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# Current Control by a Homopolar Machine with Moving Brushes

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# CURRENT CONTROL BY A HOMOPOLAR MACHINE WITH MOVING BRUSHES

by

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## ABSTRACT

The equation for TNS Doublet's E-coil circuit with moving brush homopolar machine is integrated in the flux of the homopolar for a monotonically increasing current function extending beyond the current reversal into the burn period. The results show that the moving brush feature is not useful for controlling the burn.

## I. INTRODUCTION

The differential equation for the series LCR circuit with variable brush homopolar machine is solved in LA-7053-MS (January 1978) for the current when the flux function describing the variable position of the brushes is given. In this report we try to answer the question whether there is any continuous flux function that can provide for a monotonically increasing current over the period that includes current reversal and burn.

## II. DEFINITION OF THE PROBLEM

We write Eq. (5) from Paper I of LA-7053-MS, which is the circuit equation, second order in the current i and first order in the flux $\phi$,

$$L\ddot{i} + \left(R - L\frac{\dot{\phi}}{\phi}\right)\dot{i} + \left(\frac{\phi^2}{(2\pi)^2 J} - R\frac{\dot{\phi}}{\phi}\right) i = 0 . \tag{1}$$

We define the monotonically rising current by

$$i = I \cos\left\{\zeta t \exp\left[-(t/T)^2\right] + \alpha\right\} + I_{bias} . \tag{2}$$

The argument function $\zeta t \exp[-(t/T)^2]$ may be rewritten as $T\zeta x \exp(-x^2)$ where $x \equiv t/T$. The function $x \exp(-x^2)$ is plotted in Fig. 1a and $\cos[x \exp(-x^2)]$ is plotted in Fig. 1b. The maximum of $x \exp(-x^2)$ is 0.428882 for $x = 0.707$.

Conservation of energy requires that $I$ and $I_{bias}$ be chosen so the current $i = I_o$ at $t = 0$ and $i = I \cos \pi + I_{bias}$ be related with $R\int i^2 dt$. For this purpose we introduce the factor $\lambda$ so $I_o \leq i \leq -\lambda I_o$ where $I_o = -3$ MA and $-\lambda I_o$ yields

$$\lambda^2 = 1 - \frac{2RT}{L}\int_{x=0}^{0.707} \cos^2(\zeta T x e^{-x^2} + \alpha)\, dx \quad . \tag{3}$$

We can thus write Eq. (2) in the two limits $x = 0$ and $x = 0.707$, i.e.,

$$kI_o \cos \alpha + I_{bias} = I_o \qquad \text{for } x = 0$$

$$-kI_o \qquad + I_{bias} = -\lambda I_o \qquad \text{for } x = 0.707 .$$

Hence, we get

$$k = \frac{1+\lambda}{1+\cos \alpha} \tag{4}$$

$$I_{bias} = \frac{1-\lambda \cos \alpha}{1+ \cos \alpha} I_o , \tag{5}$$

where we have substituted for the current amplitude in Eq. (2) the identity

$$I \equiv kI_o \quad . \tag{6}$$

It is easy to see that the time constant $T$ is related with the time $t = t_m$ at which the maximum current corresponding to $x = 0.707$ is reached. This relationship is given by

$$T = t_m/0.707 \quad . \tag{7}$$

Similarly, we obtain the defining expression for $\zeta$ from the current when its argument function equals $\pi$, i.e.,

$$\zeta = (\pi-\alpha)/0.428882\, T . \tag{8}$$

Equations (2) through (8) define a consistent set in which $I_o$, $t_m$, $\alpha$, $R$, and $L$ are free variables. We integrate Eq. (1) numerically where the following two variables have been fixed to agree with the TNS Doublet circuit, i.e., $I_o = -3$ MA; $L = 86.1$ μH.

## III. RESULTS

The integral of Eq. (1) is plotted in Figs. 2 through 5, where in each figure a current plot, a plot for the voltage on the E-coil and homopolar, and a flux plot are shown. The initial flux $\phi_o$ corresponds to the voltage $V = 419$ V and the angular velocity $\omega$ at the full energy of 387 MJ.

