered by a \(^{31}\text{P}\) target from a 35 MeV beam, showing 2.5 keV resolution (Ref. 15).
Fig. 7 Spectrum of $^{24}\text{Mg}({}_3\text{He},{}^6\text{He})^{24}\text{Mg}$. Resolution is approximately 30 keV FWHM (16).

Tuning for $K=0$ (Au)

$^{12}\text{C}$ elastic scattering on $^{12}\text{C}$ and $^{197}\text{Au}$

$E_1=72.2\text{MeV}$ $\theta_{\text{lab}}=7^\circ$

Fig. 4 Spectres en position obtenus derrière le spectromètre avec une ouverture angulaire très étroite ($\pm 1.5\text{ mrad}$) à l'entrée, pour deux réglages des éléments de transport du faisceau. On notera l'amélioration spectaculaire de la résolution du pic de diffusion sur le $^{12}\text{C}$ avec le second réglage. PSD spectra obtained behind the spectrometer with two tunings of the beam transport system. Both are taken with a very narrow ($\pm 1.5\text{ mrad}$) entrance aperture of the spectrometer. A drastic improvement of the resolution is observed for the elastic scattering on $^{12}\text{C}$ with the tuning adjusted to its actual value $K=0.12$. (ref. 7).
Abstract

The problem of designing an ion optical system with the minimum line width is addressed in this paper. The importance of the effects of the details of the first-order layout on the higher order aberrations is discussed. Procedures for minimizing the induced third and higher order aberrations are presented. We also present a technique for testing if these higher order aberrations are important when only second order intrinsic aberration matrices are available.
Introduction

At Michigan State University we are designing two quite different electromagnetic devices for analysis of charged particle spectra. One is a large solid angle (20 msr), high dispersion (10 cm/%) magnetic spectrograph for momentum measurements of reaction products. The other is a small solid angle (1 msr) low dispersion (1 cm/%) mass separator for fast heavy ion reaction products. In the course of the optical designs for these devices we have found that correction of second order aberrations can easily create higher order aberrations which are larger than the original second order aberrations. A systematic investigation of the higher order aberrations was thus begun in order to determine the source of the induced higher order terms and to understand how to minimize them.

Throughout this paper we use the matrix notation as used in the computer codes TRANSPORT and TURTLE\(^{(1)}\). For the purposes of this paper, a ray entering the optic system can be specified by 5 parameters: 1) \(x\) = displacement from the optic axis in the radial plane, 2) \(\theta = \frac{dx}{dz}\) where \(z\) is the coordinate along the optic axis, 3) \(y\) = displacement from the optic axis in the axial direction, 4) \(\phi = \frac{dy}{dz}\), and \(\xi = \frac{\Delta p}{p}\) where \(\Delta p\) is the momentum difference from the central ray and \(p\) is the momentum of the central ray. The trajectory of the ray is assumed to depend on these parameters.
and the properties of the optic system.

**Test System**

In order to generalize the results of our investigations, we have chosen not to use either of the designs being worked on at MSU as our test system. We use a modified version of the Brown achromat. The system, shown in Fig. 1, consists of four segments, with each segment consisting of a quadrupole triplet with simultaneous point-to-parallel and parallel-to-point focusing in both x and y directions. The entrance and exit drifts for the triplets are about 3 m. A dipole is situated with its center point on the boundary between the first and second segments and between the third and fourth. These two dipoles have identical bend angles and bend in the same direction. The resulting system is achromatic; its first and second halves are also identical.

Each quadrupole also has a sextupole component. The x-focusing quadrupoles have x-focusing sextupole components and the y-quads have y-sextupole components. By setting all x-sextupoles at the same strength and all y-sextupoles at the same strength and use multipole coupling coefficients of Brown one can show that the \((x/\theta \delta)\) and \((y/\phi \delta)\) aberrations can be corrected at the end of the system without affecting the other second order aberrations. Since the uncorrected system has essentially no second order aberrations besides \((x/\theta \delta)\) and \((y/\phi \delta)\), the corrected system has no second
order aberrations of the position of the rays at the final focus. (There are some unimportant second order angular aberrations remaining at the final image in this system.) The dispersion \( \frac{x}{\delta} \) at the intermediate focus can be easily varied by changing the bend angle of the dipoles. Small adjustments are made in the drifts before and after the triplets to maintain the perfect negative-unit first order monochromatic transfer matrix for each half of the system.

**Optimization of Line Width**

The test system was evaluated using the computer codes TRANSPORT and TURTLE. These codes are described in detail in the contribution to this conference by D. C. Carey\(^1\) and, therefore, will not be described in detail here. In brief, TRANSPORT is a second order matrix optics fitting program and TURTLE does pseudo-ray-tracing using locally second order matrices. TURTLE lets one investigate so-called "induced" third and higher order aberrations.

The first step was to determine whether there is an optimum dispersion for the system for a given geometrical acceptance and range of transmitted momenta. To test this, a series of calculations were done with different dispersions, different solid angles, and different momentum spreads. For each dispersion the sextupoles were adjusted to correct the second order aberrations. The resulting line widths for a 10% momentum range are shown in figure 2. Similar
results were obtained for 5% and 20% ranges. The abscissa parameter is the magnetic field in the dipoles which is linearly proportional to the dispersion. The line width for each solid angle has a minimum. The value of $B$ at the minimum ($B_{\text{min}}$) changes monotonically with solid angle as shown in Figure 3 for momentum ranges of 5, 10, and 20%. Each curve is approximately straight and the three lines are parallel; therefore the positions of the minima appear to be quite systematic. Our conclusion is that for a given solid angle and momentum acceptance there is an optimum dispersion for a minimum spot size. We point out that the important aberrations in these line widths are of third and higher order since the second order displacement aberrations are all zero. Thus, we find that it is possible to minimize the effects of third and fourth order aberrations by choosing the proper first order layout and that the choice depends upon the size of the phase space that the device should accept. The use of higher order multipoles to correct the induced higher order aberrations is beyond the scope of this paper. However, it seems intuitively obvious that a second order solution which is optimized to induce a minimum amount of higher order effects, will permit the multipole correction series to converge most rapidly.

For the present system, the systematic interdependence of angular acceptance, momentum acceptance and $B_{\text{min}}$ can
be expressed much more simply: the minimum line width is obtained when the monochromatic envelope is equal in size to the zero-solid-angle, nonmonochromatic envelope, i.e. 
\[ |(x/\theta)_{\text{max}}| = |(x/\delta)_{\text{max}}|, \]
in the region of the sextupoles. The authors have observed this relation to hold true for a variety of systems and would suggest using it as a rule-of-thumb when \((x/\delta)\) is the primary aberration to be corrected, eventhough a first principles explanation for the rule-of-thumb has not been derived.

Sextupole Scaling

It was shown above that by choosing the proper first order layout the induced higher order aberrations can be minimized. There are many situations where this is not possible, however. It would be advantageous if there was some simple technique for improving the line width for a fixed first order layout. One possible solution is described below.

The usual procedure of aberration correction is to get a perfect second order system, then correct third order, then fourth order, etc. Attaining the perfect second order system can create very large higher order aberrations, as was shown earlier. The true goal of the corrections is to minimize the line width regardless of the order of the aberrations. Thus, if the second order is not quite perfect and the higher order terms are small, the system may be better than if the second order terms are zero but the
higher order terms are large. One simple technique for relaxing the second order constraints is to change all sextupole strengths by the same ratio relative to the value needed for "perfect" correction. This ratio (or scaling factor) reflects the "amount" of correction. If the scaling factor is unity, then the second order system is fully corrected; if the scaling factor is zero, the system is uncorrected. By using the same scaling factor for all sextupoles we guarantee that all second order aberrations will change linearly with the scaling factor. In particular we ensure that if an aberration is approximately zero in both the uncorrected system and the corrected system, then it will be approximately zero for all values of the scaling factor between zero and one. To illustrate how scaling can be used, the results of a series of calculations are shown in figure 4. Here the line width is shown as a function of the scaling factor for the test system using three different dispersions. Again we define the scaling factor such that a value of zero has all sextupoles turned off and a value of one yields a perfect second order system. We observe that the B=5.0 kG curve (B is the value of the field in the dipoles; the dispersion and bend angle are proportional to this field.) has a minimum at a scaling factor of one and that the curve is approximately symmetric above this value. The B=8.0 kG curve, however, has its
minimum at a scaling factor of about 0.9 and is not symmetric about this value. Thus we see that by relaxing the second order constraints we can improve the total line width, in this case by about 30%. We also observe that the $B=3.0$ kG system is not improved by scaling. This implies that for this field the higher order aberrations are insensitive to the scaling factor. Why this is true is dealt within the next section.

**Calculation of Induced Aberrations**

All of the effects and systematics described above are the results of calculations of a system which only involve locally second order matrices. All of the third and higher order aberrations were "induced" by the second order aberrations. We shall now discuss how the aberrations are "induced", explain the sources of the systematics, and show how one can look for the "induced" aberrations when all that are available are the second order matrices.

First, the principles of matrix optics should be examined. The usual design technique is to make a first order design that accomplishes the goals one has set (resolving power, solid angle, achromaticity, etc.), next correct the important second order aberrations, then third order, and so forth to whatever order is necessary. This technique is possible because a multipole field of multipolarity $2(N+1)$ affects aberrations of order $N$ and higher.
but does not affect aberrations of order $N-1$ or lower.\(^{(3)}\) As such, designers usually do not worry about the higher order aberrations created by a correcting element of some given order since they can in principle correct these aberrations with higher order multipoles.

Nothing, however, guarantees that the above series is convergent. Each order of correction can create aberrations of the next order that are more harmful than the aberrations being corrected. This is the danger of matrix optics. Matrix optics intrinsically truncates the space in which one is calculating. Any parameter ($x, \theta, y, \phi$) at the end of the system is assumed to be expandable in a Taylor series of the parameters of the system at the beginning (e.g. $x_o, y_o, \theta_o, \phi_o$, and $\delta$). The Taylor expansion is usually truncated at first, second or third order terms. Because the transfer matrix of the system is calculated as the product of the transfer matrices of the individual elements of the system, the system's matrix is calculated only to the order that the matrices are known for each element.

It is possible to calculate, however, higher order "induced" aberrations even if the "intrinsic" higher order aberrations are not known. An "intrinsic" aberration is an aberration that a given device has independent of the presence of other devices in the system. An "induced" aberration arises when one device follows another and an $M$-th order
aberration of the first element can combine with an N-th order aberration of the second element to produce an aberration of the combined two elements of order as high as M x N. To illustrate this, consider a system with intrinsic aberrations of second order. Assume that the system is divided into two parts, A and B. Assume that A, the first part, has point-to-point and parallel-to-parallel focussing in both planes and has no second order aberrations except for a large $\theta/\delta^2$ term. Now assume that B, the second part, has point-to-point focussing in both planes and that B has a large $x/\theta^2$ aberration but all other second order aberrations are zero.

Now we can calculate the overall dependence of x on the initial parameters, i.e. to $x_o$, $\theta_o$, $y_o$, $\phi_o$, and $\delta$.

Let $x_2 = x$ at the end of the system, then

$$x_2 = \left(\frac{x}{x}\right)_B x_1 + \left(\frac{x}{\delta}\right)_B \delta + \left(\frac{x}{\theta^2}\right)_B (\theta_1)^2$$

(1)

where $x_1$ and $\theta_1$ are the values of x and $\theta$ at the beginning (end) of B (A) and where the subscript on the matrix element indicates from which part of the system that term comes.

Thus to get $x_2 = F(x_o, y_o, \theta_o, \phi_o, \delta)$ we need to know $x_1$ and $\theta_1$.

$$x_1 = \left(\frac{x}{x}\right)_A x_o + \left(\frac{x}{\delta}\right)_A \delta$$

(2)

$$\theta_1 = \left(\frac{\theta}{\theta}\right)_A \theta_o + \left(\frac{\theta}{\delta}\right)_A + \left(\frac{\theta}{\delta^2}\right)_A \delta^2$$

(3)

Therefore, combining eqns. 1, 2, and 3 and rearranging terms, one gets
IV. D-11

\[ x_2 = \{ (x/x)_A (x/x)_B \} x_0 + \{ (x/x)_B (x/x)_A \} \frac{x}{\delta} + \{ (x/\delta^2)_B (x/\delta^2)_A \} \delta^2 + \{ (x/\delta^3)_B (x/\delta^3)_A \} \delta^3 + \{ (x/\delta^4)_B (x/\delta^4)_A \} \delta^4 \]

The result is that the expression for \( x_2 \) involves not just first and second order terms but also third and fourth order terms, i.e., \( (x/\delta^2) \), \( (x/\delta^3) \), and \( (x/\delta^4) \). These third and fourth order terms are the "induced" aberrations. There are also induced second order aberrations, e.g. \( (x/\theta^2) \), but we do not concern ourselves here with those terms because they would be dealt with properly in a second order matrix calculation. This is obviously not true for the third and fourth order terms. The above analysis can be generalized to the following: if a calculation is done involving \( N \) elements with \( M \)th order intrinsic aberrations, then the system can have induced aberrations of order \( M, M+1, \ldots, M^N \).

Now that the origin of induced aberrations is understood, it is possible to investigate the source of the systematics found earlier for line-width versus dipole field. Table 1 gives the values of most of the nonzero \( x \) and \( \theta \) aberrations at the center of the corrected test system for dipole fields of 3, 5, and 8 kilogauss. Since the first and second halves of the test system are identical.
then these aberrations are those of both halves; the entire system has no second order aberrations of displacement however. Clearly the aberrations change with the dipole fields. This is because the sextupole strengths needed to correct the intrinsic aberrations (i.e. \(x/\theta \delta\) and \(y/\phi \delta\)) are dependent upon the dispersion which, in turn, depends upon the dipole field. The multipole coupling coefficients of Brown\(^{(3)}\) describe the dependence of a multipole's effect on a given aberration. The effect of a sextupole on the \((x/\theta \delta)\) aberration is given by

\[
\Delta(x/\theta \delta) = +B_3 M_x \int (x/\theta)^2 (x/\delta) \text{dz},
\]

where the integration is over the length of the sextupole, \(M_x\) is the magnification at the final focus, and \(B_3\) is the sextupole pole-tip field divided by the rigidity of the central ray and the square of the sextupole aperture radius. Since we are keeping the monochromatic optics fixed, the only thing that changes in the expression for \(\Delta(x/\theta \delta)\) is \((x/\delta)\) which is proportional to the dipole's field \(B_1\). Therefore, since \((x/\theta \delta)\) of the uncorrected system is also constant, we see that \(B_3 \approx 1/B_1\).

To explain how the minimum in the line width versus \(B_1\) can occur we need only look at the two aberrations, \((x/\delta^3)\) and \((x/\delta^2 \delta)\). Using the second order terms listed in Table 1 and following the prescription outlined earlier for calculating the induced aberrations one gets
IV. D-13

\[(x/\delta^3) = (x/x\delta)_B (x/\delta^2)_A \]  \hspace{1cm} (6)

\[(x/\theta^2\delta) = (x/x\delta)_B (x/\theta^2)_A \]  \hspace{1cm} (7)

as the dominant terms for \(x/\delta^3\) and \(x/\theta^2\delta\). Here we have already included the first order effects. It is possible to calculate what the value of \(x/\delta^3\) and \(x/\theta^2\delta\) will be. For simplicity, we assume that there is one sextupole in each half of the system; the two are energized identically and are placed equidistant from the intermediate focus. Using the coupling coefficients it is possible to find \((x/x\delta)\), \((x/\delta^2)\), and \((x/\theta^2)\) as a function of \(B_1\), assuming that they are negligibly small if \(B_3=0\). The result of applying the coupling coefficient equations and inserting the results into eqns 6 and 7 give the following:

\[(x/\delta^3) = \frac{1}{4}a^2 (x/x)C_1B_1 (x/\theta)^2 \]  \hspace{1cm} (6')

\[= \text{const} \cdot B_1 \]

\[(x/\theta^2\delta) = \frac{1}{4}a^2 (x/x) / C_1B_1 \]  \hspace{1cm} (7')

\[= \text{const./}B_1 \]

The "a" in eqns 6' and 7' stands for the uncorrected value of \((x/\theta\delta)\), and \((x/\delta)\) at the sextupole is given by \(C_1B_1\).

To test the above predictions, a comparison to the results for the test system can be made. Table 2 lists the values of \((x/\delta^3)\) and \((x/\theta^2\delta)\) calculated from the values in Table 1 for the three values of \(B_1\). The aberration \((x/\delta^3)\) increases by a factor of 8 while \((x/\theta^2\delta)\) decreases by a factor of 2 when \(B_1\) changes from 3 kG to 8 kG. These
two aberrations do not account for all of the line width at all values of $B_1$, but their dependences on $B_1$ show how a minimum line width can occur, i.e., where the contributions of $(x/\theta^2 \delta)$ and $(x/\delta^3)$ are roughly equal. For lower $B_1$ fields $(x/\theta^2 \delta)$ increases the line width, and for higher $B_1$ fields $(x/\delta^3)$ increases the line width. The predictions are thus consistent with the calculations. The dependences of the minimum's position on solid angle, momentum acceptance, and dipole field are now understandable. For each combination of solid angle and momentum acceptance there is a certain optimum ratio of geometric and chromatic aberrations. If the solid angle is increased then the amount of geometric aberration must be decreased. Thus the minimum shifts to a higher value of $B_1$ since $(x/\theta^2 \delta)$ decreases as $B_1$ increases. The inverse is true for an increase in the momentum acceptance, which increases the importance of the chromatic aberrations. The chromatic aberrations increase with $B_1$, therefore if the momentum acceptance is increased, then the minimum line width occurs at a lower value of $B_1$.

Finally, to determine if the system one is investigating has large induced aberrations, one need only carry out calculations similar to those described for $(x/\theta^2 \delta)$ and $(x/\delta^3)$. Such calculations would be quite tedious if done manually. However, with a computer code which is in some way coupled to a second order optics code the calculations become rather simple and the 19 third order aberrations can
be quickly calculated. Extension to fourth order terms would certainly increase the complexity of the computer code but could well be useful. For example, the dominant aberration for the test system with $B_1 > 6$ kG was not $(x/\delta^3)$ but $(x/\delta^4)$, which can be induced as $(x/x^2)_B(x/\delta^2)_A^2$ and is proportional to $B_1^4$.

The above discussion serves to explain the systematics of the induced aberrations in our test system. This procedure should not replace a full evaluation of any given system with a full raytracing code, however. For example, even if the induced aberrations are estimated to be small by this method, other problems may exist. Firstly, the system may have significant higher order intrinsic aberrations which are not treated at all by this procedure. Secondly, large induced aberrations may arise from terms within each portion of the segmented system rather than by couplings between the segments.

**Summary**

In this paper, we have investigated the role of induced aberrations in ion optical designs. We have not only shown how the usefulness of a "perfect" second order design can be completely destroyed by induced aberrations but also how to go about trying to minimize their effects, i.e. by optimizing the ratio of the monochromatic envelope to zero-solid-angle, chromatic envelope or by scaling the sextupole strengths.
We have also outlined how to estimate if the induced aberrations will be significant by taking the products of appropriate second order aberrations. This procedure should prove useful to people who have easy access to only a second order matrix code. If a third order code, such as GIOS\(^{(4)}\) is available then the induced third order aberrations are included in the code's results; an induced fourth order aberration is obviously not included but can be calculated as a product of three second order terms or as a product of one second order term and one third order term.

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References

* Permanent address SLAC, Stanford, CA 94305.

1 D. C. Carey, this conference.
4 J. Brezina, K. Becker, and H. Wollnik,
Values of the Second Order Aberrations for the First Half of the Test System at Three Different Dipole Fields

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IV. D-19

Table 2.

Values of Two Induced Aberrations in the Test System

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IV. D-20

Fig. Captions

1) Schematic of first-order optics of the test system. The $y/y$ and $y/P$ rays are similar to the $x/x$ and $x/\theta$ rays since the focussing conditions are the same for both planes. The labels under the elements indicate how each is energized. Any two elements with the same label are energized identically. A, B, and C are combined quadrupoles and sextupoles, and D is dipole. A and C have the same sextupole components.

2) Line width versus dipole field is shown for the corrected test system. The various curves are for different solid angles. The momentum acceptance is held constant at 10%. Note that there is a minimum in each curve. The $\theta$ and $\phi$ ranges are equal for each curve and are: a) 1 mr, b) 2 mr, c) 5 mr, d) 7.5 mr, e) 10 mr, and f) 20 mr.

3) The positions of the minima in the line width versus dipole field are shown. These positions were obtained from figure 2 and other plots for 5% and 20% momentum ranges.

4) The effect of scaling the sextupoles is shown. Note that the curves for $B_1 = 3$ kG and 5 kG have minima at a scaling factor of 1.0. The $B_1 = 8$ kG curve is minimum at a scaling factor of 0.9.
B₁ for minimum line width

\[ -2.5 \leq \delta \leq 2.5 \]
\[ -5.0 \leq \delta \leq 5.0 \]
\[ -10.0 \leq \delta \leq 10.0 \]
Integrated Magnet, Optics and Detector Designs for a Spectrograph*

J.A. Nolen, Jr.
National Superconducting Cyclotron Laboratory
Michigan State University
East Lansing, Michigan 48824

ABSTRACT

This report illustrates how the physics measurement goals, practical considerations and/or restraints, beam properties, and detector limitations can all be integrated to lead to an optimized spectrograph design. The various measurement goals are considered iteratively or interactively with the magnetic and detector limitations and specifications to achieve a cost effective design with an enhanced probability of success.

*This material is based upon work supported by the National Science Foundation under Grant No. PHY-22696 and DOE-DEAC-02-80-ER 10579.
The route to an optimized spectrograph design requires a large degree of flexibility and interaction between the measurement goals and the achievable magnetic and detector properties. The starting point is always the goals of the physics measurements to be carried out with the instrument. However, these goals must be matched to what can be reasonably expected with modest advancements in state-of-the-art magnet and detector design. For example, larger solid angle devices are clearly better. However, energy and angular resolution requirements usually dominate the specifications. Hence, a compromise may be to build a device which easily achieves design resolution at one half the full solid angle, but still has reasonable resolution at full solid angle. To be very rigid in the specifications is likely to greatly increase the cost of the project.

This report will not cover the basic design considerations of how to achieve the required first order resolving power or how to optimize the solid angle through a given magnet gap. These points are covered very nicely in the classic paper by Karl Brown\textsuperscript{1}, and illustrated in detail in a paper by S. Martin, \textit{et al}\textsuperscript{2} on the design of the Big Karl spectrograph in Julich. Here we address some of the other options which are available in optimizing a spectrograph system design.

The procedure used in the design of the 1.2 GeV/c high resolution spectrograph for heavy ion research at the National Superconducting Cyclotron Laboratory is outlined in this report. It is impossible to detail all of the considerations which went into the final choice of optical mode, zero-order spectrograph
component layout, etc., so only the most important goals, constraints, and guidelines are spelled out below. The resulting beam analysis system and spectrograph designs are presented in more detail in separate reports at this conference.

The first order parameters of the system are largely determined by a combination of the physics goals (table I) and the practical constraints (table II). Typical guidelines to be kept in mind while striving to meet the physics goals are listed in table III. The practical constraints are simply that the spectrograph must meet specifications while accepting all or most of the beam from the accelerator, the beam spot should be reasonably small, the system must physically fit within certain building constraints and the design should be cost effective.

The physics goals were determined by the types of experiments planned for the system. These goals were tempered somewhat by what was considered achievable with a state-of-the-art system. For example, a momentum resolution significantly greater than 20,000 would be useful for resolving states in two body reactions with beam energies of up to 8 GeV. However, excellent angular resolution (~1 mr) and large solid angle (>10 msr) are also essential in heavy ion reactions. This combination of goals was then accepted as meeting the basic needs of the experiments while at the same time representing a combination of momentum resolution, and solid angle not previously achieved in nuclear physics.

The unusual physical layout chosen for this spectrograph conveniently satisfies several of the goals, constraints and
and guidelines. Firstly, vertical dispersion and horizontal scattering essentially decouples the momentum and scattering angle measurements. This is most relevant to beam phase space matching to the spectrograph. For example, in the dispersive direction it is desirable to have a small incoherent spot size on target, with the implied relatively large divergence, whereas in the scattering direction small beam divergence, and the consequent relatively large spot size, are dictated.

Once vertical dispersion was chosen, it was most convenient to place the spectrograph in a pit with the target several meters below grade level. The analysis system then is the same set of magnets which delivers the beam to the lower level. This layout also fit very nicely into the planned building addition.

The spectrograph mode was chosen to be the normal point-to-point imaging in the dispersive direction, but parallel-to-point in the axial direction to facilitate accurate scattering angle determinations. This is the system in use at the LAMPF HRS in Los Alamos. Thus, in first order, the focal plane is two-dimensional with $x$ determining the particle's momentum and $y$ its scattering angle. Kinematic effects, i.e. momentum variation with scattering angle, are easily compensated for via software with this system. Hence, once the beam line and spectrograph parameters are optimized, e.g. at zero degrees, no retuning is necessary as scattering angle is varied.

The choice of first order resolving power, or $D/M$, for the system is intimately related to the resolution specification.
Actually it is $D/Mx_0$, where $x_0$ is the incoherent spot size, which determines the first order resolution. It is common to assume $x_0 = 1\text{ mm}$ at the target so that $D/M = 20 \text{ cm/\%}$ implies a resolution of 20,000. However, in the present design a less conventional choice has been made, i.e. a spectrograph $D/M = 10 \text{ cm/\%}$ combined with an analysis system $D/M = 20 \text{ cm/\%}$. This yields a factor of two larger solid angle for a given dipole width in the spectrograph while still permitting an overall momentum resolution of 20,000 at the spectrograph focal plane. In this system it is assumed that there is an incoherent spot size $x_0 = 1 \text{ mm}$ at the object slits of the beam analysis system. The dispersion matching condition specifies that the dispersion of the beam at the target must equal the $D/M$ of the spectrograph or $D = 10 \text{ cm/\%}$ on target in this case. This implies $M = \frac{1}{2}$ at the target since $D/M = 20$ for the beam, yielding an incoherent spot size of $0.5 \text{ mm}$ at the target. With the spectrograph $D/M = 10$ and $M = 1$, the line width in the focal plane is also $0.5 \text{ mm}$ with a dispersion of $10 \text{ cm/\%}$, giving a momentum resolution of 20,000.

