THREE-DIMENSIONAL MISE-A-LA-MASSE MODELING APPLIED TO
MAPPING FRACTURE ZONES

by

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ABSTRACT

A numerical scheme applying the method of integral equations has been developed for borehole-to-borehole and borehole-to-surface modeling of the apparent resistivity response of a thin conductive body in a half-space; the inhomogeneity simulates a fracture zone. The algorithm is applicable for the direct-current case when the buried electrode is either inside (*mise-à-la-masse*) or outside (*near-miss*) the body. In implementing the scheme, the integral equation is transformed into a matrix equation as a result of discretizing the inhomogeneity into rectangular subcells. All properties are assumed to be constant within each subcell. The rectangular subcells are used throughout execution of the algorithm.

The computed surface and subsurface apparent resistivity responses are examined for similar bodies with different orientations: (1) vertical, (2) horizontal, (3) dipping at 60 degrees, and (4) dipping at 30 degrees. The different bodies produce apparent resistivity cross-section plots which differ little from each other with the exception of orientation. In plan view, estimates of orientation, areal extent, and dip can often be made. The maximum depth at which a body can be located and still produce a detectable surface anomaly is dependent upon the position of the buried electrode and upon the contrast in conductivity. Locating the buried electrode just outside of the body does not significantly alter the results from those of when the electrode is embedded in the inhomogeneity. However, the

similarity between the results of these two cases decreases as the distance between the electrode and the body is increased.

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INTRODUCTION

To date, electrical borehole geophysical techniques have been used mostly in the mining industry. However, the geoelectrical setting of a geothermal fracture zone (i.e., a thin conductive target in a more resistive host) suggests that these techniques may be of use in detecting and/or delineating fracture zones. A numerical modeling study of the direct-current borehole-to-borehole and borehole-to-surface techniques applied to this problem was undertaken. For both methods, consideration was given to the case when the buried electrode is inside the body (mise-à-la-masse) as well as when it is outside the body (near-miss). The results of this study are presented herein.

The mise-à-la-masse method was first developed and used by Schlumberger (1920). The method utilizes a buried current electrode within a conductive body of interest and the other current electrode is placed a large distance away from the anomalous mass and at the surface. If the conductivity of the anomalous body is very high, there will be relatively little potential drop across the body itself, and the buried body (presumably) can be mapped at the surface of the earth or in boreholes as a zone of low potential. The near-miss technique utilizes a buried current electrode that is outside the conductive body of interest and a second current electrode at the surface and at a large distance from the anomalous mass. Potentials may be

measured either at the surface or in one or more boreholes with either method.

Ross and Ward (1984) reviewed and compiled a list of the literature regarding the *mise-à-la-masse* technique. Examination of the papers comprising their list shows that the literature is dominated by mining case histories. McMurray and Hoagland (1956), Parasnis (1967, 1974), Pelton and Hallof (1971), and Ketola (1972) reported typical case studies which integrate both surface and subsurface *mise-à-la-masse* data. More recently, Mansinha and Mwenifumbo (1983) examined a surface *mise-à-la-masse* study carried out at the Cavendish Geophysical Test Site in Ontario, Canada.

In geothermal exploration, the borehole-to-surface method has primarily been executed using a well casing in an existing hole as the buried current electrode. Sill and Ward (1978) discussed the utilization of this method at Roosevelt Hot Springs KGRA, Utah. Kauahikaua et al. (1980) used a well casing as the buried current electrode in a study aimed at defining the boundaries of the HRP-A geothermal reservoir, Hawaii. Tagomori et al. (1984) reported on the direct detection of the Hatchobaru geothermal field using this method. Each of these papers reported at least moderate success in determining the location of the geothermal reservoir and/or the location of fault zones that bound them. This technique was also applied both before and after injection tests at Raft River KGRA, Idaho (Sill, 1983).

Mwenifumbo (1980) used the finite-element technique to compute surface potential and apparent-resistivity profiles over various two-dimensional (2-D) bodies for multiple electrode configurations. Eloranta (1984) presented an integral equation technique for computing

mise-à-la-masse anomalies in the case of high contrasts in conductivity. A method for computing three-dimensional (3-D) terrain corrections for mise-à-la-masse surveys was presented by Oppliger (1984). An integral equation technique by Hohmann (1975) was used by Newkirk (1982) to generate a catalogue of apparent resistivity responses for 3-D bodies in a half-space for the buried electrode case. Although the actual measured quantity in a mise-à-la-masse survey is potential difference, these measurements are readily converted to apparent resistivities; this has been the custom in recent studies.

The work herein is an extension to that of Newkirk (1982), though the computer algorithm has subsequently been modified. Rather than discretizing the body into cubes, as was previously done, the algorithm now permits discretization of the body into rectangular prisms whose size may vary in all three principal directions. At no point are the rectangular cells further discretized into cubes; this reduces computing time considerably when modeling elongated bodies. Using this algorithm, a series of theoretical model responses, in both plan and section views, for a variety of models and buried electrode positions, have been computed. These results should prove useful for data interpretation, not only for detection of fracture zones in geothermal systems, but for any geologic problem where a thin, tabular conductor in a more resistive host is the target of exploration.

THEORY

Hohmann (1975) developed an integral equation technique for modeling the electromagnetic responses of 3-D bodies in a half-space. The algorithm used in this study is an adaption of that of Hohmann (1975) to the case of a direct-current source. In the modified algorithm, the inhomogeneity is discretized into rectangular subcells, rather than in cubic subcells.

Electric-Field Integral Equation

The problem under consideration is a 3-D body in a half-space (Figure 1). The algorithm allows the conductivity in the body to vary as a function of position, though this feature is not utilized in this study. The total electric-field integral equation corresponding to the situation in Figure 1 is (Hohmann, 1975)

$$\underline{E}(\underline{r}) = \underline{E}_i(\underline{r}) + \int_V \frac{[\sigma_2(\underline{r}) - \sigma_1]}{\sigma_1} \underline{G}(\underline{r}, \underline{r}') \cdot \underline{E}(\underline{r}') dv'. \quad (1)$$

$\underline{E}_i(\underline{r})$ is the incident electric field. $\underline{G}(\underline{r}, \underline{r}')$ is the half-space dyadic Green's function, which for the static case is

$$\underline{G}(\underline{r}, \underline{r}') = -\nabla \nabla' G(\underline{r}, \underline{r}'), \quad (2)$$

where

$$G(\underline{r}, \underline{r}') = \frac{1}{4\pi R} + \frac{1}{4\pi R_s} \quad (3)$$

is the scalar half-space Green's function with

Figure 1. Three-dimensional body. The depth to the top of the body is D .

$$R = [(x-x')^2 + (y-y')^2 + (z-z')^2]^{1/2} \quad (4)$$

and

$$R_s = [(x-x')^2 + (y-y')^2 + (z+z')^2]^{1/2}. \quad (5)$$

The second term on the right-hand side of equation (3) arises because fictitious image sources are needed to satisfy boundary conditions at the air-earth interface.

