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CALCULATIONS OF THE DOPPLER COEFFICIENT OF
LARGE CERAMIC-FUELED FAST REACTORS

by

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SUMMARY

Calculations of the Doppler coefficient of large, ceramic-fueled fast reactors containing plutonium have been made by means of the ELMOE program to provide accurate flux calculations. The temperature-dependent cross sections of P. Greebler et al., for U^{238} and Pu^{239} were used. Doppler coefficients of the order of $10^{-5} \delta k/^{\circ}C$ were obtained, in agreement with Greebler's results. Coefficients of carbide fueled reactors are about 0.7 of those of oxide-fueled reactors at the same fuel enrichment. Effective coarse-group elastic-removal cross sections for light elements are tabulated. It appears that, with tabulations of this sort as a guide, coarse-group sets of cross sections can be constructed to give adequate accuracy in calculations of Doppler coefficients without use of ELMOE.

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I. INTRODUCTION

A knowledge of the Doppler coefficients of large, ceramic-fueled reactors is potentially of great importance for safety, as there is no other reliable, prompt-acting, negative reactivity feedback available. The only other feedback available instantaneously with rise of fuel temperature, namely, fuel expansion, is not dependable enough to be regarded as a safeguard in the event of an excursion. A Doppler coefficient $-\delta k/\delta T$ of the order of $5 \times 10^{-6} \text{ }^{\circ}\text{C}$ or larger could be very valuable, whereas a coefficient of 1×10^{-6} or smaller would be unimportant.

The importance of the Doppler coefficient provides an incentive for developing more accurate means for calculating it. The effect is due almost entirely to neutrons with energies below about 10 kev; these are present in appreciable quantity in the reactors of concern because of the moderating effect of light elements. Because of uncertainty in the accuracy of calculations of the neutron flux, it seemed appropriate to use the ELMOE program⁽¹⁾ for the IBM-704. This program treats moderation and scattering by light elements in fast reactors much more accurately than has previously been possible.

Even with this more elaborate flux calculation there are still serious uncertainties resulting from a lack of knowledge of resonance parameters, particularly for the fissile isotopes. Questions as to the validity of the one-level formula and of the overlap of resonances for the fissile isotopes also present complications. The level spacings for fertile isotopes are sufficiently large that these problems do not arise. In spite of these inadequacies in the nuclear data, the results presented here should be sufficiently reliable to serve as a useful guide in planning experiments for measurement of the Doppler coefficient.

II. METHOD OF CALCULATION AND INPUT PREPARATION

In the ELMOE code fluxes are calculated for a single spatial Fourier component, and the calculations of the Doppler coefficient are made for this fundamental mode. This procedure involves the assumption that the spectrum in the entire core is identical with that of the

fundamental mode and that the Doppler effect in the blanket is negligible. It is believed that these assumptions should be quite good for a large reactor, particularly since the Doppler effect has its greatest importance at the center of the core. Correction factors of the order of 10 to 20% would need to be applied to take account of reactor temperature distributions.⁽²⁾ These corrections are not important for the purposes of the present study.

Calculated reactivity changes on heating were obtained by first running the problem with room-temperature cross sections, to obtain the critical value of the buckling B^2 , and then rerunning the problem with cross sections for higher temperatures and the same value of B^2 . The new value of reactivity is the ratio of iterated to original fission source. The reactivity change can also be obtained over a limited temperature range by using perturbation theory.

The code utilizes both a coarse and a fine neutron-energy group structure. The coarse groups are those ordinarily used in fast-reactor calculations. They are used to furnish certain input data and to serve as a basis for collapsing the fine-group calculations. The fine groups are the ones actually used for the flux calculation. They are all of equal lethargy width, which in the present case was taken as 0.0174. At least one coarse group must remain undivided into fine groups at the upper and lower ends of the energy scale. The subdivided region in the present case extended from 3.67 Mev to 100 ev and encompassed 604 fine groups.

The information furnished by the coarse-group library includes fission and capture cross sections, and an inelastic transfer matrix. Removal cross sections are normally held constant over the fine groups corresponding to a given coarse group, whereas the inelastic-scattering source is distributed among the fine groups corresponding to a given coarse group according to the energy width. There is an option which permits inputting variable capture cross sections in fine groups, and it was found necessary to exercise this option for capture in the resonance in Na^{23} at 2.85 kev, as is discussed later. Otherwise, fission and capture cross sections were kept constant over coarse groups.

For the fine groups, the ELMOE library previously employed^(1,7) was used to furnish the Legendre polynomial expansion coefficients of the differential scattering cross sections of the light elements at the energy limits of the fine groups. In the case of sodium the data of Hibdon⁽³⁻⁵⁾ were used with a narrow resonance approximation treatment. The effective sodium cross sections obtained in this way were used to obtain a revised set of Legendre polynomial expansion coefficients for sodium. This calculation was based only on metal fuel compositions, but the same sodium expansion coefficients were used for the oxide and carbide cases because of the excessive hand labor required at present to incorporate the

Hibdon data correctly for each case considered. Since the narrow resonance approximation need be used only above 50 kev, the only effect of an error insofar as the Doppler coefficient is concerned is to alter slightly the slowing-down density at about 10 kev. Alterations to be made in the routine for applying the narrow resonance approximation will in the future produce a corrected calculation of the slowing-down density automatically.

