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ADDENDUM II

**AEC RESEARCH AND
DEVELOPMENT REPORT**

**SEAL-SHELL-2 — A COMPUTER PROGRAM
FOR THE STRESS ANALYSIS OF A
THICK SHELL OF REVOLUTION WITH
AXISYMMETRIC PRESSURES, TEMPERATURES,
AND DISTRIBUTED LOADS**

JUNE 1969

CONTRACT AT-11-1-GEN-14

**BETTIS ATOMIC POWER LABORATORY, PITTSBURGH, PA.,
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UC-32 Mathematics and Computers

SEAL-SHELL-2 - A COMPUTER PROGRAM FOR THE STRESS ANALYSIS OF A
THICK SHELL OF REVOLUTION WITH AXISYMMETRIC PRESSURES,
TEMPERATURES, AND DISTRIBUTED LOADS

C. M. Friedrich

June 1969

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NOTATION OF SUBSCRIPTS AND ARGUMENTS

a, b	range of segments on an input card
a, b, c	center points of fillet circles on surfaces of shell in sample problem
e	external surface of shell
i, j	index on an input card = 1, 2, 3, ...
i	inside surface of shell
k	position along x axis, 0 to n
k	segment, 1 to n
m	angular direction of a bending moment or its working deflection
max	maximum value
n	number of segments, 2 to 100
n	end point on x axis
nom	nominal stress
o	origin of x axis
oct	octahedral (von Mises) stress
p	effect of pressure
r	radial direction in meridian plane
s	stress signal on input card 90i
s	effect of shear stress
t	effect of temperature
t	effect of normal stress
w	effect of applied loads
x	direction along reference surface in meridian plane
x	reference surface of shell
z	axial direction in meridian plane
θ	circumferential direction perpendicular to meridian plane

NOTATION OF VARIABLES

A	8 x 8 flexibility matrix of a segment
A_{ij}	coefficient in row i and column j of A, where i and j = 1 to 8
D_{-}	generalized deflection corresponding to the generalized force F_{-}
D	vector of deflections corresponding to the force vector F acting on the free body of a segment
E	modulus of elasticity on card 40i
F_{-}	generalized force
F	vector of forces acting on the free body of a segment
G	magnitude of a boundary condition on card 102
h	thickness perpendicular to reference surface of shell
M_{-}	input option on an input card
P	pressure
r	radius in meridian plane, positive to the right
S	normal stress, positive tension or negative compression
SX0YMT	stress signal on card 102
T	temperature
W	applied loading on circumference of reference surface
x	distance along reference surface in meridian plane
y	distance normal to x axis, positive inward
z	axial coordinate (elevation) in meridian plane, positive upward
Δy	distance along y axis from reference surface to middle surface
τ	shear stress
ν	Poisson's ratio

A revised version of Seal-Shell-2 is described. The reference surface of the shell may be placed anywhere between the inside and outside surfaces, not just at the middle surface. Also, the program may be used to plot stress against segment index.

SEAL-SHELL-2 - A COMPUTER PROGRAM FOR THE STRESS ANALYSIS OF A
THICK SHELL OF REVOLUTION WITH AXISYMMETRIC PRESSURES,
TEMPERATURES, AND DISTRIBUTED LOADS

C. M. Friedrich

I. INTRODUCTION

The original version of SEAL-SHELL-2 (Reference 1) determines stresses, strains, deflections, and reactions in a thick shell of revolution with axisymmetric loading. The shell material is linear-elastic with tensile, bending, and shear strains. The loading consists of a temperature distribution, pressures on the inside and outside surfaces, and circumferential forces and moments applied to the reference surface midway between the inside and outside surfaces.

Version 2 (Reference 2) has two additional options. First, the shape of the shell in a meridional plane may be plotted on radial r and axial z coordinates. Second, the flexibility matrix of any segment may be input, instead of being calculated in the program. This option permits the treatment of branched shells.