To implement equation (1) numerically, the inhomogeneity is discretized into N rectangular cells with dimensions Δ_{xn} , Δ_{yn} , and Δ_{zn} , where n denotes the n th cell (Figure 2). Assuming that the electric field is constant within each cell, equation (1) can be approximated by

$$\underline{\underline{E}}(\underline{\underline{r}}) = \underline{\underline{E}}_i(\underline{\underline{r}}) + \sum_{n=1}^N \frac{[\sigma_{2n} - \sigma_1]}{\sigma_1} \int_V \underline{\underline{G}}(\underline{\underline{r}}, \underline{\underline{r}}') dv' \cdot \underline{\underline{E}}_n. \quad (6)$$

The quantities σ_{2n} and $\underline{\underline{E}}_n$ are the conductivity and electric field within the n th cell, respectively. Defining $\underline{\underline{I}}(\underline{\underline{r}}, \underline{\underline{r}}')$ as the volume integral over $\underline{\underline{G}}(\underline{\underline{r}}, \underline{\underline{r}}')$, equation (6) becomes

$$\underline{\underline{E}}(\underline{\underline{r}}) = \underline{\underline{E}}_i(\underline{\underline{r}}) + \sum_{n=1}^N \frac{[\sigma_{2n} - \sigma_1]}{\sigma_1} \underline{\underline{I}}(\underline{\underline{r}}, \underline{\underline{r}}') \cdot \underline{\underline{E}}_n. \quad (7)$$

The treatment of $\underline{\underline{I}}(\underline{\underline{r}}, \underline{\underline{r}}')$ is given below.

Equation (7) may be written so that it defines the electric field at the center of the n th cell. Doing so for all N cells results in a partitioned matrix equation (Hohmann, 1975) which is solved for the electric fields inside the body.

Figure 2. Geometric relation between cells m and n .

Computation of dyadic Green's function $\underline{\underline{r}}(\underline{r}, \underline{r}')$

In the electromagnetic case, both current and charge sources contribute to $\underline{\underline{r}}(\underline{r}, \underline{r}')$. However, in the static case charges that accumulate at cell boundaries are the sole contributors. Hence, applying the relation (Van Bladel, 1964)

$$\int_V \nabla G \, dv = \int_S G \, d\underline{s} \quad (8)$$

enables $\underline{\underline{r}}(\underline{r}, \underline{r}')$ in equation (7) to be expressed as the following surface integral:

$$\underline{\underline{r}}(\underline{r}, \underline{r}') = \sum_{j=1}^3 \sum_{\ell=1}^2 (-1)^\ell \int_{S_{jn}} [-\nabla' G(\underline{r}, \underline{r}') \cdot \underline{u}_j] ds'. \quad (9)$$

The j index represents the three components of the electric field and the three principal directions normal to the faces of a cell; the ℓ index denotes the two faces of a cell in each of the three principal directions. Hence, equation (9) represents integrations over the six faces of the n th cell. The three different integrals over the faces are given by

$$S_{jn} = \begin{cases} \int_{Y_n - \frac{\Delta_{yn}}{2}}^{Y_n + \frac{\Delta_{yn}}{2}} \int_{Z_n - \frac{\Delta_{zn}}{2}}^{Z_n + \frac{\Delta_{zn}}{2}} dy' dz', & j=1 \quad ; \\ \int_{X_n - \frac{\Delta_{xn}}{2}}^{X_n + \frac{\Delta_{xn}}{2}} \int_{Z_n - \frac{\Delta_{zn}}{2}}^{Z_n + \frac{\Delta_{zn}}{2}} dx' dz', & j=2 \quad ; \\ \int_{X_n - \frac{\Delta_{xn}}{2}}^{X_n + \frac{\Delta_{xn}}{2}} \int_{Y_n - \frac{\Delta_{yn}}{2}}^{Y_n + \frac{\Delta_{yn}}{2}} dx' dy', & j=3 \quad . \end{cases} \quad (10)$$

In equation (9),

$$\underline{r}' = \underline{r}_n + (-1)^k \frac{\Delta_{jn}}{2} \underline{u}_j, \quad j = 1, 2, \text{ and } 3, \quad (11)$$

where

$$\Delta_{jn} = \begin{cases} \Delta_{xn}, & j=1 \\ \Delta_{yn}, & j=2 \\ \Delta_{zn}, & j=3 \end{cases}, \quad (12)$$

and

$$\underline{u}_j = \begin{cases} \underline{u}_x, & j=1 \\ \underline{u}_y, & j=2 \\ \underline{u}_z, & j=3 \end{cases} \quad (13)$$

are unit vectors in the x-, y-, and z-directions.

Since in equation (3) $G(\underline{r}, \underline{r}')$ consists of two parts, the integrand in equation (9) can be split into two parts. Let

$$-\nabla'G = -\nabla'G^P - \nabla'G^{IM}, \quad (14)$$

where the P superscript denotes the primary part due to a whole space and the IM superscript represents the part required by image theory to account for the air-earth interface. The quantities on the right-hand side of equation (14) are given by

$$-\nabla G'^P = \frac{1}{4\pi R^3} \begin{bmatrix} (x-x') \underline{u}_1 \underline{u}_1 + (x-x') \underline{u}_1 \underline{u}_2 + (x-x') \underline{u}_1 \underline{u}_3 \\ + (y-y') \underline{u}_2 \underline{u}_1 + (y-y') \underline{u}_2 \underline{u}_2 + (y-y') \underline{u}_2 \underline{u}_3 \\ + (z-z') \underline{u}_3 \underline{u}_1 + (z-z') \underline{u}_3 \underline{u}_2 + (z-z') \underline{u}_3 \underline{u}_3 \end{bmatrix}, \quad (15)$$

and

$$-\nabla G^{IM} = \frac{1}{4\pi R_S^3} \begin{bmatrix} (x-x') \underline{u}_1 \underline{u}_1 + (x-x') \underline{u}_1 \underline{u}_2 + (x-x') \underline{u}_1 \underline{u}_3 \\ + (y-y') \underline{u}_2 \underline{u}_1 + (y-y') \underline{u}_2 \underline{u}_2 + (y-y') \underline{u}_2 \underline{u}_3 \\ + (z+z') \underline{u}_3 \underline{u}_1 + (z+z') \underline{u}_3 \underline{u}_2 + (z+z') \underline{u}_3 \underline{u}_3 \end{bmatrix} . \quad (16)$$

The quantities R and R_S are given in equations (4) and (5), respectively.

From equations (9) and (14), $\underline{\Gamma}(\underline{r}, \underline{r}')$ may also be written as

$$\underline{\Gamma} = \underline{\Gamma}^P + \underline{\Gamma}^{IM} , \quad (17)$$

where

$$\underline{\Gamma}^P = \sum_{j=1}^3 \sum_{\ell=1}^2 (-1)^\ell \int_{j_n} [-\nabla G^P(\underline{r}, \underline{r}') \cdot \underline{u}_j] ds' , \quad (18)$$

and

$$\underline{\Gamma}^{IM} = \sum_{\ell=1}^2 (-1)^\ell .$$

$$\begin{bmatrix} \Delta_{yn} \Delta_{zn} (x-x') \underline{u}_1 \underline{u}_1 + \Delta_{xn} \Delta_{zn} (x-x') \underline{u}_1 \underline{u}_2 + \Delta_{xn} \Delta_{yn} (x-x') \underline{u}_1 \underline{u}_3 \\ + \Delta_{yn} \Delta_{zn} (y-y') \underline{u}_2 \underline{u}_1 + \Delta_{xn} \Delta_{zn} (y-y') \underline{u}_2 \underline{u}_2 + \Delta_{xn} \Delta_{yn} (y-y') \underline{u}_2 \underline{u}_3 \\ + \Delta_{yn} \Delta_{zn} (z+z') \underline{u}_3 \underline{u}_1 + \Delta_{xn} \Delta_{zn} (z+z') \underline{u}_3 \underline{u}_2 + \Delta_{xn} \Delta_{yn} (z+z') \underline{u}_3 \underline{u}_3 \end{bmatrix} . \quad (19)$$

The integrations in equation (18) are defined in equation (10). No surface integration appears in equation (19) since the second term on the right-hand side of equation (17) arises from image sources which are treated as point sources.

