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General Reactor Sizing Techniques
Volume I
Aerothermodynamic Optimization

W.Z. Prickett

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VOLUME I
PERFORMANCE OPTIMIZATION OF NUCLEAR POWER SYSTEMS
OPERATING ON THE BRAYTON GAS CYCLE

W. Z. Prickett

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ABSTRACT

This work presents a method for the aerothermodynamic optimization of the net power and/or propulsive thrust per unit reactor free flow area of a nuclear power plant operating on the Brayton cycle. The value of this technique is that a system so optimized will translate into minimum size, therefore the minimum weight, nuclear system for any selection of reactor materials, lifetime and fuel loading. Integrated herein are the theory and development of the thermodynamic optimization process, the importance and effect of various parameters and specific methods to be employed in the optimization of the various forms of the Brayton cycle. A sample calculation for the case of the ramjet application is included.

The results of the application of these techniques to any Brayton cycle system may be used in conjunction with the nuclear sizing methods of Volume II of this work, for beryllia-moderated reactors, to determine the required reactor size as a function of fuel loading and reactivity requirements. Volume II is published under separate cover as APEX-723B (Confidential RD).

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1.0 Introduction

This work presents a method for optimizing of the net power and/or propulsive thrust per unit reactor free flow area of a nuclear power plant operating on the Brayton cycle. The value of this technique is that a system so optimized with respect to thermodynamics will translate into the minimum size, therefore the minimum weight, nuclear system for any selection of reactor materials, lifetime and fuel loading. Although this method has been developed specifically for nuclear systems, the technique is independent of whether the heat source is nuclear or electrical.

This work presents: (1) the theory and development of the thermodynamic optimization process; (2) the importance and effect of the various parameters; and (3) the specific methods to be employed in the optimization of the various forms of the Brayton cycle, whether ramjet, turbojet, turboprop, or closed cycle. A specific example is presented for the ramjet system.

This work is self-contained in that all required calculation procedures and their derivations are included. Its intended application is in the area of preliminary design as an aid for delineating the general area in which the final aerothermodynamic design should be concentrated. Its use in conjunction with Volume II of this report permits integration of the aerothermodynamic optimization with the nuclear characteristics of beryllia-moderated reactors and thus the delineation of the general area in which final nuclear analysis should be concentrated.

Familiarity with the characteristics of the Brayton cycle and with compressible fluid flow in a constant area duct with heat addition and friction as described in References 1 through 3 is assumed.

2.0 Thermodynamic Optimization

2.1 Principle

The basic concept in the optimization of the nuclear Brayton cycle is that peak performance per unit reactor flow area will be attained when the product of net specific thrust (or work) and the reactor flow rate per unit flow area is a maximum. It is a rule that this maximum does not occur when either or both components of the above defined product is a maximum. This optimization process is found to be characteristic of reactor systems which, in general, require large pressure drops relative to those encountered in the heat addition region of chemical systems.

It follows directly that once the system has been optimized with respect to its thermodynamics in terms of net power or thrust per unit flow area, the flow area requirements for a required power or thrust level is readily determined. For the straight-through uniform flow area system, this flow area may then be converted into a reactor size when the nuclear requirements and materials of the system are known.

2.2 The Nuclear Brayton Cycle

The ideal Brayton cycle consists of an isentropic compression, a constant-pressure heat addition, an isentropic expansion, and a constant-pressure heat rejection. Compression is accomplished by an axial or centrifugal compressor, aided to varying degrees by the ram effect of the kinetic energy of the incoming air. Work required for compression is extracted from the working fluid during the expansion process by means of a turbine or a nozzle or both. The remaining energy is available for useful work, and as such, may also be extracted by use of a turbine and/or a nozzle as shaft power and/or propulsive thrust, depending upon its end application. The real Brayton cycle consists of these same processes as modified by the efficiencies of the various components.

Consider the ideal Brayton cycle which is defined on a "per pound of working fluid" basis. It may be shown⁽¹⁾ that the ideal Brayton cycle efficiency is given by the relation:

$$\eta_{th} = 1 - \left(\frac{1}{r} \right)^{\frac{k-1}{k}}$$

where r denotes the cycle pressure ratio. Examination of this relation shows that the efficiency increases as the cycle pressure ratio increases. It may be further shown⁽¹⁾ that the cycle net specific work, Δh_n , is a function both of the cycle pressure ratio, r , and the cycle temperature ratio, α , and that for each value of α there is a optimum pressure ratio, r_o , which will yield the maximum net specific cycle work. The condition for this optimum for the ideal cycle is found to be:⁽¹⁾

$$(r)^{(k-1)/k} = \sqrt{\alpha}$$

Thus, it is always possible to develop a cycle with the greatest net available energy per pound of working fluid since the temperature limitations imposed by materials considerations will dictate the required pressure ratio for this optimum output. However, when the Brayton cycle is employed in conjunction with a nuclear heat source, considerations of the reactor's heat transfer and fluid dynamic characteristics readily illustrate that optimization of the net specific cycle work is not adequate to guarantee the desired maximum power per unit active-core free-flow area.

Since heat transfer and fluid dynamics considerations must enter into consideration because (1) fluid temperatures cannot be made equal to limiting materials temperatures from heat transfer considerations and (2) flow rate per unit flow area is affected by pressure level, fluid temperature rise and pressure loss allocated to the heat transfer process.

The general thermodynamic correlation (Appendix A) readily illustrates that the ratio of the fluid temperature drop to the fluid temperature rise uniquely determines the fluid flow parameter, $L/\bar{D}^{1.2}$ where L denotes the heated length and $\bar{D}$ denotes the hydraulic diameter for heat transfer. It may be further shown (Appendix B) that the actual pressure loss in the reactor heat transfer matrix is affected to a large extent by friction in the form:

$$\frac{P_3 - P_4}{\rho} \propto \frac{L}{\bar{D}^{1.2}}$$

where: P_4 denotes the exit fluid total pressure
 P_3 denotes the inlet fluid total pressure
 q denotes the average dynamic head
 f denotes the smooth-tube friction factor

Thus, noting that the reactor flow rate may be expressed in terms of the dynamic head, it is seen that this function is affected by the choice of the ratio of the film temperature drop to the fluid temperature rise, if so desired, all other factors being constant. Similarly, for a fixed selection of this temperature difference ratio, the reactor flow rate will be affected by the allowable total pressure loss, again all other factors being constant.

The reactor total pressure loss may be also readily expressed as a function of the reactor flow function, $G\sqrt{T_3}/P_3$ (Appendix B). A fixed value of the film temperature drop - temperature rise ratio and the total pressure loss will result in a unique geometry factor, $L/DH^{1.2}$, and a particular reactor flow function, $G\sqrt{T_3}/P_3$. Thus, all other parameters constant, the flow function will be constant, resulting in a direct increase in flow rate per unit flow area, G , with increasing system (reactor inlet) pressure level, P_3 , and similarly, a decrease in G as the square root of the system temperature level, T_3 .

These considerations readily illustrate the necessity of the combined optimization of the reactor aerothermal dynamics and the power cycle. The method is now presented for accomplishing this in terms of the thermodynamic boundary conditions of the power cycle as well as the interrelation between the net cycle specific work and the reactor flow rate per unit flow area as a function of these boundary conditions.

2.3 Parametric Groupings for Optimization

The choice of the parametric grouping to be optimized will depend upon whether the cycle application involves propulsion or static power supply. This differentiation is necessary because the velocity of the airframe affects the net thrust of a propulsion system, while no such effect exists in a stationary system. For example, in the operation of a ramjet, no thrust is generated either at zero velocity or when the airframe velocity is equal to the exit velocity of the exhaust gases; maximum net thrust exists at some intermediate flight speed. In the static turboshaft power systems, performance is independent of such a parameter and the optimization procedure is different from that for propulsion systems.

Hence, two separate parametric groupings have been formulated: (a) the net thrust coefficient based upon the reactor free flow area, C_{FA} , for the propulsion systems, and (b) the net power coefficient based upon the reactor free flow area, C_{PA} , for the static power supply systems. These coefficients are defined by the following relations:

$$2.1 \quad C_{FA} = \frac{F_n}{A_{ff} q_0}$$

$$2.2 \quad C_{PA} = \frac{W}{A_{ff}} \Delta h_n$$

where:

F_n denotes the net thrust, lbs
 A_{ff} denotes the reactor free flow area, inch²
 q_0 denotes the free stream dynamic head, lbs/inch²
 W denotes the reactor airflow rate, lbs/sec
 Δh_n denotes the cycle net specific work, Btu/lb

While the thrust coefficient is dimensionless, the power coefficient has the units of Btu/sec-inch² or power per unit flow area. It has been found convenient to formulate the thrust coefficient in terms of the free-stream dynamic head, q_0 , since it then has the same parametric and altitude dependence as the drag coefficient and is therefore equal to the drag coefficient except for the difference in area base. The base area of a cylindrical body drag coefficient is the maximum cross sectional area of the body, which, for a nuclear system, will be generally proportional to (1) the reactor free flow area as dictated by thermodynamic considerations and desired thrust level and (2) the required solid fraction of the system as dictated by nuclear considerations. Thus, when the net thrust per unit reactor flow area based on the dynamic head is optimized, the power plant diameter and therefore the power plant drag based upon the same dynamic head will be a minimum. It must be understood that in general, other airframe drag contributions such as wing drag, with other base areas such as wing area, will exist and that therefore the net thrust coefficient must always exceed the body drag coefficient.