Version 3 allows the middle surface to be shifted away from the reference surface of the shell. Preparation of input data is thereby greatly simplified for shells with varying thicknesses and sharp bends in the reference surface.

In version 4, any stress may be plotted against segment index. Also the program may list maximum and minimum stress components along with the segments containing these components. All the options of the previous versions are included in version 4.

All options in version 4 are discussed in Section II. In Section III the required input cards are explained. In Section IV a sample problem is included to show how version 4 of SEAL-SHELL-2 may be used. Finally, stresses calculated for the sample problem by SEAL-SHELL-2 are compared with photoelastic test data.

II. OPTIONS IN SEAL-SHELL-2

The following four options are in version 4 of SEAL-SHELL-2:

- A. Option to Plot Shell Geometry;
- B. Option to Input Segment Flexibility;
- C. Option to Shift Middle Surface from Reference Surface; and
- D. Option to Plot Stresses.

Options A and B are discussed in Reference 2. However, for completeness the four options are explained in the following.

A. Option to Plot Shell Geometry

The program may be instructed to make a plot of the shell geometry in a meridional plane with radial r and axial z coordinates. A CDC-6600 microfilm plotter is directed by a set of FORTRAN subroutines (Reference 4).

Each segment is plotted as in Figure 1. The outside, reference, and inside surfaces are drawn as straight lines, even though the segment may be curved; however, the actual straight or curved geometry is used in all calculations. Boundary surfaces at positions $k-1$ and k on each segment k are plotted correctly as straight lines. If the slope of the reference surface is not continuous at any position k , two boundary surfaces are drawn (Figure 1).

Figure 1. Plot of Shell Geometry

The input signal for the plot is -110.0 in place of +110.0 in columns 61 to 70 of the title card.

B. Option to Input Segment Flexibility

Consider segment k as a free body with external pressure P_e , inside pressure P_i and boundary loads at positions $k-1$ and k (Figure 2). The flexibility of the free body may be specified by the matrix equation

$$D(k) = A(k)F(k)$$

where $D(k)$ is a vector of 8 generalized deflections corresponding to the force components of the vector $F(k)$, $A(k)$ is the 8×8 flexibility matrix, and the 8 forces of the vector $F(k)$ are, in order:

- F_t , a unit load representing thermal effects;
- F_p , a unit load representing effect of pressure on the inside and outside surfaces;
- F_w , a unit load representing effects of circumferential loads externally applied to position k on each segment k ;
- F_{zk} , the circumferential force (pounds) applied by segment $k+1$ to segment k in the negative axial direction at position k on the reference surface;

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Figure 2. Free-Body Sketch of Segment k

- F_{rk-1} , the circumferential force (pounds) applied by segment $k-1$ to segment k in the negative radial direction at position $k-1$ on the reference surface;
- F_{mk-1} , the circumferential bending moment (inch-pounds) applied by segment $k-1$ to segment k in the clockwise direction at position $k-1$ on the reference surface;
- F_{rk} , the circumferential force (pounds) applied by segment $k+1$ to segment k in the positive radial direction at position k on the reference surface; and
- F_{mk} , the circumferential bending moment (inch-pounds) applied by segment $k+1$ to segment k in the counterclockwise direction at position k on the reference surface.

The remaining load F_{zk-1} in Figure 2 is calculated from an equation of axial equilibrium for the segment, and it is not listed separately in the force vector $F(k)$. Therefore the axial deflection D_{zk} at position k is relative to the axial deflection at $k-1$.

For each force component $F_{_}$, there is a corresponding deflection component $D_{_}$ in the deflection vector $D(k)$. Since D_t , D_p , and D_w , the first three deflections, are not used in the theory of SEAL-SHELL-2, the first three rows of the flexibility matrix $A(k)$ may contain all zeroes.

