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**The Potential for Efficient Frequency Conversion
at
High Average Power
Using
Solid State Nonlinear Optical Materials**

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ABSTRACT

High-average-power frequency conversion using solid state nonlinear materials is discussed. Recent laboratory experience and new developments in design concepts show that current technology, a few tens of watts, may be extended by several orders of magnitude. For example, using KD*P, efficient doubling ($> 70\%$) of Nd:YAG at average powers approaching 100 KW is possible; and for doubling to the blue or ultraviolet regions, the average power may approach 1 MW. Configurations using segmented apertures permit essentially unlimited scaling of average power. High average power is achieved by configuring the nonlinear material as a set of thin plates with a large ratio of surface area to volume and by cooling the exposed surfaces with a flowing gas. The design and material fabrication of such a harmonic generator is well within current technology.

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1. INTRODUCTION

Frequency conversion is a useful technique for extending the utility of high power lasers. It utilizes the nonlinear optical response of an optical medium in intense light fields to generate new frequencies. It includes both elastic (optical-energy-conserving) processes, such as harmonic generation, and inelastic processes (which deposit some energy in the medium), such as stimulated Raman or Brillouin scattering.

There are several commonly used elastic processes. Frequency doubling, tripling, and quadrupling generate a single harmonic from a given fundamental high-power source. The closely related processes of sum and difference frequency generation also generate a single new wavelength, but require two high-power sources. These processes have been used to generate high-power radiation in all spectral regions, from the ultraviolet to the far infrared. Optical parametric oscillators and amplifiers generate two waves of lower frequency. They are capable of generating a range of wavelengths from a single frequency source, in some cases spanning the entire visible and near-infrared regions. The materials used for frequency conversion are often also suitable for other devices, such as electrooptic switches. A number of excellent reviews of frequency conversion can be found in the literature [K172, Sh76, Hu73, Ze73, Ko76, By74, Sm72, Ba79].

These techniques are common for single-pulse, or low-average-power applications, but there has been comparatively little interest in scaling them to high average power. Broadly speaking, this is because the average power in the converted beam is limited primarily by the average

power capability of the laser drive. Increasing the average power of a laser generally causes the beam quality to deteriorate, or results in damage to the laser due to the thermal load. Then the primary requirement on the harmonic generator is that it maintains high conversion efficiency for a laser with poor spatial quality. The highest conversion efficiency is then obtained in materials with a large acceptance angle and damage threshold. Although the linear optical absorption in the nonlinear crystal gives rise to thermal gradients which destroy phase-matching, managing the heat deposition has been important only if the acceptance angle is very large, that is, under conditions of noncritical phasematching. A second reason for the comparative lack of interest in high average power is that it has been widely assumed that solid state materials do not handle thermal loads efficiently. Therefore, it has been argued, even if diffraction-limited, high-average-power lasers were available, frequency conversion using solid state materials would not be practical.

All the research to date on average power frequency conversion has addressed frequency-doubling second-harmonic generation of Nd:YAG lasers [Ho79]. Two techniques have been used: intracavity doubling [Sm70, Ba75], in which the doubler is internal to the optical cavity of the laser, and extracavity doubling, where the doubler is external to the cavity.

In intracavity doubling, the 1064-nm beam makes many passes through the doubler, and experiences a small conversion efficiency on each pass. One mirror of the cavity is dichroic, and the 532-nm beam passes out of the cavity. Geusic [Ge68] reports using (noncritically phase-matched)

$\text{Ba}_2\text{NaNb}_5\text{O}_{15}$ to convert effectively 100% of the fundamental to the second harmonic. The average power was nominally 1 W. More recently, Perkins [Fa84, Pe84] reports using KTP for intracavity doubling, obtaining 25 W of average power.

The first report of average power harmonic generation external to the cavity is by Kogan and Crow [Ko77], who used KD*P in a type II configuration to double a high-quality 10-Hz, 1-J, 17-ns Nd:YAG laser. The beam divergence was 0.4 mr and the aperture was 12 mm. They reported an average power of 10.5 W and a conversion efficiency of about 50%. Their system delivered over 20 million shots without any damage to the crystal. (However, the flashlamps were replaced every 5 million shots.) Note that KD*P is not noncritically phasematched at this wavelength. More recently, Hon et al. [Ho76, Ho77a, Ho77b] report 35 W of average power using CD*A, which does show noncritically phasematching at about 112°C. Thermal effects are important in CD*A because of its comparatively small temperature bandwidth. The thermal gradients were managed using beam shaping and electrooptic or piezoelectric tuning of the crystal. Hon and Moses [Mo78] also report a beam fanning or scanning technique for spreading the thermal load out over a larger volume of crystal, thereby avoiding the large thermal gradients associated with tightly focused beams.

This summarizes the current state of the art in high-average-power frequency conversion. The average power is relatively modest in part because without a high-average-power Nd:YAG laser, there is little or no imperative to develop the frequency conversion techniques to handle higher average power. Although the development of high-average-power

solid state lasers has languished during the last two decades (with CO_2 being the prime average power laser), these lasers have recently begun to enjoy a renewed interest. One reason for this is the recent appearance of new applications for lasers, including those associated with military programs, and the Strategic Defense Initiative [Em84]. The average power requirement of these applications varies up to several orders of magnitude beyond the current state of the art in average-power frequency conversion. Also, fusion research at LLNL [Kr79] has shown that commercial power plants based on inertial confinement fusion will require lasers delivering over 1 MJ of short wavelength light at repetition rates of a few hertz. The average power requirement is approximately six orders of magnitude beyond the current state of the art in frequency conversion. This has lead to a re-examination of high-average-power frequency conversion to increase the average power to the required levels.

As part of an overall program addressing the laser for the commercial application of laser fusion, the potential of solid state lasers to achieve high average power was re-examined recently at LLNL [Em84]. New design concepts and recent developments in materials and techniques provided a focus for that study. Particular attention was paid to thermal management and optical distortion, and especially to efficient utilization of the pump energy. The conclusion was that solid state lasers are capable of unconstrained scaling of average power while maintaining high beam quality. Moreover, it was found that efficiencies of over 10% should be achieved at high power and irradiance. The study showed how this is possible with careful attention to conserving the pump energy and with intelligent thermal management. In parallel with this

study, and using experience gained in frequency conversion of a 100-KJ, 100-TW laser [Su81, Si83], the potential of solid state nonlinear materials to reach high average power while maintaining high conversion efficiency was also thoroughly evaluated. The conclusion was that solid state nonlinear materials also support unlimited average power, at conversion efficiencies in excess of 70%. This is possible with careful attention to the thermal gradients and the beam shape and by balancing the various sources of phase mismatch. For example, using KD*P in a quadrature configuration to double a 10-ns Nd:YAG laser permits over 50 KW of average power in a single crystal segment to be doubled with an efficiency in excess of 70%.

This paper reports the results of the study of high-average-power frequency doubling. Single-pulse frequency conversion is summarized in Sec. 2, which includes some perhaps unfamiliar results on the role of dephasing at high conversion efficiency. Thermal gradients are analyzed in Sec. 3. The performance of the scalable design (the N-plate design) is fully analyzed in Secs. 4, 5, and 6, and design procedures are given in Sec. 7. For each of the important effects which enter into the design, a thorough analysis is given and carried through in detail for one particular system: the doubling of a high-average-power neodymium laser by KD*P in a type II configuration. A list of symbols is included preceding the Bibliography.

Although this paper treats frequency doubling, by a trivial extension it also addresses third harmonic generation, sum and difference frequency generation, and optical parametric amplifier (OPA) design. It appears that all these other processes are scalable in average power just as

second-harmonic generation is. Consequently, high-average-power solid state laser systems are just as capable of frequency versatility as are single-pulse systems.

2. SINGLE-PULSE PERFORMANCE

This section discusses the origin and basic physics of harmonic generation in solids and reviews in detail their design for applications. Several reviews of the technique are available; only the main features will be repeated here.

Harmonics are generated through a nonlinear material response to a strong electromagnetic wave. A commonly used model invokes an electron in a potential well which is both anharmonic and asymmetric about the electron's equilibrium position. The response of the electron can be written as a Fourier sum over all harmonics of the driving field. The radiation emitted by the oscillating charge therefore contains all the harmonics of the driving field, with an intensity in each harmonic given by the dipole moment of the electron at that frequency. For second-harmonic generation, the material is therefore characterized by a nonlinear coefficient which gives the dipole moment density at the harmonic in terms of the driving field:

$$P_i = \epsilon_0 d_{ijk} E_j E_k \quad (1)$$

A crystal may be envisioned as a sum over many such electrons spread out over the volume of the crystals, each one being driven, and therefore radiating, with a well-defined phase relative to the driving field. It is useful to arrange these electron oscillators into planes perpendicular to the direction of propagation of the fundamental driving wave. All the electrons in each plane therefore oscillate in phase, and the dipole moments at each harmonic are therefore also in phase. Each plane of this

phased array of oscillating dipole moments will generate highly directional radiation, and the radiation at the second harmonic is essentially a plane wave parallel to the fundamental.

Of course, the second-harmonic radiation from a given plane of atoms is not in phase with the radiation from another plane, in general. The reason is that the phase between the oscillating dipole moments on two different planes is given in terms of the wavelength of the fundamental. On the other hand, the radiated wave travels forward to the boundary of the crystal, acquiring a phase given by the wavelength of the harmonic itself. The phase of the radiated wave from the plane of atoms at location z , relative to the phase of the radiated wave from the first plane of atoms, is

$$\delta\phi = [\phi_1(z) + \phi_2(1 - z)] - \phi_2(1) = \phi_1(z) - \phi_2(z) \quad . \quad (2)$$

Here ϕ_1 is the phase accumulated by the fundamental, and ϕ_2 is the phase accumulated by the harmonic. The only way for all the planes to radiate in phase is for the velocity of the fundamental and second-harmonic waves to be equal. This condition is called phase-matching and is not satisfied in general, due to the wavelength dispersion of the index of refraction.

The conservation of energy requires that the sum of the intensities of the fundamental and second harmonics be constant (in the absence of absorption). Now the rate of transfer of power between the fundamental and the harmonic is given by the scalar product of the dipole moment density and the electric field, both at the second harmonic. If the

dipole moment and the radiated wave have different velocities, then this scalar product oscillates in sign at their beat spatial frequency. Within one beat wavelength, as much energy is transferred back from the harmonic as was transferred to it, and therefore after one beat wavelength, the harmonic intensity is zero. The maximum power in the harmonic is therefore achieved after $N + 1/2$ beat wavelengths, where N is an integer. This implies that the harmonic power is always less than the power generated from a limited number of atomic planes. The coherence length of the medium is defined as the length at which the power is maximum, that is, it is $1/2$ the beat wavelength between the dipole moment density and the electric field. Clearly, efficient conversion is possible only if the coherence length is sufficiently long that enough planes can contribute.

The coherence length in most materials is typically tens of microns, far too short to allow efficient conversion. The most common way to arrange for a long coherence length is to use birefringent media. In these crystals, the wave velocity depends on the direction of propagation and on the polarization. In all crystals there are two (orthogonal) polarizations for which the wave has a unique velocity. (For intermediate polarizations, the wave behaves as a sum of two waves of different velocities.) By arranging for the fundamental's polarization to correspond to the lower velocity, and the harmonic's polarization to the higher velocity, the effects of dispersion can be counterbalanced by the birefringence. Because the velocity depends on the direction of propagation, it is possible to arrange for the birefringence and dispersion to cancel exactly, so that the fundamental and harmonic waves

have exactly the same velocity, and the coherence length is effectively infinite. To put this another way, the orientation of the crystal relative to the beam direction gives limited control over the wave velocities. The degree of variation is

$$\Delta v/c = \Delta n/n^2 . \quad (3)$$

If the birefringence is large enough, adjusting the crystal orientation will match the wave velocities at the two frequencies exactly.

Because the coherence length is a sensitive function of the crystal orientation, that orientation must be held to within tight tolerances to maintain phasematching and efficient frequency conversion. In general, the beat wavelength is given by the wavevector mismatch. For a process in which waves with indices n_1 and n_2 combine into a wave at the second harmonic with index n_3 , the wavevector mismatch is

$$\Delta k = (2\omega n_3 - \omega n_2 - \omega n_1)/c , \quad (4)$$

where the n_j are the (orientation-dependent) refractive indices. This has a zero at the phasematching angle; if $n_1 = n_2$, this is the same condition in which the velocities are equal at infinite coherence length. For small deviations from that orientation, the wavevector mismatch is

$$\Delta k = \beta_\theta \delta\theta , \quad (5)$$

where $\delta\theta$ is the effective angular misalignment and β_θ is a material constant. For real laser beams, $\delta\theta$ receives contributions from the

whole beam divergence and also from the local phase and amplitude variations that are always present. In fact, any effect which disturbs the indices can alter the coherence length. Thus, not only the crystal orientation and the laser beam quality, but also scattering, diffraction, linear optical aberrations and defects, thermal gradients and stresses can all contribute to the dephasing. Understanding and evaluating these dephasing effects is extremely important in designing both single-shot and average-power systems; it represents an essential and significant part of this study.

For discussions of phase-matching and angular sensitivity in the literature, the reader is referred to Ho67, Wa69, Ch68, Or69, Ir75, Sh82, Ze73, Ar62.

The coupling between the waves ($\vec{P} \cdot \vec{E}$) clearly depends on the polarizations and the tensor components of the nonlinear coupling. If these are properly taken into account, the polarization at the second harmonic can be written in terms of an effective coupling:

$$P(2\omega) = \epsilon_0^d \text{eff} E(1\omega)^2. \quad (6)$$

From Maxwell's equations, the growth of the harmonic wave inside the crystal can be found. It is convenient to normalize the fields in units of $[GW^{1/2}/cm]$ so that the intensity is E^2 with no other factors. Then

$$\frac{\partial E_2}{\partial z} = CE^2(1\omega) e^{i\Delta kz}, \quad (7)$$

where

$$C = 2.75 d_{\text{eff}}(\text{pm/V})/\lambda_3(\text{um})(n_1 n_2 n_3)^{1/2}$$

is the nonlinearity expressed in units of $\text{GW}^{-1/2}$. Along with the equations for the fundamental, these equations have been solved by Armstrong et al. [Ar62]. For a crystal of thickness w , the solution is

$$\eta = \tanh^2 \left[\frac{1}{2} \tanh^{-1}(\text{sn}[2n_0^{1/2} 1 + \delta^2/4n_0]) \right] , \quad (8)$$

where sn is a Jacobi function, and

$$n_0 = C^2 I \ell^2 , \quad (9)$$

$$\delta = 1/2 \Delta k \ell .$$

These two parameters are the nonlinear drive and the phase between the second harmonic waves from the first and last planes of atoms. A very useful reference for comparison purposes is doubling 1064 nm using type II phasematching in KD*P. The material constants are

$$C = 0.97 \text{ GW}^{-1/2} \quad \text{and} \quad (10)$$

$$\beta_8 = 2.5 \text{ cm}^{-1}/\text{mrad} .$$

Thus the drive in 1-cm crystals experiencing 1 J/cm^2 in 1 ns is 1; and the low power acceptance angle of the crystal for angular misalignments (see below) is $\sim 1 \text{ mrad}$.

If $n_0 < 1$, then the conversion efficiency is

$$\eta = n_0 (\sin \delta / \delta)^2 . \quad (11)$$

If the drive is small, then it is also a direct measure of the phase-matched conversion efficiency. For larger drives the fundamental pulse is more strongly depleted, and the point $\eta_0 = 1$ can be considered as the point where saturation begins. For example, the conversion efficiency for $\delta = 0$ is

$$\eta = \tanh^2(\eta_0^{1/2}) \quad (12)$$

and η approaches 1 as the drive tends to infinity.

A second and less well-known effect is that the sensitivity of the conversion process to angular misalignments increases as the drive increases [Ec84, Ei81, Ho82]. The effective detuning δ_{eff} is defined as follows:

$$\delta_{\text{eff}} = \delta \exp(\sqrt{\eta_0}) > \delta \quad (13)$$

The saturating efficiency η , for $\eta_0 \gg 1$, and $\delta_{\text{eff}} \ll \pi$, is approximately

$$\eta = (\tanh(\sqrt{\eta_0}) \sin \delta_{\text{eff}} / \delta_{\text{eff}})^2 \quad (14)$$

This is a simple extension of Eqs. (11) and (12).