Figures 2 and 3 are the plots for $t_m = 10$ s and phase angles $\alpha = 15°$, 30°, 45°, and 90°. Figure 2 is for $R \approx 0$ and Fig. 3 for $R = 6.1$ μΩ. The $R \approx 0$ plots have no practical significance and are shown as an indication of the significance of the circuit resistance in these calculations. In all the plots, the curves are discontinued at the argument for which the flux function becomes discontinuous, which occurs between t = 3 s and t = 6 s. It should be noted that the flux should not exceed the value $\phi_o$ in a practical machine.

Figures 4 and 5 are the plots for small $\alpha$ values, where Fig. 4 is plotted again for $R \approx 0$ and Fig. 5 for $R = 6.1\ \mu\Omega$. The reference times $t_m$ are 10 s and 25 s in Fig. 4. It can be seen that for the degenerated case of $R \approx 0$ and $\alpha \approx 0$, control over relatively long time intervals is theoretically feasible. It should be noted that in Fig. 5 the minimum angle $\alpha$ for which a solution may be obtained is approximately 12.5°. At angles $\alpha \lesssim 12.5°$, the initial flux derivative $|\dot{\phi}\ (t=0)|$ becomes very large. Figures 4 and 5 demonstrate the great significance of R in rendering control by means of the moving brush feature impossible during the burn period.

The peculiar behavior of the homopolar machine exhibited by the flux functions in Figs. 2b through 5b is explained by the equivalent circuit elements in Eq. (1), i.e.,

$$L_{equiv} = L \tag{9}$$

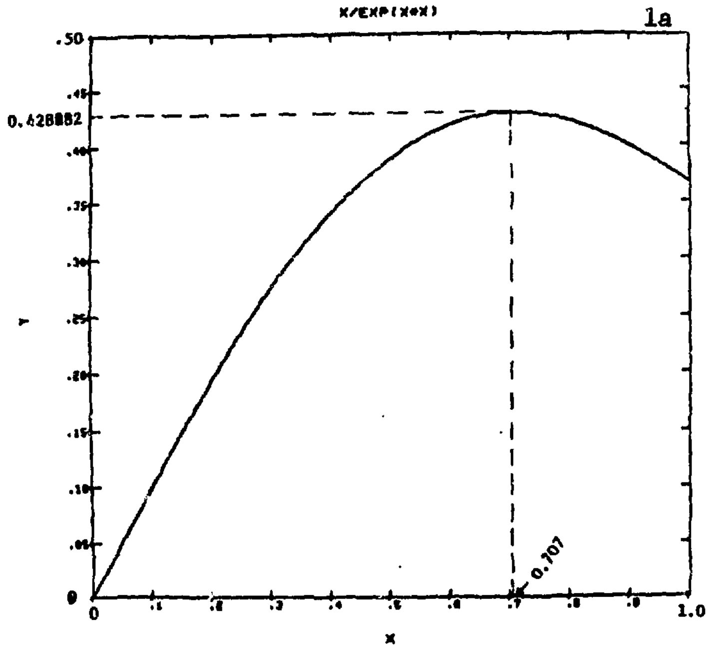
$$R_{equiv} = R - L\,\frac{\dot{\phi}}{\phi} \tag{10}$$

$$1/C_{equiv} = \frac{\phi^2}{(2\pi)^2 J} - R\,\frac{\dot{\phi}}{\phi}$$

or

$$C_{equiv} = \frac{C}{1 - RC\,\frac{\dot{\phi}}{\phi}} \quad . \tag{11}$$

The flux changes during the first fraction of a second to a steady value that corresponds to the natural frequency of the circuit. This change is finite because $\phi \to 0$ is a feasible operating condition of the machine. The change from $\phi_o$ to the steady $\phi$-value is less significant for the largest $\alpha$ values plotted because larger $\alpha$ values imply greater $\dot{i}\ (t=0)$. Because the frequency of the LC-oscillation must subsequently decrease for the given current function, $C_{equiv}$ must increase, which requires $\dot{\phi}$ to become positive in Eq. (11). The implication from $\dot{\phi} > 0$ is that $\phi$ increases, whereas smaller values would be required for a new steady state in $\phi$. Hence, $\dot{\phi}$ has basically the wrong sign and a steady state can never be reached again, once $\dot{\phi} > 0$ because it could be reached only by way of $\phi = \infty$. The moving brush feature is, therefore, not useful for controlling the burn period.



