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TURBULENT INTERCHANGE IN TRIANGULAR
ARRAY BARE ROD BUNDLES

by

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M. Yeung, L. Wolf; "Effective Conduction Mixing Lengths for Subchannel Analysis of Finite Hexagonal LMFBR Bundles," ANS Meeting, New York, June 1977.

C. Chiu and N. Todreas; "Flow Split Measurements In An LMFBR Radial Blanket Assembly," ANS Meeting, New York, June 1977.

ABSTRACT

Bulk mixing coefficients were measured for single plane water flow in a simulated rod bundle with a pitch to diameter ratio of 1.10. A tracer technique employing Rhodamine B as the tracer and measuring fluorescence was used. Isokinetic sampling was achieved by using a pressure balance method. The results were corrected for both entrance effects and diversion crossflows.

Our results showed a change in Reynolds number behavior as the laminar sublayer began to "choke" the turbulent mixing. This, and a review of other mixing experiments, suggested that secondary flows do not compensate for laminarization and that turbulent mixing decreases as the pitch to diameter ratio decreases for values of P/D less than 1.05 in a manner similar to that predicted by Ramm et al.

Concentration profiles were measured through the clearance gap and the values of the gradient were used to calculate the gap averaged circumferential eddy diffusivity for mass.

A discussion of the eddy diffusivity concept and its applicability to turbulent mixing is presented.

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NOTATION USED

- A - subchannel cross-sectional area (Appendix C)
 - constant defined by equation (D-14b)
 - with numerical subscript, constant defined by (D-20)
- a_t - thermal diffusivity
- B - constant, $(\frac{W'}{m})$, equation (D-15b)
- C - dye concentration, when used with a subscript,
 indicates subchannel average value
- c - clearance gap (ft.)
- c_p - specific heat (BTU/lbm-°F)
- D - rod diameter (ft)
 - constant, equation (D-15c), (Appendix D only)
- D_h - hydraulic diameter (ft)
- E - constant, equation (D-15d)
- f - designates "function of"
- G - mass flux (lbm/hr-ft²)
- h - enthalpy ($\frac{\text{BTU}}{\text{lbm}}$)
- k - thermal conductivity ($\frac{\text{BTU}}{\text{hr-ft-}^\circ\text{F}}$)
- L - length (ft)
 - mixing length scale

- M - nondimensional mixing quantity, equation (C-6)
- m - mass flowrate ($\frac{\text{lbm}}{\text{hr}}$)
 - coefficient, equation (C-6)
- n - coefficient, equation (C-6)
- O - designates "is of the order of"
- Pr - Prandtl number
- P - pitch (ft)
 - pressure (PSI)
- q' - linear heat rate ($\frac{\text{BTU}}{\text{hr-ft}}$)
- q'' - heat flux ($\frac{\text{BTU}}{\text{hr-ft}}$)
- R - roots of a differential equation, (Appendix D)
- Re - Reynolds number
- r - radius (ft)
 - coordinate in the radial direction
- s - Laplace transform operator
- T - temperature ($^{\circ}\text{F}$), when accompanied by a subscript represents subchannel bulk value
- t - time (hr)
- u - velocity, ($\frac{\text{ft}}{\text{hr}}$), Appendix F
- v - velocity, ($\frac{\text{ft}}{\text{hr}}$)

- v - velocity, ($\frac{ft}{hr}$)
- W' - turbulent mixing coefficient
- W - diversion cross-flow
- x - x coordinate (axial)
- Y - distance between subchannel centroids
- y - Y coordinate (transverse)
- Z - thermal mixing distance
- z - z coordinate (axial in cylindrical coordinates)
- rod displacement, (Appendix G)

Greek Letters:

- ϵ - eddy diffusivity
- λ - mean free path
- ξ - nondimensional radius ($\frac{r}{R}$)
- ϕ - circumferential coordinate
- ρ - density (lbm/ft^3)
- τ - shear stress
- μ - viscosity ($lbm/hr.ft$)
- ν - kinematic viscosity (ft^2/hr)

Subscript

C	- calculated value
	- corrected value
H	- energy
i	- subchannel i
j	- subchannel j
L→R	- left to right
m	- measured
	- momentum
R→L	- right to left
r	- in the radial direction
x	- in the axial direction
y	- in the transverse direction
z	- in the axial direction (cylindrical coordinates)
φ	- in the circumferential direction
0	- initial value
1	- traced central subchannel
2	- central receptor subchannel

- 3 - peripheral donor subchannel
- 4 - peripheral receptor subchannel
- 5 - peripheral donor subchannel
- 6 - peripheral receptor subchannel

Superscripts

- 1 - fluctuating component
- corrected value
- * - nondimensionalized
- - time averaged
- gap averaged
- subchannel averaged
- Laplace transform

CHAPTER I

INTRODUCTION

The internal geometry of the majority of nuclear reactors is composed of a group of rod bundles. Each rod bundle is a cluster of fuel pins arranged in either of two configurations, the square array (prevalent in LWR's) or the triangular (proposed for the LMFBFR). Figure 1-1 illustrates these two geometries. A designer faces the task of predicting the rod and coolant temperatures subject to flowrates and heat fluxes which can vary radially, axially, and temporally. The standard method of attacking this problem is subchannel analysis, which is a lumped parameter, slug flow approach with the subchannel being the fundamental unit cell.

Subchannel analysis requires the use of mathematical expressions to describe the transport phenomena of mass, momentum and energy. These phenomena, as defined by Rogers and Todreas [1], are listed in Table 1-1. Some of these processes can be described exactly, but others must be approximated by empirical correlations. It is one of the latter phenomena, turbulent interchange, which is the topic of this thesis. This work was undertaken to contribute to the understanding of turbulent mixing and the buildup of data necessary for design criteria.

Square Array

Triangular Array

Subchannels

- 1. Central
- 2. Edge
- 3. Corner

FIGURE 1-1: Rod Bundle Geometries

TABLE 1-1
TRANSPORT PHENOMENA

	Natural Mechanisms	Forced Mechanisms
Molecular Effects	Conduction/Diffusion	
Nondirectional Flow Effects	Turbulent Interchange	Flow Scattering
Directional Flow Effects	Diversión Cross Flow	Flow Sweeping

1.1 Natural Transport Phenomena

Natural transport mechanisms consist of those processes which are operable in the absence of flow perturbations such as spacer grids.

1.1.1 Molecular Diffusion of Heat and Mass

In fluids, heat is conducted by an energy diffusion process which involves nearly-elastic molecular collisions. The macroscopic theory of heat conduction, based on the definition of thermal conductivity, quantifies this process by the Fourier Conduction Equation.

$$q'' = -k \left(\frac{\partial T}{\partial x_1} + \frac{\partial T}{\partial x_2} + \frac{\partial T}{\partial x_3} \right)$$

Heat conduction in LWR's is important for low Reynolds number conditions ($Re < 10,000$), and is important in LMFBR's even for moderately higher Reynolds number conditions.

In tracer experiments, such as ours, the tracer concentration is used as an analog for temperature and mass transfer processes simulate heat transfer processes. Thus, in our experiment, mass diffusion replaces heat conduction but is of little importance. This is demonstrated in Figure 1-2, a graph of calculations made by Ramm et al [2] showing the dependence of the mixing coefficient upon Reynolds and Prandtl number. A Prandtl number of unity corresponds to an LWR, $Pr = 0.007$ to an LMFBR, and $Pr = 1000$ to a tracer experiment using water and dye having a Schmidt number one thousand. In Figure 1-2, the coefficient, W_{1j}^2/μ , includes the transport due to conduction thereby inflating its value at low Reynolds and Prandtl number.

FIGURE 1-2: W'_{ij} vs. Pr and Re ²¹

1.1.2 Turbulent Interchange

Turbulent interchange is the name given to a form of mass transfer which enhances the diffusion of both heat (or dye concentration) and momentum. This mass transfer is due to the fluctuating pressures and velocities which occur in turbulent flow and is characterized by a zero net flow between communicating subchannels.

Since prediction of local temperatures and transverse velocities within a subchannel is quite complicated, the subchannel approach uses a macroscopic approach which requires expressions for the transport of momentum and energy between the lumped regions (i.e. subchannels). The heat transfer rate between two subchannels, i and j, is given by:

$$q' \Big|_{\text{gap}} = c_p W'_{ij} (T_i - T_j)$$

where W'_{ij} is the mixing coefficient. The quantity W'_{ij} has no real physical significance but is treated as a lateral flowrate through the clearance gap which, if it existed, would produce the same energy exchange rate as the turbulence actually produces.

1.1.3 Diversion Cross-flow

Diversion cross-flow is the directional flow between subchannels due to a redistribution of the axial flow split. This redistribution is the result of radial pressure gradients across the clearance gaps. In a reactor, these gradients can be established by either of two types of phenomena - geometry variations or non-uniform changes in fluid density. Geometry variations include such things as fuel rod

bowing and swelling, while density changes encompass the small differences across a bundle due to radial heat flux variations and the large local differences due to the onset of boiling.

In a tracer experiment, diversion cross-flow will be present if the test section geometry varies along its axis due to machining tolerances. The calculations in Appendix I show that for a subchannel whose area changes from its nominal value along the length of the bundle to a value $1/4$ of 1% less, the induced cross-flow will be of the same order of magnitude as the turbulent interchange. The presence of diversion cross-flow in a mixing experiment will lead to larger mixing rates, and the inferred value of the mixing coefficient will be higher than the true value.

1.2 Forced Mixing

Forced mixing occurs when structural elements are arranged to divert or disturb the fluid flow. Grid spacers, axial and radial fins cause random fluid dispersal where flow redistribution and wake effects enhance the mixing rates. This form of interchange is classified as "flow scattering". Wire-wrap spacers, helical fins and mixing vanes divert the fluid in preferred directions and is referred to as "flow sweeping".

CHAPTER 2

MIXING THEORY

2.1 The Macroscopic Energy Equation

In many heat transfer calculations, the energy equation has a differential form and contains the local values and local derivatives of enthalpy, velocity, and pressure. This approach is impractical for the analysis of large rod bundles, so an energy equation based on bulk subchannel properties has been adopted for subchannel analysis. The resultant "global" energy equation, using the notation of COBRA [3], for the i th subchannel is:

$$m_i h_i + \frac{\partial}{\partial x}(m_i h_i) dx + W_{ij}' h_i dx + W_{ij} h^* dx = q_i' dx + m_i h_i + W_{ji}' h_j dx \quad (2-1)$$

where h^* is the effective enthalpy transferred across the clearance gap by diversion cross-flow. h^* has a directional dependence, which is accounted for in its definition:

$$h^* = \begin{cases} h_i & ; \quad W_{ij} > 0 \\ h_j & ; \quad W_{ij} < 0 \end{cases}$$

By definition, turbulent interchange incurs a zero net flow, or

$$W_{ij}' = W_{ji}'$$

Substituting this into equation (2-1) and simplifying yields

$$\frac{\partial}{\partial x}(m_i h_i) = q_i' + W_{ij}'(h_j - h_i) - W_{ij} h^* \quad (2-2)$$

FIGURE 2-1: Schematic Energy Eqn.

Considering all channels adjacent to the i th, then

$$\frac{dm_i}{dx} = - \sum_{j=1}^N W_{ij}$$

and equation (2-2) becomes

$$n_i \frac{dh_i}{dx} = q'_i + \sum_{j=1}^N W'_{ij} (h_j - h_i) + \sum_{j=1}^N W_{ij} (h_i - h^*) \quad (2-3)$$

The term q'_i is the linear heat rate added to the i th subchannel. For "plane-source" types of tracer experiments this term is zero and equation (2-3) can be restated in terms of concentrations

$$n_i \frac{dC_i}{dx} = \sum_{j=1}^N W'_{ij} (C_j - C_i) + \sum_{j=1}^N W_{ij} (C_i - C^*) \quad (2-4)$$

2.2 Turbulent Interchange

2.2.1 Momentum Transport by Molecular Diffusion

The transverse transport of momentum in laminar flow can be described as a continuum process by the application of Newton's equation of viscosity, which states that the shear stress is proportional to the local velocity gradient. This transfer, however, is the result of a molecular process. Consider a gas flowing between parallel plates with a fully developed velocity profile. Individual molecules have, at a given instant, a random distribution of velocities superimposed upon the local macroscopic axial velocity. Consequently, transverse velocities are present in the flow, and any axial plane (except the boundaries) is penetrated by molecules from both directions in such a manner that there is a zero time averaged flow across the plane.

Due to these transverse velocities, molecules move randomly from a higher-velocity region into a lower velocity region and vice versa causing a transfer of momentum. If the length scales associated with the flow (λ , the molecule's mean free path) are small compared to the characteristic dimension of the flow passage, the axial velocity difference between the particle's "original" and "new" position is small and can be approximated by

$$v_x(\lambda) - v_x(0) = \frac{\Delta v_x}{\Delta y} \lambda \approx \lambda \frac{dv_x}{dy} \quad (2-5)$$

shown schematically in Figure 2-2.* For a Newtonian fluid, the statistical average of these molecular interactions results in a local momentum transport rate that is directly proportional to the velocity gradient and is a restatement of Newton's equation of viscosity.

2.2.2 Turbulent Transport and Eddy Diffusivities

In turbulent flow, there are two mechanisms for the transverse transport of momentum. This is illustrated by the time averaged Navier-Stokes equation (for a 2-D flow in X direction)

$$\begin{aligned} \frac{\partial \bar{v}_x}{\partial t} + \bar{v}_x \frac{\partial \bar{v}_x}{\partial x} + \bar{v}_y \frac{\partial \bar{v}_x}{\partial y} = & \left(\frac{1}{\rho} \left(\mu \frac{\partial \bar{v}_x}{\partial y} - \overline{\rho v'_x v'_y} \right) \right. \\ & \left. - \frac{\partial}{\partial x} (\overline{\rho v'_x}) \right) - \frac{1}{\rho} \frac{\partial \bar{P}}{\partial x} \end{aligned} \quad (2-6)$$

where

$$\begin{aligned} v_x &= \bar{v}_x + v'_x \\ v_y &= \bar{v}_y + v'_y \end{aligned}$$

*Figure 2.3 of Reference 4

FIGURE 2-2
MOLECULAR MOTION IN A SHEAR FLOW

and the barred quantities represent mean components independent of time, and the primed quantities, the fluctuating components. For steady, fully developed flow, equation (2-6) reduces to

$$\bar{v}_y \frac{\partial \bar{v}_x}{\partial y} = \frac{1}{\rho} \left[\frac{\partial}{\partial y} \left(\mu \frac{\partial \bar{v}_x}{\partial y} - \overline{\rho v'_x v'_y} \right) \right] - \frac{1}{\rho} \frac{\partial \bar{P}}{\partial x} \quad (2-7)$$

The expression in parentheses is the momentum flux in the Y direction and can be rewritten in terms of an apparent shear stress

$$(\tau_{yx})_{APP} = - \left(\mu \frac{\partial \bar{v}_x}{\partial y} - \overline{\rho v'_x v'_y} \right) \quad (2-8)$$

This apparent flux is comprised of a molecular diffusion flux (as in laminar flow) and an eddy flux which is responsible for the increased transport rates that characterize turbulent flow.

These eddies are "patches" of fluid which have a randomly fluctuating velocity distribution superimposed upon the local average axial velocity ($\bar{v}_x$). As such, these "patches" or "moles" are somewhat analogous to the molecules in the momentum diffusion process for laminar flow. Figure 2-3* shows that the steeper the velocity gradient ($\partial \bar{v}_x / \partial y$), the greater will be the v'_x induced by a v'_y , and hence the product $\overline{v'_x v'_y}$ will be larger. This suggests that the turbulent shear stress be treated in a manner similar to the molecular shear stress and the eddy diffusivity for momentum, ϵ_m , is defined for this purpose, such that

$$\overline{v'_x v'_y} = - \epsilon_m \frac{\partial \bar{v}_x}{\partial y} \quad (2-9)$$

*Figure 8.1 of Reference 5

FIGURE 2-3
 V'_x Induced By V'_y

Scalar properties, such as temperature, are also transported by turbulent eddies and the time averaging of the energy equation (subject to the same restrictions as the momentum equation) yields an apparent heat flux in the Y-direction:

$$(q'')_{y,APP} = - \left(k \frac{\partial \bar{T}}{\partial y} - \rho c_P \overline{v'T'} \right) \quad (2-10)$$

As before, the turbulent transport term is related to the temperature gradient by an eddy diffusivity, and the eddy diffusivity for energy, ϵ_H , is introduced, such that

$$\overline{v'T'} = - \epsilon_H \frac{\partial \bar{T}}{\partial y} \quad (2-11)$$

2.2.3 The Validity of the Eddy Diffusivity Concept

Although widely used, the eddy diffusivity approach is neither entirely correct nor as straightforward as it appears. First, it must be noted that eddy diffusivities are not properties of the fluid (like the viscosity) but rather are properties of the flow. As such, these diffusivities have, at least, the functional dependencies noted below:

$$\epsilon_m = f(\text{Re}, \text{Geometry}, \text{Position})$$

$$\epsilon_H = f(\text{Re}, \text{Geometry}, \text{Position}, \text{Pr})$$

Consequently, the desired simplification of the Navier-Stokes and the energy equation is not realized. References [6] and [8] contain several graphs showing the parametric dependence of the eddy diffusivities mentioned above.

Important discrepancies between the concept and the physical situation it describes arise when the values predicted near the flow boundaries and on the zero shear stress axis are examined. On the axis where the velocity gradient vanishes, the eddy diffusivity approach states that the turbulent transfer term will also vanish. Bulëev [7], notes that turbulence does exist along this axis and that the turbulent transport cannot be zero. Near the channel boundaries, the velocity gradient is steepest - hence, this theory predicts the largest amount of transport in this region. However, approximately thirty percent of this boundary layer region is contained in the laminar sub-layer and makes no contribution to the eddy fluxes.

More fundamentally, the "statistical exactness" which existed for transport by molecular diffusion does not exist for turbulent eddy diffusion. The greater length scales of turbulent flow preclude this "exactness" by undermining two basic assumptions. First, in laminar flow, the molecules retain their individual axial velocities as they migrate from one region to another. If this were true for turbulent flow, the fluctuating velocity at the point where the eddy "breaks-up" would simply be the difference of the axial velocities, namely

$$v'_x(\lambda) = \bar{v}_x(0) - \bar{v}_x(\lambda) \quad (2-11)$$

which is similar to the laminar result. However, eddies are governed by continuum rather than particle mechanics and therefore have interactions before they reach the "break-up" point. These interactions result in a decaying of the eddy and lower the absolute value of $v'_x(\lambda)$. The decay process for momentum is more pronounced than that for energy (not necessarily true for low Pr).

Also, the lateral distance the eddies traverse is of the same order of magnitude as the channel's characteristic dimension. The consequent axial velocity difference (Equation 2-11) can be so large that it cannot be reduced to the differential format (equation 2-5) employed in molecular diffusion. This is especially true for regions where significant velocity (or temperature) changes occur over a small part of the channel.

2.2.4 Eddy Diffusivities and Turbulent Interchange

In a rod bundle, eddies are responsible for two types of turbulent energy transport - that within a subchannel and that between subchannels. Within a subchannel, the eddy transport enhances the heat transfer from the fuel rod surface resulting in higher heat transfer coefficients and a lower clad temperature for the same heat flux and subchannel bulk temperature. This phenomena is the same as that which occurs in turbulent pipe flow and can be handled (except for liquid metals) using the "hydraulic diameter-friction factor" approach common to all of turbulent flow heat transfer.

Eddy transport through the clearance gap can increase the inter-subchannel energy transfer by several orders of magnitude over the heat conduction value for water, and one order of magnitude for liquid metals (see Figure 1-2). This tends to even out the bundle's temperature distribution due to the radially varying heat flux and reduces the associated hot spot factor.

In subchannel analysis, the desired output is the axial variation of every subchannel's bulk temperature. This requires detailed knowledge of the linear heat generation rate of the fuel rods and the interchange between subchannels. The latter is quantified by performing a heat balance across the clearance gap. From the time-averaged energy equation the heat transferred between subchannels is:

$$q_{1j}''(r) = \frac{q_{1j}^i(r)}{P-D} = - \left[\frac{k}{r} \frac{\partial T(r)}{\partial \phi} + \rho c_P \overline{v_\phi'(r) T'(r)} \right]_{\phi=0} \quad (2-12)$$

where the first term on the right-hand side is the conductive heat transfer and the second is the convective transfer. Equation (2-12) is valid for steady-state, fully developed, incompressible flows, with fluid properties that are independent of temperature, and velocity fields which do not possess either diversion cross-flows or secondary flows. These restrictions will be present throughout this analysis.