The exact saturating behavior, as a function of η_0 and δ , is presented as a contour map in Fig. 1. The lines connect points (η_0, δ) which give the same saturating efficiency; there are nine contours corresponding to the efficiencies 0.1, 0.2, 0.3, ..., 0.8, 0.9

($\eta = 1$ does not exist). It is clear that any energy in the laser pulse which meets the crystal with an η_0 and δ for which the conversion efficiency is low is not converted. Therefore, high conversion efficiency requires that all the energy in the laser pulse meet the crystal at or above a high efficiency contour. Efficient conversion requires using either the central lobe of Fig. 1, or the secondary lobe. For the central lobe, efficient conversion requires that

$$|\delta| \ll \pi/10 \quad (15)$$

and

$$1 < \eta_0 < 5 \quad (16)$$

The secondary lobe requires that

$$\delta = 1.5 \pm \pi/10 \quad (17)$$

and

$$1.5 < \eta_0 < 40 \quad (18)$$

Clearly, the maximum efficiency in the secondary lobe is less than that in the central lobe. The efficiency achieved depends on the details of the spatial and temporal character of the drive. At the edges of the pulse the intensity drops to zero. The central lobe converts this light moderately efficiently,

Figure 1. Contour plot of the conversion efficiency η as a function of the (dimensionless) drive $C^2 l^2$ and the detuning $(1/2)\Delta k l$. The line of zeros between the first and second lobes follows the approximate curve $\delta = 8\exp(-\eta_0^{1/2})$. Conversion using the first lobe becomes increasingly sensitive to detuning at higher drive.

but the secondary lobe is bounded by a line of zeroes (compare the point $\eta_0 = 12$, $\delta = 1.5$). Consequently, using the secondary lobe necessarily implies poor efficiency for the spatial and temporal edges of the beam.

For Gaussian pulses, this causes a significant reduction in conversion efficiency. Only for pulses which are flat in space and time will the conversion efficiency be high in the secondary lobe. While for some pulse shapes this secondary lobe may be more efficient than the central lobe, for most applications the central lobe will give higher efficiency. Therefore, without dismissing the secondary lobe, we focus on the central lobe in this study.

The choice of crystal thickness is illustrated in Fig. 2. If the intensity and the angular divergence of the beam are considered to be fixed, then as the crystal thickness is varied, a curve is mapped out in the drive/detuning plane of Fig. 1. The drive varies quadratically with the crystal thickness, whereas the detuning varies linearly; the curve is therefore a parabola. For high intensity beams, the parabola lies close to the drive axis, and as the intensity is lowered it becomes more vertical, as shown. The conversion efficiency varies under the curve as indicated by the contours. As the crystal length is increased, the efficiency peaks and then oscillates between the peak and zero, in a nonsinusoidal fashion. The optimum crystal length is the lowest length for which the efficiency peaks. As discussed above, this is always in the central lobe of the efficiency contour. Thus, for a given intensity and angular divergence, there is an optimum crystal length, which depends on the trade-off between drive and detuning.

Figure 2. The crystal thickness is chosen by optimizing the trade-off between drive and detuning. For fixed intensity and beam divergence, the system variables lie on parabolas in the drive/detuning plane. Each point on a parabola corresponds to a particular crystal thickness. The crystal thickness is chosen to give the peak efficiency, which requires operating in the central lobe close to the origin, as discussed in the text. By optimizing the crystal length this way, the conversion efficiency is determined entirely by the total peak power in the laser pulse and the beam quality. The efficiency is not optimized by maximizing the intensity, but by optimizing the laser brightness. See text for details.

It is still true that even for crystals whose length is optimal, high efficiency requires that the intensity be larger than some value defined by the angular divergence of the beam. It has often been argued in the literature that the efficiency will be maximized by going to as high an intensity as possible without damaging the crystal. This leads to system designs which focus the beam into the nonlinear crystal. However, proper optimization of the crystal and system parameters shows that this argument is incorrect, and that the performance is not in fact optimized by maximizing the intensity. We shall now prove this somewhat counter-intuitive result.

Consider a laser beam with a fixed peak power (pulse energy divided by pulse width), but whose area can be adjusted using telescopes. As the (de)magnification m is changed, the intensity and angular divergence change, and therefore the optimal crystal thickness changes, also. If the drive is telescoped down by m to increase the intensity, then the intensity increases by m^2 , and the optimum crystal length decreases by m . The drive to the crystal remains unchanged and is independent of the demagnification. Moreover, the angular divergence increases by m also, so that the product $L\theta$ is independent of m . Telescoping the beam has no effect on either the drive or the detuning, and the conversion efficiency is independent of the demagnification. Telescoping merely changes the crystal size; it has no effect on the performance. On the other hand, the fluence on the crystal increases as m^2 . There is, therefore, a minimum beam area or crystal size fixed by the optical damage threshold of the crystal. So long as the crystal size and beam

area are large enough to keep the fluence below the damage limit, all beam areas are equivalent in performance; there is no advantage whatsoever in maximizing the intensity.

Mathematically, this is expressed as follows. The drive on the crystal is

$$\eta_0 = C^2 L^2 I = C^2 (L^2 / HH') P_{pk} \quad , \quad (19)$$

where the beam area is HH' (see Fig. 3) and P_{pk} is the peak power in the laser pulse. Clearly, doubling all the dimensions of the crystal and beam area (i.e., L , H , and H') leaves the drive unchanged. Also the detuning may be expressed in terms of the beam quality, Q , as follows:

$$\delta = \frac{1}{2} \Delta k L = \frac{1}{2} (\beta_0 \lambda) Q (L/H) \quad , \quad (20)$$

where the beam quality Q is given by

$$\delta \beta = Q(\lambda/H) \quad . \quad (21)$$

Q is roughly the ratio of the beam divergence in the sensitive direction of the crystal to the beam divergence of a diffraction-limited beam of the same area, and it is independent of the beam dimensions. Clearly, doubling all the dimensions (i.e., L and H) leaves the detuning unchanged. Therefore, the drive, detuning, and efficiency are independent of the beam size. By eliminating the crystal thickness, the condition for high efficiency is obtained.

$$E/\tau \geq (\beta_0 \lambda / C)^2 (\eta_0 / 4\delta^2) Q^2 / (H/H') \quad (22)$$

Figure 3. The crystal dimensions of the harmonic generator. The beam area is HH' , and the sensitive direction of the crystal for angular misalignments is the (LH) plane. The performance of the generator is independent of the size of the crystal and depends only on its shape.

where E is the pulse energy, and τ is the pulse width. From Figs. 1 and 2, η_0 and δ are chosen to give high efficiency. Then Eq. (22) gives the peak power required to reach high conversion efficiency. It increases rapidly as the beam quality deteriorates. This condition can also be viewed as a Yes/No test for using a particular material to double the frequency of a given laser system. Note that a poor material will be poor regardless of the demagnification and cannot be "tweaked up" to give high efficiency. The use of this equation and relevant material constants are discussed in greater detail in Sec. 8.

A corollary of this analysis is that one can in principle simply design around any optical damage problem, by choosing a beam area which is sufficiently large. A material which meets the efficiency condition will still be efficient at beam sizes where the fluence is arbitrarily low. Surmounting optical damage problems only requires crystals of sufficient size. This conclusion has profound consequences for design and performance of nonlinear optical devices.

The beam quality Q is related to the number of transverse modes in the output of the laser drive. Typically, $Q \sim n^{1/2}$, although this depends on the cavity design in general. However, to the extent that $Q^2 \sim n$, the condition for high conversion efficiency is essentially a requirement on the peak power per transverse mode in the laser drive.

This treatment of system design ignores such factors as the temporal pulse shape and depolarization effects. The conversion efficiency for Gaussian pulses is limited because some of the energy of the drive meets the crystal with a low intensity. The highest conversion efficiency is obtained for temporally flat pulses.

The conversion efficiency is also reduced by depolarization of the drive. In type I phasematching schemes, any energy in the "wrong" polarization is wasted and does not participate in the conversion process. It appears as a straight loss in conversion efficiency. In type II schemes, the input to the crystal must consist of equal parts of the two eigenpolarizations. Any energy imbalance between them not only appears as a straight loss but also participates in the conversion process and reduces the conversion efficiency further. Typically the drive η_0 is moderate, and the dominant effect is the straight loss. Thus the admissible depolarization can be estimated using only the straight loss to evaluate the reduction in conversion efficiency. For type I processes, the wasted energy is

$$\Delta E/E = \sin^2(\Omega) \quad , \quad (23)$$

and for type II processes, the energy imbalance between the two eigenpolarizations is

$$\Delta E/E = \sin(2\Omega) \quad . \quad (24)$$

Clearly the type II process is more sensitive to the depolarization, Ω . For $\Delta E/E < 0.1$, the depolarization is

$$\Omega = E'/E < 0.05 \quad . \quad (25)$$

All three of these effects (temporal pulse shape, angular misalignments, and depolarization) are less deleterious on the conversion

efficiency in the two crystal quadrature schemes [Ei83, Ei84] than in the single crystal schemes just discussed. A full discussion of these schemes is beyond the scope of this paper; the relevant facts are that quadrature schemes use two crystals of different thickness, and that they are less sensitive to phase aberrations and depolarization in the laser pulse than single crystal schemes. For example, it has been demonstrated that quadrature schemes using AR-coated KD*P crystals can achieve a system conversion efficiency greater than 95% for Gaussian pulses [Ei84]. The only price to pay for this performance is the use of two crystals rather than one. High-average-power system design for quadrature schemes is essentially the same as for single crystal schemes.

3. GENERAL HIGH-AVERAGE-POWER CONSIDERATIONS: THE N-PLATE DESIGN

If the repetition rate is f , the average power per unit area is

$$\begin{aligned} I_{av} &= fJ \\ &= f\tau I \end{aligned} \quad (26)$$

Linear optical absorption in the crystals causes the temperature to rise and temperature gradients to form. These gradients cause thermal stresses and index changes. The stresses can become strong enough to fracture the crystal, and both the stresses and the temperature gradients cause index changes which dephase the waves in the crystal.

If the linear optical absorption coefficient is α_I , then the heat deposited per unit volume in the crystal is

$$W = \alpha_I I_{av} \quad (27)$$

The heat deposition is independent of position if, and only if, the optical absorption is the same at the drive and converted frequencies. This is certainly not true for most materials, but for the moment we shall continue with Eq. (27) as a useful approximation and leave the question of the frequency dependence of the absorption until later. With a uniform heat deposition, a steady state is set up in which temperature gradients are established to carry off the heat by thermal conduction.

Whatever the direction of the temperature gradients relative to the direction of the laser pulses, the temperature profile set up in the crystal will be quadratic. We have

$$\nabla^2 T = W/K \quad , \quad (28)$$

where K is the thermal conductivity (assumed isotropic). Recall that a streamline for the heat flow is a line which is always parallel to the local direction of heat flow, or $\text{grad } T$. Along any streamline the temperature is quadratic:

$$T = T_0 + sT_1 + s^2 W/2K \quad . \quad (29)$$

Here s is the distance along the streamline, and T_0 and T_1 are integration constants.

Consider the point in the crystal where the temperature in steady state is a maximum and consider the streamlines emanating from that point. For all such streamlines, T_1 is zero because the temperature gradient is zero at the maximum:

$$T(s) = T_0 + s^2 W/2K \quad . \quad (30)$$

Every streamline originates at the warmest point and eventually meets the boundary of the crystal where the heat it transports is carried off by

some agent other than thermal conduction. If a streamline has length s , then the temperature difference between its ends is simply

$$\Delta T = s^2 W / 2K \quad . \quad (31)$$

The lowest temperature in the crystal is, therefore, at the other end of the longest streamline. The temperature difference between the warmest and coolest point in the crystal is therefore minimized by arranging for the longest streamline to be as short as possible. This rather obvious conclusion has profound consequences for high-average-power lasers.

Because optical materials damage at high fluence, the only way to achieve high energy per pulse is to increase the aperture of the laser. High pulse energy and high average power will be achieved with large apertures. If the streamlines are perpendicular to the laser beam, then increasing the average power by increasing the aperture causes the length of the streamlines (and therefore the maximum temperature difference) to increase. On the other hand, if the streamlines are parallel to the laser beam, their length is fixed by the crystal thickness, and the temperature difference is independent of the aperture. The total average power is then not limited by thermal considerations, although the average power per unit area is.

With the streamlines parallel to the laser beam, the temperature is a maximum in the center of the crystal and a minimum on its faces. The temperature difference between the center and the faces is

$$\Delta T = L^2 \alpha_{I_{av}} / 8K \quad . \quad (32)$$

Typically ΔT is limited by the thermal limits discussed in the next section. Then the average power per unit area is maximized by choosing thin crystals. However, thin crystals do not convert efficiently. On comparing ΔT with the drive η_0 , we find that η_0 and ΔT are both proportional to L^2 , and that their ratio is determined solely by the product f_τ , independent of the crystal thickness,

$$\Delta T/\eta_0 = f_\tau (\alpha_L/8KC^2) \quad (33)$$

The only system variables in this equation occur in the product f_τ ; the other variables are either material constants or design parameters fixed by performance requirements.

For single crystal schemes, Eqs. (32) and (33) show that the duty cycle f_τ is fixed by the thermal limits on ΔT and high conversion efficiency and not by the crystal thickness. The reason is that ΔT determines the product $I_{av}L^2$, whereas η_0 determines the product IL^2 . These two conditions can be met only if I and I_{av} have the correct ratio; hence the condition on the duty cycle, f_τ .

The thermal streamlines can be shortened further by replacing the single crystal with a set of N thinner crystals with the same total thickness and flowing a coolant gas between them (see Fig. 4). This is equivalent to removing heat from the single crystal system on planes inside the crystal. The temperature difference within any single plane is reduced by a factor N^2 , and therefore the average power per unit area is increased by the same factor for the same limit on ΔT . If the thickness of a single plate is $l = L/N$, then

Figure 4. A high-average-power harmonic generator design with efficient scalable thermal management.

$$\Delta T = \frac{1}{2} \alpha_I^2 I_{av} / 8K \quad (34)$$

and

$$\Delta T / \eta_0 = f_T / N^2 (\alpha_I / 8KC^2) \quad (35)$$

Thus for a given limit on ΔT , the N-plate design allows the duty cycle f_T to be a factor of N^2 larger and delivers an average power per unit area which is larger by the same factor.

4. LIMITATIONS ON THE TEMPERATURE PROFILE

Temperature gradients cause thermal stresses to be established in the medium. They also give rise to inhomogeneities in the refractive indices; these arise directly, through dn/dT , and also indirectly, through the stress dependence of the indices. Both of these effects limit the harmonic generation performance of the crystals.

4.1. SURFACE FRACTURE

In Sec. 5, thermal stresses in anisotropic media are analyzed for the given temperature profiles. The principal thermal stresses in the crystal are found to be transverse to the beam direction and are largest at the crystal surface. They are parallel to the crystal surface. The surface is, of course, not precisely flat but contains pits, scratches, and other optically invisible defects which normally do not affect optical propagation. However, these surface defects will grow to macroscopic proportions if the surface stress exceeds a certain threshold. In increasing its size, the defect releases strain in the surrounding material. The defect will grow if the energy required to increase the defect size is less than the energy released by the accompanying reduction in strain. If this occurs, the crystal surface will become crazed, or it will shatter altogether.

Each surface defect is characterized by its own unique threshold, which depends on the size and shape of the defect. Typically the threshold for a defect of size a is

$$S = S_0/\sqrt{a} \quad , \quad (36)$$

where S_0 is the surface strength. Crystal failure occurs when the surface stress exceeds the lowest of all the defects' thresholds; according to Eq. (36), this is the threshold for the largest defect. In general, the surface stress is proportional to maximum temperature difference in the crystal and is independent of its thickness (see Sec. 5). For example, for isotropic media heated uniformly, the peak temperature is at the midplane of the crystal, and the temperature profile is parabolic [Em84, Bo60]. The surface stress is

$$S = 2\alpha_T Y \Delta T / 3(1 - \nu) = (2/3)\alpha_T Y \Delta T / (1 - \nu) \quad , \quad (37)$$

where Y is Young's modulus, ν is Poisson's ratio, and α_T is the thermal expansion coefficient, here assumed to be isotropic. If S_{th} is the threshold for surface failure, then

$$\Delta T < 3(1 - \nu) S_{th} / 2\alpha_T Y \quad . \quad (38)$$

Thus surface failure limits ΔT directly; this limit is independent of the crystal thickness. The fracture toughness of KD*P was determined recently [Ma85] by the direct crack method [An81]:

$$S_0 = 0.016 (E/H)^{1/2} (P/C_0^{3/2}) \quad (39)$$

where E is Young's modulus (20 GPa), H is the Vicker's hardness (1.2 GPa), P is the indenter load in newtons, and C_0 is the equilibrium post

indentation crack size. The value obtained, $S_0 = 0.09 \text{ MPam}^{1/2}$ was an average from six measurements each at 2-, 5-, and 10-N indentation loads [Ma85].

