PHOTOVOLTAIC ENGINEERING SERVICES
PERTINENT TO SOLAR ENERGY CONVERSION

R. O. Bell, J. C. T. Ho, W. Kurth, and T. Surek

Mobil Tyco Solar Energy Corporation

June 1975

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ABSTRACT

The application of the compound parabolic concentrator (CPC) for use with solar cells has been investigated. Experiments with state-of-the-art Si cells in a CPC and under solar concentration were performed. A theoretical model for calculating the behavior of Si solar cells with concentration was developed. Detailed calculations of the energy distribution in the CPC were made. Finally a cost effectiveness analysis shows that the CPC system will produce power at very much lower cost than will flat panel solar cell arrays.

INTRODUCTION

This is the final report of a program to investigate the possible use of the compound parabolic concentrator (CPC)^{1, 2, 3} with silicon solar cells for electric power generation. The program was to study how solar cells can be designed to take advantage of the unique properties of the CPC and to anticipate any unusual problems that may arise. For example with most photovoltaic systems, either with or without concentrators, the solar radiation is near to or normally incident on the cell, but with a CPC relatively significant deviations can occur. It is important to know in what ways state-of-the-art cells must be modified to operate at concentrations around 10 suns. As has rapidly become clear over the course of this effort, the series resistance must be reduced to optimize the efficiency. This was found both experimentally and theoretically, and can be done most easily by increasing the density of the current collection grid bars on the solar cell.

In addition to a complete analysis of the distribution of radiation across the exit slit of the CPC for a wide range of conditions, an economic analysis has been made to determine how costs might be expected to vary as a function of materials and as the concentration changes. From this study the overall economics look very favorable with substantial cost reductions apparently possible for concentrations in the range of 6 to 10.

The first section of this report presents the experimental results of measuring state-of-the-art commercial solar cells with various junction depths, antireflection coatings, and temperatures in a heliostat concentrator. Some of these solar cells were then measured in a nominal 8 times CPC designed and built by the Argonne National Laboratory. The second section discusses the model developed to use in the design and optimization of solar cells. The third section is an analytical study of the CPC. The distribution of energy as a function of acceptance angle, degree of truncation, incident angle, reflectivity and concentration was examined. Also the energy distribution averaged over a day was

obtained. The fourth section analyzes the cost as a function of concentration and material cost. The final section is a brief summary of the results.

The CPC is a trough-like non-imaging collector which concentrates radiation by the maximum possible amount. The general description for designing such a concentrator has been given by R. Winston of Argonne National Laboratory and University of Chicago.^{1, 2, 3}

A number of different variations are possible for use with flat solar cells. Two of the simplest are shown in Fig. 1. Figure 1 shows a simple CPC with the solar cell on the bottom. The collector might or might not be filled with a dielectric either for thermal cooling or for increasing the acceptance angle. Cooling along the bottom of the cell is the usual case.

The concentration is given by the ratio of the entrance to exit areas. If something less than the maximum acceptance angle is to be used, then the length may be truncated and in practice almost certainly will be done. It is straightforward to calculate the coordinates of the surface when the entrance width, exit width and acceptance angle (which obviously must be less than or equal to the maximum) are specified (see Appendix). Figure 1a is to scale for an acceptance angle of 7.5° and a concentration of 6. Filling the collector with a dielectric of index of refraction of 1.5 will increase the acceptance angle from 7.5° to 11.3° but will not change the concentration.

Another possibility is the use of a two stage CPC with a dielectric fluid in the bottom section. This way the cell can be cooled at the top surface where all the solar energy is deposited. Another benefit would be the improved optical match into Si. In general, the use of a 2 stage CPC will reduce the overall length and thus the area of the reflecting surface for a fixed acceptance angle. Figure 1b was calculated also for a 7.5° acceptance angle and concentration of 6. The lower section is filled with a dielectric with an index of refraction of 1.5, which allows an overall reduction of the collector length by 35%.

Although many other types are possible this report will be concerned only with these two.

Fig. 1. Two different configurations of a compound parabolic collector. (a) Simple CPC with solar cell mounted at the bottom, concentration 6 and acceptance angle 7.5° , (b) Two-stage collector with a dielectric index of refraction of 1.5 in the bottom. Also for concentration 6 and acceptance angle 7.5° .

I. EXPERIMENTAL INVESTIGATION

The 1.1 by 2.1 cm silicon solar cells were fabricated of Czochralski grown material using standard state-of-the-art commercial processing. The n^+ top layer was formed by phosphorous diffusion. The diffusion time was adjusted to give junction depths of 0.25, 0.35, and 0.5 μm . The collector grid was made by a silver over aluminum evaporation of a ten finger (5 grid/ $\square$) collection grid. The antireflection coating was an evaporated silicon oxide (SiO_x) with thicknesses of 400, 800, and 1200 $\text{\AA}$.