The primary part in equation (17) is singular and is due to sources close to the observation point, so the integrals in equation

(18) must be evaluated. Let r_{mn} denote $r(r_m, r_n)$. Then from equations (15) and (18),

$$r_{mn}^P = \sum_{j=1}^3 \sum_{\ell=1}^2 \int_{jn} \frac{(r_m - r') \cdot u_j}{4\pi |r_m - r'|^3} ds'. \quad (20)$$

This equation is best evaluated by considering the diagonal and off-diagonal cases separately.

The diagonal elements of r_{mn}^P are given by

$$r_{mn}^{P(u_j u_j)} = \frac{u_j u_j}{4\pi} \sum_{\ell=1}^2 (-1)^\ell \int_{jn} \frac{(r_m - r') \cdot u_j}{|r_m - r'|^3} ds', \quad j=1, 2, \text{ and } 3, \quad (21)$$

where r' is given in equation (11). The result of analytically performing the integration in equation (21) is the solid angle Ω_ℓ subtended by the face of the cell under consideration. Equation (21) may then be expressed as

$$r_{mn}^{P(u_j u_j)} = \frac{u_j u_j}{4\pi} \sum_{\ell=1}^2 (-1)^\ell \Omega_\ell, \quad j=1, 2, \text{ and } 3, \quad (22)$$

where

$$\Omega_\ell = \sum_{v=1}^2 \sum_{w=1}^2 (-1)^{v+w} \tan^{-1} \left[\frac{([r_{mn} - (-1)^v \frac{\Delta_{in}}{2} u_i] \cdot u_i) ([r_{mn} - (-1)^w \frac{\Delta_{kn}}{2} u_k] \cdot u_k)}{([r_{mn} - (-1)^\ell \frac{\Delta_{jn}}{2} u_j] \cdot u_j) R_{ijk}} \right], \quad i \neq j \neq k, \quad (23)$$

with

$$R_{ijk} = \{([r_{mn} + (-1)^v \frac{\Delta_{in}}{2} u_i] \cdot u_i)^2 + ([r_{mn} + (-1)^w \frac{\Delta_{kn}}{2} u_k] \cdot u_k)^2 + ([r_{mn} + (-1)^\ell \frac{\Delta_{jn}}{2} u_j] \cdot u_j)^2\}^{1/2}, \quad (24)$$

and

$$r_{mn} = r_m - r_n. \quad (25)$$

The off-diagonal terms of Γ_{mn}^P are given by

$$\Gamma_{mn}^{P(u_i u_j)} = \frac{u_i u_j}{4\pi} \sum_{\ell=1}^2 (-1)^\ell \int_{S_{jn}} \frac{(\underline{r}_m - \underline{r}') \cdot \underline{u}_i}{|\underline{r}_m - \underline{r}'|^3} ds', \quad i \neq j. \quad (26)$$

Once again, $\underline{r}'$ is given by equation (11). The integral in equation (26) is computed analytically to give

$$\Gamma_{mn}^{P(u_i u_j)} = \frac{u_i u_j}{4\pi} \sum_{\ell=1}^2 (-1)^\ell T_\ell, \quad i \neq j. \quad (27)$$

where

$$T_\ell = \ln \left\{ \frac{A \cdot B}{C \cdot D} \right\}, \quad (28)$$

with

$$A = [\underline{r}_{mn} - \frac{\Delta_{kn}}{2} \underline{u}_k] \cdot \underline{u}_k + \{([\underline{r}_{mn} + \frac{\Delta_{in}}{2} \underline{u}_i] \cdot \underline{u}_i)^2 + ([\underline{r}_{mn} - (-1)^\ell \frac{\Delta_{jn}}{2} \underline{u}_j] \cdot \underline{u}_j)^2 + ([\underline{r}_{mn} - \frac{\Delta_{kn}}{2} \underline{u}_k] \cdot \underline{u}_k)^2\}^{1/2}, \quad i \neq j \neq k,$$

$$B = [\underline{r}_{mn} + \frac{\Delta_{kn}}{2} \underline{u}_k] \cdot \underline{u}_k + \{([\underline{r}_{mn} - \frac{\Delta_{in}}{2} \underline{u}_i] \cdot \underline{u}_i)^2 + ([\underline{r}_{mn} - (-1)^\ell \frac{\Delta_{jn}}{2} \underline{u}_j] \cdot \underline{u}_j)^2 + ([\underline{r}_{mn} + \frac{\Delta_{kn}}{2} \underline{u}_k] \cdot \underline{u}_k)^2\}^{1/2}, \quad i \neq j \neq k,$$

$$C = [\underline{r}_{mn} - \frac{\Delta_{kn}}{2} \underline{u}_k] \cdot \underline{u}_k + \{([\underline{r}_{mn} - \frac{\Delta_{in}}{2} \underline{u}_i] \cdot \underline{u}_i)^2 + ([\underline{r}_{mn} - (-1)^\ell \frac{\Delta_{jn}}{2} \underline{u}_j] \cdot \underline{u}_j)^2 + ([\underline{r}_{mn} - \frac{\Delta_{kn}}{2} \underline{u}_k] \cdot \underline{u}_k)^2\}^{1/2}, \quad i \neq j \neq k,$$

$$D = [\underline{r}_{mn} + \frac{\Delta_{kn}}{2} \underline{u}_k] \cdot \underline{u}_k + \{([\underline{r}_{mn} + \frac{\Delta_{in}}{2} \underline{u}_i] \cdot \underline{u}_i)^2 + ([\underline{r}_{mn} - (-1)^\ell \frac{\Delta_{jn}}{2} \underline{u}_j] \cdot \underline{u}_j)^2 + ([\underline{r}_{mn} - \frac{\Delta_{kn}}{2} \underline{u}_k] \cdot \underline{u}_k)^2\}^{1/2}, \quad i \neq j \neq k,$$

and r_{mn} is defined in equation (25).

Computation of Potentials and Apparent Resistivities

Having found the electric field in each cell, the potential at any point can be computed from

$$V(\underline{r}) = V_i(\underline{r}) + \frac{1}{4\pi} \sum_{n=1}^N \sum_{\ell=1}^2 \frac{[\sigma_{2n} - \sigma_1]}{\sigma_1} \left[\frac{\Delta_{yn} \Delta_{zn}}{R_1}, \frac{\Delta_{xn} \Delta_{zn}}{R_2}, \frac{\Delta_{xn} \Delta_{yn}}{R_3} \right] \cdot \underline{E}_n, \quad (29)$$

where

$$R_j = |\underline{r} - \underline{r}'|, \quad j = 1, 2, \text{ and } 3, \quad (30)$$

with $\underline{r}'$ defined in equation (11). Finally, apparent resistivities are computed from the potentials via

$$\rho_a(\underline{r}) = \frac{\rho_1 V(\underline{r})}{V_i(\underline{r})}, \quad (31)$$

where ρ_1 is the resistivity of the half-space.