The 18-group coarse-group cross section set used was identical with that of Yiftah, Okrent, and Moldauer⁽⁶⁾ for the first eleven groups, those above 25 kev. Below this energy, the group structure and temperature-dependent cross sections of Pu^{239} and U^{238} given by Greebler et al.,⁽²⁾ were used, except that Greebler's Group 10, extending from 9.1 to 25 kev, was subdivided at 15 kev to form Groups 12 and 13 in the present set; the same U^{238} and Pu^{239} cross sections were used, however, in both groups. Greebler's low-energy cross sections are reproduced in Appendix A, Table A-IV. The Pu^{239} cross sections are based on a one-level formula with no overlap of resonances. Neglect of overlap would be expected to overestimate the positive contribution of Pu^{239} .

The Pu^{239} cross sections were given by Greebler et al.,⁽²⁾ for σ_p (potential scattering per reacting atom) of 247 and 370 barns. The appropriate number to be used varies with fuel-pin size and with energy, particularly in the vicinity of the 2.85-kev sodium resonance. Except in the vicinity of this resonance, a σ_p of 247 b is probably best representative of the 7:1 composition and the 370-b value for the 9:1. Because of flux depression at this resonance, events occurring in this energy range do not appear to contribute significantly to the total result. All the flux calculations in this report were performed with σ_p for $\text{Pu}^{239} = 370$ b. However, for the 7:1 case, perturbation-theory calculations were performed to evaluate the effect of using the 247-b cross sections (see Table IV). The effect on the overall Doppler coefficient was found to be unimportant.

The output from the code includes the following:

For coarse groups: flux, effective $3\Sigma_{tr}$, effective elastic-removal cross section for each material, average capture cross section.

For fine groups: flux, slowing-down density.

The normalization of the flux is such that $\sum_j (\nu\Sigma_f)_j \phi_j = 1$. The code does not currently provide for the calculation of adjoint fluxes. These are readily obtained by hand calculations for the low-energy coarse groups, transfers existing only between adjacent groups. The adjoint functions calculated here were normalized in accordance with $\sum_j \beta_j \phi_j^+ = 1$, where

β_j is the fraction of the fission spectrum born in group j . With this normalization, the expression for reactivity change due to changes in fission and capture cross sections as given by perturbation theory is

$$\delta k = \sum_j \phi_j \delta(\nu \Sigma_f)_j - \sum_j \phi_j \phi_j^+ \delta(\Sigma_f + \Sigma_c)_j \quad (1)$$

The lethargy width of 0.0174 for the fine groups is twice that used in previous calculations with the ELMOE code.^(2,7) The double width was used in order to allow the complete energy region of interest to be covered in a single calculation. Otherwise, two calculations would have had to be made with different energy regions subdivided into fine groups, allowing some overlap in the subdivided regions. The relative accuracy of the single and double lethargy-width calculations insofar as the Doppler-effect calculations are concerned can readily be determined by calculating the slowing-down density at 9.1 kev for the two cases. This energy was a convenient one to use. The difference was found not to exceed 2%.

Essentially all the Doppler effect for the cases considered here occurs below 9.1 kev, so that events above this energy serve only to determine the source of low-energy neutrons and therefore the normalization of the Doppler effect. No error is incurred by use of the double width below 9.1 kev because the fluctuations in cross section for the light elements can be described satisfactorily with this group width.

III. RESULTS

Calculations were carried out for a series of oxide- and carbide-fueled reactors, the fuel being a mixture of Pu^{239} and U^{238} . The ratios of U^{238} to Pu^{239} used were 7 and 9. Compositions are listed in Table I. Calculations were also run in each case with sodium removed, but with no other change.

These compositions correspond to core volumes in the range of several thousand liters. For the oxide in spherical geometry the 7:1 case corresponds to a core size of 1700 liters, and the 9:1 case corresponds to 5000 liters. The sizes for carbide cores have not been calculated but are estimated at 1000 and 2500 liters for the 7:1 and 9:1 cases, respectively, in spherical geometry.

Values of the total reactor Doppler coefficient for various cases are given in Table II. Also given is the critical value of B^2 at 300°K. The average coefficient for the range 300-2500°K is based on machine calculations carried out at 300 and 2500°K. The coefficients for the range 300-750°K were obtained from perturbation calculations based on the 300°K fluxes, whereas those for the range 1500-2500°K were obtained from similar calculations using the 2500°K fluxes. The results are consistent

with a temperature variation of dk/dT as $1/T$, as was found in Reference 2. The reactivity changes for carbide fuel are seen to be about 0.7 those for oxide at the same enrichment. The removal of sodium decreases the reactivity change by a factor of about 2.

Table I
COMPOSITIONS (atoms/cc $\times 10^{-24}$) USED IN CALCULATIONS

	Oxide-fueled Reactor	Carbide-fueled Reactor
Carbon	-	0.00981
Oxygen	0.01568	-
Sodium	0.01144	0.01144
Stainless Steel (Type 304)	0.01355	0.01525
U ²³⁸ 7:1	0.00686	0.008584
9:1	0.007056	0.008829
Pu ²³⁹ 7:1	0.00098	0.001226
9:1	0.000784	0.000981

Note: For the oxide-fueled reactors these compositions correspond to 52 vol-% sodium, 16 vol-% steel, and 32 vol-% fuel ($\rho = 10.9 \text{ gm/cm}^3$). For the carbide-fueled reactors the compositions correspond to 52 vol-% sodium, 18 vol-% steel, and 30 vol-% fuel ($\rho = 13.5 \text{ gm/cm}^3$).

