4/5/10/88 05 (4) PPPL-2498

DR# 0457-X
PPPL-2498

UC-427

Nonlinear Gyrokinetic Theory for Finite-β Plasmas

By

T.S. Hahm, W.W. Lee, and A. Brizard

FEBRUARY 1988

PLASMA PHYSICS LABORATORY

PRINCETON UNIVERSITY
PRINCETON, NEW JERSEY

PREPARED FOR THE U.S. DEPARTMENT OF ENERGY,
UNDER CONTRACT DE-AC02-76-CHO-3073.

NOTICE

Printed in the United States of America

Available from:

National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Road
Springfield, Virginia 22161

Price Printed Copy $ ___*___; Microfiche $4.50

*Pages	NTIS Selling Price
1-25	$7.00
25-50	$8.50
51-75	$10.00
76-100	$11.50
101-125	$13.00
126-150	$14.50
151-175	$16.00
176-200	$17.50
201-225	$19.00
226-250	$20.50
251-275	$22.00
276-300	$23.50
301-325	$25.00
326-350	$26.50
351-375	$28.00
376-400	$29.50
401-425	$31.00
426-450	$32.50
451-475	$34.00
476-500	$35.50
500-525	$37.00
526-550	$38.50
551-575	$40.00
567-600	$41.50

For documents over 600 pages, add $1.50 for each additional 25-page increment.

Nonlinear Gyrokinetic Theory For Finite-β Plasmas

T. S. Hahm, W. W. Lee, and A. Brizard

Plasma Physics Laboratory, Princeton University

Princeton, New Jersey 08544

PPPL--2498

DE88 009358

Abstract

A self-consistent and energy-conserving set of nonlinear gyrokinetic equations, consisting of the averaged Vlasov and Maxwell's equations for finite-β plasmas, is derived. The method utilized in the present investigation is based on the Hamiltonian formalism and Lie transformation. The resulting formulation is valid for arbitrary values of $k_\perp \rho_i$ and, therefore, is most suitable for studying linear and nonlinear evolution of microinstabilities in tokamak plasmas as well as other areas of plasma physics where the finite Larmor radius effects are important. Because the underlying Hamiltonian structure is preserved in the present formalism, these equations are directly applicable to numerical studies based on the existing gyrokinetic particle simulation techniques.

1. Introduction

Low-frequency microturbulence is generally believed to be closely linked to the observed anomalous transport in tokamaks.[1-3] Theoretical research in this area has been very active for more than two decades. One of the most important advances was the use of gyrokinetic ordering,[4,5] which considerably simplified the analytical aspect of the problem while keeping intact all the relevant physics. Although the ordering was originally designed to facilitate the linear analysis, Frieman and Chen[6] have successfully implemented the idea to derive nonlinear gyrokinetic equations in general geometry which can be used for studying the drift-Alfvén system. Their work was followed by the development of the new nonlinear gyrokinetic equations which are formulated in terms of the total distribution function and, therefore, can be easily studied by the particle simulation method.[7] Dubin et al.[8] have derived an energy-conserving set of gyrokinetic equations which preserves Hamiltonian symmetry by using the Hamiltonian-Lie perturbation method,[9,10] and has set Lee's equations on a firmer theoretical ground. One salient feature of their set of equations is the appearance of the ion polarization density response in the gyrokinetic Poisson's equation, which has a great impact on the numerical schemes utilized in solving these equations. The fact that the polarization drift appears not in the gyrophase-averaged Vlasov equation but rather in the gyrokinetic Poisson equation, is the natural consequence of the gyrokinetic procedure and is also desirable from a numerical viewpoint.

The feasibility and efficiency of "gyrokinetic particle simulation" in studying low-frequency phenomena in tokamaks have been discussed in a recent publication.[11] Since the fast particle gyrations are eliminated from the simulation plasma, while the all-important finite Larmor radius effects are kept intact, orders of magnitude improvement in time step, grid spacing, and noise level over the conventional particle simulation have been achieved. The simulation scheme described in Ref.11, which is based on the equations derived for the electrostatic problems in a slab geometry,[7,8] has recently been used in the investigations of drift-type instabilities in two-dimensional[12-14] and three-dimensional[15] geometries. Aside from demonstrating the versatility and superiority of the gyrokinetic approach in numerical simulation, considerable insight has also been obtained through these studies concerning the saturation and the induced transport mechanisms. Thus, it is natural at this stage to extend the present capability by including additional physics in the gyrokinetic formulation so as to better describe the phenomena of interest in a more realistic situation.

As a first step in that direction, we have developed in the present investigation a reduced set of gyrophase-averaged equations in slab geometry, which consists of the Vlasov equation as well as the associated Poisson's equation and Ampere's law, for studying low-frequency electromagnetic fluctuations in finite-β plasmas. Thus, the present paper, in which magnetic perturbations and induction electric fields are accounted for, can be considered as a natural generalization of the work by Dubin et al.[8] It differs from the existing nonlinear gyrokinetic theories for finite-β plasmas,[6] since the resulting formalism in our case preserves the Hamiltonian symmetry of the original Vlasov-Maxwell system and, consequently, it is suitable to both analytical and particle simulation studies. More specifically, without breaking up the distribution function into the unperturbed and perturbed parts as in the other formal approaches, the resulting formalism is explicitly 'phase space preserving' and, as such, conservation laws (e.g., number density and energy) can easily be constructed from these equations, which are essential for numerical purposes.

Recently, it has been shown that the numerical properties of the gyrokinetic plasma can be further improved even with a very moderate value of plasma β, e.g., $\beta \gtrsim m_e/m_i$.[16] Therefore, it is desirable from the numerical point of view to study both the predominantly electrostatic microinstabilities and those associated with the magnetic perturbations with the present set of equations. This is also the reason why it is necessary to first develop the finite-β gyrokinetic equations in slab geometry before embarking on the task of including the toroidal effects in the formulation. The latter in the electrostatic limit has been investigated earlier by the authors of Refs.17 and 18. Although they have successfully derived the gyrophase-averaged Vlasov equation which contains the lowest order nonlinearities, the energy-conserving set of Vlasov-Poisson equations which contains all the crucial polarization physics is yet to be derived. It is necessary to keep formally higher order nonlinearities to achieve that goal.[19] Another interesting aspect of our equations is that they can easily be reduced in the long wavelength limit to the "reduced MHD equations."[20] Thus, it suggests the possibility of simulating global MHD instabilities with the finite-β gyrokinetic particle code.[16] This exciting prospect, which can provide us with the opportunity to study kinetic effects on MHD modes, is also an impetus for the present investigation.

In this paper, the usual gyrokinetic ordering of $\omega/\Omega_i \sim k_\parallel/k_\perp \sim e\phi/T_e$

$\rho_i/L_n \sim O(\varepsilon)$ and $k_\perp \rho_i \sim (1)$ as well as $\delta B/B \sim O(\varepsilon)$ has been used. Here, ω and Ω_i are the characteristic fluctuation frequency and the ion cyclotron frequency, respectively; $k_\parallel$ and $k_\perp$ are the components of the wave vector in the parallel and perpendicular direction with respect to the ambient magnetic field; ρ_i is the ion gyroradius; L_n is the density scale length; φ and δB are the fluctuating electrostatic potential and magnetic field; and ε is the smallness parameter (not to be confused with the inverse aspect ratio). For simplicity, we neglect the compressional component of the magnetic perturbation, i.e., $A_\perp$, in the formulation. This component does not directly cause any radial transport and the approximation is good for the cases with a moderate β [e.g., $\beta \lesssim (a/R)^2$ for tokamaks, where a is the minor radius and R is the major radius].