If the geometry of a previous SEAL-SHELL-2 problem is to be used as a single segment in the present problem, then the 5 by 8 flexibility matrix in the output of the previous problem becomes the last 5 rows of the input flexibility matrix of the present problem. All 8 rows must be present in the input, even though the first 3 rows may contain only zeroes.

Four steps must be accomplished for each special segment whose flexibility matrix is input:

1. Set the elastic modulus $E_t = 0$ in a 40i card for each special segment.
2. Set the pressures $P_i = 0$ and $P_e = 0$ in a 60i card for each special segment.
3. Set $W_{rk} = 0$, set $W_{mk} = 0$, and calculate the total axial load W_{zk} on the free body of the segment. The effects of pressure and applied loads in the previous problem are put into the calculation of W_{zk} , under the assumption that the end load $F_{zn} = 0$ in the previous problem. In this way, the axial load F_{zk-1} will be calculated correctly in the equation of axial equilibrium.

4. Put the flexibility matrix on 8 cards after the 1001 card. Each of the 8 cards has the format 9E8.0 with the card number $100k+1$ in the ninth field, where k is the segment, 1 to n , and i is the matrix row, 1 to 8. In the first 8 fields are the coefficients $A_{i1}(k)$ to $A_{i8}(k)$. These cards must be in increasing numerical order after the 1001 card. For example, if segments 3 and 42 have $E_t = 0$, then the cards after 1001 are 301, 302, ..., 308, 4201, 4202, ..., 4208.

The input flexibility matrix may be for any axisymmetric structure, such as a tube sheet in a heat exchanger (Reference 2). In this way, different branches of shells may be connected together. Another application is to condense a group of segments into a single segment and so reduce running time on the next problem containing that group of segments. Also, using the condensed group of segments as a single segment allows more segments to be specified in a critical region of the shell.

C. Option to Shift Middle Surface from Reference Surface

In versions 1 and 2 of SEAL-SHELL-2, a shell of revolution is described geometrically in a meridional plane by a curvilinear reference line x and a distribution along x of thickness $h(x)$ which is both normal to the line and bisected by the line. Bisection makes the description difficult when the line has corners and the thickness changes.

Version 3 allows the program user to input a shift $\Delta y(x)$ along y of the middle surface relative to the reference surface of the x axis (Figure 3). Then the inside surface has the y coordinate

$$y_i(x) = \Delta y(x) + 0.5h(x),$$

the middle surface has the y coordinate $\Delta y(x)$, and the outside surface has the coordinate

$$y_e(x) = \Delta y(x) - 0.5h(x),$$

where

$$|\Delta y(x)| \leq 0.5h(x) \quad .$$

Figure 3. Shift of Middle Surface from Reference Surface

At each point k on the line x between segments k and $k+1$, the loads F_{zk} , F_{rk} , F_{mk} , W_{zk} , W_{rk} , and W_{mk} act on the circumference $2\pi r_k$ of the reference surface. Stresses, however, are still calculated at the outside surface $y_e(x)$, middle surface $\Delta y(x)$, and inside surface $y_i(x)$. Also, temperatures are specified at the five surfaces y_e , $0.5(y_e + \Delta y)$, Δy , $0.5(y_i + \Delta y)$, and y_i .

The shift Δy is input with thickness h on cards 30i, in the form

$$k_a, k_b, h_a, h_b, \Delta y_a, \Delta y_b, 30i ;$$

where h_a and Δy_a are the thickness and shift at position k_a-1 on segment k_a , h_b and Δy_b are at position k_b on segment k_b , and in between values are interpolated against index k .

III. INPUT CARDS

Figure 4 is a table of input cards. The title and comment cards have the format (6A10, E10.0, I2), but column 1 should be left blank. The regular data cards 101 to 1001 have the format (7E10.0), and the flexibility cards have the format (9E8.0). The four options are input as follows:

A. Plot Option

The program will plot shell geometry if the title card number is - 110.0, not positive as usual.