Table II
SUMMARY OF DOPPLER REACTIVITY COEFFICIENTS*

Case	300-2500°K		300-750°K		1500-2500°K		Critical B ² at 300°K ($\text{cm}^{-2} \times 10^2$)
	$-\% \Delta k$	$-\frac{\Delta k}{\Delta T} \times 10^5$	$-\% \Delta k$	$-\frac{\Delta k}{\Delta T} \times 10^5$	$-\% \Delta k$	$-\frac{\Delta k}{\Delta T} \times 10^5$	
Oxide							
1:9, Na	2.63	1.15	1.09	2.40	0.59	0.59	0.0678
1:9, No Na	1.44	0.65	0.63	1.40	0.29	0.29	0.0617
1:7, Na	1.81	0.79	0.77	1.73	0.40	0.40	0.1219
1:7, No Na	0.92	0.40	0.39	0.87	0.18	0.18	0.1011
Carbide							
1:9, Na	1.95	0.89	0.82	1.82	0.40	0.40	0.0998
1:9, No Na	1.11	0.48	0.47	1.05	0.20	0.20	0.0818
1:7, Na	1.30	0.59	0.54	1.20	0.29	0.29	0.1633
1:7, No Na	0.63	0.29	0.27	0.60	0.12	0.12	0.1338

*These calculations used $\sigma_p = 370 \text{ b/atom}$ for Pu²³⁹. The reactivity changes between 300°K and 2500°K were obtained by direct calculation. The changes between 300°K and 750°K and between 1500°K and 2500°K were obtained by perturbation theory using the 300°K and 2500°K fluxes, respectively.

Although the results are not given here, it was found that metal systems had a Doppler coefficient about 0.1 that of oxide systems at the same fuel enrichment and volume fraction.

In Table III are given values of q , the slowing-down density at 9.1 kev. Because the fission source is normalized to unity, this is the fraction of fission neutrons slowing down below 9.1 kev. The value of q as obtained with the use of the cross sections of Reference 6, denoted as YOM, is given in the last column. The ratio of δk to q is also given and is seen to be nearly constant in the various cases. Since the ratio of leakage to absorption is small in the low-energy region, the Doppler effect is due essentially to a change in effective k_{∞} with temperature. This change in effective k_{∞} depends both on the energy variation of k_{∞} and on the variation with temperature of the energy at which fissions and captures take place. The presence of sodium also affects the energy at which reactions take place mainly because of the 2.85-kev scattering resonance. None of these effects appear to be extremely large. Even the increase in reactivity loss in going from the 1:7 to 1:9 case is due mainly to an increase in low-energy flux.

Table III

RATIO OF $-\delta k$ FOR 300°K-2500°K TO q , FRACTION OF FISSION
NEUTRONS SLOWING DOWN BELOW 9.1 kev

Case	q	$-\% \delta k$ 300°K-2500°K	$-\frac{\delta k}{q}$	q YOM
Oxide 1/9; Na	0.210	2.63	0.125	0.180
Oxide 1/9; No Na	0.111	1.44	0.130	0.102
Oxide 1/7; Na	0.152	1.81	0.119	0.130
Oxide 1/7; No Na	0.0745	0.92	0.124	0.0695
Carbide 1/9; Na	0.152	1.95	0.128	0.116
Carbide 1/9; No Na	0.0775	1.11	0.143	0.0584
Carbide 1/7; Na	0.1073	1.30	0.121	0.0800
Carbide 1/7; No Na	0.0506	0.63	0.124	0.0380

These effects may be studied in more detail in Table IV, in which the spectral distribution of the Doppler coefficient is given for U^{238} and Pu^{239} . Here, the effect of using the two sets of Pu^{239} cross sections for different σ_p is compared. The shift due to the presence of sodium is evident. One expects that, with sodium present, $\delta k/q$ will be more positive because more neutrons are forced into the region of lower energy, in which the changes in Pu^{239} cross sections are larger compared with those in U^{238} than at higher energy. The overall effect of this shift is small, however. These conclusions are, of course, dependent on the assumed U^{238} and Pu^{239} cross sections and conceivably could be altered by gross errors in these.