The assumptions that the crystal is mechanically and thermally isotropic and that the heating is uniform are not valid, in general, for nonlinear materials. In the first place, they are necessarily acentric crystals and therefore anisotropic in all their properties. However, we shall show below that Eq. (38) is valid in general, even for anisotropic media, if the material constants are considered to be effective parameters, appropriate for the particular crystal orientation for harmonic generation. The linearity between ΔT and S_{th} and the independence on crystal thickness is correct, in general, for nonlinear materials.

In the second place, the optical absorption is usually frequency dependent, and the heating of the crystal is therefore necessarily non-uniform. However, it does not invalidate the relation between ΔT and S_{th} so long as effective parameters are used.

4.2. TEMPERATURE DETUNING

The second effect of temperature gradients is that they cause index inhomogeneities which destroy the phase-matching conditions. The wavevector mismatch depends linearly on the local temperature;

$$d\Delta k/dT = \beta_t$$

$$= \beta_T + \beta_S dS/dT \quad . \quad (40)$$

Here β_T describes the direct effect of temperature on the refractive indices, and β_S describes their dependence on the local stresses. The coefficient β_S is an effective parameter representing the full stress-optic effect in the crystal. Note that in a general thermoelastic problem the temperature and stress profiles are not the same, which would make β_t dependent on location in the crystal. However, in the geometry of this problem the stress and temperature profiles are in fact the same, and β_t is independent of position.

The effect of the index inhomogeneity on the conversion efficiency is given by

$$\partial E_3 / \partial z = C E_1 E_2 e^{i\chi(z)} . \quad (41)$$

The phase mismatch $\chi(z)$ for a single plate is the sum of three terms:

$$\begin{aligned} \chi(z) = & \phi_3 - \phi_2 - \phi_1 \\ & + \beta_0 \delta\theta z \\ & + \beta_t \int_0^z (T(z') - T_0) dz' . \end{aligned} \quad (42)$$

Here the crystal is taken to be oriented so that for a perfect plane wave input ($\delta\theta = 0$), the harmonic generation process is exactly phasematched at the mean operating temperature of the plate, T_0 . Then the last term in Eq. (42) includes only the effect of deviations from the mean (or operating) temperature of the plate.

4.2.1. Optical Dispersion

The phases ϕ_1 , ϕ_2 , and ϕ_3 are each the difference between the phase of the wave at the entrance surfaces of the first plate and the Mth plate. Thus, besides the phases associated with propagation and nonlinear optics, they include the phase accumulated in the gas coolant flowing between the plates. This is significant because the optical dispersion in the coolant disturbs the relative phases and can reduce the conversion efficiency. If the separation between the plates is l_g , then the phase accumulated at the entrance to the Mth plate is

$$\chi(M) = (\omega_3 n_3 - \omega_2 n_2 - \omega_1 n_1) l_g (M - 1) / c, \quad (43)$$

where the indices now refer to the coolant and not the crystal. If the phase $\chi(M)$ is π , then the harmonic generated in the Mth plate has a phase of π relative to the harmonic generated in the first plate. The harmonics from all the plates will not add constructively unless $\chi(N) \ll \pi$. Constructive interference requires that

$$l_g \ll l_c / (N - 1), \quad (44)$$

where l_c is the coherence length of the coolant.

$$l_c = c / (\omega_3 n_3 - \omega_2 n_2 - \omega_1 n_1). \quad (45)$$

The index difference between the waves in nitrogen at STP is 3.9×10^{-6} , so that the coherence length for doubling $1 \mu\text{m}$ is about 2.1 cm. A study of the literature values for the index data on gases

indicated considerable ($> 2X$) uncertainty in the coherence lengths. A recent study by Velsko and Eimerl [Ve85] measured these lengths directly using a nonlinear interferometer, to an accuracy better than 1%. These data are listed in Table 1. It is clear that coolant dispersion is a significant aspect of high-average-power design.

4.2.2. Angular Detuning

The second term in the phase-mismatch is the angle tuning term. This term originates in the angular misalignment of the plate(s) and in the spatial harmonics in the drive. It accumulates with each plate. For example, at low drive, the condition $\Delta kL = \pi$ causes the harmonic from the last plate to be almost exactly out of phase with that from the first plate. At high power, this destructive interference occurs for $\Delta kL \ll \pi$ (see Fig. 1). With regard to angular misalignment, the N plates behave like a single plate of the same total thickness. We can then estimate from Fig. 1 that the right-hand side of this inequality must be about $\pi/10$. Therefore, angular detuning limits the total plate thickness $L = Nz$:

$$\Delta kL = \beta_{\theta} N z \ll \pi/10 \quad . \quad (46)$$

Here β_{θ} contains the error in orientation of the crystal plates, as well as the beam structure effects. Other things being equal, it might be expected to increase in an rms fashion with the number of plates.

Table 1. Coherence lengths of gases.^{a,b}

Gas	l_c (cm)
He	42.76
Ne	22.90
Ar	2.305
Kr	1.031
Xe	0.4407

H ₂	3.289
N ₂	2.153
O ₂	1.787
CO	1.239
CO ₂	1.019
CH ₄	0.9773
SF ₆	0.8763

Air	2.083

^a 1.064 μm and STP.

^b Velsko and Eimerl [Ve 85].

4.2.3. Longitudinal Temperature Rise

The third term in the phase mismatch is the temperature detuning term. In this expression T_0 is the temperature at which the process is exactly phasematched, and the plate is oriented so that T_0 is also the mean temperature of the plate. Then the complete temperature detuning integral is zero for each plate:

$$0 = \int_0^L (T(z') - T_0) \delta z' \quad . \quad (47)$$

In contrast to the other two terms, the complete temperature detuning term is exactly zero for each plate; it does not accumulate with the number of plates. To ensure high conversion efficiency, it is sufficient to limit the maximum phase mismatch in a single plate.

The effect of the parabolic temperature profile on the conversion efficiency cannot be expressed in closed mathematical form. However, in the approximation that the temperature profile is two step functions, the conversion efficiency can be obtained using the effective phase mismatch, χ_t :

$$\chi_t = \frac{1}{8} \beta_t \Delta T L \quad . \quad (48)$$

High conversion efficiency requires that $\chi_t < \pi/10$, as before. The appropriate temperature detuning limit is

$$\beta_t \Delta T L < 4\pi/5 \quad . \quad (49)$$

The step function approximation certainly gives an upper limit on the effective dephasing, and Eq. (49) is a conservative limit on ΔT .

4.2.4. Transverse Temperature Rise

The temperature of the crystal faces is not strictly uniform over the entire face (Fig. 5). The face is cooled by flowing gas, which warms as it traverses the crystal and removes heat from it. In steady state flow, the coolant removes a heat flux $W\ell/2$ from each of the two faces on either side of it. The coolant temperature rises approximately linearly as it absorbs heat from the crystals, and its gradient is [Ka66]

$$dT/dx = W\ell/C_p v b \quad , \quad (50)$$

where C_p is the specific heat per unit volume of the coolant, v is its velocity, and b is the thickness of the flow layer absorbing the heat.

The temperature of the faces and the coolant is determined by the heat flow

$$W\ell/2 = h(T_c - T_f) \quad , \quad (51)$$

where h is the convective heat transfer coefficient and T_c and T_f are the temperatures of the crystal faces and the mixed mean temperature of the coolant. Now the heat removed per unit area ($W\ell$) is independent of x , and if h is independent of x , the temperature gradients of both the coolant and the crystal in the z -direction are also independent of x .

Figure 5. The temperature profile in a single plate. The longitudinal temperature profile (the parabola) conducts the absorbed heat out to the surfaces. The coolant temperature rises as it carries away the heat, causing a transverse temperature gradient. Both temperature differences are important in optimizing the performance.

This implies that if the temperature of the coolant rises by ΔT_{tr} as it traverses the crystal face, then the surface temperature of the crystal also rises by the same amount, to maintain the same rate of heat removal from the crystal. Now the transverse temperature difference ΔT_{tr} must be small enough to lie within the temperature bandwidth of the phasematching condition. If this condition is not met by ΔT_{tr} , then it will not be possible to achieve phasematching over the entire aperture.

The difference between the absolute maximum and minimum of the temperature in the crystal is $\Delta T + \Delta T_{tr}$. The full temperature detuning limit is therefore

$$\beta_L \lambda (\Delta T_{tr} + \Delta T) < \pi/10 \quad . \quad (52)$$

Now the transverse temperature rise ΔT_{tr} is obtained by integrating Eq. (50) over the aperture, H , and is simply related to the longitudinal temperature difference, ΔT , given by Eq. (34). Their ratio is essentially the ratio of the heat flow resistances of the crystal and the gas coolant:

$$\Delta T_{tr}/\Delta T = (8K/C_p v b)(H/\lambda) \quad . \quad (53)$$

The transverse temperature rise will be larger unless

$$C_p v b/K > 8H/\lambda \quad . \quad (54)$$

This is not a trivial condition to meet, as the following estimate shows. The ratio H/λ is fixed by mechanical considerations to be not

more than about 15, and the specific heat C_p can be estimated using the perfect gas result at STP, $C_p = 555 \text{ J/m}^3/\text{K}$. For the crystal conductivity, we use the value for KDP, 2 W/m/K . Then the inequality [Eq. (54)] states that

$$vb > 0.43 \text{ m}^2/\text{s} . \quad (55)$$

The length b must be less than or equal to the separation between the crystals, and if this is taken to be 2.2 mm the flow velocity is 200 m/s , which is comparable to the velocity of sound. The flow must be turbulent to achieve this value for b . If the flow is laminar, then b is the boundary layer thickness (determined by the thermal conductivity of the gas) and is much smaller.

The flow velocity can be reduced somewhat by increasing the pressure, and reducing the temperature of the gas. However, it appears certain that reducing the transverse temperature rise below the longitudinal rise requires substantial mass flow cooling.

Secondary aspects of such high mass transfer flows include the possibility of exciting acoustic modes in the crystals and the possibility of damaging the crystal surfaces by impact of solid microparticles in the coolant gas. These will cause the fracture strength to decrease and may reduce the optical quality of the output.

Finally we note that the transverse temperature gradient generates a transverse flow of heat. The ratio of the transverse and longitudinal heat currents is

$$q_x/q_z = (\Delta T_{tr}/\Delta T) (k/H) . \quad (56)$$

Clearly the transverse temperature gradient is smaller. Therefore, under appropriate flow cooling conditions, the temperature gradient is indeed essentially parallel to the laser beam.

4.2.5. Cooling Rate Limits

The temperature difference between the crystal and the coolant is determined by the convective heat transfer coefficient h , which is discussed in some detail in Sec. 9:

$$T_c - T_f = Wl/2h = \Delta T_{bl} \quad (57)$$

This temperature difference is limited only by the thermal or chemical stability regions of the crystal. As examples, consider the most common materials available in large sizes. KD*P and lithium iodate begin to decompose below 200°C, whereas lithium niobate and CD*A are stable up to the maximum temperature at which they can be phasematched, around 120°C. Consequently, the crystal temperature and the cooling rate are limited. The average power density is

$$I_{av} \leq h(T_c - T_f)/\alpha_I l \quad (58)$$

Typical values for the parameters in this equation are $h \sim 1 \text{ KW/m}^2/\text{K}$, $\alpha \sim 1 \text{ m}^{-1}$, $l = 1 \text{ mm}$, $T_c - T_f = 100 \text{ K}$; this implies that $I_{av} < 10 \text{ KW/cm}^2$. Under the expected operating conditions, flow cooling is easily capable of removing the heat deposited in the crystals.

4.2.6. Beam Quality Requirements

High conversion efficiency for single-pulse harmonic conversion requires that the polarization and wavelength of the drive be within the conversion bandwidth of the crystal. Also, its spatial structure, with respect to both amplitude and phase variations, must be such that most of the energy lies in the high conversion region of Fig. 1. We have also seen that the temperature profile of the crystal must lie entirely within its temperature bandwidth. These requirements also apply to high-average-power conversion.

The temperature profile of the crystal is determined both by the heat loading W and the flow cooling condition. The heat loading is due entirely to optical absorption

$$W = \alpha_I I_{av} \quad (59)$$

and therefore has the same spatial structure as the average power density. If this is spatially nonuniform, the temperature of the crystal will tend to have the same spatial variation as average power density.

In this way spatial variations in the beam profile translate directly into a transverse temperature variation. This argument applies to any spatial variation in intensity which is systematically repeated from pulse to pulse. It therefore applies to the energy at the edge of the beam where the intensity is dropping to zero, as well as to small-scale variations.

The transverse heat flow can be calculated by separation of variables in the diffusion equation, letting the longitudinal temperature profile be determined by the deviations of the heat loading from its average value. Then, in the transverse direction

$$\Delta T_{xx} = W/K(\Delta I(x)/I) \quad , \quad (60)$$

where W and I are spatial averages. Decomposing T and ΔI into Fourier components, we find

$$\Delta T(\Lambda) = (W\Lambda^2/4K\pi^2)(\Delta I(\Lambda)/I) \quad , \quad (61)$$

where Λ is the spatial wavelength of the heat loading. In terms of the longitudinal temperature difference ΔT ,

$$\Delta T(\Lambda)/\Delta T = (2\Lambda^2/\pi^2)(\Delta I(\Lambda)/I) \quad . \quad (62)$$

So long as the periodicity Λ is comparable to the plate thickness, the transverse variation in temperature will be small compared to the longitudinal temperature variation.

The problem of the temperature profile at the edges of the beam can be solved in at least two ways. One way is to arrange for the fill factor of the crystal aperture to be as close to unity as possible and to thermally insulate the edges. The temperature drop at the edge will be smaller than the longitudinal ΔT , if the beam intensity maintains its average value to within about one plate thickness of the edge.

Alternatively, a small heater at the edge can be used to maintain temperature uniformity. It must deliver a very small power, WA_2 , where A is the crystal surface area not filled by the laser pulses. This will cause the heat loading to remain uniform out to the edge and eliminate any temperature drop there.

We conclude that there are no extra beam quality requirements for high-average-power harmonic generation beyond those given above for the single-pulse case.

4.3. TURBULENT DEPHASING

In a turbulent gas there are fluctuations in the local velocity, temperature and density, and the index of refraction fluctuates with the density variations as

$$n = 1 + K\rho \quad . \quad (63)$$

The beam therefore acquires a phase which fluctuates with the density,

$$p = K\rho\Delta z \quad . \quad (64)$$

The spatial variation of this phase over the aperture of the beam causes a reduction in the beam quality. The associated beam divergence causes the beam to spread in the far field, and can introduce, through propagation, an intensity fluctuation. It also increases the angular dephasing δ .

The effect of random medium fluctuations on light propagation has been studied by several authors [Ch65, Ta65, Ho73]. A part of the beam is scattered into near-forward angles. The beam divergence of the scattered wave is

$$\theta = \langle (dp/dx)^2 \rangle^{1/2} \\ = AK\rho_0(\delta\rho/\rho)(l_l l_g/l_t)^{1/2} \quad (65)$$

Here l_l is the longitudinal correlation length, l_t is the transverse correlation length, and $\delta\rho/\rho$ is the fluctuation of the density in the turbulent gas. A is a numerical factor of order unity, depending on the statistics and mean distribution of the density, and ρ_0 is the mean mixed density of the gas. Not all the beam experiences this increase in beam divergence. There is the unscattered part which remains unaffected by the turbulence; only the scattered part experiences any degradation. The scattered part of the wave will be small if the mean square phase fluctuations are small; otherwise the entire beam experiences the scattering [Ho73].

The flow patterns in the turbulent boundary layer have been studied using Schlieren methods to photograph the gas density [Hi75, Sc62]. The experiments were carried out under adiabatic conditions, where the gas carried no heat away from the wall. Under these conditions the density fluctuations are related to the turbulence intensity as follows:

$$\delta\rho/\rho = M^2 \delta u/u , \quad (66)$$

where M is the Mach number. The velocity fluctuation reaches a maximum at the edge of the boundary layer. It is also largest for Reynolds numbers in the range 2000 to 4000. The maximum value is about 0.1, and the mean fluctuation across the flow channel is about 0.05. The density fluctuations are therefore bounded by 0.1 for subsonic flow and are probably somewhat less:

$$0.1 > \delta\rho/\rho > 0.01 . \quad (67)$$

Note that the turbulent Prandtl number in a gas is close to unity. Therefore the thermal contribution to $\delta\rho/\rho$ is expected to be of the same order as the momentum contribution.

The turbule size in the wall region is proportional to the distance from the wall. In the wake region, the turbule size varies little with location in the flow channel. It can also be shown that the shear forces in the boundary layer give rise to a longer correlation length parallel to the flow direction than perpendicular to the wall. We may therefore estimate

$$l_t > l_s \quad (68)$$

and

$$l_s = c_1 , \quad (69)$$

where c is a numerical constant approximately equal to the ratio of the wall region thickness to the channel width. This is typically about 0.1.