A. Solar Concentration Measurements

All the experimental measurements were made outside between 10:00 AM and 2:00 PM. In order to make measurements as a function of solar concentration, a heliostat was constructed using up to 20 mirrors that could be adjusted to illuminate the solar cell holder. Figure 2 shows the mirrors mounted on the base plate and the cell mount at the top. Figure 3 shows the water cooled solar cell test fixture under illumination. Also mounted in the fixture was a calibrated photodiode (EG & G, Model SFD 160). The intensity was adjusted to be that of about ten suns and the lower levels were obtained with a set of neutral density filters. The test fixture was a copper block which used a series of spring fingers along the entire length to minimize the contact resistance. Figure 4 shows typical results of the short circuit current, fill factor and open circuit voltage as a function of concentration. This is for 20°C, 1200 $\text{\AA}$ AR coating and a 0.25 μm junction depth. The short circuit current, I_{sc} , is linear with intensity but the open circuit voltage, V_{oc} , increases from 0.59 to about 0.63 volts at a concentration of ten. The fill factor, (FF), shows a steady decrease with increasing concentration though, which is almost certainly due to series resistance either in the cell or test setup. The power output, $P = I_{sc} V_{oc} (FF)$ at one sun is 12.5 mW/cm^2 which is quite reasonable for a commercial cell. Since the incident energy for AM1 is about 100 mW/cm^2 , this is an actual efficiency obviously greater than 12%.

Fig. 2. Twenty mirror heliostat with the water cooled solar cell test fixture at the top.

Fig. 3. Solar cell test fixture while being illuminated with 10 suns. The round, calibrated Si photodiode is just above the solar cell.

Fig. 4. The short circuit current, fill factor and open circuit voltage as a function of concentration. The temperature was 20°C , the AR coating was $1200 \text{ \AA}$, and the junction depth was $0.25 \mu\text{m}$.

Figure 5 shows the power output versus concentration for the three different junction depths. Identical outputs were obtained for all depths, but close examination of the data indicates that the deepest junction had a slightly lower current, caused by the thickest dead layer, but the highest fill factor, caused by the lower series resistance, just compensating to produce a constant output. The overall shape of the power versus solar concentration is in excellent agreement with that calculated theoretically (see Section II).

The effect of AR coating is shown in Fig. 6. The 1200 Å and 800 Å give identical results but 400 Å was evidently too thin. The temperature dependence is shown in Fig. 7 at 20° and 78°C where again the power per unit area is plotted as a function of concentration. The decrease in efficiency of 0.4%/°C is independent of concentration and is also in good agreement with theory.

We had originally hoped to examine a finer collection grid structure but delays in obtaining the masks prevented us from fabricating any such cells.

B. Compound Parabolic Collector (CPC) Measurements

The solar cells, on which the tests reported in the foregoing, were mounted in an untruncated CPC designed to give a concentration of 8. Figure 8 shows the cells in the collector. Note how the cells appear to fill almost the entire aperture. The entrance width is 7.8 cm and where the cells were mounted the exit width was 1.3 cm thus the expected concentration is only 6. A silver epoxy was used to make thermal and electrical contact of the cells to a water cooled brass block. This proved to be relatively unsatisfactory as cell performance on the block was reduced to less than 10% compared to 12% in the other test fixture, probably due to additional series resistance. Because of the extremely limited time period of the contract and the large number of cloudy days, no design changes were possible.

Fig. 5. Power output versus concentration for three different junction depths 0.25, 0.35, and 0.50 μm . The temperature was 20°C and the AR coating 1200 Å.

Fig. 6. Power output versus concentration for three different AR coating thicknesses; 400, 800, and 1200 Å. The temperature was 20°C and the junction depth 0.25 μm .

Fig. 7. Power output versus concentration for 20° and 78°C. The average decrease in efficiency was 0.4%/°C. The AR coating was 1200 Å and the junction depth was 0.25 μm .

Fig. 8. View along the axis of a nominal 8 times CPC. Note how the solar cell image nearly fills the entire collector. The actual cells are located to the right of the center of the collector.

Figure 9 shows the I-V characteristics for a typical solar cell while under illumination in the CPC. The collector was aligned with the zero degree angle aimed directly at the sun and then curves were recorded as the angle was varied.

Figures 10 and 11 show the short circuit current versus the angle. The width of the response curves where I_{sc} has fallen to half of its maximum is about 15° which is very close to the ideal 14.36° for a times 8 collector. The slightly double peaked angular distribution may be related to the deviation from the ideal geometry. The deeper junction depth ($0.5 \mu m$) shows slightly less output than the shallower ($0.25 \mu m$) but whether this is due to experimental errors or is real is not clear (Fig. 10). The $400 \text{ \AA}$ AR coating is definitely not as good as the $1200 \text{ \AA}$ one (Fig. 11).