It appears that this scaling down of the spectrograph resolving power could be carried to extremes, but there are other considerations which limit it. Firstly, for a given beam momentum spread $\delta_B$, the dispersed spot width at the target is $x_t = (D/M)_s \delta_B$, i.e. larger spectrograph $D/M$ implies large target spot width and conversely for small $D/M$. In the present case, the choice of $(D/M)_s = 10 \text{ cm/\%}$ combined with $\delta_B = 0.1\%$ gives a target spot width of 1 cm. This is a reasonable size for rare isotope target materials. A smaller $D/M$ value would reduce the spot size even further,
but for a given beam current the power density, due to energy loss in the target, would be increased. With the large energy losses of heavy ion beams this is an important consideration, establishing a lower limit for acceptable values of \((D/M)_s\). Secondly, as the \(D/M\) of the spectrograph is reduced and the \(D\) and \(M\) of the beam line reduced to match it, the angular divergence of the beam on target is increased. The angular divergence with \(D_B=10\text{cm/}\%\) and \(M_B=0.5\) will be \(2\text{mr}\) if the incoherent beam emittance is \(1\ \text{mm-mr}\) or \(10\text{mr}\) if the emittance is \(5\ \text{mm-mr}\). When the spectrograph is near zero degrees this term directly limits the accuracy of the scattering angle determination. Hence, to reduce \((D/M)_s\) still further would increase the beam angular divergence beyond acceptable values.

The previous considerations helped in the selection of \((D/M)_B, D_B, \) and \((D/M)_s\), but they do not determine the spectrograph focal plane dispersion \(D_s\). This term and the associated magnification \(M_s\) determine the spatial line width of the beam at the focal plane and the length of the focal plane for a given spectrograph momentum range \(\delta_s\). For the present spectrograph \(D_s=10\text{cm/}\%\) and \(M_s=1\) were chosen, giving a line width of \(0.5\text{mm}\) for momentum resolution of \(20,000\), and a detector length of \(50\ \text{cm}\) for a momentum range of \(5\%\). It is assumed that a detector resolution of \(0.2\text{mm}\) for a \(50\text{cm}\) long detector is attainable, so that detector resolution should not limit the system resolution. Detectors with these specifications for high energy protons and pions have been developed and are in use in the EPICS
and HRS systems at LAMPF in Los Alamos. Decreasing $M_s$ would begin to push the detector technology beyond proven limits, while increasing $M_s$ would result in an unnecessarily larger detector system and vacuum box.

The above discussion was limited to first order considerations, but to achieve high resolution at large solid angle also requires careful consideration of the effects of optical aberrations in the system. Orbit calculations are carried out using the programs RAYTRACE or MORDER to evaluate the magnitudes of the aberration terms. In the present system with no edge curvatures or other correcting elements the dominant aberrations are $(x/\theta^2)$ and $(y/\theta^6)$, with the former limiting momentum resolution and the latter angular resolution. In a system such as this one, where angular resolution is as important as momentum resolution, it seems to be impossible to fully correct both of these terms without introducing new second and higher order terms in the process. However, since $\theta$ in the focal plane must be measured for accurate scattering angle determination, this information can also be used for aberration correction.

The degree to which aberrations can be corrected via focal plane angle measurements depends on the accuracy possible in these measurements. For example, in the uncorrected system $x_f$ vs. $\theta_f$ is a parabola with very steep slopes at the ends:

$$\frac{dx_f}{d\theta_f} \approx 0.8 \text{ mm/mr}$$

at $\theta_{\text{max}}$. So this term in the uncorrected system is about a factor of 2 larger than is correctable by focal plane measurements with
IV. E-8

1 mr uncertainties. Similarly \( \phi_f \) measurement gives some information about \( y_t \) and can, therefore, be used for partial correction of the \( (y/y_\theta) \) term. Hence, with two two-dimensional detectors in the focal plane most aberration effects can be corrected with software. It is only necessary to partially correct the aberrations with edge curvatures or sextupoles, in order to bring them down to a size which is correctable within the accuracy of the focal plane measurements.

The specified resolving power, momentum range, and solid angle of the spectrograph, combined with the chosen first order layout, determines the volume of "good" magnetic field required in the dipoles. The choice of dipole design then to a large extent determines the cost of the system. If the dipoles are over designed, they can easily cost two to three times more than necessary. However, if they are under-designed or poorly constructed the system will not meet the resolution specifications at the designed solid angle. [It will have good resolution if the solid angle aperture and/or the beam phase space are stopped down far enough.]

In a system, such as the present one, where focal plane measurements of \( x_f, y_f, \theta_f, \) and \( \phi_f \) are made for physics reasons and for aberration correction, the dipole field flatness specifications can be greatly reduced. For example, it is certainly not necessary to specify a field flatness of 1 in 20,000 in order to obtain a momentum resolution of 20,000. It is essential that the dipole fields not have small scale or fine structure, but slowly changing fields across the dipole widths can be accounted for via the focal plane measurements in the same way as the intrinsicic aberrations.
discussed above.

Quantitative consideration must be given to the upper limits of field gradients which can be accommodated with the focal plane measurements. For example, the following analysis shows that in the present design field gradients of up to 1 gauss per cm in the radial direction can be corrected via focal plane angle measurements. Consider a ray with the central momentum which leaves the target with angle $\theta_t$ and hits the focal plane at position $x_f$ with angle $\theta_f$. This ray passes through the dipole with an average radial displacement $\bar{x}_D$ given by:

$$\bar{x}_D = (x/\theta) \theta_t,$$

when $(x/\theta)$ is the average value of the matrix element $(x/\theta)$ within the dipoles. Since a change or error in the magnetic field is equivalent to the same percentage change in momentum, then a field variation $\Delta B$ changes the $x$-coordinate at the focal plane by:

$$\Delta x_f = (\Delta B/B)(x/\delta)$$
$$= (B - B_0)/B_0 (x/\delta).$$

Assume the actual dipole field $B$ is given by

$$B = B(x_D),$$

such that

$$\frac{dB}{d\theta} = \frac{dB(x_D)}{dx} (x/\theta).$$

These equations can be combined to give the uncertainty in focal plane $x$-coordinate $\delta x_f$ as a function of the uncertainty $\delta \theta_t$ in the initial angle at the target:
\[ \delta x_f = \left( \frac{1}{B} \frac{dB}{dx} \right) (x/\delta) (x/\delta) \delta \theta_t. \]

Finally, the uncertainty in \( x_f \) is converted to momentum uncertainty \( \delta (\Delta p/p) \) by dividing by the dispersion \( (x/\delta) \), and the uncertainty in \( \theta_t \) is related to the uncertainty in the measured quantity \( \theta_f \) by the magnification \( M_\theta = M^{-1}_x \), yielding:

\[ \delta (\Delta p/p) = M_x (x/\delta) \left( \frac{1}{B} \frac{dB}{dx} \right) \delta \theta_f. \]

For our system \( M_x = 0.8 \) and \( (x/\delta) = 0.7 \, \text{cm/mr} \), so if the angular measurement uncertainty \( \delta \theta_f \) is 1 mr, then a gradient of 1 G/cm at 15 kG yields:

\[ \delta (\Delta p/p) = \frac{0.8 (0.7) (1.\text{mr})}{15,000} \approx 1/27,000. \]

In other words, field gradients of as much as 1 part in 15,000 per cm can be tolerated in the dipoles if the focal plane angle is measured to 1 mr uncertainty and the magnitudes of these gradients are measured.

In conclusion, a few of the many considerations which lead to a final spectrograph design have been discussed. Many of the choices are directly related to the physics goals of the system, others are related to the assumed beam quality of the accelerator, and others are to some extent determined by the taste of the designer. The main message is that in an optimized system each component should be designed with the performance of the overall system in mind. For example, the overall cost may be reduced by requiring more from the detectors and less from the dipoles, etc.
IV. E-11

Such optimization can also lead to increased probability of success through the increased flexibility introduced into the system.

References

3. The idea of placing the spectrograph below grade level in order to optimize space in the existing building plan was suggested by Edwin Kashy.
4. The idea of using a larger resolving power in the beam analysis system than in the spectrograph itself came from discussions with P. Roussel of Orsay.
TABLE I.—PHYSICS GOALS

1. Momentum Resolution (800 keV at 8 GeV) 20,000
2. Angular Resolution (diffraction structure) 1 mr
3. Solid Angle (event rate) "Large" (10-20 msr)
4. Momentum Range (100 MeV at 1 GeV) 5%
5. Particle Rigidity (accelerator range) 0.4 GeV/c to 1.2 GeV/c

TABLE II.—PRACTICAL CONSTRAINTS

1. Beam phase space (accelerator property) 5mm-mr, 0.1% Δp/p
2. Target size (rare isotopes) 1 cm x 1 cm
3. Must fit in reasonable space
4. Budget $2,000,000 (1983-84)

TABLE III.—GUIDELINES

1. No detectors can be in the beam or in the spectrograph before the focal plane because of multiple scattering effects.
2. Δθ and Δφ must be measured for physics reasons, hence, this information can also be used for aberration correction, and dipole magnetic fields do not have to be perfectly "flat" or invariant with field strength.
3. Assume 1 mm object spot size for beam analysis system. This system must have a resolving power of 20,000 to achieve an overall momentum resolution of 20,000. The spectrograph itself can have a smaller resolving power (i.e. 10,000) without limiting the resolution.
4. It should not be necessary to retune the system as the scattering angle is changed.
5. Focal plane detectors with position resolution of 0.2 mm FWHM and lengths ≤ 60 cm are feasible.
6. Superconducting coils can be used to reduce initial and operational cost.
The MSU 1.6 GeV/c Beam Analysis System*

J.A. Nolen, Jr.
National Superconducting Cyclotron Laboratory
Michigan State University
East Lansing, MI 48824

ABSTRACT

This report describes the design of the beam analysis system which is to prepare the beam for the large high-resolution spectrograph to be used with the superconducting heavy ion cyclotrons at MSU. The system has a momentum resolving power of over 20,000 with a 1 mm object slit and is of a very flexible, modular design which is readily adaptable to other applications.

*This material is based upon work supported by the National Science Foundation under Grant #NSF78-22696 and DOE-02-80-ER-10579.
The design of the MSU large solid angle, high resolution, 1.2 GeV/c spectrograph has been described in a separate paper at this conference. This paper reports on the design of the beam analysis system which prepares the heavy ion beam so that the spectrograph system operates in the dispersion-matched or energy-loss mode. This beam analysis system must operate at beam momenta of up to 1.6 GeV/c even though stripping of the heavy ion at the target limits the maximum rigidity requirement of the spectrograph to 1.2 GeV/c.

The plan and elevation views of the beam analysis and spectrograph system are shown in figure 1. The design of the beam analysis system was somewhat constrained by the physical layout of the laboratory. The spectrograph disperses vertically and is about 8 m tall. Hence, a convenient, space-efficient solution was to put the spectrograph in a deep pit and disperse the beam vertically via two 45° bends which also serve to bring the beam to the lower level. The other specifications for the system are a momentum resolving power of at least 20,000 with a 1mm object slit, a phase space acceptance of 5 mm-mr (base-to-base) in both the radial and axial directions, and a range of ±0.05% Δp/p.

The optics of the resulting analysis system are illustrated in figure 2 where the beam envelopes from a TRANSPORT calculation with above mentioned phase spaces are given. The specified resolving power requirement of 20,000 combined with the two 45° bend angles determined that the average value of the (x/θ) matrix element in the dipoles must be 1.2 cm/mr. The other constraints put on the system are that there be a y-waist and
IV. F-3

(θ/θ)=0 at the center of each 45° bend, and there be a y-waist and x-image at the slit between the 45° bends. There is also an x-image and a y-waist at the target.

These optical conditions were met by the following fitting procedure. Each 45° bend is made via the modular units illustrated in figure 3, which consist of two 22.5° rectangular dipoles, two quadrupole doublets, and a sextupole. For an arbitrary initial drift distance the y-waist and (θ/θ)=0 conditions at the center of the first 45° bend can be obtained by adjusting the quadrupole doublet. The first quadrupole is x-defocussing to increase (x/θ) in the dipoles. However, for short initial drift distances the value of (x/θ) is still too small. The initial drift is then increased and the quads readjusted until (x/θ) is large enough and the other two conditions are still satisfied. In the case illustrated in figure 2 the resulting initial drift distance is 8 m. The six meter long five-quadrupole phase space rotator, described in a separate report, fits conveniently into this space. The remaining distances are all fixed by building constraints. The x-image and y-waist at the intermediate slit are easily obtained via adjustment of the second quadrupole doublet. One-half of the resolving power of the complete system exits at this slit, so it can be used to limit the beam momentum spread to ~ 0.1% if necessary, or to monitor beam energy variations. By symmetry, the three conditions at the center of the second 45° bend are met if the third quadrupole doublet parameters are the same as the second.
The energy-loss or dispersion-matching condition requires that the dispersion value \((x/\delta)_B\) of the beam analysis system at the target position be numerically equal to the resolving power, \(D/M\), of the spectrograph, i.e. \((x/\delta)_B = -11.57 \text{ cm/}\%\) in our case. [The \(x\)-direction is actually vertical in the lab due to the vertical dispersion of the beam analysis system and the spectrograph.] Also the size of the \(y\)-waist on target determines the \(y\)-divergence and the related angular uncertainty in the scattering angle measurement. The four constraints of \(x\)-image, \(y\)-waist, \(x\)-magnification (dispersion), and \(y\)-size on target are achieved by proper adjustment of the four quadrupole singlets in the last leg of the analysis system. With these four quadrupoles a range of \(x\)-magnifications and \(y\)-sizes are achievable. The \(x\)-magnification variations are required to dispersion match the system for nuclear reaction \(Q\)-values different from zero, while the \(y\)-size variation permits optimization of angular resolution vs target spot size.

An additional parameter which was not mentioned above is the value of \((\theta/\delta)\) at the target position\(^2\). This parameter is related to kinematic compensation and scattering angle determination when the spectrograph is positioned near 0°. The spectrograph has a 7° acceptance angle in the dispersion direction and a 10° acceptance angle in the axial direction. The spectrograph mode is parallel-to-point in the axial direction so the angle within the aperture in this direction is determined to within 1 mr by a \(y\)-position measurement in the focal plane. However, when the spectrograph is near 0° the reaction scattering
angle is also determined by the ray angle in the dispersion direction. The spectrograph mode is point-to-point in this direction, so $\theta_{\text{focal plane}}$ must be measured to determine $\theta_{\text{target}}$.

However, for rays which do not originate from $x=0$ at the target due to the dispersion matching condition, the scattering angle inferred from $\theta_{\text{focal plane}}$ will be erroneous. This is illustrated in figure 4. Rays 1 and 1' represent the central rays of the beam and the spectrograph, respectively. If ray 1 passes through the target without angle change then $\theta_{\text{focal plane}}=0$. If ray 1 does change angle by an amount $\theta_{\text{target}}$ then $\theta_{\text{focal plane}}=(\theta/\theta)\theta_{\text{target}}$, and the scattering angle is properly inferred from the focal plane measurement if $(\theta/\theta)$ is known. Now consider ray 2 which has a beam momentum offset $\delta_B$, and hits the target at the position $x$ determined by the dispersion matching condition. This ray also hits the target with a finite angle $\theta$ determined by $(\theta/\delta)$ of the beam line, which is $-50$ mr/% for the case illustrated in figure 2. On the other hand, ray 2' is the ray which will hit the focal plane at the position of ray 1' (dispersion matched) and will also have $\theta_{\text{focal plane}}=0$. For $\delta=0.05\%$, ray 2 hits the target with $\theta=-2.5$ mr while ray 2' has $\theta=+3.8$ mr. Hence a scattering which changes ray 2 to 2' corresponds to $\Delta\theta=2.5+3.8=6.3$ mr, but yields a $\theta_{\text{focal plane}}=0$, i.e. the focal plane measurement misinterprets the scattering angle by about $1/3^\circ$. This may be significant in certain critical applications such as with
large kinematic shifts or with very rapidly changing cross sections.

The beam analysis system can be dispersion matched in angle ($\theta/\delta$), as well as in position ($x/\delta$), by adding one or two quadrupoles to the system shown in figure 2. The beam envelopes for such a design are shown in figure 5, which also includes the beam envelope through the spectrograph. A single quad has been added at the position of the slits between the 45° bends and the fourth quad on the last section before the target has been shifted towards the target. The quad at the intermediate slit position could be a "split singlet" to allow access to the slits. The quad at the intermediate slits changes ($\theta/\delta$) without affecting other fit conditions. And the final quad near the target permits additional fine tuning of ($\theta/\delta$) with little effect on other parameters.

The beam half-widths in the x and y-directions for the complete dispersion matched system are given in figure 5. The x-spot size at the target due to a beam momentum spread of 0.1% is 11.6 mm whereas this is reduced to 0.45 mm at the focal plane due to the dispersion matching. A single ray with a momentum offset of 0.1% is traced through the system in figure 6. Here $x_{\text{initial}}$ and $\theta_{\text{initial}} = 0$, and $x_{\text{focal plane}}$ as well as $\theta_{\text{focal plane}} = \cdot\cdot\cdot$, i.e. the combined beam analysis - spectrograph system is fully achromatic.

The matrix optics details of the beam line, the spectrograph, and the combination are given in the table. The assumption about the initial beam vector and the resulting beam vector
IV. F-7

at the target position are also included.

The discussion thus far has been limited to first order considerations. Second order TRANSPORT calculations have indicated that the only two second order intrinsic matrix elements which are significant even at the 0.1 mm level in the beam line are $(x/\theta\delta)$ and $(x/\theta^2)$. Two sextupoles, one at the center of each 45° bend unit, easily correct these terms. The matrix element $(x/\theta)$ is large in both bend units, but $(x/\delta)$ is only large in the second. Hence S2 corrects $(x/\theta\delta)$, while S1 corrects the intrinsic and induced $(x/\theta^2)$ terms. Variable sextupoles at the positions indicated are preferable to fixed value edge curvatures because this permits the dipoles to have sextupole components of their own which vary with magnetic field level.

It is possible that third order effects will require the addition of octupole components to the beam line also. Detailed raytracing will be done on the system to determine the extent of third and higher order effects.

The beam line analysis system described in this report is very flexible and readily adaptable to other applications. The fact that the dispersion is vertical is not an important feature of the design, it could equally well be horizontal. The final beam direction is parallel to, but displaced vertically (or horizontally) from the original direction. The "bend unit" module could be almost any angle, and the $(x/\theta)$ matrix element adjusted to achieve the desired resolving power. Likewise, the quadrupole doublets at the entrance and exit of each bend
unit allow large variations in the distances between magnets, i.e. building constraints can easily be worked around. Furthermore, other transport modes such as dispersionless and fully achromatic images at the target position can also be achieved by retuning the quadrupoles of this system with the appropriate fitting constraints.

Footnotes

1. The concept of putting the spectrograph in a pit and using a vertically dispersing analysis system to transport the beam to the lower level was developed in conversations with Edwin Kashy.

2. The issue of the relevance of this term was raised at the conference by David Hendrie. This paragraph is in answer to his question.
Table I. The vectors and matrices representing the beam, the analysis system, the spectrograph, and the combination. The notation is as in the code TRANSPORT.

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<thead>
<tr>
<th></th>
<th>x (cm)</th>
<th>θ (mr)</th>
<th>y (cm)</th>
<th>φ (mr)</th>
<th>δ (%)</th>
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<tr>
<td><strong>Beam</strong></td>
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<td>±0.25</td>
<td>±5</td>
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<td>φ</td>
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<td>-11.</td>
<td>0.</td>
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<tr>
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<td>±0.18</td>
<td>±0.68</td>
<td>±0.05</td>
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<td>-0.074</td>
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<td>φ</td>
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<td>-11.</td>
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</table>
IV. F-10

Figure Captions

1. Plan and elevation views of the beam analysis system and the spectrograph. The object slits for the beam analysis system are at the left, followed by the 5 quad drift-twister, and the first 45° bend unit. Beam energy measurement and/or defining slits are halfway down the 45° slope. The spectrograph pit is about 7m deep and the target is at the center of a 7m radius circle. The spectrograph has an angular range from about -15° to +150°.

2. The x- and y- envelopes calculated by TRANSPORT for the beam analysis system of figure 1. Here the 45° bends are done via two 22.5° dipoles each. There is a sextupole between each pair of dipoles.

3. A schematic sketch of the modular 45° bend unit. Each unit contains two 22.5° rectangular 1.2m long window frame dipoles with 5cm gaps and 15cm useful field width at 17.5kG magnetic fields. There are also two quadrupole doublets, each quad having a 10cm aperture and being 40cm in length. The sextupole will have a useful aperture approximately 10 cm radially x 5cm axially.

4. A schematic ray diagram at the target position. The central rays are 1 and 1', while the dispersed rays are 2 and 2'. Ray 2 is matched for angular as well as linear dispersion.
IV. F-11

5. The x- and y- envelopes from a TRANSPORT calculation for a modified version of the beam analysis system of figure 2. In this system the angular dispersion of the beam is also matched to the spectrograph. In this case the beam envelope is also shown all the way to the spectrograph focal plane.

6. The beam envelope of a single ray through the same system as figure 5. This ray starts with $x=0$ and $\theta=0$ at the object slits, but has $\delta=0.1\%$. Note that the combined beam analysis and spectrograph system is fully achromatic, i.e. $x_{\text{focal plane}}$ and $\theta_{\text{focal plane}}=0$. 
SBOO BEAM ANALYSIS SYSTEM

TOTAL LENGTH = 23.8 m
45° BEND MODULE
IV. F-15

BEAM SIDE

SPECTROGRAPH SIDE

TARGET

1

2

2''

1'

2'

\theta

\theta''

\alpha
The layout of the 500 MeV (max) linear electron accelerator facility at NIKHEF-K is shown in fig. 1. The facility contains the following beam lines:
- 000: the accelerator itself and the tune-up line for accelerator control and tuning
- 100: beam line to the low energy physics (LEF) station, with a 180° scattering facility (140 MeV max.).
- 200: beam line to the low energy chemistry (LECH) station
- 300: Pion-muon production channel (PIMU)
- 400: Deflecting system A, which contains the energy-defining slits
- 500: Deflecting system B, which transports the beam to the electron-scattering facility (EMIN)
- 600: beam line to the high-energy chemistry (HECH) station.

In this paper only the 400 and 500 line will be discussed. The layout of this system is shown in fig. 2. The kicker magnets K002 and K401 will allow beam sharing by different experiments. Since at present only the two low-energy stations and the EMIN-facility are operational, this kicker-option has not been used yet. The deflecting system should be capable of delivering a non-dispersive beam (Normal Mode, NM) at the target as well as a vertically dispersed beam (Dispersion Matching, DM). In this last mode of operation the dispersion at the target can be varied between 5 and 13 cm%/%. In order to have a good control on the tuning, the deflecting system has been designed in a modular way, with orthogonal control of the various functions. The first part of the system (B401 uptill B402) is an achromatic deflecting system; it contains the energy-defining slits, and it delivers a beam of well-known energy and energy spread to the second part of the deflecting system.
The slits are (as well as the beam dumps) made by SLAC, and can handle a power of 100 kW. At the very end of the accelerator there is a singlet quadrupole which enlarges the beam at the slits in the vertical direction in order to diminish the local power deposition. This quad also increases the sensitivity of the second quadrupole of the 500-beam line (Q502) which acts as "x-width control". This quadrupole is located at a position, where $R_{12} = 0$; variation of the field of this quadrupole therefore only affects the spot-size in the non-dispersive direction (which is at the target, after the rotation, the $x$-direction). In front of this quadrupole, there is another one, labelled Q501, the so-called "resolution quad"; the function of this quadrupole is to increase the value of $R_{12}$ inside the deflecting magnets B501 and B502; since the resolution of a system is proportional to the Karl Brown Integral:

$$\int_0^t R_{12}(\tau) h(\tau) d\tau,$$

this quadrupole controls the resolution of the deflecting system. The next part of the system between the quadrupoles Q503 and Q505, forms what is known as a "-I-cell", an invention by Karl Brown. In Normal Mode the quadrupole Q504 between the two dipoles is switched on to such a value, that another achromatic deflecting system is formed. For the Dispersion Matched Mode, however, this central quadrupole is switched off; the transformation matrix between the two "dispersion control"-quadrupoles Q503 and Q505, is given by:

$$\begin{pmatrix} -1 & 0 & \delta \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

vertical direction, respectively. If the two "dispersion control" quadrupoles have equal but opposite strength, the transformation matrices become:
IV. G-3

\[
\begin{pmatrix}
1 & 0 & 0 \\
1/f & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
-1 & 0 & \delta \\
0 & -1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 \\
-1/f & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
= \begin{pmatrix}
-1 & 0 & \delta \\
0 & -1 & -d/f \\
0 & 0 & 1
\end{pmatrix}
\]

and

\[
\begin{pmatrix}
1 & 0 \\
-1/f & 1
\end{pmatrix}
\begin{pmatrix}
-1 & 0 \\
0 & -1
\end{pmatrix}
\begin{pmatrix}
1 & 0 \\
1/f & 1
\end{pmatrix}
= \begin{pmatrix}
-1 & 0 \\
0 & -1
\end{pmatrix}
\]

for the horizontal and vertical direction, respectively.