As we have mentioned earlier, the methodology used in our derivation is based on the Hamiltonian formalism and Lie transformation, which has been used extensively by Littlejohn[9,10] in his investigation of guiding center drifts in a specified (non-self-consistent) electromagnetic field in the drift-kinetic limit ($k_\perp\rho_i$<<1) and, subsequently, by Dubin et al.[8] in their derivation of a self-consistent set of gyrokinetic equations ($k_\perp\rho_i$ 1). This approach is systematic and requires much less algebra compared to the conventional derivation which involves direct averaging of the Vlasov equation.[6,7] One important issue for our case is the choice of the coordinates in the phase space. We have decided that there are several advantages in using canonical momentum p_z as an independent variable rather than the velocity v_z as in the previous analyses.[6] (Here, the subscript z designates the direction of the equilibrium magnetic field.)

For example, by doing so, we can introduce fluctuating magnetic field δB explicitly in the Hamiltonian. This fact has been utilized in both the study of test particle transport [21] in tokamaks and the derivation of the linear relativistic gyrokinetic equation.[22] One important consequence is that the resulting formalism becomes covariant in that the fluctuating electromagnetic fields appear as a particular combination of potentials, i.e., the generalized potential $\Psi = \varphi - v_z A_z$. Furthermore, the induction electric field, which involves the partial time derivative, does not appear explicitly in the gyrokinetic equation. Therefore, the equation is in a form most suitable for renormalization. Since the numerical schemes for calculating the induction field are quite involved for finite-β particle simulations in which the transverse displacement current is neglected,[16,23] this choice of variables is also advantageous computationally. The formulation in terms of v_z has actually been performed in the present investigation. However, the resulting equations are much more complicated than those of the formulation in terms of p_z. We will simply present the results in terms of v_z in the Appendix for the interested readers. The details of the derivation involving the action-variational Lie perturbation theory[24,25] will be given in a future publication where general geometry is considered.[19]

The organization of the paper is as follows. In Sec. II we use the Hamiltonian formalism and Lie transformations to derive a gyrophase-averaged single-particle Hamiltonian in the presence of an electromagnetic field. In Sec. III we consider the Vlasov-Maxwell equations and use the gyrophase-averaging procedure developed in Sec. II to derive a self-consistent, energy-conserving set of gyrokinetic equations for the Vlasov-Maxwell system. In Sec. IV various limiting cases are explored. Our conclusions are drawn in Sec. V. Finally, in the Appendix, an alternative formulation in terms of v_z is presented.

II. Derivation of Gyrokinetic Hamiltonian

In this section, we derive the gyrophase-averaged Vlasov equation using the Hamiltonian formalism and the Lie perturbation method. The mathematical foundation of the theory can be found elsewhere.[9,10]

We set m=c=1 for simplicity. The equations of motion (or Hamilton's equations) in an arbitrary phase coordinate system z take the form

$$dz/dt = \{z,H\}, \tag{1}$$

where H is the single-particle Hamiltonian and { , } is the Poisson bracket. The Poisson bracket of two functions f(z) and g(z) is defined by

$$\{f,g\} \equiv \partial f/\partial z \cdot J \cdot \partial g/\partial z, \tag{2}$$

where J is the Poisson tensor which is antisymmetric and covariant. In particular, in the extended canonical coordinates, $z_c = (q,p,w,t)$, this tensor takes the form

$$J(z_c) \equiv \begin{bmatrix} 0 & -I \\ I & 0 \end{bmatrix} . \tag{3}$$

where 0 and I are the 4×4 null and unit matrices, q and p are the usual canonical coordinates, and time t is now a coordinate conjugate to energy w in an extended eight-dimensional phase space. It has been shown previously that the averaging procedure is facilitated by the introduction of the extended phase space.[8] In this investigation, we are interested in

$z = (x, v_\perp, p_z, \theta, t, w)$ where x is the position, $v_\perp$ is the perpendicular component of particle velocity, and p_z is the canonical momentum along the z direction (i.e., along b). Since J transforms contravariantly, we can find its form in any set of coordinates $\bar{z}$ connected via a diffeomorphism to canonical coordinates z by

$$J(\bar{z}) = \partial\bar{z}/\partial z \cdot J(z) \cdot \partial\bar{z}/\partial z. \tag{4}$$

The single-particle (zeroth-order) Hamiltonian H_0 for a charged particle in a uniform magnetic field is given by

$$H_0(x, v_\perp, p_z, \theta, t, w) = (p_z^2 + v_\perp^2)/2 - w, \tag{5}$$

where $B_0 = B_0 b$. In our coordinates $(x, v_\perp, p_z, \theta, t, w)$, the nonvanishing elements of J derived from Eq.(4) are:

$$\{x, v_\perp\} = c, \tag{6a}$$

$$\{x, p_z\} = b, \tag{6b}$$

$$\{x, \theta\} = -a/v_\perp, \tag{6c}$$

$$\{\theta, v_\perp\} = \Omega/v_\perp, \tag{6d}$$

$$\{w, t\} = 1, \tag{6e}$$

where $\theta = \tan^{-1}(v \cdot e_1 / v \cdot e_2)$. e_1 and e_2 are arbitrary orthogonal unit vectors in the plane perpendicular to b, and $a = e_1 \cos\theta - e_2 \sin\theta$, $c = -e_1 \sin\theta - e_2 \cos\theta$, $\Omega = eB_0/mc$.

As shown in Eqs.(5) and (6), the θ dependence is in the Poisson bracket, while the Hamiltonian H_0 is θ-independent. Now we want to remove the θ dependence from the zeroth-order equations of motion by finding an appropriate set of coordinate transformations. This preparation can be achieved by transforming z to a new set of "gyrocenter" coordinates. In the absence of perturbed electromagnetic fields δE and δB, the gyrocenter variables are well known, i.e.,

$$Z = (X, \mu, \theta, p_z, t, W)$$

with $X = x - \rho$, $\rho = b \times v/\Omega$, and $\mu = v_\perp^2/2\Omega$. Here we would like to point out that there is an alternative way, that is, to define the gyrocenter coordinates in terms of the *total* electromagnetic field (i.e., in the presence of δE and δB). This can be done by using "Darboux Theorem"[24] which ensures the existence of the local canonical variables. The advantage of the Darboux transformation is that it gives considerable insight into single-particle dynamics beyond the capability of more conventional approaches. This is one of the reasons why the Darboux theorem has been widely used in the calculation of higher-order guiding center drifts in the drift-kinetic regime.[9,10] One may want to adopt that procedure directly for a gyrokinetic problem. However, in general (except for the case of the electrostatic problem in slab geometry considered in Ref.8), the transformation from the particle to the Darboux coordinates becomes quite complicated even after truncation to some order. Consequently, Poisson's equation and Ampere's law expressed in these coordinates are not practical at all for numerical simulation purposes. Since the main aim of this work is to derive a useful self-consistent set of gyrokinetic equations for the investigation of plasma microturbulence and transport which arise from collective fluctuations, we believe the optimal way is to define gyrocenter variables in terms of the unperturbed magnetic field B_0 only ($E_0 = 0$ is assumed throughout this paper) and introduce δE, δB later on as perturbations using the Lie transformation techniques. By doing so, the transformation from particle to gyrocenter coordinates becomes simpler and the resulting equations are more suitable for gyrokinetic particle simulation. Furthermore, the physics contents in these equations are more transparent, which make the comparisons to the existing theories[6] easier. The usefulness and simplicity of the derivation given here are even more apparent when we consider the problem in general geometry.[19]