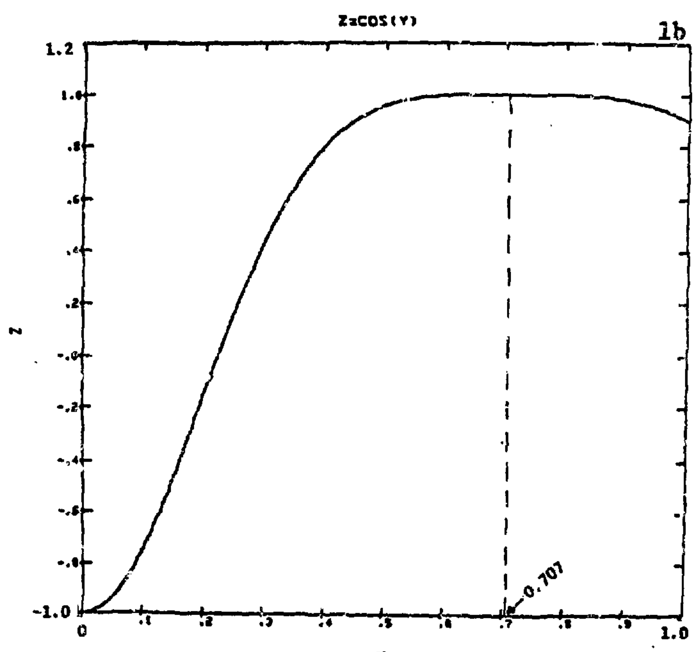
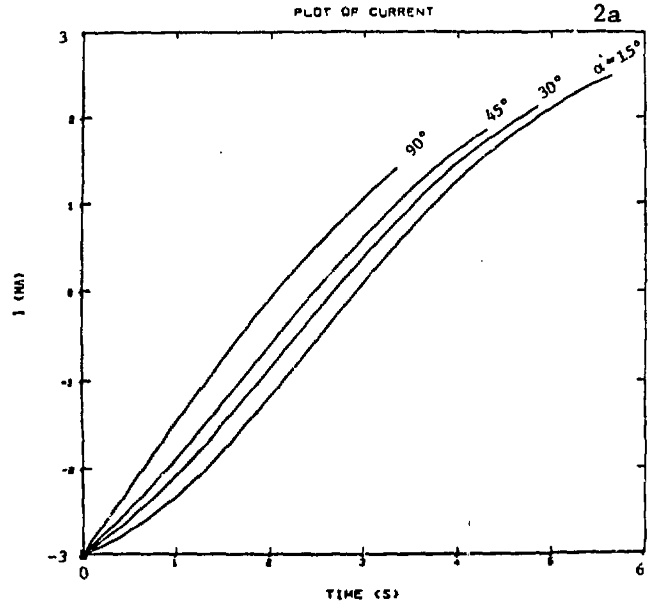




Fig. 1. Plots of the functions a) $y = x \exp(-x^2)$, and
b) $z = -\cos[y(x) \cdot \pi/0.429]$.




$R = 1\ n\Omega$; $L = 86.1\ \mu H$;

$t_m = 10$ s

$\left(\omega_{rotor}\right)_{ref} = 200$ rad/s;