The most significant effect of the dynamic head is that it introduces an altitude dependence into the computation of the net thrust per unit flow area (which is not, in general, required for stationary closed cycle systems employing the power coefficient for optimization purposes). This is illustrated by rearranging equation 2.1 as follows:

$$C_{FA} = \frac{F_n}{A_{ff} q_0} = \frac{W}{A_{ff}} \cdot \frac{I_{sp}}{\frac{1}{2} \rho V_0^2} = \frac{\rho V_0 I_{sp}}{\frac{1}{2} \rho V_0^2} = \frac{2 I_{sp}}{V_0} = \frac{2 I_{sp}}{\sqrt{\gamma g R T_0}}$$

If, for simplicity, the case of the fully expanded nozzle is considered, the net thrust coefficient is independent of ambient pressure and varies only inversely as the square root of the ambient temperature. Thus, the net thrust coefficient is seen to be a function of altitude except in the region of the tropopause where it will be a constant for any given set of boundary conditions. It is in this region that the effect of dynamic head upon the net thrust per unit flow area is best demonstrated. As will be shown in the example problem, one of the results of the optimization study, in addition to the optimum net thrust coefficient, will be a fixed value of the free stream Mach number. Consideration of equation 2.1 for constant C_{FA} and M_0 shows that the net thrust per unit flow area will decrease with increasing altitude in the tropopause because the ambient pressure and therefore the free-stream dynamic head ($k p_{s0} M_0^2/2$) decreases. In the tropopause and other regions of the atmosphere where the ambient temperature is not constant, both the net thrust per unit flow area and the optimum net thrust coefficient will depend upon the altitude under consideration.

It is now found to be convenient to introduce the reactor flow function, $G\sqrt{T_3}/P_3^*$ into equations 2.1 and 2.2 and the specific net thrust parameter, specific impulse, I_{sp} , into equation 2.1:

$$2.3 \quad C_{FA} = \left(\frac{G\sqrt{T_3}}{P_3} \right) \frac{I_{sp}}{q_0} \frac{P_3}{\sqrt{T_3}}$$

$$2.4 \quad C_{PA} = \left(\frac{G\sqrt{T_3}}{P_3} \right) \Delta h_n \frac{P_3}{\sqrt{T_3}}$$

where: $G = W/A_{ff}$, the reactor airflow rate per unit flow area.
 $I_{sp} = F_n/W$, pounds of thrust (net) per pound of airflow per second.

At this point, it is also noted that specific impulse may be expressed by the relation:⁽¹⁾

$$2.5 \quad I_{sp} = \frac{V_8 - V_0}{g} + \frac{A_8}{W} (P_{s8} - P_{s0})$$

where stations 0 and 8 refer to free stream and nozzle exit plane conditions respectively. For the case of the fully expanded nozzle ($P_{s8} = P_{s0}$) equation 2.5 reduces to:

$$2.6 \quad I_{sp} = \frac{V_8 - V_0}{g}$$

Examination of equations 2.3 and 2.4 in light of 2.5 reveals that both the thrust and power coefficients are formulated as the product of separate parametric groupings which may be independently associated with the reactor or the cycle thermodynamics.

Specifically, $G\sqrt{T_3}/P_3$ is a function of: reactor coolant properties; pressure ratio, P_4/P_3 ; reactor temperature ratio, T_4/T_3 ; and the geometry parameter, $KL/DH^{1.2} G^{0.2}$. The geometry parameter is a function only of: the reactor inlet and exit coolant temperatures, T_3 and T_4 ; the maximum temperature of the average fuel element, T_{sm} ; and the longitudinal power distribution. This functional dependence is clearly illustrated in Appendices A and B. The computation of the reactor flow function is therefore independent of the application of the power cycle but dependent only on the choice of coolant.

Both the net specific power Δh_n and the specific impulse (specific net thrust) I_{sp} are noted to be a function of the cycle only in that they are independent of the method of heat addition. The computational techniques to be employed for these parameters differ, but both are found to be a function of the specific work available from the expansion process after the required specific compression work has been allocated.

In summary, the thermodynamic optimization process is based upon the facts that the reactor flow is independent of the power cycle, while the specific thrust or specific power is a function of the power cycle, but is independent of the flow rate per unit flow area and therefore the method of heat addition. Finally, it is the product of these two parameters which yields the net thrust per unit flow area, and it is the maximization of this product which will yield the minimum total reactor flow area for any required total net thrust or shaft power.

* Station 3 denotes reactor inlet.

2.4 The Optimization Process

At this point, the desirability of a thermodynamic optimization has been shown as well as the independence of the two primary parametric groupings to be employed in the optimization technique. Prior to presenting real examples of the calculation procedure, it remains to be demonstrated that an optimization will indeed occur. This is accomplished in the following qualitative discussion.

It is first necessary to examine the thermodynamic properties and boundary conditions which permit the calculation of the two primary parametric groupings of the optimization, the reactor flow function and the net specific power or thrust. Consideration of the relations between these parameters is sufficient to prove the existence of the optimization. The parameters required for the calculation of the reactor flow function $G\sqrt{T_3}/P_3$ are the inlet and exit conditions of temperature and pressure, the maximum surface temperature of the average fuel element T_{sm} and the longitudinal power distribution, which will be denoted hereafter by LPD. The net specific thrust or power will depend upon (1) the temperature and pressure level of the system, usually dictated by the environmental conditions; (2) the compression work required; (3) the temperature and pressure level at the start of the expansion process, a function of the specific heat addition but not the method of heat addition; and (4) the efficiencies of the various components of the system.

Heat addition in a gas-cooled (Brayton cycle) nuclear reactor is accomplished by convection. The general description of the process is that of heat addition in a constant-area duct with friction. Considerations of the effect on fluid properties of this process⁽²⁾ show that the larger the temperature rise and pressure drop required in the heating process, the lower the permissible airflow rate per unit flow area of the system and vice versa. Similarly, considerations of convective heat exchanger design illustrate that the heat transfer area and therefore the reactor size approach infinity as the exit coolant air temperature is required to approach the wall temperature, independent of inlet conditions and LPD. Conversely, the minimum but redundant size reactor will exist for the case of no heat addition.

The effect of LPD is to decrease the size of the reactor for any given set of boundary conditions as the wall temperature distribution approaches the isothermal, corresponding to a highly forward-peaked LPD. Since the shape of the LPD is primarily a nuclear rather than thermodynamic parameter, it will not be considered further except to here note that in the final design process, care must be exercised to guarantee that the LPD as determined by nuclear methods is identical with that employed in the thermodynamic sizing.

It is now possible to draw the following general conclusions with respect to the reactor flow function and therefore the reactor flow rate per unit flow area (equations 2.3 and 2.4). The reactor flow function will increase as (1) required temperature rise and pressure drop decrease and (2) the difference between the required coolant exit temperature and the maximum average wall temperature increases, and vice versa.

In consideration of the net specific thrust and power, it is first noted that since the compression and expansion processes are not 100% efficient, a certain amount of energy must be added to the working fluid to operate on the cycle even if no net useful energy is extracted. Consideration of a hypothetical cycle requiring a fixed amount of specific compression work illustrates that the net useful specific energy, which may be translated into net power or thrust, will increase directly with the amount of specific energy added to the working fluid and **inversely with the** pressure loss incurred in the process. That is, the higher the temperature rise and the lower the pressure drop, the greater the net specific thrust or power for a fixed amount of specific compression work, and vice versa.

These trends are seen to be directly opposite to that of the reactor flow per unit flow area.

A further extension of the line of thought illustrates that if by some means the available expansion work is fixed, the net specific thrust or power will decrease as the required specific compression work is increased. However, at the same time, due to the increased compression, the required temperature rise and pressure drop across the reactor will decrease, resulting in higher allowable flow rates per unit flow area, surface temperature remaining constant. Again, the contrary trends are noted.

The preceding discussion may be summarized as follows:

- (1) The net thrust or power coefficient will always increase with increasing maximum surface temperature of the average fuel element, T_{sm} , all other boundary conditions being constant.
- (2) For constant T_{sm} and specific compression work, the specific impulse and the specific power increase with increasing reactor exit coolant temperature and decreasing pressure drop while the reactor flow function (proportional to flow per unit flow area) decreases.
- (3) For constant reactor T_{sm} , exit coolant temperature and pressure, and environmental conditions, the specific impulse and the specific power decrease with increasing specific compression work while the reactor flow function decreases.
- (4) The product of net specific thrust or power and the flow rate per unit flow area is not a maximum when either of these components is a maximum.
- (5) The free stream dynamic head, q_0 , occurring as it does in the denominator, **affects** the above maximization process of the net thrust coefficient only in **that it results in a lower absolute value of the maximum at a lower flight speed.**

3.0 General Cycle Processes

This section presents a brief discussion of the various processes which must be computed in the application of the Brayton cycle to different power plants. Included are the ramjet, the turbojet, the open cycle turboshaft (turboprop), and the closed cycle turboshaft.

3.1 The Ramjet

The ramjet, with its absence of turbomachinery, represents the simplest application of the Brayton cycle. Compression is accomplished by a supersonic-subsonic diffuser and the expansion process by a nozzle, usually of the convergent-divergent type. The sole application of the ramjet is the air breathing propulsion of aircraft and missiles at supersonic speeds. Subsonic ramjets, although inefficient, are theoretically possible.