Now, in terms of our gyrocenter variables, the zeroth order Hamiltonian is

$$H_0 = \mu B_0 + p_z^2/2 - W. \tag{7}$$

We note that $A_{0z}=0$ is consistent with the assumption of the uniform equilibrium magnetic field $\mathbf{B}_0 = B_0\hat{z}$. The Poisson brackets for gyrocenter variables are

$$\{\theta,\mu\} = 1, \tag{8a}$$

$$\{\theta,Z\} = 0, \text{ for } Z \neq \mu, \tag{8b}$$

$$\{\mu,Z\} = 0, \text{ for } Z \neq \theta, \tag{8c}$$

$$\{X,X\} = b \times I/\Omega, \tag{8d}$$

$$\{X,p_z\} = b, \tag{8e}$$

$$\{W,t\} = 1, \tag{8f}$$

where I is the unit dyadic. Now, the full Hamiltonian written in terms of the gyrocenter variables becomes

$$\begin{aligned}\hat{H}(Z) &= \mu B + (p_z - eA_z(X+\rho,t))^2/2 - W + e\varphi(X+\rho,t)\\ &= \mu B + p_z^2/2 - W - \varepsilon(ep_zA_z - e\varphi) + \varepsilon^2(eA_z)^2/2,\end{aligned} \tag{9}$$

where the gyro-angle dependence appears only in the first and second order Hamiltonians which are tagged by ε's, i.e.,

$$\hat{H}_1 = -\varepsilon\,(ep_zA_z - e\varphi), \tag{10a}$$

$$\hat{H}_2 = \varepsilon^2(eA_z)^2/2. \tag{10b}$$

Now, we can systematically remove the θ dependence from the perturbed Hamiltonian by transforming it to a set of new variables using the Lie perturbation theory.[9,10] Denoting the transformation by T, we have

$$\overline{Z} = TZ\ ,\quad \overline{H} = T^{-1}\hat{H}, \tag{11}$$

where $T = \exp[-\int^{\varepsilon} d\lambda\ L(\lambda,\varepsilon)]$. $\overline{Z}$ and $\overline{H}$ are the gyrophase-averaged coordinates and Hamiltonian, L is the Lie operator with $L(\lambda,\varepsilon) \equiv \sum \lambda^{n-1}L_n(\varepsilon)$, $L_n(\varepsilon) = \{G_n,\ \} \equiv L_{n0} + \varepsilon L_{n1}$, and G_n's are the generating functions of the transformation. Based on the gyrokinetic ordering, the Lie operator can be ordered as

$$L_{n0} = \partial G_n/\partial\theta\cdot\partial/\partial\mu - \partial G_n/\partial\mu\cdot\partial/\partial\theta + 1/\Omega\nabla G_n\cdot b\times\nabla, \tag{12a}$$

$$L_{n1} = b\cdot\nabla G_n\partial/\partial p_z - \partial G_n/\partial p_z b\cdot\nabla - \partial G_n/\partial t\cdot\partial/\partial W, \tag{12b}$$

where $\nabla\equiv\partial/\partial X$ and $\partial G_n/\partial W = 0$, since we do not transform in time. Expanding Eq.(11) in ε, we have

$$\overline{H}_0 = \hat{H}_0, \tag{13a}$$

$$\overline{H}_1 = \hat{H}_1 + L_{10}\hat{H}_0, \tag{13b}$$

$$\overline{H}_2 = \hat{H}_2 + L_{10}\hat{H}_1 + (L_{20} + L_{10}^2 + 2L_{11})\hat{H}_0/2. \tag{13c}$$

At each order, we demand that $\overline{H}_n$ be θ-independent and the generating functions contain no θ-independent part to avoid secularity in θ. The averaged Hamiltonian up to second order in ε then becomes

$$\begin{aligned}\overline{H} = \overline{\mu}B + \overline{p_z}^2/2 - \overline{W} + \varepsilon e(\langle\varphi\rangle - \overline{p_z}\langle A_z\rangle) + \varepsilon^2 e^2\langle A_z^2\rangle/2 \\ -(\varepsilon^2 e^2/2\Omega)[\partial/\partial\overline{\mu}\langle(\tilde{\varphi} - \overline{p_z}\tilde{A}_z)^2\rangle \\ +(1/\Omega)\langle\overline{\nabla}(\tilde{\Phi} - \overline{p_z}\tilde{\alpha}_z)\cdot b\times\overline{\nabla}(\tilde{\varphi}-\overline{p_z}\tilde{A}_z)\rangle],\end{aligned} \tag{14}$$

where $\langle\varphi\rangle \equiv 1/2\pi \oint d\overline{\theta}\ \varphi(\overline{X}+\overline{\rho},t)$, $\tilde{\varphi} \equiv \varphi - \langle\varphi\rangle$, $\tilde{\Phi} \equiv \int^{\theta}\tilde{\varphi}\ d\theta$, $\tilde{\alpha}_z \equiv \int^{\theta}\tilde{A}_z d\theta$, $\overline{\rho} = (2\overline{\mu}/\Omega)^{1/2}\overline{a(\theta)}$, and the superscript bar denotes the transformed variables. The generating function G_1 is

$$\partial G_1/\partial\theta = -(e/\Omega)(\tilde{\varphi} - p_z\tilde{A}_z). \tag{15}$$

whereas the formula for G_2 is not of interest at this order. The explicit relationship between the transformed variables and the original variables is not usually needed for practical purposes. This is because the functional form of the Poisson's tensor remains the same under the Lie transformation. From Eq.(14), the equations of motion are

$$d\overline{X}/dt = \{\overline{X},\overline{H}\} = \{\overline{X},\overline{X}\}\cdot\overline{\nabla H} + \{\overline{X},\overline{p_z}\}\partial\overline{H}/\partial\overline{p_z} = b\times I/\Omega\cdot\overline{\nabla H} + b\partial\overline{H}/\partial\overline{p_z}$$

$$= e/\Omega\, b\times\overline{\nabla}\Psi + b\,\overline{u_z}, \tag{16a}$$

$$d\overline{p_z}/dt = \{\overline{p_z},\overline{H}\} = \{\overline{p_z},\overline{X}\}\cdot\overline{\nabla H} = -b\cdot\overline{\nabla H} = -eb\cdot\overline{\nabla}\Psi, \tag{16b}$$

and

$$d\overline{\mu}/dt = 0, \tag{16c}$$

where the renormalized effective potential is

$$\Psi\equiv\langle\varphi\rangle-\overline{p_z}\langle A_z\rangle+e\langle A_z{}^2\rangle/2-(e/2\Omega)[\partial/\partial\overline{\mu}\langle(\tilde{\varphi}\ -\overline{p_z}\tilde{A}_z)^2\rangle+1/\Omega\langle\overline{\nabla}(\tilde{\Phi}-$$

$$\overline{p_z}\tilde{\alpha}_z)\cdot b\times\overline{\nabla}(\tilde{\varphi}-\overline{p_z}\tilde{A}_z)\rangle] \tag{16d}$$

and the effective parallel velocity is

$$\overline{u_z}\equiv\partial\overline{H}/\partial\overline{p_z}=\overline{p_z}-e\langle A_z\rangle-(e^2/2\Omega)\partial/\partial\overline{p_z}[\partial/\partial\overline{\mu}\langle(\tilde{\varphi}-\overline{p_z}\tilde{A}_z)^2\rangle$$

$$+(1/\Omega)\langle\overline{\nabla}(\tilde{\Phi}-\overline{p_z}\tilde{\alpha}_z)\cdot b\times\overline{\nabla}(\tilde{\varphi}-\overline{p_z}\tilde{A}_z)\rangle]. \tag{16e}$$

As a result of the Lie perturbation theory, the effective potential becomes rather complicated in the new coordinates at the expense of the removal of the gyrophase dependence while preserving the Hamiltonian symmetry.