Using the eddy diffusivity approach, equation (2-11) can be rewritten in cylindrical coordinates

$$\overline{v_\phi'(r) T'(r)} = - \epsilon_{H\phi}(r) \left(\frac{1}{r} \right) \left(\frac{\partial \overline{T(r)}}{\partial \phi} \right) \quad (2-13)$$

and substituted into equation (2-12) to give:

$$\frac{q_{1j}^i(r)}{P-D} = - \left[\frac{k}{r} \frac{\partial T(r)}{\partial \phi} + \rho c_P \epsilon_{H\phi}(r) \left(\frac{1}{r} \right) \frac{\partial \overline{T(r)}}{\partial \phi} \right]_{\phi=0} \quad (2-14)$$

Overbars were included in equations (2-13) and (2-14) to stress that these are time-averaged values of the temperature gradient, henceforth the bars will be omitted. To proceed towards attaining a macroscopic expression for the interchange rate, the spatial average of (2-14) is taken:

$$\frac{q_{1j}^i}{P-D} = - \frac{\int_{D/2}^{P/2} [k + \rho c_P (\epsilon_{H\phi}(r))_{\phi=0}] \left(\frac{1}{r} \right) \frac{\partial T(r)}{\partial \phi} \Big|_{\phi=0} dr}{\int_{D/2}^{P/2} dr} \quad (2-15)$$

Introducing the following definitions

$$Pr = \frac{c_p \mu}{k}$$

$$\xi = \frac{r}{R}$$

and simplifying equation (2-15)

$$q'_{1j} = - 2\mu c_p \int_1^{P/D} \left| \frac{1}{Pr} + \left(\frac{\epsilon_{H\phi}(\xi)}{\nu} \right)_{\phi=0} \right| \left| \frac{1}{\xi} \frac{\partial T(\xi)}{\partial \phi} \right|_{\phi=0} d\xi \quad (2-16)$$

Equation (2-16) is the exact expression for q' , the power crossing the gap per unit axial length. Now, making the assumption that the temperature gradient $\left| \frac{1}{\xi} \frac{\partial T(\xi)}{\partial \phi} \right|_{\phi=0}$ is constant over ξ ,

$$q'_{1j} = - 2\mu c_p \left| \frac{1}{\xi} \frac{\partial T(\xi)}{\partial \phi} \right|_{\phi=0} \int_1^{P/D} \left| \frac{1}{Pr} + \left(\frac{\epsilon_{H\phi}(\xi)}{\nu} \right)_{\phi=0} \right| d\xi \quad (2-17)$$

This assumption may seem somewhat arbitrary, but it is part of the usual procedure for deriving the relation between the bulk mixing coefficient and the eddy diffusivity for energy. Also, Ramm et al [2] performed calculations using a phenomenological turbulence model which demonstrated that the circumferential temperature gradient at the gap was nearly constant radially.

The gap average of the eddy diffusivity is given by

$$\left(\frac{\epsilon_{H\phi}}{\nu} \right)_{\phi=0} = \frac{\int_1^{P/D} \left(\frac{\epsilon_{H\phi}(\xi)}{\nu} \right)_{\phi=0} d\xi}{\int_1^{P/D} d\xi}$$

and substitution into equation (2-17) yields

$$q'_{1j} = - 2\mu c_P \left(\frac{P}{D} - 1 \right) \left| \frac{1}{\xi} \frac{\partial T(\xi)}{\partial \phi} \right|_{\phi=0} \left| \frac{1}{Pr} + \left[\frac{\overline{\epsilon_{H\phi}}}{\nu} \right] \right|_{\phi=0} \quad (2-18)$$

In the absence of diversion cross-flows, which if present would alter the temperature gradient, the energy transported across the gap per unit axial length, q'_{1j} , can be equated to the turbulent mixing rate of the macroscopic energy equation, thus

$$q'_{1j} = W'_{1j} c_P (T_1 - T_j) \quad (2-19)$$

From Eqs. (2.18) and (2.19), the definition of the bulk mixing coefficient becomes

$$W'_{1j} = \frac{- 2\mu \left(\frac{P}{D} - 1 \right) \left| \frac{1}{\xi} \frac{\partial T(\xi)}{\partial \phi} \right|_{\phi=0} \left| \frac{1}{Pr} + \frac{\overline{\epsilon_{H\phi}}}{\nu} \right|_{\phi=0}}{(T_1 - T_j)} \quad (2-20)$$

where W_{1j}^* includes both molecular and eddy effects. An examination of the functional dependence of the terms in equation (2.20) will reveal the parameters of which the nondimensional mixing coefficient, (W_{1j}^*/μ) , is a function.

$$1) \left(\frac{P}{D} - 1\right) = f\left(\frac{P}{D}\right),$$

$$2) \frac{1}{Pr} = f(Pr); \text{ temperature dependence is neglected by one of our initial assumptions,}$$

$$3) \left| \frac{\bar{\epsilon}_{H\phi}}{v} \right|_{\phi=0} = f(\text{Geometry, Re, Pr, Axial Position (Z)});$$

If the velocity profile is fully developed, another of our assumptions, the dependence on Z can be neglected and the asymptotic value used. Also, geometry can be replaced by subchannel type (i.e., central, corner or edge), array type (square, triangular), and P/D.

$$4) \frac{\left| \frac{1}{\xi} \frac{\partial T(\xi)}{\partial \phi} \right|_{\phi=0}}{(T_1 - T_j)} = f(\text{Boundary Conditions, Geometry, Re, Pr, Z});$$

Boundary conditions specify inlet temperatures and temperature gradients at the rod surfaces. An important distinction is whether the case under consideration is of the "plane-source" or the "heated-rod" type. Geometry dependence is upon the same factors as $(\bar{\epsilon}_{H\phi}/v)$. This ratio of gradient to bulk difference varies along the length of the rod bundle due to the development of the temperature profile. An illustration of

this variation, and its convergence to an asymptotic value is given in Appendix J for a plane-source, rectangular channel, slug flow example. In situations where the boundary conditions are not functions of Z , this ratio will approach a constant value and after a, "developing length", and can be treated as a constant thus eliminating axial dependence.

The above leads to:

$$\frac{W'_{ij}}{\mu} = f(\text{subchannel type, array type, } \frac{P}{D}, Z, Re, Pr)$$

and, if fully developed

$$\frac{W'_{ij}}{\mu} = f(\text{subchannel type, array type, } \frac{P}{D}, Re, Pr)$$

Figure 2-4 is a graph of calculations made by Ramm et al [2] showing the axial variation of the mixing coefficient.

To complete this mixing length analysis, the thermal mixing length scale, L_{ij} , will be derived. The thermal mixing length scale is formed by dividing the effective thermal mixing distance, Z_{ij} , by the distance between adjacent subchannel centroids, Y_{ij} , yielding

$$L_{ij} = \frac{Z_{ij}}{Y_{ij}} \quad (2-22)$$

The effective mixing distance is defined as that distance which, when divided into the subchannel bulk temperature difference yields the gap averaged circumferential temperature gradient, or

$$\frac{T_i - T_j}{Z_{ij}} = - \frac{2}{(P-D)} \int_1^{P/D} \left| \frac{1}{\xi} \frac{T(\xi)}{\partial \phi} \right|_{\phi=0} d\xi \quad (2-23)$$

FIGURE 2-4: Axial Variation of W'_{ij}

Using the assumption of a radially constant temperature gradient,
the effective mixing distance becomes

$$Z_{ij} = \frac{-D(T_i - T_j)}{2 \left[\frac{1}{\xi} \frac{\partial T(\xi)}{\partial \phi} \right]_{\phi=0}} \quad (2.23)$$

and the mixing scale is given by

$$L_{ij} = \frac{-D}{2Y_{ij}} \frac{(T_i - T_j)}{\left[\frac{1}{\xi} \frac{\partial T(\xi)}{\partial \phi} \right]_{\phi=0}} \quad (2.24)$$

Or, in terms of the mixing coefficient defined in equation (2-21)

$$L_{ij} = \frac{(P-D)}{Y_{ij}} \frac{\left[\frac{1}{Pr} + \frac{\bar{\epsilon}}{v} \right]}{\left(\frac{1}{u} \right)} \quad (2.25)$$

where for square arrays

$$Y_{ij} = P,$$

and for triangular arrays

$$Y_{ij} = \frac{\sqrt{3}}{3} P$$

2.2.5 Comments on the Validity of a Mixing Coefficient Based on the Eddy Diffusivity Concept

From section 2.2.3 it would appear that the mixing length theory could have little significance in the study of turbulent mixing. However, the comments made there were in reference to momentum mixing within a channel and do not necessarily apply to thermal mixing between channels. By noting the difference between these situations, the two assumptions necessary for "statistical exactness";

- 1) The particle (or mole) retains its characteristic axial velocity (or temperature) along its path, and
- 2) The mean free path (or associated length scale) is small compared to the distance over which the gradient occurs,

can be shown to be plausible.

First, the temperature of a mole is more nearly conserved than its momentum [4] and hence condition (1) is more nearly satisfied. Also, in turbulent flow, the major portion of the velocity gradient occurs in a boundary layer which occupies a small portion of the channel cross-section. Whereas the temperature gradient between subchannels occurs over a much larger distance, compared to which the mean free path (which is the same for momentum and energy transport) can be small. Figure 2-5* shows the radial distribution of velocity for a triangular array rod bundle with $P/D = 1.10$, at a Reynolds number of 50,000, and Figure 2-6** shows a calculated temperature gradient for the same

*Figure 9 of Reference 9

**Calculations received in a personal communication with K. Johannsen, April, 1975.

FIGURE 2-5
Radial Velocity Distribution

NONDIMENSIONAL TEMPERATURE

FIGURE 2-6 FULLY DEVELOPED TEMPERATURE PROFILE

Geometry for Figure 2-5

Geometry for Figure 2-6

FIGURE 2-7

geometry, "plane-source" boundary conditions, a Reynolds number of 50,410, a Prandtl number of 1,000, and an axial position where the mixing coefficient is fully developed.

Assuming the eddy diffusivity approach is a valid method for describing turbulent mixing, the relation between $\overline{\epsilon_{H\phi}}$ and W'_{ij} (Eqn. 2-20) could be experimentally verified (or disproved). To check this relation, both the bulk subchannel temperatures and the temperature gradient at the gap would have to be measured. It is probable that this relation is an adequate description of the transfer rate in a fully developed "plane-source" experiment. However, in a situation where the turbulent length scales are large and steep temperature gradients exist within or between subchannels this relation can break down due to the use of the gap averaged temperature gradient and eddy diffusivity. In such a situation, detailed knowledge of the eddy diffusivities within the channel would be necessary to predict the subchannel interchange rate. If a computer code (such as Rasmussen et al.'s) which uses predicted local values of the eddy diffusivity could be "calibrated" using measured temperatures and gradients from both "plane-source" and "heated-rod" experiments, then subchannel interchange rates could be predicted for any flow or heat flux conditions with a reasonable confidence. This would allow prediction of bulk mixing coefficients for the particular flow and heat flux conditions of each channel, rather than an experimental correlation which depends only on flow conditions. The relation between $\overline{\epsilon_{H\phi}}$ and W'_{ij} for a "plane-source" experiment is investigated in this thesis.

CHAPTER 3

EXPERIMENTAL APPARATUS AND PROCEDURE

Mixing tests were conducted using a simulated triangular array rod bundle with a $\frac{P}{D}$ of 1.10 for single phase water flow. In this experiment, both subchannel bulk concentrations (from which the bulk mixing coefficient can be inferred) and the gap concentration profile (from which, when combined with the bulk concentrations the value of the circumferential eddy diffusivity at the gap can be estimated) were measured. The flow loop and its major components are detailed in section 3.1 while the experimental procedures for the bulk and local measurements are described in sections 3.2 and 3.3 respectively.

3.1 EXPERIMENTAL APPARATUS

A schematic of the test rig is given in Figure 3.1 and its components are discussed below.

3.1.1 The Test Section

In designing the test section, several objectives were of primary concern. These were:

1. Maintenance of design tolerances and subchannel symmetry over the entire length of the test section,
2. Fluid boundary conditions of sampled subchannels should be the same as those of a central subchannel in a rod bundle,
3. Dye concentration should be uniform in the donor channel before mixing begins,

FIGURE 3-1: SCHEMATIC OF THE TEST RIG

4. Sufficient cross sectional area, so that the gradient across the gap could be measured,
5. Machining simplicity,
6. Ability to change P/D so that test section can be re-used,
7. Reasonable cost, and
8. Corrosion resistance.

To achieve objectives 1 and 2, we decided on a variation of the two subchannel type of test section rather than a mock-up of a rod bundle. The simple two channel section used by Walton (25) is illustrated in Figure 3.2. This approach was rejected because we felt the presence of the wall would cause the point of maximum velocity to move closer to the clearance gap. This would reduce the effective mixing distance and lead to higher values for the measured mixing rates.

For our new test section we chose the six channel design of Figure 3.3. We sacrificed the shape of the peripheral channels in order to achieve objectives 5, 6 and 7. The rods were centerless ground to the correct diameter, then flats were milled onto them, and they were bolted and pinned into place with spacers (which were later removed) in the rod bundle.

Since this experiment necessitates the use of an open flow loop, water from the city water service must be used and corrosion resistance becomes important. Plexiglas was ruled out because of the machining problems encountered in a former test section. Finally, brass was chosen over stainless steel as the construction material due to its machinability. The individual sections will be explained below.

FIGURE 3-2
Walton's Test Section

FIGURE 3-3

CROSS-SECTION OF TEST SECTION

Pitch To Diameter Ratio	1.10
Rod Diameter	1.50 in.
Hydraulic Diameter	0.5103 in.

a. Entrance Section (Fig. 3-4a)

The purpose of this section is to let the flow split naturally and then inject the dye into the right or the left section and the developing sections to provide further assurance that the dye concentration is uniform.

b. Developing Sections (Fig. 3-4b)

These two sections have the same geometry as the mixing sections with the addition of a 0.020 in. separator strip to prevent mixing until after the velocity profile has a chance to develop.

c. Mixing Sections (Fig. 3-4c)

Mixing occurs in these two sections, and data runs can be taken using either one or two sections so that entrance effects present in the first section can be subtracted. A complete listing of relevant dimensions is given in Table 3-1.

3.1.2 Water Supply (Figure 3-5)

A 350 gallon tank fed by the city water service with hot and cold water, (for temperature regulation), is the supply for two pump-motor combinations. These pumps can be used separately or in series to provide flows of 2 to 40 GPM. Pump no. one is a Worthington D-820 centrifugal pump, coupled with a General Electric 5 horse power electric motor. Pump no. two is an Ingersoll-Rand 1-1/2 RVH 7-1/2 centrifugal pump, coupled with a 7-1/2 horsepower electric motor.

a) Entrance Section

b)

FIGURE 3-4: Test Section Schematic

FIGURE 3-5
Water Supply

FIGURE 3-6
Tracer Injection System

TABLE 3-1
TEST SECTION SPECIFICATIONS

Triangular array	60°	
Pitch-to-diameter ratio	1.10	
Pitch	1.650"	
Rod diameter	1.500"	
Gap	.150"	
Hydraulic diameter	0.5103	
Length	(in)	(L/D _H)
Total	94.9	188.49
Developing sections	48.12	95.99
Mixing section #1	22.31	44.50
Mixing section #2	24.06	48.00
Mixing section total	46.37	92.50
Window thickness	.020"	

3.1.3 Input Flow Regulation

To measure the input flowrates a matched pair of Fisher and Porter rotameters, model no. 10A3567, were used. These meters each have two sets of tubes and floats so that their maximum rating can be set to 20.0, 9.5, or 6.0 GPM (see Appendix A for calibration curves). Regulation is provided by 3/4" globe valves.

3.1.4 Tracer Injection System (Figure 3-6)

Bulk Mixing Experiments

The tracer injection system utilizes a 15 gallon galvanized pressure tank as a reservoir. This tank was statically charged with air to provide the necessary pressure head for injection. Pressure control was accomplished with a conventional regulator supplied by service air at 100 psig. Flow control was established by a 1/8" needle point metering valve downstream of a Fisher and Porter, model no. 10A1017-A, "Tri-Flat" rotameter rated at 200 cc/min.

Local Concentration Experiments

The measurement of local concentrations necessitated a dye injection system that was more precise and stable than the above. To this end, a fixed displacement pump combined with a speed controlled electric motor, Masterflex model #7015, was used. This system was capable of maintaining a specific dye flowrate over a longer period of time, so that a set of local concentration measurements could be taken without variation of the inlet concentration. Unfortunately, this pump could not provide enough head to be used in the bulk experiments.

3.1.5 Exit Flow Regulation

In the Bulk Mixing Experiments, exit flow rates were measured and each exit line was regulated by a set of two valves placed in parallel. The large valves, 1/2" globe valves, were for coarse control, while the small valves, 1/8" brass needle point, served the dual purpose of fine control and sample withdrawal.

3.1.6 Isokinetic Sampling System

In the bulk mixing tests, it was desired to sample the bulk concentration of the central receptor channel alone, rather than the average concentration of all three receptor channels. To do this, probes shaped to fit the subchannel cross section, (see Figure 3-7a) were inserted into the rod bundle's central channels at the sampling plane. The exit flow from each of these subchannels was "pulled" using two small pumps (Oberdorfer - 1/4 HP) to make up the difference in flow resistance. Flow measurement was provided by either one of two matched pairs of Fisher and Porter rotameters, model nos. 10A3567 and 10A1027, with maximum rated flows of 6.5 and 2.7 GPM. Flow regulation was provided by the valves described in section 3.1.5. Isokinetic sampling was achieved by re-establishing the pressure differences that existed in the bundle when the sampling probes were not in position.

During local concentration measurements, samples were withdrawn through a pitot-static tube of 1/16" OD and .020" ID. This probe was connected to a carriage which could be positioned to within $\pm .0002$ " using a micrometer head, Mitutoya #153-206, with a non-rotating spindle.

FIGURE 3-7: Probe and Sampling System

A standard laboratory vacuum pump was used to "pull" the sample as is shown in Figure 3-8. Flow metering was provided by four Fisher and Porter rotameters, model #10A1017, placed in parallel and having maximum flowrates of 1.05, 3.20, 10.3, and 18.6 cc's per minute. Flow regulation was provided by 1/8" needle valves. Isokinetic sampling was achieved by setting the sampling flowrate so that the average velocity of the fluid entering the probe was equal to the velocity at that position in the flow field.

3.1.7 Pressure Measurement

A Datametrics Barocel "Electric Manometer" (sensor type 517 - signal processor 1013) adapted for water service was used to measure differential pressures. This instrument has seven scale ranges providing full scale readings from $\pm .0002$ PSI to ± 20.0 PSI with an accuracy of $\pm 0.1\%$ of full scale. Also, a ± 1 volt DC output voltage, directly proportional to pressure, is provided by the signal processor. This signal was fed to a TSI integrating voltmeter to filter out turbulent fluctuations and read the time averaged differential pressure.

3.1.8 Instrumentation System

In our experiments, Rhodamine B was used as the tracer and a fluorometric technique was used to determine sample concentrations. A Beckman Ratio Fluorometer, model no. 772, with a flow through cell was used. This instrument compares the fluorescence of the sample to a standard solution and records the percentage of fluorescence. Unfortunately, dye concentration is not linear with fluorescence. If it were, the input solution could be used as the standard solution and the

FIGURE 3-8: POINT CONCENTRATION SAMPLING SYSTEM

nondimensional outlet concentration read from the meter. Consequently a calibration curve was generated (Appendix B) and all samples had to be converted to PPM. The useful operating range with this dye was from .005 to .1 PPM with an error of $\pm 0.2\%$ of full scale

3.1.9 Description of the Flow Loop

Figure 3-9 is a schematic of the flow loop. Essential features are:

1. Input flow measurement - the total mass flowrate can be measured by either or both of two rotameters. By using different combinations of floats and tubes the error can be kept as small as possible ($\pm 1\%$ of total rated flow),
2. Tracer injection - dye is injected into either channel in the entrance section (Fig. 3-5a) just upstream of the plenum,
3. Flow split - the inlet flow splits "naturally" via a plenum in the afore-mentioned entrance section,
4. Sampling planes - the bulk concentration of the central subchannels can be sampled at two axial locations (in different test runs) by the removal of the second mixing section.
5. Exit flow measurement and regulation - the flowrate of each central subchannel is monitored by either a low range (maximum flow = 2.7 GPM) or a high range (max = 6.5 GPM) rotameter. Valves for both coarse and fine control are provided as described in Section 3.1.5. A valve is provided on the exit line for the other four channels to create a back pressure to damp out variations, and,

FIGURE 3-9: Flow Loop

6. Instrumentation - the exit concentration of the central receptor channel can be continuously monitored using the Ratio Fluorometer.