$V_{ref} = 419$ V

$W_{ref} = 387$ MJ

$\lambda = 0.99993$

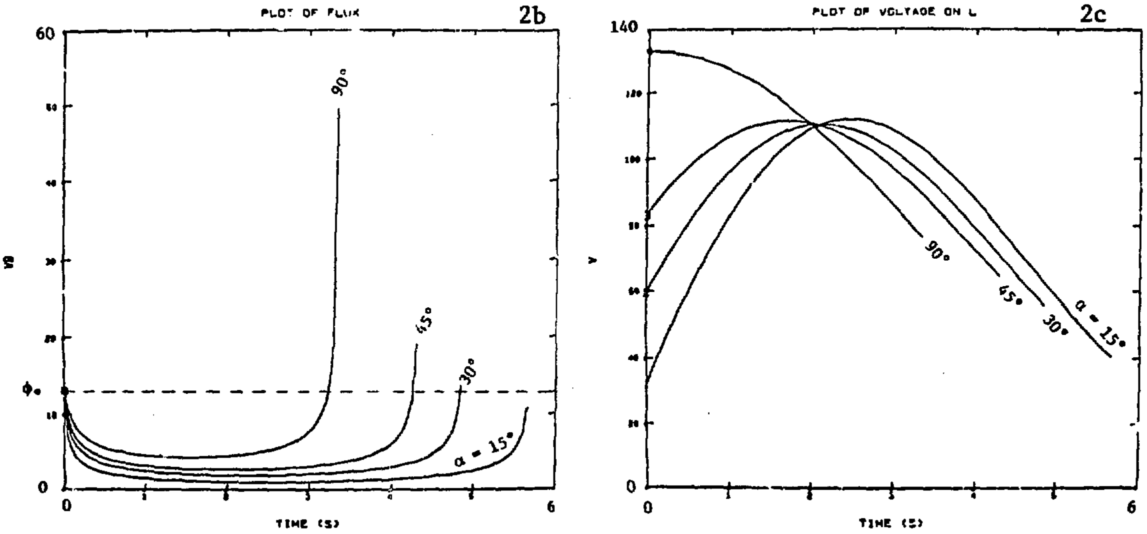
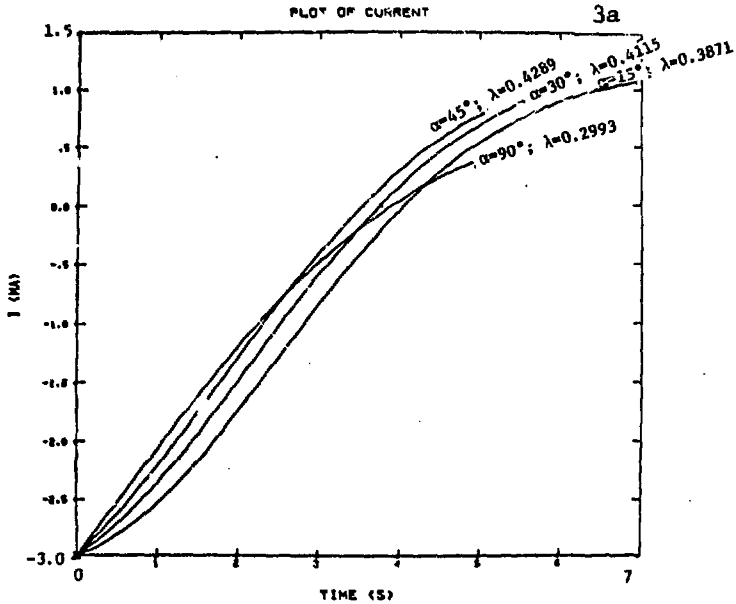



Fig. 2. Plots of a) current, b) homopolar flux, and c) voltage on homopolar and E-coil; for the constants (i.e., negligible resistance). (See above list.)



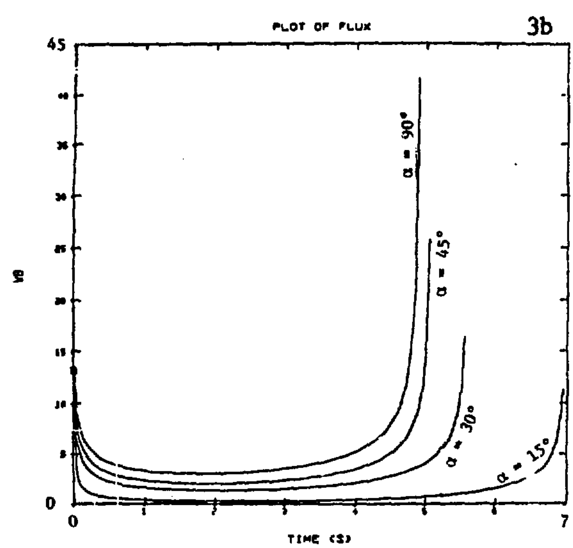

$R = 6.1\ \mu\Omega$; $L = 86.1\ \mu H$

$t_m = 10$ s

$\left(\omega_{rotor}\right)_{ref} = 200$ rad/s;