III. The Gyrokinetic Vlasov-Poisson-Ampere System

In this section we derive a reduced Vlasov equation for the ion distribution function F_i in the gyrophase-averaged coordinates, using the

Hamiltonian derived in the previous section. The reduced electron Vlasov equation can be obtained by taking the drift kinetic limit of Eq.(16). Although there exist some cases where a gyrokinetic description is also necessary for the electrons[25] ($k_\perp \rho_e \sim 1$), we can gain more physical insights by considering the case where two species are governed by two different equations. To recover the gyrokinetic description for the electrons is rather straightforward. Self-consistency relations (Poisson's equation and Ampere's law) are then presented in terms of F_i and f_e. Although their derivation is straightforward, the enforcement of self-consistency is quite important and sometimes leads to a conclusion which is drastically different from that of a non-self-consistent theory. From this section on, we restore the proper dimensions by writing m and c explicitly. The Vlasov-Poisson-Ampere system consists of

$$\{f_i, \hat{H}(Z)\} = 0, \tag{17}$$

$$\nabla^2 \varphi(x,t) = -4\pi e\{\int f_i(Z)\delta(X-x+\rho)d^6Z - \int f_e d^3v\}, \tag{18}$$

$$\begin{aligned}\nabla_\perp{}^2 A_z(x,t) &= -4\pi e\{\int(p_z-(e/m_ic)A_z)f_i(Z)\delta(X-x+\rho)d^6Z \\ &\quad -\int(p_z+(e/m_ec)A_z)f_e d^3v\} \\ &= (\omega_{pe}/c)^2 A_z - 4\pi e\{\int p_z f_i(Z)\delta(X-x+\rho)d^6Z - \int p_z f_e d^3v\},\end{aligned} \tag{19}$$

where $d^6Z \equiv \|\partial z/\partial Z\| d^3X d\mu dp_z d\theta$, p_z is the momentum per unit mass, and $m_e/M_i \ll 1$ has been used.

The collisionless skin depth appears explicitly in Ampere's law because we use p_z as an independent variable. Now, we apply the averaging transformation T^{-1} in Eq. (17) to get[8]

$$\{F_i, \overline{H_i}\} = 0, \tag{20}$$

where $F_i = T^{-1} f_i$, and we have used the fact that the form of the Poisson tensor remains the same under T^{-1}. Equation (20) indicates that F_i is a function of the integrals of motion of H and hence θ-independent. In terms of the gyrophase-averaged variables, Poisson equation and Ampere's law become

$$\nabla^2\varphi(x,t) = -4\pi e\{\int TF_i(\overline{Z})\delta(\overline{X}-x+\overline{\rho})d^6\overline{Z} - n_e\} \tag{21}$$

and

$$(\nabla_\perp{}^2-(\omega_{pe}/c)^2)A_z(x,t) = -4\pi e\{\int\overline{p_z}TF_i(\overline{Z})\delta(\overline{X}-x+\overline{\rho})d^6\overline{Z} - \int p_z f_e d^3v\}, \tag{22}$$

respectively.

Equations (20-22) constitute the gyrokinetic Vlasov-Poisson-Ampere system. From Eq.(16), the explicit form of the gyrokinetic Vlasov equation, accurate to $O(\varepsilon^3)$, becomes

$$\partial F_i/\partial t + \overline{u_e}b\cdot\overline{\nabla}F_i - (e/\Omega_i m_i)\overline{\nabla}\Psi\times b\cdot\overline{\nabla}F_i - eb\cdot\overline{\nabla}\Psi\partial F_i/\partial\overline{p_z} = 0, \tag{23}$$

where $d\overline{\mu}/dt = 0$ and $\partial F/\partial\overline{\theta} = 0$ have been used. Up to $O(\varepsilon^2)$, the corresponding Poisson's equation and Ampere's law are, respectively,

$$\nabla^2\varphi(x,t)=-4\pi e[\int(F_i(\overline{Z})+(\varepsilon e/\Omega)\{(\tilde{\varphi}-\overline{p_z}\tilde{A}_z/c)\partial F/\partial\overline{\mu}+(1/\Omega)\overline{\nabla}(\tilde{\Phi}-\overline{p_z}\tilde{\alpha}_z/c)\cdot b$$
$$\times\overline{\nabla}F_i\})\delta(\overline{X}-x+\overline{\rho})d^6\overline{Z} - n_e], \tag{24}$$

and

$$(\nabla_\perp{}^2-(\omega_{pe}/c)^2)A_z(x,t)= -4\pi e\{\int\overline{p_z}\,(\,F_i(\overline{Z}) +(\varepsilon e/\Omega_i)\{(\tilde{\varphi}-\overline{p_z}\tilde{A}_z/c)\partial F_i/\partial\overline{\mu}$$
$$+(1/\Omega)\overline{\nabla}(\Phi-\overline{p_z}\tilde{\alpha}_z/c)\cdot b\times\overline{\nabla}F_i\})\,\delta(\overline{X}-x+\overline{\rho})d^6\overline{Z} - \int p_z f_e d^3v\}. \tag{25}$$

The reason we are keeping $O(\varepsilon^2)$ terms only in Eqs.(24-25) is related to energy conservation. We will discuss this later. For completeness, we also write the electron drift kinetic equation,

$$\partial f_e/\partial t + u_e b\cdot\nabla f_e - c/B_0 \nabla\Psi_e \times b\cdot\nabla f_e + e b\cdot\nabla\Psi_e \partial f_e/\partial p_z = 0, \qquad (26)$$

where $\Psi_e \equiv \langle\varphi\rangle - p_z\langle A_z\rangle + e\langle A_z^2\rangle$, and $u_e \equiv \partial H/\partial p_z = p_z + e\langle A_z\rangle$. Equations (23-26) constitute a closed set of equations describing low frequency electromagnetic (loosely termed as drift-Alfvén) plasma fluctuations. Here, we remark that the acceleration term containing the induction electric field $\partial A_z/\partial t\cdot\partial f/\partial v_z$ is hidden in the first term of Eq.(26), i.e.,

$$\partial/\partial t\,|_p f = \partial/\partial t\,|_v f + (e/c)\partial A_z/\partial t\cdot\partial f/\partial v_z.$$

Similarly, the electrostatic field along the perturbed magnetic field line is contained in the third term of Eq. (26).