This model of the turbulence leads to the following estimate of the angular divergence of the scattered wave.

$$\theta = AK\rho_0(\delta\rho/\rho)c^{-1/2} . \quad (70)$$

Note that because the turbule size is always a fixed fraction of the channel width, the angular divergence of the scattered wave is independent of the channel width, but is proportional to $M^{1/2}$, where M is the mean number of turbules (i.e. scatterers) a ray bundle passes through. The fraction of the beam which experiences this scattering is [Ho73]

$$I/I = \exp(-s^2) , \quad (71)$$

where s is the rms phase variation at a single point. It is linearly related to the scattering angle as

$$s = \theta/[c^{1/2}/k_0 l_B] \quad (72)$$

If the scattering angle θ exceeds $c^{1/2}/k_0 l_B$, then essentially all the beam experiences the scattering; otherwise a portion of the beam remains unaffected by the turbulence.

Using the above estimates for the maximum density fluctuation and the scale lengths, the turbulent dephasing is typically less than 0.1

mrad/atm for nitrogen and 0.01 mrad/atm for helium. Estimates for other gases can be scaled from this using the data in Table 1. The divergence for $N - 1$ channels is

$$\theta_N = \theta(N - 1)^{1/2} . \quad (73)$$

The total beam divergence is therefore about 0.1 - 1 mrad for nitrogen and 0.01 - 0.1 mrad for helium, at 1 atmosphere.

The effect of turbulent dephasing on harmonic generation is most easily expressed in terms of the beam quality parameter, Q . This is increased by turbulence by an amount which depends on the beam dimension H .

$$Q' = Q + \theta_N H/\lambda . \quad (74)$$

The deleterious effects of the turbulence are therefore minimized by designing with small apertures, where necessary.

For helium-cooled KD*P, the increase in the beam divergence is small enough to be insignificant in high average power performance. It will become more important for other materials whose figure of merit P_{th} is lower. The relationship between beam quality and performance is discussed in Sec. 8.

5. THERMAL STRESS IN ANISOTROPIC CRYSTALS

The relationship between thermal stress and temperature used above [(Eq. (38))] does not take into account the anisotropy of the crystal. In an anisotropic medium the relation between stress and strain is [Bo60]

$$\eta_{ij} = s_{ijkl} \sigma_{kl} + \alpha_{ij} T, \quad (75)$$

where s is the elastic compliance tensor of the crystal and T is its temperature. In the usual stress formulation of a thermoelastic problem, the stress field must satisfy both the equations of equilibrium of the solid and also the so-called compatibility equations. The equations of equilibrium are

$$s_{ij,j} = 0, \quad (76)$$

where $,_j$ indicates differentiation with respect to x_j . The compatibility equations are due to the form of the strain tensor. The strain-displacement relation is

$$\eta_{ij} = u_{i,j} + u_{j,i}. \quad (77)$$

Operating on the strain tensor with antisymmetric derivatives (curl operators) gives

$$\epsilon_{ijk}\epsilon_{rst}\eta_{kt',js} = 0 \quad , \quad (78)$$

where ϵ_{ijk} is the fully antisymmetric tensor of rank three. Equation (78) is an identity satisfied by the strain tensor. On inserting the generalized Hooke's law [Eq. (75)] into Eq. (78), the compatibility equations for the stresses are obtained:

$$\epsilon_{ijk}\epsilon_{rst}(s_{ktmn}S_{mn,j's} + \alpha_{kt}T_{,js}) = 0 \quad . \quad (79)$$

There are three equilibrium equations (76) and six compatibility equations (79), giving nine equations for the nine stress variables.

The stress formulation is completed by specifying the normal component of the stress on the boundary:

$$S_{ij}n_j = G_i \quad . \quad (80)$$

In many problems, the applied stress G is zero.

To apply this to the current problem, Cartesian axes are defined in which z is the beam direction and is, of course, perpendicular to the crystal surfaces, which are therefore parallel to the (x,y) plane. The (xyz) frame is the laboratory frame and is related by a three-dimensional rotation to either the optical $(\alpha\beta\gamma)$ frame or the crystallographic (XYZ) frame. In the thin plate geometry, we take the temperature distribution to be independent of x and y , and therefore we are looking for a solution in which the stresses are also independent of x and y [Ho55, Ho57]. Then the equations of equilibrium, together with the

boundary conditions, imply that all the stress components are zero except S_{xx} , S_{xy} and S_{yy} . After some algebra, the compatibility equations become

$$s_{xxmn} S_{mn}'' + \alpha_{xx} T'' = 0 \quad (81)$$

$$s_{xymn} S_{mn}'' + \alpha_{xy} T'' = 0 \quad (82)$$

$$s_{yy mn} S_{mn}'' + \alpha_{yy} T'' = 0 \quad (83)$$

The sum over mn is restricted to the terms (xx) , (xy) , and (yy) , because all other stresses are zero. These are, therefore, three linear equations for the three remaining stresses in terms of the given temperature distribution, T . Assuming that the determinant of the appropriate compliances is not zero, the solution is obtained by standard linear algebra and is linear in T'' :

$$S_{mn}'' + \xi_{mn} T'' = 0 \quad (84)$$

The solution of Eq. (84) is

$$S_{mn} = -\xi_{mn} T + A_{mn} + B_{mn} z, \quad (85)$$

where the constants A and B are to be determined from the boundary conditions. A good approximation in thin plates is to choose the constants to give zero force and moment over the edges of the plate. Then

$$S_{mn} = \xi_{mn} [-T(z) + U_0 + zU_1] , \quad (86)$$

where

$$U_0 = (1/l)_0 \int_0^l T(z') dz' \quad (87)$$

$$U_1 = (12/l^3)_0 \int_0^l z' T(z') dz' . \quad (88)$$

Clearly this approximation to the solution does not have completely traction-free edges. However, at distances from the edges greater than about one plate thickness, Saint-Venant's principle implies that this solution is an accurate approximation even for traction-free boundary conditions. It clearly satisfies the boundary conditions on the planes $z = 0$ and $z = l$; it is, therefore, a good approximation to the unique solution to the anisotropic thermoelastic problem.

For temperature distributions symmetric about the midplane $z = l/2$, the moment term U_1 is zero. Also, the term U_0 is the mean temperature of the plate. Thus for these conditions the stresses are proportional to the temperature different ΔT :

$$S_{mn}(z) = \xi_{mn} \Delta T(z) . \quad (89)$$

The stresses in the plate surface are given by ΔT and ξ_{mn} , which is a function only of the 27 compliances and of the orientation of the crystallographic axes (XYZ) relative to the laboratory frame (xyz). The

general expression for the ϵ_{mn} is very cumbersome; as an example, we will evaluate them below for KD*P.

5.1 AN EXAMPLE: THERMAL STRESSES IN KD*P

As an example of an anisotropic medium, we treat the case of KD*P. KD*P is a uniaxial crystal, and in the crystallographic frame (XYZ) its thermal conductivity (K) and thermal expansion (α) tensors are diagonal with the XX and YY components equal. For example

$$K = \begin{bmatrix} K_{11} & 0 & 0 \\ 0 & K_{11} & 0 \\ 0 & 0 & K_{33} \end{bmatrix}. \quad (90)$$

The compliance tensor is symmetric under interchange both of its first and second indices and of its third and fourth indices; it is usually expressed as a 6x6 matrix. For KD*P, which is in the crystal class $\bar{4}2m$, it has the form

$$s = \begin{bmatrix} s_{11} & s_{12} & s_{13} & & & \\ s_{12} & s_{11} & s_{13} & & & \\ s_{13} & s_{13} & s_{33} & & & \\ & & & s_{44} & & \\ & & & & s_{55} & \\ & & & & & s_{66} \end{bmatrix}. \quad (91)$$

The most believable material constants for KD*P are given in Table 2 [LB79, Ma85].

In second harmonic generation from 1064 nm to 532 nm in a type II scheme, the crystal is oriented so that the 1064-nm drive makes an angle θ with the Z-axis, as in Fig. 6. In the crystallographic frame, the beam direction is

$$\mathbf{k} = (S_c, S_s, C) \quad , \quad (92)$$

where

$$S = \sin\theta, \quad C = \cos\theta \quad (93)$$

and

$$s = \sin\phi, \quad c = \cos\phi \quad . \quad (94)$$

The angles θ and ϕ are shown in Fig. 6. For KD*P they are

$$\theta = 54^\circ \quad (95)$$

$$\phi = 45^\circ \quad . \quad (96)$$

The surface of the crystal is perpendicular to $\mathbf{k}$. We can take the laboratory x- and y-directions to be the polarization vectors for the ordinary and extraordinary ray in the crystal. Thus

Table 2. Material constants for KD*P.

Nonlinear coefficient	(Type II, 1.064 μm)	C	0.97 GW ^{-1/2}
Angular sensitivity		β_θ	2.5 10 ⁵ m ⁻¹ /rad
Temperature sensitivity		β_T	47 m ⁻¹ /K
Absorption (1.064 μm)		α_0	1.2 m ⁻¹
		α_e	< 0.1 m ⁻¹
	(0.532 μm)	α_0, α_e	< 0.1 m ⁻¹
Thermal conductivity		K ₁₁	1.86 W/m/K
		K ₃₃	2.09 W/m/K
Thermal expansion		α_{11}	2.49 10 ⁻⁵
		α_{33}	4.40 10 ⁻⁵
Surface toughness		S ₀	0.09 MPa m ^{-1/2}
Compliance tensor		S ₁₁	15.8 (TPa) ⁻¹
		S ₁₂	2.1 (TPa) ⁻¹
		S ₁₃	-4.0 (TPa) ⁻¹
		S ₃₃	20.1 (TPa) ⁻¹
		S ₄₄	79.0 (TPa) ⁻¹
		S ₆₆	168 (TPa) ⁻¹
Stress-optic tensor ^a		q ₁₁	4.15 (TPa) ⁻¹
		q ₁₂	4.08 (TPa) ⁻¹
		q ₁₃	1.12 (TPa) ⁻¹
		q ₃₁	3.60 (TPa) ⁻¹
		q ₃₃	0.44 (TPa) ⁻¹
		q ₄₄	-1.47 (TPa) ⁻¹
		q ₆₆	-10.26 (TPa) ⁻¹

^a Data reported here are for KD*P.

Figure 6. The o - and e - vectors associated with a ray with wavevector k in an anisotropic crystal. In uniaxial media, these are the polarizations of the two eigenmodes of electromagnetic waves in the crystal. In biaxial media, the eigenpolarizations are rotated relative to the (oe) basis set around the k -vector. The (oe) vectors lie in the crystal surface if the ray enters normally and provide a useful basis set for thermooptic and stressoptic calculations such as those in Secs. 5 and 6.

$$x = "o" = (-s, c, 0) \quad (97)$$

and

$$y = "e" = (-Cc, -Cs, S) \quad (98)$$

It is now straightforward to evaluate the relevant piezoelectric constants in the laboratory frame. For example, in terms of Cartesian components in the (XYZ) frame, the xxxx component in the laboratory frame is

$$s_{xxxx} = o_i o_j s_{ijkl} o_k o_l \quad (99)$$

or, referring to the 6x6 matrix representation,

$$s_{xxxx} = (oo)_U (oo)_V s_{UV} \quad (100)$$

Similarly,

$$s_{xyxy} = (oe)_U (oe)_V s_{UV} \quad (101)$$

In general, each laboratory frame constant is the sum over all the crystallographic frame constants. Table 3 catalogs the coefficients of each of the q_{UV} in the expressions for each of the s_{ijkl} .

The relevant thermal conductivity is obtained in the same way,

$$\begin{aligned} K_{zz} &= k_i K_{ij} k_j \\ &= K_{ZZ} C^2 + K_{XX} S^2, \end{aligned} \quad (102)$$

and the thermal expansion coefficients are

$$\alpha_{xx} = \alpha_{XX}$$

$$\alpha_{xy} = 0 \quad (103)$$

$$\alpha_{yy} = \alpha_{ZZ}C^2 + \alpha_{XX}S^2 .$$

Using these formulae, we can now evaluate the parameters ϵ_{mn} which determine the stresses. At the standard orientation for doubling,

Table 3. Factors for evaluating piezoelectric terms in KD*P.

	q_{11}	q_{33}	q_{44}	q_{66}	q_{12}	q_{21}	q_{13}	q_{31}
$s_{xxxx} = (eeeee)$	$C^4(s^4 + c^4)$	S^4	$S^2 C^2$	$C^4 c^2 s^2$	$C^4 c^2 s^2$	$C^4 c^2 s^2$	$C^2 S^2$	$C^2 c^2$
$s_{xxxy} = (eeqoe)$	$2C^3(s^3 + c^3)$	0	0	$C^3 cs(s^2 - c^2)$	$-2C^3 c^3 s$	$2C^3 cs^3$	0	0
$s_{xxyy} = (eeqoo)$	$2C^2 s^2 c^2$	0	0	$-C^2 c^2 s^2$	$C^2 c^4$	$C^2 s^4$	0	S^2
$s_{xyxx} = (oeqee)$	$2C^3(c^3 s - cs^3)$	0	0	$C^3 cs(s^2 - c^2)$	$2C^3 cs^3$	$-2C^3 c^3 s$	0	0
$s_{xyxy} = (oeqoe)$	$8C^2 c^2 s^2$	0	S^2	$C^2(s^2 - c^2)^2$	$-4C^2 c^2 s^2$	$-4C^2 c^2 s^2$	0	0
$s_{xyyy} = (oeqoo)$	$2Csc(s^2 - c^2)$	0	0	$-Csc(s^2 - c^2)$	$2Cc^3 s$	$-2Ccs^3$	0	0
$s_{yyxx} = (ooqee)$	$2C^2 s^2 c^2$	0	0	$-C^2 c^2 s^2$	$C^2 s^4$	$C^2 c^4$	S^2	0
$s_{yyxy} = (ooqoe)$	$2Csc(s^2 - c^2)$	0	0	$-Csc(s^2 - c^2)$	$-2Ccs^3$	$2Cc^3 s$	0	0
$s_{yyyy} = (ooqoo)$	$s^4 + c^4$	0	0	$s^2 c^2$	$s^2 c^2$	$s^2 c^2$	0	0

where $\phi = 45$ deg, the piezoelectric constants with an odd number of x's are zero, and the remaining coefficients are symmetric. That is, the remaining components are

$$\begin{bmatrix} s_{xxxx} & - & s_{xxyy} \\ - & s_{xyxy} & - \\ s_{yyxx} & - & s_{yyyy} \end{bmatrix} \quad (104)$$

and

$$s_{xxyy} = s_{yyxx} \quad (105)$$

On substituting these expressions into the compatibility equation, it is immediately clear that the (xy) shear stress is zero. That is

$$s_{xyxy} \frac{\partial^2}{\partial x^2} + \alpha_{xy} T'' = 0 \quad (106)$$

and $\alpha_{xy} \approx 0$. Therefore, there are no shear forces introduced by the thermal gradients, and

$$\epsilon_{xy} = 0 \quad (107)$$

The two remaining parameters are

$$\epsilon_{xx} = \frac{-\alpha_{xx} s_{yyyy} + \alpha_{yy} s_{xxyy}}{s_{xxyy} - s_{xxxx} s_{yyyy}} \quad (108)$$

$$\text{and} \quad \xi_{yy} = \frac{\alpha_{xx} s_{xxyy} - \alpha_{yy} s_{xxxx}}{s_{xxyy}^2 - s_{xxxx} s_{yyyy}} \quad (109)$$

Using the material constants in Table 2, the effective moduli ξ are

$$\xi_{xx} = -1.32 \quad \text{MPa/K}^{-1} \quad (110)$$

$$\xi_{yy} = -0.781 \quad \text{MPa/K}^{-1} \quad (111)$$

From these parameters the maximum longitudinal temperature rise can be determined in terms of the assumed surface strength. The fracture toughness of KD*P has been measured recently [Ma85]:

$$S_0 \approx 0.09 \text{ MPa(m}^{1/2}\text{)} \quad (112)$$

For an assumed surface defect size of 0.1 mm, the surface strength is

$$S_{th} = 9 \text{ MPa} \quad (113)$$

and the maximum temperature difference is obtained using Eq. (99)

$$\Delta T = 7 \text{ K} \quad (114)$$

In this way all the parameters are determined by the standard crystal orientation for harmonic generation. Other things being equal, fracture resistance is greatest when the ratio ξ/K_{zz} is smallest. The phasematching condition allows ϕ to be adjusted a little, but not θ . However, the nonlinear coupling C is proportional to $\sin(2\phi)$, and the performance will deteriorate if ϕ deviates from 45 deg.

It is interesting to note that the fracture limit in anisotropic crystals depends on the material constraints for specific directions in the crystal. It is sometimes possible, by adjusting the composition of the crystal, to arrange for zero thermal expansion along certain directions. It is also sometimes possible to directionally adjust the thermal conductivity. In particular, a material with zero expansion perpendicular to the beam (i.e., in the crystal surfaces) will generate no thermal stress. There are several known acentric materials whose thermal expansion can be made to vanish or be slightly negative. These include the Eucryptite-Spodumene class and a new class CTP [Ro84], of which NaYSiO_4 is an example and is acentric. These possibilities may eventually be exploited to engineer materials with higher resistance to thermal fracture.