The achromatic terms are unchanged, only the angular dispersion term, which finally determines the dispersion at the target, is changed. Therefore equal, but opposite changes of the two "dispersion control" quadrupoles determine the dispersion at the target, without any effect on monochromatic spots size. Since the dispersion at the target has to be adjusted during inelastic electron scattering experiments this feature is extremely useful. The quadrupole triplet just behind the wall services to image the beam at the target. The five quadrupole rotator is a standard one, with an optical length equal to its physical length. Therefore the rotator only rotates the beam, without effect on the other beam properties. Finally the system contains two sextupole-magnets around deflecting magnet B502. These magnets are needed for second order corrections:

\[ T_{316} = 0 \text{ and } T_{366} = 1.54 \text{ cm/}(\%)^2 \] 

at the target. This last term is required since \(<x|x^2>\) of the spectrometer turned out to be hard to correct; it was simpler to add a quadratic dispersion into the deflecting system. This once more shows the importance to design a completely integrated system from the accelerator uptill the detectors: aberrations of one part of the total system can sometimes be corrected rather easily in another part.

Experience so far with the system shows that the modular design allows a rather fast and easy tuning; the beam in the first part of the system is tuned with the aid of (non-interfering) travelling wave monitors; in the second part,
where beam dimensions are rather large, view screens can be put into the beam. The monochromatic properties and the dispersion can be controlled by inserting a tantalum mask with 5 small (0.6 mm wide) slots into the beam. This allows proper setting of the x-knob, the triplet and the rotator, as well as coarse tuning of the dispersion. Fine tuning is done by measuring the elastic peak of electrons scattered from a horizontal strip of Mo-foil; by variation of the position one can scan the momentum distribution of the beam spot. Experience has shown that one such a series of measurements allows a proper setting of the dispersion control quadrupoles and the sextupoles without the need of further iterations.

Following this procedure, an overall experimental resolution of better than $10^{-4}$ has been obtained, showing that the intrinsic properties of the deflecting system meet the high resolution requirements of the whole setup. More details of the system are given in: J.C. Bergström and R. Maas, Design and study of the Amsterdam 500 MeV beam handling system; InterIKO7615; Internal Report.

Figure captions:

fig. 1 Layout of the intermediate energy physics facility
fig. 2 Layout of the 500 MeV beam handling system.
IV. G-6

Fig. 2
Presumably, I was asked to talk to you because many of you are members of that vanishing species known as nuclear physicists, who may be asking "what's in it for me?" Specifically, one may wonder whether there are any sufficiently significant possibilities here that might reward the perspicacious with yet another new toy. Judging by the number of current projects at higher energy, it would seem to be a good bet. In fact, it's hard to find a project within the past few years that hasn't involved at least one ring. For instance, Cern's p-p project has 3 rings with 7 or 8 different damping processes to stochastically cool the momentum and transverse phase space of the tail and core of the beam. Generally, the higher the energy, the greater the number of rings. This is not surprising since the most important parameters of storage rings and such are the energy and the beam current.

Since I am supposed to speak your language, I will try to "answer" based on a Hamiltonian approach. In fact, one can develop a suitable approach based on Feshbach's projection operator formulation. This technique is useful because we are primarily interested in only a small and relatively simple part of the overall problem (sound familiar?). Bunches of charged particles must be confined in stable 3-dimensional potential wells. In the rest frame of the bunch, a transverse electric potential results from the transverse magnetic guide field and the longitudinal well results from the lab rf field. The relativistic equation of motion of charged particles in an electromagnetic field in Hamiltonian form i.e. the total energy as a function of the usual canonical variables (q,p) e.g. the transverse displace-
ment from the equilibrium orbit (i.e. $x, x'$ etc.) is given by

\[ H = i\hbar \dot{\mathbf{r}} = (H_{\text{rad}} + H_{\text{part}} + H_{\text{int}}) \]

\[ H_{\text{rad}} = \frac{1}{8\pi} (E^2 + B^2) d\mathbf{r} ; \quad H_{\text{part}} + i\hbar \dot{r} = \frac{\hbar}{i} \]

\[ H_1(r, \mathbf{r}) = e_1 \phi_1(r_1) + \left[ c^2 (\mathbf{p}_1 - (e_1/c) \mathbf{A}(r_1))^2 + m_1^2 \right]^{1/2} \]

where $\mu_1 = m_1 c^2$ and $A_\mu = (\mathbf{A}, i\phi/c)$ is the external field from the magnets, atoms, or lasers as well as the fields produced by the charges themselves.

If the momentum is small compared to $\mu_1$ then

\[ H_{\text{rel}} = -\mu_1 \left( \mathbf{p}_{\text{rel}} - (e_1/c) \mathbf{A}(r_{\text{rel}}) \right)^2/(2m_1) + e_1 \phi(r_{\text{rel}}) + V_0. \]

Notice that we have used the subscript denoting the Hamiltonian in the "rest" frame. For a pure electrostatic field ($\mathbf{A} = 0$) this gives the familiar nonrelativistic expression for the energy. Neither $H$ nor $H_1$ includes interaction between particles so far unless we either add another term such as $V$ with subscripts $ij$, $ijk$ etc. which then gives coupled equations. However, if we are interested in beam dynamics such as coherent effects within a beam bunch, or various excitation modes in a laser medium, crystal lattice, atom or even an "elementary" particle we must consider such terms.

The fields $\mathbf{A}$ and $\phi$ are generally nonlinear due to magnet errors and end fields, the sinusoidal character of the rf as well as the fields induced by the beam through self forces or wake fields (interaction with the rest of the external world exclusive of guide fields). Such fields can couple the degrees of freedom of the single particle e.g. provide transverse-transverse (1) ($x$-$y$) and transverse-longitudinal (2) ($x$-$z$) coupling. Further-

(1) Skew quads and solenoidal magnetic fields in the lab will also do this.

(2) Static, transverse, crossed electric and magnetic fields in the lab or an rf cavity will also do this e.g. in the lab, $E_y = ay$ and $B_y = ay$, whereas in the rest frame $A_{oy} = -az/2$ and $A_{oz} = ay/2$ in Eq. 2.
more, since wake fields can be either fast or slowly decaying ($\tau \ll 1/\omega_z$ or $1/\omega_{x,y}$ for strong Fourier components $\omega \gtrapprox c/L$), one expects both single and multibunch instabilities.

Even assuming only one beam and one bunch, there are a number of current-dependent effects which can cause beam blowup and subsequent particle loss by leakage from the potential well. The best general reference to study the single-particle effects is ref. 3. Collective effects have been discussed by Pelligrini in ref. 4. They may be broken down into coherent and incoherent depending on whether there are phase relations between individual particles or not. When there are, one can think of modes of motion like that of the incompressible liquid drop of Bohr and Mottelson etc. i.e. one has dipole, quadrupole, etc. motion that can be quite dramatic. There are many ways to both induce and cure such coherent effects. Thus, as the bunch oscillates, the potential well dynamically distorts which can produce an oscillating force back on the beam that can drive or damp it. Similarly, the external potential well can be made to act the same way - usually via negative electronic feedback that senses and then feeds back to damp the instability.

How might one damp incoherent, longitudinal motion i.e. decrease the energy spread in the beam? More specifically, is it possible to use scattering foils to reduce phase space density e.g. energy spread and beam size? Similarly, the question has been raised whether the world wouldn't be a better place for having a thin target in a storage ring in conjunction with a high-resolution spectrometer - the subject of your meeting. My own interest in this problem has been to ask, but not answer, whether it would be possible to put a thin, single-crystal in a storage ring to produce coherent, channeling radiation. This is a somewhat harder problem because of the thickness

required ($t \gg 1000$ atom diameters) which approaches self destructive proportions for both target and beam unless it is somehow integrated into the optics.

The most obvious problem here is that we have now essentially eliminated our single-beam and bunch simplification, i.e. in an appropriate reference frame we now have unbalanced, less-relativistic, counter-rotating beams - the bane of all storage rings. Thus, a particle moving in the same direction as its bunch sees a direct EM force from the other electrons reduced by $(1/\gamma^2)$ due to a near cancellation between the repulsive electric and attractive magnetic forces. As long as the particle is in the bunch it is continuously acted on by this weak force so that this effect is relatively free of spatial harmonics. On the other hand, the effective counter-rotating beam of the thin target produces fields in which the electric and magnetic forces add, producing a strong, localized effect. One then expects such things as tune shifts and spreads dependent on beam size, target thickness and time since the beam phase space i.e. size is being perturbed by the target.

Furthermore, since the beam current is being reduced by scattering and other loss mechanisms, we know the tune will also be changing from this as well. If you inject into the ring to "top-up", this generally perturbs the stored beam via kicker action etc. unless one has a very elaborate partitioned vacuum chamber. Of course, one can remove the target or stop experiments during injection so this would just represent some overhead.

Thus, from all of the above, I do not believe it is appropriate to consider the scattering target like a "missing" dipole magnet i.e. that it is essentially equivalent to damping from synchrotron radiation. However, let's suppose the target is so thin that we can ignore all of the above.
IV. H-5

This is then presumably equivalent to a very short magnet and we can therefore expect significant quantum fluctuation effects which can "blow" the beam up if the single or plural (not multiple in the Moliere sense) energy loss is not sufficiently small. However, when the energy quanta $\delta E$ are small, it takes many revolutions to damp $(v \epsilon_{beam}/\delta E)$. This is just another way of saying that the scattering target doesn't help us damp very much unless we have no other means. In other words, it is conceivable this could be used for protons because they don't radiate appreciably and also because they don't get as much angle-spread from the scattering due to their larger mass ($\delta p_s/p^p << \delta p_s/p^e$).

Finally, insofar as I know, there is essentially only one published work(5) that bears on this question. However, it is a very important point to keep in mind. Simply stated, the result is that any electromagnetic field which is independent of a particle's motion can only couple its various degrees of freedom but cannot change its total damping rate. For instance, if $\psi(q,p,z)$ in Eq. 1 is an ensemble or "dust-cloud" in phase space, its trajectory over time is determined only by $H$ and its initial conditions. Since

$$\dot{q}_i = \frac{\partial H}{\partial p_i} \text{ and } \dot{p}_i = -\frac{\partial H}{\partial q_i} \quad (3)$$

one has that

$$\frac{\partial}{\partial q_i} \left( \frac{\partial H}{\partial p_i} \right) - \frac{\partial}{\partial p_i} \frac{\partial H}{\partial q_i} = 0.$$  

This is equivalent to saying that both the summed projections over all coordinate planes $\sum_{i} q_i, p_i$ is an invariant for every particle $i$ as well as the sum of all particles projected on all planes which is just the Liouville theorem. Wigglers are a good example of this works. Thus, I would conclude that such a scheme doesn't buy you much except possibly on the power bill.

Important studies are now undertaken to develop continuous wave high energy electron accelerators with energy ranging from 1 to 4 GeV. So, very important effort must be now put on the development of the experimental set-up matching the performances expected from the electron beam. Major steps in the understanding of the nuclear systems will come from more and more exclusive experiments where well defined mechanisms will be selected.

I. Exclusivity

There are two classical ways to make exclusive experiments: High resolution single arm experiments and coincidence experiments. In elastic scattering or for some particular selected levels in inelastic scattering, *exclusivity comes from the resolution*. Generally, no electron energy can be higher than the incident electron energy. So, high performances magnetic spectrometers and fast detectors, if they are well shielded from all unwanted particles, are most suitable to provide extremely clean nuclear information. When the background rejection is adapted, single-armed experiments have no fundamental limits on the beam intensity as long as the target can hold it. This will be the purpose of another paper to try to figure out what could be the aspect of the best-suited spectrometer for those single-armed experiments. Let me presently focus on the second type of experiments for which 100% duty cycle represents a major breakthrough: the coincidence experiments.
IV. I-2

When the nucleus emits particles carrying an important amount of the momentum of the system, exclusivity cannot be obtained without detecting and identifying all the products. This is done through coincidence experiments. Provided that the incident beam intensity and the target thickness are sufficiently low, all the particles detected within a narrow window of time have a great probability of coming from the same nucleus. However, if in order to get a larger counting rate the beam intensity or the target thickness is increased, most of the coincidences become accidental, thus merging the real ones into a sea of non-information. This fundamental limitation put severe restrictions on the size of measurable cross sections which are usually very small in electron scattering. See, for example, the cross sections measured at Saclay (1) in $D(e,e',p)$ ranging from:

$$8 \times 10^{-33} \text{ to } 0.015 \times 10^{-33} \text{ cm}^2/\text{steradian}^2/\text{MeV}.$$ 

For this purpose, CW electron accelerators represent a major improvement. The gain in duty cycle is directly a gain on smallest measurable cross section. This is why coincidence experiments will be the first priority of this coming generation of accelerators.

2. A proposed set-up

However, this improvement can and must be coming in pair with an equivalent step in the domain of detection efficiency, I mean the detection solid angle. The only geometry that provides the maximum solid angle have the symmetry of the incident beam, i.e. the
IV. I-3

axial symmetry. A magnetic field covering almost 4 pi must be looked for, because it is the best way to get particle identification and momentum determination. It is interesting to point out that, apart from the classical solution of a solenoid which provides a field parallel to the beam axis, a toroid has been several times proposed by high energy physicists.

I will report here on some results that have been obtained from calculations. It has been considered (2) as possible to build a toroid providing a field induction integral of 0.85 Tesla meter with the help of eight radially disposed superconducting coils (see fig.1). Those coils would let a free space for the particles of about 80% of the total azimuthal angle.

The field would not have a perfect axial symmetry due to the finite number of coils as shown on fig. 2. However, the differences compensate when integrating radially. There is also a non zero radial field around the coils, which sweeps out of the radial plane the particles travelling close to them. But, this effect tends also to compensate for the full trajectory. A complete set of detectors would have to be installed all around the toroid: wire chambers, zero time detectors, Cherenkov detectors and neutral particles detectors. A resolution of 1 MeV should be obtained on 1 to 2 GeV charged particles considering the present accuracy of wire chambers. The price of the coils, refrigeration system and vacuum chamber is considered to be of the order of magnitude of what has been spent for the "CELLO" experiment at DESY in Germany.
IV. 1-4

Such a toroidal field could be the magnetic basis of a large variety of coincidence experiments. We must keep in mind that the coincidence experiments can be sketched along many extremely different ways. This has been shown by L. Cardman in his pioneering work on the 100 % duty cycle beam of MUSL 2 at the University of Illinois.

3. Background

When the solid angle is so widely opened, the detectors are necessarily in direct view of the target. No shielding can protect them from unwanted particles emerging from the target. So, in selecting particular mechanism, careful identification is the only solution.

However, the two other major sources of background are the beam entrance port and the beam dump. Extremely careful attention must be paid to the study of all the devices which can clean up the beam and shield the beam dump. This has been a successful experience in Saclay to show the importance and the efficiency of those studies on the high energy electron scattering set-up.

4. Conclusion

It must be pointed out that with an almost full spatial efficiency, the reals to accidentals ratio is greatly enhanced with respect to the present set-ups of Saclay or Amsterdam. With their very small apertures, when a particle have been detected, there is
a very small probability that its counterpart would have found its way through the entrance slit of the second spectrometer. This happens only in perfectly correlated kinematics. Thus, generally, most of the detected particles are singles. The situation is clearly better in 4π geometry. An other important advantage of this geometry is to allow straightforwardly out-of-plane coincidences. Moreover, it is certainly the only way to measure more than two particles coincidence cross-sections.

Finally, together with the expected gain of duty cycle of the next generation accelerators, we can hope to multiply solid angles and real to accidentals ratio by a very large factor. This would allow the measurement of cross sections that are presently far out of reach. They represents a really interesting field of the coincidence experiments in electron scattering.

5. References

1— M. Bernheim et al., Nuclear Physics, A365(1981) 349,370.

2— H. Desportes, private communication.
Fig 1. Toroidal solution
**Fig 2** The field of eight lumped coils
Spectrometer Requirements For (e,e'2N) Studies

J. W. Lightbody, Jr.
National Bureau of Standards
Washington, D.C., 20234

One specific experiment that may be performed with a future CW accelerator is a study of (e,e'2N) reactions through which we may learn details of the short range interaction of two nucleons within nuclear matter. It is suspected that the only mechanism which can lead to the observed high momentum components in the single nucleon momentum distribution (above ~ 400 MeV/c) inferred from (e,e'p) and (γ,p) measurements is the presence of short-range few-body correlations in the many-body nuclear wave function. It is expected that the explicit pair correlation function may be inferred from relative two-nucleon momentum distributions measured in (e,e'2N) experiments. It is therefore interesting to estimate counting rates using measured one-body momentum distributions to see what types of spectrometers are required.

Figure 1 shows just how severely the count rates vary with momentum transfer using even very large acceptance spectrometers. Also shown in Figure 1 is a schematic showing the relative directions of reaction products which indicates both nucleons emitted along the same direction. This particular choice of kinematics comes from a simple analysis using pre-existent, high relative momentum nucleon pairs. Figure 2 indicates that in an antiparallel mode (case 1), if the virtual photon couples to one nucleon, then there are two initial relative momenta leading to a given final state. In fact the process will be dominated by the low relative momentum component of the two competing amplitudes. In the parallel mode (case 2) this ambiguity can be avoided, and the possibility exists of detecting both nucleons with the same spectrometer.
Figure 3 shows the one-body momentum distribution for $^{16}O$ and exactly why the $(e,e'2N)$ cross section falls so rapidly with increasing momentum transfer, $q$. High momentum components in the wave function decrease by a decade for every 100 MeV/c increase in nucleon momentum. It turns out that the momentum ($\vec{P}_{\text{rel}}$) conjugate to the internucleon spacing is $1/2(\vec{p}_1 - \vec{p}_2)$. One can show that the values for $P_{\text{rel}}$ and electron energy loss, $\omega$, are related as follows:

$$P_{\text{rel}} = q/2 \pm \sqrt{m\omega - (q/2)^2}.$$  

where $m$ is the nucleon mass. For equal energy protons $\omega = q^2/4m$ and $P_{\text{rel}} = q/2$. For unequal energy protons there is a continuous distribution in $\omega$, from $q^2/4m$ up to $q^2/2m$, and a corresponding distribution in $P_{\text{rel}}$, from 0 to $q$, where for a given $\omega$ there are two relative momenta probed. Since the low momentum components in the wave function far outweigh those of high momentum, we are sensitive largely to the $P_{\text{rel}}$ range from 0 to $q/2$. Furthermore, since we more or less run out of counts in this reaction at $q = 1500$ MeV/c, we are only able to probe the pair correlation function out to relative momenta of ~750 MeV/c, which corresponds to separations no smaller than 0.3 fm. Detection of 750 MeV/c protons requires a relatively modest spectrometer - 1.5T fields and 1.5m bending radii - on the scale of machine energies being contemplated.

What then are the actual spectrometer requirements? In Figure 1, I indicated an electron spectrometer of 30 msr solid angle. This solid angle is essentially state-of-the-art and should be more or less a minimum requirement on any new system for use with a CW machine. Momentum acceptance of order $\pm 15\%$ (with resolution between $10^{-3}$ and $10^{-4}$) is also possible now and I see no need to back off too far from this value. With this acceptance we can cover an electron energy loss interval of roughly 600 MeV for a 3 GeV accelerator,
and 300 MeV for a 2 GeV accelerator, permitting probes of $P_{rel}$ over roughly the entire range from 0 to $q/2$ in a few field settings of the electron spectrometer.

Turning to the nucleon arm of this experiment, the first requirement is large solid angle. Because of heavy counting rate losses due to high rates of true events and because of high backgrounds, I do not consider 4π or other bizarre spectrometers to be realistic. The 120 msr solid angle figure used in deriving the counting rates shown in Fig. 1 is four times larger than present technology, but with software corrections there is hope that this can be achieved at only modest loss of resolution. It should also be pointed out that for a 120 msr solid angle ($\pm 10^\circ \times \pm 10^\circ$), below momentum transfers of ~ 400 MeV/c the effect of Fermi motion begins to wash out the angular correlation about the momentum transfer direction and therefore limits the low-$P_{rel}$ end of any probes of the pair momentum distribution.

The momentum acceptance of the proton spectrometer should ideally be large enough to cover the entire range of single proton momenta corresponding to the electron spectrometer energy loss interval ($\sim q^2/4m$ to $q^2/2m$). Unfortunately this range can be as large as 200 MeV/c to $q - 200$ MeV/c, and at $q$ near the counting rate limit of 1500 MeV/c, the acceptance required is clearly impractical. However, simultaneously detecting the minimum and maximum energy protons corresponds to studying a low $P_{rel}$ process. This can more easily be done by detecting more nearly equal energy protons at a much lower value of $q$. The momentum acceptance of the proton spectrometer is somewhat difficult to arrive at from the standpoint of physics. With increasing proton momentum acceptance, coverage of the corresponding electron energy loss interval unfortunately
increases very slowly. Therefore, a modest value of ± 5% would appear adequate. The choice of proton momentum acceptance a factor of three smaller than the electron momentum acceptance comes about by requiring the (solid angle) x (momentum acceptance) product to be roughly constant.

Resolution is a less serious consideration than solid angle for this experiment. The electron energy resolution in this case should correspond to a proton energy range such that variations in the momentum distribution $|\phi(p)|^2$ over this range are of minor importance. A 1 MeV energy resolution on both electron and proton spectrometers is more than adequate. This corresponds to a $\Delta p/p$ of $3 \times 10^{-4}$ and $1 \times 10^{-3}$, for electrons up to 3 GeV/c and protons up to 750 MeV/c, respectively. In fact the count rate estimates were based on energy intervals of 10 and 5 MeV for the electron and proton detection systems, respectively. These are more reasonable values for the interval over which variations in the momentum distributions are of minor importance; however it is unlikely that such limited resolutions would be of value in other spectrometer applications.

Finally, it would be valuable for the spectrometer to have a straight-through port for neutron coincidence studies [i.e., $(e,e'np)$ reactions] since the quasi-deuteron process is the dominant mechanism for photo-absorption above the giant resonance. The solid angle subtended by such a port should be similar to the spectrometer solid angle.

There have been reported in this meeting the details of a low energy pion spectrometer (LEP) for use at Los Alamos. (See Figure 5.) It appears that this type of spectrometer, or a multigap version scaled up in field, could be used for the proton arm of this experiment. It would have to be determined to whether or not the solid angle in a single gap device could be increased by a factor of four over the LEP design of 30 msr, possibly by
extensive use of software corrections, considering the modest resolution requirements. Finally, I think it would be a mistake to design spectrometers for this (or any) type of study which went far beyond the kinematic range where counting rates are sensible (~1 count in 10 hours), or which have resolution far exceeding dictates of the physics.
Large Bore Spectrometers

Specific interest in \((e,e'N)\) reaction

Two nucleons moving in same direction (nearly along \(q\)) with about the same momentum

\(\phi_e = 30^\circ\)

\[\begin{array}{c|c|c}
\phi_e & \text{Cts/sec} \\
250 & 5 \times 10^4 \\
500 & 1.5 \times 10^3 \\
750 & 1.7 \times 10^1 \\
1000 & 1.9 \times 10^{-1} \\
1200 & 2.0 \times 10^{-3} \\
1500 & 1.5 \times 10^{-4} \\
\end{array}\]

\(\Delta \Omega_e = 30\, \text{msr}\)

\(\Delta E_e = 10\, \text{MeV}\)

\(\Delta \Omega_{p_1p_2} = 120\, \text{msr}\)

\(\Delta E_{p_1p_2} = 5\, \text{MeV} \) (same spectra)
IV. J-7

Kinematics \((e,e'2n)\)

Case 1

\[
\text{Initial relative pair momentum not unique}
\]

Case 2

\[
\text{Final relative pair momentum uniquely determined}
\]
Why does cross section vary so fast with $q$?

$$\frac{d^6 \sigma}{d^6 q} \sim |\Phi(P)|^2$$

- High momentum components in single nucleon momentum distribution result from short range collisional process involving two-nucleons.
- Two-nucleon relative momentum distribution sought to find $\Phi(r_1 - r_2)$
- $\Delta R \sim \frac{1}{|P_{rel}|}$, $P_{rel} = \frac{1}{2}(P_1 - P_2)$
IV. J-9

- Needed large solid angle for counting rate and to capture correlated pairs (loss of angular correlation \( \theta > 20^\circ \) for \( q \leq 400 \text{ MeV/c} \)).

120 \( \omega \theta = \pm 10^\circ \times \pm 10^\circ \)

(four times larger than Bates pion spectrometer)

- Resolution \( \leq 1 \text{ MeV} \) at electron energies up to approximately \( 3 \text{ GeV} \)

\[
\frac{\Delta P_e}{P_e} \leq 10^{-4}
\]

For protons with \( P = \frac{1}{2} \) have \( E_{\text{max}} \approx 250 \text{ MeV} \)

and for same 1 MeV energy resolution

\[
\frac{\Delta P_p}{P_p} \leq 10^{-3}
\]

- Requirement on magnetic spectrometer for particles not a 0°. port assist for neutron studies (i.e., \( (e,e'np) \) reactions) which should have a similarly large solid angle
Ideal spectrometer for detecting both protons.

- Can solid angle be increased by 4x?
- Can higher fields be used (was 1.6 T)?
- How low in energy before software corrections become impractical?
 Systems Considerations for Large Solid Angle Detection of Coincident Reaction Products from Nuclear Electroexcitation

L. S. Cardman and C. N. Papanicolas
Nuclear Physics Laboratory
University of Illinois
Champaign, Illinois 61820

Abstract:
Large solid angle detection systems for (e,e'x) measurements involving the coincident detection of nuclear decay products from either the giant multipole resonances or bound states are discussed. Particular emphasis is given to systems considerations relevant to a broad variety of electron-induced coincidence studies.