3.2 The Turbojet

The application of the Brayton cycle to the turbojet, as well as all remaining applications to be considered herein, results in the introduction of turbomachinery. The turbojet is similar to the ramjet in that it has diffuser and nozzle components but differs significantly in that the compression work is primarily mechanical, requiring that mechanical work be removed from the cycle by a turbine to drive the compressor. For any flight point, the net specific thrust may be computed only after the required specific compression work has been subtracted from the available specific expansion work, both subject to the component efficiencies.

3.3 The Open Cycle Turboshaft

This system differs from the turbojet primarily in that the net specific expansion work is supplied to a shaft by use of a power turbine in addition to the turbine required to drive the compressor. In general, this power turbine cannot take advantage of the entire available net specific expansion work, and therefore a certain fraction of it must either be rejected to the environment in the case of a stationary power supply system, or a portion of it may be converted to net specific thrust in the case of the turboprop. The case of the turboprop introduces a complication into the optimization procedure in that both the specific shaft work and the specific nozzle thrust must be optimized. Since it is impossible to translate the specific nozzle thrust into equivalent specific shaft work, the specific shaft work must be translated into specific propeller thrust by considerations of gearing and propeller efficiency.⁽³⁾ This component is then optimized in terms of the net thrust coefficient.

3.4 The Closed Cycle Turboshaft

The primary difference between the open and closed cycle systems is that the coolant for the closed cycle system is not limited to air and a radiator is required to provide for heat rejection to the environment. Other differences are the complete absence of any ram compression and the lack of a nozzle for any portion of the net specific expansion work conversion. As such, this system is optimal with respect to the net power coefficient.

3.5 Summary

The existence of the various components of each of the preceding applications is presented in Table 1.1. Whether or not the particular component receives or delivers specific work is also noted. The net specific work of each cycle is defined as the work delivered minus the work required.

TABLE 1.1

SUMMARY OF SPECIFIC WORK DISTRIBUTION FOR
VARIOUS BRAYTON CYCLE APPLICATIONS

Cycle Application	Ramjet	Turbojet	Open Cycle Turbo- machinery	Closed Cycle Turbo- shaft
Component				
Diffuser	I	I	I(2)	
Compressor		I	I	I
Compressor Turbine		O	O	O
Shaft Power Turbine			O	O
Nozzle	O	O	O(2)	

Notes:

(1) I denotes work required; O denotes work delivered.

(2) Applicable to turboprop only.

4.0 Specific Application: The Ramjet

This section illustrates the mechanics of the optimization technique by use of a sample calculation for the ramjet application. Sample calculations are presented, and the results of the complete study are tabulated and graphed.

4.1 Introduction

This section presents the optimization of a nuclear-powered ramjet. The primary results of this work are the free stream Mach number, M_0 , the boundary conditions of temperature and pressure for the reactor, and the net thrust coefficient. These results may be further expressed in terms of the reactor geometry parameter, $E/D_H^{1.2} G^{0.2}$, the reactor flow function, $G\sqrt{T_3/P_3}$, and the net specific thrust, I_{sp} . Since the thrust coefficient is independent of the environmental (atmospheric) ambient pressure but is a function of the ambient temperature, the optimization is conducted at 40,000 feet and is therefore valid for all altitudes between 36,089 feet and 82,021 feet, the ARDC Model Atmosphere, in which interval the ambient temperature is constant.

After the optimization study is completed, a certain thrust level and certain reactor nuclear characteristics are arbitrarily assumed in order to illustrate the application of the results to reactor sizing.

4.2 Assumptions

This section presents and discusses the assumptions employed in the aerothermodynamic optimization study. These assumptions are:

- (1) The vehicle is always at its design point such that the inlet shock wave always intersects the inlet lip, and thus there is no spillage drag.
- (2) All air taken on board exits through the nozzle, i.e., there is no diffuser bleed.
- (3) The diffuser pressure recovery is given by the relation:

$$4.1 \quad P_3/P_0 = 1.0 - 0.075 (M_0 - 1.0)^{1.35}$$

- (4) There is no total pressure loss from diffuser exit to reactor inlet.
- (5) The total pressure loss resultant from expansion at the reactor exit is given by $P_5/P_4 = 0.99$.
- (6) Only 96% of the total reactor power is transmitted to only 92% of the nozzle airflow.
- (7) The nozzle velocity coefficient, C_v , the square root of the nozzle efficiency, is 0.98.
- (8) The nozzle is fully expanded.

The assumption of design point operation follows directly from the requirement that the design point operation is to be optimized. It is necessary to assume a realistic inlet-diffuser total pressure recovery as a function of the free stream

Mach number since one of the main objectives of this optimization study is to determine the free stream Mach number. Since the pressure recovery, equation 4.1, is a function only of the Mach number, a unique optimization with respect to Mach number is possible. That is, the specific compression work is a unique function of Mach number in that a mechanical compressor is not involved.

The assumption of no diffuser bleed serves to make the optimization independent of the diffuser design.

Assumptions (4) and (5) both facilitate the handling of the reactor aerothermodynamics and introduce a required degree of conservatism into the work. The entrance losses to the reactor may be almost completely negated by rounding the entrances to the coolant channels thus eliminating the abrupt area contraction. Since certain long-range particles such as gamma rays are products in the fission process, not all of the reactor power generated will reach the primary coolant flow. Coolant air will be required for secondary reactor components such as structure and control rods and eight percent (8%) of the primary coolant air is arbitrarily allocated for this purpose. Since no secondary cooling requirement analysis may be accomplished at this stage of the design, it is not possible to determine the temperature rise of the secondary coolant air; therefore assumption (5) also defines that no heat is added to this flow. The terminal result of this assumption is that the mixed nozzle inlet air temperature of the primary (92%) and secondary (8%) coolant flows will decrease below that of the exit temperature of the primary reactor coolant, resulting in a net penalty in specific impulse. Assumptions (6) and (7) are concerned primarily with the nozzle design. Assumption (6) represents an arbitrary assumption of nozzle efficiency, while assumption (7), regarding complete expansion, greatly simplifies the nozzle analysis. This is valid since at this point in the design, the degree of nozzle expansion is unknown and thus the assumption presents a consistent, relative basis for performing the work.

4.3 Analytical Procedure

This section presents in outline form, the analytical procedure to be employed in the optimization study. With reference to Section 3.0 of this work, it is noted that selection of free stream Mach numbers, ambient conditions, reactor temperature and pressure ratios, and the maximum average fuel element surface temperature are sufficient to permit the optimization study since Mach number in conjunction with equation 4.1 may be translated into reactor inlet conditions. This analytical procedure is as follows:

- (1) Select a free stream Mach number, M_0 , maximum surface temperature of the average fuel element, T_{sm} , a reactor total temperature ratio, T_4/T_3 , a reactor total pressure ratio, P_4/P_3 , and a longitudinal power distribution.
- (2) With standard gas tables, compute the free stream stagnation conditions of temperature and pressure from the Mach number.
- (3) Compute the reactor inlet total pressure, P_3 , from the free stream stagnation pressure, P_0 , and equation 4.1.
- (4) Note that the reactor inlet temperature, T_3 , is the free stream stagnation temperature, T_0 , since the diffusion process is adiabatic and the reactor inlet Mach number is small.

(5) Since the parameters, T_{sm} , T_3 , P_3 , P_4/P_3 and T_4/T_3 (and therefore T_4 and P_4) are known, the reactor flow function, ϕ , is now computed by the methods of Appendices ~~XXXXXX~~.

(6) The temperature-mixing process occurring between reactor exit and nozzle inlet may be expressed by the relation:

$$4.2 \quad H_5 = \frac{0.92}{0.96} (H_4 - H_3) + H_4$$

and since T_3 and T_4 are known, T_5 , the nozzle inlet temperature, may be computed from the gas tables.

(7) The nozzle inlet pressure, P_5 , is now computed from the known value of P_4 and assumption (5) above.

(8) The real nozzle exit velocity, V_8 , is now computed from the relation:

$$4.3 \quad V_7 = C_v [2g J(H_5 - h_8)]^{\frac{1}{2}}$$

where h_8 is the static enthalpy at the nozzle exit computed for complete expansion in terms of the nozzle inlet total pressure and temperature and the ambient static pressure by use of the gas tables.

(9) The free stream velocity and dynamic head are now computed from the free stream Mach number as a function of the ambient conditions.

(10) The specific impulse (net specific thrust) is now computed for the defined fully expanded nozzle from equation 2.6.

(11) The net thrust coefficient is now computed from equation 2.3 for the results of steps (5), (9) and (10) and is valid only for the assumed boundary conditions of step (1).

(12) The process is repeated for other assumed conditions until sufficient data is generated to illustrate the optimization.

4.4 Selection of T_{sm} , M_o , T_4 , and P_4/P_3

The following range and values of these parameters are selected for investigation.

T_{sm} :	2550, 2650, 2750°F
M_o :	3.0, 4.0, 5.0
T_4 :	2060, 2220, 2370°F
P_4/P_3 :	0.5, 0.6, 0.7, 0.8

It is noted that both the surface temperatures and reactor exit air temperatures preclude the use of metallic reactor materials and therefore require a ceramic material such as beryllia. The type and limitations of potential reactor materials must be considered in selecting the boundary conditions of temperature.

