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EQUIPMENT AND TECHNIQUES FOR MEASUREMENTS OF THERMAL CONDUCTIVITY,
THERMAL EXPANSION AND ASSOCIATED PROPERTIES ABOVE 1000°C*

by

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INTRODUCTION

Modern engineering technology has always directed its efforts to higher efficiency, higher speed, better materials, and so on. Many of the engineering goals associated with reactors, aerospace, and supersonics depend on successful fabrication and use of both well known and experimental refractory materials. This in turn has led the materials scientist into the region of high temperatures where familiarity with fabrication and testing techniques are necessary for understanding and application of new ideas and materials. Measurements of thermo-physical properties at temperatures in excess of 1000°C and to temperatures which may approach 3000°C are discussed in this work. Some of the material is new and has not previously appeared in the literature, however this paper is largely a review of equipment and methods which have been successfully used in the temperature regime of interest and whose descriptions have previously been published by the author. (1-5)

THERMAL EXPANSION

Past experience in making a variety of mechanical, physical, and thermal measurements to temperatures which approach 3000°C has led to a desire to build a furnace in which any necessary pre-experimental preparations could be performed and completed in a convenient manner before the system was to be taken to the desired temperature. With this in mind the concept of a furnace built in the form

of a box wherein one or more walls could be moved or removed for easy access to the interior was adopted. A typical furnace of this nature is shown in Fig. 1. This particular furnace was primarily designed for use in measurements of linear thermal expansion over the temperature interval 800 - 2700°C, and the window arrangement, heater configuration, etc. are arranged accordingly. The box is hinged so that a single panel is fixed and the five other walls of the box comprise the movable section. It may be seen from the figure that this arrangement completely exposes the heating assembly, sample suspension system, radiation shielding, and any insulation that might be installed. When swung shut, a seal between the two sections of the furnace is effected by a Neoprene ring and the system is ready to be evacuated (or flushed) in preparation for an experiment. While this furnace is designed to operate at or below atmospheric pressure, a modest redesign of the windows, electrodes, and door clamps would permit use of the furnace at pressures above atmospheric. The vacuum system is quite conventional, it consists of a mechanical and a diffusion pump. Without cold trapping, pressures of 10^{-5} Torr are readily attained.

The walls of the furnace are water cooled. In the furnace shown in Fig. 1 coolant passages were fabricated by drilling through the walls in a direction parallel to the long dimension of the furnace. This was done with a drill designed for use in drilling large length-to-diameter ratio holes. Before this type drilling device was made available for this purpose, coolant passages were fabricated by milling coolant slots in a flat plate and silver soldering an appropriately shaped closure

Figure 1. Box furnace designed for high temperature thermal expansion measurements on graphite.

plate in position to form the desired hollow wall geometry. The coolant passages are arranged such that all coolant flow is in series; this has proved to be a more effective method than that in which parallel flow of the coolant is maintained by means of a large pressure drop across the coolant passages. Apparently in the parallel flow systems minute hot spots may develop which can lead to localized two phase flow which in turn has the potential of introducing a flow instability which could result in melting of the wall at the hot spot. In addition, an interlock between the flow and the power would not be sensitive to development of two phase flow in the parallel system, but it would in the series flow system. Thus far, no symptoms of hot spotting have appeared in the series flow design.

For work which is primarily concerned with graphitic materials, the heater components are made of graphite, although replacement of the graphite parts with metal parts is perfectly straightforward. Details of the heater are shown in Fig. 1; the tubular graphite resistance heater is held at the top and at the bottom by a pair of graphite grips. These grips are in the form of cylinders with a slight taper on the inside diameter; a series of saw cuts made parallel to the axis of the cylinder results in a finger-like appearance. Because of this fabrication process these fingers are actually slightly flexible so that when the heater end is forced into the grips and the tapered end is spread to accommodate the heater, a forceful contact over a considerable area is achieved. Because of practical considerations of assembly, the upper and lower grips are of different dimensions, this permits the operator to simply drop the heater tube through the upper grip and force the two ends of the