NUMERICAL CHECKS

Verification of the accuracy of any numerical solution is essential. Direct comparison of two different types of numerical solutions applied to a given problem is a good check of a solution. Results from the volume integral-equation algorithm used in this study were compared to those of the surface integral-equation technique of Yang and Ward (1985) which computes the subsurface ρ_a response of thin ellipsoids. The algorithm used in this study cannot model ellipsoids, but ellipsoids can be approximated by polygons of similar shape and volume. Though the test results (not shown) for the near-miss case are not exactly the same for both methods, they are encouraging in that they both exhibit the same features and show anomalies of approximately the same magnitude.

Internal convergence tests were also performed as a check on the solution. The different discretizations used in the convergence test are shown in Figure 3. In Figure 3a, the cell containing the electrode was not broken into cubes because the algorithm gives the best results when the electrode is in the center of a cell, rather than at a cell boundary.

Cross-sectional apparent resistivity (ρ_a) contour plots through the center and perpendicular to the strike of the bodies in Figures 3a and 3b were computed (not shown). The bodies were at a depth of 8 units (dimensionless). These plots display ρ_a values both inside and

Figure 3. Different body discretizations for convergence check.
Dimensionless presentation is used.

(a) 97 cells

(b) 25 cells

(c) 9 cells

SCALE
1 Unit $\longleftrightarrow$

outside of the body. Using the plot for the more densely discretized body (97 cells) as the standard, ρ_a values inside the 25 cell body are always within 18 percent of those of the 97 cell model. The external ρ_a values are always within 2 percent of those of the standard. Although the internal ρ_a values differ substantially, the external values, which are the values of importance to exploration geophysicists, are in excellent agreement. Surface ρ_a plots (not shown) over the bodies ($D=1$ unit) in Figures 3a and 3b are also in excellent agreement since they consist of external values. Increasing the depth to the body will improve convergence since the observation points become farther from the source.

Although there is a 2 percent discrepancy between the external ρ_a values for the 97 cell model as compared to the 25 cell model, it is compensated for by the reduction in computation time in going to less cells. Computing a subsurface ρ_a plot consisting of 255 observation points for the 97 cell body takes approximately 35 minutes of CPU time on a PRIME 400 computer. A similar plot for the 25 cell body takes only approximately 3 minutes of CPU time.

Decreasing the number of cells to 9 (Figure 3c) further decreases the computation time, but a loss of accuracy in the solution also ensues. The internal ρ_a values in a cross-section (not shown) through the center and perpendicular to the strike of the body ($D=8$ units) in Figure 3c differs by up to 40 or 50 percent, depending on whether the 25 or 97 cell body is used as the standard, respectively. The external ρ_a values for the 9 cell model differ by a maximum of 15 percent from both the 25 and 97 cell model. The surface values (not shown) over the 9 cell model exhibit the same discrepancy. (Note:

the percent differences in the surface ρ_a values given above do not hold for all body geometries and electrode positions since the magnitude of the surface response is dependent upon these parameters, as is illustrated in the following sections.) Hence, for best results the length of any one side of a cell should not exceed approximately twice that of the smallest side.

Additional support for the validity of the algorithm used in this study lies in the solution checks presented by Hohmann (1975), Ting and Hohmann (1981), and Wannamaker et al. (1984), of which the latter two are also extensions of the former. Although each of these papers presents solution checks for the electromagnetic case, the fundamental basis of each algorithm is the inhomogeneous vector Fredholm integral equation of the second kind, which is also the basis of the algorithm used in this study.

MODEL RESULTS

The following sections present the results of modeling the geothermal fracture zone with four bodies: (1) vertical, (2) horizontal, (3) dipping at 60 degrees, and (4) dipping at 30 degrees (Figure 4). Each of the bodies contains the same number of cells and the same volume. In addition, all *cells* are of equal volume and have two dimensions which are 1 unit; the third is .5 units. The *bodies* are designed so that two dimensions are equal or approximately equal to ten times that of the third dimension. Unless specified otherwise, the contrast in conductivity between the body and the half-space is 10:1, with the body being more conductive (i.e., $\rho_1=100 \Omega\cdot m$ and $\rho_2=10 \Omega\cdot m$).

Whenever an electrode is placed in the body, it is centered within a cell. This buried electrode is always the positive current source. The negative current source is the distant electrode, which is placed on the surface 50 units away from a point (on the surface) directly above the center of the body. All model results are presented in contour plots of apparent resistivities ($\Omega\cdot m$). These contour plots are constructed from computed data points at each node on either a 15*15 unit grid (plan view) or a 15*17 unit grid (section view).

Mise-à-la-masse Models

Vertical body.

(a) Section view. - The depth to the body is found to have little

Figure 4. Four different bodies modeled in this study.

a) Vertical

b) Horizontal

c) Dipping (60 degrees)

d) Dipping (30 degrees)

effect upon the overall pattern of the ρ_a plots in section view, as shown by comparison of results when the electrode is in the center of bodies with $D=1$ unit (Figure 5) and $D=8$ units (Figure 6). When $D=1$ unit, some asymmetry is present due to the influence of the surface of the earth, while for $D=8$ units, the contours are essentially symmetrical. Since surface effects are negligible when $D=8$ units, this depth is used in subsequent modeling of downhole responses for the vertical body.

The position of the electrode within the body greatly affects the pattern of the contours. When the electrode is not in the center of the body, the symmetry observed in Figure 6 disappears, though the following two features remain: (1) ρ_a lows surround the electrode, and (2) ρ_a highs occur at edges of the body which are away from the electrode. Figure 7 illustrates these features for the case when the electrode is near the bottom of the body. Symmetry of the body requires that when the electrode is near the top of the body, the ρ_a plot is that of Figure 7, only reflected over the x-y plane bisecting the body.

Comparison of Figures 7 and 8 shows that moving the electrode laterally within the body does not significantly change the ρ_a plot, provided that the plane of view contains the electrode. The plane of view is found to be important in determining what the ρ_a plot looks like. Figures 9 and 10 illustrate how the ρ_a plot changes from that of Figure 8 when the plane of view no longer contains the electrode. As the plane of view is moved away from the electrode, the ρ_a low surrounding the electrode is lost, but the high ρ_a values at the top of the body and the thinning of the contours toward the bottom of the

Figure 5. Section view ρ_a plot through a vertical body with $D=1$ unit. Shaded region in inset shows plane of view. Contours in $\Omega \cdot m$. Electrode position given by *.

MISE-À-LA-MASSE:

VERTICAL BODY

APPARENT RESISTIVITY-FROM V

SECTION VIEW $\rho_1 = 100 \Omega \cdot m$

$D = 1 \text{ UNIT}$ $\rho_2 = 10 \Omega \cdot m$

SCALE: 1 UNIT

Figure 6. Section view ρ_a plot through a vertical body with $D=8$ units. Shaded region in inset shows plane of view. Contours in $\Omega\cdot m$. Electrode position given by *.

Figure 7. Section view ρ_a plot through a vertical body with $D=8$ units. Shaded region in inset shows plane of view. Contours in $\Omega \cdot m$. Electrode position given by *.