Table IV
SPECTRAL DISTRIBUTION OF DOPPLER COEFFICIENT BY PERTURBATION THEORY

1/7 Oxide, Sodium					1/7 Oxide, Sodium					1/9 Oxide, Sodium					1/9 Oxide, Sodium				
Group	E _L , kev	% δk 3000-7500K			Group	% δk 15000-25000K			Group	% δk 3000-7500K			Group	% δk 15000-25000K					
		U238	Pu239 σ _p = 370 b	Pu239 σ _p = 247 b		U238	Pu239 σ _p = 370 b	Pu239 σ _p = 247 b		U238	Pu239 σ _p = 370 b	Pu239 σ _p = 247 b		U238	Pu239 σ _p = 370 b	Pu239 σ _p = 247 b			
12	15-25	-0.0409	-	-	12	-0.0083	-	-	12	-0.0518	-	-	12	-0.0105	-				
13	9.1	-0.0296	-	-	13	-0.0059	-	-	13	-0.0385	-	-	13	-0.0076	-				
14	4.0	-0.1184	0.0043	0.0043	14	-0.0502	0.0087	0.0051	14	-0.1586	0.0055	0.0051	14	-0.0660	0.0101				
15	1.0	-0.3821	0.0098	0.0040	15	-0.2000	0.0141	0.0141	15	-0.5413	0.0526	0.0526	15	-0.2776	0.0179				
16	0.3	-0.2795	0.0823	0.1067	16	-0.1476	0.0224	0.0328	16	-0.4446	0.1240	0.1240	16	-0.2308	0.0330				
17	0.1	-0.0724	0.0512	0.0598	17	-0.0431	0.0121	0.0214	17	-0.1366	0.0949	0.0949	17	-0.0818	0.0222				
18	0.03	-0.0037	0.0068	0.0058	18	-0.0017	0.0017	0.0017	18	-0.0088	0.0161	0.0161	18	-0.0041	0.0040				
		-0.9266	0.1544	0.2406		-0.4569	0.0590	0.0751		-1.3802	0.2931	0.2931		-0.6784	0.0872				
% δk = -0.7722 (σ _p = 370 b) % δk = -0.6860 (σ _p = 247 b)					% δk = -0.3979 % δk = -0.3818					% δk = -1.0871					% δk = -0.5912				
1/7 Oxide, No Sodium					1/7 Oxide, No Sodium					1/9 Oxide, No Sodium					1/9 Oxide, No Sodium				
12		-0.0313	-	-	12	-0.0065	-	-	12	-0.0422	-	-	12	-0.0086	-				
13		-0.0215	-	-	13	-0.0043	-	-	13	-0.0301	-	-	13	-0.0059	-				
14		-0.0878	0.0033	0.0033	14	-0.0372	0.0064	0.0038	14	-0.1283	0.0046	0.0046	14	-0.0529	0.0083				
15		-0.2203	0.0243	0.0377	15	-0.1099	0.0079	0.0069	15	-0.3516	0.0353	0.0353	15	-0.1696	0.0114				
16		-0.0770	0.0224	0.0290	16	-0.0370	0.0055	0.0081	16	-0.1426	0.0395	0.0395	16	-0.0664	0.0095				
17		-0.0170	0.0118	0.0129	17	-0.0095	0.0025	0.0044	17	-0.0371	0.0253	0.0253	17	-0.0201	0.0054				
18		-0.0008	0.0014	0.0012	18	-0.0004	0.0004	0.0004	18	-0.0021	0.0038	0.0038	18	-0.0009	0.0009				
		-0.4557	0.0632	0.0841		-0.2048	0.0227	0.0236		-0.7340	0.1085	0.1085		-0.3244	0.0355				
% δk = -0.3925 % δk = -0.3716					% δk = -0.1821 % δk = -0.1812					% δk = -0.6255					% δk = -0.2889				
1/7 Carbide, Sodium					1/7 Carbide, Sodium					1/9 Carbide, Sodium					1/9 Carbide, Sodium				
12		-0.0336	-	-	12	-0.0069	-	-	12	-0.0437	-	-	12	-0.0088	-				
13		-0.0241	-	-	13	-0.0048	-	-	13	-0.0323	-	-	13	-0.0063	-				
14		-0.0956	0.0035	0.0035	14	-0.0404	0.0070	0.0041	14	-0.1321	0.0047	0.0047	14	-0.0547	0.0085				
15		-0.2856	0.0310	0.0616	15	-0.1472	0.0105	0.0105	15	-0.4202	0.0415	0.0415	15	-0.2112	0.0139				
16		-0.1785	0.0518	0.0857	16	-0.0913	0.0136	0.0200	16	-0.2936	0.0525	0.0525	16	-0.1487	0.0213				
17		-0.0391	0.0270	0.0385	17	-0.0228	0.0059	0.0106	17	-0.0782	0.0330	0.0330	17	-0.0444	0.0293				
18		-0.0017	0.0031	0.0020	18	-0.0008	0.0008	0.0008	18	-0.0042	0.0075	0.0075	18	-0.0019	0.0018				
		-0.6582	0.1164	0.1913		-0.3142	0.0238	0.0460		-1.0094	0.1892	0.1892		-0.4760	0.0748				
% δk = -0.5418 % δk = -0.4669					% δk = -0.2904 % δk = -0.2682					% δk = -0.8202					% δk = -0.4012				
1/7 Carbide, No Sodium					1/7 Carbide, No Sodium					1/9 Carbide, No Sodium					1/9 Carbide, No Sodium				
12		-0.0238	-	-	12	-0.0050	-	-	12	-0.0330	-	-	12	-0.0067	-				
13		-0.0165	-	-	13	-0.0033	-	-	13	-0.0239	-	-	13	-0.0047	-				
14		-0.0665	0.0025	0.0025	14	-0.0280	0.0048	0.0028	14	-0.0990	0.0036	0.0036	14	-0.0408	0.0064				
15		-0.1503	0.0166	0.0258	15	-0.0735	0.0053	0.0053	15	-0.2456	0.0025	0.0025	15	-0.1164	0.0079				
16		-0.0462	0.0132	0.0172	16	-0.0217	0.0032	0.0047	16	-0.0882	0.0242	0.0242	16	-0.0398	0.0057				
17		-0.0090	0.0061	0.0072	17	-0.0050	0.0013	0.0023	17	-0.0203	0.0136	0.0136	17	-0.0107	0.0028				
18		-0.0004	0.0007	0.0006	18	-0.0002	0.0002	0.0002	18	-0.0010	0.0018	0.0018	18	-0.0004	0.0004				
		-0.3127	0.0391	0.0533		-0.1367	0.0148	0.0153		-0.5110	0.0457	0.0457		-0.2195	0.0232				
% δk = -0.2736 % δk = -0.2594					% δk = -0.1219 % δk = -0.1214					% δk = -0.4653					% δk = -0.1963				

In Table V the spectral distribution of absorptions, $\delta q_j/q$, and the group reactivity change per neutrons disappearing, $\delta k_j/\delta q_j$, are given for the oxide cases for a temperature change from 300°K to 750°K. It is evident from this why $\delta k/q$ is not more sensitive to the presence or absence of sodium, as the main effect is to shift neutrons among Groups 14, 15, and 16, for which $\delta k_j/\delta q_j$ does not vary drastically. Few neutrons survive to Groups 17 and 18 in any case.