heater into their respective grips at the same time. The grips themselves are screwed directly on to the portion of the electrode which juts into the furnace body. It is immediately clear that such an arrangement makes for very large conductive heat losses through the grips to the water cooled electrodes (the heater tube itself is within 12.7 mm of the water) and as a result there are large temperature gradients along the axis of the heater. In an effort to flatten the temperature profile of the heater tube, it was machined with thinned down wall sections near where the grips and the heater met (see Fig. 1) so that the higher temperatures in these sections would help counteract the large heat leak at the ends by flattening the axial temperature distribution in the heater. The radiation shield was lined with pyrographite foil in such a manner that the axis of the heater and the direction of maximum thermal conductivity in the foil were parallel. Since the thermal anisotropy of pyrographite is so pronounced, (the ratio of thermal conductivities in the a-b plane to the c axis direction may be as large as 100), practically no temperature gradient can be maintained parallel to the heater (in the radiation shield), that the heater is surrounded by a relatively isothermal environment. This too reduces the axial temperature gradient since the hottest parts of the heater radiates energy at the highest rate. As a test of these ideas, a graphite specimen was suspended in the furnace (Fig. 1) and the temperatures at the ends and at the middle of the specimen were measured using an optical pyrometer; the results are shown in Table I. When it is noted that the specimen length was 10 cm and the length of the heater was 22 cm, it must be concluded that the relatively

TABLE I

<u>Top</u>	<u>Middle</u>	<u>Bottom</u>
1026°C	1030°C	1021°C
1278	1282	1279
1550	1556	1551
1802	1808	1805

No detectable differences at higher temperatures.

uniform heating section in the furnace is surprisingly large. Use of an artificial graphite radiation shield without the pyrographite approximately doubles the measured temperature gradient.

Figure 2 is a detailed isometric sketch of the electrode design. In the main the material used is brass with all insulating members being fabricated from nylon. Coolant passages are arranged so that all entering water must travel to the hot end of the electrode (see arrows on Fig. 2) before being permitted to escape. Water seals are made on the inner circumference of a carefully fitted tube which slips over the end of the electrode; the coolant inlets and outlets are attached to this tube. The tube is held in place with a snap ring; removal of the snap ring permits removal of the tube and immediate access to the coolant channels (for purposes of cleaning, etc.). The nylon insulator is threaded into the furnace body and the electrode slipped into the insulator; a clamp is then used to keep the electrode in place. Vacuum seals are made by means of Neoprene rings which fit into machined grooves (see Fig. 2.). The end of the electrode is threaded to receive the graphite grip; however, the majority of the electrical contact is made on the flat end of the electrode itself. The thread in the grip is made so that the electrode can bottom; after positioning the grip the entire electrode assembly may be rotated so as to insure proper alignment of the upper and lower heater grips and still maintain good electrical contact between the electrode and the grip. It is obvious from Fig. 2 that the structure of the electrode does not make for a particularly sturdy system so it is necessary that a light weight conductor be used in carrying power to the electrode. Bringing a heavy duty conductor to within

Figure 2. Detail of a typical box furnace electrode.

a few centimeters of the electrode and carrying the power for this short distance with a section of water cooled flexible copper tubing meets the requirement that the conductor be light weight and yet carry considerable power with little loss. (For example, with the power supply delivering 1800 A at 16 V, less than 5% of this power is lost in the water cooled conductors).

As the furnace is shown in Fig. 1, with a single graphite radiation shield, it can attain a temperature of 2500°C with a power input of about 30 KVA. Addition of thermal insulation, variations in the cross sectional area and length of the resistance heated element, etc. can result in large differences in power requirements.

The windows were designed for measurement of linear thermal expansion over the operating temperature of the furnace. For this measurement two windows are used; one for the observation along the length of the specimen and measurement of the dilatation, the other for temperature measurement. For length measurements, a slot 9 cm long x 0.95 cm in width was milled in the movable wall in the position shown in Fig. 1. The fused quartz window which was somewhat larger than the stated dimensions made a vacuum seal by seating on a Neoprene ring. The window mount is completely removable and may be replaced by windows in different positions or by windows of different geometries. For thermal expansion measurements, a heater and radiation shield with slots matched to the window dimensions are used. With this arrangement the observer can look directly at the specimen. A small, round window mounted at 90° to the measurement window was employed for measure-