Presented at the Argonne Workshop on High Resolution, Large Acceptance Spectrometers, September 8-11, 1981.
IV. K-2

1. Introduction

The availability of high duty cycle electron beams has facilitated the extension of electron scattering techniques to experiments in which a broad range of nuclear decay products are observed simultaneously with electroexcitation. Heretofore, coincident electron scattering has been limited to the study of reactions, such as quasi-elastic (e,e'p), for which there are strong angular correlations between the reaction products. This paper focuses on the experimental techniques developed at Illinois for electron scattering coincidence studies of the giant multipole resonances and nuclear bound states. While the details of the apparatus we have built were dictated by the experimental problems associated with the region of nuclear excitation under study, the solutions also reflect general considerations relevant to a broad variety of electron-induced coincidence reactions. Most important among these considerations is the need for systems engineering of the common experimental apparatus—the spectrometer and its readout electronics and scattering chamber—in order to accommodate a broad variety of decay product detectors with a minimum of effort. Also of fundamental importance is the need to recognize the differences between singles and coincidence experiments. For example, the coincidence requirement can dramatically improve both signal-to-noise ratios and energy resolution. Finally, we find that the use of large solid angle detectors and good background rejection techniques are crucial ingredients for coincidence experiments involving giant resonance and bound state decays.

In the sections that follow we first discuss (Section 2) the differences between coincidence and singles experiments and the special problems associated with low excitation energy studies. Then the apparatus common to both classes of experiments is discussed (Section 3). We describe there the electron spectrometer, the scattering chamber, and the generalized coincidence electronics hardware. Finally, (Section 4) a variety of decay particle detectors either in use or under development at Illinois are presented.

2. General Considerations

a. Coincident (e,e'x) vs Inclusive (e,e') Measurements

Elastic electron scattering studies extended to high momentum transfer have determined the ground state charge and current densities to an accuracy of a few percent throughout the nuclear volume. Inelastic electron scattering to isolated excited nuclear states can provide similar information about the
transition charge and current densities. The continuous beams provided by the MUSL-2 accelerator at Illinois have permitted the extension of electron scattering techniques to include experiments in which nuclear decay products, either from bound states or from the giant multipole resonances, are observed simultaneously with electroexcitation. This capability has created new possibilities for experiments which cannot be performed with lower duty factor beams.

The simplest coincidence experiment involves only the detection and identification of the nuclear decay particle with no energy or angular definition. In this case the coincidence requirement essentially eliminates the main background present in inelastic electron scattering, namely the radiative tail associated with elastic scattering. This background is particularly troublesome in studies involving continuum excitations such as the giant multipole resonances. These resonances are broad and overlapping, and the experimental separation of the nuclear excitation cross sections from the radiation tail background is a major difficulty. The detection of a particular species of nuclear decay particle can also be used to select or emphasize a particular reaction channel among the large number which can occur in an inclusive, single-arm experiment. Both of these possibilities are illustrated in the spectra shown in Figure 1. The upper spectrum shows a typical, low-q $^{238}$U(e,e') energy loss spectrum, dominated by the elastic peak and its radiation tail. Only about 1% of the counting rate observed above electron energy loss $\omega > 7$ MeV is due to nuclear excitation. The lower spectrum shows the same data, but with the requirement of coincident detection of a fission fragment in a 500 msr thin film scintillation detector located opposite to the momentum transfer direction. The opening of the fission channel and the effect of neutron competition above excitations corresponding to the neutron binding energy, $B_n$, are clearly visible because the radiation tail is absent. Also visible is a rather smooth and featureless excitation in the region where the giant quadrupole resonance is predicted.

If the coincidence detection system also measures the energy of the decay particle, additional possibilities emerge. For example, the energies of giant resonance decay products provide important clues as to the nature of the decay (statistical or semi-direct) and are sensitive tests for nuclear models describing the microscopic structure of the excited resonances and their coupling to the continuum.$^3$ Another possibility is the use of decay product
Figure 1. Electron Scattering Spectra from $^{238}$U Observed with and without the Requirement of Coincident Detection of a Fission Fragment

\[ E_e = 67.11 \text{ MeV} \]
\[ \theta_e = 60^\circ \]
\[ \theta_f = 180^\circ \]
energy measurement to improve the energy resolution of conventional \((e,e')\) experiments. This is feasible whenever the excitation energy resolution of the decay particle detector exceeds the absolute energy resolution of the electron spectrometer at the beam energy of interest. For example, the use of a Ge(Li) detector for the detection of bound state decay gamma rays in coincidence with the inelastically scattered electrons would provide an overall excitation energy resolution of a few keV, independent of the electron beam energy. This represents an order of magnitude improvement over the resolution currently available at Intermediate energy facilities. Such possibilities are particularly relevant when the traditional trade-offs between high resolution and large solid angle for an electron spectrometer are under consideration.

More information can be obtained by a careful measurement of the angular distribution relative to the momentum transfer ax's of the coincident nuclear decay products. These angular distributions provide a model-independent means for determining the spin and parity of the state excited. In the case of mixed transitions (excitation from a non-zero spin ground state) it should be possible to separate the contributing multipole form factors without a nuclear model for the first time. In single-arm \((e,e')\) measurements a sum over all multipoles is observed. In the case of broad, overlapping states of different spin and parity, such as the giant multipole resonances, the detection of semi-direct decay products will make it possible, model-independently, to separate the multipole excitations and to investigate the relative phases of the nuclear matrix elements.

The formalism necessary for the interpretation of electron scattering coincidence data has been discussed in general terms by deForest and Pritchett et al., among others. The specific case of photon decay has been developed in detail by Rose and his collaborators, while Drechsel and Uberall have examined the case where the reaction proceeds through a well-defined intermediate resonance. In contrast to hadronic \((h,h'x)\) coincidence studies, the theory of the \((e,e'x)\) reaction is well-understood. The difficulties in successfully realizing these measurements lie in experimental problems, such as small cross sections, rather than in the interpretation of the data.

b. Experimental Difficulties

None of the advantages discussed above are available without a substantial increase in effort relative to conventional electron scattering
IV. K-6

experiments. The most obvious difficulty is the small magnitude of the cross section. The already large difference between the two-fold (single-arm) and three or four-fold (coincidence) differential cross sections is aggravated for low nuclear excitation energies because there is no kinematic correlation, such as occurs in the \((e,e'p)\) quasi-elastic knockout reaction, to concentrate reaction products into a relatively modest solid angle. The only solutions for this problem are patience (and a great deal of beam time) and/or the development of complex, large solid angle detection systems for the nuclear decay products. If the experiment under consideration includes angular distribution measurements for spin and parity determination, the detector must be segmented to provide the angular resolution necessary for the highest multipolarity of interest. Detector segmentation can also help reduce pile-up when high counting rates are a problem. An additional complication arises from the necessity to provide for measurements out of the scattering plane. Such measurements are required if all four nuclear form factors involved in coincidence measurements are to be uniquely determined.

The above mentioned difficulties are inherent to all coincidence measurements. Two additional problems arise with electrons. The first is the large electron, gamma ray, and neutron background present in the vicinity of the scattering target. The electron background includes two components; incident electrons elastically and inelastically scattered from nuclei in the target, and atomic electrons knocked out of the target by electron-electron (Møller) scattering. The reduction of these backgrounds to tolerable levels requires careful design of shielding arrangements and the use of a variety of techniques ranging from magnetic filters to pulse shape discrimination. The second problem is due to the strong forward peaking of electroproduction \((e,x)\) cross sections. As the reaction products from forward angle electroproduction are indistinguishable from those of interest (the same nuclear excitation energy is involved) they result in a high singles counting rate in the reaction product detector. A beam of high duty factor (which reduces the random coincidence rate for a given average beam current) is an important factor in reducing this background. Equally important is excellent timing. The random coincidence rate is inversely proportional to the duty factor and directly proportional to the coincidence resolving time. Additional help can be obtained via a large solid angle and momentum bite for the electron
spectrometer, which increases the true coincidence event rate for a given beam current.

Specific examples of the techniques we have developed to overcome these experimental difficulties are presented in Section 4. The impact of an optimized electron spectrometer on our research program is discussed in Section 3.

3. Apparatus Common to All (e,e'x) Studies

a. The Electron Spectrometer

An electron spectrometer is, of course, crucial to all (e,e'x) coincidence studies. We are presently using a 15 year old, 0.5 meter radius of curvature, n=1/2 spectrometer obtained from Yale University. This device subtends a solid angle of 5 msr and has a momentum bite of only 4.5%. The intrinsic resolution of the spectrometer is $5 \times 10^{-4}$ but the current focal plane array limits the experimental resolution to $2 \times 10^{-3}$. While this instrument has proven adequate for preliminary (e,e'x) experiments, substantial improvements could be obtained with a modern spectrometer design. In coincidence applications, the spectrometer solid angle and momentum bite are very important because experiments will frequently be limited either by the beam current sustainable on the nuclear target or by the singles counting rates in the reaction product detectors. As mentioned earlier, most of the events in the reaction product detectors come from electroproduction associated with very small angle scattering (very low three-momentum transfer). As a result, the coincidence counting rates tend to be proportional to the electron spectrometer solid angle. Spectrometer requirements include: (1) as large a momentum bite as possible, to permit study of a large range of energy loss simultaneously; (2) moderate to good energy resolution ($\lesssim 5 \times 10^{-4}$); (3) a large range of accessible scattering angles (from about $20^\circ$ to $160^\circ$) to permit low q measurements at high beam energies (requiring forward scattering angles) and to permit the enhancement of the transverse form factor (requiring backward scattering angles); and (4) the capability of precisely measuring the scattering angle from the target (i.e., of subdividing the spectrometer solid angle) for measurements where the counting rate varies rapidly with scattering angle.

Two spectrometer designs which are well matched to these requirements are the Bates Medium Energy Pion Spectrometer, described by Blomqvist at this conference, and the "clamshell" spectrometer presented by Kowalski. Either of
these spectrometers would provide over a factor of 50 increase in the solid angle-momentum bite product relative to our present instrument. The resulting improvement in the experimental data is obvious.

b. The Scattering Chamber

Single arm electron scattering experiments require rather sophisticated target chambers in order to accommodate special targets (e.g., gas, oxidizable, or radioactive). Coincidence experiments use similar targets but also require the detection of a second particle from the target. The nature of that particle may vary from experiment to experiment. Sometimes the detectors must be mounted close to the target within the vacuum (e.g., fission detectors). At other times they must be distant (e.g., neutron time-of-flight detectors). To make full use of all the information available, the secondary particle must be frequently observed in directions out of the scattering plane.

We have constructed a modular scattering chamber with a minimum of fixed elements to provide the flexibility necessary for coincidence experiments. The basic chamber is just a skeletal frame, as shown in Figure 2. The target holding mechanism is supported from the bottom of the chamber, leaving the top free to support an out-of-plane counter array. The standard side panels for the chamber are cylindrical sections which bolt in place over the large openings shown. They are provided with three vertical rows of ports at 20° angular increments centered from 25° to 145° relative to the beam direction. By installing these side panels upside down, the port centers are offset by 10°, and are therefore located at 20° angular increments centered from 35° to 155°. The ports have an angular opening of ± 7.75° relative to the port center. Using this technique, all angles from 20° to 160° relative to the beam are accessible. Secondary particle detectors may be suspended from the top of the scattering chamber (as is the case for the solid state detector array and conversion electron spectrometer described below) or mounted on the cover plates for the side ports. Alternately, for large secondary particle detectors requiring a vacuum connection, an entire side of the chamber may be removed and replaced with a large vacuum extension.

c. Generalized Coincidence Electronics

The difficulty and complexity of coincidence experiments is directly reflected in the required electronic counting chains. Despite the fact that each specific experiment has its own peculiarities which must be addressed
Figure 2. \((e,e'x)\) Scattering Chamber
separately, a large fraction of the electronic circuitry is common to all measurements. In an effort to minimize duplication and to maximize the effectiveness of our electronics pool, we have developed a Generalized Coincidence Electronics (G.C.E) system which can service most of these experiments using a common counting chain. A software package has also been developed which takes advantage of the flexibility inherent in the hardware standard. The hardware-software system offers a number of advantages. Principal among them are standardization and ease of operation; the same programs, and practically the same hardware, are used for all experiments. Provision has also been made for extensive on-line diagnostics, which need not be reinvented for each new experiment. Finally, systematic errors are minimized because the G.C.E. permits the simultaneous accumulation of coincidence, electron single, and secondary single events at adjustable ratios using the identical hardware.

The generalized coincidence electronics allows up to 32 detectors to be coupled to each other logically or to the electron spectrometer logic signals. Different decay product detectors behave differently, but once the output of each has been reduced to a timing and an analog output signal, the system takes over the processing. The operational philosophy of generalized coincidence electronics can be understood from the simplified block diagram in Figure 3. The signals from the electron spectrometer have been divided into two groups; the trigger logic, which provides a fast logic pulse indicating the arrival of any electron (e), and the track definition signals which provide the information necessary to reconstruct the electron trajectory through the spectrometer to determine the momentum, scattering angle, and point of origin of the scattered electron. The leading edge of the trigger logic signal indicates the time of arrival, \( t_e \), of the electron in the spectrometer focal plane. In the diagram each of the \( m \) reaction product (x) detectors provides an analog signal, \( a^x \), and a logical timing signal, \( t^x \), which indicates the arrival time of the reaction product at the detector. The timing signals are logically ORed at the gate \( OR_1 \) to provide a single pulse indicating the arrival of a reaction product at any one of the \( X \) detectors. This pulse is shaped by a monostable (MS) whose width determines the overall coincidence time window, and placed in coincidence with the electron trigger at the gate \( AND_1 \). By adjusting the \( X \) signal monostable to provide a pulse width substantially larger than the width of the electron signal, the output
Figure 3. \((e,e'x)\) Generalized Coincidence Electronics
of AND₁ reflects the timing of the electron signal. It is used to start a bank of TDC's which measure the precise time difference between the electron signal and each X detector. True and accidental coincidence events are accumulated simultaneously and separated in the data analysis software based on the TDC measured time differences. The output of AND₁ is also shaped and placed in coincidence with the OR₁ X detector signal at the gate AND₂ to provide a new signal which indicates a coincidence event (e•X) but with the timing of the X detector signal. The resulting signal strobes a latch which identifies the X detector associated with the event. It is also convenient as a strobe for the analog signal measurement system (ADC's, QDC's, etc.) because it is synchronized with the X detector signals.

The GCE also provides for the simultaneous accumulation of singles spectra from both the electron and X detector systems. This feature serves two purposes; it provides an on-line diagnostic tool for evaluating the individual detector performance, and it permits random coincidence background subtraction without the systematic errors that might be introduced if separate analog conversion systems were required for the singles spectra. The two gates, OR₂ and OR₃ mix (possibly) rate-divided pulses, synchronized with the X detector and electron trigger respectively, with the coincidence pulses, thus permitting the identical hardware to be used for both singles and coincidence spectrum accumulation. The event interrupt to the computer is derived at OR₄ by the logical sum of coincident, electron singles, and X-detector singles pulses.

When different research groups use the same basic instrument (the spectrometer) for a variety of unrelated experiments, the further generalization of the coincidence electronics shown in Figure 4 is desirable. (In this figure the analog signal processing has been omitted for simplicity.) The timing signals from each X detector are now sent to a multiplexer which can select among eight possibilities for the 32 input channels to the GCE. This arrangement isolates experiments from each other by separating the X detector signals experiment by experiment. The coincidence gate AND₁ in Figure 3 has been replaced by a memory logic matrix. This device is basically a fast lookup table; the m+1 inputs are stored at the time of the strobe and used as the address lines for fast ECL memories—one per output. Using such a scheme all 2^{m+1} logical possibilities may be realized. The OUT₁ output of this device performs the function of the gate AND₁ in Figure 3. To achieve
Figure 4. Expanded \((e,e'x)\) Generalized Coincidence Electronics
identical functionality, the memory for this output would be programmed to
\[ \text{OUT}_1 = (\text{IN}_1 + \text{IN}_2 + \ldots + \text{IN}_n) \cdot \text{IN}_{m+1}. \]
The strobe input to the memory unit is chosen between the electron and X detector OR signals at the strobe multiplexer. The gate AND\textsubscript{3} is added to ensure that the strobe signal for the spectrometer readout is synchronized with the electron trigger logic signal. The OR\textsubscript{2}, OR\textsubscript{3}, OR\textsubscript{4}, and AND\textsubscript{2} gates perform the same function as those in Figure 3, as do the TDC's and the LATCH\textsubscript{1} register. Finally, additional (programmable) outputs of the memory logic matrix (OUT\textsubscript{2}, \ldots, OUT\textsubscript{n}) are strobed into LATCH\textsubscript{2} to identify and characterize the event for the computer software, thereby speeding event processing. Omitted from both of these figures, but crucial to the sanity of the physicist, are a variety of calibration and timing diagnostic signals which are used to verify the correct operation of the GCE.

4. Secondary Particle Detectors

a. Thin Film Scintillator Fission Detectors

Our first (e,e'x) experiment\textsuperscript{2} involved the simple coincident detection of fission fragments from the decay of the giant multipole resonances in \textsuperscript{238}U. To achieve a large solid angle and high count-rate capability two thin film scintillator detectors were placed inside the scattering chamber near the target (figure 5) resulting in a solid angle of 500 msr per detector. These detectors were constructed by pressing 500\mu g/cm\textsuperscript{2} films of NE-102 scintillator onto 1/32" thick UVT lucite. The light output was then coupled to two 8575 photomultiplier tubes by aluminized light guides as shown in figure 6. The scintillator was sufficiently thin that the detectors operated in the dE/dx mode, distinguishing fission fragments from all other charged particles on the basis of pulse height. Coincidence resolving times of 4 nsec were achieved. Typical spectra obtained using one of these detectors with a source and with the electron beam are shown in figure 7. The resolution degradation and the large number of small pulses seen with the electron beam are probably due to low energy Möller electrons emerging from the target. These spectra could be improved by the addition of a magnetic filter between the target and the detector. The data shown in figure 1 were obtained using one of these detectors.

b. Photon Detection

The large γ-ray and neutron backgrounds present near an electron scattering target result in a particularly hostile environment for photon
Figure 5. Experimental Geometry for (e,e') Measurements Using Thin Film Detectors
**Figure 6. A Thin Film Fission Fragment Detector**

- **THIN FILM DETECTOR**
  - O-Ring
  - Light Shield
  - Photomultiplier Tubes
  - Lucite Window
  - Hollow Lucite Light Guide
  - Aluminized Interior
  - Scintillator on \( \frac{1}{32} \)" Lucite Sheet
  - Aluminized Surfaces

- 500 \( \mu \text{gm/cm}^2 \)
Figure 7. Pulse Height Spectra Obtained using a Thin Film Detector
detectors. The problems are illustrated by the spectrum shown in figure 8 using both linear and logarithmic scales. This spectrum was obtained from a $^{12}\text{C}$ target with a 10" x 12" NaI crystal. It is dominated by the neutron capture line at 6.8 MeV. The 4.4 MeV gamma ray from the $^{12}\text{C}(\text{e},\gamma)$ reaction is clearly visible in the linear plot, while the 15.1 MeV gamma ray from the $^{12}\text{C}(\text{e},\gamma)$ reactions is most easily seen in the logarithmic plot. The counting rate observed above 10 MeV is consistent with the calculated bremsstrahlung spectrum. These spectra were obtained using the shielding arrangement shown in figure 9. Lead and borafin were used to reduce the gamma and neutron backgrounds, and a plastic scintillator was used to veto events in which elastically scattered electrons entered the detector. Without this anticoincidence condition the counting rate from elastically scattered electrons would be roughly comparable to the rate observed from neutron capture. Neutron backgrounds were further reduced by a layer of $^{6}\text{Li}_2\text{CO}_3$. The dramatic effect of the coincidence requirement on the data is illustrated in figure 10. Three pulse height spectra obtained in coincidence with inelastically scattered electrons which lost 4.4 MeV in a $^{12}\text{C}$ target are displayed; coincidences (trues plus accidentals), accidentals, and trues. The accidentals spectrum is proportional to the singles spectrum in the crystal (although reduced by about six orders of magnitude), while the trues spectrum displays a shape which reflects the response function of the detector to 4.4 MeV gamma rays. The time resolution of the NaI-electron coincidence was about 8 nsec for these measurements.

The effect of the coincidence requirement on the observed electron scattering spectra can be seen in figure 11. Shown are spectra with and without the requirement of coincident observation of a "4.4 MeV" gamma ray (2.4 MeV < E < 6.4 MeV). The improvement in the signal to noise ratio is evident, as is the sacrifice in overall counting rate due to the small (~ 50 msr) solid angle of the gamma detector used. This apparatus has been used in a study\textsuperscript{10} of the 4.4 MeV ($J^\pi = 2^+$) and 15.1 MeV ($J^\pi = 1^-$) states in $^{12}\text{C}$ to demonstrate the feasibility of the \((\text{e},\text{e}'\gamma)\) technique.

An improved version of the gamma ray detection system which can simultaneously measure three photon angles is under construction at our laboratory. This system, which is shown in figure 12, incorporates custom made shielding blocks and a permanent magnet sweeping field to block elastically scattered electrons. We are also evaluating the performance of
Figure 8. Linear and Logarithmic Plots of the Pulse Height Spectrum Obtained from a $^{12}$C Target Using a 10" x 12" NaI Crystal. 
$E_0 = 67.35$ MeV, $\theta_\gamma = 130^\circ$
Figure 9. NaI Detector Shielding, $\theta_y = 90^\circ$
Figure 10. True, Accidental, and Total Coincident Photon Pulse Height Spectra Observed in a NaI Crystal in Coincidence with Inelastically Scattered Electrons which Lost 4.4 MeV in a $^{12}$C Target. $E_0 = 67.35$ MeV, $\theta_e = 60^0$, $\theta_\gamma = 130^0$.
Figure 11. The Spectra of Scattered Electrons Observed from $^{12}\text{C}$ with and without the Requirement of Coincident Detection of a 4.4 MeV gamma-ray.
Figure 12. Improved Shielding for an Array of NaI Detectors
IV. K-24

Ge(Li) and HPGe detectors in the radiation environment of our electron scattering spectrometer for the study of states where the ultimate resolution is necessary. Preliminary results are encouraging, although the beam currents available from our accelerator (< 0.5 μA) are too low for coincidence experiments using these detectors.

c. Neutron Detectors

We are presently measuring the $^{208}$Pb (e,e'ν) cross section in an effort to determine the quadrupole strength in the isoscalar giant resonance region. For heavy nuclei (A > 70) the dominant decay mode is neutron emission. Neutron detection is difficult due to the intense neutron and γ-ray backgrounds present near electron scattering targets. The use of pulse shape discrimination techniques, excellent timing, and large area neutron detectors are essential.

Figure 13 shows an exploded picture of one of the two neutron detectors in use at Illinois. It consists of a 12" diameter flask containing a scintillator (NE-213) viewed by three phototubes. Both analog and digital pulse-shape discrimination (PSD) techniques are used to eliminate γ-ray backgrounds. The analog PSD provides for fast particle identification in order to filter γ-ray events before interrupting the computer. The digital PSD improves the γ-ray rejection through software processing. The separation between γ's and neutrons produced by our locally constructed analog PSD module can be seen in Figure 14, while a time of flight spectrum (between neutrons and electrons) is shown in Figure 15. The intrinsic resolution of the detector, as measured by observing the γ-flash, is about 1.2 nsec. Most of the (random coincidence) background in this spectrum is due to electroproduced neutrons (not in coincidence) which emerge from the target into the detector solid angle. An expanded array of similar detectors is envisaged for use with the next generation of experiments.

d. Solid State Detector Array

We have developed a large solid angle solid state detector array for use in our scattering chamber in an effort to improve the detection of charged particles. The performance of solid state detectors close to electron scattering targets is seriously degraded by the presence of low energy Möller electrons, particularly when light fragments such as alphas or protons are being detected. The insertion of strong magnetic fields (of about a few kG) between the detectors and the target significantly improves the quality of the
Dry Nitrogen Inlet Valve

Expansion Chamber

(3) XP2041 PMT'S

Glass Flask With NE-213

Figure 13. Liquid Scintillator Neutron Detector
Figure 14. Neutron Detector Pulse Shape Discrimination
The ratio between the short- and long-lived decay components of the light from NE-213 scintillator liquid is measured by a zero crossing network. The lower figure shows count rate vs zero crossing time from an artificial neutron-γ source. The upper figure shows the separation attained in an actual (e,e'ν) experiment. Neutrons and γ-rays are clearly distinguished.
$E_0 = 67.0 \text{ MeV}$
$
\theta_e = 60^\circ$

$\theta_n = 131^\circ$

$9.5 \text{ MeV} < \omega < 15.5 \text{ MeV}$

Flight Path = 1 m

Figure 15. $^{208}\text{Pb}(e,e'n)$ Time of Flight Spectrum
signals, particularly at the low end of the spectrum and at forward angles. To realize such a filter for a large number of solid state detectors (or avalanche counters) simultaneously, the device shown in Figure 16 was constructed. A magnetic field perpendicular to the scattering plane is provided at all angles not seen by the primary or scattered beam. The gap in the field for the spectrometer exit can be placed at an arbitrary angle by removing one of the 20 pole segments.