4.5 The Meaning of a 2:1 Longitudinal Power Distribution

This example will assume a flat radial power distribution achieved in an undefined method and a 2:1 longitudinal power distribution as given by a sine curve; this shape results from neutron leakage out of either end of the reactor under the assumption of an unspecified but uniform fuel loading. Since the air flows longitudinally, the heat transfer in any one channel will not be affected by the shape of the radial power distribution, but will depend greatly upon the longitudinal power distribution. Since a 2:1 longitudinal power distribution defined by a sine curve has been postulated, it is well to present exactly what this means. The following argument is, in general, valid for a 3:1, 4:1, etc., power distribution. Note that no power is generated in the unfueled reflectors.

Consider Figure 1.1 which presents a complete sine curve superimposed upon a typical LPD for a reflected active core. The end reflectors are not shown. This analysis will illustrate analytically the amount that must be chopped off either end of the sine curve proportional to lengths a and b , such that the LPD will be both 2:1 and approximated by the sine curve. Note the following items:

- (a) $S = 1 + a + b$ and is proportional to 180 degrees.
- (b) The power density, Q'' , is proportional to the $\sin \theta$.
- (c) The parameter, X/L , is zero at the front of the active core and unity at the rear.
- (d) The power distribution is assumed to be symmetrical such that $a = b$.

By definition, a symmetrical 2:1 LPD is one in which the power density at a and b is one-half that at the center of the reactor ($\theta = 90^\circ$ $\sin \theta = 1.0$). Noting that:

$$4.4 \quad \sin^{-1} 0.5 = 30^\circ,$$

it is seen that the lengths a and b are both proportional to 30° . Since these lengths represent the portion of the sine curve to be chopped off, then:

$$4.5 \quad (X/L)_{\max} = 1.0 \text{ at } 180^\circ - 2(30^\circ) = 120^\circ$$

By direct proportion then, it is seen that:

$$4.6 \quad \frac{30^\circ}{a} = \frac{120^\circ}{1} = \frac{120^\circ}{(X/L)_{\max}} = \frac{30^\circ}{b},$$

the solution of which yields $a = b = 0.25$ and $s = 1 + a + b = 1.50$.

Thus, it is seen that the 2:1 symmetrical LPD is expressed by $\sin \theta$ where $30^\circ \leq \theta \leq 150^\circ$. It is apparent that an LPD where the minimum power is, for example, one-third the maximum, could be expressed in a similar manner. The range on θ would simply be different.

Data developed in this section will be employed in the aerothermodynamic analysis of this system at the design point.

FIGURE 1.1
ILLUSTRATED 2:1 LONGITUDINAL
POWER DISTRIBUTION AND
DESCRIPTIVE NOMENCLATURE

4.6 Sample Calculation

A sample calculation is now presented for an altitude of 40,000 feet, at which the ambient temperature and pressure are 390.0°R and 2.72 psia respectively. This calculation will follow the general procedure outlined in Section 4.3 and will be subdivided into four distinct areas. First, the calculation of the reactor boundary conditions will be presented which will be followed by the calculation of the reactor flow function. Finally the specific impulse and the net thrust coefficient will be calculated.

4.6.1 Reactor Boundary Conditions

- (a) The following boundary conditions selected from the range defined in Section 4.4 are assumed:

$$\begin{aligned} M_0 &= 4.0 \\ T_4 &= 2220^\circ\text{F} \\ T_{sm} &= 2650^\circ\text{F} \\ P_4/P_3 &= 0.70 \\ LPD &= 2:1 \text{ sine} \end{aligned}$$

- (b) The free stream velocity at Mach 4.0 is given by the relation:

$$V_0 = M_0 \sqrt{\gamma g R t_0}$$

which, for $t_0 = 390^\circ\text{R}$ yields a free stream velocity of 3872 feet/sec.

- (c) The total energy of the flow is given by the relation:

$$H_0 = h_0 + V_0^2/2gJ$$

where $h_0 = 93.13 \text{ Btu/lb}^{(4)}$ for $t_0 = 390^\circ\text{R}$. Substitution from step (b) yields:

$$H_0 = 93.13 + 299.5 = 392.63 \text{ Btu/lb}$$

which yields:

$$T_0 = T_3 = 1588^\circ\text{R}$$

- (d) The pressure ratios corresponding to $H_0 = 392.63 \text{ Btu/lb}$ and to $h_0 = 93.13 \text{ Btu/lb}$ are 69.66 and 0.4447 respectively. The isentropic free-stream stagnation pressure then is computed from the relation:

$$P_0 = P_{so} \left(\frac{69.66}{0.4447} \right)$$

which becomes 426.06 psia for $P_{so} = 2.72 \text{ psia}$.

- (e) The ram recovery, P_3/P_0 , as computed from equation 4.1 is 0.66, which yields, in conjunction with step (c), a reactor inlet total pressure, P_3 , of 281.20 psia.

2. Reactor Flow Function

- (a) The determination of the reactor flow function, $G \sqrt{T_3/P_3}$, requires solution of equation A14 of Appendix 1, which is repeated below:

$$\left(\frac{G \sqrt{T_3}}{P_3} \right)^2 \left(\frac{KL}{T_H^{1.2} G^{0.2}} + \psi \right) = \xi$$

The parameters ψ and ξ are seen to be functions only of reactor temperature and pressure ratio, T_1/T_3 and P_1/P_3 (equations E15 and E16). The parameter $L/D_H^{1.2} G^{0.2}$ may be determined from the methods of Appendix 1, specifically equation A17 where L denotes the heated length (length of the active core) and D_H denotes the heat transfer hydraulic diameter of the coolant channel. The parameter K denotes the product of two quantities. The first is the ratio of the rough-tube to the smooth tube friction factor, while the second is the ratio of total length, end reflectors plus active core, to the heated length. This adjustment is required since there is little heat addition in the unfueled end reflectors, which generate no power; but friction does exist which increases the pressure drop for a given mass flow.

Note that once the parameters $L/D_H^{1.2} G^{0.2}$ and G have been determined in terms of the aerothermodynamic boundary conditions, the selection of any length from nuclear considerations will result in a corresponding and unique coolant channel diameter, D_H .

- (b) Determination of $L/D_H^{1.2} G^{0.2}$: The equations of Appendix A are employed in the computation of $L/D_H^{1.2} G^{0.2}$. Significant space can be saved by noting that the parameters $\sin(\pi b/s)$, $\sin(\pi a/s)$, $\cos(\pi b/s)$, and $\cos(\pi a/s)$ occur frequently in the equations of Appendix A and evaluating them at this time for the 2:1 LPD. Thus, for $a = b = 0.25$ and $s = 1.50$, it is seen that:

$$\begin{aligned} \sin(\pi a/s) &= \sin(\pi b/s) = 0.50 \\ \cos(\pi a/s) &= \cos(\pi b/s) = 0.866 \end{aligned}$$

The desired result, $L/D_H^{1.2} G^{0.2}$, is found in equation A15, which is seen to be:

$$\frac{\pi}{s} \frac{1}{\sqrt{\alpha^2 - 1}} = 3.278 \times 10^{-3} E \frac{L}{D_H^{1.2} G^{0.2}} \frac{T_b^{0.67}}{T_f^{0.56}}$$

Smooth tube heat transfer coefficient is now assumed and therefore the parameter E equals unity. The subscripts b and f refer to bulk and film properties respectively. The bulk/film temperature ratio is given by equation A22 as:

$$\frac{T_b^{0.67}}{T_f^{0.56}} = (T_{sm})^{0.11} \frac{\left[1 - \frac{T_{sm} - T_H}{T_{sm}} \left\{ \frac{\alpha^2 - 1}{\alpha^2 - \cos(\pi b/s)} \right\} \right]^{0.67}}{\left[1 - \frac{1}{2} \left(\frac{T_{sm} - T_H}{T_{sm}} \right) \left\{ \frac{\alpha^2 - 1}{\alpha^2 - \cos(\pi b/s)} \right\} \right]^{0.56}}$$

and the parameter, α , by equation A9 as:

$$\alpha = \left(\frac{T_{sm} - T_L}{T_L - T_3} \right) \cos(\pi a/s) + \left(\frac{T_{sm} - T_L}{T_L - T_3} + 1 \right) \cos(\pi b/s)$$

Since the temperature boundary conditions and the parameters, s , a , and b are known, the problem becomes one of the solution of three equations in three unknowns,

$$\alpha, T_b^{0.67}/T_f^{0.56}, L/D_H^{1.2} G^{0.2}$$

Substitution of $T_{sm} = 3110^\circ R$, $T_3 = 1588^\circ R$, and $T_L = 2680^\circ R$ yields, in the following order of calculation:

$$\alpha = 1.548$$

$$T_b^{0.67}/T_f^{0.56} = 2.3976$$

$$L/D_H^{1.2} G^{0.2} = 225.5$$

4.6.3 Calculation of Reactor Flow Function

In this work it will be assumed that the friction factor is 1.15 times the smooth tube value and that the ratio of the core length plus end reflector length to the core length is 1.14. Thus, the parameter K becomes $1.14 \times 1.15 = 1.31$. The parameter $KL/D_H^{1.2} G^{0.2}$ then becomes $1.31 \times 225.5 = 295$. The parameters, ψ and ξ , are given by the relations (Appendix B):

$$\psi = \frac{101 (T_L/T_3 - P_L/P_3)}{(P_L/P_3 - 0.1) (T_L/T_3 - 0.111)}$$

$$\xi = 73.6 \frac{1 - (P_L/P_3)^2}{0.542 + T_L/T_3}$$

These parameters are seen to be a function of the temperature and pressure ratio only. The pressure ratio has been assumed to be 0.70 for this example, while the temperature ratio is computed as $T_L/T_3 = 2680/1588 = 1.688$. Substitution of these two values into the above relations yields:

$$\psi = 105.5$$

$$\xi = 16.83$$

The relation

$$\left(\frac{G \sqrt{T_3}}{P_3} \right)^2 \left[\frac{KL}{D_H^{1.2} G^{0.2}} + \psi \right] = \xi$$

may now be solved for the reactor flow function, which solution yields:

$$\frac{G \sqrt{T_3}}{P_3} = 0.2018$$

4.6.4 Calculation of the Specific Impulse

The specific impulse for the assumed fully expanded nozzle is given by the relation:

$$I_{sp} = (V_8 - V_0)/g$$

where V_8 denotes the nozzle exit velocity given by the relation:

$$V_8 = \left[\{ C_v 2gJ(H_5 - h_{s8}) \} \right]^{1/2}$$

H_5 denotes the total enthalpy of the nozzle inlet flow equal to the total enthalpy of the mixed primary and secondary reactor coolant flows.