Figure 8. Section view ρ_a plot through a vertical body with $D=8$ units. Shaded region in inset shows plane of view. Contours in $\Omega\cdot m$. Electrode position given by *.

Figure 9. Section view ρ_a plot through a vertical body with $D=8$ units. Shaded region in inset shows plane of view. Contours in $\Omega \cdot m$. Electrode position given by *.

Figure 10. Section view ρ_a plot through a vertical body with $D=8$ units. Shaded region in inset shows plane of view. Contours in $\Omega\cdot m$. Electrode position given by *.

SURFACE

Not to Scale

ISOMETRIC

MISE-À-LA-MASSE:

VERTICAL BODY

APPARENT RESISTIVITY: FROM V

SECTION VIEW $\rho_1 = 100 \Omega \cdot m$ D = 8 UNITS $\rho_2 = 10 \Omega \cdot m$ SCALE : 1 UNIT $\longleftarrow$

body indicate that the electrode is located somewhere along the bottom of the body.

(b) Plan View. - Varying the position of the buried electrode within the body has great effect upon the ρ_a plots in plan view. Figures 11, 12, and 13 show the dramatic differences in computed apparent resistivities for three different buried electrode positions in a body with $D=1$ unit. The anomaly is considerably larger when the electrode is near either the top (Figure 11) or bottom (Figure 13) of the body, rather than in its center (Figure 12). Consideration of the relative locations of subsurface ρ_a highs and lows for different electrode positions, as previously discussed and illustrated in the section-view plots (Figures 6 and 7), helps in understanding why in plan view a high or low in ρ_a results when the electrode is near either the bottom or top of the body, respectively. Despite the differences in Figures 11, 12, and 13, one feature is common to each of them: the ρ_a plot is symmetric about the electrode position, implying that the electrode is in some manner symmetrically placed in the body.

The symmetry observed in the previous plan view plots is lost when the electrode is near an edge of the body. Figure 14 illustrates the surface response when the electrode is near the top corner of a body with $D=1$ unit. Note the bipolar pattern consisting of a low in ρ_a marking the electrode position and a high in ρ_a approximately defining the location of the edge of the body opposite the electrode. The orientation of the body is that of a line connecting the two poles. If the electrode is placed in the bottom corner of the same body, the bipolar pattern is lost (Figure 15). However, the elongation of the contours still enables determination of the strike of the

Figure 11. Plan view ρ_a plot over a vertical body with $D=1$ unit.
Contours in $\Omega \cdot m$. Electrode position given by *.

Not to Scale

ISOMETRIC

MISE-À-LA-MASSÉ:

VERTICAL BODY

APPARENT RESISTIVITY: FROM V

PLAN VIEW $\rho_1 = 100 \Omega \cdot m$

$D = 1 \text{ UNIT}$ $\rho_2 = 10 \Omega \cdot m$

SCALE: 1 UNIT $\longrightarrow$

Figure 12. Plan view ρ_a plot over a vertical body with $D=1$ unit.
Contours in $\Omega\cdot m$. Electrode position given by *.

Not to Scale

ISOMETRIC

MISE-À-LA-MASSÉ:

VERTICAL BODY

APPARENT RESISTIVITY: FROM V

PLAN VIEW $\rho_1 = 100 \Omega \cdot m$

D = 1 UNIT $\rho_2 = 10 \Omega \cdot m$

SCALE : 1 UNIT

Figure 13. Plan view ρ_a plot over a vertical body with $D=1$ unit.
Contours in $\Omega \cdot m$. Electrode position given by *.

Not to Scale

ISOMETRIC

MISE-À-LA-MASSE:

VERTICAL BODY

APPARENT RESISTIVITY: FROM V

PLAN VIEW $\rho_1 = 100 \Omega \cdot m$ D = 1 UNIT $\rho_2 = 10 \Omega \cdot m$

SCALE : 1 UNIT

Figure 14. Plan view ρ_a plot over a vertical body with $D=1$ unit.
Contours in $\Omega \cdot m$. Electrode position given by *.

Not to Scale

ISOMETRIC

MISE-À-LA-MASSÉ:

VERTICAL BODY

APPARENT RESISTIVITY: FROM V

PLAN VIEW $\rho_1 = 100 \Omega \cdot m$

D = 1 UNIT $\rho_2 = 10 \Omega \cdot m$

SCALE : 1 UNIT $\longrightarrow$

Figure 15. Plan view ρ_a plot over a vertical body with $D=1$ unit.
Contours in $\Omega \cdot m$. Electrode position given by *.

Not to Scale

ISOMETRIC

MISE-À-LA-MASSÉ:

VERTICAL BODY

APPARENT RESISTIVITY: FROM V

PLAN VIEW $\rho_1 = 100 \Omega \cdot m$

D = 1 UNIT $\rho_2 = 10 \Omega \cdot m$

SCALE : 1 UNIT $\longleftarrow$

body.

The response in plan view of the vertical body changes as the depth to the body is varied. For a given electrode position within the body, the surface response decreases as the depth to the body is increased. Since, as previously shown, the anomaly amplitude is dependent upon the position of the embedded electrode, so will be the magnitude of surface anomalies for different depths to the body. For $D=1$ unit, the surface anomalies are largest when the electrode is near either the top or bottom of the body (Figures 11 and 13, respectively). It follows, that these electrode positions also produce the largest surface anomaly when the depth to the body is greater than one. With the electrode in the center of the body, the small anomaly in Figure 12 is all but gone when $D=4$ units (not shown). When $D=4$ units and the electrode is near either the top or bottom of the body (not shown), the overall ρ_a patterns observed in Figures 11 and 13 are unchanged, but the anomalies are reduced from approximately 25 percent to approximately 13 percent, relative to the background resistivity.

Horizontal body.

(a) Section view. - The subsurface ρ_a response of a horizontal body with $D=7.75$ units is shown in Figure 16. (Note: all non-integer depths in subsequent models arise from necessary considerations of the numerical scheme.) Comparison of Figures 6 and 16 shows that for a given electrode position within a deeply buried body, the subsurface response of a horizontal body is markedly similar in both amplitude and contour pattern to that of a vertical body. This is to be expected since the two bodies are identical with the exception of their orientation. It is found that subsurface modeling of the horizontal

Figure 16. Section view ρ_a plot through a horizontal body with $D=7.75$ units. Shaded region in inset shows plane of view. Contours in $\Omega\cdot m$. Electrode position given by *.

body for various electrode positions and planes of view produces results that correspond directly to the analogous cases for the vertical body, only rotated by 90 degrees.

(b) Plan view. - As with the vertical body, increasing the depth to the body decreases the surface response. When $D=1$ unit, the areal extent of the body is well defined (Figure 17). Increasing the depth to 2 units does not affect the concentric ring pattern, but does result in a decreased resolution of the extent of the body (not shown). For $D=4$ units, the maximum anomaly is only approximately 5 percent relative to the background resistivity, indicating that the ability to detect a body at depths greater than 4 units is questionable.

The concentric ρ_a contours observed in Figure 17 disappear when the electrode is moved to an edge of the body (Figure 18). For this electrode position, the areal extent of the body is approximately outlined by inflection points in the contours, the ρ_a low above the electrode, and the ρ_a high overlying the edge of the body away from the electrode. These diagnostic features persist for the case when the embedded electrode is near a corner of the body (Figure 19).