This device has been installed in our present scattering chamber, and is now being used in an \( (e,e'f) \) experiment. Solid state detectors were placed as far forward as 30° relative to the beam direction with excellent results. Detectors can be placed as close as 5 cm to the target, yielding a total solid angle for an array of such detectors approaching 1 sr. The detector mounting plate can be cooled to liquid nitrogen temperatures for use with HPGe detectors or for improving the quality of the signals from silicon detectors. The device is suspended from the lid of the scattering chamber, which carries all the vacuum feedthroughs. This arrangement is especially useful because it allows for the installation of a complicated multi-detector system in the scattering chamber with only one vacuum seal. Figure 17 shows a typical time of flight spectrum between a fission detector mounted in this array and inelastically scattered electrons. The singles spectrum acquired at the same time from this detector is also shown. The substantial improvement in signal quality achieved by this arrangement relative to the thin film detector presented above is obvious. It is due both to the use of the sweeping field and to the fact that the solid state detector measures the energy rather than the energy loss of the fission fragment.

e. Conversion Electron Orange Peel Spectrometer

The final device we shall discuss is a conversion electron spectrometer. For low excitation energies in high Z nuclei (for example rotational levels in rare-earth nuclei) a bound state decay is more likely to result in conversion electron emission than in gamma ray emission. In such cases, the \( (e,e'\gamma) \) reaction will be more useful than the \( (e,e'f) \) reaction. To examine the experimental difficulties of such measurements, we have constructed a small magnetic spectrometer similar in field configuration to our tagged photon spectrometer.12 It has a radius of curvature of 9.5 cm, a large solid angle (~75 msr), and is capable of detecting conversion electrons with energies up to about 750 keV. This spectrometer has been designed as one
IV. K-29 To Spectrometer

Section of Return Yoke Removed
- Pole Tip Removed
- To Faraday Cup
Magnetic Field (⊥ to plane)

Collimator (typical)

Detector Telescopes (typical)

Incoming Electron Beam

Pole Tip Removed

Target

TOP VIEW
(Scattering Plane)

Removeable Return Yoke

Removeable Pole Tip

Magnetic Field

Detector Telescope

Target Ladder

LN Cooled Plate

CROSS SECTION
(Beam ⊥ to page)

Figure 16. Solid State Detector Array
Figure 17. Fission Spectra from Inelastic Electron Scattering

The lower diagram shows the time-of-flight spectrum between the fission fragment and the inelastically scattered electron. The upper diagram shows a typical broad, double peaked, pulse height spectrum of (e,f) singles counts characteristic of asymmetric fission.
sector of an "orange peel" array (see Figure 18). It acts as a magnetic filter, focusing a narrow range of conversion electron energies on a well shielded Si(Li) detector which provides adequate energy resolution. If our tests prove successful, we shall expand the array and try to measure the angular distribution of conversion electrons in coincidence with inelastically scattered electrons.

5. Conclusions

As the above examples have shown, it is now possible to perform inelastic electron scattering experiments in which a broad variety of reaction products from the decays of the giant multipole resonances and nuclear bound states are detected in coincidence with the inelastically scattered electron. Large solid angle and good background rejection are crucial characteristics of the decay particle detectors. The development of generalized coincidence electronics and the related software has greatly simplified the installation, testing, and use of many different secondary particle detectors with our electron spectrometer. The ability of coincidence techniques to dramatically improve signal to noise ratios in electron scattering has been demonstrated. With the construction of a large solid angle, large momentum bite electron spectrometer and a high duty factor intermediate energy electron accelerator, electron scattering coincidence experiments can be expected to provide new insights into the structure of nuclei.

The systems described in this paper are the result of a collaborative effort including many of our colleagues at Illinois and visiting scientists. Among them are G. Bolme, P. T. Debevec, D. H. Dowell, R. W. Downing, L. J. Koester, P. Leconte, H. Rønnaas, V. J. Simaitis, and S. E. Williamson. We also thank Prof. M. K. Brussel for a careful reading of the manuscript. The research program at the Nuclear Physics Laboratory is supported, in part, by the National Science Foundation.
IV. K-32

TYPICAL SEGMENT

CONVERSION ELECTRON SPECTROMETER ARRAY

Figure 18.
References
A. Introduction

The important design parameters, as discussed in the initial proposal, were for: large solid-angle (~36 msc), large momentum bite (±15%), good resolution (~0.1%), and a short path length (~2.5 m). The type of spectrometer envisaged was QQUO, similar to the recent projects at Indiana and MIT. When the design study was given to Prof. Harald Enge (MIT), he proposed a simpler, single-dipole solution. The dipole would have non-parallel pole faces, producing weak transverse focusing along the path length. This field configuration produces both a large solid-angle acceptance and a reduced value for one of the usually bothersome third-order aberration terms.

B. Spectrometer

The optical design studies are now being actively pursued by Professors Harald Enge and Stanley Kowalski at MIT. The non-parallel concept looks very promising, with two different versions having reached "intermediate" states of optimizations. Both versions achieve a large solid angle with a short path length.

A layout of the second spectrometer design, which we will call B, (the first to be called A), is shown in Fig. 1. It features a single dipole, with a gap that varies from 10 cm to 18 cm in a geometry transverse to particle trajectories. A cross-sectional view, showing the pole gap variation, is given in Fig. 2. The pole faces, however, are still flat and no special machining or construction will be necessary. Since there are no quadrupoles in the system, the path lengths are very short: 1.9 m to 1.4 m for design A, and 2.9 m to 2.1 m for design B, with a ±15% variation in momentum.

Professors Enge and Kowalski have analyzed the proposed designs with ray-tracing programs. A summary of their results is given in Table I. It is seen from the values in the table that the criteria for momentum bite, resolution, solid angle and path length have all been met or exceeded. Design "A" features a very short path length of about 1.7 meters within a compact magnet weighing 12 tons; design "B" features better intrinsic
resolution with a longer path length, about 2.5 meters, and a larger, 25-ton magnet.

With the ray-tracing programs, it is possible to investigate second- and higher-order aberrations. Those aberration terms contributing to the spot size in the focal plane are exhibited in Table II. When they are all added together in quadrature, a minimum spot size for design "A" of \( x = 0.12 \) cm results. This gives an intrinsic spectrometer resolution \( \frac{0.12}{1.23 \times 100} = 10 \times 10^{-4} \). (It should be noted that this is an uncorrected value. Since we measure \( x, \theta \) and \( y, \phi \) in the focal plane, we can trace back to reconstruct \( \theta_0, \phi_0 \) and reduce aberration terms.) For design "B", the intrinsic resolution is \( 3 \times 10^{-4} \). Aberrations and resulting intrinsic spot sizes along the focal plane for design "B" are given in Table III.

The spectrometer resolution, however, will be limited by the beam spot size on the scattering target. We are considering two modes of channel operation: The standard acromatic beam with a 0.4-cm spot and a dispersed beam with a 0.3-cm spot. The resulting system resolution for design A would be:

\[
\text{Resolution (achromatic)} = \frac{(0.4)(0.60)}{(1.54 \times 100)} = 1.6 \times 10^{-3}
\]
\[
\text{Resolution (dispersed)} = \frac{(0.3)(0.60)}{(1.54 \times 100)} = 1.2 \times 10^{-3}
\]

At a pion energy of 50 MeV, with higher order aberrations folded in, this would result in an energy resolution of 165 keV at 50 MeV.

These resolutions are obtainable at the center of the focal plane without corrections, but the aberration corrections would be needed at the low-momentum end of the focal plane. Similar results are obtained for design A.

The effective solid angle of the design has been investigated for the central momentum by looking at the beam envelope. In Fig. 3 the beam envelope, for a maximum extension in the transverse angle \( \phi \), is shown with the pole gap as a function of the distance along the trajectory.
Transmission factors for the extreme rays that would make up the whole beam envelope are listed in Fig. 4. A crude "integration" of the separate effects, as shown on Fig. 4, leads to an overall transmission efficiency \( t = 0.98 \). This implies, at the central momentum, a solid angle \( \omega = 0.98 \times 40 \times 39 \text{ mrad} \).

C. Spectrometer Comparison

There are a number of recent, or current, projects for the design and construction of magnetic spectrometers for low-energy pions. Five such designs from Indiana, MIT, SIN, TRIUMF, and our proposal to LAMPF are compared in Table IV. The larger solid-angle and shorter path length of our design will be especially valuable in experiments involving very low-energy (<20 MeV) pions.

D. Spectrometer System

Both designs "A" and "B" can be mounted on the present Bicentennial Spectrometer turntable and stand. A schematic layout, with the larger design "B" magnet on the BCS stand, is shown in Fig. 5. Also shown, in obvious schematic form, are a number of items that together will make a spectrometer: scattering chamber, pion monitor, slit system, detector mount. All of these are suitable topics for discussion at this workshop.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dispersion (x/δ, cm/%)</td>
<td>1.23</td>
<td>1.54</td>
</tr>
<tr>
<td>Magnification (x/x)</td>
<td>.54</td>
<td>.60</td>
</tr>
<tr>
<td>Momentum Bite (Δp/p)</td>
<td>±15%</td>
<td>±15%</td>
</tr>
<tr>
<td>Maximum Momentum (p max)</td>
<td>250 MeV/c</td>
<td>250 MeV/c</td>
</tr>
<tr>
<td>Intrinsic Resolution (δp/p)</td>
<td>10 x 10^-4</td>
<td>3 x 10^-4</td>
</tr>
<tr>
<td>Solid Angle (Ω)</td>
<td>40 msr</td>
<td>40 msr</td>
</tr>
<tr>
<td>Path Length (L)</td>
<td>1.4 - 1.9 m</td>
<td>2.1 - 2.9 m</td>
</tr>
<tr>
<td>Weight</td>
<td>12 tons</td>
<td>25 tons</td>
</tr>
<tr>
<td>Gap</td>
<td>10 - 14 cm</td>
<td>10 - 18 cm</td>
</tr>
</tbody>
</table>
### TABLE II

SPECTROMETER ABERKATIONS FOR X

<table>
<thead>
<tr>
<th>Term</th>
<th>Coefficient (cm - mr)</th>
<th>RMS value (cm; $\nu = 40$ m$r$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x/\theta^2 + x/\theta^4$</td>
<td>$4.1 \times 10^{-6}$</td>
<td>$0.018$</td>
</tr>
<tr>
<td>$x/\theta^2$</td>
<td>$-4.3 \times 10^{-6}$</td>
<td>$0.019$</td>
</tr>
<tr>
<td>$x/\theta^3 + x/\theta^5$</td>
<td>$-2.9 \times 10^{-7}$</td>
<td>$0.084$</td>
</tr>
<tr>
<td>$x/\theta^2 \phi^2$</td>
<td>$2.7 \times 10^{-7}$</td>
<td>$0.078$</td>
</tr>
<tr>
<td>$x/\theta^2 \phi^2$</td>
<td>$-1.4 \times 10^{-9}$</td>
<td>$0.027$</td>
</tr>
<tr>
<td>$x/\phi^4$</td>
<td>$-4.1 \times 10^{-10}$</td>
<td>$0.008$</td>
</tr>
</tbody>
</table>
TABLE III
FOCAL PLANE
PION MOMENTUM

<table>
<thead>
<tr>
<th>Aberration</th>
<th>170</th>
<th>185</th>
<th>200(MeV/c)</th>
<th>215</th>
<th>230</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1^2 + y_1^4$</td>
<td>-2.7</td>
<td>-1.1</td>
<td>$0.39 \times 10^{-5}$</td>
<td>2.0</td>
<td>3.9</td>
</tr>
<tr>
<td>$x_1^2$</td>
<td>-2.3</td>
<td>-0.50</td>
<td>$0.53 \times 10^{-5}$</td>
<td>0.80</td>
<td>0.39</td>
</tr>
<tr>
<td>$x_1^3 + y_1^5$</td>
<td>-1.1</td>
<td>-0.20</td>
<td>$-1.2 \times 10^{-7}$</td>
<td>-0.85</td>
<td>-0.80</td>
</tr>
<tr>
<td>$x_1^2 \phi^2$</td>
<td>0.49</td>
<td>0.12</td>
<td>$-0.85 \times 10^{-7}$</td>
<td>-1.9</td>
<td>-2.9</td>
</tr>
<tr>
<td>$x_1^2 \phi^2$</td>
<td>-7.3</td>
<td>-2.7</td>
<td>$-9.4 \times 10^{-10}$</td>
<td>-5.3</td>
<td>8.4</td>
</tr>
<tr>
<td>$x_1^4 \phi^2$</td>
<td>-3.6</td>
<td>-4.8</td>
<td>$-7.9 \times 10^{-10}$</td>
<td>-9.5</td>
<td>-12</td>
</tr>
</tbody>
</table>

INTRINSIC RESOLUTION, FOR $\phi = \theta = 66$ mr

(Corrections can be made to reduce $\sigma_x$ by factors of 3-5)

$\sigma_x$  | 0.16  | 0.06  | 0.05  | 0.11  | 0.19  |
<table>
<thead>
<tr>
<th></th>
<th>IUCF</th>
<th>BATES</th>
<th>SIN</th>
<th>TRIUMF*</th>
<th>LAMPF*</th>
</tr>
</thead>
<tbody>
<tr>
<td>α (msr)</td>
<td>16-35</td>
<td>35</td>
<td>20</td>
<td>20</td>
<td>40</td>
</tr>
<tr>
<td>Δp/p (%)</td>
<td>± 25</td>
<td>± 15</td>
<td>± 25</td>
<td>± 20</td>
<td>± 15</td>
</tr>
<tr>
<td>Δp/p (10^{-4})</td>
<td>5</td>
<td>10</td>
<td>6</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>L (meters)</td>
<td>3</td>
<td>3.3</td>
<td>4.1</td>
<td>2.4</td>
<td>1.7</td>
</tr>
<tr>
<td>D (cm/2)</td>
<td>1.1</td>
<td>2.0</td>
<td>1.3</td>
<td>1.0</td>
<td>1.2</td>
</tr>
<tr>
<td>M</td>
<td>0.5</td>
<td>0.4</td>
<td>0.8</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>P_{max} (MeV/c)</td>
<td>225</td>
<td>250</td>
<td>170</td>
<td>250</td>
<td>250</td>
</tr>
<tr>
<td>σ_{max} (kg)</td>
<td>16</td>
<td>-</td>
<td>11</td>
<td>-</td>
<td>16.3</td>
</tr>
<tr>
<td>Optics</td>
<td>QQDD</td>
<td>QQDD</td>
<td>QQQDD</td>
<td>QQD</td>
<td>a</td>
</tr>
</tbody>
</table>

*Proposed designs
Fig. 1

\[ \theta = \pm 100 \text{ mr} \]
\[ \phi = \pm 100 \text{ mr} \]

Airgap = 10 cm, \( B = 1.5 \text{ kg} \)

Airgap = 18 cm, \( B = 8.33 \text{ kg} \)

170 MeV, 200 MeV, 230 MeV
Fig. 3

\[ y = 0, \ \theta = 0, \ \phi = 100 \text{ mrad} \]

For central ray (200 MeV/c)

DISTANCE (cm) ALONG TRAJECTORY

TRANSVERSE SIZE (cm)

POLE GAP

BEAM ENVELOPE
<table>
<thead>
<tr>
<th>( d )</th>
<th>( \Theta )</th>
<th>( \Phi )</th>
<th>Survival Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100</td>
<td>100</td>
<td>1.00</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>100</td>
<td>1.00</td>
</tr>
<tr>
<td>0</td>
<td>-100</td>
<td>100</td>
<td>0.97</td>
</tr>
<tr>
<td>.9</td>
<td>0</td>
<td>100</td>
<td>1.00</td>
</tr>
<tr>
<td>-1.9</td>
<td>0</td>
<td>100</td>
<td>0.98</td>
</tr>
<tr>
<td>.9</td>
<td>100</td>
<td>100</td>
<td>1.00</td>
</tr>
<tr>
<td>-1.9</td>
<td>100</td>
<td>100</td>
<td>1.00</td>
</tr>
<tr>
<td>.9</td>
<td>-100</td>
<td>100</td>
<td>1.00</td>
</tr>
<tr>
<td>-0.9</td>
<td>-100</td>
<td>100</td>
<td>0.97</td>
</tr>
<tr>
<td>-0.9</td>
<td>-100</td>
<td>100</td>
<td>0.88</td>
</tr>
</tbody>
</table>

\[ \Xi = .99 \]  
\[ \Xi = .993 \]  
\[ \Xi = .993 \]  
\[ \Xi = 1.00 \]  
\[ \Xi = .981 \]  
\[ \Xi = .987 \]  
\[ \Xi = .95 \]  

\[ \Xi = .98 \]  

**Fig. 4**
Fig. 5
The MSU 1.2 GeV/c Spectrograph*
National Superconducting Cyclotron Laboratory
Michigan State University
East Lansing, MI 48824

The philosophy and principles outlined in previous papers (see other conference contributions from these authors) were the basis on which the 1.2 GeV/c spectrograph was designed. The basic layout of the MSU spectrograph is shown in figure 1, complete with the beam analysis system. Figure 2 shows an enlarged side view of the spectrograph. Figures 3-6 are engineering sketches of the pit and support structure needed to convert the magnets into a working spectrograph.

Design goals and physical parameters for the system are listed in Table 1. It should be noted that the final magnet weights are not completely determined since we are still investigating alternatives to the somewhat complex saddle coil approach. The total costs (1981$) will be approximately $1.7 million, including the beam analysis system.

*Research supported by the NSF, Grant No. PHY 80-17605 and DOE #DEAC-02-80-EI-10579
The first order optics of the spectrograph were chosen to satisfy a complicated set of arguments related to physics measurements, physical constraints, matching to the accelerator beam, simplicity of tuning, and monetary considerations. These general considerations were discussed earlier at this conference, under the topic of "integrated design". The resulting first order optics solution is illustrated in figures 7 and 8. Shown in figure 7 are the beam envelopes in both the radial and axial directions for the cases of 20 msr solid angle with a δ=0 bandpass and for 10 msr with the maximum δ=± 2.5%. Figure 8 is a schematic view of a two dimensional focal plane detector. The optical mode is point-to-point in the dispersive direction (x) and parallel-to-point in the axial direction (y). The ray's angle in the axial direction (φ) is also in the reaction plane, i.e. φ = scattering angle if not too close to zero degrees. [Near zero degrees scattering angle, the scattering angle is related to both θ and φ (radial and axial angles).] Kinematic effects, variations of outgoing particle energy with scattering angle, are accounted for in this design as indicated in figure 8. No retuning of the spectrograph or beam line parameters are required for a change of scattering angle.

The momentum resolution goal for this spectrograph system is \( \frac{p}{\Delta p} = 20,000 \). Normally, such a specification is coupled with the assumption of an incoherent spot size \( 2x_0 = 1 \) mm on target. In this case we assume \( 2x_0 = 1 \) mm at the entrance slits of the transport system and therefore can use a spectrograph with a smaller resolving power [see discussion in contribution on MSU transport system].
In order to produce a magnet design that minimized the dipole masses, a gradient in the dipole fields was allowed. Of course, you don't get something for nothing so the savings in lighter magnets is partially offset by additional measurements required at the focal plane \((x_f, y_f, \theta_f, \phi_f)\). Hence, more complex detectors, i.e. 2 x-y position sensitive ones, are needed, as well as increased computer capabilities. The additional advantage in doing ray tracing is that aberrations do not have to be completely eliminated, only reduced to an acceptable level. As discussed in our previous contribution on "integrated design" the information deduced from the additional focal plane measurements is also essential for the physics of the heavy ion reactions studied.

Figure 9 shows ray positions at the focal plane for \(\delta=0\) and \(\pm 2.5\%\) for the system before the use of second or higher order curvatures. The "+'s are for \(\theta > 0\), the "*"'s are for \(\theta < 0\) and the triangles are for 0.25 cm of y spot size on target. The rays are 0 to \(\pm 60\) mr for \(\theta\) and 0 to \(+90\) mr in \(\phi\). This is one of a pair of detector positions which will be used to measure the ray divergences at the focal plane. The detectors are to be used at "normal entrance", although the actual momentum focal plane has a 15\(^\circ\) tilt which is corrected in the ray tracing. The x size of the image is approximately 3 cm, caused mainly by the \((x/\theta^2)\) aberration. Looking at a plot of \(x_f\) vs \(\theta_f\) as in figure 10 (for the case of \(\phi=0\)), when \(\theta_1\) goes from \(-60\) mr to \(+60\) mr for \(\delta=0\) and \(\pm 2.5\%\) gives us an idea of what ray tracing must correct. From the design goals listed in Table 1, a resolution
of 0.45 mm is required for an energy resolution of $10^{-4}$. If the angle is measured with an uncertainty of 1 mr, the slope of the x vs. $\theta$ curve for $\epsilon=0$ in figure 9 are 0.46 and 0.63 mm/mr for the negative and positive ends, respectively. So while the smaller thetas (or solid angles) have slopes better than those required, it is obvious that the system needs aberration corrections to give a $dx/d\theta$ of $\leq 0.45$ mm/mr at the maximum solid angle.

To first order, the y measurement in the focal plane ($y_f$) determines the angle $\phi$ at the target ($\phi_i$) as indicated in figure 8. However, as indicated in figure 9, there are spot size ($y_i$) aberrations, mainly ($y/y_0$) which also affect $y_f$. This effect is partially correctable, because $\phi_f$ is related in first order to $y_i$, ($\phi/y$) = 1.4 mr/mm. However, quantitative analysis of the y-size aberrations shown in figure 9 indicates that a measurement of $\phi_f$ with an uncertainty of 1 mr is not sufficient to reduce this aberration to the desired level.

The above analysis indicates that second order corrections are necessary to achieve the desired resolutions in momentum and in scattering angle, even with focal plane angle measurements having 1 mr uncertainties. The second order corrections need not be exact, but only sufficient to reduce the aberrations to a level correctable with focal plane measurements of the specified accuracy.

Acceptable second order corrections can be made by putting edge curvatures at the entrances and exits of the two dipoles. The two dominant aberrations in the uncorrected spectrograph
(figures 9 and 10) are \((x/\theta^2)\) and \((y/\theta)\). Using the sensitivity equations of K.L. Brown (SLAC-75), leads to the following combination of curvatures. A large positive curvature is needed on the first dipole (D1) entrance to reduce \((x/\theta^2)\), while a negative curvature at D1 exit reduces \((y/\theta)\). This latter correction induces \((y/\theta \delta)\) and \((x/\theta \delta)\) terms which are corrected with positive and negative curvatures at the entrance and exits of D2, respectively. The magnitudes of these corrections were not chosen to completely eliminate the second order terms, but only to reduce them to tolerable values. Attempts to eliminate these terms completely resulted in other second and higher order terms which were worse than the original aberrations.

Table II lists the resulting curvatures for D1 and D2, defined in the usual way, that provide a good solution. The values in parenthesis are an alternate set which gives essentially identical results. In addition, a single very small 3rd order correction, to reduce \(x/\theta^3\), is used, as indicated. This correction represents only a 2.5 mm deviation from a circular entrance curvature on D1 at a distance of 20 cm from the axis.

Shown in figures 11 and 12 are the plots for the system when the curvatures are used. Either set of curvatures from Table II provides essentially the same solution. For \(\delta=0\) the x size is only reduced to 2 cm, but the slopes of the \(\theta_{\text{final}}\) curves are now a maximum of 0.3 mm/mr at either end. It is also evident that there remain uncorrected, but greatly reduced y-size related aberrations. These terms are now correctible via \(\phi_f\) measurements. It should be noted that the rays in
figure 11 which display the worst y resolution do not actually come through the spectrograph, i.e. $\theta=+60\text{ mr}$, $\delta=+2.5\%$ and $\theta=-60\text{ mr}$, $\delta=-2.5\%$. This is related to the fact that the solid angle is 20 msr for the central momentum, but falls to 15 msr at either end of the focal plane (figures 7 and 13). If the acceptance aperture is reduced to $\theta=\pm30\text{ mr}$ then a bandpass of 10 msr over the entire momentum range is transmitted and the remaining aberrations are considerably reduced (also figures 7 and 13).