3.2 EXPERIMENTAL PROCEDURE - BULK MIXING TESTS

The primary objective of this investigation is the determination of bulk mixing coefficients for a triangular array rod bundle. To accomplish this, a tracer-type, plane-source mixing experiment was conducted. The distinguishing features of this type experiment are:

1. A tracer is used as an analog for heat, and hence mass transfer processes replace heat transfer processes,
2. The experiment is "adiabatic" and the boundary conditions are thereby reduced to initial values which are further simplified by making the initial concentrations uniform over a subchannel.

The basic format of the experiment is outlined below:

1. The inlet flow is split into two parts in the entrance section,
2. One half of the flow is then traced with a dye,
3. Both flows enter the "development" sections where they remain separated while the velocity profile develops ($96 L/D_h$'s),

4. The flows enter the "mixing" sections and the two flows come in contact with each other and mixing begins,
5. Along the length of the mixing section (44.5 or $92.5 L/D_h$'s) the initially infinite concentration gradient between the traced and untraced (receptor) channels develops as the concentration of the receptor channel increases,
6. At the sampling plane, the entire flow from each of the two central subchannels is withdrawn so that their bulk concentrations can be measured,
7. A sample is also taken from the traced inlet flow and its concentration is used to non-dimensionalize the exit concentration.

From these nondimensional exit concentrations the mixing rate can be found.

Concurrent with the turbulent transport, mass is also transferred through the clearance gap by both diversion crossflow and entrance effects. The presence of these mechanisms in a mixing test will result in higher values for the measured exit concentration (receptor channel) which, if not analytically compensated for, will produce mixing rates higher than the turbulent mixing rate. Diversion crossflows can be caused by either geometrical asymmetries in the test section or by withdrawing the exit flows of the two central subchannels in a non-isokinetic manner. The latter is a function of the sampling flowrate which can be controlled by the experimenter and will be discussed later. The crossflows caused by geometrical variations, however,

cannot be eliminated and since the concentration change they cause is an integrated effect they cannot be "balanced" out either. If the axial variation of the crossflow is known (or can be assumed) and mixing experiments are conducted so that exit concentrations are recorded when both sides of the test section are alternately traced (i.e. mixing from right to left and left to right) then the magnitude of the diversion crossflow can be calculated and its influence on the mixing coefficient negated. The discussion in Appendix I postulates that the most likely geometrical asymmetry would cause an axially constant diversion crossflow. This assumption together with the method of data analysis detailed in Chapter 4 are used to subtract the crossflow mixing rate from the total mixing rate. This procedure requires making data runs in which the central subchannels are alternately traced.

Entrance effects are comprised of three phenomena:

1. After leaving the development sections, the fluid velocity profile is not fully developed due to the separator strip,
2. Wake effects will be present at the trailing edge of the separator strip, and
3. The mixing coefficient has an axial dependence in its developing region.

All three of these phenomena lead to mixing rates higher than the asymptotic value of the turbulent mixing coefficient. To subtract out these effects, mixing data was taken at two different axial

locations (44.5 and 92.5 L/D_n 's). Then the exit concentrations from the shorter mixing section were used as the inlet concentrations to a computer code which used the exit concentrations of the longer section as the exit concentrations of a mixing section 48 L/D_n 's long. This procedure subtracts the entrance effects in the first section from our results but requires taking a complete set of data for each mixing length.

To isokinetically sample the two central subchannels, it is necessary to withdraw the entire flow of each channel in such a manner that there is no redistribution of the flow field. With the total flowrate set by the inlet flow, there are two flow splits that must be maintained, that between the two central subchannels and the four peripheral channels and that between the two central subchannels. The former can be maintained by adjusting the total exit flowrate of the four peripheral channels so that the test sections axial pressure drop is equal to what it would be if the sampling probes were not in the rod bundle. Then the flow split between the two central subchannels can be maintained by adjusting their flowrates to re-establish the undisturbed pressure difference between these two subchannels (henceforth referred to as the "radial" pressure drop) while keeping the axial pressure drop at its undisturbed value.

The testing procedure that results from the above is:

1. Pressure Tests - both the axial and radial pressure drops are measured for a set of Reynolds numbers when the sampling probes are not in place,

2. Mixing Tests (Long Section) - a complete set of exit concentrations versus Reynolds number is taken for $L/D_h = 92.5$ and for both mixing directions;
 - i) Mixing right to left - the three right channels are traced
 - ii) Mixing left to right - the three left channels are traced,
3. Mixing Tests (Short Section) - the second mixing section is removed and exit concentrations are measured at $L/D_h = 44.5$, for both steps i and ii.

The sequential procedure followed in steps 2 and 3 is given below:

1. Tracer Solution - the tracer was mixed to a concentration (400 PPM) just beyond the saturation limit of Rhodamine B in water. This insured a constant tracer concentration since the dye will always be at saturation and eliminates errors due to weighing the dye or measuring the water.
2. Pressure Transducer Adjustment - all the pressure lines were purged of air bubbles and the transducer was zeroed and calibrated.
3. Fluorometer Adjustment - a sample of water from the main storage tank was used as the background sample, and the fluorescence of this sample was adjusted to read zero percent. Then a standard solution was used and the reading was adjusted to 100%.

4. Inlet Flow Adjustment - valves #1 and #2 are used to set the total mass flowrate, and hence Reynolds number, for the data run.
5. Preliminary Outlet Flow Adjustment - either valves #3 and 4 or 5 and 6 are used to control the exit flowrates of the central subchannels. Initially these valves are set so that the sub-channel mass flowrate which corresponds to the total mass flowrate ($M_c = .1688M_T$) is withdrawn. Valve #7 is gradually closed until it has an appreciable effect on the inlet pressure.
6. Axial Pressure Drop Adjustment - the axial pressure drop is monitored and valve #7 is adjusted until the measured pressure drop is the same as in the undisturbed case.
7. Subchannel Differential Pressure Adjustment - the differential pressure between the two central subchannels at the sampling plane is monitored and valves #3 and 4 or 5 and 6 are adjusted until the undisturbed pressure distribution is re-established. Since this step may change the axial pressure, steps 6 and 7 are repeated until both pressures are within ± 5 percent of their undisturbed values.
8. Tracer Injection - the donor channel is selected and the tracer flowrate is set so that the fluorescence of the central receptor channel is in the fluorometer's mid range. Then steps 6 and 7 are repeated to eliminate any crossflow that might result from the dye injection.

9. Concentration Measurement - the fluorescence of the central receptor channel is recorded and a sample from the donor channel's inlet is taken. After dilution its fluorescence and concentration are determined and used to nondimensionalize the exit concentration of the central receptor channel.
10. Steps #8 and #9 are repeated with the opposite central channel traced.

3.3 EXPERIMENTAL PROCEDURE - POINT CONCENTRATION MEASUREMENTS

In order to determine the concentration gradient across the gap, a series of local concentration measurements were made along the symmetry line shown in Figure 3-10. Since the concentration could not be remotely determined, samples were withdrawn through a pitot-static tube. Isokinetic sampling required that fluid be withdrawn at such a rate that the fluid streamlines just upstream of the probe remain undisturbed. In a single phase flow, this can be accomplished by setting the probe flowrate equal to the product of the probe's cross-sectional area and the velocity at that point in the flow field.

An effort was made to measure the local velocity field, using a pitot tube, however, the pressure fluctuated in such a manner that these measurements were unreliable. Consequently, these velocities were calculated using the method of reference 9 and graphs of the velocity profile are given in Appendix F. The sequential procedure used in these measurements is outlined below:

FIGURE 3-10

LOCATION OF POINT SAMPLING TRAVERSE

1. Inlet Flow Adjustment - the Reynolds number is set by adjusting the inlet flow.
2. Initial Probe Positioning - the probe is positioned in the donor channel at the measurement point farthest from the clearance gap.
3. Sampling Flowrate Adjustment - the flowrate of the sampling probe is set to the calculated flowrate.
4. Tracer Flowrate Adjustment - the flowrate of the tracer is adjusted so that the fluorescence of the sample being withdrawn is nearly full scale on the fluorometer.
5. Probe Traverse - the probe is then moved to the next sampling location (usually an increment of .05") and step 3 is repeated. Then keeping the tracer flowrate the same as in Step 4, the sample's fluorescence is measured. This step is repeated until the traverse is completed.
6. Inlet Sample Withdrawal - a sample of the traced fluid is taken at the test section inlet and its concentration is used to non-dimensionalize the local concentrations.
7. The donor channel is switched, and steps 2 through 6 are repeated.

To check whether these samples were being taken isokinetically, a curve of sample concentration versus sampling flowrate was generated. The results of this are given in Section 6.3.2 and used to estimate the error incurred in the sampling procedure.

CHAPTER 4

DATA ANALYSIS AND RESULTS

4.1 DATA ANALYSIS - BULK MIXING TESTS

The objective of this analysis is to determine the bulk turbulent mixing coefficient with suitable care taken to adjust for the presence of entrance effects and diversion crossflow in the mixing experiment. There are two types of assumptions necessary for our analysis, those which are common to most mixing experiments, and those which derive from the analytic procedure we employ.

4.1.1 Necessary Assumptions

The assumptions which are common to most mixing experiments are listed below;

1. The flow split can be calculated using a method which assumes the axial pressure drop is the same for all subchannels (Appendix C),
2. The tracer is thoroughly mixed with the subchannel fluid, such that all eddy transport is assumed to have a concentration equal to the subchannel bulk concentration at that axial location. Note, this is equivalent to saying that the macroscopic energy (concentration) equation (2-4) can be used.
3. The tracer effect on fluid properties is negligible.

The assumptions which derive from our analytic procedure are;

4. The turbulent mixing coefficient is a constant and doesn't vary axially.
5. The diversion crossflow mixing is in one direction and is axially constant.
6. Diversion crossflow is present only between the central-central pair of subchannels.
7. The turbulent mixing coefficient between a peripheral-central pair is equal to that of a central-central pair.

4.1.2 The Mixing Equations

Using the terminology of Figure 4-1, a tracer concentration-mass balance can be written for all six subchannels:

$$m_1 \frac{\partial C_1}{\partial x} + W'_{12} (C_1 - C_2) + W'_{15} (C_1 - C_5) + W'_{13} (C_1 - C_3) + W_{12} (C^* - C_1) = 0$$

$$m_2 \frac{\partial C_2}{\partial x} + W'_{21} (C_2 - C_1) + W'_{26} (C_2 - C_6) + W'_{24} (C_2 - C_4) + W_{21} (C^* - C_2) = 0$$

$$m_3 \frac{\partial C_3}{\partial x} + W'_{31} (C_3 - C_1) = 0$$

$$m_4 \frac{\partial C_4}{\partial x} + W'_{42} (C_4 - C_2) = 0$$

$$m_5 \frac{\partial C_5}{\partial x} + W'_{51} (C_5 - C_1) = 0$$

$$m_6 \frac{\partial C_6}{\partial x} + W'_{62} (C_6 - C_2) = 0$$

(4-1)

FIGURE 4-1

where,

$$C^* = \begin{cases} C_1; W_{12} > 0 > W_{21} \\ C_2; W_{12} < 0 < W_{21} \end{cases} \quad (4-2)$$

From symmetry,

$$m_1 = m_2 = m$$

$$m_3 = m_4 = m_5 = m_6 = m_{II}$$

$$C_3 = C_5$$

$$C_4 = C_6$$

$$W'_{12} = W'_{21} = W'_I$$

$$W'_{15} = W'_{13} = W'_{24} = W'_{26} = W'_{32} = W'_{42} = W'_{31} = W'_{51} = W'_{II}$$

$$W_{12} = -W_{21} = W$$

From assumption (1) and Appendix C,

$$m_{II} = (.981)m$$

and assumption (7) yields

$$W'_{II} \approx W'_I$$

Employing these simplifications; the set of equations (4-1) can be reduced to a set of four linear differential equations;

$$m \frac{dC_1}{dx} + W'_1 (C_1 - C_2) + (2) W'_1 (C_1 - C_3) + W(C^* - C_1) = 0$$

$$m \frac{dC_2}{dx} + W'_1 (C_2 - C_1) + (2) W'_1 (C_2 - C_4) - W(C^* - C_2) = 0$$

$$(.981)m \frac{dC_3}{dx} + W'_1 (C_3 - C_1) = 0$$

$$(.981)m \frac{dC_4}{dx} + W'_1 (C_4 - C_2) = 0 \quad (4-3)$$

From a single mixing test, the inlet concentrations of all four channels and the outlet concentration of the central receptor channel are known. If the odd numbered channels are traced, then the knowns are;

$$C_{1I}, C_{2I}, C_{3I}, C_{4I} \text{ and } C_{2O}$$

where the subscript I denotes an inlet condition and the subscript O denotes an outlet condition. The unknowns are;

$$C_{1O}, C_{3O}, C_{4O}, W'_1, \text{ and } W$$

Since there are only four equations, this system is indeterminable (NOTE: C_{1O} isn't measured because the associated error is proportional to the concentration and the quantity of interest is actually the difference between C_{1I} and C_{1O} which is more than a magnitude smaller than the concentration C_{1O} . Hence a negligible error in the measurement of the concentration C_{1O} can be significant when compared to the difference). To resolve this difficulty, two mixing tests at the same flow conditions are run, one in which the odd numbered channels are traced (mixing from right to left) and one in which the even numbered channels are traced

(mixing from left to right). Then an additional set of four differential equations (listed below) is added to (4-3) resulting in a set of eight equations.

$$\begin{aligned}
 m \frac{dC_5}{dx} + W_I' (C_5 - C_6) + (2)W_I' (C_5 - C_7) + W(C^* - C_5) &= 0 \\
 m \frac{dC_6}{dx} + W_I' (C_6 - C_5) + (2)W_I' (C_6 - C_8) - W(C^* - C_6) &= 0 \\
 m \frac{dC_7}{dx} + W_I' (C_7 - C_5) &= 0 \\
 m \frac{dC_8}{dx} + W_I' (C_8 - C_6) &= 0
 \end{aligned} \tag{4-4}$$

Now, the knowns are;

$$C_{1I}, C_{2I}, C_{3I}, C_{4I}, C_{5I}, C_{6I}, C_{7I}, C_{8I}, C_{20} \text{ and } C_{50}$$

and there are eight unknowns;

$$C_{10}, C_{30}, C_{40}, C_{60}, C_{70}, C_{80}, W_I' \text{ and } W.$$

Since there are eight equations and eight unknowns, this system is determinable.

4.1.3 Solution Procedure

A digital computer program (WMIX) was written to solve this set of equations. The solution procedure is as follows;

1. A guess is made of the values of W_I' and W .
2. The equations are then simultaneously solved for their exit concentrations.

3. The computed values, C_{20} and C_{50} , are compared to the experimentally determined values, C_{2m} and C_{5m} . If the differences are less than 0.5% the calculation stops, otherwise W_1' and W are adjusted and steps 2 and 3 are repeated.

The logic which determines whether W_1' and W are increased or decreased is illustrated in Figure 4-2. (NOTE: Diversion crossflow (W) is considered positive when it acts right to left).

4.1.4 Entrance Effects Subtraction Method

After a development length of $50 L/D_h$'s, all of the entrance effects should be negligible. Hence, we wish to consider only the mixing which occurs in the second mixing section. To accomplish this we consider two pairs of mixing tests, one pair is taken using only the short mixing section, the other is taken using both mixing sections (a pair consists of a right to left and a left to right mixing test). The procedure is as follows;

1. The results of a "short" mixing test are nondimensionalized using the inlet concentration of the central donor channel.
2. These concentrations are input to the program WMIX, which outputs the remaining concentrations;

$$C_{10S}^*, C_{20S}^*, C_{30S}^*, \dots, C_{80S}^*$$

3. The results of a "long" mixing test are nondimensionalized (as before), which yields;

$$C_{20L}^* \text{ and } C_{50L}^*$$

FIGURE 4-2

ITERATION SCHEMATIC

4. A mixing section which consists of the distance between the short and long sampling planes (henceforth called the "subtracted" mixing section) is considered. The inlet concentrations are those listed in Step 2, the known exit concentrations are those listed in Step 3. These values are inputted to WMIX which returns the value of the mixing coefficient for the "subtracted" mixing section.

This procedure requires four data runs and consequently incurs four times the experimental error of a simple mixing test/data reduction procedure but results in the elimination of entrance effects and at least partial cancellation of diversion crossflow's effect on the mixing coefficient.

4.2 RESULTS - PRESSURE TESTS

With the rod bundle "clean" (sampling probes not in position) the pressure characteristics of the bundle were determined. This entailed measurement of both axial and radial (at the sampling plane) pressure differences for both the short and long mixing sections. Appendix E tabulates the measurements. From the axial pressure drop data, the friction factor was calculated and is plotted in Figure 4-3. The radial pressure differences are plotted in Figure 4-4.

4.3 RESULTS - BULK MIXING TESTS

The exit concentrations of the central receptor channel, nondimensionalized by the inlet concentration of the donor channel, are plotted in Figures 4-5 and 4-6. Figure 4-5 contains curves for the

FIGURE 4-3: EXPERIMENTALLY DETERMINED FANNING FRICTION FACTORS

FIGURE 4-4: Subchannel Pressure Difference

FIGURE 4-5: MIXING OCCURS FROM RIGHT TO LEFT

case in which the right channels are traced and mixing occurs from right to left for both the long and short mixing sections. Figure 4-6 contains the same curves for mixing from left to right. Appendix E contains all of the mixing data in tabulated form.

a. Data Analysis Results - Short Mixing Section

The exit concentrations measured at the $44.5 L/D_h$ sampling plane are combined to form pairs (a pair consists of a data point from a right to left mixing test and a left to right mixing test) in such a manner that at a certain Reynolds number all of the possible combinations are made. For example, at a Reynolds number of 2164, there are two exit concentrations from right to left tests and two from left to right tests, these are combined for form four pairs of concentration data. These pairs are then input to the program WMIX which returns a value for the tabulated mixing coefficient for each pair. These results are plotted in Figure 4-7 along with a curvefit.

b. Data Analysis Results - Long Mixing Section

The same procedure is followed for the exit concentrations of the $92.5 L/D_h$ sampling plane and the mixing coefficient is plotted in Figure 4.8.

c. Data Analysis Results - Entrance Effects Subtracted

The procedure outlined in Section 4.1d is followed and a set of four concentrations is used in each calculation. These sets represent all the possible combinations (for a specific Reynolds number) of the

FIGURE 4-6: MIXING OCCURS FROM LEFT TO RIGHT

FIGURE 4-7: RESULTS - SHORT SECTION

FIGURE 4-8: RESULTS - LONG SECTION

FIGURE 4-9: RESULTS - ENTRANCE EFFECTS SUBTRACTED

pairs from parts (a) and (b). For example, at a Reynolds number of 2164, there are twelve pairs at $L/D_h = 92.5$ and four pairs at $L/D_h = 44.5$ which result in 48 sets of concentration data. The output from WMIX (810 sets of concentrations) is plotted in Figure 4-9 along with a curvefit of these points.

4.4 DATA ANALYSIS - LOCAL CONCENTRATION MEASUREMENTS

a. Local Concentrations

The measured concentrations are nondimensionalized by the inlet concentration of the central donor channel.

b. Concentration Gradient

To obtain a value for the concentration gradient, the local concentrations are curvefitted. Then the value of the gradient of the curvefit, at the point where the nondimensional concentration equals 0.5, is calculated. The resulting gradient is of the form:

$$\left| \frac{1}{\xi} \frac{dG^*(\xi)}{d\phi} \right|_{\phi=0}$$

c. Eddy Diffusivities

Equation (2-21) defines the mixing coefficient as

$$W'_{1j} = \frac{-2\mu(\frac{P}{D} - 1) \left[\frac{1}{\xi} \frac{\partial T(\xi)}{\partial \phi} \right]_{\phi=0} \left[\frac{1}{Pr} + \frac{\bar{\epsilon}_{H\phi}}{\nu} \right]}{(T_1 - T_j)}$$

Solving for the circumferential eddy diffusivity

$$\left| \frac{\bar{\epsilon}_{H\phi}}{\nu} \right|_{\phi=0} = \frac{W'_{1j}(T_1 - T_j)}{-2\mu(\frac{P}{D} - 1) \left[\frac{1}{\xi} \frac{\partial T(\xi)}{\partial \xi} \right]_{\phi=0}} + \frac{1}{Pr} \quad (4-6)$$

FIGURE 4-10 CONCENTRATION PROFILE FOR $RE = 2164$

FIGURE 4-11 CONCENTRATION PROFILE FOR $RE = 4054$

FIGURE 4-12 CONCENTRATION PROFILE FOR RE = 8239

FIGURE 4-13 CONCENTRATION PROFILE FOR $RE = 12218$

FIGURE 4-14 CONCENTRATION PROFILE FOR $RE = 14347$

FIGURE 4-15 CONCENTRATION PROFILE FOR $RE = 16864$

FIGURE 4-16 CONCENTRATION PROFILE FOR $RE \approx 19448$

FIGURE 4-17 CONCENTRATION PROFILE FOR $RE = 23896$

Putting this expression in terms of concentrations, neglecting $(\frac{1}{Pr})$ since $Pr = 1000$, and substituting in for $\frac{P}{D}$ yields;

$$\left| \frac{\bar{\epsilon}_{H\phi}}{\nu} \right|_{\phi=0} = \frac{(-5)W_1' (C_1^* - C_1^*)}{\nu \left[\frac{1}{\xi} \frac{\partial C^*(\xi)}{\partial \phi} \right]_{\phi=0}} \quad (4-7)$$

The quantities in this equation can be evaluated using the results of sections 4.3 and 4.5 so that the magnitude of the eddy diffusivity can be calculated.