5.2 HEAT FLOW IN ANISOTROPIC MEDIA

In anisotropic media the direction of heat flow is not parallel to the gradient of the temperature. The thermal conductivity is a tensor,

$$q_i = K_{ij} T_{,j} . \quad (115)$$

In the thin plate problem, the temperature gradient is perpendicular to the crystal surface only if the surfaces are isothermal. This is true only in the approximation that the transverse temperature rise due to flow cooling is small. In an anisotropic medium there is a transverse heat flow in addition to that due to flow cooling. In the body of the plate this presents no problem, but the transverse heat flow can affect the temperature distribution near the edges. If the edges of the plate are thermally insulated, then the heat flow at the plate edge is parallel to the edge. In order to avoid transverse temperature gradients at the edge, it should be cut so that it is parallel to the heat flow vector in the center of the plate. This applies regardless of the origin of the transverse temperature gradient.

The longitudinal temperature distribution in the plate is again quadratic:

$$T(z) = (W/8K_{zz}) (z - l/2)^2 . \quad (116)$$

The effective thermal conductivity depends on the orientation of the crystal. It is clearly advantageous to arrange for K_{zz} to be as large as possible.

6. THE STRESS-OPTIC EFFECT

6.1 PROPAGATION IN A STRESS-FREE MEDIUM

Maxwell's equations in a dielectric medium are [Bo70]

$$\begin{aligned} \text{curl } \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\ \text{curl } \vec{H} &= \frac{\partial \vec{D}}{\partial t} \end{aligned} \quad (117)$$

For a plane wave traveling in the direction of the unit vector $\mathbf{k}$, with index of refraction n , these lead to

$$D_i = \epsilon_{ij} n^2 (-k_j (k_m E_m) + E_j) \quad (118)$$

where ϵ is the dielectric tensor of the medium. In general it is anisotropic:

$$D_i = \epsilon_{ij} E_j \quad (119)$$

These equations give the eigenvalue equation for the polarization vector $\mathbf{D}$ and the refractive index:

$$(1 - k k)_{ij} (1/\epsilon)_{jk} \Delta_k = (1/n^2) D_i \quad (120)$$

Clearly $\mathbf{k} \cdot \mathbf{D} = 0$ and $\mathbf{D}$ lies in the plane perpendicular to $\mathbf{k}$. It may

therefore be expressed as a sum over any two vectors which span the two dimensional space perpendicular to k :

$$D_i = o_i \cos \phi + e_i \sin \phi .$$

For the basis vectors, we choose the o and e vectors defined in Fig. 6.

These vectors are the usual ordinary and extraordinary polarization vectors for uniaxial media. In biaxial media the eigenmodes are obtained by rotating the (oe) basis set about the ray direction by the angle ϕ .

Using the expression for D_i , the eigenvalue equation becomes

$$\begin{bmatrix} D_{oo} & D_{oe} & 1 \\ D_{eo} & D_{ee} & \tan \phi \end{bmatrix} = (1/n^2) \begin{bmatrix} 1 \\ \tan \phi \end{bmatrix}, \quad (121)$$

where

$$\begin{aligned} D_{ee} &= C^2 c^2 / \epsilon_{xx} + C^2 s^2 / \epsilon_{yy} + 2 / \epsilon_{zz} , \\ D_{oo} &= s^2 / \epsilon_{xx} + c^2 / \epsilon_{yy} , \end{aligned} \quad (122)$$

$$D_{eo} = Ccs (1/\epsilon_{xx} - 1/\epsilon_{yy}) ,$$

$$D_{oe} = D_{eo} .$$

The eigenvalues are the solutions of

$$(D_{oo} - 1/n^2) (D_{ee} - 1/n^2) - 4D_{oe}^2 = 0 , \quad (123)$$

and the rotation angle is

$$\tan \phi = (1/n^2 - D_{oo})/D_{oe} . \quad (124)$$

The rotation of the eigenvectors from the uniaxial vectors represented by ϕ may vary from 0 to $\pi/2$, depending on the ray direction and the refractive indices. In general, the only cases where ϕ vanishes are restricted to the uniaxial cases where two of the three dielectric constants are equal, or to the case where the ray propagates perpendicularly to a dielectric axis. There is no simple closed form for the refractive indices in the general case.

We write the polarization states of the propagating modes and their associated indices as follows:

$$a_i = o_i \cos \phi + e_i \sin \phi; \quad n = n_a \quad (125)$$

$$b_i = -o_i \sin \phi + e_i \cos \phi; \quad n = n_b . \quad (126)$$

6.2 PERTURBATIONS OF THE EIGENMODES

The effect of stress on the eigenmodes is most easily calculated using perturbation theory. The dielectric tensor is slightly perturbed by the stresses. It can be written as the sum of a diagonal term and a perturbation whose form is arbitrary except that it is symmetric in its two indices:

$$(1/\epsilon)_{ij} = (1/\epsilon_0)_{ij} + \Delta(1/\epsilon)_{ij} . \quad (127)$$

We define two matrices, M and V:

$$M_{ij} = (1 - kk)_{ip} (1/\epsilon_0)_{pq} (1 - kk)_{qj} \quad (128)$$

$$V_{ij} = (1 - kk)_{ip} \Delta(1/\epsilon)_{pq} (1 - kk)_{qj} , \quad (129)$$

where k is the ray direction in the crystal. In the absence of a perturbation, the polarization vectors a_i and b_i of the propagating modes are the eigensolutions of Maxwell's equations:

$$M_{ij} a_{0j} = a_0 a_{0i} \quad (130)$$

$$M_{ij} b_{0j} = b_0 b_{0i} , \quad (131)$$

where $a_0 = 1/n_a^2$, etc. In the presence of the perturbation they are

$$(M + V)_{ij} a_j = a a_i \quad (132)$$

$$(M + V)_{ij} b_j = b b_i , \quad (133)$$

where $a = 1/n_a^2$, etc. The interaction V perturbs both the eigenvectors and the eigenmodes. That is,

$$a = a_0 + \delta a \quad (134)$$

$$a_{ij} = a_{0ij} + \delta a_{ij}$$

and similar expressions for the other mode. Using these equations in the eigenvalue equation and retaining only the lowest order terms, we find

$$\delta a = a_{0i} V_{ij} a_{0j} = V_{aa}$$

$$a_{0i} \delta a_i = 0 \quad (135)$$

$$b_{0i} \delta a_i (b_0 - a_0) = -b_{0i} V_{ij} a_{0j} = V_{ba} ,$$

and

$$\delta b = b_{0i} V_{ij} b_{0j} = V_{bb}$$

$$b_{0i} \delta b_i = 0 \quad (136)$$

$$b_{0i} \delta a_i = -a_{0i} \delta b_i .$$

The perturbation of the eigenvectors is perpendicular to the unperturbed eigenvector. Under the action of the perturbation, its length is preserved, and it simply rotates about the ray direction. The rotation angle is given by

$$x = a_{0i} \delta b_i \quad (137)$$

The perturbations in the refractive indices are

$$\delta n_a = (1/2) n_{0a}^3 v_{aa} \quad (138)$$

and a similar equation for δn_b .

This is the general solution to the problem of perturbations of the dielectric tensor in biaxial media, except for the case where the ray direction is close to an optic axis. The rotation of the eigenmodes χ is essentially the ratio of the induced birefringence to the birefringence appropriate to the direction of the propagation, $n_a - n_b$. For propagation along an optic axis, the expression for the rotation angle is inconsistent with the assumption that the perturbation is small. The problem is solved by choosing the unperturbed eigenvectors to make the perturbation diagonal. Let the new basis vectors A and B be obtained by a rotation γ around the beam direction, k. Then

$$A_i = a_i \cos \gamma + b_i \sin \gamma \quad (139)$$

$$B_i = -a_i \sin \gamma + b_i \cos \gamma \quad (140)$$

and the angle γ must be chosen so that

$$V_{AB} = A_i B_{ij} B_j = 0 \quad (141)$$

That is

$$\tan 2\gamma = -2V_{ab}/(V_{bb} - V_{aa}) , \quad (142)$$

and the new eigenvalues are

$$1/n_A^2 = 1/n^2 + V_{ab} \pm \Delta V , \quad (143)$$

where

$$\Delta V = \sin 2\gamma (1/2) (V_{aa} + V_{bb}) . \quad (144)$$

The unperturbed eigenvalues $1/n^2$ are the same for each wave. In a general biaxial medium, this index is the intermediate index n_β ($n_\alpha < n_\beta < n_\gamma$). The optic axes lie in the $(\alpha\gamma)$ plane.

The vectors a and b may be chosen arbitrarily so long as they are perpendicular to the optic axis. A convenient choice is to choose a to be the (β) axis, and b to lie in the $(\alpha\gamma)$ plane perpendicular to the optic axis.

Note that there is no physical reason to suppose that the $(\alpha\beta\gamma)$ axes coincide with the (XYZ) crystallographic axes. In general, they will not, and care must be taken to correctly identify the $(\alpha\beta\gamma)$ axes in applications.

6.3 STRESS OPTICS OF THIN PLATES

The stress-optic effect is described by a tensor which relates the perturbation in $(1/\epsilon)$ to the stress tensor [LB79]:

$$\Delta(1/\epsilon)_{ij} = q_{ijkl} S_{kl} \quad . \quad (145)$$

The perturbations are

$$\Delta(1/n_a^2) = a_i a_j q_{ijkl} S_{kl} \quad (146)$$

$$\Delta(1/n_b^2) = b_i b_j q_{ijkl} S_{kl} \quad (147)$$

$$\chi = \frac{n_a^2 n_b^2 a_i a_j q_{ijkl} S_{kl}}{n_a^2 - n_b^2} \quad . \quad (148)$$

The rotation of the eigenvectors due to the stresses is essentially the ratio of the stress-induced birefringence to the natural birefringence along the given direction of propagation, k . The induced birefringence is small for most media, around 10^{-5} or 10^{-6} . Except for p -propagation close to an optic axis, the natural birefringence is much larger, around 10^{-2} or 10^{-3} , and the energy rotated into the other polarization is 10^{-6} . Therefore, we ignore the stress-induced polarization rotation in all harmonic generation media. We note in passing, however, that this is not always possible for other media, such as laser host materials or electro-optic media.

The solution of the thermoelastic problem shows that all the stresses are zero except the stresses in plane parallel to the crystal surfaces. In the thermal stress problem, the stresses were written in the "laboratory" frame (xyz), and the basis vectors were chosen to be the (oe) basis set. In the stress-optic problem, the basis vectors are chosen to be the (ab) basis set. The polarization eigenmodes are the natural variables in which to formulate the problem. In this basis set the stress tensor is the sum of three terms:

$$\begin{aligned}
 S_{ij} = & S_{aa} a_i a_j \\
 & + S_{ab} (a_i b_j + b_i a_j) \\
 & + S_{bb} b_i b_j .
 \end{aligned}
 \tag{149}$$

In terms of these stresses the index changes are

$$\begin{aligned}
 \delta n_a = & -(1/2) n_a^3 \{ (aa) q(aa) S_{aa} \\
 & + 2(aa) q(ab) S_{ab} \\
 & + (aa) q(bb) S_{bb} \}
 \end{aligned}
 \tag{150}$$

$$\begin{aligned}
\delta n_b = & -(1/2)n_b^3[(bb)q(aa)s_{aa} \\
& + 2(bb)q(ab)s_{ab} \\
& + (bb)q(bb)s_{bb}] .
\end{aligned}
\tag{151}$$

The stress-optic tensor q_{ijkl} is symmetric under interchange of ij and kl . Like the compliance tensor, it is usually expressed as a 6×6 matrix. Then the tensor products become

$$(aa)q(bb) = (aa)_U q_{UV} (bb)_V . \tag{152}$$

There is an interesting sum rule concerning the phases accumulated by the two polarization states in completely traversing the crystal. For each stress component the integral over the crystal depends only on the moment of the temperature distribution. For example

$$\int_0^L s_{aa} dz = \epsilon_{aa} U_1 L^2 / 2 . \tag{153}$$

For symmetric temperature distributions, the moment is zero, and consequently the phase accumulated due to stress-induced index changes is also zero:

$$\int_0^L \delta n_a dz = \int_0^L \delta n_b dz = 0 . \tag{154}$$

This shows that stress-induced index changes do not accumulate from one crystal to the next. They affect only the phasematching condition in

a single plate, just as the temperature profile does. This is the real justification for including both direct temperature detuning and the indirect effect through the induced stresses in the single parameter β_t [Eq. (40)]. In fact, ignoring the rotation of the eigenvectors,

$$dn_a/dT = \partial n_a / \partial T - (1/2)n_a^3 a_i a_j q_{ijkl} \xi_{kl} \quad , \quad (155)$$

where ξ_{kl} is taken from the solution to the thermoelastic problem, Eqs. (81-83). There is an analogous equation for the other index.

6.4 RESULTS FOR KD*P

The stress-optic tensor for KD*P is listed in Table 2. It has the same structure as the compliance tensor, with the exception that it is not a symmetric 6x6 matrix, whereas the compliance tensor is symmetric. Because KD*P is a uniaxial crystal, the polarization eigenvectors (ab) reduce to the (oe) basis set. Using Tables 2 and 3, the temperature derivative of the ordinary and extraordinary indices arising from the induced stresses are

$$\partial n_o / \partial T = 7.50 \times 10^{-6} \text{ K}^{-1} \quad (156)$$

$$\partial n_e / \partial T = 6.65 \times 10^{-6} \text{ K}^{-1} \quad , \quad (157)$$

and the associated contribution to the temperature derivative of the phase mismatch is

$$\partial(\Delta k)/\partial T = -0.05 \text{ cm}^{-1}/\text{K} . \quad (158)$$

These derivatives are to be compared with the stress-free temperature derivatives of the indices.

$$\partial n_o/\partial T = 2.86 \times 10^{-5} \text{ K}^{-1} \quad (159)$$

$$\partial n_e/\partial T = 2.0 \times 10^{-5} \text{ K}^{-1} . \quad (160)$$

The stress-free and stress-induced contributions to the phase mismatch have the same sign, and so the total temperature derivative of Δk is

$$\partial(\Delta k)/\partial T \approx \beta_t = -0.53 \text{ cm}^{-1}/\text{K} . \quad (161)$$

7. MAXIMUM AVERAGE POWER IN A SINGLE APERTURE

The optimized system design must achieve both high conversion efficiency and efficient heat removal. High conversion efficiency requires sufficient drive, such that

$$1 < n_0 < 5 \quad (162)$$

where

$$n_0 = c^2 N^2 \epsilon^2 I_{pk} \quad (163)$$

and also that the total detuning be small. The total detuning includes the effects of the coolant dispersion, the beam divergence and bandwidth, the longitudinal temperature profile, the transverse temperature profile, and index inhomogeneities in the crystal. Thus

$$\delta = \delta_g + \delta_{tr} + (1/8)\delta_T + \delta_\theta + \delta_\lambda + \delta_n, \quad (164)$$

where

$$\delta_g = (N - 1)\epsilon_s / \epsilon_c \quad (165)$$

$$\delta_\theta = \beta_\theta \Delta \theta NL \quad (166)$$

$$\delta_T = \beta_T \Delta T L \quad (167)$$

$$\delta_{Tr} = \beta_t \Delta T_{tr} l \quad (168)$$

$$\delta_\lambda = \beta_\lambda \Delta \lambda \quad (169)$$

$$\delta_n = 2\pi \Delta n l / \lambda \quad , \quad (170)$$

and the temperature differences are

$$\Delta T = \alpha_I I_{av} l^2 / 8K \quad (171)$$

$$\Delta T_{tr} = \Delta T (8K / C_p v l_s) (H/l) \quad . \quad (172)$$

(Recall that β_t includes both the direct and the stress-induced temperature dependence of the indices.) To achieve high efficiency (Fig. 1) the total detuning δ must be constrained, such that

$$\delta < \pi/10 \quad . \quad (173)$$

There is also a requirement on the polarization of the beam. Any depolarized light does not participate in a Type I conversion process and will reduce the conversion efficiency for Type II processes. This is a constraint on the drive beam quality such that

$$E'/E < 0.1 \quad (174)$$

The remaining constraints are mechanical and chemical. In the first place, the fluence per pulse must be less than the damage threshold of the crystal:

$$I_{pk}\tau < J_d(\tau) \quad . \quad (175)$$

Recall that J_d is the n-on-1 damage fluence and may be quite different from the single pulse values commonly used for this property. In the second place, there are two mechanical or chemical limits involving the crystal temperature. The temperature profile in the crystal is limited by fracture and the temperature at the surface by the requirement of either chemical stability or phasematching. Thus the surface fracture limit is

$$\Delta T < \Delta T_F \quad (176)$$

or

$$I_{av} < 8K\Delta T_F / \alpha_I \ell \quad , \quad (177)$$

and the cooling rate limit is

$$I_{av} < 2h\Delta T_{bl} / \alpha_I \ell \quad , \quad (178)$$

where h is the convective heat transfer coefficient, discussed in Sec. 9,

and ΔT_{bl} is the boundary layer temperature rise of the coolant.