From equation 4.2, H_5 is given as:

$$H_5 = \frac{92}{96} (H_4 - H_3) + H_3$$

For T_3 and T_4 equal to 1588°R and 2680°R respectively, H_3 and H_4 are found to be 392.54 Btu/lb and 697.56 Btu/lb respectively. The nozzle inlet total enthalpy, H_5 , is now readily computed as 684.85 Btu/lb, which corresponds to a pressure ratio, Pr_5 , of 544.1 and a temperature, T_5 , of 2636°R. The nozzle inlet pressure, P_5 , is readily computed from the known value of $P_3 = 281.20$, the assumed reactor pressure ratio of $P_4/P_3 = 0.70$, and the assumed reactor exit loss $P_5/P_4 = 0.99$ as $P_5 = 426.06 \times 0.70 \times 0.99 = 194.87$ psia.

For the fully expanded nozzle the exit static pressure p_{s8} must equal the ambient static pressure of 2.72 psia. Thus, the nozzle pressure ratio becomes $P_5/p_{s8} = 194.87/2.72 = 71.64$. The pressure ratio at the nozzle exit, Pr_8 , is then computed to be 7.595 from the relation:

$$Pr_8 = Pr_5 (p_{s8}/P_5)$$

which is seen to correspond to a nozzle exit static enthalpy h_{s8} of 210.12 Btu/lb and a static exit temperature, t_{s8} , of 875°R.

For the assumed nozzle velocity coefficient $C_v = 0.98$, the real nozzle exit velocity, V_8 , is computed to be 4750 feet/sec. The free stream velocity, V_0 , has been computed as 3872 feet/sec., and thus, the specific impulse becomes:

$$I_{sp} = (4750 - 3872)/32.17 = 27.32 \text{ sec.}$$

4.6.5 Calculation of Net Thrust Coefficient

From equation 2.3, the net thrust coefficient based upon the active core free flow area is given by:

$$C_{FA} = \frac{G \sqrt{T_3}}{P_3} \frac{I_{sp}}{q_0} \frac{P_3}{\sqrt{T_3}}$$

However, this relation is valid only for the case in which the active core flow equals the nozzle flow. In this example, 8% of the nozzle flow has been bypassed around the active core to provide for secondary cooling. Thus, since

the reactor flow function, $G\sqrt{T_3}/P_3$ is associated only with the active core flow, the active core airflow rate, W ($G = W/A_{ff}$), must be appropriately increased before multiplying by the specific impulse, which parameter is associated with the total flow, both primary and secondary. This is readily accomplished by division by the quantity $1.0 - 0.08$ or 0.92 where the 0.08 denotes the secondary coolant fraction of the total flow. Thus, for this example, the net thrust coefficient is given by the relation:

$$C_{FA} = \frac{G\sqrt{T_3}}{0.92P_3} \frac{I_{sp}}{q_0} \frac{P_3}{\sqrt{T_3}}$$

All values are known with the exception of the free stream dynamic head, q_0 , which is given by the relation:

$$q_0 = k P_{so} M_o^2/2$$

as 30.46 psia for $p_{so} = 2.72$ psia at 40,000 feet, $M_o = 4.0$. Thus, the net thrust coefficient based on the active core free flow area becomes:

$$C_{FA} = \frac{0.2018 \times 27.32 \times 281.20}{0.92 \sqrt{1588}} = 1.395$$

4.7 Summary of Results

By repeating the preceding calculation, data may be generated for as many points as desired. This has been accomplished and is presented in Tables 1.2, 1.3, and 1.4 and is partially graphed upon Figures 1.2 through 1.5. Tables 1.2 through 1.4 summarize the specific impulse, I_{sp} , reactor flow function $G\sqrt{T_3}/P_3$, and the free flow area net thrust coefficient, C_{FA} , as a function of T_{sm} , T_4 , P_4/P_3 , and M_o . Examination of Table 1.4 shows that maximum thrust coefficient occurs near a reactor pressure ratio of 0.60. Thus, neglecting the effect of pressure ratio, Figures 1.2 through 1.4 were constructed which clearly show the optimization of the net thrust coefficient at Mach 4.0 for the higher surface temperature. Figure 1.5 drawn for Mach 4.0 shows the change in thrust coefficient with reactor exit air temperature as a function of pressure ratio and surface temperature. Note that there is no optimization with respect to surface temperature and that, for the fixed Mach number, net thrust coefficient always increases with surface temperature.

4.8 Selection of Design Point

The design point selected from this optimization study is Mach 4.0 at $T_4 = 2300^\circ\text{F}$, $T_{sm} = 2750^\circ\text{R}$, and $P_4/P_3 = 0.60$ based upon the results of the optimization study. Interpolation of the data of Tables 1.2 through 1.4 results in a specific impulse of 28.50 seconds and a reactor flow function, $G\sqrt{T_3}/P_3$, equal to 0.212. It is noted that these values, as well as the parameter $G\sqrt{T_3}/P_3$, for this design point may be computed directly by the method of the sample calculation. However, the interpolation was used for this example.

TABLE 1.2

Specific Impulse

M_0	P_0/P_3	0.5	0.6	0.7	0.8
	$T_{11}, ^\circ F$	Specific Impulse, I_{sp} , sec.			
3.0	2060	33.45	36.02	38.30	40.38
	2220	37.40	40.32	42.74	44.54
	2370	41.22	44.39	46.69	48.06
4.0	2060	17.59	19.40	20.92	21.82
	2220	21.28	23.31	24.32	25.00
	2370	24.85	26.68	27.79	28.73
5.0	2060	-0.716	0.55	1.55	2.61
	2220	3.96	5.25	6.43	7.34
	2370	8.70	10.21	11.35	12.37

TABLE 1.3
REACTOR FLOW FUNCTION

M_0	$T_{sm},$ $^{\circ}F$	P_4/P_3	0.5	0.6	0.7	0.8
		$T_{4,}^{\circ}F$	Reactor Flow Function, $G\sqrt{T_3}/T_3$			
3.0	2550	2060	.190	.184	.169	.146
		2220	.161	.154	.141	.121
		2370	.120	.113	.102	.0867
	2650	2060	.195	.189	.175	.151
		2220	.173	.166	.153	.131
		2370	.144	.136	.124	.106
	2750	2060	.208	.203	.188	.163
		2220	.182	.175	.161	.139
		2370	.159	.152	.139	.119
4.0	2550	2060	.244	.238	.222	.193
		2220	.210	.202	.186	.160
		2370	.159	.151	.137	.116
	2650	2060	.255	.251	.233	.202
		2220	.224	.217	.202	.174
		2370	.188	.180	.165	.141
	2750	2060	.263	.259	.242	.210
		2220	.235	.229	.213	.185
		2370	.206	.198	.183	.157
5.0	2550	2060	.373	.380	.365	.327
		2220	.309	.306	.288	.256
		2370	.239	.230	.212	.180
	2650	2060	.386	.396	.384	.346
		2220	.325	.316	.307	.275
		2370	.269	.262	.243	.207
	2750	2060	.398	.412	.401	.364
		2220	.340	.342	.326	.293
		2370	.289	.284	.266	.227

TABLE 1.4
NET THRUST COEFFICIENT

M_0	T_{sm} °F	P_1/P_3	0.5	0.6	0.7	0.8
		$T_{4.5}$ °F	Thrust Coefficient, C_{fA}			
3.0	2550	2060	.994	1.036	1.016	.922
		2220	.944	.974	.944	.842
		2370	.774	.786	.748	.652
	2650	2060	1.022	1.066	1.048	.952
		2220	1.014	1.050	1.022	.914
		2370	.928	.948	.908	.796
	2750	2060	1.088	1.144	1.130	1.032
		2220	1.064	1.106	1.080	.968
		2370	1.026	1.058	1.018	.896
4.0	2550	2060	1.082	1.164	1.168	1.062
		2220	1.282	1.326	1.280	1.170
		2370	1.156	1.164	1.132	.988
	2650	2060	1.132	1.224	1.228	1.112
		2220	1.372	1.430	1.390	1.274
		2370	1.366	1.390	1.360	1.198
	2750	2060	1.166	1.264	1.272	1.156
		2220	1.438	1.506	1.466	1.350
		2370	1.498	1.530	1.510	1.334
5.0	2550	2060	-.108	.088	.222	.338
		2220	.518	.642	.724	.734
		2370	.814	.920	.924	.870
	2650	2060	-.112	.092	.232	.358
		2220	.546	.682	.774	.790
		2370	.916	1.050	1.062	1.004
	2750	2060	-.116	.096	.244	.274
		2220	.570	.718	.820	.840
		2370	.984	1.138	1.162	1.100