4.5 RESULTS - LOCAL CONCENTRATION MEASUREMENTS

a. Local Concentrations

Point concentrations were measured in the gap region for two cases for each Reynolds number, one where the right channels were traced and the other where the left channels were traced. Appendix G contains a table of these measurements. The resulting concentration profiles are plotted in Figures 4-10 through 4-17.

b. Concentration Gradient

The gradients calculated from these profiles are tabulated in Appendix G and plotted in Figure 4-18.

c. Eddy Diffusivities

Using equation (4-7) the circumferential eddy diffusivity $(\bar{\epsilon}_{H\phi})_{\phi=0}$ was calculated and is illustrated in Figure 4-19 along with a curve that shows its possible behavior with Reynolds number.

FIGURE 4-18; GAP AVERAGED CONCENTRATION GRADIENT

FIGURE 4-19

GAP AVERAGED CIRCUMFERENTIAL EDDY DIFFUSIVITY

CHAPTER 5

LITERATURE SURVEY

This chapter presents a brief synopsis and critique of experiments similar to ours with whose results our data will be compared. The pertinent details of each experiment are presented in a tabular format.

5.1 WALTON'S EXPERIMENT

The first triangular array bare rod mixing tests were done by Walton [26] and are detailed below.

TEST SECTION TYPE: Two subchannel simulated triangular array rod bundle (Figure 5.1).

P/D RATIO: 1.052

EXPERIMENTAL TECHNIQUE: Plane source, tracer, bulk concentration measurements. In air tests methane was used as the tracer, while potassium nitrate was used in water tests.

EXPERIMENTAL PROCEDURE: Bulk subchannel exit concentrations were measured at two axial locations, $L/D_h = 57$ and 115. In air experiments, the exit flowrates were adjusted so that no transverse subchannel pressure difference existed. In water experiments, the exit flowrates were measured using rotometers and adjusted to be equal ($\pm 1\%$ accuracy). Mixing tests were done for both directions,

FIGURE 5-1: WALTON'S TEST SECTION

FIGURE 5-3: PETRUNIK'S TEST SECTION

(vis. right to left and left to right) during each data run. If the results of these two tests varied by more than 10%, then it was assumed that the flowrates were maladjusted and that data run was deleted.

ANALYTICAL PROCEDURE: A mass balance between the two channels was performed with the assumption that all tracer transport was due to turbulent interchange.

COMMENTARY:

WATER EXPERIMENTS: The "absolute" mixing (lbm/hr) that occurred in these experiments was of the order of 4-5% of the total flowrate. However, the accuracy of the rotometers was only $\pm 1\%$. Therefore, errors in the exit flowrates of 2% (since there are two measurements) could be expected and would lead to errors in the mixing rate of 40-50%.

The procedure of neglecting data runs that showed a directional variation of more than 10% limits the above error but ignores the possibility that significant geometry induced crossflows could be the cause of this discrepancy. For example, suppose that a crossflow exists from right to left, then the exit flowrates will be adjusted so that a diversion crossflow

from left to right occurs just upstream of the sampling plane. The results of this are that the mixing rates for each channel are within 10% of each other and are higher than the turbulent mixing rate.

In an earlier test, we measured the "balanced" mixing results along with the results when the exit flowrates were adjusted using rotometers along for three different Reynolds numbers. The results are plotted in Figure 5-2. If the right to left results are considered correct, and the left to right are enhanced by diversion crossflow, then the balanced mixing procedure overestimates the mixing by 19-85%. This occurs because it is not physically possible to compensate for integrated diversion crossflow over the entire test section length by adding only an effective exit resistance to a subchannel. The inflation of the mixing rates measured using this procedure would be even more significant when laminarization begins to affect the mixing coefficient.

At low Reynolds numbers, the measured mixing rate could be primarily due to diversion crossflow (section 6.2.1) and turbulent exchange coefficients calculated in this manner could be grossly inflated.

FIGURE 5-2: RESULTS FROM PRELIMINARY TESTS USING THE MIXING BALANCE TECHNIQUE

AIR EXPERIMENTS:

The accuracy of the radial pressure measurements was $\pm .04$ in. of water. This should be more than adequate for controlling the exit flow split.

ENTRANCE EFFECTS:

Although mixing tests were performed at two different axial locations, no effort was made to subtract entrance effects. However, the experimental errors were larger than the differences in the mixing rates for the two mixing lengths and entrance effects may be negligible.

DIVERSION CROSSFLOW:

No detailed checks were performed to see if crossflows existed. As mentioned above, the results at low Reynolds numbers are particularly sensitive to this.

GEOMETRY:

The presence of boundaries where clearance gaps would be in a rod bundle could significantly affect the results. Two likely effects are:

1. The turbulent intensities would increase, and
2. The high velocity core would be translated towards the clearance gap.

Both of these effects would increase the mixing rate.

CONCLUSION: Considering the probable mechanisms for experimental errors and the possible influence of two subchannel geometry, the results from this experiment should reflect higher than actual mixing rates. In particular, the low Reynolds no. water results may be considerably over estimated.

5.2 PETRUK'S EXPERIMENTS

TEST SECTION TYPE: Six subchannel, simulated rod bundle (Fig. 5.2)

EXPERIMENTAL TECHNIQUE: Plane source, tracer, bulk concentration measurements, using Rhodamine B in water.

EXPERIMENTAL PROCEDURE: The bulk exit concentration of the three receptor channels was measured at one mixing length. The flow split was governed by differential pressure measurements so the subchannel transverse pressure was balanced to within $\pm .05''$ of water. However, if any directional mixing effects existed, a mixing balance procedure was used so that exit concentrations differed by no more than 10%.

ANALYTICAL PROCEDURE: A set of six differential equations were solved for the mixing coefficient using the entrance and exit concentrations of a group of three channels as the boundary conditions. A relation between the mixing coefficient of a central to central subchannel pair and a central to peripheral pair was assumed.

COMMENTARY:

DIVERSION CROSSFLOW: same as in Walton's experiment.

ENTRANCE EFFECTS: Mixing tests were conducted at one mixing length only, so no effort was made to either subtract the entrance effects or show that they were negligible.

EXPERIMENTAL ERRORS: The accuracy of the pressure measurement system would be sufficient if the working fluid was air (see Section 6.2.5), however, for water service the stated accuracy would produce errors on the order of 10 - 35% (these numbers come from our results for a test section of $P/D = 1.10$) for moderate Reynolds numbers and $P/D = 1.13$. For test sections of smaller P/D and especially at low Reynolds numbers, this accuracy may not be adequate. Also, as discussed with regard to Walton's experiment, the mixing balance procedure could inflate the measured mixing rates.

CONCLUSION: Same as in Walton's experiment.

5.3 ROIDI et al [31]

- TEST SECTION TYPE: Triangular array rod bundle, 47 rods.
- P/D RATIO: 1.246
- EXPERIMENTAL TECHNIQUE: Point source, tracer, local concentration measurements, using ethane in air.
- EXPERIMENTAL PROCEDURE: Quantities measured were;
1. Velocity profiles,
 2. Subchannel cross-sectional area (3 axial locations), and
 3. Local tracer concentrations for the traced channel and its three neighbors for two axial locations.

To measure local concentrations, air samples were extracted through a pitot-static tube at a flowrate calculated from the measured velocity.

ANALYTICAL PROCEDURE: A set of transport equations were written with turbulent interchange (eddy diffusivity) and diversion crossflow as the mixing mechanisms. Using the results from the two axial locations (37 and 40 L/D_h 's), the concentration fluxes across the clearance gaps were calculated. Then the diversion crossflow was estimated from the subchannel area measurements. Substituting these results the transport equations yielded a value for the eddy diffusivity, which when integrated with the concentration gradient

produced a value for the mixing coefficient.

COMMENTARY:

EXPERIMENTAL ERRORS: Since this experiment does not require bulk sampling, it isn't necessary to regulate the subchannel exit flow rates. This is an advantage since the primary cause of experimental error in most mixing experiments is improper sampling of the subchannel flowrate. Still, non-isokinetic sampling could be a problem in local measurements, but it is not as critical. Many local measurements are needed, however, and this could lead to a different problem. The collection of a sample requires a finite amount of time and this time multiplied by many sampling locations can be of considerable duration. Maintaining both tracer and air flowrates over a long period of time is difficult and small changes could adversely affect the measurements.

ENTRANCE EFFECTS:

Since the concentration fluxes are calculated by subtracting the concentrations at one axial location from another the entrance effects present before the first sampling plane are subtracted. The mixing length in this experiment, however, was only $37 L/D_h$'s, and hence the mixing might not be fully developed at the first sampling plane.

DIVERSION CROSSFLOW: From three measurements of the subchannel area, a polynomial fit was obtained and used to describe the area variation over the length of the rod bundle. A flow split type analysis was then performed and used to estimate the diversion crossflow.

CONCLUSION: Their experimental method is a different and probably better technique for determining bulk mixing coefficients than the usual procedure where bulk exit concentrations are measured. The large reported experimental errors ($\pm 50\%$) are probably due to inaccurately estimating the diversion crossflow (the calculated values were 1.5-5 times greater than the turbulent mixing rate). An experiment in which these crossflows are absent or more precisely corrected for should yield excellent results.

5.4 TAHIR[23]

TEST SECTION TYPE: Triangular array rod bundle, 18 rods.

P/D RATIO: 1.40

EXPERIMENTAL TECHNIQUE: Plane source, tracer, bulk concentration measurements using methane and air.

EXPERIMENTAL PROCEDURE: A central subchannel is traced and the exit concentration of an adjoining channel is measured (at $84 L/D_h$'s). This process is repeated with the channels switched. The entire flow from the sampled subchannel is extracted

through a probe shaped to fit the subchannel. The exit flowrate of this channel is adjusted so that the radial pressure distribution is unchanged from that measured when the probe was not in the test section.

ANALYTICAL PROCEDURE: A system of differential equations is solved for the exit concentration of the measured channel for an assumed value of the mixing coefficient. When the resulting value of the calculated exit concentration is within 1% of the measured exit concentration the assumed value of the mixing coefficient is taken as the result. Turbulent interchange is the only mixing mechanism present in this system of equations.

COMMENTARY:

EXPERIMENTAL ERROR: Extreme care was used in the fabrication of the test bundle and the resulting tolerances should cause little diversion crossflow at moderate and high Reynolds numbers. The isokinetic sampling procedure followed depended on radial pressure measurements (accuracy $\pm .05$ inches of water) and should be sufficient to insure minimum error in the sampling procedure.

ENTRANCE EFFECTS

Since their measurements were made at L/D_h of 84, entrance effects were assumed to have become negligible. It is true that most of the entrance effects have decayed by then, but the effect of inflated mixing rates over the first part of a bundle produces higher measured exit concentrations. These concentrations, in turn, produce higher values for the calculated mixing rates.

This process is a result of the fact that the quantity being measured is the exit concentration, not the mixing coefficient. Consequently, the measurements reflect the axial average of the mixing rate rather than its value at $L/D_h = 84$. Not correcting for this effect could possibly result in mixing rates up to 20% higher than the fully developed mixing rates. (Based on the results of our experiments)

DIVERSION CROSSFLOW:

No analytical provisions are made to include diversion crossflow in the mixing equations. However, the results when alternate channels were traced showed a variation of less than 4% and hence diversion crossflow was negligible in this experiment.

CONCLUSION:

The results of this experiment are very good but may be inflated due to entrance effects. Unfortunately, data was not taken in the region where laminarization begins to affect the turbulent mixing so its low Reynolds number behavior cannot be compared with our results.

5.5 SINGH:

TEST SECTION TYPE: Two subchannel simulated square array rod bundle.

P/D RATIO: 1.02, 1.04, and 1.10

EXPERIMENTAL TECHNIQUE: Plane source, tracer, bulk concentration measurements. Air tests were conducted using methane and water tests used potassium nitrate.

EXPERIMENTAL PROCEDURE: Same as Walton.

ANALYTICAL PROCEDURE: Simple mass balance.

COMMENTARY:

AIR TESTS:	}	Same as Walton
WATER TESTS:		

ENTRANCE EFFECTS: Measurements taken at only L/D_h , no effort made to subtract effects.

DIVERSION CROSSFLOW: Used a mixing balance technique. (Same as Walton)

GEOMETRY: Same as Walton.

CONCLUSIONS: Same as Walton

5.6 GALBRAITH

TEST SECTION TYPE: Six rod, two complete, and six partial square array subchannels.

P/D RATIO: 1.011, 1.028, 1.063, 1.127 and 1.228.

EXPERIMENTAL TECHNIQUE: Plane source, tracer, bulk concentration measurements. Used Rhodamine B in water.

EXPERIMENTAL PROCEDURE:	}	Same as Petrunik.
ANALYTICAL PROCEDURE:		

COMMENTARY: Same as Petrunik.

FIGURE 5-4: SINGH'S TEST SECTION

FIGURE 5-5: GALBRAITH'S TEST SECTION

5.7 OTHER EXPERIMENTS

Recently, the results of Kjellström's experiment has come to our attention through a secondary source. Unfortunately, there was not enough time to procure first hand documentation, so this experiment will not be commented on here, but his results will be presented for comparison in Chapter 6. This experiment, with those described above, is summarized in Table 5.1.

TABLE 5-1. Recent Experimental Data

Experimentor	Reference	P/D	(inch)	Subchannel Type	Fluid	Reynolds No. Range	Experimental Technique	Equivalent Pr
Walton	26	1.05	0.796	Two T-T	Water	$1.9 \times 10^3 - 5.7 \times 10^3$	Tracer (KNO_3 & Mixing Balance)	~ 1000
Walton	26	1.05	0.796	Two T-T	Air	$4.8 \times 10^3 - 9.1 \times 10^4$	Tracer (Methane) & Pressure Balance	~ 1
Petrunik	45	1.033 1.068 1.13	0.78	Six T-T	Water	$1.35 \times 10^3 - 1.08 \times 10^4$ $3.9 \times 10^3 - 2.5 \times 10^4$ $4.4 \times 10^3 - 3.8 \times 10^4$	Tracer (Rhodamine B) & Mixing Balance	~ 1000
Petrunik	45	1.13	0.78	Six T-T	Genetron	$7 \times 10^3 - 4.5 \times 10^4$	Temperature Measurements	3.5
Tahir	23	1.40	1.0	T-T 6 Rods & 12 Semi Rods	Air Air	$8.1 \times 10^3 - 4.95 \times 10^4$	Tracer (Methane) & Pressure Balance	~ 1
Roidt	31	1.246	2.5	T-T 39 Rods & 8 Semi Rods	Air	7×10^4	Tracer (Ethane) & Calculated Diversion Cross plains	~ 1
Kjellstrom	8	1.217	6.16	T-T 6 Rods	Air	$1.5 \times 10^5 - 3.6 \times 10^5$	Hot Wire Anemometer (Turbulence Measurement)	~ 1
Singh	21	1.02 1.04 1.10	0.84 0.82 0.78	Two S-S	Water & Air	$1.3 \times 10^3 - 3.8 \times 10^4$	Tracer: Water: KNO_3 - Mixing Balance Air: Methane - Pressure Balance	~ 1000 ~ 1
Galbraith	33	1.011 1.028 1.063 1.127 1.228	1.0	S-S 6-Rods	Water	$8 \times 10^3 - 3 \times 10^4$	Tracer (Rhodamine B) & Pressure Balance	~ 1000
Kelly	This Paper	1.10	1.5	Six T-T	Water	$2 \times 10^3 - 2.4 \times 10^4$	Tracer (Rhodamine B) & Pressure Balance	~ 1000

CHAPTER 6

DISCUSSION OF RESULTS

6.1 Pressure Test Results

From the friction factor curve (Figure 4-3) it can be seen that laminarization influences the friction factor up to a Reynolds number of approximately 4000. A curvefit of the high Reynolds number data yields

$$f = (.0780)(Re)^{-.228} \quad Re > 8000 \quad (6-1)$$

and the low Reynolds number data yields

$$f = \frac{28.5}{Re} \quad 1000 < Re < 4100$$

For comparison, Tahir (23) reports this relation for a bundle with $\frac{P}{D} = 1.4$.

$$f = (.044)(Re)^{-.194} \quad (6-2)$$

and Walton (26) reports

$$f = (.094)(Re)^{-.28} \quad (6-3)$$

for a simulated rod bundle with $\frac{P}{D} = 1.05$.

These correlations for the Fanning friction factor are plotted in Figure 6-1. Tahir's results are essentially the same as the smooth tube friction factor curve. The 20% difference between our results and Tahir's is probably due to surface roughness. Since we used water from the city water service, the brass rods quickly became tarnished. Consequently, our surface was rougher than those of Walton (Plexiglass) and Tahir (centerless ground metal) and hence, our friction factor would be larger.

6.2 Bulk Mixing Results

6.2.1 Low Reynolds Number Behaviour: Laminarization

As the Reynolds number decreases, the laminar sublayer-through which turbulent diffusion is negligible - increases and occupies a significant fraction of the clearance gap. The approximate thickness of the laminar sublayer in the gap, δ , was calculated using the relation [22]:

$$y^+ = \frac{\delta}{D_n} \text{Re} \sqrt{f/2} \quad (6-4)$$

where the sublayer extends to a value $y^+ = 5$. The fraction of the gap which is laminar increases from 5.2% to 29% as the Reynolds number decreases from 10,000 to 1,000, as is shown in Figure 6-2. This result is conservative since equation (6-4) strictly applies to the layer in smooth tubes but also describes the sublayer in the interior of the channel reasonably well and the laminar sublayer will increase in thickness from the interior of the channel to the gap region.

FIGURE 6-1
Comparison of Friction Factors

FIGURE 6-2

PERCENTAGE OF THE CLEARANCE GAP BLOCKED BY THE LAMINAR SUBLAYER

The growth of the sublayer severely restricts turbulent diffusion and, in the absence of significant molecular diffusion, causes the mixing coefficient to decrease sharply. If, however, molecular diffusion is significant then the mixing coefficient will approach a constant value at low Reynolds number. Figure 1-2 illustrates this Prandtl number dependence of the mixing coefficient as calculated by Ramm et al [2]. Mixing experiments can be classified by "Prandtl" number as follows;

- 1) $Pr \approx 1000$ - water experiments that use a tracer,
- 2) $Pr \approx 1$ - air experiments that use a tracer,
 - experiments that use temperature measurements.

Table 5-1 lists the experimental conditions and techniques used in the experiments whose results will be presented for comparison. The divergence of the mixing coefficient curves for $Pr = 1$ and $Pr = 1000$ should be present in the experimental results and has already been demonstrated in a square array rod bundle by Singh and St. Pierre [22]. Figure 6-3 shows the existing triangular array experimental results along with Ramm's prediction for Reynolds number up to 10,000. All the data curves in Figure 6-3 are from water-tracer experiments and therefore have negligible molecular diffusion. Laminarization is evident only in Ramm's calculations and our results and is characterized by a decrease in the mixing coefficient that is roughly proportional to $(Re)^{2.9}$.

Ramm et al [6] attribute this discrepancy between their calculations and the experimental results to two factors, secondary flow and experimental

errors. Their turbulence model makes no provision for the effect of secondary flows on the mixing rate and consequently could underestimate the mixing at low P/D 's and low Reynolds numbers. However, they felt that the omission of secondary flow effects would only lead to a small discrepancy and not the large difference shown in Figure 6-3. Also, our results very closely approximate the predicted low Reynolds number behaviour (curve slope) from which it can be concluded that for a P/D of 1.10, secondary flow effects are negligible and the probable cause for the qualitative difference between other experiments and Ramm's model is due to experimental error.

In any test-section, small geometrical imperfections will exist that will lead to small diversion crossflows. These crossflows, as is proposed in Appendix I, have a Reynolds no. dependence of $(Re)^{-9}$ and are not significantly affected by gap laminarization. Consequently, diversion crossflows that constitute only a fraction of the mixing rate at high and moderate Reynolds nos. will decrease with Reynolds no. at a much slower rate than the turbulent mixing and become dominant in the low Reynolds no. region. This behaviour is shown in Figure I-2. Therefore, only slightly imperfect test conditions can result in the measurement of larger than actual mixing rates when laminarization affects the turbulent mixing.