One last point is to illustrate the need for very good angular resolution in heavy ion reactions at high energies. Shown in figure 14 is a calculated angular distribution for 60 MeV per nucleon $^{16}\text{O}$ on $^{28}\text{Si}$ (using optical model parameters from J.G. Cramer et al, Phys. Rev. C15, 693(1977) ). In this case the angular resolution is as important as the energy resolution since a $\Delta E/E$ of only $2\times10^{-3}$ is needed whereas an angular resolution of 2.5 mr is needed to resolve the predicted angular structure.
Figure Captions

1. Spectrograph and beam analysis system
2. Side view of spectrograph
3. Engineering sketch of spectrograph
4. Engineering sketch of spectrograph
5. Engineering sketch of spectrograph
6. Engineering sketch of spectrograph
7. Spectrograph beam envelopes
8. Focal plane detector illustration
9. Uncorrected rays at the focal plane
10. Uncorrected solution in the x-θ plane at the focal plane.
11. The same as 9, but with edge curvatures
12. The same as 10, but with edge curvatures
13. Solid angle vs. band pass
14. Calculated $^{16}\text{O} + ^{28}\text{Si}$ elastic scattering
### Table I

**PARAMETERS OF THE MSU 1.2 GeV/c SPECTROGRAPH**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ENERGY RESOLUTION:</strong></td>
<td>( \Delta E/E = 10^{-4} ) WITH 1MM RADIAL OBJECT SIZE FOR BEAM ANALYSIS SYSTEM</td>
</tr>
<tr>
<td><strong>ENERGY RANGE:</strong></td>
<td>( \Delta E/E = 10% )</td>
</tr>
<tr>
<td><strong>SOLID ANGLE:</strong></td>
<td>( \Omega = 10-20 \text{ MSR} )</td>
</tr>
<tr>
<td><strong>RESOLVING POWER:</strong></td>
<td>( D/M = 11.7 )</td>
</tr>
<tr>
<td><strong>RADIAL DISPERSION:</strong></td>
<td>( D = 9 \text{ cm} )</td>
</tr>
<tr>
<td><strong>RADIAL MAGNIFICATION:</strong></td>
<td>( M = 0.73 )</td>
</tr>
<tr>
<td><strong>AXIAL DISPERSION:</strong></td>
<td>( R_{34} = 0.74 \text{ mm/MR} )</td>
</tr>
<tr>
<td><strong>ANGULAR RESOLUTION:</strong></td>
<td>( \Delta \theta \leq 2\text{mr} ) (TOTAL OF BEAM PLUS SPECTROGRAPH CONTRIBUTIONS)</td>
</tr>
<tr>
<td><strong>FOCAL PLANE SIZE:</strong></td>
<td>50 cm (RADIAL) x 15 cm (AXIAL)</td>
</tr>
<tr>
<td><strong>FOCAL PLANE TILT:</strong></td>
<td>15°</td>
</tr>
<tr>
<td><strong>MAGNETIC RIGIDITY:</strong></td>
<td>( B' = 4 \text{T-M} )</td>
</tr>
<tr>
<td><strong>DIPOLE FIELDS:</strong></td>
<td>( B = 1.5 \text{T} (\rho = 2.7 \text{M}) )</td>
</tr>
<tr>
<td><strong>DIPOLE GAP:</strong></td>
<td>( D = 15 \text{ cm} )</td>
</tr>
<tr>
<td><strong>DIPOLE SIZE:</strong></td>
<td>2 OF 3.5 m LONG x 100 cm WIDE (75° BEND)</td>
</tr>
<tr>
<td><strong>WEIGHT OF DIPOLES:</strong></td>
<td>APPROX. 50 TONS EACH</td>
</tr>
<tr>
<td><strong>QUAD SIZES:</strong></td>
<td># 1 20cm ID x 40 cm LONG</td>
</tr>
<tr>
<td></td>
<td># 2 35cm x 17cm x 40cm</td>
</tr>
<tr>
<td><strong>DETECTOR REQUIREMENTS:</strong></td>
<td>TWO 2-DIMENSIONAL DET., 1m SEPARATION</td>
</tr>
<tr>
<td></td>
<td># 1 50cm x 15cm</td>
</tr>
<tr>
<td></td>
<td># 2 62cm x 16cm</td>
</tr>
<tr>
<td><strong>RESOLUTION:</strong></td>
<td>RADIAL 0.2 mm</td>
</tr>
<tr>
<td></td>
<td>AXIAL 0.5 mm</td>
</tr>
</tbody>
</table>
### Table II
Magnet Parameters for the MSU
1.2 GeV/c Spectrograph

<table>
<thead>
<tr>
<th>Element</th>
<th>Central or Pole Tip Field (kG)</th>
<th>Entrance Edge Angle (deg)</th>
<th>Exit Edge Angle (deg)</th>
<th>2nd Order Curvatures (m⁻¹)</th>
<th>3rd Order Coefficients*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1</td>
<td>21</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q2</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D1</td>
<td>15</td>
<td>0</td>
<td>29.7</td>
<td>1.2(1.0)⁺</td>
<td>-0.36(-0.39)</td>
</tr>
<tr>
<td>D2</td>
<td>15</td>
<td>29.7</td>
<td>0</td>
<td>0.21(0.24)</td>
<td>-0.13(-0.12)</td>
</tr>
</tbody>
</table>

* Amplitude of 3rd order correction: \( \Delta z = \rho S \left( \frac{x}{\rho} \right)^3 \)

⁺ Parameters in parenthesis are for the alternate solution, giving an indication of tolerances.
MSUX-80-496

TWISTER (DRIFT)

DET 2
DET 1

VERTICAL HEIGHT ABOVE TARGET (METERS)

HORIZONTAL DISTANCE (METERS)

SLITS

SLITS

GRADE LEVEL

ROOF SHIELDING

PLATFORM

D2
D1

TGT

Qx
Qy
K=800 Spectrograph  Focal Plane Detector

208\text{Pb}(^{40}\text{Ca},^{40}\text{Ca})

15 cm wide (axial)
0.7 mm/mr

50 cm tall (radial)
8.9 cm/\% \frac{\Delta p}{p}
M_x = 0.87
WITHOUT CURVATURES

\[ x \text{ [CM]} \]

\[ \text{THETA FINAL [MR]} \]
WITH CURVATURES
Asymmetric Acceptance

Symmetric Acceptance

\[ \Delta P/P \text{ (\%)} \]

\[ d\Omega \text{ (msr)} \]
A Superconducting Multigap Spectrograph
Harald A. Enge

Fig. 1 shows a sideview of the original multigap spectrograph, design initiated at MIT in 1955. Fig. 2 shows a top view. The magnet is a complete toroid with 13 gaps spaced 7.5 degrees from 0 to 90° in one quadrant and 12 gaps from 90 to 172.5 degrees in the opposing quadrant. There is also a larger gap at 45 degrees as shown. Coils are placed between the gaps to produce ampereturns where it is needed.

We have now made a preliminary optical design of a multigap for hypernuclear studies intended to be used in conjunction with a kaon beam at the Brookhaven AGS. Fig. 3 shows a top view - very preliminary, of course - and Fig. 4 shows a cross section of the quarter-toroid. The optics of one gap is similar to the Clamshell spectrometer designed for LAMPF, indeed this design predates the Clamshell by a year. In order to produce the desired field distribution, namely that of a conventional magnet with a wedge-shaped gap, some of the conductors in a coil are returned along slanted straight lines as shown in the figure. The whole coil would be imbedded in an aluminum box with liquid-helium cooling channels.

This particular design is made for a primary beam of kaons which with its low intensity does not pose any problems of disposal. For a high-intensity beam a compensating field would have to be produced at zero degree to allow for proper beam
disposal. There are substantial design difficulties with any pro-
ject of the kind dealing with magnetic forces and heat insulation
simultaneously. None of these problems have been looked into. The
proposed maximum field (at entrance and exit) is about 30 kG, but
the average field does not exceed 20 kG.
Fig. 2
Beam direction for $\theta = 0^\circ - 45^\circ$

Beam direction for $\theta = 0^\circ - 90^\circ$

Fig. 3
A VERY LARGE SOLID ANGLE SPECTROMETER FOR SINGLE ARM ELECTRON SCATTERING EXPERIMENTS

Ph. Leconte
CEN. SACLAY

Contents

1. Introduction 1

2. How to use a non-focussing spectrometer? 2

3. Two possible spectrometers 3

   3.1. A dipole spectrometer 4

   3.2. Critics on the dipole spectrometer 4

   3.3. An other geometry: The TT spectrometer 5

4. Conclusion 6

5. References 7
V. D-2

1. Introduction

Major informations about short range behaviour of nuclear forces should be obtained through electron scattering experiments at high momentum transfer. Cross sections will be very low as it is usually the case in electron scattering. See, for example, the lowest cross section measured at Saclay on Lead 209 at Saclay (1):

$1 \times 10^{-7} \text{cm}^2/$steradian

In order to reach them, the solid angle of the detection system will have to be enlarged by an extremely important factor. Traditional optics cannot give correct answer to the problem. For very large apertures, it is impossible to obtain good focusing properties which provide accurate momentum/position correlation with no dependence on the entrance angles. Furthermore, the experiment will require anyway the measurement of these angles. It means that the final system will be equipped with a complete set of position sensitive detectors able to measure positions and angles of trajectories in both planes. Then, the question arises: is it really necessary to provide good focusing? or more precisely: is it possible to get all the required informations without the help of a sophisticated predetermined magnetic optics? We shall try to answer this question and then to sketch from a new point of view the best spectrometer we could think of.
2. How to use a non-focussing spectrometer?

The answer comes in three steps:

1- Assume that the magnetic map is reproduceable within the required accuracy.

2- Make a table containing all information about entrance and exit coordinates of a complete set of trajectories filling the whole acceptance phase space.

3- Use first order beam optics theory to interpolate and find with the required accuracy, the momentum and the entrance angles of the measured trajectory.

Let us show that this interpolation is in principle possible. Four quantities have been measured: positions and angles in the image space: \( x, \theta, y, \phi \). There are five unknowns: \( x, \theta, y, \phi, d \). There are four linear equations:

\[
\begin{align*}
x &= f_x(x_1, \theta_1, y_1, \phi_1, d) \\
\theta &= f_{\theta}(x_1, \theta_1, y_1, \phi_1, d) \\
y &= f_y(x_1, \theta_1, y_1, \phi_1, d) \\
\phi &= f_{\phi}(x_1, \theta_1, y_1, \phi_1, d)
\end{align*}
\]

There is one more unknown than the number of equations so that to solve the system, we must prescribe the value of the \( x \) coordinate.
This means that the beam spot size must be small enough to match the required resolution. The preceding set of equations can then be inverted to provide four linear equations as the following one for the momentum:

\[ d = \alpha x + \beta x + \gamma^* y + \epsilon^* \xi + \zeta^* \rho^* \]

The resulting resolution can be expressed as a function of the various accuracies in the measured quantities:

\[ \Delta d = \sqrt{\alpha^2 (\Delta x)^2 + \beta^2 (\Delta x)^2 + \gamma^2 (\Delta y)^2 + \epsilon^2 (\Delta y)^2 + \zeta^2 (\Delta \xi)^2 + \zeta^* (\Delta \rho)^2} \]

If the resolving power is about 10 000, and the solid angle about 1 steradian, the number of reference trajectories may be of the order of magnitude of ten to the six. However, improvements of this brute force method may probably reduce the figure by a considerable amount.

3. Two possible spectrometers
3.1. A dipole spectrometer

In a proceeding work (2), I have tried to push to its furthest limits, the classical solution of the dipole spectrometer as shown in fig. 1.

To obtain the largest possible angular acceptance, it is necessary to channel the particles sending them into the dipoles by the help of a point to parallel optics of two large quadrupoles. Their fields reach 0.8 to 0.9 Tesla. In order to shield the detectors from the background originating in the beam area, a large shielding is provided. A second dipole improve background rejection by elimination of the particles scattered on the wall of the vacuum chamber.

The total weight of iron would be larger than 3000t in such a device.

3.2. Critics on the dipole spectrometer

Several critics can be made on such a project:

1- The field in the quadrupoles would be extremely large.

2- Gaps of the quads are as big as their depths. Field map is only made of fringing fields. Their properties would be far from the prescribed ones.
3- Preliminary calculations have been done at first order only. Higher order will show explosion of the phase space along the magnet, requiring accurate compensation or much larger acceptance of the dipoles, thus increasing the already huge dimensions of the dipoles.

4- The magnetic field which has been produced, is not used in an efficient way. All the bending force used to channel the electrons is not worth to measure the momentum, which is the most important role of the magnetic field.

3.3. An other geometry: The TT spectrometer

Axial symmetry is the only good answer to the problem. Moreover, as in electron scattering, the counting rate vary very quickly with the scattering angle. This implies that it is not worth opening widely the solid angle in the scattering direction. The only efficient way is then to get an acceptance as large as possible in the azimuthal direction, which implies an axial symmetry.

This why the following "TT" spectrometer (TT holds for Toroid Toroid) is proposed (fig. 2). Each toroidal field is obtained by the help of 8 superconducting coils which do not occupy more than 20 % of the azimuthal angle.

The first toroid helps to define the scattering angle and channel the particles into cylindrical slits through a large cylindrical shielding. The target is moved along the beam, as proposed by Blomqvist (3), in order to change the scattering angle.
A solid angle of 1 steradian at 90 degrees of scattering angle can be obtained. One can hope to have a resolution close to 200 KeV between 1 to 2 GeV.

Some focussing property can be looked for to reduce the size of the wire chambers. Indeed, they may be the most expensive part of the whole set-up.

No energy loss compensation can be implemented unless a complementary detection is provided between the two toroids as proposed by H. Enge. But, this complementary detection would be exposed to a very large counting rate and background problem due to the lack of shielding.

4 Conclusion

This proposal is not yet a realistic proposal in the sense that all technical problems are far to be solved. The price of such a device can be large and may be comparable with the price of the accelerator itself. It is very important to point out that in the present experimental facilities, the detection efficiency is very low due to the small solid angle subtended by their spectrometers (typical values range from 5 to 10 millisteradians). Remarkable improvements presented in this workshop show that it is possible to reach 30 to 40 millisteradians. But it is not enough. With such spectrometers, the experimentally observed counting rate will include only a small fraction of the actual nuclear reactions induced by the beam in the same range of momentum transfer. One steradian spectrometers are the challenge.
5 References


2-- C. Grunberg and Ph. Leconte, "A spectrometer of extremely large aperture for 1 to 2 Gev electron scattering experiments", in Prospects for new facilities on intermediate energy electron machines. Saclay internal report, DPhN/HE 81-2.

3-- I. Blomquist, "Some wild ideas about multiparticles coincidence spectrometers" Symposium on coincidence spectrometers. Mainz, Feb. 81
Fig 1. Dipole Spectrometer.
T.T Spectrometer

Side View
Axial Symmetry

Fig. 2. The Toroidal Solution
THE QDD AND QDQ SPECTROMETERS AT NIKHEF-K

H. de Vries

Nationaal Instituut voor Kernfysica en Hoge-Energiefysica,
div. K, Amsterdam, The Netherlands

The experimental program of the electron scattering group at the 500 MeV, 2½-10% duty cycle accelerator in Amsterdam contains high-resolution single arm experiments as well as coincidence experiments. The requirements for the detection equipment for these two classes of experiments are somewhat conflicting; a good compromise has been found in the construction of two magnetic spectrometers:
1) a high-resolution ($\Delta p/p \leq 10^{-4}$) QDD-spectrometer, and
2) a large solid angle ($\Delta \Omega = 17.2$ msr) QDQ spectrometer.

The QDD is used in high-resolution single-arm experiments, the QDQ with its more moderate resolving power ($\leq 5 \times 10^{-4}$) can be used for single arm experiments where resolution is not so important (Giant Resonance (G.R.), Quasi-elastic (Q.E.)). In coincidence experiments the QDD is used for the observation of the scattered electron, the QDQ for the detection of the knocked-out ($p, d, a$) or produced ($\pi^\pm$) particle.

For both spectrometers we have chosen for the "hardware" design, which means that as many optical aberrations as possible have been corrected by the application of curved polefaces of the magnets and the addition of higher order multipole components in the quadrupole fields. This facilitates the data-reduction, since no important software corrections are required. In particular both spectrometers have a flat focal plane, reducing the need for a high spatial resolution (0.25 mm) to one wire chamber.

The function of the entrance quadrupoles is two-fold:
1. they limit the dimension of the accepted rays in the non-dispersive direction, thus reducing the required gap of the dipoles by a factor of two;
2. they enlarge the value of the Karl-Brown integral
\[ \int_0^t R_{12}(\tau) h(\tau) d\tau \]
(see H. Enge's contribution to this conference), thus improving the resolution of the spectrometers. The optical design of both spectrometers has been made in collaboration with H. Enge. The high-resolution QDD has point-to-point \((<x|0>=0)\) focussing in the dispersive direction, and parallel-to-point \((<y|y>=0)\) focussing in the \(y\)-direction. This last condition facilitates the determination of the scattering angle, which is needed for the application of kinematic corrections.

Furthermore there is a cross-over (point-to-point, \(<y|\phi>=0\), in the non-dispersive direction between the two dipoles. At this place we have installed a simple multipole magnet; due to the fact that the dimensions in the \(y\)-direction of the accepted rays are very small at this position, only \(x\) and \(\phi\)-terms are affected by this multipole. Therefore unforeseen aberrations of the instrument can be eliminated without the introduction of cross-terms between \(x\) and \(y\).

The strong resolution requirements demanded corrections of high order to be applied; both entrance and exit faces of the dipoles are described by a 7th order polynomial, and the quadrupole contains higher order multipole terms up till dodecapole. The resulting main properties are listed in Table I.

The QDQ spectrometer also requires rather strong entrance and exit curvatures; in order to create another place in the magnet where corrective curvatures can be applied, the dipole has an insert in the middle where the pole has partially been cut-away. The focussing conditions of the QDQ are point-to-point in both planes, i.e. \(<x|\theta>=0\) and \(<y|\phi>=0\). The main properties are listed in table I.

In order to obtain a resolution of about \(10^{-4}\) the field homogeneity has to be of the same order of magnitude. This imposes strong requirements on the quality of the steel and the flat-
ness of the poles. Both the poles and the yokes have been made from forged low-Carbon steel, with a Carbon content < 0.005% for the poles and < 0.01% for the yokes. The achieved flatness of the poles is better than 7 μm. A schematic cross section of the poles is shown in fig. 1. The poles have an approximate Rogowski profile in order to prevent saturation of the edges. The poles are separated by titanium (QDD) or beryllium-copper (QDQ) spacers, shaped such that they also act as baffles for particles with momenta or directions outside the acceptance range. The vacuum chamber has been made from stainless steel; it is welded to the poleshoes such that the poleshoes form a package serving at the same time as vacuum chamber. This whole package is separated from the yoke by a Purcell gap. This not only improves the homogeneity of the magnet by re-aligning the flux lines in the poles, but it also decouples the poles from the yoke, which might be deformed by mechanical and magnetic forces. A prevacuum in the Purcell gap eliminates the deflection of the poles due to air pressure. The rigidity requirements of the poles as well as the high additional costs prevented the use of separate pole caps of extremely high quality steel (as has been used for the M.I.T. DD spectrometer). As a result it is necessary to use well-defined cycling procedures for the setting of the magnetic fields. The final field setting is reached after an (empirically determined) overshoot in current. Following these cycling procedures a field homogeneity of better than 2x10^-4 can be obtained, with a good reproducibility. A typical longitudinal homogeneity is shown in fig. 2. The dashed window represents the ± 2x10^-4 region. Also the shapes of the effective field boundaries agreed well with the design as can be seen in fig. 3. The figure shows the deviation of the position of the effective field boundary of the entrance of the first dipole of the QDD from the design as a function of the excitation of the magnet; the deviations in shape are less than 0.2 mm, the shift in the position of the EFB is less than 0.5 mm between 0.13 and 1.2T.
Only at the exit field of the first dipole of the QDD position variations up to 1 mm occur; however, this effect has somewhat been expected, since the limited space available caused a slightly underdimensioned design of the field clamp. These deviations can, however, easily be corrected with the multi-pole magnet.

The homogeneity of the QDD shows, of course, a dip around the split region as shown in fig. 4. The corresponding shift in the effective field length of the magnet agreed well with the design.

Magnetic field measurements of the quadrupoles included effective length measurements as well as an analysis of the higher order multipole components. The last measurement has been done with the aid of a harmonic analysis of the signal induced in a rotating coil. The largest deviations showed up in the sextupole components (5% too low), resulting in larger $<x|0\rangle^2$ values. This does not deteriorate the resolution drastically. Software corrections can even completely eliminate this effect. A schematic drawing of the two-spectrometer setup is shown in fig. 5. Both spectrometers can rotate around the scattering chamber over an angular range of $25^\circ$ to $155^\circ$. The scattering chamber has been equipped with a sliding foil construction for both spectrometers. Around the focal planes of both spectrometers shielding with a total weight of 220 tons and consisting of heavy concrete (s.g. 3.5 g/cm$^2$) with layers of boron-loaded polyethylene and lead on the inside has been applied. A special support construction carries the shielding of the QDD, thus preventing the heavy shielding to influence the mechanical properties and tolerances of the spectrometer.

The central pivot point is a spherical bearing. The spectrometers rotate along a roller bearing, consisting of a track with 80 cylinders of 100 mm height and 100 mm diameter for each spectrometer. Once set at the required angle the spectrometer is lifted from the roller bearing by means of two hydraulic jacks and levelling takes place. The whole process
of rotating the spectrometer, including the synchronous rotation of the sliding foils and the levelling, is controlled by a micro-processor. The angular setting and read-out is done with an accuracy of 0.01°.

Each spectrometer has a number of movable tantalum plates with fixed apertures for the definition of the solid angle. A vacuum valve behind this slit system allows the replacement of a complete set of plates without breaking the vacuum of the spectrometers.

The detector system of both spectrometers consists of a series of Multi Wire Drift Chambers (MWDC) backed by scintillators and a Cerenkov counter. The wire chambers contain 280 wires of 110 mm length. The gap as well as the wire spacing are 4 mm. The use of an internal clock for drift time measurements allows a spatial resolution of 0.25 mm, which corresponds with $2.5 \times 10^{-5}$ momentum resolution. By using a "normal" chamber (x-chamber) followed by a chamber with slanted wires (y-chamber) it is possible to determine the x and y position; the use of two of those sets, separated by 20 cm, allows the reconstruction of the complete particle track in the focal plane.

A schematic view of the detector system is shown in fig. 6.

The first results obtained with the electron scattering setup are shown in fig. 7. A so-called "magic target", consisting of Carbon, Aluminium and Niobium, was used for proper tuning of the deflecting system and the spectrometers. A proper dispersion matching yields about equal peak width for the Carbon (central part of the target) and Niobium (outermost parts of the target) peaks. The observed peak width can almost completely be attributed to kinematic broadening effects and straggling in the target and the exit window of the spectrometer, thus indicating that the intrinsic resolution of the spectrometer is better than $10^{-4}$. A typical spectrum of electrons of 205 MeV, scattered through 80° from $^{58}$Ni, is shown in fig. 8. Without any software correction a resolution of $1.5 \times 10^{-4}$ has been obtained. Further tests will be undertaken in the near future; these tests include the so-called missing-energy matching in coincidence experiments.
In this mode the beam dispersion at the target is matched to the dispersion of the spectrometers in such a way, that its contribution to the missing energy is zero in first order (see for further details L. Lapikás and P.K.A. de Witt Huberts, Proc. Symposium on Perspectives in Electro- and Photonuclear Physics, 1980, Saclay).

Using this technique we expect to obtain an overall missing energy resolution of about 150 keV.
Table I.

<table>
<thead>
<tr>
<th></th>
<th>ODD</th>
<th>ODDQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius of curvature</td>
<td>1.40 m</td>
<td>1.60 m</td>
</tr>
<tr>
<td>Max. particle momentum</td>
<td>600 MeV/c</td>
<td>750 MeV/c</td>
</tr>
<tr>
<td>Max. field strength</td>
<td>1.43T</td>
<td>1.56T</td>
</tr>
<tr>
<td>Deflecting angle</td>
<td>2 x 75°</td>
<td>90°</td>
</tr>
<tr>
<td>Gap of the dipoles</td>
<td>7 cm</td>
<td>12.8 cm</td>
</tr>
<tr>
<td>Momentum acceptance</td>
<td>± 5%</td>
<td>± 5%</td>
</tr>
<tr>
<td>Angular acceptance Δθ</td>
<td>± 40 mr</td>
<td>± 70 mr</td>
</tr>
<tr>
<td>Δφ</td>
<td>± 40 mr</td>
<td>± 70 mr</td>
</tr>
<tr>
<td>Ω</td>
<td>5.6 msr</td>
<td>17.2 msr</td>
</tr>
<tr>
<td>Focussing conditions</td>
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<td>θ&gt;=0</td>
</tr>
<tr>
<td></td>
<td>&lt;y</td>
<td>θ&gt;=0</td>
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<tr>
<td>Angular magnification</td>
<td>&lt;θ</td>
<td>θ&gt;=-1.67mr/mr</td>
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<tr>
<td></td>
<td>&lt;y</td>
<td>φ&gt;=.087cm/mr</td>
</tr>
<tr>
<td>Dispersion</td>
<td>6.78 cm/%</td>
<td>7.28 cm/%</td>
</tr>
<tr>
<td>Focal plane angle</td>
<td>41°</td>
<td>50°</td>
</tr>
<tr>
<td>Resolution</td>
<td>&lt;1x10^-4 (Δp/p = ± 1%)</td>
<td>&lt;5x10^-4 (Δp/p = ± 5%)</td>
</tr>
<tr>
<td></td>
<td>&lt;3x10^-4 (Δp/p = ± 5%)</td>
<td></td>
</tr>
<tr>
<td>Focal plane</td>
<td>straight</td>
<td>straight</td>
</tr>
<tr>
<td>Weight</td>
<td>120 ton</td>
<td>145 ton</td>
</tr>
<tr>
<td>Weight shielding</td>
<td>128 ton</td>
<td>94 ton</td>
</tr>
</tbody>
</table>

*) In dispersion matching mode for incoming beam spread of ± 1.5x10⁻³.
Figure captions.

fig. 1 Schematic cross section of a spectrometer dipole

fig. 2 Typical longitudinal homogeneity of the first dipole of the QDD

fig. 3 Deviations of entrance profile of the first dipole from the design positions as a function of excitation.

fig. 4 Longitudinal homogeneity measurement of the dipole of the QDQ, showing the EFB-shift in the split-region

fig. 5 Layout of the two-spectrometer setup

fig. 6 Schematic view of the detector system in the QDD

fig. 7 Elastic peaks from $^{12}$C, $^{27}$Al and $^{93}$Nb as determined with the "magic target". The equal widths of the peaks show proper functioning of the deflecting system and the spectrometer

fig. 8 Spectrum of 205 MeV electrons scattered through 80° from $^{58}$Ni. Spins and parities of known levels are indicated.
Fig. 1
Fig. 2
Fig. 3
Split Region of Dipole 3

\[ B_0 = 0.30 \, T \]

Track nr. 6

Input files

File DS6030.DAT

Number of datapoints 406

EFB shift

19.92 mm

Stepsize 2.50 mm
Fig. 5
Fig. 6
Fig. 7

MAGIC TARGET (e,e')
E_0 = 116.1 MeV
\(\theta = 63.9^\circ\)

- C 23.78 mg/cm²
- Al 26.67 mg/cm²
- Nb 34.49 mg/cm²
$^{58}\text{Ni} \ (e, e')$

$E = 204.7 \text{ MeV}$

$\theta = 79.8 \text{ Deg}$

Channels:
- $6^+_1$: 5.13
- $4^+_1$: 4.755
- $3^-_1$: 4.475
- $4^+_3$: 4.405
- $4^+_1$: 3.62
- $O^+_3$: 3.53
- $2^+_4$: 3.265
- $2^+_3$: 3.038
- $4^+_1$: 2.460
- $2^+_1$: 1.454
- $O^+_1$: elastic
Design Considerations for a Coincidence Arrangement of two Magnetic Spectrometers with Large Solid Angle and Moment Acceptances*)

Reiner Neuhausen
Institut für Kernphysik, Universität Mainz, 6500 Mainz, Germany

The c.w. electron accelerator MAMI, which is under construction at the university of Mainz, consists of a cascade of three race track microtrons and is designed to deliver a 100 μA electron beam at a maximum energy $E_0 = 820$ MeV with an energy spread $\delta E_0 = 120$ keV. Because of the duty factor close to 100 % ideal conditions will be provided for electroproduction coincidence experiments which will be an essential part of the future experimental program with MAMI. For these experiments, a coincidence arrangement consisting of two magnetic spectrometers is proposed.