ment of the temperature. This window was matched by small holes in the heater and radiation shield, respectively, and permitted a direct view of the specimen by an optical pyrometer. Because of the geometry of the system the effective spectral emissivity of the sample could be taken to be unity. The prism arrangement shown in Fig. 1 was used purely as a matter of convenience. Dilatation was measured with an optical extensometer by sighting on machined fiducial marks on the specimen, the distance between the fiducial marks was 8.8 cm; the large spacing between the fiducial marks was intended to minimize errors due to edge diffraction and image-shifting over the temperature interval. No variations could be noted when the system was tested at elevated temperature with helium, argon and vacuum. It was of utmost importance in this regard that there be no foreign materials (filters, partly silvered mirrors, etc.) in the optical path; to insure good precision all filtering was done in or behind the micrometer eyepieces. Measurements are readily reproducible to .003 mm which means that for a $\Delta L/L_0$ of about 0.5%, the uncertainty in the value of $\Delta L/L_0$ is about 1% for the size specimen which was used.

THERMAL CONDUCTIVITY

The general equation for flow of heat in an electrically conducting solid is given by

$$\nabla^2 T + A/k = (1/a) (\partial T / \partial t), \quad (1)$$

where T is the temperature; t the time; a the thermal diffusivity; k the thermal conductivity; and A the energy generation rate per unit volume and where it is assumed that k is independent of T . For the steady state $\partial T / \partial t = 0$ and Eq. (1) simplifies to

$$\nabla^2 T + A/k = 0. \quad (2)$$

In the case of a resistance heated right circular cylinder of radius r radiating freely in a vacuum

$$A = V_i / \pi r^2 l. \quad (3)$$

and

$$k = V_i / A r l (T_{\text{axis}} - T_{\text{surface}}). \quad (4)$$

It is seen from the above that a measurement of the temperature gradients and a measurement of V/l in the rod is sufficient to determine k . Optical methods may be employed to get at the temperature gradients, but instrumentation inside

the furnace is required to make a measurement of the voltage drop over the distance 1. In this situation, the ability to expose the innards of the furnace in order to position the necessary voltage probes and measure the appropriate distance is a great convenience, the furnace shown in Fig. 3 has been used to perform this particular measurement. The details of the design and construction of this furnace are quite similar to those already described for the thermal expansion furnace (Figure 1) with the exception that the box is hinged somewhat differently (see Figure 3). This furnace was designed for making measurements of thermal conductivities and associated properties (electrical resistivity, spectral and total emissivity) and as such required voltage taps inside the furnace which could be passed through the walls, as shown, and appropriately instrumented externally. The fixed-wall geometry which was used fulfilled this requirement without sacrificing any of the conveniences associated with the "box" furnace. To perform the experiment, the solid cylindrical specimen was positioned in the furnace as shown, power was obtained from a 50 KVA, three-phase supply which was converted to a direct current source by means of six silicon rectifiers. The sample was resistance heated and the thermal conductivity computed from knowledge of the heat generation within the specimen, the temperatures on the axis and on or near the surface of the specimen, from Eq. (4). In practice the test specimens were about 15 cm long and about 1.25 cm in dia. At the midpoint of the rod a flat-bottomed hole 0.75 mm in dia. was drilled to the center line; a concentric hole 1.50 mm in diameter was drilled half-way to the center line for purposes of pyrometer alignment; these holes were used for measurement of the temperature

Figure 3. High temperature radial heat flow furnace.

at the axis of the specimen, T_{axis} . measurements of T_r were made by sighting the optical pyrometer on a small hole drilled to within a distance of r_o from the specimen axis. The spectral emissivities for these holes were calculated by the method of DeVos.⁽⁶⁾ This is done in order to get the best values of the temperature drop in the specimen, and also to permit a more certain calculation of the surface temperature which may be used for computation of both spectral and total emissivities. A is determined from a voltage-drop (V) measurement over the middle 1.70 cm length (l) of the sample; this is accomplished by using two sharpened voltage probes which make contact at the specimen surface and bringing them out through the water-cooled furnace wall to a recording D.C. voltmeter. The current (i) is measured by inserting a precision, air-cooled shunt in series with the power train and measuring the voltage drop across the shunt. For the system described, power generation calculated from $A = Vi/\pi r_o^2 l = i^2 \rho_R/\pi r_o^2 l$ - assuming that ρ_R (electrical resistance) is constant over the temperature interval T_{axis} to T_i is accurate to a fraction of a per cent.