6.2.2 Dependence on Pitch to Diameter Ratio

6.2.2.1 Assessment of Data Using the Stanton Number Representation

Figure 6-4 shows the dependence of the mixing Stanton number on both Reynolds no. and pitch to diameter ratio. It is clear that the Stanton no. varies inversely with P/D . This result is not surprising

since the Stanton no. is defined as

$$N_{ST} = \frac{W'_{1j}}{G_1 c} = \left(\frac{W'_{1j}}{\mu} \right) \left(\frac{1}{Re} \right) \left(\frac{D_h}{c} \right) \quad (6.5)$$

and by definition is inversely proportional to the gap width.

The results in Figure 6.4 show discrepancies at both low ($P/D = 1.03 - 1.05$) and moderate P/D ($1.10 - 1.13$). These discrepancies will be explained by recognition of the following two sources of experimental error.

1. Mixing balance technique - as noted in Section 5.3, this procedure can induce diversion crossflows at the sampling plane which would inflate the mixing rate. This effect exists for all P/D ratios but is especially significant at low P/D 's where laminarization has begun to influence the mixing rate.
2. Wake effects - the predominant entrance effect is the generation of a wake at the trailing edge of the separator strip which would cause mixing rates significantly higher than turbulent mixing. Walton's experiment contained enough scatter that any entrance effects were obscured. However, his results do suggest that his entrance effects would be small compared to the ones we experienced. Petrunik only made measurements at one sampling plane, so significant wake effect could have occurred and gone undetected.

For the low P/D range, Walton's results ($P/D = 1.05$) are low compared to Petrunik's results ($P/D = 1.03$ and 1.07) and no satisfactory explanation has been given for this. Tahir suggests that Walton's results contain a systematic error, however, we have

FIGURE 6-4: Comparison of Stanton No.

found no evidence of an error which would cause an underestimate of the mixing in either his experimental or analytical procedure. Barring a malfunction of the testing equipment used to determine the exit concentrations, it would seem that, if anything, Walton's results are higher than the actual mixing.

As pointed out in Chapter 5, Walton when using air as the working fluid, did not employ the mixing balance technique whereas Petrunik did employ this technique. However both experimenters may suffer from entrance effects although in Walton's case this was checked by measurements at two planes whereas Petrunik made measurements at only one plane. Therefore, we conclude that Petrunik's results, especially for $P/D = 1.03$, might overestimate the mixing and that Walton's results are close to correct.

For the moderate P/D range another discrepancy occurs between Petrunik's results (for $P/D = 1.13$) and our results (for $P/D = 1.10$). This might be the result of the same two factors cited above. However, our data is corrected for entrance effects and diversion crossflow due to the actual test geometry while Petrunik's data are not corrected for either entrance effects or mixing balance technique effects. Therefore, his results should be higher than ours. To check this, Petrunik's results and our results for the long mixing section (i.e. no subtraction of entrance effects) are plotted in Figure 6-5. Since our uncorrected results are higher than Petrunik's, it seems that his results ($P/D = 1.13$) are not inflated by diversion crossflow but simply are not corrected for the entrance effects. As a consequence of this, it becomes important to know whether reported mixing results are corrected for

FIGURE 6-5: STANTON NO. COMPARISON - ENTRANCE EFFECTS
NOT SUBTRACTED

entrance effects and if not, whether significant entrance effects (specifically wake effects) are present in the test section. Conclusions (or correlations) which are based on a group of experiments, some of which have been corrected for entrance effects (or have shown these effects to be negligible) and some which have not been corrected, could be subject to a degree of uncertainty.

6.2.2.2 Mixing Coefficient Representation

Figure 6-6 plots the experimentally determined and predicted (Ramm et al) mixing coefficients. From the experiments, it would seem that the mixing coefficient is weakly dependent on and inversely proportional to P/D . Ramm et al, as is shown in Figure 6-7, predict the opposite behaviour. This discrepancy is further illustrated by Figures 6-8(a) and (b), which are graphs of the mixing coefficient for eight different P/D 's at Reynold's nos. of 10,000 and 20,000 (Roïdt's and Kjellstrom's data were extrapolated to these Reynolds' nos. assuming $W'_{ij} = (Re)^{-9}$) for bare rod triangular arrays. Figure 6-9 plots the mixing coefficient versus P/D for $Re = 20,000$ for bare rod square arrays.

As noted previously, the mixing balance technique can overestimate the mixing coefficient, particularly at low P/D 's. This is shown in Figure 6-9 where, if Singh's data (for $P/D = 1.02$ and 1.04) is neglected, the square array results approximate the P/D behaviour that Ramm predicts. This may also be the case for triangular arrays, but not enough data is presently available.

There are four competing phenomena that affect the P/D dependence of the mixing coefficient. As the pitch to diameter ratio decreases:

FIGURE 6-6 : COMPARISON OF MIXING COEFFICIENTS

FIGURE 6-7:

RAMM'S PREDICTION: DEPENDENCE OF THE MIXING COEFFICIENT ON $\frac{P}{D}$ RATIO

FIGURE 6-8.

EXPERIMENTALLY DETERMINED MIXING COEFFICIENTS (TRIANGULAR ARRAY)

FIGURE 6-9

EXPERIMENTALLY DETERMINED MIXING COEFFICIENTS VERSUS $\frac{P}{D}$

- 1) The gap clearance, and hence the probability of an eddy crossing the gap decreases.
- 2) The structure of the turbulent flow changes and the turbulent intensities increase.
- 3) The relative magnitude of secondary flows increases.
- 4) The laminar sublayer occupies a larger portion of the gap and can cause the mixing coefficient to rapidly decrease towards its "conduction only" value at ever increasing values of the Reynolds number.

At moderate and high values of P/D and Re , the first two phenomena determine the behaviour of the mixing coefficient. Unfortunately, the circumferential eddy diffusivity ($\bar{\epsilon}_{R\phi}$) is very difficult to measure [46] and hence its P/D dependence is not clearly defined. Turbulent intensities however, can be measured and Figure 6-10, reproduced from reference [47], shows the dependence of the axial turbulent intensity on P/D . From which it can be inferred that the eddy diffusivity is also inversely proportional to P/D . Based on a study where turbulence parameters were measured for two square array rod bundles, Rowe et al [18] state, "From these experimental results it can be concluded that the reduction of rod gap spacing leads to more dominant periodic flow pulsations in the region adjacent to the rod gap. This together with secondary flows, increased turbulence intensity and increased macroscale indicate an enhancement of turbulent mixing to help compensate for a decrease in rod gap spacing". The weak dependence of the mixing coefficient on P/D is a result of this phenomena.

At either low Reynolds numbers or low P/D ratios, gap laminarization

FIGURE 6-10
AXIAL TURBULENT INTENSITIES

becomes important and causes a characteristic change in slope of the mixing coefficients' Reynolds no. dependence (as was discussed in Section 6.2.1). This phenomena is responsible for the low P/D behaviour predicted by Ramm. (Figure 4 of Ramm's paper (Ref. 2) shows the relationship between P/D Reynolds number and array type regarding onset of laminarization.) There are two examples of experimental verification for this, our results for P/D = 1.10 (triangular) and low Re (which were discussed earlier) and Galbraiths' results for P/D = 1.011 (square) for all Reynolds numbers which are given in Figure 6-11.

If the triangular array results, Figure 6-8, are considered and Petrunik's results are neglected, then the mixing coefficient appears almost independent of P/D for P/D > 1.1 and moderate to high Reynolds numbers. To determine the exact dependence, a number of additional experiments will have to be conducted.

6.2.3 Comparison with Mixing Correlations

Rogers and Tahir [23] present this correlation for the mixing coefficient in bare rod triangular arrays;

$$\frac{W'_{11}}{\mu} = (.0018) (Re)^{0.9} (P/D - 1)^{-.4}$$

which for our geometry reduces to,

$$\frac{W'_{11}}{\mu} = (.0045) (Re)^{0.9} \tag{6.6}$$

A correlation of our experimental results for the region $8000 < Re < 24000$ yields:

FIGURE 6-11

CALBRAITH'S EXPERIMENTAL RESULTS

$$\frac{W_{11}}{\mu} = (.0021)(Re)^{0.935} \quad (6.7)$$

Figure 6-12 plots these two correlations.

6.2.4 Entrance Effects

Figure 6-13 is a plot of our results for the short section ($L/D_h = 44.5$), the long section ($L/D_h = 92.5$) and the subtracted results. It is clear from this figure that the measured mixing coefficient can be inflated by 35% at $L/D_h = 92.5$ if the entrance effects are not subtracted. For future experiments, either a subtraction procedure or mixing lengths of 150-200 L/D_h 's should be used.

6.2.5 Experimental Error

The primary cause of error in this experiment is diversion crossflow due to an improper balance of the subchannel radial pressure difference at the entrance of the sampling probes. An improper pressure balance leads to a redistribution of the flow split and non-isokinetic sampling. Figure 6-14 is a graph of the dependence of the exit concentration on the pressure balance for a Reynolds number of 12218. Figure 6-15 is a graph of the dependence of the mixing coefficient on pressure imbalance and Reynolds number from Galbraith's thesis [33]. These graphs show that pressure imbalances as small as .005 PSI can cause mixing rates that are inflated by 30 - 100% (inflation decreases with Reynolds number). Both of these experiments used water as the fluid and consequently experienced this sensitivity of the results on the pressure balance. Appendix I shows that for a given diversion crossflow, the pressure difference is inversely proportional to the fluid density. Therefore,

FIGURE 6-12

COMPARISON WITH RODGERS' MIXING CORRELATION

FIGURE 6-13: MIXING COEFFICIENT VS. RE

FIGURE 6-14: DEPENDENCE OF THE EXIT CONCENTRATION ON BIAS PRESSURE

FIGURE 6-15

DEPENDENCE OF THE MIXING COEFFICIENT ON BIAS PRESSURE (GALBRAITH'S RESULTS)

with W_c fixed,

$$\frac{\Delta P_{\text{air}}}{\Delta P_{\text{H}_2\text{O}}} = \frac{\rho_{\text{H}_2\text{O}}}{\rho_{\text{air}}}$$

$$\frac{\Delta P_{\text{air}}}{\Delta P_{\text{H}_2\text{O}}} = 816.5$$

What this means is that a pressure imbalance of $(816.5 \times .005) = 4.1$ PSI would be needed to cause the same 30-100% inflation of the mixing rate if air were used instead of water. Or, conversely, if the same .005 PSI pressure imbalance were imposed, the inflation of the mixing rate would only be 1-3% for air mixing tests. Consequently, in all future bulk mixing tests, where a pressure balance is necessary for isokinetic sampling, the working fluid should be air.

The best method of calculating our experimental error is by examining the actual variations in the data. Table 6.1 lists the maximum error present in the concentration measurements. Assuming a simple two channel analysis, these concentration errors can be directly translated to errors in the mixing coefficient. Hence, the implied error for the mixing coefficient is $\pm 20\%$ for the short section and $\pm 13\%$ for the long section. A curvefit of the data for these sections had the following deviations:

$$\sigma(\text{short}) = 12\%$$

$$\sigma(\text{long}) = 12\%$$

TABLE 6.1
EXPERIMENTAL ERROR (CONCENTRATION MEASUREMENTS)

	Mixing Occurs Right to Left	Mixing Occurs Left to Right
Long Section	$\pm 6.5\%$	$\pm 13\%$
Short Section	$\pm 20\%$	$\pm 20\%$

The standard deviation of the subtracted results, using the correlation given in Section 6.2.3, is 35%. Figure 6-16 presents our subtracted results along with curves representing $\pm \sigma$ and $\pm 2\sigma$.

6.3 Local Concentration Measurement Results

6.3.1 Low Reynolds Number Measurements: Laminarization

The low Reynolds number concentration profiles (Figure 4-10 through 4-12) are not symmetric about the center of the clearance gap. Instead, the profile measured when mixing is from left to right is displaced to the left and its slope (at the point where the nondimensional concentration equals 0.5) is steeper than that of the profile measured when mixing is from right to left. Diversion crossflow would have an effect on the concentration profile that is dependent on the mixing direction and would probably be as follows:

- 1) Diversion crossflow is from the traced channel to the receptor channel,
 - a) The mass transfer is enhanced and the concentration gradient becomes less steep, just as it does for higher mixing rates.
 - b) As the high concentration fluid from the traced channel moves across the gap, the "contact point" (where $C^* = 0.5$) will be displaced in the direction of the crossflow.
2. Diversion crossflow is from the receptor channel to the donor channel,
 - a) As the low concentration fluid from the receptor channel moves across the gap, the "contact point" will be displaced in the direction of the crossflow.

FIGURE 6-16: STANDARD DEVIATION OF THE MIXING DATA

- b) Mass (dye) transfer is not enhanced and the gradient should retain its turbulence only value or become steeper.

The effects described above are similar to our observations, and hence we believe that diversion crossflow occurs from right to left in our test section. However, the effects described above are based on conjecture (even though they were predicted before the measurements began), and that alone doesn't justify our conclusion that the lack of symmetry is due to diversion crossflow rather than experimental error.

As the Reynolds number increases, the turbulent mixing increases and the contact point moves towards the center of the clearance gap and symmetry returns. From the discussion in section 6.2.1 the ratio of diversion crossflow to turbulent mixing is approximately constant in the absence of laminarization. During laminarization, the crossflow maintains its Reynolds no. dependence ($Re^{.9}$) but the mixing coefficient decreases rapidly and its behaviour is roughly proportional to $(Re)^{2.9}$. Then, if the displacement of the contact point is proportional to the ratio of the diversion crossflow to the turbulent mixing, at high Reynolds numbers the displacement should maintain a constant value (this value is smaller than our experimental error), and at low Reynolds numbers the displacement should decrease with increasing Reynolds no. at a rate proportional to $(Re)^{2.0}$. Figure 6-17 is a graph of the "left to right" contact point displacement versus Reynolds number, for comparison, a line with a slope of $(Re)^{-2.0}$ is included. From this it is apparent that laminarization is the cause of the profile asymmetry.

Also, the low Reynolds number behaviour of the concentration

FIGURE 6 - 17

DISPLACEMENT OF THE CONTACT POINT

gradient (Figure 4-18) reinforces this conclusion. At $Re = 2164$, the "left to right" gradient has a value of 2.22 which rapidly decreases with Re to values of about unity. This exponential type decrease is similar to the behaviour of Ramm et al's predicted gradients. Concurrently, the "right to left" gradient has a value of 0.44 which rapidly increases to about unity as the laminarization decreases and turbulent mixing dominates diversion crossflow. This increase is proportional to $(Re)^{1.7}$.

6.3.2 High Reynolds Number Behaviour

From Figure 4-18, it appears that the gradient might decrease slightly with increasing Re . However, the experimental error is of the same magnitude as the differences between the measurements and nothing conclusive can be said about this behaviour.

Also plotted in Figure 4-18 are two measurements of Roidt et al for $Re = 70,000$ and $L/D_h = 36.9$ and 40.2 . For a development length of $92.5 L/D_h$'s, these gradients would be smaller (since the gradient decreases as the concentration profile develops) and would have approximately the same magnitude as our measurements if they were extrapolated with Re . This shows that the values for the gradient that we measured are similar to those of other experimenters.

6.3.3 Experimental Error

As in the bulk mixing tests, the significant errors are the result of non-isokinetic sampling. Figure 6-18 shows the dependence of the measured point concentration at $Re = 12218$ and $\phi = 0^\circ$ on sampling flowrate. From this it can be seen that the probable error is about $\pm 2.5\%$ of the point concentration. This error is fairly small, however, the nondimensional

FIGURE 6-18: Conc. Vs. Sampling Flowrate

concentration differences between sampling locations is only of the order 0.05 to 0.1. Assuming that the errors add, and that the point concentration is 0.5, the experimental error is between 25 - 50%.

6.4 The Circumferential Eddy Diffusivity

Using the values of the eddy diffusivity $(\bar{\epsilon}_{H\phi}|_{\phi=0})$ calculated in section 4.4.3 and presented in Figure 4-19, a curvefit for the region $8000 < Re < 24000$ is given by;

$$\frac{\bar{\epsilon}_{H\phi}}{\nu} \Big|_{\phi=0^\circ} = (.0021)(Re)^{1.03} \quad (6-8)$$

Hetsroni [48] correlated his measurements in a square array rod bundle ($P/D = 1.25$) with:

$$\frac{\bar{\epsilon}_{H\phi}}{\nu} \Big|_{\phi=0} = .0061(Re)^{.98} \quad (6-9)$$

Nijsing and Eifler [49] recommend;

$$\frac{\bar{\epsilon}_{H\phi}}{\nu} \Big|_{\phi=0} = (.154) Y_\phi (\tau_w/\rho)^{1/2}$$

where

$$Y_\phi = \frac{P/D - R}{\cos \phi} - R = .075 \text{ (in)}$$

and using our friction factor (equation 6-1) to evaluate τ_w , this reduces to;

$$\left. \frac{\bar{E}}{v} \frac{H\phi}{v} \right|_{\phi=0} = (.0045)(Re)^{.89} \quad (6-10)$$

Van Der Ros [51] suggests;

$$\left. \frac{\bar{E}}{v} \frac{H\phi}{v} \right|_{\phi=0} = (.009)(Re) \quad (6-11)$$

for a square array.

These correlations, along with the value $Roidt$ measured are plotted in Figure 6-19.

The error involved in our measurements was of the order $\pm 50\%$ and in view of this and the lack of other experimental data, much work needs to be done, not only to determine gap average values, but to determine local values within the subchannel and their variation with design parameters (P/D , Re , Pr).

FIGURE 6-19

COMPARISON OF CORRELATIONS FOR THE GAP AVERAGED CIRCUMFERENTIAL
EDDY DIFFUSIVITY

CHAPTER 7

CONCLUSIONS

7.1 Experimental Approach

From the discussion in Chapter 6, we conclude that:

1. The use of a mixing balance technique to control the exit flowrates does not insure iso-kinetic sampling and tends to overestimate the amount of mixing, especially at low Re or low P/D .
2. In a mixing experiment where iso-kinetic sampling of a sub-channel is required, a pressure balancing procedure should be used. The working fluid in such an experiment should be air rather than water.
3. Using a pressure balance technique, channels should be alternately traced to see if diversion crossflow exists due to geometric asymmetries in the test section.
4. The most dominant entrance effect is due to a generation of a wake and this phenomena varies depending on the test section geometry. In some experiments it may not be important but in others, such as ours, the mixing rate can be inflated by 35%. Future experiments should subtract entrance effects or show that they are negligible.
5. The experimental procedure used by Roidt et al has several advantages over "bulk" experiments. Entrance effects are automatically subtracted and the experimental results yield values for the eddy diffusivity as well as the mixing coefficient.

7.2 Experimental Results

6. Laminarization can drastically affect the mixing coefficient causing a distinct change in slope with Reynolds number in a manner similar to that predicted by Ramm et al. Consequently, secondary flows at low Reynolds numbers may not play as large a role in mixing as has been suggested.
7. The dependence of the mixing coefficient on P/D ratio is very weak for P/D's greater than 1.1 and moderate to high Re.
8. For small P/D ratios, the mixing coefficient may decrease sharply and approach its "conduction only" value rather than increase as predicted by some experiments.
9. Diversion crossflow will cause displacement and distortion of the concentration profiles between subchannels at low Reynolds number conditions.

7.3 Design Correlations

10. The correlation proposed by Rogers and Tahir for the dependence of the mixing coefficient on Re and P/D should be used with caution for P/D's less than 1.2 since it depends on Petrunik's data for its low P/D behaviour, and the trend predicted by that data may be in error.
11. To insure conservative estimates of the gap averaged eddy diffusivity, the correlation suggested by Nijssing and Eifler (or one that produces similar values) should be used.

In summary, a great deal of work needs to be done in this field to determine both the mixing coefficient and the circumferential eddy diffusivity and their parametric dependence. An approach that might be

especially fruitful would be to use an experimental procedure similar to Roidt's and from the measured concentration fluxes calculate local value for $\epsilon_{H\phi}$ and ϵ_{Hr} . Then, by comparing results for different P/D 's the effect that secondary flows and other macroscopic flow processes have on mixing could be determined. Using these results, a computer code could be developed which would use the measured turbulence parameters to predict both bulk mixing rates and subchannel temperature profiles for any combination of heat fluxes and flow rates.