Dipping body - 60 degrees.

(a) Section view. - Despite the staggered cell discretization, subsurface modeling of the body dipping at 60 degrees produces results very much like those of the vertical and horizontal bodies, again with the exception of orientation. This is illustrated by comparison of the subsurface ρ_a response for the dipping body at a depth of 7.78 units (Figure 20) with that of the vertical (Figure 6) and horizontal (Figure 16) bodies at a similar depth and having the electrode in the same relative position. Changing the electrode position and plane of

Figure 17. Plan view ρ_a plot over a horizontal body with $D=1$ unit.
Contours in $\Omega \cdot m$. Electrode position given by *.

Not to Scale

ISOMETRIC

MISE-À-LA-MASSE:

HORIZONTAL BODY

APPARENT RESISTIVITY-FROM V

PLAN VIEW $\rho_1 = 100 \Omega \cdot m$

$D = 1 \text{ UNIT}$ $\rho_2 = 10 \Omega \cdot m$

SCALE: 1 UNIT

Figure 18. Plan view ρ_a plot over a horizontal body with $D=1$ unit.
Contours in $\Omega \cdot m$. Electrode position given by *.

Not to Scale

ISOMETRIC

MISE-À-LA-MASSE:

HORIZONTAL BODY

APPARENT RESISTIVITY-FROM V

PLAN VIEW $\rho_1 = 100 \Omega \cdot m$

$D = 1 \text{ UNIT}$ $\rho_2 = 10 \Omega \cdot m$

SCALE : 1 UNIT $\longleftarrow$

Figure 19. Plan view ρ_a plot over a horizontal body with $D=1$ unit.
Contours in $\Omega \cdot m$. Electrode position given by *.

Not to Scale

ISOMETRIC

MISE-À-LA-MASSE:

HORIZONTAL BODY

APPARENT RESISTIVITY-FROM V

PLAN VIEW $\rho_1 = 100 \Omega \cdot m$

$D = 1 \text{ UNIT}$ $\rho_2 = 10 \Omega \cdot m$

SCALE : 1 UNIT $\longleftarrow$

Figure 20. Section view ρ_a through a dipping body (60°) with $D=7.78$ units. Shaded region in inset shows plane of view. Contours in $\Omega \cdot m$. Electrode position given by *.

view within the dipping body produces subsurface ρ_a plots illustrating the same features as those already discussed for the vertical body.

(b) Plan view. - The surface response of the body changes dramatically as the position of the electrode is varied, as indicated by the following results for a body with $D=1$ unit. When the electrode is centered near the top of the body (Figure 21), the response is difficult to distinguish from that of the vertical body having the same relative electrode position (Figure 11), indicating that the body is steeply dipping. Sharp inflections in the contours do, however, indicate the location of the updip edge of the body. Similarly, when the electrode is near the top corner of the body (Figure 22), the response looks like that of the vertical body in Figure 14, though the dip is marginally indicated by the direction of elongation of the closed ρ_a high.

The dip and areal extent of the body are better defined when the electrode is in its center (Figure 23). The extent of the body is approximately defined by the centers of the closed ρ_a contours, while the direction of dip is given by the elongation of the contours forming the ρ_a low. Figure 24 illustrates the response when the electrode is centered along an edge of the body. Inflection points and the closed ρ_a contours combine to outline approximately the extent of the body.

When the electrode is near the bottom of the body, only the updip edge of the body is well defined, as shown in Figure 25. However, the absence of a low in ρ_a over the electrode, coupled with the similarity of Figure 25 with that of the vertical body for this electrode position (Figure 13) indicate that the body is steeply dipping. The steep

Figure 21. Plan view ρ_a plot over a dipping body (60°) with $D=1$ unit. Contours in $\Omega \cdot m$. Electrode position given by *.

Not to Scale

ISOMETRIC

MISE-À-LA-MASSE:
DIPPING BODY (60 DEGREES)
APPARENT RESISTIVITY: FROM V

PLAN VIEW $\rho_1 = 100 \Omega \cdot m$

$D = 1 \text{ UNIT}$ $\rho_2 = 10 \Omega \cdot m$

SCALE : 1 UNIT

Figure 22. Plan view ρ_a plot over a dipping body (60°) with $D=1$ unit. Contours in $\Omega \cdot m$. Electrode position given by *.

Not to Scale

ISOMETRIC

MISE-À-LA-MASSÉ:

DIPPING BODY (60 DEGREES)

APPARENT RESISTIVITY: FROM V

PLAN VIEW $\rho_1 = 100 \Omega \cdot m$

D = 1 UNIT $\rho_2 = 10 \Omega \cdot m$

SCALE : 1 UNIT ———

Figure 23. Plan view ρ_a plot over a dipping body (60°) with $D=1$ unit. Contours in $\Omega\cdot m$. Electrode position given by *.

Figure 24. Plan view ρ_a plot over a dipping body (60°) with $D=1$ unit. Contours in $\Omega \cdot m$. Electrode position given by *.

Not to Scale

ISOMETRIC

MISE-À-LA-MASSE:
DIPPING BODY (60 DEGREES)
APPARENT RESISTIVITY: FROM V

PLAN VIEW $\rho_1 = 100 \Omega \cdot m$

D = 1 UNIT $\rho_2 = 10 \Omega \cdot m$

SCALE : 1 UNIT $\longleftarrow$

Figure 25. Plan view ρ_a plot over a dipping body (60°) with $D=1$ unit. Contours in $\Omega\cdot m$. Electrode position given by *.

dip of the body is also indicated when the electrode is near a bottom corner of the body (Figure 26), since no ρ_a low occurs over the electrode and only slight asymmetry is present as compared to the corresponding case for the vertical body (Figure 15).

Dipping body - 30 degrees.

(a) Section view. - Modeling of the body dipping at 30 degrees further reinforces the previous findings that for a given body geometry, electrode position, and plane of view, subsurface ρ_a plots retain approximately the same contour patterns and amplitudes while taking on the same orientation as the body. Since these results only confirm what has already been discussed, they are not included here.

(b) Plan view. - In plan view, when the electrode is centered along the top of a body with $D=1$ unit, the location and lateral extent of the updip edge are given by a low in ρ_a above the electrode and sharp inflections in the ρ_a contours (Figure 27), as is also the case when the electrode is in this position for the 60 degree dipping body (Figure 21). The dip of the body is indicated by the direction of contour elongation.

If the electrode is in the center of the body, the dip is given by the direction of contour elongation (Figure 28). The similarity between Figure 28 and that of the case when the electrode is in the same relative place in a horizontal body (Figure 17) indicates that the dip of the body is shallow. The ρ_a plot resulting when the electrode is centered along the bottom of the body (Figure 29) is nearly identical to that of the horizontal body in Figure 18, once again indicating that the dip of the body is only moderate. The surface ρ_a plots for the cases when the electrode is near an edge of

Figure 26. Plan view ρ_a plot over a dipping body (60°) with $D=1$ unit. Contours in $\Omega\cdot m$. Electrode position given by *.