FIGURE 1.2

THRUST COEFFICIENT VS. MACH NUMBER
AS A FUNCTION OF REACTOR EXIT AIR
TEMPERATURE, T_4

P_1/P_3 0.60
 T_{sm} 2550°F

Thrust Coefficient Based on Reactor Free Flow Area, C_{FA}

1.4

1.2

1.0

.8

.6

.4

.2

0

$T_4 = 2370^\circ\text{F}$

$T_4 = 2220^\circ\text{F}$

$T_4 = 2060^\circ\text{F}$

3.0

4.0

5.0

Free Stream Mach Number, M_o

FIGURE 1.3

THRUST COEFFICIENT VS. MACH NUMBER
AS A FUNCTION OF REACTOR EXIT AIR
TEMPERATURE, T_4

P_4/P_3 0.60
 T_{sm} 2650°F

Thrust Coefficient Based on Reactor Free Flow Area, C_{fA}

1.6

1.4

1.2

1.0

.8

.6

.4

.2

0

3.0

4.0

5.0

Free Stream Mach Number, M_0

$T_4 = 2370^\circ\text{F}$

$T_4 = 2220^\circ\text{F}$

$T_4 = 2060^\circ\text{F}$

FIGURE 1.4

THRUST COEFFICIENT VS. MACH NUMBER
AS A FUNCTION OF REACTOR EXIT AIR
TEMPERATURE, T_L

P_L/P_3 0.60
 T_{am} 2750°F

Thrust Coefficient Based on Reactor Free Flow Area, C_{FA}

1.6

1.4

1.2

1.0

.8

.6

.4

.2

0

$T_L =$
2370°F

$T_L =$
2220°F

$T_L =$
2060°F

3.0

4.0

5.0

Free Stream Mach Number, M_o

FIGURE 1.5

THRUST COEFFICIENT VS. REACTOR EXIT AIR TEMPERATURE
AS A FUNCTION OF REACTOR PRESSURE RATIO AND THE
MAXIMUM SURFACE TEMPERATURE OF THE AVERAGE
FUEL ELEMENT, T_{sm}

Thrust Coefficient Based on Reactor Free Flow Area, C_{FA}

1.5
1.4
1.3
1.2
1.1
1.0

P_1/P_3

$T_{sm}, ^\circ F$

.6
.7
.5

2750

.6
.5
.7

2650

.5
.6
.7

2550

Mach Number 4.0

2000

2100

2200

2300

2400

Reactor Exit Air Temperature $T_4, ^\circ F$

4.9 Typical Application of Results

The application of these results to a hypothetical design problem will now be illustrated with the aid of some basic assumptions with respect to the nuclear characteristics of the reactor. It shall be assumed that the active core is 50% solid and that its geometry is a right circular cylinder with its length equal to its diameter, the standard condition of minimum neutron leakage. It shall be further assumed that the desired net thrust level is 40,000 lbs. From Figure 1.5 at $T_4 = 2300^\circ\text{F}$, $P_4/P_3 = 0.60$ and $T_{sm} = 2750^\circ\text{F}$, the net thrust coefficient is seen to be 1.395. Consideration of equation 2.1 shows that multiplication of this value by the free stream dynamic head, 30.46 psia, at 40,000 feet, Mach 4.0, will yield the net thrust per unit flow area. Thus, for this example, $F_n/A_{ff} = 1.395 \times 30.46 = 42.50 \text{ lbs/inch}^2$. Division of the desired 40,000 lbs net thrust by this quantity yields a required active core free flow area of 941.10 inch^2 . For the assumed 50% solid active core, this free flow area translates to an active core frontal area of 1882.2 inch^2 corresponding to a diameter of 49.92 inches. Under the length equal to diameter assumption, this quantity is also equal to the active core length.

It is to be noted that this final calculation does not guarantee a nuclear critical system with some particular desired lifetime nor does it consider fuel inventory or materials properties. Its sole purpose is to illustrate the translation of the thrust coefficient to a reactor size once the required solid fraction and geometry have been determined from other considerations.

The thrust coefficient data calculated herein is again noted to be valid over the entire tropopause since it is independent of ambient pressure. However, in calculating the required free flow area to attain a given thrust level, it must be realized that, for the same optimum Mach number the dynamic head, and therefore the net thrust per unit free flow area, will decrease with increasing altitude. Thus, the net thrust delivered by a fixed system will decrease with altitude. It is indeed fortunate that the drag and therefore thrust required also decreases.

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APPENDIX A

A GENERAL METHOD FOR COMPUTING REACTOR GEOMETRY IN TERMS OF POWER DISTRIBUTION AND TEMPERATURE REQUIREMENTS

The complete derivation of the equations employed in the thermal design of the reactor of Section 4.0 is presented. The assumptions of the analysis are constant temperatures and fully developed turbulent flow.

(I) hA_H/WC_p Development

- A1 The longitudinal power distribution as presented on Figure 6.5.3 is represented by the general relation:

$$\dot{Q}'''_x = \dot{Q}'''_{\max} \sin \left[\frac{\pi}{s} \left(\frac{x}{L} + a \right) \right] \quad \frac{\text{BTU}}{\text{sec-in}^3}$$

- A2 The fluid temperature rise through the system is:

$$\begin{aligned} (T - T_3)_x &= \frac{\dot{Q}'''_{\max} A_F}{WC_p} \int_0^x \sin \frac{\pi}{s} \left(\frac{x}{L} + a \right) dx \\ &= \frac{\dot{Q}'''_{\max} A_F L s}{\pi WC_p} \left[\cos \left(\frac{\pi a}{s} \right) - \cos \frac{\pi}{s} \left(\frac{x}{L} + a \right) \right] \end{aligned}$$

- A3 The temperature difference between the cooling channel surface and fluid is:

$$(T_s - T)_x = \frac{\dot{Q}'''_x V_F}{nA_H} = \frac{\dot{Q}'''_{\max} V_F}{nA_H} \sin \left[\frac{\pi}{s} \left(\frac{x}{L} + a \right) \right]$$

- A4 Noting that $V_F = A_F L$ and expressing $\dot{Q}'''_{\max} V_F$ as:

$$\left(\frac{\dot{Q}'''_{\max}}{\dot{Q}'''_{\text{avg}}} \right) WC_p (T_4 - T_3) = \Psi_{\max} WC_p (T_4 - T_3)$$

and combining A2 and A3 yields:

$$\begin{aligned} T_{sx} - T_3 &= \Psi_{\max} (T_4 - T_3) \left[\frac{s}{\pi} \left\{ \cos \left(\frac{\pi a}{s} \right) - \cos \frac{\pi}{s} \left(\frac{x}{L} + a \right) \right\} \right. \\ &\quad \left. + \frac{WC_p}{nA_H} \sin \frac{\pi}{s} \left(\frac{x}{L} + a \right) \right] \end{aligned}$$

- A5 Differentiating A1.4 with respect to x yields:

$$\tan \Theta_m = \frac{-\pi}{s} \left(\frac{WC_p}{nA_H} \right)$$

where Θ_m is the value of $\frac{\pi}{s} \left(\frac{x}{L} + a \right)$ where $T_s = T_{sm}$.

A6 Substitution of A5 into A4 and re-arranging yields:

$$\frac{T_{sm} - T_3}{T_4 - T_3} = \Psi_{\max} \frac{s}{\pi} \left[\cos \frac{\pi a}{s} - \frac{1}{\cos \Theta_m} \right]$$

or

$$\cos \Theta_m = \frac{1}{\cos \frac{\pi a}{s} - \frac{T_{sm} - T_4}{T_5 - T_4} \left(\frac{\pi}{s} \right) \frac{1}{\Psi_{\max}}}$$

A7 $\Psi_{\max}$ may be expressed in terms of a, b, and S by integration of A1 to yield:

$$\Psi_{\max} = \frac{\pi}{s} \left[\frac{1}{\cos \frac{\pi a}{s} + \cos \frac{\pi b}{s}} \right]$$

A8 Substitution of A7 into A6 yields:

$$\cos \Theta_{\max} = \frac{1}{\cos \left(\frac{\pi a}{s} \right) - \left(\frac{T_{sm} - T_4}{T_5 - T_4} \right) \left[\cos \frac{\pi a}{s} + \cos \frac{\pi b}{s} \right]}$$

A9 For Utility, $\frac{T_{sm} - T_3}{T_4 - T_3}$ is expressed as its equivalent $\frac{T_{sm} - T_3}{T_4 - T_3} + 1$,

and the parameter α is defined as:

$$\begin{aligned} \alpha &= -1/\cos \Theta_m \\ &= \left(\frac{T_{sm} - T_4}{T_4 - T_3} \right) \cos \frac{\pi a}{s} + \left(\frac{T_{sm} - T_4}{T_4 - T_3} + 1 \right) \cos \frac{\pi b}{s} \end{aligned}$$

A10 From A9 and trigonometric identities:

$$\frac{\sin \Theta_m \sqrt{1 - \cos^2 \Theta_m}}{\cos \Theta_m} = \tan \Theta_m = -\sqrt{\alpha^2 - 1}$$

A11 Substituting A10 and A5 yields:

$$\frac{L_{AH}}{WC_p} = \frac{\pi}{s} \left(\frac{1}{\sqrt{\alpha^2 - 1}} \right)$$

Examination of equations A9 and A11 indicate that the parameter h_{AH}/WC_p is defined solely by terminal temperatures and power distributions.