The apertures H and H' are constrained by mechanical stability:

$$H/\ell, H'/\ell < \kappa \quad , \quad (179)$$

where κ is the maximum aspect ratio, typically around 15 for brittle materials. Also, we show in the Appendix that in one direction the aperture is constrained by diffractive contributions to $\Delta\theta$:

$$\Delta\theta > 2\lambda/\pi H^{\dagger} . \quad (180)$$

The constrained aperture is the one in the angularly sensitive direction of the crystal (see Fig. 3).

Lastly, recall that the average power density is related to the peak intensity by the duty cycle f_T :

$$I_{av} = I_{pk} f_T . \quad (181)$$

This connects the conversion efficiency with the thermal constraints and completes the set of equations determining the average power density.

7.1. THE DETUNING BUDGET

The dependence of the detuning δ on the plate spacing z_g appears in the first two of the six terms in δ . The coolant dispersion term is linearly proportional to z_g , whereas the transverse temperature term is inversely proportional to z_g . There is therefore an optimum plate separation which minimizes the total detuning. This is

$$z_g^2 = (\delta_T/Em) z_c^2/4(N-1) , \quad (182)$$

where the parameter Em is

$$Em = C_p v t_c / 32K \kappa' \quad (183)$$

Here κ' is the aspect ratio in the direction of flow. Numerical values for Em range from about 100, obtained for helium-cooled KD*P at a Reynolds number of 10^4 , to about 10, obtained for nitrogen-cooled $LiNbO_3$. For this plate separation the total detuning is

$$\delta = [(N - 1)\delta_T/Em]^{1/2} + (1/8)\delta_T + \delta_\theta + \delta_\lambda + \delta_n \quad (184)$$

The detuning constraint limits δ and therefore also limits δ_T and the average power. Solving for δ_T , we obtain

$$\delta_T = 16(N - 1/Em) \left\{ -1 + \left(1 + \frac{Em(\delta_\theta + \delta_\lambda + \delta_n)}{2(N - 1/Em)} \right)^{1/2} \right\}^2 \quad (185)$$

and the average power density is limited by thermal dephasing to

$$P_{av} = 8K\delta_T\kappa'/\alpha_I\beta_t^2 \quad (186)$$

The parameter Em is the ratio of the thermal resistances of the crystal plates and the coolant gas, using the optical coherence length to determine the width of the flow channel. It is a thermo-optical property of the flow, analogous to the Prandtl number of convective transport theory. Em determines the dependence of δ_T on δ , and the relative importance of the transverse contributions to the thermal detuning, and is a transverse figure of merit. For low Em , the plate separation is the

most important parameter in the detuning; this is associated with the coolant dispersion and the transverse temperature rise. For high Em , the longitudinal temperature rise is dominant. For all gases except helium, and most nonlinear crystals, Em will be in the moderate regime. However, for helium-cooled KD*P, Em is high (~ 100) and the longitudinal temperature rise dominates.

2.2 AVERAGE POWER

The system constraints divide naturally into two groups.

- (1) Surface temperature, surface fracture, and thermal detuning.
- (2) Damage, angular detuning, and drive (n_0).

The first set is associated with average power and the second set with single pulse performance, as discussed in Sec. 2. Figure 7 plots the average power constraints against the inverse of the thickness of a single plate, $1/l$. These constraints are

- Thermal dephasing:

$$P_{av} = \delta_T \left(\frac{8K\kappa^2}{\beta_t \alpha_I} \right) (1/l) \quad (187)$$

- Surface fracture:

$$P_{av} = \frac{8K\Delta T_F \kappa^2}{\alpha_I} \quad (188)$$

- Cooling rate:

$$P_{av} = \frac{2h\Delta T_b l \kappa^2}{\alpha_I} [1/(1/l)] \quad (189)$$

Here $\kappa^2 = \kappa \kappa'$, the ratio of beam area to l^2 . The slopes of the average power constraints are -1, 0, and 1 for the surface temperature, surface fracture, and thermal detuning, respectively. The only system variable which affects these bounds is N, the number of plates, and it affects only the thermal detuning bound for low Em. The thermal detuning is the most severe constraint for thick plates, and for thin plates the cooling rate limit is the most restrictive. In the intermediate region, the surface fracture limit may or may not be relevant, depending on the material parameters and cooling rate. For KD*P, it is apparently more restrictive than the other two constraints. The plate thickness at the intersections of the surface fracture and cooling rate bound is

$$l_1 = (4K/h)(\Delta T_F / \Delta T_S) \quad , \quad (190)$$

the surface fracture and thermal detuning bounds intersect at

$$l_2 = \delta_T / \beta_t \Delta T_F \quad , \quad (191)$$

and the thermal detuning and surface temperature bounds intersect at

$$l_3 = (l_1 l_2)^{1/2} \quad)$$

$$= 2(\delta_T K / h \beta_t \Delta T_S)^{1/2} \quad . \quad (193)$$

Figure 7. The effect of the transverse temperature rise on the required number of plates or plate thickness for low PRF or damage-limited operation.

The surface fracture limit applies only if

$$l_1 < l_2 \quad (194)$$

and for this situation represents the maximum average power per aperture. The average power is independent of l for $l_1 > l > l_2$. This gives a range of l for which P_{av} is maximum. Because the mechanical strength decreases with $1/l$, the more robust designs have the thickest plates. The optimum plate thickness therefore occurs at the intersection of the thermal fracture boundary and the thermal detuning boundary. That is, the plate thickness is l_1 , and the optimum average power per unit aperture is

$$P_{max} \approx 8K\Delta T_F \kappa^2 / \alpha_l \quad (195)$$

On the other hand, for tough materials the surface fracture limit is not relevant. The optimum plate thickness is l_3 , and the maximum average power occurs for a unique plate thickness. Then the average power is

$$P_{max} = (4\kappa^2 / \alpha_l) (Kh\Delta T_S \delta_T / \beta_t)^{1/2} \quad (196)$$

According to Fig. 7, the appropriate limit for $KD*P$ is the stress fracture limit. The surface fracture limit is about 45 KW, whereas the surface temperature limit is about 90 KW. The intersections are at $l_1 = 0.6$ mm, $l_2 = 2.5$ mm and $l_3 = 1.1$ mm, so the appropriate plate thickness is l_2 , and its aperture is 37.5 mm.

To achieve this average power in the frequency converted beam, it is necessary for the pulse length, repetition rate, and the number of plates to be compatible with the single pulse constraints. The condition that there is sufficient drive n_0 is essentially a condition on the number of plates. With

$$I_{pk} = I_{av}/f\tau \quad (197)$$

we find

$$N^2 = n_0 f\tau \kappa^2 / C^2 P_{av} . \quad (198)$$

To the extent that P_{av} depends on N through ΔT , this is a nonlinear or implicit equation for N . This will be the case if the average power is limited by thermal detuning ($\ell = \ell_1$), rather than surface fracture ($\ell = \ell_3$). The nonlinear equation for N is an eighth-order polynomial and must be solved numerically. With N determined, the total thickness is

$$L = N\ell \quad (199)$$

and the angular divergence of the drive must satisfy

$$\delta\theta < \delta_0/B_0 L . \quad (200)$$

This condition will be satisfied automatically because of the way the pulse width has been chosen.

7.3 THE ROLE OF THE DAMAGE THRESHHOLD

We have already seen that on a single-pulse basis the problem of optical damage is solved by scaling up the crystal and beam dimensions (see Sec. 2). However, thermal effects are increased as the crystal size increases; therefore, maximum average power in a single aperture is obtained with the smallest crystal dimensions compatible with avoiding optical damage. For fixed pulse energy, this is equivalent to a maximum pulse repetition frequency (PRF). At fixed average power, increasing the PRF reduces the pulse energy, and at some PRF, the pulse energy is so small that damage is no longer an issue. In this way, the role of the damage threshold is to define the PRF below which the system is damage limited, and above which it is limited by the cooling rate or fracture limits. The average power can be expressed in terms of the fluence per pulse.

$$P_{av} = f J_e (\kappa l)^2 \quad (201)$$

Then inserting the damage limit in Eq. (201) we obtain a limiting PRF f_0 :

$$f > f_0(l) \quad (202)$$

$$= P_{max} / (\kappa l)^2 J_d(\tau) \quad (203)$$

(Here l is either l_1 or l_3 , whichever is the appropriate plate thickness for the system.) At repetition rates below f_0 , the average power is determined by optical damage and not by the power-handling limits of the crystal and coolant. To illustrate this, the damage limit is shown in Fig. 7 for repetition rates of 0.1 Hz, 10 Hz, 1 kHz, and 100 kHz. Its slope is -2, and in general its location depends on f . At low f , the optimal plate thickness is given by the intersection of the damage limit with the thermal detuning bound. (This is the situation in most average power experiments to date.) This plate thickness is

$$l_4 \approx (8K\delta_T/B_t\alpha_I J_d f)^{1/3} . \quad (204)$$

As f increases, the damage limit moves upward, allowing greater average power, and l_4 decreases. At a sufficiently high repetition rate, l_4 is smaller than the larger of l_3 or l_1 , and the average power is limited by the thermal efficiency rather than the optical damage threshold. At this point, the system has changed from what is essentially a single-pulse system operated so as to deliver several pulses per second to a true average power system. In this sense, the damage limit defines the boundary between single pulse and average power systems. According to Fig. 7, this transition occurs for KD*P at $f_0 \sim 1$ kHz.

Finally, we discuss the diffraction limit on the total aperture, derived in the Appendix:

$$H' > 2\lambda\beta_\theta L/\pi\delta_\theta . \quad (205)$$

Expressing H' and L in terms of δ , this becomes a condition on $N\delta_\theta$:

$$N\delta_\theta < \kappa_s \pi \delta_\theta / 2\lambda \quad (206)$$

$$\sim 22.5 \text{ cm}^{-1} / \text{mrad} \quad (207)$$

For KD*P, this limits N to about 10. This limit (on N) is in opposition to the minimum limit on N derived from the drive condition, which gave $N \sim 5$ to 10. For situations involving other materials, these limits may be in conflict. The only way to resolve the conflict while simultaneously satisfying the damage limit on f is to decrease the pulse width τ .

8. SYSTEM DESIGN

Typically a laser is characterized by its pulse width (τ), pulse energy (E), and repetition rate (f). Thus, the total average power is

$$P_{av} = fE \quad . \quad (208)$$

If the total beam area is A , then the average power density is

$$I_{av} = fE/A \quad . \quad (209)$$

The laser beam may be either elliptical or rectangular in shape, and its size in either of the two spatial directions can be scaled up or down independently using cylindrical telescopes. Under this scaling, its angular spread is inversely proportional to the beam size. For each direction, it is characterized by a parameter Q , as follows:

$$\delta\theta = Q (\lambda/H), \quad (210)$$

where H is the beam dimension. Q is approximately the number of times the beam divergence is greater than a diffraction-limited beam, and is sometimes called the étendu of the beam. There is no reason to suppose that Q_x and Q_y are equal, in general. In the following, the spatial dimensions of the drive are referred to axes which are parallel to the angularly sensitive and insensitive directions of the crystal. The subscripts s and i are used to denote the sensitive and insensitive directions.

8.1. CONVERSION EFFICIENCY

For fixed pulse energy the peak drive intensity is inversely proportional to the square of the transverse dimension. Therefore, to maintain high conversion efficiency (i.e., maintaining η_0), the total thickness of the crystals must increase linearly with the transverse dimension. The increase of L with size is expressed as follows:

$$\begin{aligned} N^2/\kappa_s \kappa_i &= \eta_0 f_T / C^2 P_{av} \\ &= \eta_0 \tau / C^2 E \end{aligned} \quad (211)$$

Here $\kappa_s = H/l$ and $\kappa_i = H'/l$ (see Fig. 3) are the aspect ratios in the sensitive and insensitive directions, for N^2 is proportional to the beam area, $\kappa_s \kappa_i$.

The angular detuning is determined by the angular divergence in the sensitive direction. That is,

$$\begin{aligned} \delta_\theta &= (N/\kappa_s) \beta_\theta \lambda Q_s \\ &= (\eta_0 \tau / C^2 E)^{1/2} (\kappa_i / \kappa_s)^{1/2} \beta_\theta \lambda Q_s. \end{aligned} \quad (212)$$

The only variables in this expression not fixed by the laser are κ_s and κ_i ; as discussed in Sec. 2 this condition therefore fixes the beam shape (but not its size) at the harmonic generator in terms of the angular detuning. Note that the plate thickness does not appear in δ_θ .

The harmonic generator must be capable of efficient conversion for the parameters of each pulse. The detuning condition gives

$$\langle \eta_0 / \delta_\theta^2 \rangle P_{th} (\kappa_i / \kappa_s) < E / \tau Q_s^2 \quad (213)$$

where

$$P_{th} = (g_\theta \lambda / C)^2 \quad (214)$$

This parameter has the significance of a threshold. It is listed for some common materials at 1.064 μm in Table 4.

Table 4. Material figures of merit at 1.064 μm .

Material	P_{th} (MW)	J_0 (kJ/m ²)	Wm (W-mm)	ΔT_F
KD*P	76	70	500	7
LiIO ₃	7	10	[100]	
MgLiNbO ₃	1	100	21	3.8
KTP	< 1	{100}	{500}	
CD*A	1	70	50	

It is essential that the threshold power of the material be low enough to match the peak power of the laser drive.

If the peak power in the pulse is sufficient to satisfy this condition, efficient conversion is possible. The performance as regards

the angular detuning is independent of the beam size; this was discussed in Sec 2. Telescoping the beam down with the same (de)magnification m in both directions leaves the angular detuning δ_θ unchanged. If thermal effects are disregarded, the crystal size is thereby set by optical damage, and the conversion efficiency is independent of m .

On the other hand, the thermal effects are not independent of m . The fluence and heat loading increase as m^2 , the longitudinal temperature rise is independent of m , and the longitudinal dephasing decreases as $1/m$. Thus, by telescoping the beam down, a higher longitudinal temperature rise is possible, and the average power increases as m (the average power density increases as m^3). At some magnification the damage limit is reached; this is the operating point at low PRF, $t = t_d$. Table 4 lists the single pulse damage thresholds in KJ/m^2 for 1-ns pulses. The beam area must be large enough to reduce the fluence below this threshold:

$$H_{s,i} > E/J_d \quad . \quad (215)$$

Let us illustrate this using KD^*P as the material. For small-scale, commercially available lasers, where the aperture is about 1 cm, good conversion efficiency requires the peak power to be about 250Q^2 MW. Thus, with $E = 1\text{J}$ and diffraction-limited beams, a pulse length of a few nanoseconds is short enough to give high efficiency. A beam area of 1 cm^2 is sufficient to avoid damaging the crystal. For fusion lasers such as Nova with an aperture of 76 cm, the beam divergence is about $100\text{ }\mu\text{rad}$, and the étendue Q is about 70. Very high ($> 70\%$) conversion

efficiency is required, and the required power is about $1Q^2$ GW. For a pulse length of 1 ns, the required energy is 5.0 kJ, well within Nova's capability, and the beam fluence is about 10 K J/m^2 , well below the damage threshold.

8.2. APPROXIMATE SIZING OF THE HARMONIC GENERATOR

The average power density is limited by thermal dephasing on the one hand, and damage, surface fracture, or the boundary layer temperature rise on the other. The particular limit which applies will be evident from a graph such as Fig. 7. The thermal dephasing limit on this graph can be obtained approximately using

$$\delta_T = 8\delta \quad . \quad (216)$$

This is an upper bound on δ_T because all other contributions to the detuning δ have been neglected. The neglected terms all depend on the beam shape and the transverse figure of merit Em ; this approximation implies that the beam shape and other parameters have been chosen so that the other terms are small. The details of this trade-off between the various terms in the detuning will be discussed below. For the moment we will use Eq. (216) to develop an approximate procedure to size the harmonic generator.

With δ_T fixed at some fraction of δ , a graph of the average power limits defines the maximum power ($P_{\max}$) and the corresponding plate thickness ($l_{\max}$), for a given material. By reducing the average

power below $P_{\max}$, some safety margins can be introduced between the operating point (P'_{av}, l') and the regions where the material will fail. Also the plate thickness can be adjusted away from $l_{\max}$; this might be desirable in order to ease fabrication requirements.