Not to Scale

ISOMETRIC

MISE-À-LA-MASSE:

DIPPING BODY (60 DEGREES)

APPARENT RESISTIVITY: FROM V

PLAN VIEW $\rho_1 = 100 \Omega \cdot m$

D = 1 UNIT $\rho_2 = 10 \Omega \cdot m$

SCALE : 1 UNIT

Figure 27. Plan view ρ_a plot over a dipping body (30°) with $D=1$ unit. Contours in $\Omega\cdot m$. Electrode position given by *.

Not to Scale

MISE-À-LA-MASSE:
DIPPING BODY (30 DEGREES)
APPARENT RESISTIVITY-FROM V

PLAN VIEW $\rho_1 = 100 \Omega \cdot m$

D = 1 UNIT $\rho_2 = 10 \Omega \cdot m$

SCALE: 1 UNIT

Figure 28. Plan view ρ_a plot over a dipping body (30°) with $D=1$ unit. Contours in $\Omega\cdot m$. Electrode position given by *.

Not to Scale

MISE-À-LA-MASSE:
DIPPING BODY (30 DEGREES)
APPARENT RESISTIVITY-FROM V

PLAN VIEW $\rho_1 = 100\Omega \cdot m$

D = 1 UNIT $\rho_2 = 10\Omega \cdot m$

SCALE: 1 UNIT

Figure 29. Plan view ρ_a plot over a dipping body (30°) with $D=1$ unit. Contours in $\Omega \cdot m$. Electrode position given by *.

Not to Scale

ISOMETRIC

MISE-À-LA-MASSE:
DIPPING BODY (30 DEGREES)
APPARENT RESISTIVITY-FROM V

PLAN VIEW $\rho_1 = 100 \Omega \cdot m$

D = 1 UNIT $\rho_2 = 10 \Omega \cdot m$

SCALE: 1 UNIT

the 30 degree dipping body (not shown) further indicate that the surface response of a body dipping at a shallow angle looks similar to that of a horizontal body.

Effect of conductivity contrast.

(a) Section view. - For a horizontal body ($D=7.75$ units) with an electrode in its center and a contrast in conductivity of 30 (i.e., $\rho_1=300 \Omega\cdot m$ and $\rho_2=10 \Omega\cdot m$), the contour pattern of the subsurface ρ_a response (Figure 30) is that of the same situation with a contrast in conductivity of 10 (Figure 16). In both figures, the amplitudes of the contours reflect the background resistivity. However, examination of the figures shows that the percent anomaly relative to the background resistivity is higher for the larger contrast in conductivity. The maximum percent anomaly (the ρ_a low about the electrode) increases from approximately 35 percent (Figure 16) to approximately 45 percent (Figure 30) as the contrast in conductivity is increased from 10 to 30, respectively.

The percent anomaly about the electrode reaches its saturation value of approximately 50 percent for contrasts in conductivity of greater than 100 (not shown). The saturation value of the percent anomaly of the ρ_a high at the edge of the body is approximately 25 percent (not shown); saturation is reached for contrasts in conductivity of greater than 100. Subsurface modeling of the vertical body (not shown) reinforces these results.

(b) Plan view. - Surface modeling of a horizontal body ($D=1$ unit) with an electrode in its center and a contrast in conductivity of 30 further confirms that increasing the contrast in conductivity results in an increase in the percent anomaly, as is illustrated by comparison

Figure 30. Section view ρ_a plot through a horizontal body with $D=7.75$ units. Shaded region in inset shows plane of view. Contours in $\Omega\cdot m$. Electrode position given by *.

of Figures 31 and 17. Increasing the contrast in conductivity to values greater than 100 (not shown) saturates the maximum percent anomaly over the horizontal body ($D=1$ unit) at approximately 40 percent. In plan view, the saturation percent anomaly decreases as the depth to the body is increased (not shown). Since in plan view, the maximum percent anomaly is often a function of the electrode position within the body (see for example Figures 11, 12, and 13), a saturation percent anomaly cannot be given for an arbitrary body and arbitrary electrode position. Eloranta (1984) states this conclusion also. In addition, the value of the contrast in conductivity at which saturation occurs does not remain constant for all body orientations and electrode positions.

Near-Miss Models

The previous sections indicate that for a given plane of view and relative electrode position within the body, the four bodies produce subsurface ρ_a responses which differ little from each other with the exception of orientation. This also holds true for the case when the electrode is outside of the body. For this reason, subsurface ρ_a responses are presented here only for the vertical body. The responses of the other bodies are approximately given by rotating those of the vertical body by the appropriate amount.

Vertical body.

(a) Section view. - Placing the electrode just outside of the body produces subsurface ρ_a plots which are very similar in general character to those of when the electrode is in the body. As for the embedded electrode case, increasing the depth to the body has the effect of increasing the symmetry of the ρ_a plot. By the time $D=8$

Figure 31. Plan view ρ_a plot over a horizontal body with $D=1$ unit.
Contours in $\Omega \cdot m$. Electrode position given by *.

Not to Scale

ISOMETRIC

MISE-À-LA-MASSE:

HORIZONTAL BODY

APPARENT RESISTIVITY-FROM V

PLAN VIEW $\rho_1 = 300 \Omega \cdot m$

$D = 1 \text{ UNIT}$ $\rho_2 = 10 \Omega \cdot m$

SCALE : 1 UNIT

units, the subsurface ρ_a plot has essentially attained symmetry (Figure 32).

Comparison of Figure 32 with Figure 6 yields the following two features that persist whether or not the electrode is in the body: (1) a low in ρ_a exists within the body and is centered either around or directly across from the electrode, depending on whether the electrode is inside or outside the body, respectively, and (2) a high in ρ_a exists at edges of the body which are away from the electrode. These features diminish as the distance between the electrode and the body is increased. The low in ρ_a observed in Figure 32 is reduced to a 5 percent anomaly, relative to the background resistivity, when the distance between the electrode and the center of the body is increased from 1 to 4 units (not shown).

The two features listed above are present and create a bipolar pattern when the electrode is outside and near the bottom of the body (Figure 33). If the electrode is moved vertically up to the same relative position near the top of the body, the plot is that of Figure 33, only reflected over the x-y plane bisecting the body.

If the electrode is placed outside and near an edge of the body, and the plane of view contains the electrode, the subsurface response (not shown) changes little from that of the previous two figures, as is also the case when the electrode is embedded in the body (Figures 7 and 8). In addition, if the plane of view does not contain the electrode in the near-miss case, the resulting plots parallel those previously discussed for the situation when the electrode is in the body (see Figures 8, 9, and 10).

(b) Plan view. - As in the case of the subsurface near-miss

Figure 32. Section view ρ_a plot through a vertical body with $D=8$ units. Shaded region in inset shows plane of view. Contours in $\Omega \cdot m$. Electrode position given by *.

Figure 33. Section view ρ_a plot through a vertical body with $D=8$ units. Shaded region in inset shows plane of view. Contours in $\Omega \cdot m$. Electrode position given by *.

plots, the surface near-miss plots vary little in general character from those of when the electrode is embedded in the body. However, one deviation does occur when the electrode is directly above the top of the body and near the surface: a high in ρ_a centered about the electrode position. This is illustrated in Figure 34 for the case when the electrode is midway between the surface and a body with $D=1$ unit. This ρ_a high also occurs when the electrode is above the body near an edge (not shown), though the overall pattern is still bipolar (in a manner analogous to Figure 14).