(II) Translation to Geometric Dimensions

A12 The basic heat transfer equation to be employed (5) (6) is given below as:

$$\frac{hD_H}{k_f} = 0.0205 \left(\frac{D_{HG}}{\mu_f} \right)^{0.8} \left(\frac{C_p}{k} \right)_f^{0.4} \left(\frac{T_b}{T_f} \right)^{0.0}$$

where subscripts b and f refer to bulk and film properties respectively.

A13 The specific heat C_p , the viscosity μ and the thermal conductivity k for air may be expressed empirically (7) by the following relations. The accuracy of these relations is $\pm 5\%$ for temperatures from room temperature up to 2500°R .

$$C_p = 0.103 T^{0.13} \text{ BTU/lbs-}^\circ\text{R}$$

$$\mu = 1.481 \times 10^{-8} T^{0.68} \text{ lbs/inch-sec}$$

$$k = 0.371 \times 10^{-8} T^{0.73} \text{ BTU/sec-inch}^2\text{-}^\circ\text{R/inch}$$

The use of this technique for other coolants will require the re-evaluation of the temperature dependence of these parameters.

A14 Substitution of these relations in A12 and re-arranging yields:

$$h = 8.44 \times 10^{-5} E \left(\frac{G^{0.8}}{D_H^{0.2}} \right) \frac{T_b^{0.8}}{T_f^{0.56}} \frac{\text{BTU}}{\text{sec-in}^2\text{-}^\circ\text{R}}$$

where E is the smooth tube heat transfer coefficient multiplier.

A15 The heat transfer area A_H is given by the number of tubes, $4A_{ff}/\pi D_H^2$, times the surface area per tube, $\pi D_H L$, or $A_H = \frac{4A_{ff}L}{D_H}$.

A16 Combining the specific heat relation of A13 and equation A15 with A14 yields:

$$\frac{hA_H}{WC_p} = \frac{4A_{ff}L}{D_H WT_b^{0.13}} \left[8.44 \times 10^{-5} E \left(\frac{G^{0.8}}{D_H^{0.2}} \right) \frac{T_b^{0.8}}{T_f^{0.56}} \right]$$

Noting that $W/A_{ff} = G$ yields the final result, which in conjunction with A11 is seen to be:

$$A17 \quad \frac{hA_H}{WC_p} = \frac{\pi}{s} \left(\frac{1}{\sqrt{\alpha^2 - 1}} \right) = 3.278 \times 10^{-3} E \left(\frac{L}{D_H^{1.2} G^{0.2}} \right) \frac{T_b^{0.67}}{T_f^{0.56}}$$

Equation A17 defines the geometry requirements to yield a stated value of T_{sm} with a particular power distribution and T_4 and T_3 values given.

(III) $(T_b^{0.67}/T_f^{0.56})$ Generalization

A18 From A3 and A4

$$T_{sm} - T_b = \frac{\psi_{\max} \sin \theta}{hA_H/WC_p} (T_4 - T_3)$$

A19 From A7 and A9

$$\psi_{\max} = \frac{\frac{\pi}{s} \left(\frac{T_{sm} - T_4}{T_4 - T_3} \right)}{\alpha - \cos(\pi b/s)}$$

A20 From A9

$$\sin \Theta = \sqrt{\alpha^2 - 1} \quad \alpha$$

A21 Substituting A20, A19, and A11 into A18 yields:

$$\frac{T_{sm} - T_{bm}}{T_{sm} - T_4} = \frac{\alpha^2 - 1}{\alpha^2 - \alpha \cos(\pi b/s)}$$

where equation A21 defines the value of T_b to be used in the term $(T_b^{0.67}/T_f^{0.56})$.

A22 Since $T_f = (T_s + T_b)/2$, equation A21 may also be written as:

$$\frac{T_{bm}^{0.67}}{T_f^{0.56}} = \frac{(T_{sm})^{0.11} \left[1 - \left(\frac{T_{sm} - T_5}{T_{sm}} \right) \left(\frac{\alpha^2 - 1}{\alpha^2 - \alpha \cos(\pi b/s)} \right) \right]^{0.67}}{\left[1 - 1/2 \left(\frac{T_{sm} - T_5}{T_{sm}} \right) \left(\frac{\alpha^2 - 1}{\alpha^2 - \alpha \cos(\pi b/s)} \right) \right]^{0.56}}$$

This form is employed in the aerothermodynamic reactor sizing of Section 4.0.

A23 The limit of applicability such that T_{sm} does not occur beyond $(1+a)L$; i.e., the end of the active core, is derived from A8 by setting $\Theta = (\pi b/s)$. Thus

$$\frac{T_{sm} - T_4}{T_4 - T_3} \geq \frac{\sin^2(\pi b/s)}{\cos(\pi b/s) [\cos(\pi a/s) + \cos(\pi b/s)]}$$

for T_{sm} to occur within limits.

APPENDIX B

A GENERAL METHOD FOR COMPUTING PRESSURE LOSS FOR AIR FLOWING IN CONSTANT AREA DUCTS WITH HEAT ADDITION AND FRICTION

Introduction

This section presents generalized data applicable to determining pressure losses for air flowing in constant area ducts with heat addition and friction.

The relations evolved are based on empirical correlation of an approximate total pressure drop relation with the results of digital computer analysis. It is the purpose of this section to illustrate that the correlation has a firm analytical basis. The empirical correlation itself, based on several thousand separate digital computer analyses, is not shown.

The equations characterizing the influence of friction and heat addition on flow characterization of a compressible fluid are not amenable to closed form integration without the introduction of qualifying assumptions. These may range from assumed identity to compressible systems to integration with the assumption of average properties. These restrictions tend toward inadequacy when any of the following conditions exist:

Required accuracy of determination	- + 5% or less
System Mach number	- 0.3 or greater
System pressure ratio	- less than 0.9
System fluid temperature rise ratio	- greater than 1.6

Where several of these items coexist, as is common in high-performance heat-exchanger designs, the determination of over-all system pressure drop requires incremental integration of the following differential equation.

$$B1 \quad d \ln P = \frac{-k M^2}{2} \left[\frac{4f dL}{D} + d \ln T \right]$$

This section presents an empirical generalization based on the incremental integration of equation B1 with the aid of a digital computer.

Analytical Model and Limitations

- (1) The duct geometry is a constant flow area system characterized by a length, L, and a constant hydraulic diameter, D.
- (2) The duct friction factor, f, is expressed by a relation of the form:

$$B2 \quad f = 0.046 K (N_R)^{-0.2}$$

i.e., as a multiple of smooth tube values and for assumed turbulent flow.

- (3) Duct entrance and exit losses are not included.
- (4) Heat addition is along its total length, and the power distribution is symmetric and given by a sine function.

-) The heated fluid is non-flammable air limited to a temperature range of 50 to 2500°F with inlet temperatures of 800 to 2500°F, or up to 2500°F with inlet temperatures in the range of 800 to 1000°F. These restrictions represent the range of digital computer data used in the correlation.
- (6) The analytical empirical data correlations yield results within + 2% of those obtained by digital computer wherein the heated path is treated as a series of 10-20 increments.

Development of Correlation

The method employed is the integration of the pressure loss relation to determine the most probable form of the correlation with computer data. Once the form of the equation is known, it is fitted to the computer data by appropriate adjustment of constants such as to be valid within the limitations discussed above.

Equation B1, taken from the influence coefficients of reference 2, although useful for step-by-step integration, is not so amenable to the desired correlation as is the basic momentum equation. It is therefore this form that will be employed. Under the assumptions of no drag force, constant area, and constant flow, the momentum becomes:

$$B3 \quad pA - (p + dp) A - \tau_w dA_w = \frac{W}{g} (V + dV) - \frac{WV}{g}$$

where τ_w represents the frictional shearing stress on the tube wall area dA_w and W represents the airflow rate. Thus,

$$dp + \tau_w \frac{dA_w}{A} + \frac{W}{Ag} dV = 0$$

The following substitutions are valid:

$$B4 \quad \tau_w = \frac{f \rho V^2}{2g} = \frac{fV^2}{2gw} = \frac{fG^2 w}{2g} \quad \text{where: } G^2 = \frac{W^2}{A^2} = \frac{V^2}{w^2}$$

$$B5 \quad dA_w/A = 4 dx/D$$

$$B6 \quad V = W/\rho A = \frac{W}{\rho A} = wG$$

$dV = Gdw$ since G is a constant for constant flow and area.

The substitution of these relations in the momentum equation yields:

$$dp + \left(\frac{fG^2 w}{2g} \right) \left(\frac{4dx}{D} \right) + \left(\frac{G}{g} \right) Gdw = 0$$

Finally, with the aid of the gas law:

$$B7 \quad \frac{pdp}{RT} + \frac{4fG^2}{2gD} dx + \frac{G^2}{g} \frac{dw}{w} = 0$$

It is noted that this relation will not reduce to equation B1 since the pressure here is static pressure.