Table V

SPECTRAL DISTRIBUTION OF ABSORPTIONS, $\delta q_j/q$, AND OF
 REACTIVITY CHANGE PER ABSORPTION, $\delta k_j/\delta q_j$, FOR
 $\Delta T = 750-300^\circ\text{K}$

Oxide 9:1; Na				Oxide 7:1; Na	
Group	E_L , kev	$\sigma_p = 370 \text{ b}$		$\sigma_p = 247 \text{ b}$	
		$\delta q/q$	$\delta k/\delta q$	$\delta q/q$	$\delta k/\delta q$
14	4.0-9.1	0.195	-0.0375	0.222	-0.0338
15	1.0	0.390	-0.0600	0.408	-0.0513
16	0.3	0.276	-0.0554	0.262	-0.0433
17	0.1	0.117	-0.0170	0.093	-0.0089
18	0.03	0.021	+0.0165	0.013	+0.0109
Oxide 9:1; No Na				Oxide 7:1; No Na	
14	4.0-9.1	0.306	-0.0364	0.343	-0.0331
15	1.0	0.460	-0.0619	0.450	-0.0331
16	0.3	0.166	-0.0560	0.157	-0.0411
17	0.1	0.060	-0.0178	0.044	-0.0124
18	0.03	0.009	+0.0164	0.005	+0.0107

The positive contribution of Pu^{239} to the Doppler effect relative to the negative one of U^{238} is comfortably low, of the order of 25%, with sodium present, and this is reduced by the order of 5% of the U^{238} effect by the removal of sodium. It would seem that rather large errors in values of cross section would be required to alter these conclusions substantially.

The low-energy flux in a fast reactor, of course, attenuates rapidly with decreasing energy because of the large ratio of capture to elastic moderation. A characterization of this attenuation is the ratio of the coarse-group elastic-removal cross section of an element with constant scattering cross section to what it would be in a $1/E$ spectrum. Such ratios are given in Table VI. They tend to be lower at lower energies, as one would expect. Note that in Group 15 the 2.85-keV sodium resonance strongly affects the elastic-removal cross sections.

Table VI

RATIOS OF GROUP REMOVAL CROSS SECTIONS TO $\xi\sigma_s/\Delta u$

Oxide Fuel - Ratio for Oxygen					
E, kev	Group	300°K 1/7 Na	300°K 1/7 No Na	2500°K 1/7 Na	2500°K 1/7 No Na
4-9.1	14	0.7188	0.8922	0.7126	0.8790
1-4	15	0.9902	0.4727	0.9315	0.4214
0.3-1	16	0.5141	0.5156	0.4480	0.4618
0.1-0.3	17	0.3694	0.3109	0.3024	0.2562
		300°K 1/9 Na	300°K 1/9 No Na	2500°K 1/9 Na	2500°K 1/9 No Na
	14	0.7243	0.9143	0.7228	0.9007
	15	1.0334	0.5096	0.9719	0.4538
	16	0.5574	0.5520	0.4862	0.4911
	17	0.4129	0.3478	0.3361	0.2831
Carbide Fuel - Ratio for Carbon					
		300°K 1/7 Na	300°K 1/7 No Na	2500°K 1/7 Na	2500°K 1/7 No Na
	14	0.7634	0.9104	0.7562	0.8966
	15	0.9471	0.4529	0.8847	0.4022
	16	0.5008	0.5229	0.4362	0.4747
	17	0.3657	0.3174	0.3076	0.2686
		300°K 1/9 Na	300°K 1/9 No Na	2500°K 1/9 Na	2500°K 1/9 No Na
	14	0.7750	0.9326	0.7676	0.9181
	15	0.9911	0.4889	0.9253	0.4324
	16	0.5430	0.5560	0.4724	0.5001
	17	0.4045	0.3508	0.3361	0.2924

In Table A-I, Appendix A, are given values of coarse-group elastic-removal cross sections for light elements as obtained from the ELMOE calculation. In Table A-II representative values of the effective coarse-group values of $3\Sigma_{tr}$ are given. In Table A-III are given effective values of the capture cross section of sodium in Group 15, extending from 1 to 4 kev, obtained by varying σ_c over the 2.85-kev resonance. The average value is seen to be in the vicinity of 50 mb, and is, of course, considerably

lower than one would obtain at infinite dilution because of flux depression in the resonance. In Table A-IV the Pu^{239} and U^{238} cross sections taken from Reference 2, which were used in the calculation, are reproduced.

In Table B-I of Appendix B, coarse-group fluxes are given, and in Table B-II adjoint fluxes are tabulated. Using these, one can carry out perturbation calculations for various assumed temperature variations of fission and capture cross sections.