For dielectrics the test specimen takes the form of a hollow right circular cylinder. In this case the energy is supplied by a resistance heated rod which lies on the axis of the hollow cylinder and is isolated electrically from it. Since the outer wall of the test specimen loses energy to the environment by radiation, a temperature gradient is set up across the wall of the hollow cylinder. Since the energy generated in the interior rod must be known for the determination of the thermal conductivity in the described geometry, the voltage probes must pass not only through the walls of the furnace but also through the walls of the test cylinder

and be electrically isolated from both. Without complete access to the interior of the furnace this is an extremely difficult experiment to set up. This experiment was performed in the furnace shown in Fig. 4. This furnace was constructed in the form of a split box with each half of the box containing five sides. The electrodes, heater element, and voltage probes are all affixed to the same, immovable half of the furnace. This arrangement permits the complete assembly of an experiment and assures that no relative motion can take place during the closure, evacuation, and heating of the furnace (excluding differential thermal expansion effects).

The solution to Eq. (2) for this geometry is

$$k = \left[\frac{V_i}{2\pi\ell (T_{\text{inner}} - T_{\text{outer}})} \right] \left[\frac{\ln r_{\text{outer}}}{\ln r_{\text{inner}}} \right] \quad (5)$$

where inner and outer refer to the test specimen. An even more interesting system is shown in Fig. 5. Here again a measurement of the thermal conductivity along the radius of a hollow walled right circular cylinder was required. There was however an additional requirement that the temperature difference across the wall be controllable or that the temperatures themselves be fixed beforehand. Solution of Eq. (2) for this case of the hollow cylinder yields

$$k = \left[\frac{\Delta (V_i)}{2\pi\ell (T_{\text{inner}} - T_{\text{outer}})} \right] \left[\frac{\ln r_{\text{outer}}}{\ln r_{\text{inner}}} \right] \quad (6)$$

Figure 4. Box furnace radial heat flow thermal conductivity arrangement for a tube.

Figure 5. Setup for measurement of radial heat flow thermal conductivity with internal and external environmental temperature control.

Clearly, for the solution to this problem, two heaters are required; one inside the cylinder along the axis of the test cylinder, and one concentric with and exterior to the test cylinder. The experiment proceeds by setting up a steady state with $T_i = T_o$ in the test specimen. An increase of the power in the amount $\Delta(V_i)$ in the inner heater such that T_i and T_o are set at their prescribed values then permits a determination of k . Getting the source term by means of a voltage drop measurement where the voltage probes must pass through a hot, resistance heated cylinder and through the test cylinder proved to be an experimental problem which required considerable care for the correct solution.

The critical problem area associated with these thermal conductivity measurements is that which is concerned with the measurement of relatively small temperature differences at elevated temperatures. This may be demonstrated rather simply by means of an error analysis. From Eq. (4), the error δ , associated with a determination of k for the symmetric case is given by

$$\delta_k = \pm \sqrt{\left(\frac{\partial k}{\partial V}\right)^2 \delta_v^2 + \left(\frac{\partial k}{\partial i}\right)^2 \delta_i^2 + \left(\frac{\partial k}{\partial (\Delta T)}\right)^2 \delta_{\Delta T}^2} \quad (7)$$

Since the uncertainty in the measurement of ΔT is much greater than that for V and i combined, the above reduces to

$$\delta_k \cong \pm \frac{\partial k}{\partial (\Delta T)} \delta_{\Delta T} \cong \mp \frac{V_i}{4\pi l (\Delta T)^2} \delta_{\Delta T} \cong \mp k \left(\frac{\delta_{\Delta T}}{\Delta T} \right) \quad (8)$$

It is clear from the argument that the ratio $\delta_{\Delta T}/\Delta T$ is all important; for low values of V_i , ΔT will be small and the relative value of $\delta_{\Delta T}/\Delta T$ will be large. For example, using a 1.25 cm diameter polycrystalline graphite specimen, the uncertainty in k due to $\delta_{\Delta T}$ is about 50 per cent of the observed value of k at 950°C, whereas at 1500°C the uncertainty is more like 20 per cent. With increasing temperature, this relative error becomes less important, and even though other indeterminate errors exert an increased influence, the data is quite reliable. The hyperbolic error envelope associated with the effect of $\delta_{\Delta T}$ on the measurement of k does not lend itself to a simple averaging process although a certain amount of success has been achieved in fitting the data to a curve of the form $k = A/T(^{\circ}\text{K}) + B$ by the method of least squares where the values of k are weighted according to the absolute value of ΔT .