B. Flexibility Option

For each segment whose flexibility matrix $A(k)$ is input, the tensile modulus $E_t = 0$. All 8 rows must be input for each matrix, even though the contents of the first 3 rows are all zero.

C. Shift Option

On cards 30i, Δy_a is the middle-surface shift thickness h_a and Δy_b is the shift of h_b , positive towards the inside surface.

Regular Input

10	20	30	40	50	60	70	72	Columns
Title						$\pm 110.$	Nc	Title
Comments						Ic.		Comments, Ic = 1 to Nc.
N.	Mzn.	Mro.	Mmo.	Mrn.	Mmn.	101.		M = O(D) or 1(F).
SX0YMT.	Gzn.	Gro.	Gmo.	Grn.	Gmn.	102.		G = size of D or F.
	Ro.	Zo.				201.		X Axis.
Kb.	Rb.	Zb.	Rc.	Zc.	Se.	20i.		
Ka.	Kb.	Ha.	Hb.	ΔY_a .	ΔY_b .	30i.		H and ΔY .
Ka.	Kb.	α .	Et.	Es.	v.	40i.		Material.
Ka.	Ta(1).	Ta(2).	Ta(3).	Ta(4).	Ta(5).	50i.		Temperatures.
Kb.	Tb(1).	Tb(2).	Tb(3).	Tb(4).	Tb(5).	50j.		
Ka.	Kb.	Pi.	Pe.			60i.		Pressures.
Ka.	Kb.	Wr.	Wm.	Wz.		70i.		External Loads.
Ka.	Kb.	Mffd.	Ka.	Kb.	Mffd.	80i.		F and D Output.
Ka.	Kb.	Mss.	Ka.	Kb.	Mss.	90i.		S Output.
						1001.		

Flexibility Input

8	16	24	32	40	48	56	64	72	Columns
A_{i1}	A_{i2}	A_{i3}	A_{i4}	A_{i5}	A_{i6}	A_{i7}	A_{i8}	koi	Row i of $A(k)$.

Figure 4. Table of Input Cards

D. Stress-Plot Option

The stress plot option is controlled by the number SXØYMT in the first field of card 102, where

$$SXØYMT = 10^5 M_s + 10^4 M_x + 10^3 M_\theta + 100 M_y + 10 M_m + M_t$$

and

$$\begin{aligned} M_s &= \text{control for shear stress } \tau_{xy} \\ M_x &= \text{control for tensile stress } S_x \\ M_\theta &= \text{control for hoop stress } S_\theta \\ M_y &= \text{control for thickness stress } S_y \\ M_m &= \text{control for von Mises stress } S_{\text{oct}} \\ M_t &= \text{control for Tressa stress } 2\tau_{\text{max}} \end{aligned}$$

Each control digit M_- is formed as on card 90i; that is

$$M_- = 4(M_-)_e + 2(M_-)_{\Delta y} + (M_-)_i$$

where

$$\begin{aligned} (M_-)_e &= \text{option for external surface, 0 or 1,} \\ (M_-)_{\Delta y} &= \text{option for middle surface, 0 or 1,} \\ (M_-)_i &= \text{option for internal surface, 0 or 1.} \end{aligned}$$

For example, if the program is to plot S_x and S_θ at the outside and inside surfaces,

$$\begin{aligned} (M_x)_e &= 1, (M_x)_{\Delta y} = 0, (M_x)_i = 1, \\ M_x &= 4(1) + 2(0) + 1(1) = 5, \\ M_\theta &= M_x = 5, \text{ and} \end{aligned}$$

$$SXØYMT = 10^5(0) + 10^4(5) + 10^3(5) + 100(0) + 10(0) + (0) = 55000$$

In the stress plots, the largest discontinuity in stress between segments and the differences between S_y and the nominal surface values ($-P_i$ and $-P_e$) indicate the accuracy of the stress calculations for each segment.