The energy conservation is important not only as one of the basic properties of a Hamiltonian system, but also as a stringent test of the computational scheme. The total energy of the Vlasov-Poisson-Ampere system expressed in terms of the original particle coordinates is

$$E = (1/2)\int m_i v^2 f_i d^6z + (1/2)\int m_e v^2 f_e d^6z + (1/8\pi)\int |E|^2 + |B|^2 d^3x. \qquad (27)$$

Using the averaging transformation T, the ion kinetic energy can be written in terms of F_i as

$$E_i \equiv (1/2)\int m_i v^2 f_i d^6z = \int m_i(\mu B + (p_z - (e/m_i c)\,A_z)^2/2) T F_i d^6Z. \qquad (28a)$$

Upon integrating by parts, we can write it as

$$E_i = \int F_i(Z) T^{-1} m_i(\mu B + (p_z - (e/m_i c)\,A_z)^2/2) d^6Z. \qquad (28b)$$

For the electron kinetic energy, we need to consider only the contribution from the parallel motion, since electron dynamics are governed by the drift-kinetic equation. For the magnetic field energy, we neglect the equilibrium part which is constant. Thus, the total energy becomes

$$E = \int F_i(Z)\, T^{-1} m_i(\mu B + (p_z - (e/m_i c) A_z)^2/2)\, d^6Z$$

$$+ (1/2) \int f_e m_e (p_z + (e/m_e c) A_z)^2 d^6 z$$
$$+ (1/8\pi) \int |\delta E|^2 + |\delta B|^2 d^3 x = \text{const}, \qquad (29)$$

or performing T^{-1} explicitly,

$$E = \int F_i m_i (p_z^2/2 - (e/m_i c) p_z \langle A_z \rangle + (e/m_i c)^2 \langle A_z^2 \rangle /2 + \mu B) d^6 z$$
$$+ (1/2) \int f_e m_e (p_z + (e/m_e c) A_z)^2 d^6 z + (1/8\pi) \int |\delta E|^2 + |\delta B|^2 d^3 x$$
$$+ (e^2/2\Omega_i) \int [\partial/\partial\mu \langle \tilde{\phi}^2 - p_z^2 \tilde{A}_z^2/c^2 \rangle$$
$$+ (1/\Omega_i) \langle \nabla\tilde{\Phi}\cdot b \times \nabla\tilde{\phi} - p_z^2 \nabla\tilde{\alpha}_z \cdot b \times \nabla \tilde{A}_z/c^2 \rangle] F_i d^6 Z = \text{const}, \qquad (30)$$

where subscripts for species have been ignored for simplicity. This conservation property can be verified by taking the partial time derivative of Eq. (30) and by using Eqs. (23-26).

IV. Limiting Cases and Applications

In the preceding section, a set of gyrokinetic equations [Eqs. (23-26)] has been derived, in which fast gyrophase dependence has been completely removed. Let us now examine a number of limiting cases of the equations to gain more physical insights into these equations. From the gyrokinetic particle simulation point of view, further simplification of the equations is also desirable.

First, we note that Eqs. (23-26) recover the results of Dubin et al. in the electrostatic limit where $A_z \to 0$. Next, as mentioned before, our equations contain higher-order nonlinear terms which were not kept in the previous publication.[6] Therefore, it is enlightening to discuss relationships between our equations and those of Ref.6, and to examine whether their results can be systematically recovered from our equations. By dropping the nonlinear terms in Ψ and u_z and by linearizing Poisson's equation and Ampere's law, assuming a Maxwellian background distribution function in μ and a linear background density profile, we have

$$\partial F/\partial t + u_z b\cdot\nabla F - c/B_0 \nabla\Psi\times b\cdot\nabla F - eb\cdot\nabla\Psi\partial F/\partial p_z = 0, \tag{31}$$

$$-k^2\varphi = -4\pi e(\hat{N}_i - n_e) + 1/n_0\lambda_{Di}{}^2[n_0(1-\Gamma_0)\varphi - J_{i0}(1-\Gamma_0)A_z + (\Gamma_1-\Gamma_0)i\rho_i{}^2 k_\perp\cdot\{(\nabla_\perp n_0)\varphi - (\nabla_\perp J_{i0})A_z\}], \tag{32}$$

$$-(k_\perp{}^2+(\omega_{pe}/c)^2)A_z = -4\pi e(\hat{J}_i - j_e) - 1/n_0\lambda_{Di}{}^2[J_{i0}(1-\Gamma_0)\varphi - \Pi_0(1-\Gamma_0)A_z + (\Gamma_1-\Gamma_0)i\rho_i{}^2 k_\perp\cdot\{(\nabla_\perp J_{i0})\varphi - (\nabla_\perp\Pi_0)A_z\}], \tag{33}$$

where the Poisson-Ampere equations are Fourier-transformed to the k space (the subscript k for the perturbed quantities is omitted for simplicity), $\Psi = \langle\varphi\rangle - p_z\langle A_z\rangle$, $u_z = p_z - e\langle A_z\rangle/c$, $\Pi_{i0} = \int\overline{p_z}{}^2F_i(\overline{Z})\delta(\overline{X}-x+\overline{\rho})d\overline{Z}$, $\Gamma_n(b)=I_n(b)e^{-b}$, I_n is the modified Bessel function of order n, $b=k_\perp{}^2\rho_i{}^2$, and $\lambda_{Di}{}^{-2} = 4\pi n_0 e^2/T_i$. Here, we emphasize that in the reduced Vlasov equation [Eq. (31)], the $E\times B$ convective nonlinearity and the magnetic nonlinearity (streaming along tilted magnetic field) as well as the velocity space nonlinearity are retained. In this sense, Eq.(31) is essentially the equation of Ref.6 in slab geometry written in different variables (p_z instead of v_z). Since we are dealing with the total distribution function, the following difference still exists between our results and those of Ref.6. Since in Ref. 6 the distribution function is broken up into unperturbed and perturbed parts, the velocity space nonlinearity $b\cdot\nabla\Psi\partial\delta F/\partial v_z$ has been dropped in their formulation, based on their orderings. However, in our equations, the velocity nonlinearity is kept naturally without introducing any complications. Furthermore, our equations conserve energy while some higher order terms should be added to the Frieman-Chen equation to achieve energy conservation. The energy invariance for our system can be written as

$$E = \int F_i(p_z{}^2/2-(e/m_ic)p_z\langle A_z\rangle+(e/m_ic)^2\langle A_z{}^2\rangle/2+\mu B)d^6Z$$
$$+\ (1/2)\int f_e(p_z+(e/m_ec)A_z)^2 d^6z+(1/8\pi)\int |\delta E|^2+|\delta B|^2 d^3x$$
$$+(e^2/2T_i)(2\pi)^{-3}[n_0\int d^3k(1-\Gamma_0)|\varphi_k|^2-(1/c)^2\Pi_{i0}\int d^3k(1-\Gamma_0)|A_{zk}|^2]. \tag{34}$$

Here, we keep the terms proportional to $k_\perp\cdot\{(\nabla_\perp n_0)\varphi_k - (\nabla_\perp J_{i0})A_{zk}\}$ and $k_\perp\cdot\{(\nabla_\perp J_{i0})\varphi_k - (\nabla_\perp\Pi_{i0})A_{zk}\}$ in Eq.(32) and Eq.(33), respectively, in order to conserve energy exactly. These terms are smaller than the other terms by order of ε, and have been neglected by most authors except for those of Ref.8.