The exact effect of an ELMOE flux calculation compared with an ordinary coarse-group calculation, of course, depends on what coarse-group cross sections are used in the comparison. If collapsed cross sections derived from an ELMOE calculation for the same composition and temperature are used, there will be no difference. Use of some other set of coarse-group cross sections can make two important types of differences in the calculations. One is to change the slowing-down density at 9 kev, and the other is to change the low-energy spectrum, thus affecting the relative absorption of neutrons in U^{238} and Pu^{239} . With regard to the first type, a comparison has already been presented in Table III of q as calculated by ELMOE and as calculated by the cross sections of Yiftah *et al.*⁽⁶⁾ The Yiftah cross sections give lower q values than ELMOE, the maximum discrepancy being 34%. This is not a serious error, considering the other uncertainties in the calculation. With regard to second type of possible error, that concerned with the low-energy spectrum, it can be seen from Table V that the most likely error would be to overestimate the positive reactivity contribution of Pu^{239} by computing too soft a spectrum because of an overestimate of the light-element elastic-removal cross sections. This is, of course, not serious from a safety standpoint and will be easy to avoid when sufficient calculations of the ELMOE type are available as a guide. A study of the elastic-removal cross sections given in Table A-I shows that the variation from one case to another is not so large as to make a selection of such coarse-group cross sections for a new problem overly difficult.

One advantage of ELMOE that has not yet been exploited is that it is possible to insert resolved resonances at the proper energy location. This will allow more accurate flux calculations. Unfortunately, resonance parameters for fissile materials are not available in the energy range of interest, which limits the possible gain from this procedure.

In conclusion, it may be said that the calculated Doppler effect for dilute ceramic-fueled reactors is quite large and could be of great importance for safety. The results are similar to those obtained by Greebler *et al.*⁽²⁾ with the same low-energy cross sections for U^{238} and Pu^{239} . The more elaborate flux calculation employed here does, however, give more confidence in the accuracy of the results. It appears that

ordinary coarse-group calculations of the Doppler effect will give adequate accuracy with the help of ELMOE-type calculations as a guide in selecting group cross sections.

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APPENDIX A
EFFECTIVE COARSE GROUP CROSS SECTIONS

Table A-I

ELASTIC-REMOVAL CROSS SECTIONS FOR LIGHT ELEMENTS, IN BARNS

E _L , kev	Group	300°K - 1/7 - Oxide - Na			300°K - 1/7 - Carbide - Na			300°K - 1/7 - Oxide - No Na		300°K - 1/7 - Carbide - No Na	
		O	Na	S.S.	C	Na	S.S.	O	S.S.	C	S.S.
3668	1										
2225	2	0.249	0.318	0.100	0.683	0.303	0.099	0.247	0.097	0.681	0.097
1350	3	0.561	0.276	0.101	0.616	0.272	0.099	0.568	0.103	0.618	0.101
825	4	1.108	0.584	0.200	0.796	0.497	0.154	1.128	0.217	0.792	0.147
500	5	0.764	0.649	0.210	1.105	0.674	0.221	0.648	0.166	0.976	0.172
300	6	1.360	0.804	0.161	1.198	0.681	0.134	1.398	0.171	1.170	0.130
180	7	0.907	0.682	0.243	1.230	0.652	0.233	0.885	0.222	1.200	0.210
110	8	0.820	0.706	0.250	1.345	0.690	0.239	0.816	0.226	1.351	0.211
67	9	0.827	0.755	0.232	1.392	0.761	0.236	0.820	0.235	1.394	0.242
40.7	10	0.864	0.665	0.340	1.471	0.659	0.329	0.765	0.292	1.333	0.284
25	11	0.885	0.878	0.323	1.504	0.920	0.345	0.940	0.370	1.594	0.404
15	12	0.644	0.571	0.540	1.190	0.556	0.527	0.596	0.493	1.130	0.483
9.1	13	0.732	0.840	0.451	1.271	0.838	0.449	0.726	0.442	1.261	0.439
4.0	14	0.398	1.297	0.321	0.650	1.287	0.318	0.494	0.422	0.811	0.409
1.0	15	0.325	0.188	0.233	0.501	0.172	0.211	0.155	0.110	0.240	0.099
0.3	16	0.193	0.111	0.163	0.305	0.103	0.150	0.194	0.163	0.318	0.158
0.1	17	0.153	0.087	0.129	0.244	0.080	0.119	0.129	0.107	0.212	0.102
	Group	300°K - 1/9 - Oxide - Na			300°K - 1/9 Carbide - Na			300°K - 1/9 - Oxide - No Na		300°K - 1/9 - Carbide - No Na	
		O	Na	S.S.	C	Na	S.S.	O	S.S.	C	S.S.
	12	0.652	0.578	0.549	1.202	0.563	0.535	0.604	0.501	1.141	0.490
	13	0.741	0.850	0.457	1.284	0.849	0.456	0.737	0.450	1.278	0.447
	14	0.404	1.317	0.326	0.690	1.390	0.324	0.506	0.434	0.830	0.422
	15	0.339	0.197	0.244	0.525	0.181	0.222	0.167	0.119	0.259	0.108
	16	0.210	0.121	0.178	0.330	0.113	0.165	0.208	0.176	0.338	0.169
	17	0.171	0.097	0.145	0.270	0.089	0.133	0.144	0.121	0.234	0.114
		2500°K - 1/7 - Carbide - Na			2500°K - 1/7 - Oxide - Na			2500°K - 1/7 - Oxide - No Na		2500°K - 1/7 - Carbide - No Na	
		O	Na	S.S.	C	Na	S.S.	O	S.S.	C	S.S.
	12	0.644	0.570	0.539	1.189	0.555	0.526	0.595	0.492	1.129	0.482
	13	0.732	0.839	0.450	1.270	0.837	0.449	0.725	0.441	1.259	0.438
	14	0.394	1.284	0.318	0.673	1.273	0.314	0.486	0.415	0.798	0.401
	15	0.306	0.177	0.217	0.468	1.595	0.194	0.138	0.097	0.213	0.087
	16	0.168	0.096	0.140	0.265	0.089	0.129	0.174	0.145	0.289	0.142
	17	0.125	0.071	0.104	0.205	0.067	0.099	0.106	0.088	0.179	0.086
		2500°K - 1/9 - Oxide - Na			2500°K - 1/9 - Carbide - Na			2500°K - 1/9 - Oxide - No Na		2500°K - 1/9 - Carbide - No Na	
		O	Na	S.S.	C	Na	S.S.	O	S.S.	C	S.S.
	12	0.651	0.578	0.548	1.201	0.563	0.534	0.603	0.500	1.140	0.490
	13	0.739	0.849	0.457	1.282	0.847	0.455	0.735	0.449	1.276	0.446
	14	0.400	1.305	0.323	0.633	1.295	0.320	0.498	0.427	0.818	0.413
	15	0.319	0.185	0.228	0.490	0.167	0.205	0.149	0.105	0.229	0.094
	16	0.183	0.105	0.153	0.287	0.097	0.141	0.185	0.155	0.304	0.150
	17	0.139	0.079	0.117	0.224	0.074	0.109	0.117	0.097	0.195	0.094