For each of the six stress components, the program lists the minimum stress with its segment index and the maximum stress with its segment index.

IV SAMPLE PROBLEM

The geometry of the sample problem is a cylindrical nozzle on a sphere, as given for sphere N8F in Reference 3 and as shown in Figure 5. The sphere has an inside radius of 7.1665 inches and a thickness of 0.606 inch; the nozzle has an inside radius of 0.9225 inch and a thickness of 0.163 inch; and fillets of 0.188-inch radius are on the interior and exterior surfaces. Segment positions are arbitrarily partitioned into 30 segments in the nozzle, 20 segments in the nozzle bend, 25 in the upper sphere, and 25 in the lower sphere, for a total of 100 segments.

The loading is internal pressure. Photoelastic stress data in Reference 3 are normalized by the nominal spherical stress

$$S_{\text{nom}} = P_i(2r_i + h)/(4h) \quad .$$

In SEAL-SHELL-2, $P_i = 1,000$ psi, so that

$$S_{\text{nom}} = (1000 \text{ psi})(2 \times 7.1665 + 0.606)/(4 \times 0.606) = 6163 \text{ psi} \quad .$$

Geometry of segments is first determined. Then the input cards are listed. Finally, the output stresses of SEAL-SHELL-2 are compared with the photoelastic results of Reference 3.

A. Geometry of Segments

As shown in Figure 6, the curvilinear x axis arbitrarily is placed on the middle surface of the cylinder, on a fillet circle with a radius equal to the thickness of the sphere, and on the middle surface of the sphere. Coordinates r and z are first determined for the fillet centers a, b, and c.

Point a is at the radius

$$r_a = 0.9225 + (0.5 \times 0.163) + 0.606 = 1.61$$

and the elevation

$$z_a = \sqrt{(7.1665 + 0.303 + 0.606)^2 - (r_a)^2} = 7.913381;$$

point b is at the radius

$$r_b = 0.9225 + 0.163 + 0.118 = 1.2035$$

and the elevation

$$z_b = \sqrt{(7.1665 + 0.606 + 0.118)^2 - (r_b)^2} = 7.798178;$$

Figure 5. Geometry of Sample Problem

Figure 6. Nozzle Bend

and point c is at the radius

$$r_c = 0.9225 + 0.118 = 1.0405$$

and the elevation

$$z_c = \sqrt{(7.1665 + 0.118)^2 - (r_c)^2} = 7.209806$$

Also shown in Figure 6 are the nine lines L_1 to L_9 which form the inside, reference, and outside surfaces of the shell. The equations for these lines are as follows:

$$0 = r - 0.9225 \quad (L1)$$

$$0 = r - (0.9225 + 0.5 + 0.163) \quad (L2)$$

$$0 = r - (0.9225 + 0.163) \quad (L3)$$

$$0 = (r - 1.0405)^2 + (z - 7.209806)^2 - (0.118)^2 \quad (L4)$$

$$0 = (r - 1.61)^2 + (z - 7.913381)^2 - (0.606)^2 \quad (L5)$$

$$0 = (r - 1.2035)^2 + (z - 7.798178)^2 - (0.118)^2 \quad (L6)$$

$$0 = r^2 + z^2 - (7.1665)^2 \quad (L7)$$

$$0 = r^2 + z^2 - (7.1665 + 0.303)^2 \quad (L8)$$

$$0 = r^2 + z^2 - (7.1665 + 0.606)^2 \quad (L9)$$

Figure 7 indicates the arbitrary placement of segments in the nozzle bend. Each boundary k has the inside point k_i , the reference point k , and the outside point k_e . Coordinates for each point are determined by an intersection of two lines, as indicated by the following table:

Point	Line	Intersecting Line
30i	L_1	$0 = z - z_a$
30	L_2	$0 = z - z_a$
30e	L_3	$0 = z - z_a$
34e	L_3	$0 = z - z_b$
34	L_5	$0 = (r - r_a)(z_{34e} - z_a) - (z - z_a)(r_{34e} - r_a)$
34i	L_1	$0 = (r - r_a)(z_{34e} - z_a) - (z - z_a)(r_{34e} - r_a)$
37e	L_6	$0 = (r - r_b) - (z - z_b)$
37	L_5	$0 = (r - r_a)(z_{37e} - z_a) - (z - z_a)(r_{37e} - r_a)$
37i	L_1	$0 = (r - r_a)(z_{37e} - z_a) - (z - z_a)(r_{37e} - r_a)$

Point	Line	Intersecting Line
39e	L ₉	$0 = (r)(z_b) - (z)(r_b)$
39	L ₅	$0 = (r - r_a)(z_{39e} - z_a) - (z - z_a)(r_{39e} - r_a)$
39i	L _i	$0 = (r - r_a)(z_{39e} - z_a) - (z - z_a)(r_{39e} - r_a)$
43i	L ₁	$0 = z - z_c$
43	L ₅	$0 = (r - r_a)(z_{43i} - z_a) - (z - z_a)(r_{43i} - r_a)$
43e	L ₉	$0 = (r - r_a)(z_{43i} - z_a) - (z - z_a)(r_{43i} - r_a)$
44i	L ₄	$0 = (r - r_c) - (z - z_c)$
44	L ₅	$0 = (r - r_a)(z_{44i} - z_a) - (z - z_a)(r_{44i} - r_a)$
44e	L ₉	$0 = (r - r_a)(z_{44i} - z_a) - (z - z_a)(r_{44i} - r_a)$
45i	L ₇	$0 = (r)(z_c) - (z)(r_c)$
45	L ₅	$0 = (r - r_a)(z_{45i} - z_a) - (z - z_a)(r_{45i} - r_a)$
45e	L ₉	$0 = (r - r_a)(z_{45i} - z_a) - (z - z_a)(r_{45i} - r_a)$
50i	L ₇	$0 = (r)(z_a) - (z)(r_a)$
50	L ₈	$0 = (r)(z_a) - (z)(r_a)$
50e	L ₉	$0 = (r)(z_a) - (z)(r_a)$

Note that the segment boundaries are normal to L₅. The resulting coordinates are listed next:

k	r _{ki}	z _{ki}	r _k	z _k	r _{ke}	z _{ke}
30	0.922500	7.913381	1.004000	7.913381	1.085500	7.913381
34	0.922500	7.762376	1.018109	7.783376	1.085500	7.798178
37	0.922500	7.634640	1.048403	7.685686	1.120061	7.714739
39	0.922500	7.537931	1.078142	7.622928	1.185502	7.681559
43	0.922500	7.209806	1.186473	7.479951	1.355883	7.653322
44	0.957061	7.126367	1.223066	7.446993	1.389274	7.647331
45	1.023649	7.093016	1.257619	7.420367	1.416297	7.642373
50	1.428774	7.022630	1.489183	7.319547	1.549591	7.616464

The r and z coordinates of points ki, k, and ke are used to determine the y coordinates

$$y_{ki} = +\sqrt{(r_{ki} - r_k)^2 + (z_{ki} - z_k)^2}$$

$$y_{ke} = -\sqrt{(r_{ke} - r_k)^2 + (z_{ke} - z_k)^2}$$

the thickness

$$h_k = y_{ki} - y_{ke}$$

and the middle-surface shift

$$\Delta y_k = 0.5(y_{ki} + y_{ke})$$

The resulting data are as follows:

<u>k</u>	<u>y_{ki}</u>	<u>y_{ke}</u>	<u>h_k</u>	<u>Δy_k</u>
30	.081500	-.081500	.163000	.000000
34	.097888	-.068997	.166885	.014446
37	.135858	-.077324	.213182	.029267
39	.177339	-.122326	.299665	.027506
43	.377704	-.242398	.620103	.067653
44	.416604	-.260309	.676913	.078148
45	.402371	-.272884	.675254	.064743
50	.303000	-.303000	.606000	.000000

The geometric data have now been calculated for use on input cards.