With l' determined, the segment area is

$$A_0 = \kappa_s \kappa_i l'^2 . \quad (217)$$

and the average power density is

$$I'_{av} = P'_{av} / A . \quad (218)$$

The number of plates is

$$N = (n_0 f \tau \kappa_s \kappa_i / C^2 P'_{av})^{1/2} . \quad (219)$$

and the number of segments is

$$n_p \approx fE / P'_{av} . \quad (220)$$

We illustrate this procedure by determining an approximate design for a KP*P type II doubler for a 5-kW, 1.064- μm laser operating at 100 Hz. Its pulse width is 5 ns and it is twice diffraction-limited. Then the pulse energy is 50 J, and the peak power is 10 GW. The angular dephasing is

$$\delta_{\theta} = (\eta_0 \kappa_i / 42 \kappa_s)^{1/2} . \quad (221)$$

With $\kappa_i' / \kappa_s' = 4$ the angular dephasing is about 0.08, and most of the detuning budget δ is available for δ_T , which we take to be 0.5. These parameters will give good efficiency (50 to 70%) for Gaussian pulses.

The average power limit due to thermal dephasing is

$$P = 113 \text{ kW-mm/l} , \quad (222)$$

and at 100 Hz the maximum average power in a single segment is determined by the damage limit (see Fig. 7). It is about 25 kW, and occurs at a plate thickness of 4.5 mm. To operate at 5 kW we can choose a plate thickness of 5 mm. The number of plates is 7. This approximate calculation indicates that a single aperture of $KD \cdot P$ comprising about 5 plates about 1/2 cm thick and 40x160 mm in area will convert the given laser source with > 70% efficiency.

8.3. AVERAGE POWER DESIGN: TRANSVERSE EFFECTS

In the absence of transverse contributions to the detuning, the longitudinal dephasing δ_T is limited to δ_{T0} , where

$$\delta_{T0} = 8(\delta - \delta_{\theta} - \delta_{\lambda} - \delta_n) . \quad (223)$$

Transverse effects reduce δ_T even further by reducing that part of the total detuning available for longitudinal dephasing. The ratio $u = \delta_T / \delta_{T0}$ is the solution of

$$u + 8[u(N-1)Em/\delta_{TO}]^{1/2} - 1 = 0 \quad (224)$$

and is always less than unity. The second term depends on the number of plates and is related to u through the choice of plate thickness. The plate thickness is always determined by the intersection of the thermal dephasing limit with another constraint; that is, the damage, the surface stress, or the cooling rate boundary. Therefore, the average power density is

$$I_{av} = Wm\delta_{TO} u/l^3 \quad (225)$$

where

$$Wm = 8K/\alpha_I \beta_c \quad (226)$$

The total thickness of the plates is

$$L = (\eta_0 \delta_T / C^2 I_{av})^{1/2} \quad (227)$$

and the number of plates is

$$N^2 = (L/l)^2 = (\eta_0 \delta_T / C^2 Wm\delta_{TO})(l/u) \quad (228)$$

Depending on which other constraint is involved (besides the thermal dephasing constraint), a different set of simultaneous equations for

I_{av} , n , l , and N is obtained. The system behavior is most easily understood in terms of the motion of the thermal dephasing line in Fig. 7 as the transverse contribution varies. To illustrate this, we define the intersections of the thermal dephasing line with the other (secondary) constraints.

- Damage limit

$$l_d = (Wm\delta_{TO}/f\xi J_d)^{1/2} u^{1/2} \quad (229)$$

- Boundary layer temperature rise

$$l_{bl} = (4K\delta_{TO}/h\beta_t \xi \Delta T_{bl})^{1/2} u^{1/2} \quad (230)$$

- Surface fracture

$$l_F = (\delta_{TO}/\beta_t \xi \Delta T_F) u \quad (231)$$

These are the appropriate plate thicknesses corresponding to each limit.

In each expression a factor ξ has been introduced ($\xi < 1$), which indicates how close the operating point will be to the failure point. The appropriate plate thickness is the largest of these three lengths

$$l = \max (l_d, l_{bl}, l_F) \quad (232)$$

Each of these lengths has a different (power) dependence on δ_T .

Therefore, as δ_T varies over the range of values available to it (0

to ~ 1), different limits will be operative. For small $\delta_T \sim 0$, the damage limit applies. As δ_T increases into the higher average power region $\delta_T \sim 1$, the appropriate limit may become the boundary layer limit and eventually the surface fracture limit. Therefore, in general the plate thickness is not described by a single power law over the full range of δ_T . However, δ increases monotonically with δ_T . The situation can be envisioned by allowing the thermal dephasing line in Fig. 7 to translate parallel to the vertical axis. Of its intersections with the other three lines, the one closest to the origin gives the appropriate plate thickness as δ_T is varied. Depending on the location of the lines and the budget available to δ_T , any one of these three limits can be the most strict. As δ_T increases from zero, the strictest limit is damage, and for large enough δ_T , it is surface stress. The boundary layer limit may or may not apply in the intermediate region, depending on the flow cooling rate (given by the convection constant h) and the material constants. It becomes important for low flow velocity or high optical dispersion in the coolant.

8.4 LOW PRF OR DAMAGE-LIMITED OPERATION

In the low PRF region, by definition the average power is limited by optical damage. Inserting $\delta = \delta_d$ into the equation for N , we find

$$N \delta_T^{2/3} = (\eta_0 \epsilon_T / C W_m) (W_m / \epsilon \xi J_d)^{1/3} \quad (233)$$

$$= N_0^{2,2/3} \delta_{TO}^{2/3},$$

where N_0 is defined by this expression, and ξ (< 1) is the ratio J/J_c . Clearly, N_0 is the number of plates appropriate in the limit where transverse effects are negligible, $Em \rightarrow \infty$. The transverse effects reduce the detuning budget available to thermal dephasing; they are described by the ratio

$$y = N/N_0 \tag{234}$$

$$= (1/u)^{1/3}.$$

Because $u < 1$, N is always greater than N_0 . Inserting this into the detuning equation, we find that y is a function only of N_0 and a single combination of parameters B .

$$(1/y)^3 + 8B(1/y)^{3/2}(yN_0 - 1)^{1/2} - 1 = 0 \tag{235}$$

$$B = (1/Em\delta_{TO})^{1/2}.$$

This function is plotted in Fig. 8. For weak transverse effects, $B^2 \ll 1/N_0$, and the increase in N (or the corresponding decrease in δ), is approximately

$$N = N_0 (1 + 8BN_0^{1/2}/3) \tag{236}$$

Figure 8. The effect of the transverse temperature rise on the required number of plates (or the plate thickness) for high PRF operation where the average power is limited by the cooling rate. $B = (1/Em\delta_{T0})^{1/2}$ where Em is the transverse figure of merit.

For transverse effects to be small,

$$N_0^{1/2} B \ll 3/8 \quad (237)$$

8.5. HIGH PRF OPERATION

If the PRF is high, then optical damage is not the limiting failure mechanism. The average power is limited either by the boundary layer temperature rise, or surface fracture.

If ΔT_{b1} is the appropriate limit, then we take $t = t_{b1}$, and find

$$\begin{aligned} N_{\delta T}^{2 1/2} &= (\eta_0 f \tau / C_{wm}^2) (4K / h \beta_t \xi \Delta T_{b1})^{1/2} \\ &= N_{b1}^2 \delta_{TO}^{1/2}, \end{aligned} \quad (238)$$

where N_{b1} is defined by this expression, and ξ (< 1) describes how close the actual boundary layer temperature rise is to the limit ΔT_{b1} . The transverse effects are described by the ratio

$$\begin{aligned} y &= N / N_{b1} \\ &= (1/u)^{1/4} \end{aligned} \quad (239)$$

which is the solution of the detuning equation;

$$(1/y)^4 + 8B(1/y)^2(yN_{bl} - 1)^{1/2} - 1 = 0 . \quad (240)$$

This function is plotted in Fig. 9. For $B \ll 1$

$$N = N_{bl} (1 + 2B N_{bl}^{1/2}) \quad (241)$$

If surface fracture is the appropriate limit, then we take $l = l_F$, and find

$$N^2 = (n_0 f_{\tau a_L} / 8KC^2 \xi \Delta T_F) \quad (242)$$

$$= (n_0 f_{\tau} / C^2 W_m) (l \beta_T \xi \Delta T_F) \quad (243)$$

$$= N_F^2 , \quad (244)$$

where ξ (< 1) describes how close the surface is to fracture. There are no transverse effects in this case.

Figure 9. The effect of the transverse temperature rise on the required number of plates (or the plate thickness) for high PRF operation where the average power is limited by the cooling. $B = (1/E_m \delta_{TO})^{1/2}$ where E_m is the transverse figure of merit.

9. FLOW COOLING

The choice of coolant is determined by two parameters: the transverse temperature rise, and the surface temperature of the crystals. The transverse temperature rise is controlled by Em , and the surface temperature is determined by coefficient h , or equivalently by the Nusselt number, Nu . The performance improves as Em and Nu increase.

If the flow is laminar, then only the boundary layer contributes to the heat removal, whereas for turbulent flow, the entire volume of gas can carry heat away [Ka66]. It is, therefore, necessary that the flow be turbulent.

To be most efficient the flow must develop maximal thermal gradients to conduct heat from the crystal surfaces into the body of the gas. If these gradients are not already present in the gas as it comes into contact with the crystal, then some of the heat flow into the gas is used to establish the thermal gradients. The heat necessary to do this varies as the gas traverses the crystal surface and the gradients develop. Consequently, the temperature at the crystal surface rises faster than the mixed mean temperature of the gas. This results in nonuniform conductance into the gas and increases the transverse temperature difference across the crystal face. The thermal efficiency is, therefore, maximized for fully developed flows.

The Reynolds number is

$$Re = \frac{\rho v l}{\mu}$$

(245)

$$= \frac{16 K k}{k Pr} t [Em(N-1)\delta_T]^{1/2},$$

where k is the thermal conductivity of the gas and Pr is the Prandtl number. For KD*P cooled with helium at 1 atm pressure flowing at 220 m/s, the Reynolds number is 10^4 . For nitrogen at 1 atm, the velocity required to reach a Reynolds number of 10^4 is 410 m/s. Hydrogen is similar to helium, and most other simple gases are similar to nitrogen.

The Nusselt number for heat transport between parallel plates has been determined by Kays and Leung [Ka63]. For $Pr = 0.7$, and Reynolds numbers near 10^4 , they found

$$Nu = 36 [Re/10^4]^q$$

(246)

where q is approximately 0.72. The heat transfer coefficient is

$$h = kNu/l_s$$

(247)

and for helium cooled KD*P, at 1 atm and $Re=10^4$, we find

$$h = 1 \text{ KW/m}^2/\text{K}.$$

(248)

The heat transfer coefficient varies slowly with the material constants.

$$h \approx (k^{0.28}) (c_p^{0.86}) (1_c^{-0.14}) . \quad (249)$$

Thus, for a given flow velocity and pressure, the heat transfer is only weakly dependent on the optical and thermal constants of the gas. Under these conditions the cooling rate is behaving approximately as though it depended only on the number of molecules per unit volume, and not on their individual optical and thermal properties.

The power required to set up the flow depends on the mass density and the viscosity. The friction factor f_r relates the pressure head to the energy density of the gas,

$$\Delta P = f_r \frac{1}{2} \rho v^2 . \quad (250)$$

The friction factor depends on the surface roughness of the flow channel, and on the Reynolds number [Mo44]. For turbulent flow in the region $2000 < Re < 10000$,

$$f_r = f_0 Re^{-1/4} . \quad (251)$$

The dependence on Reynolds number is clearly weak. On the other hand, depending on the surface roughness, f_0 can vary by several orders of magnitude [Mo44, Kr63]. It is typically less than 1; it is the key parameter in reducing the power required to maintain the flow.

9.1. DEPENDENCE OF AVERAGE POWER ON FLOW VELOCITY

The conductance h is comparatively insensitive to the coolant gas parameters but is quite sensitive to other system variables, such as the pressure and flow velocity. The cooling rate depends on the plate separation, which is determined by the trade-off between the optical dispersion in the gas, and the transverse temperature rise. The plate separation, and therefore the coolant rate h , will therefore depend on the thermal dephasing δ_T . Inserting the expression for the plate separation into h , we find that

$$h = h_0 ((N-1)\delta_{TO}/\delta_T)^{0.14} (v/v_0)^{0.86} \quad (252)$$

where v_0 is the velocity of sound in the gas. Typically

$$h_0 = 1 \text{ KW/m}^2/\text{K} . \quad (253)$$

The dependence of h on the thermal dephasing is weak, but it is almost linear in the flow velocity. Therefore, as the flow velocity is reduced, two effects combine to reduce the average power. The transverse temperature rise increases as $1/v$, and the cooling rate decreases as $v^{0.86}$. At some velocity, the Reynolds number drops below 2000, and the flow is no longer turbulent; this represents a sharply defined lower bound on the flow velocity.

9.2 LOW PRF OPERATION

For low PRF, the average power density is limited by damage. As v is reduced, the transverse parameter B increases as $1/v$, and the number of plates increases. For $B \ll 1$

$$N/N_0 = 1 + 8B/3 + O(B^2) \quad (254)$$

whereas for $B \gg 1$

$$N/N_0 \sim 8B \quad (255)$$

Thus N increases approximately as $1/v$. The plate thickness decreases in exactly the same manner, so that the total crystal thickness is constant. The average power density is determined by the fluence, J .

$$I_{av} = f\xi J \quad (256)$$

The flow velocity can be reduced without reducing the average power density. The plate thickness decreases with v because less of the detuning budget is available to longitudinal thermal dephasing. The area also decreases with v . Therefore as v decreases, the average power through the segment decreases, but the power density remains constant.

9.3 HIGH PRF OPERATION

For surface-fracture-limited operation, the number of plates required is independent of the transverse temperature rise, and is therefore independent of the flow velocity. Then $N = N_F$ and the plate thickness is

$$l = (\delta_{TO}/\beta_t \Delta T_F)u \quad , \quad (257)$$

where

$$u + 8B [(N_F - 1)u]^{1/2} - 1 = 0 \quad . \quad (258)$$

As v is reduced, B increases and for $B \gg 1$, both u and l are proportional to v . The average power density is

$$\begin{aligned} I_{av} &= 8K\Delta T_F/\alpha_l l^2 \\ &= I_{av}(B=0)u^{-2} \quad . \end{aligned} \quad (259)$$

For $B \gg 1$ the average power density is proportional to $1/v^2$. The total average power through the segment is independent of l , but l decreases with v because less of the detuning budget is available for longitudinal thermal dephasing. Thus, the power density increases, whereas the power through the entire aperture remains the same.

As v is reduced the boundary layer temperature rise increases to preserve the rate of heat removal. At some point, the boundary layer temperature rise reaches its limit, and the average power density is limited by the cooling rate rather than damage or surface fracture.

The equation for N under cooling-rate-limited operation is

$$N^2 \delta_T^{1/2} = (\eta_0 f \tau / C^2 W m) (4K / h \beta_t \xi \Delta T_{bl})^{1/2} \quad (260)$$

$$= N_{bl}^2 \delta_{TO}^{1/2},$$

where N_{bl} is defined by this expression, and ξ (> 1) describes how close the actual boundary layer temperature rise is to the limit ΔT_{bl} . The plate thickness is

$$L = (4K \delta_{TO} / h \beta_t \xi \Delta T_{bl})^{1/2} u^{1/2} \quad (261)$$

and average power density is

$$I_{av} = W \delta_{TO} (h \beta_t \xi \Delta T_{bl} / 4K \delta_{TO})^{3/2} u^{-1/2} \quad (262)$$

where

$$u + 8Bu^{1/2}(u^{-1/4}N_{bl} - 1)^{1/2} - 1 = 0 \quad (263)$$

If the transverse thermal dephasing is relatively small, so that $B \ll 1$, then $u = 1$ and, using

$$h = h_0 v^2 (u/N - 1)^p \quad (264)$$

we find

$$L \sim v^{-0.415} \quad (265)$$

$$I_{av} \sim v^{1.246} \quad (266)$$

and

$$P_{av} \sim v^{0.415} \quad (267)$$

The average power per aperture drops relatively weakly with v , and the plate thickness (or beam aperture) increases. The power density therefore varies quite rapidly. In the low velocity limit where transverse effects are dominant, $B \gg 1$ and therefore

$$u = (8B)^{-8/3} \quad (268)$$

Then

$$L \sim v^{0.476} \quad (269)$$

$$I_{av} \sim v^{-0.098} \quad (270)$$

and

$$P_{av} \sim v^{1.049}.$$

(271)

In this case, the total power per segment varies as almost linearly with v .