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APPENDIX A
FLOWMETER CALIBRATION CURVES

1. Curve No. One - Starboard Inlet Flow (6.0 GPM)
 $m = (.486) X + .316$
2. Curve No. Two - Port Inlet Flow (6.0 GPM)
 $m = (.486) X + .042$
3. Curve No. Three - Starboard Inlet Flow (9.6 GPM)
 $m = (.820) X - 1.02$
4. Curve No. Four - Port Inlet Flow (9.5 GPM)
 $m = (.790) X + 1.53$
5. Curve No. Five - Starboard Inlet Flow (20 GPM)
 $m = (.689) X - 4.312$
6. Curve No. Six - Port Inlet Flow (20 GPM)
 $m = (1.661)X - 3.7368$
7. Curve No. Seven - Starboard Sample Flow (6.5 GPM)
 $m = (.5105)X - .0689$
8. Curve No. Eight - Port Sample Flow (6.5 GPM)
 $m = (.5063)X - .2437$
9. Curve No. Nine - Starboard Sample Flow (2.7 GPM)
 $m = (.2235)X + .0346$
10. Curve No. Ten - Port Sample Flow (2.7 GPM)
 $m = (.2357)X - 1.0238$

where,

$m = \text{lbm/min}$

$X = \text{meter reading (\%)}$

FIGURE A-1 FLOWMETER CALIBRATION CURVE ONE

FIGURE A-2 FLOWMETER CALIBRATION CURVE T O

FIGURE A-3 FLOWMETER CALIBRATION CURVE THREE

FIGURE A-4 FLOWMETER CALIBRATION CURVE FOUR

FIGURE A-5 FLOWMETER CALIBRATION CURVE FIVE

FIGURE A-6 FLO METER CALIBRATION CURVE SIX

FIGURE A-7 FLOWMETER CALIBRATION CURVE SEVEN

FIGURE A-8 FLOWMETER CALIBRATION CURVE EIGHT

FIGURE A-9 FLOWMETER CALIBRATION CURVE NINE

FIGURE A-10 FLOWMETER CALIBRATION CURVE TEN

TABLE A-2
POINT SAMPLING FLOWMETER CHART

Flowmeter Reading	1.05(cc/min) maximum	3.20 maximum	10.3 maximum	18.6 maximum
2	0.005	0.05	0.13	0.27
3	0.0336	0.10	0.22	0.50
4	0.093	0.18	0.40	0.85
5	0.17	0.29	0.65	1.4
6	0.275	0.43	0.96	2.15
7	0.415	0.60	1.35	3.0
8	0.59	0.79	1.80	4.0
10	1.05	1.25	2.9	6.1
12		1.81	4.2	8.4
14		2.46	5.6	10.75
16		3.20	7.2	13.5
18			8.75	16.0
20			10.3	18.6

FIGURE B-1: FLUOROMETER CALIBRATION CURVE

APPENDIX C FLOW SPLIT ANALYSIS

The flow split between subchannels will be calculated by assuming that the axial pressure drop is the same for all subchannels. Using a friction factor formulation the pressure drop for the i th subchannel is given by:

$$\Delta P_i = f_i \frac{L}{D_{hi}} \frac{\rho \bar{v}_i^2}{2 g_o} \quad (C-1)$$

and the pressure drop for the entire test section is given by:

$$\Delta P_T = f_t \frac{L}{D_{hT}} \frac{\rho \bar{v}_T^2}{2 g_o} \quad (C-2)$$

Equating these pressure drops:

$$f_i \frac{\bar{v}_i^2}{D_{hi}} = f_T \frac{\bar{v}_T^2}{D_{hT}} \quad (C-3)$$

The friction factor can be described by:

$$f = \frac{C}{\left(\frac{\rho \bar{v} D_h}{\mu} \right)^n} \quad (C-4)$$

and substituting this into equation (C-3) yields,

$$\frac{\bar{v}_i^{2-n}}{D_{hi}^{n+1}} = \frac{\bar{v}_T^{2-n}}{D_{hT}^{n+1}} \quad (C-5)$$

Rearranging this gives;

$$\left(\frac{\bar{v}_1}{\bar{v}_T} \right)^{2-n} = \left(\frac{D_{h1}}{D_{hT}} \right)^{n+1} \quad (C-6)$$

$$\frac{\bar{v}_1}{\bar{v}_T} = \left(\frac{D_{h1}}{D_{hT}} \right)^{\frac{n+1}{2-n}} \quad (C-7)$$

Roidt [31] states that the mean velocity in any channel is a function of only the local subchannel hydraulic diameter and gives the equation;

$$\frac{\bar{v}_1}{\bar{v}_T} = \left(\frac{D_{h1}}{D_{hT}} \right)^{.6284} \quad (C-8)$$

From this, it can be seen that the value of the exponent in the friction factor equation that Roidt uses is;

$$n = .158$$

The test section dimensions relevant to this analysis are:

	Central Channel	Peripheral Channel	Overall
Hydraulic Diameter (ft)	4.178×10^{-2}	4.178×10^{-2}	4.178×10^{-2}
Subchannel Area (ft ²)	2.051×10^{-3}	2.011×10^{-3}	1.215×10^{-2}

Since the hydraulic diameter is the same for both peripheral and central subchannels, the mean velocities are the same,

$$\bar{v}_C = \bar{v}_P = \bar{v}$$

and the subchannel mass flowrates are directly proportional to the ratio of the subchannel area to the total area. Then,

$$m_C = m_T \left(\frac{A_C}{A_T} \right) = (.1688) m_T$$

$$m_P = m_T \left(\frac{A_P}{A_T} \right) = (.1656) m_T$$

and

$$\frac{m_P}{m_C} = .981 \quad (C-5)$$

APPENDIX D

THE COMPUTER PROGRAM WMIX

The computer program used in the reduction of our bulk mixing data is presented in the following pages, along with a sample of the output. This program makes use of the subroutine DQSYK to solve a set of linear differential equations. DQSYK is part of the Applications Programming Package of the Information Processing Center at MIT.

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SUBROUTINE WMIX (AMS,SF,W2W1,X1R,X1L,Y1R,Y1L,Y2R,Y2L,X3R,X3L,Y4R,
1Y4L,W,C,C5R,C5L,C6R,C6L,X2R,X2L)
DOUBLE PRECISION T,Y(8),F(8),M,DELT,TFINAL,YSAVE(8),OTABLE(17,8)
DIMENSION KCRDER(8),AW(2),NC(2)
DATA KCRDER/6,4/
DATA NTS/1/,N/8/,KREG/1/,EPS/.00005/,HMIN/.005/,HMAX/0.5/,NXSTEP/
*1000/
GO TO (1,2,3),AMS
1 TFINAL=1.7758
DELT=1.9
GO TO 4
2 TFINAL=3.7808
DELT=3.9
GO TO 4
3 TFINAL=2.005
DELT=2.1
4 WR=0.0-SF*ALOG(1.0-2.0*(Y2R-Y1R)/X1R)/(12*TFINAL)
WL=0.0-SF*ALOG(1.0-2.0*(Y2L-Y1L)/X1L)/(12*TFINAL)
W1=WR
WC=WR-WL
W2=W2W1*W1
IF (W1-ABS(WC)) 34,34,35
34 WC=WC+W1/ABS(WC)
35 CONTINUE
J=0
M=1
5 Y(1)=X1R
T=0.0
J=J+1
H=0.2
Y(2)=X1L
Y(3)=Y1R
Y(4)=Y1L
Y(5)=X3R
Y(6)=X3L
Y(7)=Y4R
Y(8)=Y4L
I=1
6 IF (WC) 70,70,71
70 C1=Y(1)-Y(3)
C2=0.0
C3=0.0
C4=Y(4)-Y(2)
GO TO 72
71 C1=0.0
C2=Y(4)-Y(2)
C3=Y(1)-Y(3)
C4=0.0
72 F(1)=(W1*(Y(3)-Y(1))+2*W2*(Y(5)-Y(1))+WC*C1)/SF
F(2)=(W1*(Y(4)-Y(2))+2*W2*(Y(6)-Y(2))+WC*C2)/SF
F(3)=(W1*(Y(1)-Y(3))+2*W2*(Y(3)-Y(7))+WC*C3)/SF
F(4)=(W1*(Y(2)-Y(4))+2*W2*(Y(4)-Y(8))+WC*C4)/SF
F(5)=W2*(Y(1)-Y(5))/SF

```

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```

      F(6)=W2*(Y(2)-Y(6))/SF
      F(7)=W2*(Y(3)-Y(7))/SF
      F(8)=W2*(Y(4)-Y(8))/SF
      GO TO (7,8),1
7     CALL DCSYK (N,T,Y,F,KDEG,CPS,IFLAG,H,HMIN,HMAX,DELT,IEINAL,MXSTEP
      L,KSTEP,KERMAX,EKRMAX,KCROER,YSAVE,DTABLE)
      J=2
      GO TO 9
8     CALL DCSYK1
9     GO TO (6,6,6,11),IFLAG
11    DR=Y2R-Y(3)
      DL=Y2L-Y(4)
      DRR=ABS(DR)/Y2R
      DLL=ABS(DL)/Y2L
      IF (J-1) 12,12,13
13    IF (DRR-0.005) 19,19,20
19    IF (DRR+0.005) 20,21,21
21    IF (DLL-0.005) 22,22,20
22    IF (DLL+0.005) 20,23,23
20    IF (J-2) 10,10,14
10    SDW1=SDW1*EXP(Q2-(J-1)/10.)
      SOC1=SDC1*EXP(Q2-(J-1)/10.)
      M=2
      GO TO 31
14    IF (J-12) 18,18,90
18    IF (NW(2)-NW(1)) 28,29,28
28    SDW1=SDW1*EXP(Q2-(J-1)/10.)
29    IF (NC(2)-NC(1)) 30,17,30
30    SOC1=SDC1*EXP(Q2-(J-1)/10.)
17    NW(1)=NW(2)
      NC(1)=NC(2)
      GO TO 31
12    SDW1=C.15+W1
      SOC1=C.15*ABS(WC)
31    IF (SDW1-W1/200.) 80,80,81
80    SDW1=W1/200.
81    IF (SOC1-ABS(WC)/200.) 82,82,56
82    SDC1=ABS(WC)/200.
56    IF (DR) 50,50,51
51    IF (DL) 52,52,53
50    IF (OL) 54,54,55
53    NW(M)=1
25    IF (DR-DL) 60,60,64
60    NC(M)=2
      GO TO 51
52    NC(M)=1
      IF (WC) 38,38,24
38    NW(M)=1
      GO TO 91
24    NW(M)=2
      GO TO 91
64    NC(M)=1
      GO TO 91

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```

55      NC(M)=2
      IF (WC) 24,24,38
54      NW(M)=2
      GO TO 25
91      NN=NW(M)
      GO TO (92,93),NN
92      W1=W1+SDW1
96      NM=NC(M)
      GO TO (94,95),NM
93      W1=W1-SOW1
      GO TO 96
94      WC=WC+SDC1
      GO TO 27
95      WC=WC-SDC1
      GO TO 27
27      IF (W1=0.0) 26,26,37
26      W1=0.0
37      CONTINUE
      W2=W2+W1*W1
      GO TO 5
41      CONTINUE
90      WRITE (6,103)
      WRITE (6,100) CR,CL,CRR,OLL
      WRITE (6,100) W1,WC,SOW1,SDC1
      WRITE (6,104) NM(1),NM(2),NC(1),NC(2)
23      W=W1
      C=WC
      CSR=Y(5)
      CSL=Y(6)
      C6R=Y(7)
      C6L=Y(8)
      X2R=Y(1)
      X2L=Y(2)
101      FORMAT (14,5X,14)
103      FORMAT (14 J HAS EXCEEDED 12 --- RETURN AND CONTINUE *****
      [*****]
104      FORMAT (14(15,10X))
100      FORMAT (14(F10.5,10X))
      RETURN
      END

```

MIXING RESULTS FOR SHORT SECTION

TEST NUMBER = 1

REYNOLDS NUMBER = 24520.0

SUBCHANNEL FLOWRATE = 2839.82 LBS/HR

RATIO W2/W1 = 1.000

MIXING COEFFICIENT = 119.1115 LB/HR-FT

NONDIMENSIONAL MIXING COEF. = 50.48608

DIVERSION CROSSFLOW = 4.2071 LB/HR-FT

MIXING OCCURS FROM RIGHT TO LEFT

	DONOR	RECEPTOR
INLET CONCENTRATION		
NORMALIZED	1.0000	0.0
OUTLET CONCENTRATION		
NORMALIZED	0.9358	0.0774
PERIPHERAL (NORM.)	0.9975	0.0028

MIXING OCCURS FROM LEFT TO RIGHT

	DONOR	RECEPTOR
INLET CONCENTRATION		
NORMALIZED	1.0000	0.0
OUTLET CONCENTRATION		
NORMALIZED	0.9335	0.0748
PERIPHERAL (NORM.)	0.9975	0.0027

MIXING RESULTS FOR LONG SECTION

TEST NUMBER = 1

REYNOLDS NUMBER = 24520.0

SUBCHANNEL FLOWRATE = 2839.82 LBS/HR

RATIO V2/V1 = 1.000

MIXING COEFFICIENT = 72.6318 LB/HR-FT

NONDIMENSIONAL MIXING COEF. = 30.78537

DIVERSION CROSSFLOW = 29.6754 LB/HR-FT

MIXING OCCURS FROM RIGHT TO LEFT

DONOR

RECEPTOR

INLET CONCENTRATION

NORMALIZED

1.0000

0.0

OUTLET CONCENTRATION

NORMALIZED

0.9217

0.1334

PERIPHERAL (NORM.)

0.9951

0.0063

MIXING OCCURS FROM LEFT TO RIGHT

DONOR

RECEPTOR

INLET CONCENTRATION

NORMALIZED

1.0000

0.0

OUTLET CONCENTRATION

NORMALIZED

0.8895

0.0952

PERIPHERAL (NORM.)

0.9945

0.0045

MIXING RESULTS WITH ENTRANCE EFFECTS SUBTRACTED

TEST NUMBER

REYNOLDS NUMBER = 24520.0

SUBCHANNEL FLOWRATE = 2839.82 LBS/HR

RATIO-W2/W1 = 1.000

MIXING COEFFICIENT = 29.8722 LB/HR-FT

NONDIMENSIONAL MIXING COEF. = 12.66150

DIVERSION CROSSFLOW = 60.0529 LB/HR-FT

MIXING OCCURS FROM RIGHT TO LEFT

	DONOR	RECEPTOR
INLET CONCENTRATION		
NORMALIZED	1.0000	0.0
OUTLET CONCENTRATIONS OF FIRST SECTION		
NORMALIZED	0.9359	0.0774
PERIPHERAL (NORM.)	0.9975	0.0028
OUTLET CONCENTRATIONS OF SECOND SECTION		
NORMALIZED	0.9213	0.1334
PERIPHERAL (NORM.)	0.9961	0.0050

MIXING OCCURS FROM LEFT TO RIGHT

	DONOR	RECEPTOR
INLET CONCENTRATION		
NORMALIZED	1.0000	0.0
OUTLET CONCENTRATIONS OF FIRST SECTION		
NORMALIZED	0.9335	0.0748
PERIPHERAL (NORM.)	0.9975	0.0027
OUTLET CONCENTRATIONS OF SECOND SECTION		
NORMALIZED	0.8850	0.0952
PERIPHERAL (NORM.)	0.9956	0.0044

APPENDIX E

BULK MIXING DATA

TABLE E-1: PRESSURE TEST RESULTS

Reynolds No.	ΔP^* _{radial} (First Mixing Section)	ΔP _{radial} (Second Mixing Section)	ΔP _{axial} (To First Sampling Plane)	P _{axial} (To Second Sampling Plane)	f
1083.8			.426 .001	.589 .001	.02591
1880.3	.241 .001	.224 .001	.322 .003	.455 .003	.01539
3522.6	.272 .001	.279 .001	.804 .003	.348 .01	.01143
7160.1	.094 .003	.070 .003	.936 .01	.439 .03	.009863
10617.	.203 .003	.103 .003	.628 .03	.884 .03	.009282
12268.	.238 .003	.120 .003	.651 .03	.281 .1	.00936
13558.	.258 .003	.174 .003	.751 .03	.324 .1	.00911
14656.	.420 .003	.250 .003	.322 .1	.451 .1	.00841
16786.	.308 .003	.124 .003	.415 .1	.590 .1	.00832
16901.	.345 .003	.136 .003	.437 .1	.618 .1	.00864
20766.	.131 .01	.394 .003	.697 .1	.329 .3	.00816
32014.			.551 .1	.789 .1	.0074

* All pressures are given in TORR $\times 10^{-3}$

** All measurements taken at 60°F except for Re = 32014 which was taken at 94°F.

TABLE E-2
BULK CONCENTRATION MEASUREMENTS

Reynolds No.	Mixing Right to Left		Mixing Left to Right	
	C _{20L}	C _{20R}	C _{50L}	C _{50R}
2163.7	.1357 .1383 .1386 .1480	.07192 .0770	.1048 .1050	.2783 .2718 .2345
4053.5	.1632 .1760 .1530	.1070 .0954 .1308 .1268 .1113	.1026 .1171	.1985 .2571
8239.1	.2014 .1914	.1357 .1375 .1406 .1386	.0708 .0822 .0751	.1683 .1928
12218.	.1849 .1984	.1211 .1286	.1092 .0988	.2200
14347.	.1819 .1898	.0853	.0957 .0744	.1630 .1869 .1642
15601.	.1683 .1634	.0790 .0910	.1091 .0951 .0817 .0716	.1825 .1767 .1495 .1912
16864	.1704 .1827	.0940 .1183 .0954 .1129	.0819 .0916 .1085 .1119	.1894 .1899
19448.	.1511 .1498	.1245 .0711 .0763	.0830 .0895 .0866 .0891 .0932	.1296 .1646 .1268 .1760
23896.	.1322 .1412 .1275	.0707 .0926	.0729 .0622 .0859 .0903	.1044

RE#	$\frac{W}{u}$	TABLE E-3 SUBTRACTED RESULTS		RE#	$\frac{W}{u}$
		RE#	$\frac{W}{u}$		
2163.7	3.597	4053.5	5.145	4053.5	6.463
2163.7	3.610	4053.5	5.281	4053.5	7.061
2163.7	3.262	4053.5	6.400	4053.5	7.788
2163.7	3.768	4053.5	6.514	4053.5	8.291
2163.7	3.730	4053.5	4.140	4053.5	5.762
2163.7	3.699	4053.5	4.290	4053.5	5.934
2163.7	3.771	4053.5	5.117	4053.5	3.630
2163.7	3.791	4053.5	5.291	4053.5	5.281
2163.7	3.721	4053.5	6.480	4053.5	4.851
2163.7	4.257	4053.5	6.595	4053.5	4.957
2163.7	4.293	4053.5	4.124	4053.5	2.200
2163.7	4.198	4053.5	4.299	4053.5	2.605
2163.7	3.355	4053.5	5.135	4053.5	3.191
2163.7	3.295	4053.5	5.241	4053.5	3.226
2163.7	3.543	4053.5	6.331	4053.5	4.444
2163.7	3.451	4053.5	6.533	4053.5	4.541
2163.7	3.459	4053.5	4.117	4053.5	2.178
2163.7	3.387	4053.5	4.173	4053.5	2.200
2163.7	3.473	4053.5	3.576	4053.5	3.158
2163.7	3.455	4053.5	3.576	4053.5	3.192
2163.7	3.409	4053.5	4.727	4053.5	4.397
2163.7	3.972	4053.5	4.854	4053.5	4.515
2163.7	3.972	4053.5	2.540	4053.5	6.426
2163.7	3.926	4053.5	2.540	4053.5	6.960
2163.7	3.356	4053.5	3.573	4053.5	3.631
2163.7	3.295	4053.5	3.670	4053.5	4.290
2163.7	3.544	4053.5	5.950	4053.5	4.801
2163.7	3.451	4053.5	6.400	4053.5	2.578
2163.7	3.460			4053.5	4.586
2163.7	3.388	4053.5	6.514	4053.5	4.740
2163.7	3.474	4053.5	2.578	4053.5	1.519
2163.7	3.456	4053.5	3.556	4053.5	3.191
2163.7	3.410	4053.5	3.668	4053.5	1.536
2163.7	3.973	4053.5	4.930	4053.5	3.152
2163.7	3.973	4053.5	4.140	4053.5	2.006
2163.7	4.199	4053.5	2.178	4053.5	1.981
2163.7	3.598	4053.5	8.047	4053.5	1.952
2163.7	3.611	4053.5	8.245	4053.5	2.009
2163.7	3.234	4053.5	5.771	4053.5	2.006
2163.7	3.769	4053.5	5.145	4053.5	3.152
2163.7	3.731	4053.5	4.817	4053.5	3.253
2163.7	3.700	4053.5	5.059	4053.5	3.139
2163.7	3.772	4053.5	2.582	4053.5	1.496
2163.7	3.792	4053.5	2.632	4053.5	1.555
2163.7	3.722	4053.5	3.630	4053.5	1.536
2163.7	4.257	4053.5	3.668	4053.5	1.519
2163.7	4.294	4053.5	4.851	4053.5	3.191
2163.7	3.925	4053.5	4.957	4053.5	1.536
4053.5	5.540	4053.5	2.592	4053.5	3.235
4053.5	5.775	4053.5	2.605	4053.5	2.006
4053.5	4.785	4053.5	3.191	4053.5	1.981
4053.5	6.951	4053.5	3.226	4053.5	1.952
4053.5	4.557	4053.5	4.444	4053.5	2.009
4053.5	4.708	4053.5	4.541	4053.5	2.006
4053.5	5.614	4053.5	2.200	4053.5	3.235
4053.5	5.823	4053.5	2.200	4053.5	3.268
4053.5	6.877	4053.5	3.170	4053.5	3.139
4053.5	7.000	4053.5	3.256	4053.5	1.496
4053.5	4.604	4053.5	4.516	4053.5	1.555
4053.5	4.723	4053.5	4.557	4053.5	1.536