B. Input

The title and data cards are listed in Figure 8.

Cards 101 and 102

There are 100 segments. The boundary conditions are:

$$F_{zn} = 0., F_{ro} = 0., D_{mo} = 0., F_{rn} = 0., \text{ and } F_{mn} = 0.$$

To plot S_x versus k on outside and inside surfaces let SXθYMT = 50000.

Cards 201 to 309

The geometric data were calculated in the preceding section of this report.

Card 401

The thermal expansion α is not needed. For plexiglass, the remaining properties are $E_t = 4300$ psi, $E_s = 1500$ psi, and $\nu = 0.45$.

Cards 501 and 502

For pressure loading only, all temperatures are zero.

00000000011111111122222222233333333344444444455555555566666666677777777778
 1234567890123456789012345678901234567890123456789012345678901234567890

CHAC110,3,007,120,0000.M0110,FRIRH,U,P.

NOZZLE ON SPHERE(TAYLOR-LIND)

							-110.
100.	1.	1.	0.	1.	1.		101.
50000.	0.	0.	0.	0.	0.		102.
0.	1.004000	10.	0.	0.	0.		201.
30.	1.004000	7.913381	0.	0.	0.		202.
34.	1.018109	7.783376	1.61	7.913381	-1.		203.
37.	1.048403	7.685686	1.61	7.913381	-1.		204.
39.	1.078142	7.622928	1.61	7.913381	-1.		205.
43.	1.186473	7.479951	1.61	7.913381	-1.		206.
44.	1.223066	7.446993	1.61	7.913381	-1.		207.
45.	1.257619	7.420367	1.61	7.913381	-1.		208.
50.	1.489183	7.319547	1.61	7.913381	-1.		209.
75.	7.469500	.0	0.	0.	+1.		210.
100.	.001000	-7.469500	0.	0.	+1.		211.
1.	30.	.163	.163	.0	.0		301.
31.	34.	.163	.166885	.0	.014446		302.
35.	37.	.166885	.213182	.014446	.029267		303.
38.	39.	.213182	.299665	.029267	.027506		304.
40.	43.	.299665	.620103	.027506	.067653		305.
44.	44.	.620103	.676913	.067653	.078148		306.
45.	45.	.676913	.675254	.078148	.064743		307.
46.	50.	.675254	.606	.064743	.0		308.
51.	100.	.606	.606	.0	.0		309.
1.	100.	0.	4300.	1500.	.45		401.
1.	0.	0.	0.	0.	0.		501.
100.	0.	0.	0.	0.	0.		502.
1.	100.	1000.					601.
1.	100.	0.	0.	0.			701.
1.	100.	0.					801.
1.	100.	5.					901.
							1001.

FIGURE 8. INPUT CARDS FOR SAMPLE PROBLEM

Card 601

The inside pressure is 1000 psi.

Card 701

There is no distributed loading.

Card 801

Forces and deflections are not wanted in the output.

Card 901

To print stresses on inside and outside surfaces, let $M_{ss} = 55$.

Card 1001

This is the last data card.

C. Output

The problem ran less than 3 minutes on the CDC-6600 computer. Figure 9 is the geometric plot, and Figures 10 and 11 are the stress plots.

Figure 12 compares stresses from SEAL-SHELL-2 with photoelastic stresses from Reference 3. Apparently, the assumption that plane cross sections remain plane is fairly reasonable for the segment geometry of Figure 7.