The author has made certain efforts to improve on this situation, these were predicated on the premise that separation of the absolute temperature determination from the temperature difference measurement would result in a simplification of the ΔT problem. Specimen temperatures were measured using a Leeds and Northrup model 8622-C optical pyrometer, but the temperature difference measurements were made using a radiation detector as a temperature level indicator. The detector that was used was a commercial unit which utilized a silicon diode in the form of a p-n junction that was situated at the focal plane of an adjustable telescope and appropriately masked so that the response of the system was effectively the response to the radiation emitted from a small area which could be pre-viewed

on the specimen through the objective lens. The photo-voltaic circuit included a resistance connected in series with the p-n junction, and since the current flow in the circuit was directly proportional to the intensity of the incident radiation, the voltage drop across the resistance was a quantitative measure of the radiation intensity and could be related to the temperature level. Voltages were measured using a high impedance digital voltmeter which had a sensitivity of 1 μ volt. The high sensitivity of the detection system was another reason for using a rectified power supply; it was estimated that a specimen at 2500°C which was being resistively heated with alternating current could have a temperature ripple of about one degree Celsius, and since this exceeded the resolution of the detection system, it was deemed necessary to smooth the power input and consequently the temperature ripple using the method already described. The power supply had a ripple of 4% which when translated into a temperature fluctuation is well under one-tenth of a degree Celsius and indeed was not observed. For a given resistance, the voltage drop across the resistance could be related to the temperature by

$$\ln V = A + BT + CT^2 \quad (9)$$

from which the sensitivity is determined by

$$\frac{dV}{dT} = (B + 2CT) V \quad (10)$$

The temperature T must be obtained using an optical pyrometer, and V is the voltage level at the point of the temperature level determination. Some typical values are shown in the table.

Temperature	Sensitivity
840°C	.066 mv/°C
1500°C	.024
2000°C	.013

The radiance level at the detector (and thus the sensitivity) is relatively low since the detection equipment subtends rather small angles; a 0.15 cm hole was normally viewed from a distance of about 15-20 cm. Actual temperature difference measurements were made by sighting the detector first on one sight hole and taking a reading, then moving to the second and repeating the observation. This was usually repeated three times and an average taken if all readings were consistent. Inconsistency in the determinations made in this manner normally indicated that the steady-state had not been attained and the specimen was undergoing dynamic change. The absolute value of the temperature was determined using an optical pyrometer. With most materials this could be done by sighting on the surface using the second window in the furnace (see Figure 3) which was a convenience since the radiation detector did not have to be moved. Use of materials where for one reason or another there was an uncertainty in the value of the spectral emissivity necessitated utilization of the black-body hole in the specimen.

OTHER MEASUREMENTS

From a knowledge of the current and voltage relationship, specimen dimensions and thermal expansion characteristics, the electrical resistivity may be obtained as a function of temperature since for a resistance heated right circular cylinder

$$\rho = \frac{RA}{\ell} = \frac{V}{i} \left(\frac{\pi}{4} \right) \frac{d^2}{\ell} \quad (11)$$

$$d = d_o (1 + \alpha (T - T_o)) \quad (12)$$

$$\rho = \frac{V\pi}{4i\ell} d_o^2 (1 + \alpha (T - T_o))^2 \quad (13)$$

where d is the specimen diameter at T and α is the linear coefficient of thermal expansion along the specimen diameter. It is clear that this is not the whole story since radial temperature gradients exist in the specimen. The relationship between the isothermal value of the resistivity and the value which is measured may be estimated as follows: the temperature dependence of ρ may be written as

$$\rho = \rho_o (1 + aT + bT^2) \quad (14)$$

which is quite good for small changes in T and all but very unusual or discontinuous

functional dependences of ρ on T are very closely approximated. The average value of ρ in a temperature gradient is given by

$$\bar{\rho} = \frac{\int_{T_1}^{T_2} \rho dT}{\int_{T_1}^{T_2} dT} = \frac{\int_{T_1}^{T_2} \rho_o (1 + aT + bT^2) dT}{\int_{T_1}^{T_2} dT} \quad (15)$$

and

$$\bar{\rho} = \rho_o \left[1 + \frac{a}{2} (T_2 + T_1) + \frac{b}{3} \left(\frac{T_2^3 - T_1^3}{T_2 - T_1} \right) \right] \quad (16)$$

For $a \approx 10^{-6}$ and $b \approx 10^{-10}$ if $(T_2 - T_1) = 100^\circ\text{C}$, (which is an extreme case) then at 2000°C , $\rho = .994 \bar{\rho}$.