$V_{ref} = 419$ V

$W_{ref} = 387$ MJ



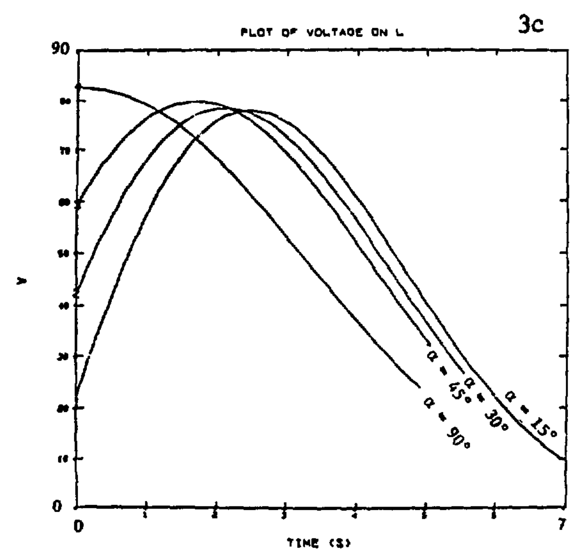
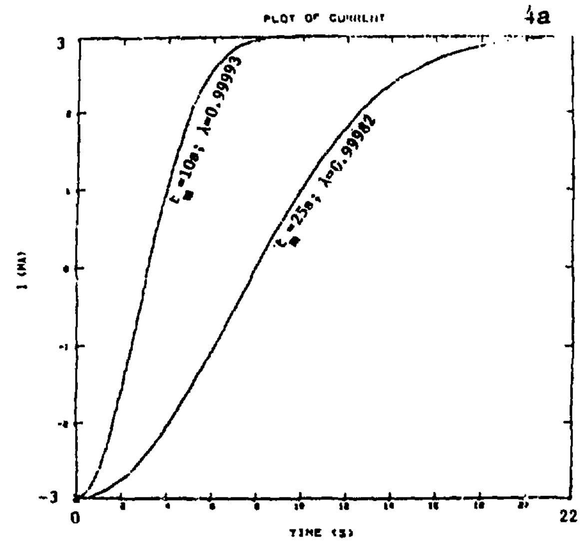




Fig. 3. Plots of a) current, b) homopclar flux, and c) voltage on homopolar and E-coil; for the constants (i.e., true resistance value). (See above list.)




$R = 1\ n\Omega$; $L = 86.1\ \mu H$

$\alpha = 0.06°$

$\left(\omega_{\text{rotor}}\right)_{ref} = 200$ rads/s;

$V_{ref} = 419$ V

$W_{ref} = 387$ MJ

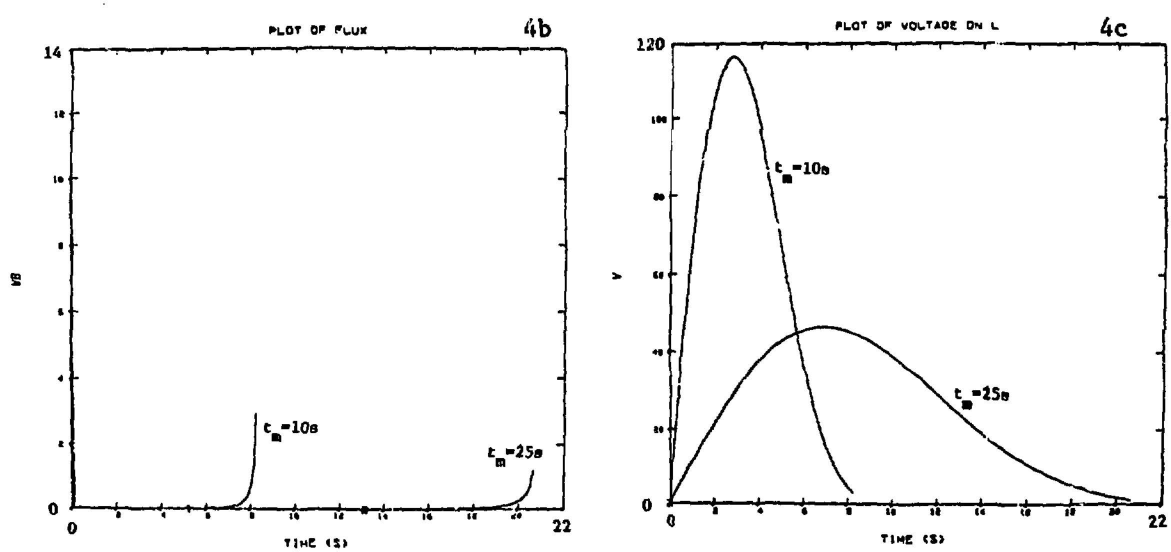
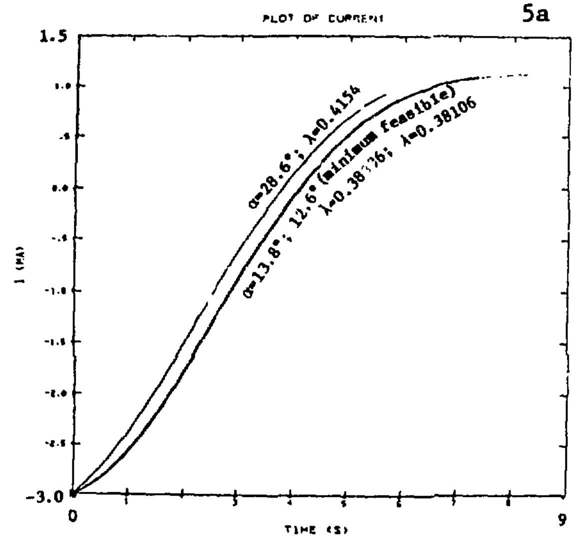