REFERENCES

1. C. M. Friedrich, "SEAL SHELL-2, A Computer Program for the Stress Analysis of a Thick Shell of Revolution with Axisymmetric Pressures, Temperatures, and Distributed Loads," WAPD-TM-398, December 1963
2. C. M. Friedrich, "SEAL SHELL-2, A Computer Program for the Stress Analysis of a Thick Shell of Revolution with Axisymmetric Pressures, Temperatures, and Distributed Loads," WAPD-TM-398, Addendum I, April 1967
3. C. E. Taylor and N. C. Lind, "Photoelastic Study of the Stresses Near Openings in Pressure Vessels," Welding Research Council Bulletin 113, Welding Research Council, New York, April 1966
4. C. J. Pfeifer, "CDC-6600 FORTRAN Programming-Bettis Environmental Report" WAPD-TM-668, January 1967

Figure 9. Plot of Geometry

Figure 10. Stress Plot S_{xe}

Figure 11. Stress Plot S_{xi}

Figure 12. Comparison of Test Results and SEAL-SHELL-2

PROGRAM ABSTRACT

1. Name or Designation of Program

The program name is SEAL-SHELL-2. The program number at BAPL is M0110.

2. Computer for Which Program is Designed and Programming Languages Used.

Except for the PLOT routine, the program is written in FORTRAN IV for a machine with 32000 words. The program has been used on PHILCO-2000, IBM-704, and CDC-6600.

3. Nature of Physical Problem Solved.

The SEAL-SHELL-2 program determines stresses, strains, deflections, and reactions in a thick shell of revolution with axisymmetric loading. The loading consists of a temperature distribution, inside and outside pressure distributions, and circumferential forces and moments applied to the middle surface. The shell is linear-elastic with tensile, bending, and shear strains.

4. Method of Solution.

The shell is divided into 2 to 100 segments. Each segment is subjected to 11 deformation shapes, whose magnitudes are calculated by the principle of virtual work. That is, for small virtual changes in magnitude, the changes in strain energy and external work are determined; then equating the energy to the work gives a set of simultaneous equations which may be solved for the magnitudes.

5. Restrictions on the Complexity of the Problem.

The basic restrictions are the limit of 100 segments, and the limit of 11 deformation shapes per segment.

6. Related and Auxiliary Programs.

The program has been changed to allow a plot of the shape of the shell, as discussed in Reference 2. The plot routine is discussed in Reference 3.

7. Typical Running Time.

The time is about 5 minutes on the PHILCO-2000.

8. References.

(1) C. M. Friedrich, "SEAL-SHELL-2, A Computer Program for the Stress Analysis of a Thick Shell of Revolution with Axisymmetric Pressures, Temperatures, and Distributed Loads," WAPD-TM-398, December, 1963.

(2) C. M. Friedrich, "SEAL-SHELL-2 A Computer Program for the Stress Analysis of a Thick Shell of Revolution with Axisymmetric Pressures, Temperatures, and Distributed Loads, WAPD-TM-398, Addendum I, April 1967

(3) C. J. Pfeifer, "CDC-6600 FORTRAN Programming-Bettis Environmental Report," WAPD-TM-668, January, 1967.

9. Unusual Features of the Program.

The program will handle thick shells of revolution, as well as the usual thin shell structures. As discussed in Reference 2, an option allows the requestor to input influence coefficients of the flexibility matrix of any segment. The coefficients may be for any axisymmetric structure, such as a tube sheet in a heat exchanger. In this way, different branches of shells may be connected together.

Another application is to condense a group of segments into a single element which may be used in a series of problems and so cut down running time on the computer. Also, using the condensed group of segments as a single segment allows the user to specify more elements in the analysis of a critical region in the shell.

10. Status

The program is in production with all options tested.

11. Machine Requirements.

The program is written in FORTRAN IV and requires 32000 words of high speed memory.

12. Operating System on Monitor Under Which Program is Executed.

Any system using standard FORTRAN-IV may be used. At BAPL, the system of Reference 3 is used.

13. Other Programming or Operating Information.

The plot routine is described in Reference 3.

14. Name and Establishment of Author.

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