SUBTRACTED RESULTS

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Re	$\frac{W'}{\mu}$	Re	$\frac{W'}{\mu}$	Re	$\frac{W'}{\mu}$
4053.5	2.720	8239.1	12.350	8239.1	9.666
4053.5	2.720	8239.1	12.539	8239.1	10.781
4053.5	3.143	8239.1	10.378	8239.1	10.127
4053.5	3.143	8239.1	10.290	8239.1	10.274
4053.5	4.466	8239.1	12.338	8239.1	10.321
4053.5	4.445	8239.1	12.474	8239.1	12.127
4053.5	2.144	8239.1	10.393	8239.1	10.158
4053.5	2.144	8239.1	10.506	8239.1	17.241
4053.5	5.879	8239.1	12.100	8239.1	12.063
4053.5	5.732	8239.1	12.400	8239.1	10.503
4053.5	8.177	8239.1	10.420	8239.1	8.459
4053.5	7.908	8239.1	10.332	8239.1	8.404
4053.5	6.665	8239.1	12.466	8239.1	10.417
4053.5	6.924	8239.1	12.776	8239.1	10.400
4053.5	6.972	8239.1	10.863	8239.1	10.916
4053.5	6.533	8239.1	10.698	8239.1	10.072
4053.5	6.410	8239.1	12.651	8239.1	11.033
4053.5	8.278	8239.1	12.855	8239.1	12.824
4053.5	5.824	8239.1	10.764	8239.1	10.839
4053.5	5.993	8239.1	10.881	8239.1	10.814
4053.5	5.717	8239.1	12.419	8239.1	12.755
4053.5	5.600	8239.1	12.728	8239.1	11.912
4053.5	6.845	8239.1	10.961	8239.1	9.861
4053.5	6.908	8239.1	10.659	8239.1	9.877
4053.5	4.576	8239.1	10.072	8239.1	11.682
4053.5	4.638	8239.1	10.860	8239.1	12.838
4053.5	7.061	8239.1	10.637	8239.1	10.713
4053.5	6.863	8239.1	12.376	8239.1	10.904
4053.5	7.788	8239.1	10.145	8239.1	12.658
4053.5	8.291	8239.1	8.454	8239.1	10.437
4053.5	5.762	8239.1	8.334	8239.1	8.451
4053.5	5.935	8239.1	10.418	8239.1	8.323
4053.5	5.823	8239.1	10.106	8239.1	10.346
4053.5	5.614	8239.1	9.892	8239.1	10.303
4053.5	6.878	8239.1	9.917	8239.1	8.372
4053.5	7.000	8239.1	12.015	8239.1	8.441
4053.5	4.604	8239.1	9.470	8239.1	10.457
4053.5	4.723	8239.1	9.924	8239.1	12.058
4053.5	5.833	8239.1	9.854	8239.1	9.996
4053.5	5.669	8239.1	11.991	8239.1	9.842
4053.5	6.454	8239.1	10.290	8239.1	12.020
4053.5	6.963	8239.1	10.854	8239.1	15.136
8239.1	12.127	8239.1	9.566	8239.1	18.949
8239.1	10.320	8239.1	11.426	8239.1	18.619
8239.1	12.107	8239.1	10.393	8239.1	17.655
8239.1	11.874	8239.1	8.418	8239.1	15.287
8239.1	12.169	8239.1	8.307	8239.1	15.060
8239.1	10.3	8239.1	10.587	8239.1	13.374
8239.1	11.758	8239.1	10.332	8239.1	16.924
8239.1	11.939	8239.1	10.032	8239.1	16.862
8239.1	9.788	8239.1	9.750	8239.1	17.032
8239.1	9.944	8239.1	11.734	8239.1	13.358
8239.1	11.697	8239.1	10.439	8239.1	13.426
8239.1	11.826	8239.1	9.862	8239.1	15.512
8239.1	9.843	8239.1	9.861	8239.1	19.120
8239.1	9.867	8239.1	11.874	8239.1	19.049
8239.1	11.710	8239.1	9.760	8239.1	19.071
8239.1	11.710	8239.1	8.402	8239.1	15.287
8239.1	9.941	8239.1	8.423	8239.1	15.196
8239.1	9.904	8239.1	10.561	8239.1	13.374

Re	$\frac{W'}{u}$	Re	$\frac{W'}{u}$	Re	$\frac{W'}{u}$
8239.1	17.156	15601.2	24.170	15601.2	32.227
8239.1	17.281	15601.2	32.227	16864.1	30.986
8239.1	17.265	15601.2	27.281	16864.1	30.986
8239.1	13.506	15601.2	15.217	16864.1	36.109
8239.1	13.426	15601.2	30.538	16864.1	36.109
12217.8	20.566	15601.2	33.794	16864.1	21.216
12217.8	20.619	15601.2	33.794	16864.1	11.351
12217.8	22.681	15601.2	29.502	16864.1	11.351
12217.8	23.062	15601.2	30.286	16864.1	0.975
12217.8	17.903	15601.2	27.426	16864.1	0.975
12217.8	18.205	15601.2	24.918	16864.1	10.746
12217.8	15.567	15601.2	29.420	16864.1	10.746
12217.8	15.695	15601.2	27.281	16864.1	3.197
14346.9	10.388	15601.2	34.097	16864.1	3.197
14346.9	36.285	15601.2	34.097	16864.1	3.189
14346.9	31.484	15601.2	25.788	16864.1	3.189
14346.9	23.372	15601.2	34.205	16864.1	10.712
14346.9	31.235	15601.2	27.765	16864.1	10.766
14346.9	23.714	15601.2	30.101	16864.1	0.972
14346.9	23.530	15601.2	25.957	16864.1	0.972
14346.9	31.379	15601.2	27.520	16864.1	11.198
14346.9	23.987	15601.2	32.521	16864.1	11.368
14346.9	31.110	15601.2	30.531	16864.1	11.194
14346.9	33.180	15601.2	20.499	16864.1	11.194
14346.9	31.520	15601.2	34.097	16864.1	0.961
14346.9	31.484	15601.2	27.703	16864.1	0.961
14346.9	22.157	15601.2	32.022	16864.1	10.606
14346.9	10.405	15601.2	20.523	16864.1	10.606
14346.9	14.406	15601.2	27.765	16864.1	3.150
14346.9	22.362	15601.2	27.168	16864.1	3.150
14346.9	13.073	15601.2	24.976	16864.1	3.126
14346.9	14.645	15601.2	15.438	16864.1	3.126
14346.9	13.175	15601.2	33.794	16864.1	10.524
14346.9	3.761	15601.2	27.281	16864.1	10.524
14346.9	11.865	15601.2	32.227	16864.1	0.953
14346.9	4.488	15601.2	15.297	16864.1	0.953
14346.9	4.117	15601.2	27.729	16864.1	11.111
14346.9	12.177	15601.2	29.381	16864.1	11.111
14346.9	4.913	15601.2	29.381	16864.1	20.883
14346.9	7.903	15601.2	29.914	16864.1	25.987
14346.9	10.803	15601.2	29.381	16864.1	25.987
14346.9	4.211	15601.2	32.022	16864.1	34.379
14346.9	3.865	15601.2	27.703	16864.1	34.379
14346.9	12.084	15601.2	20.360	16864.1	35.490
14346.9	4.353	15601.2	32.474	16864.1	35.490
14346.9	10.414	15601.2	29.495	16864.1	23.204
14346.9	10.586	15601.2	29.049	16864.1	23.574
14346.9	10.414	15601.2	26.006	16864.1	28.059
14346.9	13.295	15601.2	29.495	16864.1	28.059
14346.9	13.117	15601.2	32.474	16864.1	23.390
14346.9	13.295	15601.2	27.765	16864.1	23.390
14346.9	10.308	15601.2	25.787	16864.1	28.288
14346.9	10.411	15601.2	32.514	16864.1	28.288
14346.9	10.411	15601.2	29.109	16864.1	31.102
14346.9	13.037	15601.2	28.795	16864.1	31.102
14346.9	13.037	15601.2	28.795	16864.1	35.306
14346.9	12.759	15601.2	29.308	16864.1	35.306
15601.2	27.259	15601.2	27.237	16864.1	21.054
15601.2	25.422	15601.2	24.824	16864.1	21.391
15601.2	15.018	15601.2	27.215	16864.1	25.921

SUBTRACTED RESULTS

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Re	$\frac{W'}{\mu}$	Re	$\frac{W'}{\mu}$	Re	$\frac{W'}{\mu}$
16864.1	25.921	19447.7	35.6H1	19447.7	22.154
16864.1	31.233	19447.7	21.747	19447.7	35.594
16864.1	31.233	19447.7	21.965	19447.7	15.871
16864.1	35.911	19447.7	20.364	19447.7	35.681
16864.1	35.911	19447.7	21.205	19447.7	17.510
16864.1	31.750	19447.7	20.953	19447.7	34.920
16864.1	31.750	19447.7	37.912	19447.7	15.850
16864.1	32.670	19447.7	38.602	19447.7	35.005
16864.1	32.670	19447.7	24.450	19447.7	17.775
16864.1	21.324	19447.7	24.450	19447.7	19.328
16864.1	21.324	19447.7	17.575	19447.7	11.889
16864.1	26.259	19447.7	23.999	19447.7	18.125
16864.1	26.259	19447.7	18.396	19447.7	37.646
16864.1	31.123	19447.7	37.896	19447.7	20.347
16864.1	31.123	19447.7	38.584	19447.7	35.721
16864.1	32.922	19447.7	24.439	19447.7	35.721
16864.1	32.922	19447.7	24.439	19447.7	35.044
16864.1	23.852	19447.7	17.568	19447.7	35.044
16864.1	23.852	19447.7	23.989	19447.7	20.411
16864.1	28.663	19447.7	18.757	19447.7	36.129
16864.1	28.663	19447.7	35.273	19447.7	19.094
16864.1	23.602	19447.7	35.955	19447.7	38.464
16864.1	23.602	19447.7	21.412	19447.7	20.418
16864.1	28.050	19447.7	21.814	19447.7	36.304
16864.1	28.050	19447.7	15.680	19447.7	18.999
16864.1	31.226	19447.7	21.412	19447.7	37.777
16864.1	31.226	19447.7	17.146	19447.7	19.162
16864.1	31.482	19447.7	35.129	19447.7	35.240
16864.1	31.482	19447.7	35.808	19447.7	17.951
16864.1	21.382	19447.7	21.677	19447.7	38.018
16864.1	21.382	19447.7	21.677	19447.7	19.058
16864.1	26.331	19447.7	18.654	19447.7	34.610
16864.1	26.331	19447.7	21.278	19447.7	18.071
16864.1	31.094	19447.7	20.125	19447.7	17.783
16864.1	31.519	19447.7	38.057	19447.7	18.995
16864.1	31.307	19447.7	38.749	19447.7	17.446
16864.1	31.054	19447.7	24.087	19447.7	12.052
19447.7	21.998	19447.7	24.540	19447.7	33.317
19447.7	35.215	19447.7	15.955	19447.7	12.308
19447.7	35.895	19447.7	24.087	19447.7	22.242
19447.7	21.728	19447.7	16.962	19447.7	33.132
19447.7	21.872	19447.7	37.738	19447.7	20.906
19447.7	17.610	19447.7	38.330	19447.7	35.273
19447.7	21.328	19447.7	24.285	19447.7	19.001
19447.7	18.698	19447.7	24.446	19447.7	35.224
19447.7	37.777	19447.7	20.293	19447.7	35.808
19447.7	38.464	19447.7	24.285	19447.7	35.129
19447.7	24.366	19447.7	19.014	19447.7	37.482
19447.7	24.366	19447.7	35.282	19447.7	19.154
19447.7	18.701	19447.7	35.823	19447.7	17.652
19447.7	23.917	19447.7	35.144	19447.7	38.415
19447.7	20.251	19447.7	35.144	19447.7	18.820
19447.7	35.230	19447.7	37.746	19447.7	35.099
19447.7	35.911	19447.7	22.377	19447.7	17.639
19447.7	21.737	19447.7	37.751	19447.7	37.729
19447.7	21.881	19447.7	15.854	19447.7	18.955
19447.7	17.617	19447.7	38.330	19447.7	20.458
19447.7	21.337	19447.7	17.403	19447.7	19.281
19447.7	18.705	19447.7	37.646	19447.7	11.862
19447.7	35.005	19447.7	15.818	19447.7	17.412

SUBTRACTED RESULTS		SUBTRACTED RESULTS		RESULTS - LONG SECTION 189	
Re	$\frac{W'}{u}$	Re	$\frac{W'}{u}$	Re	$\frac{W'}{u}$
19447.7	33.301	23895.9	40.676	14346.9	30.779
19447.7	21.090	23895.9	33.153	14346.9	30.584
19447.7	38.163	23895.9	34.791	14346.9	30.533
19447.7	21.978	23895.9	35.405	14346.9	25.826
19447.7	33.091	23895.9	32.759	14346.9	25.953
19447.7	21.079	23895.9	36.521	14346.9	30.432
19447.7	35.129	23895.9	41.077	15601.2	30.201
19447.7	17.145	23895.9	33.497	15601.2	30.378
19447.7	35.267	23895.9	28.899	15601.2	33.186
19447.7	20.888	23895.9	28.797	15601.2	33.186
19447.7	35.955	23895.9	28.704	15601.2	33.186
23895.9	23.085	23895.9	20.721	15601.2	33.954
23895.9	27.708	23895.9	21.400	15601.2	33.954
23895.9	20.431	23895.9	20.736	15601.2	34.246
23895.9	23.085	23895.9	23.302	16864.1	37.273
23895.9	27.508	23895.9	28.308	16864.1	37.273
23895.9	20.195	23895.9	20.726	16864.1	26.998
23895.9	22.659	23895.9	22.872	16864.1	27.133
23895.9	27.708	23895.9	27.972	16864.1	39.737
23895.9	20.195	23895.9	20.384	16864.1	39.737
23895.9	36.702	23895.9	22.601	19447.7	32.208
23895.9	38.777	23895.9	27.972	19447.7	32.334
23895.9	33.662	23895.9	20.384	19447.7	32.836
23895.9	26.156	RESULTS LONG SECTION		19447.7	32.868
23895.9	26.295			19447.7	37.650
23895.9	26.114	Re	$\frac{W'}{u}$	19447.7	38.043
23895.9	18.404	2163.7	4.362	19447.7	38.060
23895.9	18.550	2163.7	4.362	19447.7	30.371
23895.9	18.318	2163.7	4.291	19447.7	30.371
23895.9	35.796	2163.7	4.113	19447.7	30.874
23895.9	41.297	2163.7	4.100	19447.7	31.135
23895.9	33.472	2163.7	4.100	19447.7	31.135
23895.9	36.057	2163.7	4.091	19447.7	38.174
23895.9	36.537	2163.7	4.037	19447.7	38.380
23895.9	33.075	2163.7	4.036	19447.7	38.457
23895.9	28.532	2163.7	4.036	23895.9	37.802
23895.9	29.014	2163.7	4.024	23895.9	38.031
23895.9	28.496	2163.7	3.969	23895.9	38.046
23895.9	23.542	4053.5	6.864	23895.9	39.521
23895.9	28.058	4053.5	7.050	23895.9	40.068
23895.9	20.593	4053.5	9.502	23895.9	40.867
23895.9	22.668	4053.5	9.243	23895.9	41.252
23895.9	28.263	4053.5	8.861	23895.9	42.033
23895.9	20.213	4053.5	8.611	23895.9	43.845
23895.9	21.326	4053.5	8.366		
23895.9	20.946	4053.5	8.115		
23895.9	19.973	8239.1	13.433		
23895.9	23.188	8239.1	13.479		
23895.9	28.072	8239.1	14.380		
23895.9	20.664	8239.1	14.474		
23895.9	22.912	8239.1	18.003		
23895.9	26.489	8239.1	18.100		
23895.9	19.897	8239.1	20.316		
23895.9	18.712	8239.1	20.479		
23895.9	18.761	12217.8	28.958		
23895.9	18.719	12217.8	31.574		
23895.9	36.140	14346.9	34.510		
23895.9	41.318	14346.9	33.625		
23895.9	33.153				
23895.9	35.711				

RESULTS - SHORT SECTION

Re	$\frac{W'}{\mu}$	Re	$\frac{W'}{\mu}$	190
2163.7	4.343	15601.2	34.324	
2163.7	4.656	15601.2	33.986	
2163.7	4.656	15601.2	34.324	
2163.7	4.343	15601.2	39.068	
4053.5	11.461	15601.2	35.390	
4053.5	12.036	16864.1	52.179	
4053.5	10.771	16864.1	52.112	
4053.5	13.086	16864.1	50.532	
4053.5	12.491	16864.1	50.489	
4053.5	9.341	16864.1	44.504	
4053.5	9.359	16864.1	44.504	
4053.5	9.407	16864.1	44.084	
4053.5	9.464	16864.1	43.794	
4053.5	9.483	16864.1	43.208	
4053.5	10.771	16864.1	43.109	
4053.5	10.879	16864.1	42.724	
4053.5	10.879	16864.1	42.475	
4053.5	11.527	16864.1	38.671	
4053.5	12.036	16864.1	38.592	
4053.5	12.036	16864.1	38.297	
4053.5	12.491	16864.1	38.234	
4053.5	12.491	19447.7	41.184	
4053.5	14.189	19447.7	41.184	
4053.5	14.565	19447.7	41.184	
4053.5	13.179	19447.7	41.184	
4053.5	11.660	19447.7	41.184	
4053.5	10.700	19447.7	38.541	
4053.5	11.616	19447.7	38.541	
4053.5	11.527	19447.7	38.541	
8239.1	19.221	19447.7	38.541	
8239.1	19.221	19447.7	38.351	
8239.1	16.373	23895.9	41.496	
8239.1	16.506	23895.9	48.653	
8239.1	17.307	23895.9	41.112	
8239.1	17.528	23895.9	47.069	
8239.1	18.924	23895.9	47.069	
8239.1	19.089	23895.9	59.391	
8239.1	19.182	23895.9	56.651	
8239.1	19.126	23895.9	46.603	
8239.1	17.617			
8239.1	17.591			
8239.1	17.577			
8239.1	17.565			
8239.1	16.671			
8239.1	16.563			
8239.1	16.520			
8239.1	16.483			
12217.8	36.885			
12217.8	33.652			
12217.8	33.685			
12217.8	37.123			
14346.9	29.596			
14346.9	30.453			
14346.9	34.667			
14346.9	38.785			
14346.9	59.638			
14346.9	33.986			
15601.2	39.327			
15601.2	31.032			
15601.2	31.149			

FIGURE F-1 VELOCITY PROFILE FOR $RE = 2164$

FIGURE F-2 VELOCITY PROFILE FOR RE = 4054 ..