The ρ_a high above the electrode is not present when the electrode is located at the side and below the top of the body. For this case, some asymmetry is introduced into the ρ_a plot. This is illustrated by comparing the resulting plot when the electrode is centered in the body ($D=1$ unit) .5 units below its top (Figure 11) with that of when the electrode is .5 units below the top and .5 units to the side of the body (Figure 35). In Figure 35, the closed ρ_a low still overlies the conductive body, but the contours are no longer perfectly symmetric. The points at which the contours begin to deflect outward do, however, approximately define the edges of the body.

The general character observed in Figure 35 remains as the electrode is moved further from the body (not shown), though as in section view, increasing the distance between the electrode and the body results in a decreased anomaly.

Locating the electrode at other places just to the side of the body also produces surface ρ_a plots (not shown) like those of the corresponding cases when the electrode is in the body (Figures 12-15). In addition, if the electrode is located directly below the

Figure 34. Plan view ρ_a plot over a vertical body with $D=1$ unit.
Contours in $\Omega \cdot m$. Electrode position given by *.

Figure 35. Plan view ρ_a plot over a vertical body with $D=1$ unit.
Contours in $\Omega\cdot m$. Electrode position given by *.

body, the resulting surface ρ_a plot (not shown) is difficult to distinguish from the corresponding plot when the electrode is inside and near the bottom of the body (Figures 13 and 15).

Horizontal body.

(a) Plan view. - With the exception of an ρ_a high centered directly above the electrode, the surface plots resulting when the electrode is .5 units above a horizontal body with $D=1$ unit (not shown) are essentially those previously given for the case when the electrode is in the body (Figures 17, 18, and 19). The amplitude and areal extent of the ρ_a high is approximately the same as that observed when the electrode is above a vertical body (Figure 34). The general character of the surface ρ_a responses in Figures 17, 18, and 19 is also preserved when the electrode is below the body. This situation results in a low in ρ_a above the electrode (not shown), provided the electrode is at least .5 units below the bottom of the body. Otherwise, a slight high in ρ_a still exists above the electrode.

When the electrode is to the side of the body, the surface plot differs little from when the electrode is in the body near an edge. This is illustrated by comparing Figure 36 with Figure 18. The effects in plan view of moving the electrode above and below that in Figure 36 are a slight high or low in ρ_a above the electrode, respectively.

Dipping body-60 degrees.

(a) Plan view. In general the surface responses for the body dipping at 60 degrees are very similar to those of the vertical body. With the electrode above the top of the body (not shown), the response resembles Figure 32, though a slight outward deflection of

Figure 36. Plan view ρ_a plot over a horizontal body with $D=1$ unit.
Contours in $\Omega \cdot m$. Electrode position given by *.

Not to Scale

NEAR-MISS: HORIZONTAL BODY
APPARENT RESISTIVITY -FROM V

PLAN VIEW $\rho_1 = 100 \Omega \cdot m$

D=1 UNIT $\rho_2 = 10 \Omega \cdot m$

SCALE: 1 UNIT

the contours at the updip edge does indicate that the body is not vertical. When the electrode is directly below the bottom of the body (not shown), a high in ρ_a centered over the updip edge of the body results (in the manner of Figure 13), though no indication of dip (i.e., asymmetry) is observed.

Whether the electrode is on the hangingwall or footwall side of the body has little effect upon the surface ρ_a plot, provided the electrode is near either the top or bottom of the body. However, this is not the case when the electrode is near the center of the body, as illustrated by comparison of Figure 37 (electrode on hangingwall side) with Figure 38 (electrode on footwall side).

Dipping body-30 degrees.

(a) Plan view. - The near-miss surface responses (not shown) are very similar to those of the horizontal body, as is also the case when the electrode is in the body. An ρ_a high above the electrode is observed when it is located above the top of the body. This ρ_a high transforms into an ρ_a low as the electrode is moved downdip while remaining the same distance above the body. The overall patterns are those of Figures 27, 28, and 29, though the percent anomalies relative to the background resistivity are somewhat reduced. The percent anomalies are further reduced when the electrode is on the footwall side of the body (i.e., underneath), but the overall pattern exhibits little change.

Effect of conductivity contrast. - In general, varying the contrast in conductivity has the same effects when the electrode is outside of the body as when it is inside. Varying the contrast does not alter the pattern of the ρ_a contours in either plan or section

Figure 37. Plan view ρ_a plot over a dipping body (60°) with $D=1$ unit. Contours in $\Omega \cdot m$. Electrode position given by *.

Not to Scale

ISOMETRIC

SECTION

MISE-A'-LA-MASSE:

DIPPING BODY (60 DEGREES)

APPARENT RESISTIVITY: FROM V

PLAN VIEW $\rho_1 = 100 \Omega \cdot m$

D = 1 UNIT $\rho_2 = 10 \Omega \cdot m$

SCALE : 1 UNIT $\longleftarrow$

Figure 38. Plan view ρ_a plot over a dipping body (60°) with $D=1$ unit. Contours in $\Omega\cdot m$. Electrode position given by *.

view. Increasing the contrast increases the percent anomaly relative to the background resistivity; saturation of the percent anomaly does occur.

CONCLUSIONS

Subsurface mise-a-la-masse modeling of the four bodies shows that varying the depth to the body does not affect the overall pattern of the anomaly. Effects of the surface of the earth vanish for bodies at around $D=8$ units. At depths approximately equal to this, the responses of the four bodies are essentially the same, with the exception of orientation, which is that of the body.

Moving the plane of view away from the electrode results in a decreased resolution of both the ρ_a low surrounding the electrode and the ρ_a high that occurs at edges of the body that are away from the electrode. These latter two features are also prevalent in the plan-view responses when the electrode is in the body. The closed ρ_a anomalies combine with inflection points to, in many instances, approximately define the areal extent of the body.

Estimates of the orientation and direction of dip of the body can also be made by consideration of the location and elongation of these anomalies. Though the same features in the contours appear for bodies of both shallow and steep dip, the two are discernible, provided the depth to the body is not too great. At depths greater than 4 units, the surface responses for all of the bodies decrease to levels that make detection difficult, if not impossible.

Increasing the contrast in conductivity does not alter the pattern of the anomaly, though it does result in an increase in the

percent anomaly. The percent anomaly does have a saturation value, but its value and the contrast in conductivity at which this occurs is dependent upon the body orientation and electrode position.

Near-miss model results, in both section and plan view, differ little from those of the case when the electrode is in the body. The ρ_a contour patterns are changed only slightly, if at all, though in some instances the magnitudes may be somewhat reduced. The net results are basically the same whether the electrode is above, below, or to the side of the body. Only when dealing with a steeply dipping body is there a significant difference between the surface plots for the case when the electrode is on the hangingwall versus when it is on the footwall side of the body.

Though the preceeding results suggest that borehole-to-borehole and borehole-to-surface techniques may be of use in detecting and/or delineating fracture zones in geothermal systems, they are for the ideal case of a single conductive body in a more resistive half-space. Hence, evaluations of the response of a 3-D body in the presence of conductive overburden and a layered earth still remain to be done. Use of the finite-element technique, as opposed to the integral-equation technique, will allow for modeling of more general bodies. In addition, the effects of various types of noise and evaluation of the signal-to-noise ratio have yet to be investigated. Furthermore, actual field evaluations of the techniques discussed need to be performed in order to ascertain their true applicability to exploration for geothermal fracture zones.

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