Fig. 4. Plots of a) current, b) homopolar flux, and c) voltage on homopolar and E-coil; for the constants (i.e., negligible resistance and phase angle $\alpha$, different time constants). (See above list.)



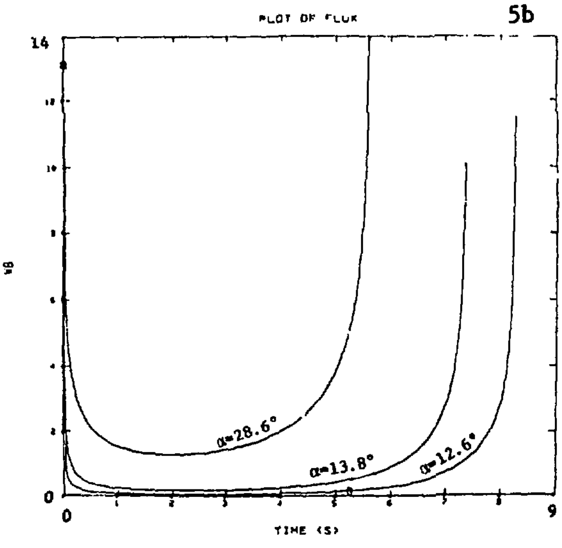

R = 6.1 μΩ; L = 86.1 μH

$t_m$ = 10 s

$\left(\omega_{rotor}\right)_{ref}$ = 200 rad/s;

$V_{ref}$ = 419 V

$W_{ref}$ = 387 MJ



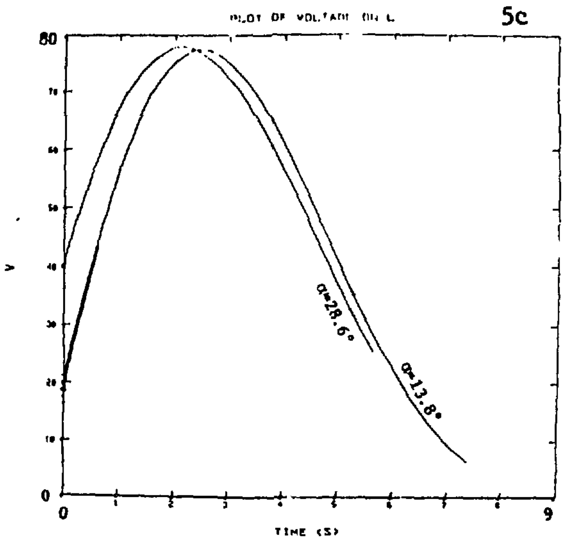




Fig. 5. Plots of a) current, b) homopolar flux, and c) voltage on homopolar and E-coil; for the constants (true resistance, minimum and near-minimum possible phase angle α). (See above list.)