Measurements of both spectral and total hemispherical emittance using either a solid rod or the Worthing tube technique. In this latter method, a thin walled tube is resistance heated and the brightness temperatures of the inside of the tube (measured through a small sight hole) and the outer surface of the tube are measured using an optical pyrometer. The interior of the tube is considered to be blackbody so that the brightness temperature corresponds to the true temperature. The outer surface of the tube radiates to a cooler environment so the brightness

temperature is in error due to the emittance characteristics of the surface. Optical pyrometers use filters with transmittance maxima at about 6500 Å, therefore the ultimate result of the error correction at the surface is a calculation of the spectral emittance of the surface for the wavelength used by the pyrometer. By using a tube with a length-to-diameter ratio of 20 or more, and by considering only a short length of the tube interior for the blackbody, good accuracy may be attained. Removal of the radiation shields and any other insulating materials from the interior of the furnace and wiping the walls with a thin layer of a light diffusion pump oil followed by a "dusting" of the walls with a fine carbon black effectively allows the outer surface of the tube to radiate to a nonreflecting surface of essentially zero temperature. If the thermal conductivity of the tube material is known, measurement of the energy generation rate by methods already described permits a calculation of the temperature drop across the wall of the test specimen. The difference between the internal temperature and the temperature drop so calculated is then the true temperature of the outer surface. The spectral emissivity is given by

$$\ln \epsilon_s = (C_2/\lambda) [(1/T) - (1/T_{\text{obs}})], \quad (17)$$

where $C_2 = 1.4388 \text{ cm } (^{\circ}\text{K})$, T is the true temperature, T_{obs} the observed temperature, λ the wavelength passed by pyrometer filter, and ϵ_s the spectral emittance for λ .

The total hemispherical emissivity is defined as being the ratio of the total energy radiated divided by the energy which would have been radiated by a black-body under identical conditions. For a resistance heated cylindrical body which is radiating to a nonreflecting environment at 0°K, this ratio is the power dissipation per unit area divided by the energy loss for the blackbody at the same temperature as calculated by the Stefan-Boltzmann law,

$$\epsilon = V_i / 2\pi r l \sigma T^4, \quad (18)$$

where ϵ is the total hemispherical emittance. The techniques already described for simulating a nonreflecting low temperature environment and for measuring power generation at elevated temperatures are also used for measurements of total hemispherical emittance.

SPECIAL PROBLEMS

Materials with anisotropic thermophysical properties (e.g. graphite, BN, etc.) give rise to special problems. The source of the difficulty is illustrated by the schematic in Figure 6 in which it is seen that preparation of a cylindrical specimen for radial heat flow measurements from graphite where the cylinder axis is parallel to the average basal plane results in a sample in which there exists a crystallographic asymmetry. Since the thermal conductivities parallel and perpendicular to the average basal plane are different the employment of the radial heat flow technique gives rise to a two-dimensional heat flow system.

Figure 6. Thermal conductivity specimens prepared from an extruded graphite showing the anisotropy with respect to the average basal plane.

In this instance, the temperature drops parallel and perpendicular to the average basal plane are different and the radiation boundary condition which is implied in the solution of Eq. (2)

$$-k \left(\frac{\partial T}{\partial r} \right)_{r=R} = \epsilon \sigma T^4 \quad (19)$$

is not met. The exact solution for the heat flow equation which corresponds to this physical situation has not been found. The differential equation for these conditions is:

$$k_1 \frac{\partial^2 T}{\partial X^2} + k_2 \frac{\partial^2 T}{\partial Z^2} = -A \quad (20)$$

where k_1 and k_2 are thermal conductivities in the X and Z directions respectively. It is assumed $\frac{\partial T}{\partial Y} = 0$ and that the k 's are constant with temperature for temperature differences existing along the rod radius. Since the surface temperature (T_3) and total emissivity will vary with an angular position, if $X^2 + Z^2 = R^2$, and the angular dependence of the emissivity is taken to be

$$\epsilon(\theta) = \epsilon_1 \cos^2 \theta + \epsilon_2 \sin^2 \theta \quad \text{where } \theta = \tan^{-1}(Z/X) \quad (21)$$