Returning to the issue of simplification for particle simulation applications, one problematic property of Eq.(31) comes from the last term of the renormalized potential Ψ. It involves both a derivative in velocity (μ) space and a convolution in k-space, which require some cumbersome and expensive computations. This nonlinear part of the renormalized potential can be greatly simplified in the long wavelength (small $k_\perp\rho_i$) limit. Therefore, we get the following limiting case of the gyrokinetic equations by simplifying only the nonlinear part of the renormalized potential. Using the small-argument expansion of the Bessel functions, it is straightforward to show that from Eq.(16d)

$$\Psi = \langle\varphi\rangle - p_z\langle A_z\rangle/c + e\langle A_z{}^2\rangle/2-(e\rho_i{}^2/2T_i)|\nabla_\perp(\varphi - p_zA_z)|^2. \tag{35}$$

Also, in this limit, we have

$$u_z = p_z - e\langle A_z\rangle/m_ic + (e^2\rho_i^2/T_i)\nabla_\perp A_z\cdot\nabla_\perp(\varphi - p_zA_z),$$

$$= P_z(1 - (\delta B/B)^2) - e\langle A_z\rangle/m_ic + (e^2/\Omega_i^2)(\nabla_\perp A_z\times b)\times\nabla\varphi\cdot b, \quad (36)$$

where the correction factor in the first term of the last line is related to the difference between $\langle A_z^2\rangle$ and $\langle A_z\rangle^2$. The last term represents the $\delta E\times\delta B$ drift in the parallel direction. We can also simplify Poisson's equation and Ampere's law by assuming that F_i is Maxwellian in μ, and making long wavelength approximations only to the nonlinear convolution terms. Then, they become

$$\nabla^2\varphi(x,t) = -4\pi e(N_i - n_e) -(1/n_0\lambda_{Di}^2)\{ n_i(1-\Gamma_0)\varphi - J_i(1-\Gamma_0)A_z$$
$$+\rho_i^2(\nabla_\perp N_i\cdot\nabla_\perp\varphi - \nabla_\perp J_i\cdot\nabla_\perp A_z)\} \quad (37)$$

and

$$(\nabla_\perp^2-(\omega_{pe}/c)^2)A_z(x,t) = -4\pi/c(J_i - j_e)$$
$$- (1/n_0\lambda_{Di}^2)\{J_i(1-\Gamma_0)\varphi-\Pi_i(1-\Gamma_0)A_z+\rho_i^2(\nabla_\perp J_i\cdot\nabla_\perp\varphi - \nabla_\perp\Pi_i\cdot\nabla_\perp A_z)\}, \quad (38)$$

where $J_i = e\int\overline{p_z}F_i(\overline{Z})\delta(\overline{X}-x+\overline{\rho})d\overline{Z}$, $j_e = e\int f_e p_z d^3v$, $N_i = \int F_i(\overline{Z})\delta(\overline{X}-x+\overline{\rho})d\overline{Z}$, $n_i = \int F_i(\overline{Z})\delta(\overline{X}-x)d\overline{Z}$, with the energy integral defined by

$$\mathcal{E}=\int F_i(1/2p_z^2-p_z\langle A_z\rangle+e^2/2\langle A_z^2\rangle+\mu B)d^6Z$$
$$+(1/2)\int f_e(p_z+(e/m_ec)A_z)^2d^6z +(8\pi)^{-1}\int|\delta E|^2+|\delta B|^2d^3x +$$
$$(e^2/2T_i)/(2\pi)^3[n_i\int d^3k(1-\Gamma_0)|\varphi_k|^2 - (1/c)^2\Pi_i\int d^3k(1-\Gamma_0)|A_{zk}|^2]. \quad (39)$$

The above set of equations, which is valid for arbitrary $k_\perp\rho_i$ linearly and for small $k_\perp\rho_i$ nonlinearly, is useful for simulation studies for which a recent numerical scheme of Ref.11 can be used.

Finally, we show that we can recover the reduced MHD (magnetohydrodynamic) equations from our gyrokinetic equations. Although it is possible to derive a variety of generalized fluid-like equations[26-29] which retain finite Larmor radius effects, detailed discussion will be given in a future publication where toroidal geometry is considered and a direct comparison would then be more meaningful. Changing the variable from p_z to v_z, we obtain in the limit of $\mu \rightarrow 0$ and neglecting A_z^2 nonlinear terms,

$$\partial f/\partial t + v_z \mathbf{b}^* \cdot \nabla f - cB_0^{-1} \nabla\varphi \times \mathbf{b} \cdot \nabla f + e/m_i(\mathbf{b}^* \cdot \nabla\varphi + c^{-1}\partial A_z/\partial t)\partial f/\partial v_z = 0, \tag{40a}$$

where $\mathbf{b}^* = \mathbf{b} + \nabla A_z \times \mathbf{b}/B_0$. By taking the density moment of Eq.(40a) and taking the density and parallel velocity moment of the electron drift kinetic equation, we obtain the following equations.

$$dN_i/dt + \mathbf{b}^* \cdot \nabla\Gamma_i = O(\varepsilon^3), \tag{40b}$$

$$dn_e/dt + \mathbf{b}^* \cdot \nabla\Gamma_e = O(\varepsilon^3), \tag{40c}$$

$$m_e d\Gamma_e/dt + \mathbf{b}^* \cdot \nabla P_e - e(\mathbf{b}^* \cdot \nabla\varphi + c^{-1}\partial A_z/\partial t)n_e = O(\varepsilon^3), \tag{40d}$$

where $\Gamma_\sigma = \int f_\sigma v_z d^3v$. Also, Poisson's equation in the quasineutral plasma limit ($\lambda_{De} << \rho_s$) becomes [from Eq. (37)]

$$\hat{N}_i - n_e + e\rho_i^2/T_i\{ \nabla_\perp \cdot n_i \nabla_\perp \varphi \} = 0. \tag{41}$$

On the other hand, for the usual cases of negligible ion contribution to equilibrium parallel current, we can ignore the last term of Eq. (38). Then, the Ampere's law simplifies to

$$\nabla_\perp^2 A_z(\mathbf{x},t) = -4\pi(J_i - j_e)/c. \tag{42}$$

Subtracting Eq. (40b) from Eq. (40c) and using Eqs. (41-42), we get the following vorticity equation,

$$4\pi m_i c B_0^{-2} d/dt\, \nabla_\perp \cdot n_i \nabla_\perp \varphi = -\, b \cdot \nabla \nabla_\perp^2 A_z. \qquad (43)$$

Equation (40d) is the Ohm's law where the first term is the electron inertia and the second term is related to the electron diamagnetic drift. Ignoring these kinetic corrections, we obtain the following induction equation,

$$dA_z/dt = -b \cdot \nabla \varphi \ (\text{ or } c^{-1} \partial A_z/\partial t + b^* \cdot \nabla \varphi = 0). \qquad (44)$$

Equations (43) and (44) form the low-β reduced MHD equations in slab geometry.

V. Conclusions

In the present paper, we have derived a self-consistent and energy-conserving set of gyrophase-averaged nonlinear equations for the Vlasov-Poisson-Ampere system. The new formulation is useful for describing low-frequency electromagnetic fluctuations in finite-β plasmas. The main difference between our equations and those of Ref. 6 is that our equations are formulated in terms of the total distribution function and explicitly phase space preserving. Furthermore, we have kept the formally higher order nonlinearities, such as those associated with parallel acceleration and polarization effects, both of which were ignored in Ref. 6. Another important aspect of our equations is that they can be readily solved by particle simulation techniques, which, over the years, have proven to be the most effective numerical tools for studying plasma instabilities when the kinetic effects, such as wave-particle interactions and finite Larmor radius effects, are important. There exists accumulating evidence that the transport properties in the confinement region (excluding the sawtooth region and the edge) of tokamaks are dominated by this type of high temperature (collisionless) microscopic phenomena.

The advantage of using the gyrokinetic equations for particle simulation instead of the original Vlasov-Poisson-Ampere system is the elimination of high-frequency space charge waves from the simulation plasma. The highest frequency normal modes for the reduced equations can be obtained from the well-known dispersion relation of

$$D \equiv 1 + [1-(\omega/k_{\parallel}v_A)^2][1+\xi_e Z(\xi_e)]/(k_{\perp}\rho_s)^2 = 0, \qquad (45)$$

for a homogeneous plasma in a shearless slab with cold ion response ($\omega << k_{\parallel}v_{ti}$) and $(k_{\perp}\rho_i)^2 << 1$, where $v_A \equiv c\lambda_{De}/\rho_s$ is the Alfvén velocity, λ_{De} is the Debye length, $\xi_e \equiv \omega/\sqrt{2}k_{\parallel}v_{te}$, and Z is the usual plasma function. Thus, for $\beta >> m_e/m_i$ (or $v_{te} >> v_A$), the kinetic shear-Alfvén waves with

$$\omega = \pm k_{\parallel}v_A[1+(k_{\perp}\rho_s)^2]^{1/2} \qquad (46)$$

now represent the oscillations with the highest frequencies in the simulation. As one can see, they are considerably smaller than, for example, the frequencies for plasma waves ω_{pe} and lower hybrid waves ω_{LH}. For the reduced MHD equations, Eqs. (43) and (44), the normal modes are the usual shear-Alfvén waves, $\omega = \pm k_{\parallel}v_A$ with magnitude not much different from that of Eq. (46).