Table A-II

VALUES OF $3\Sigma_{tr}$, cm^{-1} , 1:7, 300°K

Group	E_L , kev	Oxide		Carbide	
		Na	No Na	Na	No Na
1	3670	0.270	0.226	0.276	0.232
2	2230	0.299	0.245	0.332	0.281
3	1350	0.342	0.284	0.348	0.291
4	825	0.475	0.377	0.398	0.303
5	500	0.500	0.342	0.511	0.353
6	300	0.657	0.524	0.561	0.432
7	180	0.644	0.504	0.633	0.493
8	110	0.702	0.557	0.737	0.592
9	67	0.771	0.620	0.827	0.674
10	40.7	0.801	0.638	0.865	0.702
11	25	0.934	0.774	1.017	0.854
12	15	0.829	0.682	0.909	0.761
13	9.1	0.999	0.826	1.095	0.922
14	4.0	1.410	1.075	1.527	1.191
15	1.0	1.157	0.918	1.272	1.019
16	0.3	1.179	1.076	1.309	1.206
17	0.1	1.352	1.251	1.521	1.420
18	0.03	2.343	2.239	2.733	2.630

Table A-III

EFFECTIVE CAPTURE CROSS SECTIONS OF
Na, IN BARNS, IN GROUP 15 (1-4 kev)

		300°K	2500°K
Oxide	1/7	0.0485	0.0515
	1/9	0.0464	0.0495
Carbide	1/7	0.0517	0.0555
	1/9	0.0492	0.0530

Table A-IV

TEMPERATURE-DEPENDENT CROSS SECTIONS (IN BARNS) OF
 U^{238} AND Pu^{239} USED IN CALCULATIONS⁽²⁾

Group	E, kev	σ_{γ} of U^{238} ($\sigma_p = 40$ b)	
		300°K	2500°K
12, 13	9.1-25	0.601	0.617
14	4.0-9.1	0.399	0.487
15	1.0-4.0	0.728	1.02
16	0.3-1.0	0.850	1.35
17	0.1-0.3	1.55	2.36
18	0.03-0.1	2.31	2.91

Group	Pu^{239} ($\sigma_p = 370$ b)				Pu^{239} ($\sigma_p = 247$ b)			
	300°K		2500°K		300°K		2500°K	
	σ_f	σ_{γ}	σ_f	σ_{γ}	σ_f	σ_{γ}	σ_f	σ_{γ}
12, 13	2.25	1.06	2.25	1.06	2.25	1.06	2.25	1.06
14	2.57	1.33	2.59	1.34	2.55	1.32	2.58	1.34
15	3.83	2.14	3.93	2.20	3.73	2.09	3.88	2.17
16	7.24	4.50	7.92	4.93	6.76	4.21	7.65	4.76
17	15.2	9.86	18.2	11.9	13.4	8.73	16.9	11.0
18	13.8	17.1	18.1	22.4	11.7	14.5	15.7	19.5

APPENDIX B
COARSE-GROUP FLUXES AND ADJOINT FLUXES

Table B-I
 TABULATION OF FLUX ϕ_j (NORMALIZED TO $\sum_j (\nu \Sigma_f)_j \phi_j = 1$)