What is presented in this paper, is not considered as a final design of the coincidence arrangement, but more as a basic concept which we believe can be successfully pursued with the aim to achieve a coincidence arrangement which will meet all requirements of future coincidence experiments, not only $(e,e'p)$ experiments but also those with other charged particles. Though the requirements of the physical experiment should have highest priority, the high costs for magnetic spectrometers in medium and high energy physics impose restrictions to their design.

Objectives for the design: The objectives for the design (table 1) were strictly fixed in favour of coincidence experiments. Single arm $(e,e')$ experiments are considered to be of minor priority for our future program. The most important features are a large solid angle ($\Omega = 30$ msr) and a large momentum acceptance ($\Delta p/p = 20\%$) for both arms. To keep the pole width of the last dipole in reasonable limits the dispersion was restricted to $2-3$ cm/%. An excessively high momentum resolution is not aimed to, but $\delta p/p = 2 \times 10^{-4}$ for both arms seems to be a reasonable value for coincidence experiments.

*) Based in part on the diploma thesis of Thomas Baumgärtner
Table 1: Objectives for the design of an (e,e'p) coincidence arrangement consisting of two magnetic spectrometers

<table>
<thead>
<tr>
<th>Parameter</th>
<th>e⁻-spectrometer</th>
<th>p-spectrometer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum momentum</td>
<td>820 MeV/c</td>
<td>950 MeV/c</td>
</tr>
<tr>
<td>Maximum energy</td>
<td>820 MeV</td>
<td>400 MeV</td>
</tr>
<tr>
<td>Maximum field</td>
<td>1.2 T</td>
<td>1.36 T</td>
</tr>
<tr>
<td>Dispersion</td>
<td>2-3 cm/%</td>
<td>2-3 cm/%</td>
</tr>
<tr>
<td>Momentum acceptance</td>
<td>20 %</td>
<td>20 %</td>
</tr>
<tr>
<td>Solid angle</td>
<td>30 msr</td>
<td>30 msr</td>
</tr>
<tr>
<td>Momentum resolution</td>
<td>2x10⁻⁴</td>
<td>2x10⁻⁴</td>
</tr>
</tbody>
</table>

The quantity for which we want to have good resolution is the "missing energy" $E_M$ in (e,e'p) experiments:

$$E_M = E_0 - E_e - T_p - \frac{p_k^2}{2M_k}$$

(1)

Here, $E_e$ is the energy of the scattered electron and $T_p$ the kinetic energy of the knocked-out proton. Neglecting the recoil energy of the remaining nucleus (last term in eq.(1)) the uncertainty of the missing energy is given by the following expression:

$$\delta E_M^2 = \delta E_0^2 + \delta E_e^2 + \left(\frac{p_k^2}{m_p^2 + T_p}\right)^2 \delta p_k^2$$

(2)

From the specific example (fig.1), we conclude that a good momentum resolution is necessary for both arms and that the proposed value of 2x10⁻⁴ is sufficient to yield a missing energy resolution of approximately 200 keV, a value which allows to separate nuclear states in many cases. Until now, no decision is made for or against a dispersion matching mode. Due to the good energy resolution of the incident electron beam this mode will only bring a minor improvement.

Since the design objectives are the same for both arms (except the slightly higher field for the p-spectrometer), a symmetric design of two identical spectrometers is taken into consideration.
Fig. 1: Missing mass energy resolution versus the kinetic energy of the electron and the proton, respectively. Incident electron energy $E_e = 800\,\text{MeV}$, missing mass energy $E_M = 50\,\text{MeV}$. Dashed curves: energy spread of incident beam ($\delta E_e = 120\,\text{keV}$) is included. Solid curves: dispersion matching mode.

Philosophy of design: The ingredients of the design are quadrupoles and dipoles with homogeneous field. Entrance and exit face rotations and sometimes curvatures are primarily used to minimize the dimensions of the dipoles. To correct for aberrations the position and the angle of each particle will precisely be measured at the exit of the spectrometer in both directions and the trajectory of the particle will be ray-traced. The reasons are the following: 1) Drift chambers with high spatial resolution were developed during the last years. 2) The development of computer technology allows to handle a sufficiently large number of events, especially, if one has in mind the possibility to perform a fast ray-tracing on-line and a more precise and therefore more time consuming one after the experiment for the good events. 3) The dependence of the cross sections on the
scattering angle forces us in any case to determine the entrance angles in both directions. For example, a solid angle of 30 msr corresponds to entrance angles $\theta_0 = 2\phi_0 = 10^\circ$. 4) The use of simple magnets will hopefully keep the costs low.

The question which immediately arises is that for the unavoidable uncertainties of momentum and entrance angles determined by ray-tracing. For a given magnet configuration these uncertainties depend on the size of the beam spot and on the spatial resolution of the drift chambers. A computational test with realistic assumptions is discussed later in this paper.

**Magnet configuration:** To provide a large solid angle and to keep the gap of the following dipole(s) small the first magneto-optical element was chosen to be a quadrupole focussing in the non-dispersive plane. We investigated configurations QD, QQD and QDD and found that a QDD configuration meets our requirements best. The lay-out is shown in fig. 2 and the parameters are given in table 2. The first order imaging modes are point-to-point in the dispersive plane and point-to-parallel in the non-dispersive plane. The entrance face curvature and the exit face rotation of the first dipole are used to keep the gap of the second dipole small. The entrance face curvature of dipole 2 minimizes the second order matrix element $\langle x, x_\delta \rangle$. The exit face rotation of dipole 2 reduces the drift length between the dipole 2 and the image plane to 1.15 m and the dispersion to 2.57 cm/% while the ratio $M_x/D_x$ is staying constant.

The requirement of a solid angle of 30 msr at a momentum acceptance of 20 % is fulfilled by a dipole gap of 24.4 cm (fig. 3) and a quadrupole diameter of 30 cm. To make use of an increased solid angle of 50 msr at a momentum acceptance of 4 % a quadrupole diameter of 40 cm would be necessary.

**Computational test of ray-tracing:** To determine the momentum and angular resolutions for the proposed QDD system a computational test was performed by using the Monte Carlo method. For a fixed momentum $\delta = \Delta p/p$, the entrance variables were chosen by random, $x_0$ and $y_0$ within a Gaussian distribution with a standard deviation $\sigma_s$ and cut-off limits $|x_0| = |y_0| \leq 2\sigma_s$, and $\theta_0$ and $\phi_0$ within a uniform distribution limited by the maximum angular acceptance. The standard deviation $\sigma_s$ corresponds to the beam spot size. Then, the coordinates $x_1, y_1$ and
Fig. 2: Lay-out of proposed QDD

Fig. 3: Solid angle versus momentum acceptance. Upper curve: gap = 24.4 cm, lower curve: gap = 20.0 cm.
Table 2: Parameters of the proposed QDD-spectrometer

Quadrupole (focusing in the non-dispersive plane):

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<tr>
<td>Diameter for $\omega=30$ msr</td>
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<tr>
<td>for $\omega=50$ msr</td>
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Dipoles:

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<td>Deflection angle</td>
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<td>Maximum field</td>
<td>1.2 T</td>
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<tr>
<td>Mean radius</td>
<td>230 cm</td>
</tr>
<tr>
<td>Entrance face curvature</td>
<td>-153 cm</td>
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<tr>
<td>Exit face rotation</td>
<td>-200</td>
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<td>Pole width at entrance</td>
<td>111 cm</td>
</tr>
<tr>
<td>at exit</td>
<td>153 cm</td>
</tr>
<tr>
<td>Gap</td>
<td>24.4 cm</td>
</tr>
</tbody>
</table>

| Dipole D2                        |             |
| Deflection angle                 | $60^0$      |
| Maximum field                    | 1.2 T       |
| Mean radius                      | 230 cm      |
| Entrance face curvature          | -166 cm     |
| Exit face rotation               | -250        |
| Pole width at entrance           | 168 cm      |
| at exit                          | 180 cm      |
| Gap                              | 24.4 cm     |

Distances:

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<td>Dipole D1 - dipole D2</td>
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<td>Total length of central ray</td>
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<tr>
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<tr>
<td>Momentum acceptance $\Delta p/p$</td>
<td>20 %</td>
</tr>
<tr>
<td>Solid angle $\Omega$ for $\Delta p/p = 20%$</td>
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</tr>
<tr>
<td></td>
<td>50 msr</td>
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<tr>
<td></td>
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</table>

First order matrix elements:

| (x|x)       | -0.327 cm/cm |
|-------------|--------------|
| (x|\theta)   | 0            |
| (x|\phi)     | 2.571 cm/%   |
| (y|y)        | -9.980 cm/cm |
| (y|\phi)     | 0.032 cm/mrad|
| (\theta|x) | -18.215 mrad/cm |
| (\theta|\theta) | -3.056 mrad/mrad |
| (\theta|\phi) | 2.461 mrad/% |
| (\phi|y)     | -30.823 mrad/cm |
| (\phi|\phi)  | 0            |
$x_2, y_2$ at the positions of two x-y-drift chambers at the exit of the QDD system were calculated. These coordinates were varied by random within Gaussian distributions representing the spatial resolution of the drift chambers. With these falsified exit coordinates the "ray-traced" trajectory with the entrance variables $x_0^*, y_0^*, \phi_0^*$ and $\delta^*$ was calculated. We set $x_0^* = 0$, because only 4 variables can be determined from the 4 exit coordinates. By comparing the ray-traced values $\delta^*, \theta^*_0$ and $\phi^*_0$ with the original ones we obtain distributions for

$$\delta^* - \delta, \theta^*_0 - \theta_0, \phi^*_0 - \phi_0.$$  \hspace{1cm} (3)

The FWHMs of these distributions give the momentum and angular resolutions.

The results of the specific calculations, which were performed with 10 000 rays for each momentum, are shown in fig. 4. We took $2\sigma_s = 1$ mm (curves 1 and 3) and $2\sigma_s = 0.4$ mm (curves 2 and 4) for the beam spot size and $2\sigma_d = 0.4$ mm (curves 1 and 2) and $2\sigma_d = 0.2$ mm (curves 3 and 4) for the spatial resolution of the drift chambers. In both cases, the second value is considered to be more realistic.

The momentum resolution (fig. 4a) shows a slight dependence on the momentum, mainly due to the second order term $(x, x_0\delta)$ which is not corrected for because of the lack of knowledge of $x_0^*$. For the most realistic case 4, a momentum resolution of almost $10^{-4}$ is obtained. The Monte Carlo value, which contains the first and second order contributions, is $\delta p/p = 1.15 \times 10^{-4}$ for $\Delta p/p = 0$ and must be compared with $\delta p/p = 1.10 \times 10^{-4}$, which one obtains by only taking the first order contributions of beam spot size and drift chambers resolution into account. From the good agreement we conclude that the ray-tracing method as discussed here corrects for the second order effects almost completely.

The angular resolution in the dispersive plane (fig. 4b) is constant in the full momentum range and better than 1 mrad, also under pessimistic assumptions.

Because of the large value for the first order matrix element $(y|y)$ the angular resolution in the non-dispersive plane (fig. 4c), which is the scattering plane when using the spectrometer in the upright position, depends strongly on the accuracy with which the angle $\phi$ is measured at the exit of the spectrometer. The obtained resolution $\delta \phi_0 = 10$ mrad seems to be sufficient for determining the kinematics and the angular dependence of the cross sections, but it
Fig. 4: Results of the Monte Carlo calculations: a) Momentum resolution, b) angular resolution in the dispersive plane, c) angular resolution in the non-dispersive plane.
is certainly not sufficient for correcting for kinematic broadening which spoils the momentum resolution when investigating light nuclei. To give an example, for $^{12}$C in the worst case ($E_0 = 820$ MeV, scattering angle = 90°) the kinematic broadening is $\delta E = 560$ keV and would then be the largest contribution to the overall resolution ($\delta E/E = 6.8 \times 10^{-4}$).

**QDD-Q system:** To improve the angular resolution in the non-dispersive plane we investigated the possibility of adding a quadrupole at the exit of the QDD system. Since this quadrupole will certainly restrict the momentum acceptance and the solid angle, it should be used only for certain experiments which demand higher resolution. With this additional quadrupole the momentum resolution is $\delta p/p = 7 \times 10^{-5}$ and the angular resolution is $\delta \theta_0 = 1.1$ mrad and $\delta \phi_0 = 1.4$ mrad, respectively.
Moderate Resolution Spectrometer

R. York and R. Minehart
University of Virginia
Charlottesville, VA 22901

Abstract

A general discussion of moderate resolution ($\sim 10^{-3}$), large solid angle (> 10 msr) spectrometers for operation in the several GeV regime is presented.
The motivation for this analysis was provided primarily by the requirements specified for experiments suggested for a high duty factor, several GeV, electron accelerator. A large number of these experimental proposals will simultaneously measure the kinematics of two particles in the final state. With intrinsically low rates, many of these proposals need spectrometers with large acceptance in target length, solid angle, and momentum, whereas their requirements in momentum and angular resolution are modest. Typically $\Delta p/p = 10^{-3}$ is satisfactory. Momentum acceptance, greater than 10% of the central momentum is usually necessary, frequently the larger the better. The use of gas or liquid targets has led to the desire for large acceptance in target length as well. For 2 GeV/c incident electrons, the momentum of the scattered electron cannot exceed 2 GeV/c, but recoil hadrons can have momenta approaching 4 GeV/c. Therefore we have studied the design of moderate resolution 2 and 4 GeV/c spectrometers with large acceptances (10-30 msr and 10 cm long targets).

Although a large momentum acceptance is not hard to obtain, the combination of large solid angle and long target is virtually impossible with conventional magnets operating at practical power levels. If the spectrometer is designed to accept a large solid angle from a point target, it is possible to approach 10 msr with bending magnets whose gaps are of the order of 10 cm. Larger solid angles require larger gaps and the power consumption for conventional magnets becomes prohibitively high. Even if a solid angle of 10 msr is acceptable, it must be obtained by putting quadrupoles at the beginning of the system close to the target, so that their focal lengths will be short. The magnification will then be large so that acceptance of off-axis rays from the target dictates a very large gap in the bending magnet. Typically we have found it impossible to achieve 10 msr over target lengths.
greater than 1 cm using conventional iron core bending magnets.

A solution to the problem may be provided by the use of superconducting magnet technology. One possibility is to use superconducting quadrupole magnets with large apertures and high gradients, and superconducting magnetic dipoles with or without iron pole pieces. The superconducting magnets consume little power. With air cores they can produce high fields. Because of saturation, dipoles with iron pole tips are limited to fields of 18 to 20 kG. They have the advantage however of clamping the field to a shape determined by the geometry of the poles, and therefore improve the reproducibility in the spectrometer. When powered with superconducting coils large gaps can be used with power reduced by a factor of 20. The use of iron simplifies the field clamping, but as shown in Ref. 2, field inhomogeneities are not a significant problem for the air core dipoles, and if higher fields are needed iron cores can be left out altogether.

We have designed a QQDQ configuration which is capable of providing 22 msr for a zero length target. The momentum resolution is better than $10^{-3}$, and the momentum acceptance is $\pm 10\%$ of the central momentum. To maximize the solid angle the first two quadrupoles focus point to parallel. A procedure of K. Brown based on the thin lens approximation was used to optimize the magnet positions and lengths. A 30° bend angle in the dipole provides sufficient dispersion to match wire chamber detectors which are located on a focal plane beyond the third quadrupole. This last quadrupole focuses the beam to provide an overall point to point focus in the momentum plane, and parallel to point focus in the production angle plane.

The response of this system to a finite target length was calculated with the ray tracing program, Decay Turtle. The acceptance in target length can be increased by increasing the distance of the first quadrupole from the target, so calculations were made at a series of distances increasing from the point
target design. At each position the quadrupole radii and lengths were re-optimized. The quadrupole radii were chosen for a pole tip field of approximately 40-45 kG. In Table 1 the parameters for each setting are given, along with the solid angle for a zero length target. For each setting the same number of rays per unit length and per unit solid angle were started into the spectrometer.

A sketch of the system is given in Fig. 1, which is used to define the parameters. The number of events accepted by the spectrometer as a function of the source position along the target is shown in Fig. 2. It is apparent that as the target to quadrupole distance is increased there is a reduction in the point target solid angle that is not accompanied by a dramatic increase in the number of events accepted from points away from the target center. Experimentally the important function is the total counting rate versus target length. The relative counting rate for an experiment using a long target (assuming a constant cross section along the length of the target) is also given in Table 1. The counting rate does increase slightly as the central solid angle is reduced, but the improvement is rather small. It would seem preferable to design the experiments with a target length of 5 cm or less and adjust the spectrometer parameters to maximize the point target solid angle.

Most of the proposed experiments require two of these spectrometers, each capable of swinging from 20° to 160° in the production plane. Using standard symmetrical quadrupole designs, the dimensions of the 22 msr solution (1#) allow only an angular range of 40° to 120°. However, quadrupoles using coils of unusual shape might be used to overcome the problem, and engineering studies directed to this end are needed. The designs #4 and #5 can satisfy the angular range with quadrupoles of standard design.

It is estimated that the cost of all the spectrometer components including cryogenic support facilities will be under $0.5 M per spectrometer.
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* X(cm) = α X₀(cm) + β (θP)(cm); Y(cm) = γ φ(mr)
† as seen by zero length target
Figure 1. Moderate Resolution Spectrometer
Figure 2. Moderate Resolution Spectrometer Acceptance
References

3. S. J. St. Lorant, private communication.
High Resolution Spectrometer

R. York, R. Minehart

University of Virginia
Charlottesville, Virginia 22901

Abstract

A QDDDD design for a high resolution spectrometer is proposed. The design features an estimated resolution of $5 \times 10^{-5}$ using a versatile scheme of multiple dipoles. The cost for this spectrometer has been estimated as $1.3 \text{ M}$ in 1980.
General Discussion

Recent analyses\(^1\) of the future of nuclear science have emphasized the need for the development of a new, high duty factor electron accelerator capable of operation in the energy regime of 0.5 to several GeV. Historically, many experiments in nuclear physics for energies in the range of several hundreds of MeV have involved the measurement of cross sections requiring momentum resolution on the order of \(10^{-4}\) to \(10^{-5}\). It is expected that there will be a demand to continue these high resolution measurements to the greater energies afforded by this accelerator.

Since cost increases rapidly with the maximum momentum of the spectrometer, it is important to assess the physics needs accurately. The rates for the classical types of electron scattering experiments requiring high resolution become prohibitively low for incident energies above roughly 1.2 GeV\(^2\). Therefore, in our design the maximum momentum requirement for the high resolution spectrometer (HRS) was set at 1.5 GeV/c. A resolution of \(5 \times 10^{-5}\) provides a precision in the energy measurement of 60 keV at this maximum energy, sufficient for the needs of electron-nucleon experiments. The HRS proposed is intended to be used with an accelerator that will deliver an incident beam with an energy spread (\(\Delta E/E\)) of \(10^{-4}\) to \(10^{-3}\). Thus the beam must be dispersed in energy at the target, which led us to consider an energy-loss spectrometer as the best solution.

At the Los Alamos Meson Physics Facility (LAMPF) there already exists a spectrometer\(^3\) with nearly the desired characteristics. Therefore, it was decided to take advantage of the LAMPF experience and to modify their design in order to simplify and improve the operation.

The proposed design is a QDDD configuration with a vertical bend plane and a horizontal scattering plane as shown in Fig. 1. The quadrupole provides
a cross-over in the horizontal plane and magnifies the beam in the vertical plane in order to maximize the efficiency of the dipoles. With the spatial resolution of the detectors we expect to use, a bend angle of 120° is required to achieve the desired resolution. The bend is equally divided among the four dipoles. A multipole magnet at the midpoint between the two center dipoles allows correction of higher order effects. To maintain a minimum gap in the dipoles as well as to allow separation of the x and y corrections, a y cross-over exists at the center of this multipole. The $H_t$ windings in the LAMPF HRS design are not actually used at LAMPF and we do not expect to use them in our spectrometer. The use of four dipole magnets keeps each element of the spectrometer at a manageable size and reduces the cost of the spectrometer. In addition, the eight pole faces of these magnets provide means for making first and second order corrections.

The system was designed through second order using the computer based matrix transport program, TRANSPORT. The parameters of the proposed HRS are listed in Table I.

The design performance is enhanced by the use of two separate focal planes. To first order, the momentum and production angle may be obtained from the first and second focal planes respectively. Track reconstruction techniques would provide further information.

The focal plane detectors could be multi-wire planar proportional chambers similar to those in use at the MIT-Bates Linear Accelerator. Their inherent spatial resolution is nearly 0.15 mm. They would be used in crossed configuration at both focal planes to improve performance by providing data for track reconstruction.

Dispersion matching requirements would be satisfied by nominal momentum dispersion at the target of 15.4 cm/%. This corresponds to a vertical spot size at the target of about 31 mm for an incident beam momentum spread of ± 0.1%.
Table 1. High Resolution Spectrometer

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**Quadrupole Parameters:**

- Bore: 30.0 cm
- Effective Length: 70.0 cm
- Pole Tip Field (1.5 GeV): 7.0 kG

**Dipole Parameters:**

- Effective Length: 2.0 m
- Gap: 10. cm
- Width: 1.0 m
- Field (1.5 GeV): 13.1 kG
Production Angle Focal Plane
\(<y|\theta_o|> = -0.179 \text{ cm/mr}\)

Momentum Focal Plane
\(<x|\theta_o|> = -1.19 \\
\(<x|\Delta p|> = 18.27 \text{ cm/}\%\)

\(\alpha = \text{pole face curvature}^{-1}\)  \\
\(\theta = \text{pole face rotation}\)

\(\theta = 25.75^\circ\)  \\
\(\alpha = -1.00 \text{ m}^{-1}\)

\(\theta = 0.00^\circ\)  \\
\(\alpha = -1.04 \text{ m}^{-1}\)

\(\theta = 25.75^\circ\)  \\
\(\alpha = 0.98 \text{ m}^{-1}\)

\(\theta = 0.00^\circ\)  \\
\(\alpha = 1.26 \text{ m}^{-1}\)

\(\theta = 0.00^\circ\)  \\
\(\alpha = 0.73 \text{ m}^{-1}\)

\(\theta = 25.75^\circ\)  \\
\(\alpha = -1.00 \text{ m}^{-1}\)

\(\theta = 0.00^\circ\)  \\
\(\alpha = -0.84 \text{ m}^{-1}\)

\(\phi = 3.82 \text{ m}\)

\(r = 15 \text{ cm}\)  \\
\(l = 70 \text{ cm}\)  \\
\(\alpha = -0.30 \text{ m}^{-1}\)

Dipole Gap = 10 cm

Figure 1. High Resolution Spectrometer
References


2. Hall Crannell, private communication.


Cost Analysis

The following analysis was done by J. Spencer of S.L.A.C. Costs are given in 1980 dollars.

The beam characteristics and resulting magnet characteristics assumed a \( \sim 3 \text{ msr} \) angular acceptance and 4% momentum acceptance for full resolution. Rather than making one single, large dipole such as in a split-pole spectrometer, four 30° bends with removable pole-tips result in a more versatile system. For instance, if only two dipoles are used a useful, lower resolution spectrometer is obtained, and if more than four are used, the system could be extended to higher energies or resolution. Making a modular system with a basic unit magnet also has several other advantages such as fewer connecting pieces per magnet as well as no single piece of steel weighing as much as 15 tons so that fabrication, assembly, measurement and shimming are all considerably simplified. Furthermore, the fact that no single magnet weighs more than \( \sim 46 \) tons simplifies shipping and installation.

All magnets, including the quadrupole, were designed to have comparable voltage drops to allow use of a single, bulk power supply (presumably 600 V) with individual pulse width modulated choppers (2 kHz). It is believed that this system can be made at less cost and provide more stability than conventional SCR supplies. It is also easier and cheaper to extend in the event of addition of another quadrupole and/or two dipoles (total of 850 kW necessary as opposed to 500 kW for the present proposal).