In general, as the velocity is reduced the power per aperture decreases between $v^{0.42}$ and $v^{1.05}$. The effective power of v increases with decreasing v , depending on relative size of the transverse dephasing given by E_m . E_m also determines the behavior of the plate thickness and power density.

The behavior of the system as v is reduced can be envisioned in terms of the motion of the boundary lines in Fig. 7. Both the damage boundary and the surface fracture boundary are independent of the flow velocity. The cooling rate boundary drops as $v^{0.86}$, and the thermal dephasing boundary drop more or less rapidly depending on the extent to which transverse effects are important (i.e., the parameter B).

In summary, for low PRF the average power density is independent of the flow velocity until the cooling rate limit is reached. On the other hand, the beam area and the plate thickness decrease to avoid excessive thermal dephasing. Therefore, the power per aperture decreases as the flow rate is reduced. Asymptotically, the power per aperture is proportional to v^2 ; however, the cooling rate limit may be reached before this behavior can be seen. For surface fracture in the high PRF region, the average power is limited either by surface fracture or boundary layer temperature rise. The power per aperture is independent

of the flow velocity. whereas the power density increases as the plate thickness decreases to avoid thermal dephasing. In this case, also the asymptotic dependence of the area is v^2 .

If the flow velocity is sufficiently small that the cooling rate determines the average power, then the power per aperture drops as $v^{1.05}$, the beam area as $v^{0.95}$, and the power density as $v^{-0.10}$.

10. CONCLUSIONS

The purpose of this study was to assess solid state nonlinear materials for high-average-power applications. The current state of the art in high average power is a few tens of watts. It has been shown here that solid state nonlinear materials can in fact surpass the current state of the art by at least three orders of magnitude. Moreover, configurations exist which permit essentially unconstrained scaling of average power.

In average power operation, the linear optical absorption in the crystals causes temperature gradients which can reduce the conversion efficiency by disturbing phase-matching. If nonlinear crystals were perfectly transparent, there would be no special aspects to average power harmonic generation. Design would then proceed along the lines outlined in Sec. 2, and average power would cease to be an issue. However, most nonlinear materials are not sufficiently transparent that thermal effects can be ignored. An absorption of 0.1 m^{-1} is often large enough to be significant.

Because the interest is in high average power, intrinsically unscalable geometries such as tight focusing were not considered. A careful assessment of the heat flow in the nonlinear crystal (Sec. 3) showed that the thermal gradients must be perpendicular to the crystal faces for efficient, scalable heat removal. The optimum geometry uses plane waves and a number of crystal plates with a coolant gas flowing between them to remove the heat deposited as a result of optical

absorption. The gas flow is high velocity and turbulent for efficient heat removal. By dividing a large aperture into segments (which was successfully demonstrated on Nova)[Si84] and optimizing each segment, the total average power in the entire aperture is essentially unconstrained. Each segment consists of several plates of single crystals through which the beam passes consecutively, as shown in Fig. 4. The average power in each segment is constrained by thermal dephasing, surface fracture, surface temperature, and optical damage. Each of these effects was analyzed in detail. It was found (see Fig. 7) that a single segment of a generator using KD*P to double $1.064\text{ }\mu\text{m}$ is ultimately capable of average powers approaching 100 kW. With a relatively modest number of segments, very high average powers can be envisioned for the total aperture.

It is worth noting that for doubling wavelengths below $1.064\text{ }\mu\text{m}$, the absorption of KD*P is considerably smaller than at $1.064\text{ }\mu\text{m}$ (0.6 m^{-1}). Therefore, the ultimate average power per segment for doubling to the blue or ultraviolet regions is much higher than 100 kW. A precise value cannot be given without accurate absorption data; however, a value typical of transparent window materials, 0.03 m^{-1} , permits the ultimate average power per segment to be greater than 1 MW.

The temperature gradients cause thermal dephasing of the waves and disturb phase-matching. There are two contributions to the temperature variation in the crystal (Fig. 5). One is the longitudinal temperature rise, associated with the flow of heat outwards to the crystal surfaces. The other is the transverse temperature rise, associated with the rise in

temperature of the coolant as it traverses the crystal face. Now, the total detuning budget is constrained by the requirement of high conversion efficiency, as described in Sec. 2. It contains six terms. These are the longitudinal and transverse temperature rise, the optical dispersion in the coolant the angular divergence and linewidth of the drive, and the index inhomogeneity in the crystals. Of these, only two depend on the separation between the plates; these are the optical dispersion in the coolant, and the transverse temperature rise. The separation between the plates is therefore uniquely determined by the trade-off between these two terms. With the plate separation fixed the thermal part of the total detuning then includes only two terms: the longitudinal temperature rise and the optimized combination of the optical dispersion and transverse temperature rise terms. Because the average power is related to the longitudinal temperature rise, highest average power is obtained by minimizing the transverse terms. The transverse terms are given by the transverse figure of merit E_m :

$$E_m = C \frac{v_L}{p_c} / 32 K_{\kappa_t} \quad (261)$$

and their relative size is given by B/N , where

$$B/N = (N/E_m \delta_{T0})^{1/2} . \quad (262)$$

Here N is the number of plates that would be required if the transverse terms were zero, and

$$\delta_{TO} = \delta - \delta_{\theta} - \delta_{\lambda} - \delta_n$$

(263)

is that part of the total detuning budget available to the longitudinal dephasing, assuming that the transverse terms are zero. The effect of the transverse terms is of the order of E/N .

The transverse effects are treated quantitatively in Sec. 8, and summarized in Figs. 8 and 9. Referring back to Fig. 7, the effect of the transverse terms can be visualized as a downward translation of the thermal dephasing line.

The precision with which the flow velocity must be controlled was addressed in Sec. 9. As the velocity of the coolant is reduced, both the cooling rate h and the transverse figure of merit Em decrease. The effect can be visualized in Fig. 7 as a downward translation of both the thermal dephasing and the boundary layer temperature rise lines. These constraints depend in different ways on the flow velocity; in some situations their dependence on v can be described approximately, with an effective power law (Sec. 9). This power is generally not large; depending on which effect is limiting it, the average power is either independent of the flow velocity, or drops moderately as v is reduced. At some point the Reynolds number becomes low enough for the flow to become laminar. At this point the average power drops discontinuously with v by at least an order of magnitude.

Overall, the prognosis for nonlinear optics at high average power is very encouraging. The heat load in a nonlinear material with moderate optical absorption and nominal thermo-mechanical properties can be handled gracefully with turbulent gas cooling. This conclusion is not limited to second harmonic generation, but appears to generalize to any (optical energy conserving) process involving frequency conversion. If a nonlinear material with nominal properties is capable of high efficiency on a pulse-by-pulse basis, then it will also scale to high average power.

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LIST OF SYMBOLS

A	beam area
a	size of surface defect
a_i	eigenmode polarization
α_I	linear optical absorption coefficient
α_{ij}	thermal expansion tensor
α_T	thermal expansion coefficient
B	parameter giving transverse effect on average power
b	width of boundary layer carrying heat
b_i	eigenmode polarization
β_s	Stress sensitivity of crystal
β_t	total temperature sensitivity of crystal
β_T	intrinsic temperature
β_θ	angular sensitivity of the crystal
C	nonlinearity of the material
c	velocity of light
C_{ij}	boundary condition
C_p	specific heat per unit volume of coolant gas
δ	total detuning
δ_{eff}	effective total detuning at high drive
δ_g	detuning due to dispersion in coolant
δ_r	detuning due to angular divergence
δ_{tr}	detuning due to transverse divergence
δ_λ	detuning due to frequency bandwidth
δ_n	detuning due to index inhomogeneity
δ_T	detuning due to longitudinal temperature rise

δ_{T0}	maximum detuning available to δ_T
$\delta\theta$	beam divergence
Δk	wavevector mismatch
ΔT	longitudinal temperature rise (see Fig. 5)
ΔT_{bl}	boundary layer temperature rise
ΔT_f	temperature rise for surface fracture
ΔT_{tr}	transverse temperature rise (Fig. 5)
ϵ_{ij}	dielectric tensor
ϵ_{ijk}	antisymmetric tensor of rank 3
ϵ_0	permittivity of free space
η	conversion efficiency
η_{ij}	strain tensor
η_0	dimensionless drive intensity
E	pulse energy
Em	transverse figure of merit ($C_p V t_c / 32 K \kappa_t$)
E_1, E_2, E_3	electric fields of interacting waves
$\vec{E}, \vec{B}, \vec{D}, \vec{H}$	electromagnetic fields
f	pulse repetition frequency (PRF)
f_r	friction factor
Λ	spatial wavelength
λ	wavelength
H	transverse dimension of beam or crystal
h	convective heat transfer coefficient
I	intensity
I_{av}	average power per unit area
I_{pk}	peak intensity per pulse

J	fluence
J_d	damage fluence
J_0	damage fluence per pulsewidth τ_0
K, K_{ij}	thermal conductivity, -tensor
k	thermal conductivity of gas
κ	aspect ratio
l	thickness of individual plate (N/L)
l_c	coherence length of coolant gas
l_s	plate separation
L	total thickness of the crystal
M_{ij}	matrix for unperturbed modes
μ	viscosity of gas
N	number of plates
Nu	Nusselt number
n	refractive index
$\mathbf{o}, \mathbf{e}$	unit vectors in crystal faces
Ω	depolarization angle
ω	angular frequency
p	diffraction parameter
P_{av}	average power
P_{pk}	peak power per pulse
ϕ	angle describing eigenmodes in biaxial media
Q	beam quality (étendu)
q	heat flow vector
q_{ijkl}	stress-optic tensor
Re	Reynolds number
ρ	density

S, S_{ch}	fracture toughness, stress
S_0	surface strength
S, C	$\sin\theta, \cos\theta$
s, c	$\sin\phi, \cos\phi$
s	distance along streamline
S_{kl}	stress tensor
S_{ijkl}	compliance tensor
T	temperature
τ	pulsewidth
τ_0	reference pulsewidth
U	moments of temperature distribution
u	ratio of δ to δ_{TO}
u_i	displacement vector
ν	Poisson's ratio
V_{ij}	matrix for perturbed modes
v	flow velocity of coolant gas
v_0	velocity of sound in gas
W	volumetric heat loading in medium
w	total dephasing in the crystal
W_m	thermal dephasing parameter ($8K/\alpha_1/\delta_t$)
x	transverse coordinate
χ, ϕ	phase mismatch
ξ	safety margin
ξ_{ij}	constant relating stress and temperature profiles
Y	Young's Modulus
z	longitudinal coordinate in crystal

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APPENDIX: DIFFRACTIVE CONTRIBUTIONS TO ANGULAR DEPHASING

A beam with a finite aperture does not have a well-defined direction. Typically the diffraction angle associated with a particular aperture size can be estimated as follows:

$$\theta = 1/k_0 a \quad , \quad (A-1)$$

where a is the aperture, and k_0 is the wavevector of the light. When such a beam passes through an anisotropic crystal, it diffracts and also acquires a direction-dependent phase. When this occurs in a nonlinear crystal, both of these effects must be included to obtain the correct conversion efficiency. Although a general solution to this problem is not available, there is a simple model in which an exact and transparent solution exists.

Consider type II second harmonic generation in a crystal of thickness l using a beam with a flat spatial profile of width a . (The beam is nonzero between $x = -a/2$ and $x = +a/2$.) This beam can be expressed as a sum over plane waves:

$$E = E_0 \int d\kappa \, h(\kappa) \exp(-i\kappa x + i\kappa_z z) \quad , \quad (A-2)$$

where

$$h(\kappa) = \sin(\kappa a/2)/\pi\kappa \quad , \quad (A-3)$$

and the longitudinal wavevector is

$$k_z^2 = -\kappa^2 + (n(\kappa)\kappa_0)^2. \quad (A-4)$$

Here k_0 is $2\pi/\lambda$, the free space wavevector. Each plane wave is an exact solution of Maxwell's equations in the crystal, and its index of refraction depends on its direction. The dipole moment density in the crystal at the second harmonic is proportional to E^2 ; it is a sum over every pair of waves in E.

$$P = \chi E^2 \int d\kappa d\kappa' p(\kappa, \kappa') \exp(-i(\kappa + \kappa')x) (\sin(\kappa a)/\kappa) (\sin(\kappa' a)/\kappa') \exp(i(k_z + k'_z)z). \quad (A-5)$$

In this expression we shall take κ to refer to the ordinary wave and κ' to refer to the extraordinary wave. Each component of the dipole moment density radiates a wave at the second harmonic, with transverse wavevector $\kappa + \kappa'$. The radiated wave depends on the noncollinear phasematching for that pair of waves. Assuming that the waves $\kappa = \kappa' = 0$ are exactly phasematched, the phase mismatch for any other pair is

$$\Delta k = \kappa \frac{\partial n_o}{\partial \theta} + \kappa' \frac{\partial n_e}{\partial \theta} - (\kappa + \kappa') \frac{\partial n_e}{\partial \theta}. \quad (A-6)$$

Now the ordinary index is independent of direction and to a very good approximation the derivative of the extraordinary wave is independent of frequency. Thus

$$\Delta k = \beta_\theta (\kappa/k_0), \quad (A-7)$$

where β_θ is the angular sensitivity of the collinear type II process. Note that Δk is independent of κ' , the transverse wavevector of the extraordinary wave. The radiated second harmonic wave is then

$$E'(x) = CE(x)E_0 f(x) , \quad (A-8)$$

where

$$f = \int \frac{\delta v \sin v \sin p v \cos(2xv/a)}{p\pi v^2} , \quad (A-9)$$

and the parameter p is the ratio of the diffraction to the detuning angle.

$$p = \beta_\theta L/k_0 a \quad (A-10)$$

The integral has an exact solution (GR, p414).

$$f = 1/2p (\text{Min}(p, 1 + 2x/a) + \text{Min}(p, 1 - 2x/a)) . \quad (A-11)$$

Because the second harmonic is proportional to $E(x)$, which is zero for $|x| > |a/2|$, all the arguments in f may be treated as > 0 .

Consider the center point of the beam $x = 0$. For $p < 1$, we have $f = 1$, and the second harmonic is

$$E' = CE^2 L, \quad [x = 0, p < 1] . \quad (A-12)$$

However, for $p > 1$, $f = 1/p$, and the second harmonic is

$$E' = CE^2 L/p, \quad [x = 0, p > 1] \quad (A-13)$$

Thus, in the center of the beam, the conversion efficiency is independent of the aperture size unless the diffraction angle is greater than the angular acceptance of the crystal, when it drops linearly with the aperture size.

At the edge of the $[x = a/2, p < 2]$, the second term of f is always zero. Thus f is always less than $1/2$ there. For $p < 2$ we have

$$E' = CE^2 L/2 \quad [x = a/2, p < 1] \quad (A-14)$$

whereas for $p > 2$,

$$E' = CE^2 L/p, \quad [x = a/2, p > 2] \quad (A-15)$$

Thus near the edge of a square beam, the conversion efficiency is always $1/4$ of the efficiency in the center.

At an intermediate position in the beam, there are three regions defined by $p = 1 \pm 2x/a$. We have

$$E' = CE^2 L, \quad [p < 1 - 2x/a] \quad (A-16)$$

$$E' = CE^2 L/p, \quad [1 + 2x/a < p] \quad (A-17)$$

and in the intermediate region

$$E' = CE^2 L \frac{1}{2} (1 + (1 - 2x/a)/p) \quad [1 + 2x/a < p < 1 - 2x/a] \quad . \quad (A-18)$$

Diffraction affects some locations in the beam and leaves others unchanged. The beam profile therefore depends on the diffraction parameter, p . For $p > 2$, diffraction affects all portions of the beam the same, and again the second harmonic is flat. The conversion efficiency is reduced by $1/p^2$. For all smaller values for p , the efficiency is reduced by a factor which is bounded by both 1 and $1/p^2$, and for some parts of the beam it is less than these bounds. Recall that at the edge of the beam the efficiency is always reduced by 0.25, for any value of p other than exactly zero. For values of p less than 1, the conversion efficiency is unaffected by diffraction for

$$x < x_p = a/2(1 - p) \quad . \quad (A-19)$$

However, between x_p and $a/2$, the efficiency drops quadratically to 1/4 of its value in the center:

$$\eta = \eta(0) = C^2 I^2 L^2; \quad x < x_p \quad (A-20)$$

and

$$\eta = \eta(0)/4 [1 + (1 - 2x/a)/p]^2; \quad x > x_p \quad . \quad (A-21)$$

The average conversion efficiency is reduced by the following amount:

$$R = 1 - 5p/12 \quad .$$

(A-22)

Thus to avoid significant reduction in the conversion efficiency, the diffraction parameter must be small. Choosing $R > 0.9$, we find

$$p < 0.25 \quad . \quad (A-23)$$

In terms of the aperture a , this is

$$a > (2/\pi) \pi_0 \lambda \quad .$$

(A-24)

This is the limit used in the text.