As shown by Refs.11 and 16, the net result for the elimination of space charge waves is the tremendous increase in the time step and grid spacing used in the simulation as well as the substantial reduction of the noise level, which enables us to use considerably less number of simulation particles than we would for conventional particle codes.

Thus, with the availability of the present generation of supercomputers, we can easily simulate not only electromagnetic-type microinstabilities in tokamaks but also those that are basically electrostatic in nature by taking advantage of the numerical properties of finite-β gyrokinetic plasmas. Moreover, since the improvement of the numerical properties comes from the shear-Alfvén waves, which exist in both the gyrokinetic equations and the reduced fluid equations, the simulation of the global MHD modes with a gyrokinetic code utilizing the formulation in this paper is, therefore, a distinct possibility. It will afford us with the unique opportunity to study the kinetic effects on those modes.

Acknowledgments

The authors wish to thank Dr. J. A. Krommes for his helpful suggestions and his critical reading of the manuscript. Useful discussions with Dr. L. Chen, Dr. R. M. Kulsrud, Dr. C. Oberman, and Dr. W. Park are also gratefully acknowledged.

This work was supported by the U. S. Department of Energy under Contract No. DE-AC02-76-CHO-3073.

Appendix

In this appendix we present the results of an alternative formulation of the electromagnetic gyrokinetic equations in terms of v_z rather than p_z. Although the resulting equations are quite cumbersome because of the reasons we have mentioned earlier, they provide a useful comparison to the results presented in the main text. Also, this formulation is probably more familiar to the workers in the particle simulation field where the Darwin model is frequently used. The derivation has been carried out using the action-variational (generalization of Lagrangian approach to the phase space) method.[30,31] The Euler-Lagrange equation (equation of motion) can be derived from the following averaged fundamental 1-form (generalized Lagrangian).

$$\Gamma = \gamma_i dz^i - h dt = \mu d\theta + (e/m_i c)A_0(X)\cdot dX_\perp + (U + (e/m_i c)\alpha_z{}^*)dz$$
$$- (U^2/2 + \mu B + (e/m_i)\Phi^*)dt, \qquad \text{(A1)}$$

where

$$\alpha^*{}_z = \langle A_z\rangle - (e/2m_i\Omega_i)[\partial/\partial\mu\langle(\tilde{\varphi}-(U/c)\tilde{A}_z)\tilde{A}_z\rangle + \langle\nabla(\Phi-(U/c)\tilde{\alpha}_z)\cdot b\times\nabla\tilde{A}_z\rangle/\Omega_i],$$

$$\Phi^* = \langle\varphi\rangle - (e/2m_i\Omega_i)[\partial/\partial\mu\langle(\tilde{\varphi} -(U/c)\tilde{A}_z)\tilde{\varphi}\rangle + \langle\nabla(\Phi - (U/c)\tilde{\alpha}_z)\cdot b\times\nabla\tilde{\varphi}\rangle/\Omega_i]$$
$$+ (1/2)\langle\tilde{A}_z{}^2\rangle.$$

The resulting reduced Vlasov equation up to $o(\varepsilon^2)$ is straightforwardly,

$$\partial f/\partial t + v_z b^*\cdot\nabla f - cB_0{}^{-1}\nabla\Phi^*\times b\cdot\nabla f + e/m_i(b^*\cdot\nabla\Phi^* + c^{-1}\partial\alpha_z{}^*/\partial t)\partial f/\partial U$$
$$= 0, \qquad \text{(A2)}$$

where $v_z = U + (e/m_i)\, \partial\Phi^*/\partial U$.

Since Eq.(A2) is not useful for practical purposes due to the complication of α_z and Φ^*, the following limiting case which is appropriate for the gyrokinetic simulation studies involving the Darwin model is presented. In the following paragraph, we neglect the nonlinear corrections to the renormalized potential which contain $\tilde{A}_z$, while keeping the electrostatic corrections in the long wavelength limit. Even in this limit, all the *formally dominant nonlinearities such as the particle streaming along the tilted magnetic field are kept.* We also note that $<\varphi>$ and $<A_z>$ are still treated on an equal footing. Then, the renormalized potentials simplify and become,

$$\alpha^*_z = <A_z>, \ \Phi^* = <\varphi> - (e\rho_i^2/2T_i)|\nabla_\perp\varphi|^2. \tag{A3}$$

In this limit, the Poisson's equation and Ampere's law for Maxwellian $F_i(\mu)$ become

$$\nabla^2\varphi(x,t) = -4\pi e(N_i - n_e) - (1/n_0\lambda_{Di}^2)[n_i(1-\Gamma_0)\varphi + \rho_i^2(\nabla_\perp n_i\cdot\nabla_\perp\varphi)] \tag{A4}$$

$$\nabla_\perp^2 A_z(x,t) = -4\pi(J_i - j_e)/c - (1/n_0\lambda_{Di}^2)[J_i(1-\Gamma_0)\varphi + \rho_i^2(\nabla_\perp J_i\cdot\nabla_\perp\varphi)], \tag{A5}$$

where $J_i = \int v_z F_i(Z)\delta(X-x+\rho)d^6Z$, $j_e = \int f_e v_z d^3v$. Equations (A2-A5) conserve energy and the corresponding energy invariant is,

$$E = \int F_i(v_z^2/2+\mu B)d^6Z + (1/2)\int f_e v_z^2 d^6z + (8\pi)^{-1}\int |\delta E|^2 + |\delta B|^2 d^3x + (e^2/2T_i)(2\pi)^{-3}(n_i\int d^3k(1-\Gamma_0)|\varphi_k|^2). \tag{A6}$$

References

1. W. M. Tang, Nucl. Fusion 18, 1089 (1978).
2. P. C. Liewer, Nucl. Fusion 25, 543 (1985).
3. D. W. Ross, P. H. Diamond, J. F. Drake, F. L. Hinton, F. W. Perkins, W. M. Tang, R. E. Waltz, and S. J. Zweben, Fusion Research Center Report #295, The University of Texas (1987).
4. P. H. Rutherford and E. A. Frieman, Phys. Fluids 11, 569 (1968).
5. J. B. Taylor and R. J. Hastie, Plasma Phys. 10, 497 (1968).
6. E. A. Frieman and L. Chen, Phys. Fluids 25, 502 (1982).
7. W. W. Lee, Phys. Fluids 26, 556 (1983).
8. D. H. E. Dubin, J. A. Krommes, C. Oberman, and W. W. Lee, Phys. Fluids 26, 3524 (1983).
9. R. G. Littlejohn, J. Math. Phys. 20, 2445 (1979).
10. R. G. Littlejohn, Phys. Fluids 24, 1730 (1981).
11. W. W. Lee, J. Comp. Phys. 72, 243 (1987).
12. J. F. Federici, W. W. Lee, and W. M. Tang, Phys. Fluids 30, 425 (1987).
13. W. W. Lee and W. M. Tang, Phys. Fluids (in press).
14. W. W. Lee and T. S. Hahm, Bull. Am. Phys. Soc. 32, 1889 (1987).
15. R. D. Sydora, W. W. Lee, T. S. Hahm, H. Naitou, J. M. Dawson, and V. K. Decyk, Bull. Am. Phys. Soc. 32, 1889 (1987).
16. W. W. Lee, Bull. Am. Phys. Soc. 31, 1498 (1986).
17. S-C. Yang and D-I. Choi, Phys. Lett. A 108, 25 (1985).
18. W. K. Hagan and E. A. Frieman, Phys. Fluids 28, 2641 (1985).
19. T. S. Hahm, Bull. Am. Phys. Soc. 32, 1889 (1987).
20. H. R. Strauss, Phys. Fluids 20, 1354 (1977).
21. R. B. White and M. S. Chance, Phys. Fluids 27, 2455 (1984).
22. R. G. Littlejohn, Phys. Fluids 27, 976 (1984).
23. C. W. Nielson and H. R. Lewis, *Method of Computational Physics* (Academic Press, New York, 1976), Vol.16, p. 367.