No vacuum plumbing or pumping was included in the estimate except for the individual magnet vacuum chambers which were considered to be integral to the magnet. A control system for cycling and setting the magnets was not included since this is usually available as part of the overall experiment control system.
### Quad Characteristics:
- Max. Beam Size \( \sqrt{\frac{1}{2}} \times \sqrt{\frac{1}{2}} \) \((11 \times 6 \text{ cm})\)
- Magnetic Length: 0.7 m
- Max. Gradient: 0.6 kG/cm
- Effective Bore Radius: 13 cm
- Efficiency: > 96%
- Peak Pole Tip Field: 8 kG
- Max. Amp-Turns/Pole: 43,000
- Core Material: S.A.E. 1010
- Core Length: 64 cm
- Core Weight: 2000 kg
- Conductor Cross Section (Cu): 1 cm²
- Turns/Coil: 38
- Conductor Length/Coil: 84 m
- Resistance/Coil: 16.0 mΩ
- Total Conductor Weight: 300 kg
- Current: 1100 A
- Magnet Voltage: 70 V
- Max. Magnet Power: 77 kW

### Quad Cost:
- Fabrication of Complete Magnet (including steel and coils): $72,000
- Engineering Design and Field Simulations: 6,000
- Shipment Costs: 2,000
- Magnetic Testing and Shimming: 5,000
- Vacuum Chamber & Inst.: 2,000
- Installation and Alignment: 5,000
- Power Supply and Regulator*: 22,000*
- PS Installation and Cabling: 6,000

**Total** $120,000

*This system can be a component of that used to power the bends i.e., a single bulk supply with independent chopper regulators.*
### Bend Characteristics:

- **Max. Beam Size** \( \sqrt{\frac{1}{2} x \sqrt{\frac{1}{2} y}} \) \((29 \times 3 \text{ cm})\)
- **Magnetic Length** \(30^\circ\) bend \(@ p = 3.82 \text{ m}\) \(2.0 \text{ m}\)
- **Max. Induction Field** \(16.0 \text{ kG}\)
- **Total Gap Opening** \(8.0 \text{ cm}\)
- **Total Breadth of Pole** \(90.0 \text{ cm}\)
- **Efficiency** \(> 92.0\%\)
- **Max. Ampere-Turns/Pole** \(55,000\)
- **Core Material** S.A.E. 1010-1018
- **Pole Material** S.A.E. 1004-1008
- **Total Weight Steel** \(40,000 \text{ kg}\)
- **Conductor Cross Section (Cu)** \(2.16 \text{ cm}^2\)
- **Turns/Coil** \(46\)
- **Conductor Length/Coil** \(300 \text{ m}\)
- **Resistance/Coil** \(32.6 \text{ m} \Omega\)
- **Total Conductor Weight (one magnet)** \(950 \text{ kg}\)
- **Current** \(1200 \text{ A}\)
- **Magnet Voltage** \(78 \text{ V}\)
- **Maximum Single Magnet Power** \(94 \text{ kW}\)
- **RMS Field Uniformity (Gap Size Mesh)** \(0.5 \times 10^{-4}\)

### Unit Bend Cost:

- Fabrication of a Complete Dipole (including steel, coils & chamber) \(\$200,000\)
- Shipping Costs/Magnet \(\$10,000\)
- Magnet Testing and Shimming \(\$12,000\)
- Installation and Alignment \(\$8,000\)
- **Total/Magnet** \(\$230,000\)
V. H-10

Engineering Design and Field Simulations  $15,000
Power Supply and Regulators  80,000*
P.S. and Cabling  15,000
Misc. and Retrofit  40,000
Total  $150,000

Spectrometer Support Structure

Pivot  $10,000
Air Pad and Drive System  20,000
Guide Rails  8,000
Frame and Detector Platform  50,000
Mechanical Design  10,000
Assembly and Installation  8,000
Total  $106,000

GRAND TOTAL = $1.296 M

* This system can be a component of that used to power the bends i.e., a single bulk supply with independent chopper regulators.
I want to describe the new magnetic spectrometer system for intermediate energy physics at the Indiana University Cyclotron Facility (IUCF). The project is a joint venture of Indiana University and the University of Maryland.

Fig. 1. Floor plan showing the double magnetic spectrometer system.
The system consists of a K-600 and a K-300 magnetic spectrometer ($E_{\text{max}} = KQ^2/A\text{ MeV}$). The K-600 spectrometer has a variable dispersion and has been optimized for high energy resolution (10–20 keV) and a large energy bite ($0 < E_{\text{x}} < 20\text{ MeV}$) at a moderate solid angle (4–8 msr) for all incident energies. The K-300 spectrometer has a large solid angle (20 msr) and momentum bite ($p_{\text{max}}/p_{\text{min}} = 1.35$) at modest resolution ($\Delta E/E \sim 10^{-3}$). This double spectrometer system greatly enhances the research possibilities for single-arm measurements at IUCF and will provide an unprecedented capability to carry out research involving coincidence measurements.

The new spectrometer system will be installed at the extreme north end of the IUCF building as shown in Fig. 1. The beam line to the neutron swinger conducts the beam to the vicinity of the new system. The beam preparation line for the new spectrometers contains the present QDDM spectrometer as an analyzer, since this magnet has adequate first-order resolving power. The beam line design permits independent adjustments of the beam magnification prior to the analyzer and the target dispersion after the analyzer by six quadrupole singlets. The K-600 spectrometer is used in a horizontal configuration, whereas for the K-300 spectrometer a vertical installation was chosen in order to increase the angular range accessible by the two spectrometers. A side view of the spectrometer system is given in Fig. 2.

Fig. 2. Side view of the spectrometer system. The K-600 spectrometers is used in the horizontal configuration, whereas the K-300 spectrometer is upright oriented.
Description of the K=600 Spectrometer*

An outline drawing of the K=600 spectrometer is shown in Fig. 3. It consists of an entrance quadrupole, two dipoles with a modest (10 cm) gap, and a multipole between them. The dipole entrance and exit angles are almost normal to the beam to avoid strong coupling terms. The total bend angle is 110°. The relative strengths of the two dipoles are varied to change the dispersion. The vertical focusing gives a waist near the detector. There are three detector positions depending on the dispersion setting of low, normal or high dispersion. Some properties of the K=600 spectrometers for these three dispersion settings are given in Table 1. Further parameters are given in Table 2.

Fig. 3. Plan view of the K=600 spectrometer. The entrance region is shown in greater detail in Fig. 5. The half-section corresponds to either dipole near the midpoint.

*The final design is not yet fixed.
Table 1. K-600 Spectrometer Properties for the Three Different Dispersion Modes.

<table>
<thead>
<tr>
<th>Dispersion Setting</th>
<th>Low</th>
<th>Normal</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dipole field ratio $B_2/B_1$</td>
<td>4/3</td>
<td>1</td>
<td>3/4</td>
</tr>
<tr>
<td>Momentum bite ($P_{\text{max}}/P_{\text{min}}$)</td>
<td>1.16</td>
<td>1.10</td>
<td>1.065</td>
</tr>
<tr>
<td>Momentum dispersion along detector (cm/%)</td>
<td>4.8</td>
<td>7.4</td>
<td>9.0</td>
</tr>
<tr>
<td>Horizontal magnification</td>
<td>0.29</td>
<td>0.38</td>
<td>0.45</td>
</tr>
<tr>
<td>Vertical magnification</td>
<td>5.4</td>
<td>6.4</td>
<td>7.8</td>
</tr>
<tr>
<td>Energy dispersion (keV/mm)</td>
<td>40</td>
<td>40</td>
<td>40</td>
</tr>
</tbody>
</table>

A circular profile on the entrance of the second dipole reduces the $(x|\theta^2)$ aberration to near zero somewhere on the detector for each dispersion mode. The $(x|\phi^2)$ aberration is reduced by a small hexapole component in the entrance quadrupole. The multipole provides for kinematic compensation and for adjustments of higher order terms. Fig. 4 depicts the aberration figures from the program RAYTRACE assuming a circular entrance aperture of 100 mrad.

Fig. 4. Aberration figures for the K-600 spectrometer from the program RAYTRACE. For each of the three modes a ray bundle is traced to the detector center and to a point near the extremes of the momentum range.
A bundle of 14 rays filling uniformly half of this aperture are traced to the focal position to evaluate the aberration coefficients. A detector at this location must measure \( x, y, \theta \) and \( \phi \), and any dependence of \( x \) on variables other than momentum is removed by software. As seen in the lower part of Fig. 4, most of the spot width arises from the dependence on \( \theta \). The dependence of \( x \) on the vertical parameters \( y, \phi \) is an order of magnitude smaller than for \( \theta \). A correction expression of the form 

\[
x' = x + a\theta^2 + b\theta + cy^2 + d\phi^2
\]

is adequate for obtaining a final resolution of 10-20 keV.

Special provisions are made in the entrance section to bring the beam past the entrance quadrupole and to deflect it into a distant, well-shielded Faraday cup so that small angle cross sections may be determined without excessive room background. This is accomplished by means of an open-sided quadrupole and two dipole magnets. The right-hand side of Fig. 5 shows the arrangement of the quadrupole and the two dipole magnets; some mapping results from a half-scale model are shown on the left-hand side of Fig. 5.

![Diagram](image-url)

**Fig. 5.** The left side of the figure shows the quadrupole field from a first model of the open-sided entrance quadrupole, and the section shows the location of the 5° beam before diversion to the external Faraday cup. The right side shows a detail of the location of the two dump magnets and the small angle trajectories. The water cup is used only in zero-angle symmetry checks.
Table 2. Comparison of Parameters of the K=600 and K=300 Spectrometers.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>K=600</th>
<th>K=300</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum momentum (GeV/c)</td>
<td>1.06</td>
<td>0.75</td>
</tr>
<tr>
<td>Average dipole radius (m)</td>
<td>2.1</td>
<td>1.4</td>
</tr>
<tr>
<td>Bend angle</td>
<td>110°</td>
<td>105°</td>
</tr>
<tr>
<td>Solid angle (msr)</td>
<td>4–8</td>
<td>18</td>
</tr>
<tr>
<td>Momentum bite (p_{max}/p_{min})</td>
<td>1.07–1.16</td>
<td>1.35</td>
</tr>
<tr>
<td>First order resolving power (p/δp)*</td>
<td>40,000</td>
<td>2,000</td>
</tr>
<tr>
<td>Dispersion (cm/%)</td>
<td>5–9</td>
<td>2.8</td>
</tr>
<tr>
<td>Focal plane length (cm)/angle of incidence</td>
<td>75/45°–65°</td>
<td>107/49°</td>
</tr>
<tr>
<td>Dipole total weight (tons)</td>
<td>160</td>
<td>35</td>
</tr>
<tr>
<td>Power consumption (kW)</td>
<td>200</td>
<td>200</td>
</tr>
</tbody>
</table>

*After software corrections based on focal plane x, y, θ, φ measurements.

Description of the K=300 Spectrometer†

A plan view of the K=300 spectrometer is shown in Fig. 6. It consists of a double focusing quadrupole-hexapole-dipole (QHD) system. The poles of the dipole magnet have entrance and exit curvatures in order to reduce aberrations and adjust the focal plane. The entrance quadrupole has a large aperture to yield the large solid angle. The hexapole is necessary to reduce the aberrations further. A measurement of x, y, θ, and φ in the focal plane provides all the necessary information to obtain the desired resolution by software correcting the aberrations due to the large solid angle. Some parameters are collected in Table 2. Both spectrometers use room temperature magnets with maximum dipole fields of 1.6T.

†The final design is not yet fixed. Considerable changes might be made but without very much effecting solid angle, momentum bite and resolving power.
Fig. 6. Plan view of the K=300 spectrometer.
A DESIGN FOR THE LARGE ELECTRON SPECTROMETER (LES)

B. Zeidman
Argonne National Laboratory, Argonne, IL 60439

Part of the planning for experimental facilities associated with a projected 2 GeV electron accelerator involves consideration of a High Resolution Area intended to provide the capability for both coincidence and single arm studies with high resolution. A major component of the high-resolution facility is a large electron spectrometer of an unusual design that will be discussed.

Design Objectives

The design objective is to provide an arrangement capable of investigating the longitudinal nuclear response function for discrete states via (e,e'p) reactions. This requires coincident detection of emergent electrons with momenta of approximately 1.5 GeV/c and emergent protons with momenta of about 1 GeV/c. Moreover, the kinematic conditions demand the detection of electrons at forward scattering angles where overall flux rates are high. In addition, separation of discrete states implies a system resolution, ΔE/E, of about <10^{-4} while specification of kinematic conditions necessitates measurement of scattering angles with a precision of a few milliradians. The low cross sections involved also make large acceptance in both momentum and solid angle a highly desirable feature. An additional objective is to provide the capability for very high resolution, ΔP/P ≈ 10^{-5}, single arm studies of electron scattering. This capability is needed for resolving weak states in the presence of other nearby states or significant backgrounds arising from radiative tails.

In order to accommodate both objectives, a system involving two spectrometers is proposed. The primary spectrometer, intended for detection
of high energy electrons from the (e, ep) reaction, fulfills the requirements of high resolution for single arm experiments while providing large acceptance and high count rate capability. The second spectrometer, intended primarily for proton detection, operates over a more limited range in momentum and has somewhat lower resolution, but has larger angular acceptance. As a result of this asymmetric arrangement, the overall resolution in coincidence experiments is determined primarily by the second spectrometer, which will be adapted from the Mainz design (Ne 81).

Spectrometer Designs

The proposed accelerator provides variable energy electron beams with transverse phase space in both x and y of approximately 0.1π mm-mrad and longitudinal momentum spread, Δp/p = 10^-4. If a 100 μA beam, were tightly focused upon a target, the local heating would be high enough to rapidly burn a hole in the target. Rather than attempting to preserve the target by moving it rapidly, it is proposed to disperse the beam on target in both x and y directions — a beam spot of about 1 cm x 1 cm is sufficient. Use of a dispersed beam appropriately prepared by the beam transport system then implies that at a given location on the target, a momentum spread significantly smaller than 10^-5 can be achieved.

Dispersed beams coupled to matched spectrometers have been used to achieve resolutions much better than the beam spread in several single arm systems. In these systems, the dispersion of the beam on target is matched to the dispersion of the spectrometer so that the location on the focal plane directly measures the energy difference between the incident and final particles — hence the term, Energy-Loss Mode. An alternative method is to ascertain the location on target of the incident particle, thereby determining
the incident momentum, and to measure the final momentum so that the difference yields the energy loss. This is called the **Dispersed-Beam, Absolute-Energy Mode**. While both systems are comparable for single arm experiments, the flexibility of the Dispersed Beam mode provides advantages for coincidence experiments that are the primary experimental objective. Accordingly, a Dispersed-Beam spectrometer utilizing intermediate detectors is proposed for the LES. This provides precise knowledge of the incident electron momentum and location on the target and, as will be discussed, accurate knowledge of the scattering angle and momentum of the outgoing electron. For coincidence operation, the information provided by the LES can be utilized to allow a somewhat simpler design for the other arm, which is required only to determine the momentum (magnitude and direction) of the outgoing hadron.

The LES spectrometer essentially consists of two parts: 1. A large acceptance, low resolution spectrometer which provides information regarding target location, scattering angle and approximate momentum, $\Delta p/p \approx 1\%$; 2. A high resolution spectrometer whose primary function is to determine the magnitude of the momentum with an accuracy $\Delta p/p \approx 10^{-5}$. A schematic diagram of the spectrometer is shown in Fig. 1. The $x, \theta, y, \phi$ information provided by the intermediate detector planes, allows a trace-back to the target coordinates with an accuracy in position of $\pm 0.25$ mm and direction of $\pm 2$ mr. Momentum resolution $\Delta p/p \approx 1\%$ provides a selection of momentum range for further analysis. Since only the magnitude of the momentum is required from the second section, multiple scattering in the detector planes does not affect the measurement. Detector planes at the end of the spectrometer also measure $x, \theta, y$ and $\phi$, so that redundant information is available for optimization of the analysis.

The LES consists of a quadrupole triplet followed by a $45^\circ$ band
dipole, followed by three additional 45° bend dipoles. An intermediate image is formed in the bend (x) plane between the first and second dipoles where detector planes are placed. The optical mode is point to point in the bend plane and point to parallel in the transverse (y) plane. The relatively small dimensions of the beam envelope at this intermediate position allow selection of the momentum bite with reasonable precision and permits use of the spectrometer at forward angles with full beam intensity. The last three dipoles constitute a high resolution spectrometer where a momentum resolution of \( \sim 10^{-5} \) is feasible. To achieve this objective an additional set of detector planes is located after the last dipole, where x, y, and \( \phi \) are again measured. The small errors in these measurements, \(< 0.1 \text{mm} \) in position and \(< 1 \text{mr} \) in angle, allow dynamic compensation for aberrations. The optical mode in the rear section is point-to-point in x and parallel to point in y. While both position and angle-determination before and after the dipole triplet is not absolutely necessary to obtain good resolution, the redundancy of information should serve to reduce backgrounds and should allow a reduction in the uncertainties in the derived target coordinates. Parameters for the system are listed in the table.

The dipole fields are shaped by the iron at room temperature, but the fields are produced by superconducting coils located in a conventional manner. The coil design is thermally stable, i.e. the superconductor is embedded in copper with a cross sectional area sufficient to conduct all of the current at a temperature of 4.2°K. This arrangement assures that transient fluctuations in temperature do not result in catastrophic damage to the coils. Liquid helium flows through the coils maintaining temperature stability. The coils are thermally shielded from the environment by vacuum jackets and a heat shield cooled by liquid nitrogen. The use of superconducting coils has two major
advantages. The biggest advantage is the reduction in operating expenses due to power consumption, which, extended over the life of the spectrometer, could easily cost as much as the original construction. The second advantage is the reduction in magnet bulk that arises from the high current density in the coils. The total cross section area required for the coils is substantially smaller than needed for water cooled copper coils, even allowing for the vacuum shielding and copper conductor used for thermal stability. (At 4.2 K the conductivity of Cu is nearly two orders of magnitude greater than at room temperature.) For the large magnets proposed in the present design, the savings in iron and copper constitute a substantial reduction in weight and material cost.

The quadrupole field gradients and apertures utilized in the present design are not feasible with conventional quadrupoles. The mass of iron needed would limit the minimum scattering angle and be extremely heavy. More important, however, are the field gradients over large apertures which cannot be achieved with conventional quadrupoles. The quadrupole design proposed is iron free, the field being shaped entirely by the superconducting coils.

**Beam Transport**

The beam transport proposed for the High Resolution Area should be capable of transmitting polarized beams without depolarization while permitting adjustment of beam spot size on target to meet specific needs. A schematic diagram of the transport system is shown in Fig. 2. The first requirement is satisfied by generating dispersion with two 45 bends in opposite directions, thereby reversing the precession caused by the first bend with the second bend.

In order to achieve flexibility in beam spot size, the transport system utilizes the excellent emittance of the electron beam. The 0.1 mm mrad
emittance allows transport over large distances without the need for focusing elements and also allows extremely small virtual spot sizes without excessive divergence in the beam or the need for large apertures.

The optical mode in the bend plane is point to point at QT2 to point at QT4 to point at the target. In the transverse plane, the optical mode is point to point at QT4 to parallel at the target. The dispersion generated by the first dipole is inverted at QT2, which also produces focusing in the transverse plane, so that the dispersion generated by the two dipoles is additive. The dispersion on target is adjusted at QT4. The quadrupole triplet just before the target provides the appropriate imaging.

Both dipoles bend in the horizontal plane although a vertical dispersion is desired. Between the second dipole and QT4 there is a set of rotated quadrupoles whose transform is equivalent to a drift space except for the interchange of x and y. This twister or rotator, previously used at LAMPF and Bates, rotates the dispersion into the vertical plane.

The system allows adjustment not only of both horizontal (y) and vertical (x) dimensions of the beam spot on target, but also of the sign of the dispersion, which can be adjusted to any value between +50 and -50 cm/%. A one centimeter high spot on target corresponds to a dispersion of 50 cm/%. With this dispersion and the nearly unity magnification of the transport, the uncertainty in the position on target, deduced from an event analyzed with the LES system, corresponds to an error of $<5 \times 10^{-6} \Delta p/p$ in the determination of the momentum of the incident electron. It is therefore seen that the beam transport system contributes a negligible amount to the final resolution of the complete system.
TABLE I: Preliminary Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ap/p (2nd order)</td>
<td>$\sim 2 \times 10^{-5}$</td>
</tr>
<tr>
<td>Momentum Acceptance</td>
<td>$\pm 5%$</td>
</tr>
<tr>
<td>Solid Angle</td>
<td>$22 \text{ msr} (\pm 5%)$</td>
</tr>
<tr>
<td>(33 msr (±3%))</td>
<td></td>
</tr>
<tr>
<td>Angular Acceptance</td>
<td>$\pm 120 \text{ mr}(x), \pm 60 \text{ mr}(y)$</td>
</tr>
<tr>
<td>(±150 mr(x), ±70 mr(y))</td>
<td></td>
</tr>
<tr>
<td>$P_{\text{max}}$ (High Resolution)</td>
<td>1.5 GeV/C</td>
</tr>
<tr>
<td>$P_{\text{max}}$ (1.7 T)</td>
<td>1.9 GeV/C</td>
</tr>
<tr>
<td>$\rho$ (central)</td>
<td>3.5 m</td>
</tr>
<tr>
<td>Total Length</td>
<td>23. m</td>
</tr>
<tr>
<td>Deflection Angles</td>
<td>$4 \times 45^\circ$</td>
</tr>
<tr>
<td>(180°)</td>
<td></td>
</tr>
<tr>
<td>Dispersion</td>
<td>D1 1.47 cm/%</td>
</tr>
<tr>
<td>D2-4 9.72 cm/%</td>
<td></td>
</tr>
<tr>
<td>Magnification</td>
<td>D1 1.04 cm/cm</td>
</tr>
<tr>
<td>D2-4 1.18 cm/cm</td>
<td></td>
</tr>
<tr>
<td>Image Plane Angle</td>
<td>D1 15°</td>
</tr>
<tr>
<td>D4 46°</td>
<td></td>
</tr>
</tbody>
</table>
LARGE ELECTRON SPECTROMETER (LES)
BEAM TRANSPORT WITH DISPERSION

Fig. 2
WORKSHOP ON HIGH-RESOLUTION, LARGE-ACCEPTANCE SPECTROMETERS

SEPTEMBER 8 - 11, 1981

ARGONNE NATIONAL LABORATORY

BUILDING 203 AUDITORIUM

Tuesday, September 8

9:00 - 10:00 a.m. Registration

Session I: Chairman Garvey

10:00 - 12:15 p.m. Organizational Details
B. Zeidman - 10 min.

Introduction
G. T. Carvey - 10 min.

Spectrometers Reviewed
S. Kowalski - 30 min.

12:15 - 1:15 p.m. Lunch at the Argonne Cafeteria

Session II: Chairman Burman

1:30 - 5:30 p.m. Super-Conducting Technology
R. Smith - 30 min.

Calculational Tools --

Transport - D. Carey 10 min.
Raytrace - S. Kowalski 10 min.
POISSON - L. Harwood 10 min.

3-D Field Calculations
R. Lori and L. Turner - 30 min.

Superconducting Coils in Spectrometers
J. Nolen - 10 min.

SC Panofsky Quads
L. Harwood - 10 min.

SC Dipole Coils
A. F. Zeller - 15 min.

Homogenizing Fields in Picture-Frame Magnets
P. Debenham - 10 min.
Wednesday, September 9

Session III: Chairman Stinson

9:00 - 12:15 p.m. Detectors
C. Morris - 30 min.

Coffee Break
Beam Transport at MSU, Solenoid Focussing
J. Nolen - 30 min.

Beam Transport at NIKHEF
H. DeVries - 10 min.

Magnet Construction - Conventional
H. Enge - 30 min.

12:15 - 1:15 p.m. Lunch at the Argonne Cafeteria

Session IV: Chairman Hendrie

1:30 - 5:30 p.m. General Design Methods
H. Enge - 20 min.

Horizontal vs. Vertical Dispersion
E. Kashy - 20 min.

Optimizing Abberation Corrections
L. Harwood - 10 min.

Integrated Magnet, Optics, Detector Designs
J. Nolen - 20 min.

Coffee Break
Continuous Multipole Designs
S. Kowalski - 10 min.

Can a Large Volume Field be Used as a Spectrometer
P. Leconte - 20 min.

Large Bore Magnets
J. Lightbody - 20 min.

Large Acceptance Spectrometers for Coincidence Studies
L. Cardman - 20 min.
Thursday, September 10

Session V: Chairman Schwandt

9:00 - 12:15 p.m.
LEP - High Resolution Spectrometer at LAMPF
R. Boudrie - 15 min.

MSU - 1.2 GeV/c Spectrometer
A. Zeller - 15 min.

Coffee Break
(10:15 - 10:30)
A Super-Conduting Multigap Design
H. Enge - 15 min.

A Large Acceptance Spectrometer with Excellent Background Rejection
P. Leconte - 20 min.

High-Resolution QDD and Large Solid Angle QDQ at NIKHEF
H. DeVries - 20 min.

12:15 - 1:15 p.m.
Lunch at the Argonne Cafeteria

Session VI: Chairman LeVine

1:30 - 5:00 p.m.
Spectrometer Designs with Large Solid Angle and Momentum Acceptance at Mainz
R. Neuhausen - 20 min.

Proposed Design for a High-Resolution, Large Solid Angle Energy Loss Spectrometer
K. Blomquist - 20 min.

Coffee Break
(3:00 - 3:15)
High-Resolution Spectrometer for 1-4 GeV Electron Accelerators
R. Minchart - 20 min.

High Resolution, Superconducting Element Spectrometer Design
R. York - 20 min.

The IUCF Double Magnetic Spectrometer System
H. Nann - 20 min.

A Design for the Large Electron Spectrometer (LES)
R. Zeidman - 20 min.

5:30 - 7:00 p.m.
Workshop Social Hour at Cafeteria Dining Room A

7:00 - 8:30 p.m.
Workshop Dinner at Cafeteria Dining Room B

Friday, September 11, 1981

Session VII: Chairman Zeidman

9:15 - 11:30 a.m.
Summary, Future Plans, Organization

General Discussion
All participants
<table>
<thead>
<tr>
<th>Name</th>
<th>Affiliation</th>
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<tr>
<td>Barnes, Peter D.</td>
<td>Carnegie-Mellon University</td>
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<td>Bertozzi, W.</td>
<td>Massachusetts Institute of Technology</td>
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<td>Blomqvist, K. I.</td>
<td>Massachusetts Institute of Technology</td>
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<td>Boudrie, Richard L.</td>
<td>Los Alamos National Laboratory</td>
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<td>Los Alamos National Laboratory</td>
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<td>Carey, David C.</td>
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<td>Demos, Peter T.</td>
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<td>DeVries, H.</td>
<td>NIKHEF-K, Amsterdam</td>
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<td>Enge, Harald</td>
<td>Massachusetts Institute of Technology</td>
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<td>Hendrie, David L.</td>
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<td>Leconte, P.</td>
<td>Centre d'Etudes Nucleaires de Saclay</td>
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