FIGURE F-3 VELOCITY PROFILE FOR $RE = 8239$

FIGURE F-4 VELOCITY PROFILE FOR $RE = 12218$

FIGURE F-5 VELOCITY PROFILE FOR $RE = 14347$

FIGURE F-6 VELOCITY PROFILE FOR $RE = 16864$

FIGURE F-7 VELOCITY PROFILE FOR RE = 19448

FIGURE F-8 VELOCITY PROFILE FOR $RE = 23896$

APPENDIX G

LOCAL CONCENTRATION MEASUREMENTS

RE	MIXING RIGHT TO LEFT		MIXING LEFT TO RIGHT	
	$\frac{X}{R}$	C^*	$\frac{X}{R}$	C^*
2164	0.1360	0.6072	0.2293	0.2240
	0.0960	0.5737	0.1627	0.2432
	0.0627	0.5654	0.0960	0.2819
	0.1960	0.6384	0.0293	0.3240
	-0.0040	0.5326	-0.0373	0.3683
	0.2627	0.6818	-0.1040	0.4424
	-0.0707	0.4660	-0.1707	0.5719
	-0.1240	0.4146	-0.2373	0.6155
	-0.1707	0.4045	-0.3040	0.6431
	0.2827	0.7172	-0.3707	0.6502
	-0.2373	0.3525		
	-0.3040	0.3165		
	-0.3040	0.3007		
	-0.0800	0.4994		
	-0.3707	0.2698		
	-0.3707	0.2637		
4054	-0.3040	0.2884	-0.3040	0.7518
	-0.2373	0.3280	-0.2373	0.7011
	-0.1707	0.3735	-0.1707	0.6373
	-0.1040	0.4131	-0.1040	0.5891
	-0.0373	0.4705	-0.0373	0.5171
	0.0293	0.5405	0.0293	0.4358
	0.0960	0.5842	0.0960	0.3801
	0.1627	0.6132	0.1627	0.3202
	0.2293	0.6428	0.2293	0.2578
8239	-0.2373	0.3208	0.2293	0.2703
	-0.1707	0.3596	0.1627	0.3318
	-0.1040	0.4125	0.0960	0.3952
	-0.0373	0.4706	0.0293	0.4664
	0.0293	0.5382	-0.0373	0.5336
	0.0960	0.5904	-0.1040	0.6015
	0.1627	0.6433	-0.1707	0.6614
	0.2293	0.6976	-0.2373	0.7377
			-0.3040	0.7844

RE	MIXING RIGHT TO LEFT		MIXING LEFT TO RIGHT	
	$\frac{X}{R}$	C^*	$\frac{X}{R}$	C^*
12218	-0.2373 -0.1707 -0.1040 -0.0373 0.0293 0.0960 0.1627 0.2293	0.3246 0.3724 0.4149 0.4730 0.5275 0.5836 0.6291 0.6811	-0.2373 -0.1707 -0.1040 -0.0373 0.0293 0.0960 0.1627 0.2293	0.7617 0.6928 0.6147 0.5437 0.4665 0.3998 0.3411 0.2741
14347	-0.2373 -0.1707 -0.1040 -0.0373 0.0293 0.0960 0.1627 0.1627 0.2293	0.3129 0.3704 0.4029 0.4750 0.5334 0.6047 0.6911 0.6315 0.7249	0.2293 0.1627 0.0960 0.0293 -0.0373 -0.1040 -0.1707 -0.2373	0.2968 0.3489 0.3905 0.4698 0.5300 0.5826 0.6143 0.6575
16864	-0.2373 -0.1707 -0.1040 -0.0373 0.0293 0.0960 0.1627 0.2293	0.3375 0.3725 0.4194 0.4768 0.5295 0.5881 0.6405 0.7054	-0.2373 -0.1707 -0.1040 -0.0373 0.0293 0.0960 0.1627 0.2293	0.6663 0.6130 0.5652 0.5179 0.4767 0.4481 0.3926 0.3391
19448	0.2293 0.1627 0.0960 0.0293 -0.0373 -0.1040 -0.1707 -0.2373	0.6619 0.6123 0.5479 0.5253 0.4727 0.4431 0.4164 0.3575	0.2293 0.1627 0.0960 0.0293 -0.0373 -0.0373 -0.1040 -0.1707 -0.2373	0.3254 0.3568 0.4121 0.4837 0.5038 0.5539 0.5843 0.6144 0.6361
23896	0.2293 0.1627 0.0960 0.0293 -0.0373 -0.1040 -0.1707 -0.2373	0.7032 0.6597 0.5993 0.5340 0.4594 0.3945 0.3512 0.3242	0.2293 0.1627 0.0960 0.0293 -0.0373 -0.1040 -0.1707 -0.2373	0.2921 0.3600 0.4160 0.4729 0.5246 0.5869 0.6333 0.7544

TABLE G-2

CONCENTRATION GRADIENTS AND THE CIRCUMFERENTIAL EDDY DIFFUSIVITY

RE	$\frac{W'}{\mu}$	C_1^*		C_2^*		$\frac{1}{f} \frac{dC^*}{d\sigma}$		$\frac{\overline{\epsilon_{tp}}}{\gamma} \Big _{\sigma=0}$	
		R---L	L---R	R---L	L---R	R---L	L---R	R---L	L---R
2164	3.654	.8562	.7575	.1438	.2425	.4354	2.224	12.30	1.74
4053	8.682	.8356	.7734	.1644	.2266	1.175	1.394	10.21	7.01
8239	18.52	.825	.8025	.175	.1975	1.064	1.009	23.27	22.85
12218	26.21	.820	.8237	.180	.1763	.8443	1.105	40.86	31.58
14347	29.65	.8225	.8325	.1775	.1675	1.01	1.047	38.96	38.75
16864	33.13	.830	.840	.170	.160	.8443	.6644	53.28	69.76
19448	36.02	.8431	.8444	.1569	.1556	.7898	.7097	64.39	71.93
23896	39.40	.8775	.8462	.1225	.1538	1.025	.8153	59.72	68.84

APPENDIX H

RAMM, JOHANSEN, AND TODREAS' PREDICTIONS

TRIANGULAR ARRAY

P/D = 1.10

PR = 1000

L/D_h = 83.71

REYNOLDS NO.	C_1^*	$\frac{W^*}{u}$	$\frac{R}{r} \frac{dC^*}{d\phi}$
5860	0.955	0.951	3.61
13320	0.9318	3.566	2.66
29240	0.9202	9.525	2.35
50410	0.916	17.54	2.27

APPENDIX I

THE EFFECT OF GEOMETRY VARIATION ON DIVERSION CROSS-FLOW

In order to estimate the errors in our measurements due to diversion cross-flow it was necessary to assume a certain type of geometrical variation and calculate its effect upon the experiment. The fabrication of the new test-section involved three separate machining processes;

1. Milling the pieces which form the "box" to which the rods are bolted,
2. Centerless grinding of the rods to the correct diameter, and
3. Milling flats on to the rods.

The tolerances associated with centerless grinding ($\pm .0001$) are an order of magnitude smaller than the tolerances for milling, and the variations in rod diameter will be assumed negligible. Both processes one and three cause variations in the rod spacing, however, it is more likely for variations to exist in process three. This is a result of the difficulty encountered in fastening the rods to the bed of the milling machine without using a vise (which would cause flat spots where the rod was held). A further complication was the tendency of the rods to bow when the removal of metal on one side allowed the "built in" stresses to relax. Therefore, we will consider the crossflows induced by variations in the spacing of the rods due to

the flats being milled to different depths along the axis of the test-section.

To simplify this analysis, the following restrictions will be applied

1. Variations will occur in only one rod, the geometry of the other being assumed "perfect",
2. Only the cross-flows between the two central subchannels will be considered,
3. The variation will be a lateral displacement of rod B by an amount ΔZ , as is shown in Figure I-1,
4. This displacement will be zero at the entrance to the mixing section and increase linearly (with axial position) to a maximum value (Z_m) at the exit, and
5. A "flow-split" type of analysis will be used and hence, a lateral momentum balance will not be included (this will lead to larger than actual cross-flows).

First, let's consider the variations in the geometrical parameters associated with subchannel #1. For a given displacement, ΔZ , the subchannel cross-sectional area is given by:

$$A_1 = \frac{1}{2}(D + c - \Delta Z)(\sin 60^\circ)(P) - \frac{\pi}{2}\left(\frac{D}{2}\right)^2$$

which reduces to:

$$A_1 = A_{10} - (.4330)PAZ$$

FIGURE I-1
Defined Asymmetry (ΔZ)

where,

A_{10} = nominal subchannel area.

Since the wetted perimeter is not affected, the hydraulic diameter becomes,

$$D_h = 4(A_{10} - (.4330)P\Delta Z) / [\frac{\pi(D)}{2}] = D_{ho} - (.5513)\frac{P}{D} \Delta Z$$

where,

D_{ho} = nominal hydraulic diameter.

Introducing the linear variation,

$$\Delta Z = z_m \left(\frac{X}{L} \right)$$

where

X = axial distance

L = length of mixing section

these parameters become:

$$A_1 = A_{10} - (.4330)Pz_m \left(\frac{X}{L} \right) \quad (I-1)$$

$$D_h = D_{ho} - (.5513)\frac{P}{D} z_m \left(\frac{X}{L} \right) \quad (I-2)$$

From Roidts' paper [31];

$$\frac{\bar{\mu}_f}{\bar{\mu}} = \left(\frac{D_{hf}}{D_h} \right)^{.6284} \quad (2-3)$$

and

$$\frac{d}{dx} (\bar{\mu}A)_1 = -C(v_{1j} + v_{1k} + v_{1l}) \quad (2-4)$$

where

v_{1j} = gap averaged transverse velocity between subchannels
i and j

Using the notation of Figure 2-1, this can be rewritten,

$$\frac{d}{dx} (\bar{\mu}A)_1 = -C(v_{12} + v_{13} + v_{15})$$

The dependency of v_{13} and v_{15} upon the rod displacement is very complex (due to the geometry of the peripheral channels) and they will be ignored in this analysis, a justification for this follows:

1. Since the area of subchannels 3 and 5 decrease also, v_{13} and v_{15} will be negative.
2. The magnitudes of v_{13} and v_{15} will be smaller than v_{12} since they represent flows from one "choked" channel to another "choked" channel, while v_{12} represents a flow from a "choked" channel to an unchoked channel, and
3. The change in subchannel one's concentration due to v_{13} and v_{15} is small, since the concentration differences between these channels are small.

So we assume,

$$\frac{d}{dx} (\bar{\mu} A)_1 = - C v_{12} \quad (E-5)$$

and combining equations (E-3) and (E-5);

$$\frac{d}{dx} (\bar{\mu}_1 A_1) = \frac{d}{dx} \left[\bar{\mu} \left(\frac{D_{h1}}{D_h} \right)^{.6284} A_1 \right] = - C v_{12}$$

Substituting in the expressions for subchannel area and hydraulic diameter:

$$\frac{d}{dx} \left[\bar{\mu} \left[1 - (.5513) \left(\frac{P}{D} \right) \left(\frac{Z_m}{D_h} \right) \left(\frac{X}{L} \right) \right]^{.6284} (A_{10} - (.4330) P Z_m \left(\frac{X}{L} \right)) \right] = - C v_{12}$$

Since Z_m is small, we will drop all second order terms, and

$$\frac{d}{dx} \left[\bar{\mu} \left[A_{10} \left(1 - (.6284) \left(\frac{P}{D} \right) \left(\frac{Z_m}{D_h} \right) \left(\frac{X}{L} \right) \right) - (.4330) P Z_m \left(\frac{X}{L} \right) \right] \right] = - C v_{12}$$

Simplifying,

$$- C v_{12} = \bar{\mu} \left[A_{10} (.3464) \left(\frac{P}{D} \right) \left(\frac{Z_m}{D_h} \right) \left(\frac{1}{L} \right) - (.4330) P \left(\frac{Z_m}{L} \right) \right]$$

$$\frac{v_{12}}{\bar{\mu}} = \frac{\left(\frac{Z_m}{L} \right) \left(\frac{P}{D} \right)}{\left(\frac{P}{D} - 1 \right)} (.3650) \quad (E-6)$$

This transverse velocity can be simply related to the diversion cross flow by:

$$W_{12} = \rho C v_{12}$$

$$W_{12} = \rho \left(\frac{P}{D} \right) D \left(\frac{Z}{L} \right) \bar{u} \quad (E-7)$$

$$\frac{W_{12}}{\bar{u}} = \left(\frac{P}{D} \right) \left(\frac{Z}{L} \right) \left[\frac{1}{1.125 \left(\frac{P}{D} \right)^2 - 1} \right] Re \quad (E-8)$$

Figure E-2 is a graph of the ratio of the calculated diversion crossflow to the mixing coefficient (calculated by Roum et al [2] for a triangular array rod bundle, with a Prandtl no. of 1000, and $\left(\frac{Z}{L} \right)$ equal to 7.5×10^{-5} , (this is equivalent to a subchannel area variation of 1/2% for our test section) for several $\frac{P}{D}$'s versus Reynolds' number. Figure E-3 is a graph showing the value of $\left(\frac{Z}{L} \right)$ necessary to keep the ratio of diversion crossflow to turbulent mixing at 10% for the conditions listed above.

The tolerance on the milled flats in our rod bundle are at best $\pm .001"$ $\left(\frac{Z}{L} = 2.16 \times 10^{-5} \right)$ and at worst $\pm .005"$ $\left(\frac{Z}{L} = 1.08 \times 10^{-4} \right)$. From this, it would appear that significant diversion crossflows will always be present and the macroscopic energy (concentration) equation must contain a term for diversion cross-flow. It is interesting to note that in Roidt's experiment [31] the transport due to crossflow was 1.5 to 5.5 times the turbulent transport. This represents a $\left(\frac{Z}{L} \right)$ that varies from 9.5×10^{-4} to 3.5×10^{-3} .

FIGURE 1-2

FIGURE 1-3: Rod Displacement for $W/W' = 1$

1-2 Dependence of Diversion Crossflow on Subchannel Transverse Pressure Difference

Weisman [53] gives the steady state lateral momentum balance for diversion crossflow as;

$$\frac{\alpha(W_{ij})}{\alpha x} = C/D (P_i - P_j) - \frac{K|W_{ij}|W_{ij}}{2 C^2 \rho} (C/D)$$

Neglecting the term on the left hand side, which is negligible if W_{ij} does not vary axially;

$$W_{ij}^2 = B(P_i - P_j)$$

where B is a constant. Using the friction factor formulation for the axial subchannel pressure drops, this can be rewritten for $X = L$;

$$W_{ij}^2 = B(f_i \frac{L}{D_{hi}} \frac{\rho \bar{V}_i^2}{2g_o} - f_j \frac{L}{D_{hj}} \frac{\rho \bar{V}_j^2}{2g_o})$$

Substituting for the friction factor,

$$W_{ij}^2 = B \frac{L}{D_{hi}} \frac{\rho \bar{V}_i^2}{2g_o} \cdot \frac{C}{(\frac{\rho \bar{V}_i D_{hi}}{\mu})^n} - \frac{L}{D_{hj}} \cdot \frac{\rho \bar{V}_j^2}{2g_o} \cdot \frac{C}{(\frac{\rho \bar{V}_j D_{hj}}{\mu})^n}$$

Simplifying and combining all the constants into B;

$$W_{ij}^2 = B \frac{V_i^{2-n}}{D_{hi}^{1+n}} \frac{V_j^{2-n}}{D_{hj}^{1+n}}$$

or

$$W_{ij}^2 = B \frac{D_T^{n-2}}{D_{hi}^{1+n}} - \frac{D_T^{n-2}}{D_{hj}^{1+n}} Re_T^{2-n}$$

where,

D_T = hydraulic diameter of test section

Re_T = Reynolds no. of test section

Therefore, the Reynolds no. dependence of the crossflow is:

$$W_{ij}^2 = Re^{2-n}$$

$$W_{ij} = Re^{(2-n)/2}$$

and for $n = .2$

$$W_{ij} = Re^{.9}$$

APPENDIX J

SLUG FLOW ANALOG

To demonstrate the axial dependence of the mixing coefficient (without the complex solution of reference [2]), a solution for two rectangular channels (shown in Figure J-1) in slug flow was performed. The approach used was to solve a transient conduction problem, and then convert the time scale to axial distance and relate the thermal diffusivity to the eddy diffusivity. The conduction solution, for two semi-infinite slabs with different temperatures brought into contact at $t = 0$, is given below.

INITIAL AND BOUNDARY CONDITIONS

$$T(0,t) = \text{contact temperature} = 0 : t > 0$$

$$T_A(x,0) = 1 : 0 \leq x \leq a$$

$$T_B(x,0) = -1 : -a \leq x \leq 0$$

$$\left. \frac{\partial T}{\partial x} \right|_a = \left. \frac{\partial T}{\partial x} \right|_{-a} = 0$$

The differential equation is:

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{a_t} \frac{\partial T}{\partial t}$$

where a_t = thermal diffusivity

Using

$$T = X(x) \tau(t),$$

This equation can be separated;

FIGURE J-1
Slab Geometry

$$\frac{X''}{X} = \frac{1}{a_t} \frac{\tau'}{\tau} = -\partial^2$$

$$X'' + \partial^2 X = 0$$

$$\tau' + a_t \partial^2 \tau = 0$$

The solutions are:

$$X = A \sin \partial x + B \cos \partial x$$

$$\tau = e^{-a_t \partial^2 t}$$

Applying the boundary conditions;

$$T(0, t) = 0$$

$$B = 0$$

and,

$$\left. \frac{\partial T}{\partial x} \right|_{x=a} = 0$$

$$\cos \partial a = 0$$

$$\partial a = (2n + 1) \frac{\pi}{2}$$

$$\partial_n = (2n + 1) \frac{\pi}{2} : n = 0, 1, 2, \dots$$

Then,

$$T_A(x,0) = 1 = \sum_{n=0}^{\infty} C_n \sin(\partial_n x)$$

and,

$$\begin{aligned} C_n &= \frac{2}{a} \int_0^a (1) \sin(\partial_n x) dx \\ &= -\frac{2}{a} \left(\frac{1}{n} \right) \cos(\partial_n x) \Big|_0^a \\ &= \frac{2}{\partial_n a} = \frac{4}{(2n+1)\pi} \end{aligned}$$

Therefore,

$$T_A(x,t) = \sum_{n=0}^{\infty} \frac{4}{(2n+1)\pi} (\sin \partial_n x) e^{-a_t \partial_n^2 t} \quad (U-1)$$

The slab bulk temperature can be found,

$$\begin{aligned} \bar{T}_A(t) &= \int_0^a \sum_{n=0}^{\infty} \frac{4}{(2n+1)\pi} (\sin \partial_n x) e^{-a_t \partial_n^2 t} dx \\ &= 2 \sum_{n=0}^{\infty} \frac{e^{-a_t \partial_n^2 t}}{(2n+1) \frac{\pi}{2}} \end{aligned}$$

and,

$$\bar{T}_A - \bar{T}_B = 4 \sum_{n=0}^{\infty} \frac{e^{-a_t \frac{\pi^2}{2} n^2}}{[(2n+1)\frac{\pi}{2}]^2} \quad (V-2)$$

The temperature gradient at the contact point is given by:

$$\left. \frac{\partial T}{\partial x} \right|_0 = \frac{2}{a} \sum_{n=0}^{\infty} e^{-a_t \frac{\pi^2}{2} n^2} \quad (V-3)$$

The time scale can be transformed to a length scale by:

$$T = \frac{z}{V}$$

where

z = axial distance

V = slug flow velocity

Now the thermal diffusivity must be related to the eddy diffusivity for heat;

$$a_t = \frac{k}{\rho c_p} = \frac{v}{Pr}$$

and the analogous expression is,

$$a_t = \left(\frac{v}{Pr} + \epsilon_H \right)$$

For a tracer experiment with $Pr = 1000$, this can be approximated by:

$$\frac{a_t}{v} = \frac{\epsilon_H}{v}$$

An experimental correlation for the eddy diffusivity from reference [32] is:

$$\frac{\epsilon_H}{v} = .0113 Re^{.875} \quad (3-4)$$

The exponent of e can now be written as:

$$\exp = -a_t \frac{\partial^2 t}{\partial n^2} = -\epsilon_H \left[\frac{(2n+1)\pi}{2a} \right]^2 \frac{z}{v}$$

and setting a equal to the hydraulic diameter, D ,

$$\exp = \frac{-\epsilon_H}{v} \left[\frac{(2n+1)\pi}{2} \right]^2 \left(\frac{z}{D} \right) \left(\frac{1}{Re} \right)$$

and introducing the correlation for ϵ_H ,

$$\exp = - (.0113) \left[\frac{(2n+1)\pi}{2} \right]^2 \left(\frac{z}{D} \right) \left(\frac{1}{Re^{.125}} \right)$$

The heat flux from one slab to another, q'' , is

$$q'' = - \left(\frac{a_t}{v} \right) \mu c_P \left. \frac{\partial T}{\partial x} \right|_0$$

For slabs that are rectangular in cross-section, the linear heat rate is

$$q' = - \left(\frac{a_t}{v} \right) \mu c_P \left(\frac{1}{P-D} \right) \left. \frac{\partial T}{\partial x} \right|_0$$

Recalling the transport term from the macroscopic energy equation (2-20);

$$q'_{ij} = W'_{ij} c_P (\bar{T}_i - \bar{T}_j)$$

Equating these heat rates and solving for the mixing coefficient;

$$W'_{ij} = - \left(\frac{e_H}{v} \right) \left(\frac{\mu}{P-D} \right) \frac{\left. \frac{\partial T}{\partial x} \right|_0}{\bar{T}_A - \bar{T}_B}$$

Substituting in equations (G-2), (G-3) and (G-4):

$$\frac{W'_{ij}}{\mu} = - (.0113) R_e^{-.875} \left(\frac{1}{P-D} \right) \left| \frac{\sum_{n=0}^{\infty} e^{-B_n \frac{z}{D}}}{2D \sum_{n=0}^{\infty} \frac{e^{-B_n \frac{z}{D}}}{[(2n+1)\frac{\pi}{2}]^2}} \right|$$

(J-5)

Figure T-2 is a graph of calculations made using equation (3-5) showing the axial development of the mixing coefficient for three different Reynold's numbers.

FIGURE J-2: Axial Variation of W'