24. V. I. Arnold, *Mathematical Methods of Classical Mechanics* (New York, Springer-Verlag, 1978).

25. Y. C. Lee, J. Q. Dong, P. N. Guzdar, and C. S. Liu, Phys. Fluids 30,1331 (1987).

26. A. Hasegawa and M. Wakatani, Phys. Fluids **26**, 2770 (1983).

27. J. F. Drake and T. M. Antonsen, Phys. Fluids **27**, 898 (1984).

28. T. S. Hahm and L. Chen, Phys. Fluids **29**, 1891 (1986).

29. R. D. Hazeltine, C. T. Hsu, and P. H. Morrison, Phys. Fluids **30**, 3204 (1987).

30. R. G. Littlejohn, J. Math. Phys. **23**, 742 (1982).

31. J. R. Cary and R. G. Littlejohn, Ann. Phys. **151**, 1 (1983).

EXTERNAL DISTRIBUTION IN ADDITION TO UC-20

Dr. Frank J. Paoloni, Univ of Wollongong, AUSTRALIA
Prof. M.H. Brennan, Univ Sydney, AUSTRALIA
Plasma Research Lab., Australian Nat. Univ., AUSTRALIA
Prof. I.R. Jones, Flinders Univ., AUSTRALIA
Prof. F. Cap, Inst Theo Phys, AUSTRIA
Prof. M. Heindler, Institut fur Theoretische Physik, AUSTRIA
M. Goossens, Astronomisch Instituut, BELGIUM
Ecole Royale Militaire, Lab de Phys Plasmas, BELGIUM
Commission-European, Dg-XII Fusion Prog, BELGIUM
Prof. R. Boucique, Laboratorium voor Natuurkunde, BELGIUM
Dr. P.H. Sakanaka, Instituto Fisica, BRAZIL
Instituto De Pesquisas Espaciasi-INPE, BRAZIL
Documents Office, Atomic Energy of Canada Limited, CANADA
Dr. M.P. Bachynski, MPB Technologies, Inc., CANADA
Dr. H.M. Skarsgard, University of Saskatchewan, CANADA
Dr. H. Barnard, University of British Columbia, CANADA
Prof. J. Teichmann, Univ. of Montreal, CANADA
Prof. S.R. Sreenivasan, University of Calgary, CANADA
Prof. Tudor W. Johnston, INRS-Energie, CANADA
Dr. C.R. James, Univ. of Alberta, CANADA
Dr. Peter Lukac, Komenskeho Univ, CZECHOSLOVAKIA
The Librarian, Culham Laboratory, ENGLAND
The Librarian, Rutherford Appleton Laboratory, ENGLAND
Mrs. S.A. Hutchinson, JET Library, ENGLAND
C. Mouttet, Lab. de Physique des Milieux Ionisés, FRANCE
J. Radet, CEN/CADARACHE - Bat 506, FRANCE
Univ. of Ioannina, Library of Physics Dept. GREECE
Dr. Tom Mual, Academy Bibliographic Ser., HONG KONG
Preprint Library, Hungarian Academy of Sciences, HUNGARY
Dr. B. Dasgupta, Saha Inst of Nucl. Phys., INDIA
Dr. P. Kaw, Institute for Plasma Research, INDIA
Dr. Philip Rosenau, Israel Inst. Tech, ISRAEL
Librarian, Int'l Ctr Theo Phys, ITALY
Prof. G. Rostagni, Univ Di Padova, ITALY
Miss Clelia De Palo, Assoc EURATOM-ENEA, ITALY
Biblioteca, Instituto di Fisica del Plasma, ITALY
Dr. H. Yamato, Toshiba Res & Dev, JAPAN
Prof. I. Kawakami, Atomic Energy Res. Institute, JAPAN
Prof. Kyoji Nishikawa, Univ of Hiroshima, JAPAN
Direc. Dept. Large Tokamak Res. JAERI, JAPAN
Prof. Satoshi Itoh, Kyushu University, JAPAN
Research Info Center, Nagoya University, JAPAN
Prof. S. Tanaka, Kyoto University, JAPAN
Library, Kyoto University, JAPAN
Prof. Nobuyuki Inoue, University of Tokyo, JAPAN
S. Mori, JAERI, JAPAN
Librarian, Korea Advanced Energy Res. Institute, KOREA
Prof. D.I. Choi, Adv. Inst Sci & Tech, KOREA
Prof. B.S. Liley, University of Waikato, NEW ZEALAND
Institute of Plasma Physics, PEOPLE'S REPUBLIC OF CHINA
Librarian, Institute of Phys., PEOPLE'S REPUBLIC OF CHINA
Library, Tsing Hua University, PEOPLE'S REPUBLIC OF CHINA
Z. Li, Southwest Inst. Physics, PEOPLE'S REPUBLIC OF CHINA
Prof. J.A.C. Cabral, Inst Superior Tecnico, PORTUGAL
Dr. Octavian Petrus, AL I CUZA University, ROMANIA
Dr. Johan de Villiers, Fusion Studies, AEC, SO AFRICA
Prof. M.A. Hellberg, University of Natal, SO AFRICA
C.I.E.M.A.T., Fusion Div. Library, SPAIN
Dr. Lennart Stenflo, University of UMEA, SWEDEN
Library, Royal Inst Tech, SWEDEN
Prof. Hans Wilhelmson, Chalmers Univ Tech, SWEDEN
Centre Phys des Plasmas, Ecole Polytech Fed, SWITZERLAND
Bibliothewk, Fom-Inst Voor Plasma-Fysica, THE NETHERLANDS
Dr. D.D. Ryutov, Siberian Acad Sci, USSR
Dr. G.A. Eliseev, Kurchatov Institute, USSR
Dr. V.A. Glukhikh, Inst Electrophysical Apparatus, USSR
Dr. V.T. Tolok, Inst. Phys. Tech. USSR
Dr. L.M. Kovrizhnykh, Institute Gen. Physics, USSR
Nuclear Res. Establishment, Julich Ltd., W. GERMANY
Bibliothek, Inst. Fur Plasmaforschung, W. GERMANY
Dr. K. Schindler, Ruhr Universitat Bochum, W. GERMANY
ASDEX Reading Rm, IPP/Max-Planck-Institut fur Plasmaphysik, W. GERMANY
Librarian, Max-Planck Institut, W. GERMANY
Prof. R.K. Janev, Inst Phys, YUGOSLAVIA