then the flux at the surface of the rod is:

$$-\sigma \epsilon(\theta) [T_S^4 - T_E^4] = \left[k_1 \cos^2 \theta + k_2 \sin^2 \theta \right] \frac{\partial T}{\partial R} - \left[(k_1 - k_2) \frac{\sin \theta \cos \theta}{R} \right] \frac{\partial T}{\partial \theta} \quad (22)$$

where T_E = temperature of the environment. If k_1 is known for a set of measurements of temperatures in the direction of k_2 (i. e. $\theta = 0$), and if ϵ_1 and Q are also known, assuming the stated radiation boundary condition is met, k_2 and ϵ_2 may be computed by the appropriate iterative techniques. The total hemispherical emissivities which were needed for data inputs to the two-dimensional heat flow computer program can be obtained from the isotropic heat flow experiments by Eq. (18).

In the situation where the radial and axial components of the conductivity are different (see Figure 6), it might be necessary to make axial as well as radial heat flow measurements. This could be the case for an extruded rod made of graphite where the extrusion diameter was too small to fabricate a radial heat flow type specimen. It is, of course, possible to make the measurements simultaneously however if the two problems are approached separately, it is felt that the final results are likely to be not only simpler to obtain but also more reliable. The radial component of k may be determined from Eq. (4) by methods already described. The axial component of the thermal conductivity is best obtained by the method of Jain and Krishnan⁽⁷⁾ wherein the rod is resistance heated in vacuo and allowed to radiate freely thereby setting up an axial temperature gradient from which the thermal conductivity may be deduced. The experimental arrangement for this measurement is shown in Figure 7. The differential equation for this geometry is

$$\frac{d^2 T}{dx^2} - \frac{p\epsilon\sigma}{k\omega} (T^4 - T_E^4) + \frac{i^2 \rho}{k\omega^2} = 0. \quad (23)$$

Figure 7. Schematic of the linear heat flow experiment using the method of Jain and Krishnan.

(This assumes no temperature dependence of k , ρ , and ϵ and neglects the effects of the radial temperature gradient.)

Here p = rod perimeter, cm.

ω = rod cross-section, cm^2 .

$i^2 \rho$ = energy term, watts-cm.

For a rod of infinite length, Eq. (23) becomes

$$\frac{p\epsilon\sigma}{k\omega} (T_{\infty}^4 - T_E^4) = \frac{i^2 \rho}{k\omega^2} \quad (24)$$

which also defines T_{∞} . Adding Eqs.(23) and (24) and rearranging

$$\frac{d^2 T}{dX^2} + \frac{p\epsilon\sigma}{k\omega} \{ (T_{\infty}^4 - T_L^4) + (T_L^4 - T^4) \} = 0 \quad (25)$$

where T_L = measured temperature at the midpoint of the rod whose length is $2L$. If the rod is short, the temperature depression at $X = L$ as compared with an infinite rod will be large and the $(T_L^4 - T^4)$ term may be neglected. The remaining equation integrates directly into

$$k = \frac{p\epsilon\sigma}{2\omega} q^2 \left\{ \frac{T_{\infty}^4 - T_L^4}{T_L - T} \right\} \quad (26)$$

where $q = L - X$. When $(T_{\infty}^4 - T_L^4)$ is small, the following approximation may be used.

$$(T_{\infty}^4 - T^4) \approx 4 T_{\infty}^3 (T_{\infty} - T) . \quad (27)$$

Once again, adding Eqs. (23) and (24) and substituting (27), the differential equation becomes

$$\frac{d^2 T}{dX^2} + \frac{4 T_{\infty}^3 p \epsilon \sigma}{k \omega} (T_{\infty} - T) = 0 . \quad (28)$$

Again, this is a standard form whose solution is

$$\mu X = \beta - \ln (T_{\infty} - T) \quad (29)$$

where

$$k = \frac{4 p \epsilon \sigma T_{\infty}^3}{\mu^2 \omega} \quad (30)$$

and it is obvious for this case that a plot of $\ln (T_{\infty} - T)$ vs. X yields a slope whose value depends on k . Values for ϵ and for ρ may be determined during the course of the radial heat flow experiments. Since no two specimens are identical, making the radial and axial heat flow measurements on the same test sample is a most desirable manner in which to perform these experiments.

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