FLUXES - 1/7 OXIDE				FLUXES - 1/7 CARBIDE			
1/7 Oxide Sodium		1/7 Oxide Sodium		1/7 Carbide Sodium		1/7 Carbide Sodium	
Group	300°K	Group	2500°K	Group	300°K	Group	2500°K
1	2.20			1	1.95		
2	4.54			2	3.67		
3	7.24			3	6.59		
4	8.73			4	9.55		
5	15.85			5	13.37		
6	14.25			6	16.40		
7	17.41			7	14.85		
8	16.61			8	12.82		
9	13.45			9	9.87		
10	11.49			10	8.33		
11	8.63			11	5.94		
12	7.77	12	7.73	12	5.16	12	5.13
13	5.31	13	5.27	13	3.49	13	3.46
14	4.41	14	4.31	14	2.85	14	2.78
15	4.78	15	4.36	15	2.87	15	2.58
16	2.08	16	1.58	16	1.06	16	0.78
17	0.38	17	0.21	17	0.16	17	0.09
18	0.04	18	0.01	18	0.01	18	0.01
1/7 Oxide No Sodium		1/7 Oxide No Sodium		1/7 Carbide No Sodium		1/7 Carbide No Sodium	
1	2.49			1	2.18		
2	5.17			2	4.06		
3	7.80			3	7.13		
4	10.47			4	11.84		
5	23.15			5	18.17		
6	15.94			6	18.79		
7	19.94			7	15.84		
8	18.32			8	12.95		
9	13.69			9	9.21		
10	11.50			10	7.62		
11	7.25			11	4.65		
12	6.19	12	6.15	12	3.77	12	3.74
13	3.98	13	3.93	13	2.45	13	2.42
14	3.30	14	3.20	14	1.99	14	1.95
15	2.79	15	2.42	15	1.52	15	1.30
16	0.57	16	0.39	16	0.27	16	0.18
17	0.09	17	0.04	17	0.04	17	0.02
18	0.01	18	0.00	18	0.00	18	0.00
FLUXES - 1/9 OXIDE				FLUXES - 1/9 CARBIDE			
1/9 Oxide Sodium		1/9 Oxide Sodium		1/9 Carbide Sodium		1/9 Carbide Sodium	
Group	300°K	Group	2500°K	Group	300°K	Group	2500°K
1	2.28			1	2.02		
2	4.71			2	3.79		
3	7.61			3	6.93		
4	9.25			4	10.29		
5	17.18			5	14.69		
6	15.75			6	18.57		
7	19.67			7	17.32		
8	19.29			8	15.36		
9	16.07			9	12.14		
10	14.18			10	10.60		
11	11.00			11	7.82		
12	10.25	12	10.20	12	7.04	12	6.99
13	7.24	13	7.17	13	4.92	13	4.87
14	6.20	14	6.06	14	4.15	14	4.05
15	7.11	15	6.47	15	4.45	15	3.99
16	3.49	16	2.64	16	1.87	16	1.36
17	0.76	17	0.42	17	0.34	17	0.18
18	0.10	18	0.04	18	0.04	18	0.01
1/9 Oxide No Sodium		1/9 Oxide No Sodium		1/9 Carbide No Sodium		1/9 Carbide No Sodium	
1	2.58			1	2.26		
2	5.38			2	4.20		
3	8.19			3	7.49		
4	11.14			4	12.90		
5	25.72			5	20.54		
6	18.09			6	21.96		
7	23.21			7	19.12		
8	22.04			8	16.08		
9	17.05			9	11.78		
10	14.96			10	10.18		
11	9.82			11	6.46		
12	8.76	12	8.70	12	5.46	12	5.42
13	5.85	13	5.79	13	3.69	13	3.64
14	5.09	14	4.93	14	3.15	14	3.04
15	4.70	15	4.06	15	2.64	15	2.25
16	1.12	16	0.76	16	0.55	16	0.36
17	0.20	17	0.10	17	0.09	17	0.04
18	0.02	18	0.01	18	0.01	18	0.00

Table B-II

TABULATION OF ADJOINT FLUX ϕ_j^+ *

Oxide								
j	1:7, Na		1:9, Na		1:7, No Na		1:9, No Na	
	300°K	2500°K	300°K	2500°K	300°K	2500°K	300°K	2500°K
18	0.8008	0.8230	0.7272	0.7495	0.8040	0.8256	0.7296	0.7514
17	1.117	1.0412	1.018	0.9623	1.1232	1.0655	1.026	0.9656
16	1.086	0.9710	1.002	0.8859	1.0925	0.9783	1.005	0.8850
15	0.9561	0.8353	0.8845	0.7595	0.9447	0.8267	0.8693	0.7399
14	0.9556	0.8478	0.8850	0.7711	0.9446	0.8461	0.8710	0.7609
13	0.9018	0.8170	0.8381	0.7459	0.8750	0.8047	0.8087	0.7275
12	0.8522	0.7871	0.7958	0.7295	0.8192	0.7696	0.7595	0.7003
Carbide								
j	1:7, Na		1:9, Na		1:7, No Na		1:9, No Na	
	300°K	2500°K	300°K	2500°K	300°K	2500°K	300°K	2500°K
18	0.8056	0.8269	0.7314	0.7529	0.8084	0.8292	0.7335	0.7547
17	1.126	1.068	1.029	0.9680	1.133	1.071	1.035	0.9706
16	1.092	0.9774	1.006	0.8851	1.099	0.9815	1.010	0.8858
15	0.9518	0.8304	0.8770	0.7492	0.9442	0.8203	0.8642	0.7342
14	0.9534	0.8470	0.8787	0.7646	0.9474	0.8469	0.8681	0.7599
13	0.8949	0.8138	0.8265	0.7371	0.8739	0.8045	0.8157	0.7241
12	0.8436	0.7831	0.7815	0.7126	0.8186	0.7706	0.7613	0.6963

*Normalized to $\sum_j \beta_j \phi_j^+ = 1$, where β_j is the fraction of the fission spectrum in Group j.

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