Equation B7 is now integrated under the assumption that the temperature rise is approximated by the average temperature. Thus, letting subscript 1 denoted inlet conditions and subscript 2 denote exit (final) conditions:

$$\frac{G^2}{g} \ln \frac{w_2}{w_1} + \frac{p_2^2 - p_1^2}{2 RT_{avg}} + \frac{4f G^2 L}{2gD} = 0$$

$$\frac{G^2}{g} \ln \left(\frac{t_2 p_1}{t_1 p_2} \right) + \frac{p_2 - p_1}{RT_{avg}} \left(\frac{p_2 + p_1}{2} \right) + \frac{4f G^2 L}{2gD} = 0$$

Noting that $(p_2 + p_1)/2 = p_{avg}$ and rearranging the static pressure drop is seen to be given by:

$$B8 \quad p_1 - p_2 = \frac{G^2}{g \rho_{avg}} \left[\frac{2fL}{D_H} + \ln \frac{t_2}{t_1} + \ln \frac{p_1}{p_2} \right]$$

The log terms are now approximated by a series expansion. Thus,

$$\ln x = 2 \left[\frac{x-1}{x+1} + \frac{1}{3} \left(\frac{x-1}{x+1} \right)^3 + \frac{1}{5} \left(\frac{x-1}{x+1} \right)^5 + \dots \right] (x > 0)$$

Since both T_2/T_1 and p_1/p_2 are greater than zero, it is seen that, using the first term of the series:

$$\ln \frac{t_2}{t_1} \approx 2 \left[\frac{t_2/t_1 - 1}{t_2/t_1 + 1} \right] = \frac{t_2 - t_1}{\frac{t_2 + t_1}{2}} = \frac{t_2 - t_1}{t_{avg}}$$

$$\text{Similarly,} \quad \ln \frac{p_1}{p_2} \approx \frac{p_1 - p_2}{p_{avg}}$$

Substitution of these approximations into equation B8 and assuming identity of total and static temperatures yields:

$$B9 \quad p_{s1} - p_{s2} = \frac{G^2}{g \rho_{avg}} \left[\frac{2fL}{D_H} + \frac{T_2 - T_1}{T_{avg}} + \frac{p_{s1} - p_{s2}}{p_{avg}} \right]$$

However, an approximate relation for the total pressure is desired. From the energy relation, it is seen that:

$$P_1 = p_{s1} + q_1$$

$$P_2 = p_{s2} + q_2$$

$$B10 \quad P_1 - P_2 = p_{s1} - p_{s2} + q_1 - q_2$$

Noting that:

$$q = 1/2g \rho V^2 = \frac{G^2}{2g \rho} = \frac{G^2 R}{2g} \frac{t}{p}$$

Taking the logarithmic differential, noting that $G^2R/2g$ is a constant, yields:

$$\frac{dg}{q} = \frac{dT}{T} - \frac{dp}{p}$$

Integrating this relation and applying the same series approximations to the resultant log term yields:

$$\ln \frac{q_2}{q_1} = \ln \frac{T_2}{T_1} + \ln \frac{p_1}{p_2}$$

$$B11 \quad \frac{q_2 - q_1}{q_{avg}} = \frac{T_2 - T_1}{T_{avg}} + \frac{p_1 - p_2}{p_{avg}}$$

Noting that $q_{avg} = G^2/2g \cdot C_{avg}$ and substituting equation B9 and the negative of equation B11 into equation B10 and rearranging yields:

$$B12 \quad p_1 - p_2 = \frac{G^2}{g C_{avg}} \left[\frac{2fL}{D_H} + \frac{T_2 - T_1}{2T_{avg}} + \frac{p_{s1} - p_{s2}}{2p_{avg}} \right]$$

Equation B12 presents the total pressure loss. The approximations which have gone into it have been noted. It is considered more useful than an approximate integration of equation B1 since it presents the total pressure loss in more direct terms of system geometry and performance. However, in the form above, it is not possible to separate completely the geometric and performance requirements.

Up to this point, total and static pressures have been kept separate. The essential identity of total and static pressures shall now be assumed but not necessarily total and static pressure drops. It is to be kept in mind that a form for data correlation is desired and not an exact solution. Expanding equation B12 with the aid of equation B2 yields:

$$p_1 - p_2 = \frac{RG^2}{2g} \left(\frac{T_2 + T_1}{P_2 + P_1} \right) \left[(4) 0.046 \frac{KL}{D^{1.2} G^{0.2}} \mu^{0.2} + 2 \left(\frac{T_2 - T_1}{T_2 + T_1} \right) + 2 \left(\frac{P_1 - P_2}{P_1 + P_2} \right) \right]$$

$$\left(\frac{1 - P_2}{P_1} \right) = \frac{RG^2}{2g} \frac{T_1}{P_1^2 (P_2/P_1 + 1)} \left[\frac{CK'L}{D^{1.2} G^{0.2}} + 2 \left(\frac{T_2/T_1 - 1}{T_2/T_1 + 1} \right) + 2 \left(\frac{1 - P_2/P_1}{P_2/P_1 + 1} \right) \right]$$

which is valid under the assumption that $\mu^{0.2}$ is a constant. This is a good approximation for air in the temperature range of interest and must be re-evaluated for other coolants. Thus, the constant $c = 4(0.046) \mu^{0.2}$.

$$B13 \quad \frac{(1 - P_2/P_1)(P_2/P_1 + 1)}{R/2g (T_2/T_1 + 1)} + \frac{G^2 T_1}{P_1^2} \left[\frac{CK'L}{D^{1.2} G^{0.2}} + 2 \left(\frac{1 - P_2/P_1}{P_2/P_1 + 1} + \frac{T_2/T_1 - 1}{T_2/T_1 + 1} \right) \right]$$

This relation is now written in the form:

$$B14 \quad \xi = \frac{G^2 T_1}{P_1^2} \left(\frac{K'L}{D_H^{1.2} G^{0.2}} + \psi \right)$$

By comparison of equations B13 and B14 it is seen that there are two separate parameter groupings, ξ and ψ , which are specified only by the performance requirements of the system, one parameter grouping, $K'L/D_H^{1.2} G^{0.2}$, which describes the geometry of the system (and is identical with the geometric grouping of the heat transfer correlation of Appendix A, and the very useful function, $G^2 T_1/P_1^2$). After some further manipulation, it is seen that the form of the variables ξ and ψ is given by the relations:

$$\xi = a \left[\frac{b - (P_2/P_1)^2}{c + T_2/T_1} \right] \quad \psi = \frac{d (T_2/T_1 - P_2/P_1)}{(e + P_2/P_1) (f + T_2/T_1)}$$

where the parameters "a" through "f" are constants to be determined in the correlation with computer data.

Thus, a desirable form for data correlation has been obtained on an analytical basis. Since the analysis was approximate, only the indicated grouping is used in the correlation with the computer data. It is noted that attempted correlation with constants "a" through "f," as determined from the derivation, is not exact. This is to be expected due to the approximations of the derivation, but without which no deviation would have been possible. The values of the parameters "a" through "f" as determined from the computer data produce a quite accurate correlation over the range specified.

The final form of the correlation is found to be:

$$\xi = \frac{G^2 T_1}{P_1} \left(\frac{K'L}{D_H^{1.2} G^{0.2}} + \psi \right)$$

where:

$$B15 \quad \xi = 73.6 \left[\frac{1 - (P_2/P_1)^2}{0.542 + T_2/T_1} \right]$$

$$B16 \quad \psi = \frac{101 (T_2/T_1 - P_2/P_1)}{(P_2/P_1 - 0.100) (T_2/T_1 + 0.111)}$$

All temperatures and pressures are total.

Comments

- (1) It is noted that the hydraulic diameter, D_H , used in the above work is identical with that for the heat transfer correlation of Appendix A for the case of any passage so long as the heat transfer surface is equal to the surface wetted by the fluid.
- (2) In cases where the heated length is not equal to the pressure drop length, the above correlation is not accurate and the pressure drops in the unheated portion of the passage should be computed from the Fanning equation. In cases where the over-all system pressure drop is specified, the problem becomes an iterative one in proportioning pressure drops between heated and unheated portions of the system, since in general the length and hydraulic diameter are not known. A good first approximation to this problem is to assign an additional multiplier, K , to the $K'L/D_H^{1.2} G^{0.2}$ term which multiplier is an educated guess as to the ratio of heated plus unheated length to heated length. In general, methods will vary with geometry conditions known. (Note that for smooth tube friction factors, the multiplier, K , is unity.)

APPENDIX C

NOMENCLATURE

A_{ff}	Frontal free flow area of active core	inches ²
D_H	Hydraulic diameter	inches
F_n	Net thrust	pounds
f	Coolant channel friction factor (smooth tube)	
G	Active core airflow rate per unit free flow area	pounds/sec-inches ²
g	Gravity constant	feet/sec ²
H	Total enthalpy	Btu/pound
h	Static enthalpy	Btu/pound
I_{sp}	Specific impulse	seconds
k	Ratio of specific heats, cp/cv	
L	Active core length	inches
M	Mach number	
p_s	Static pressure	psia
q	Dynamic head	pounds/inch ²
T	Total temperature	°R or °F
t	Static temperature	°R or °F
T_b	Bulk temperature of reactor coolant	°R
T_f	Wall film temperature of reactor coolant	°R
T_{sm}	Maximum surface temperature of the average reactor fuel element	°R
W	Active core airflow	pounds/sec
μ	Viscosity	pounds/inch-sec
ξ	Empirical function of reactor temperature and pressure ratios	
ρ	Density	
ψ	Empirical function of reactor temperature and pressure ratios, different from ξ	

Subscripts

Primary Stations:

- 0 - Free stream conditions
- 3 - Reactor inlet
- 4 - Reactor exit
- 5 - Nozzle inlet
- 8 - Nozzle exit

AIRCRAFT NUCLEAR PROPULSION DEPARTMENT

GENERAL ELECTRIC