The general behavior of the solar cells in the CPC is the same as that determined with the heliostat concentrator. Unfortunately the program did not allow time for the fabrication of a more suitable solar cell mount for use in the CPC. Direct comparison of I_{sc} inside and outside the CPC showed an increase by a factor of 5 but because of the substantially poorer performance evidenced already at one sun and the errors in the positioning of the cells, 5 is just a lower limit on the concentration. The reflectivity of the aluminum in the concentrator was estimated to be 85% which would also explain part of the discrepancy in the measured and calculated concentration.

Fig. 9. Current-voltage characteristic of solar cell when mounted in the CPC. Each curve is for a different angle. The temperature is 31°C, the junction depth 0.50 μm , and the AR coating 1200 Å.

Fig. 10. The short circuit current as a function of the angle of incidence of the sun with respect to the CPC. The upper curve is for a junction depth of 0.25 μm and the lower for 0.50 μm . The AR coating was 1200 $\text{\AA}$ and the temperature was 31°C.

Fig. 11. The short circuit current as a function of the angle of incidence of the sun with respect to the CPC. The upper curve is for an AR coating of 1200 $\text{\AA}$ and the bottom for 400 $\text{\AA}$. The temperature was 31°C and the junction depth 0.25 μm .

II. THEORETICAL SOLAR CELL DESIGN FOR A CPC

Generally solar cells have been designed and fabricated to work with a unit solar flux. A cell optimized for such conditions is definitely not the best for use with concentration. It is the purpose of this section to develop a theoretical model of a Si solar cell that can be used for this optimization process. Of course, these conclusions will have to be verified experimentally, but the short duration of this contract does not allow enough time to significantly modify the fabrication process.

A. Model

If the physical properties, including electrical and optical, and the geometrical constraints are specified, it is possible to calculate the charge collection efficiency and other measureable electrical characteristics of a solar cell. Figure 12 shows the model that was used to simulate the solar cell. A n^+ on p cell was modeled, although, obviously, the reversed type could just as well be used. The substrate or base material is p doped Si with a thickness of d . The abrupt junction is a distance t_j below the surface. The doping in both the n and p region can be independently specified allowing the calculation of t_1 and t_2 . Since the p-n junction is generally formed in Si solar cells by diffusion from an unlimited source, a "dead" or heavily damaged layer, t_d , in which the lifetime of the minority carriers is essentially zero, occurs when the solubility limit of the dopant is exceeded. This is about $5 \times 10^{20} \text{ cm}^{-3}$ phosphorus atoms.⁴

Hole-electron pairs are generated by incident solar photons that have an energy greater than the energy gap of the semiconductor. The p-n junction produces an electric field, which in addition to diffusion, separates these charges. An external circuit is completed through a resistive load to extract useful work.

Fig. 12. Model assumed to simulate the n on p solar cell. The depletion region lies between t_1 and t_2 with the junction located at t_j . The dead layer is t_d thick and the total cell is d thick.

B. Optical Absorption and Charge Collection

The number of minority carriers generated as a function of depth below the top surface has been calculated using optical absorption data for Si^{5, 6} and the spectral irradiance curve of the sun at sea level⁷ (Fig. 13). Slightly different spectra are obtained depending upon the amount of water present in the atmosphere. One problem that often arises in discussing solar energy is the definition of the spectral distribution for different conditions. Unless otherwise specified in this report, we will take as the air mass one (AM1) spectra the solar irradiance curve at sea level, excluding the water bands, as shown in Fig. 13.

The high index of refraction of Si ($n = 3.42$)⁸ means that an antireflection coating is necessary if most of the optical energy is to be transmitted into the material. Figure 14 shows the reflectivity of Si as a function of angle for radiation polarized perpendicular and parallel to the plane of incidence. In this section we will assume that an AR coating is used so the reflectivity is zero but we should keep in mind especially for the next section that the reflecting power varies with angle and approaches 1 for a 90° angle of incidence.

The total number of hole electron pairs generated by the time the radiation has penetrated a fixed depth below the surface of Si is shown in Fig. 15 for three cases: (1) AM0,⁹ (2) AM1 without the H₂O bands, and (3) AM1 with H₂O bands. For all cases 50% of the pair generation has occurred within 3 μm of the surface and 90% within 100 μm. Total thicknesses of over 500 μm will add essentially nothing to the electrical performance of Si solar cells.

We have noted that this absorption curve can be generated exceedingly closely from 0.1 to 100 μm by the empirical expression

$$F/F_0 = x^n / (\beta + x^n) \quad (1)$$

where x is the depth below the surface in μm, and β and n are constants equal to 1.9 and 0.64 respectively.

Fig. 13. Spectral irradiance curve as a function of wavelength.

Fig. 14. Reflectivity of Si as a function of the angle of incidence. r_1 is for the radiation polarized perpendicularly to the plane of incidence and r_2 for the radiation parallel.

Fig. 15. Total number of hole electron pairs generated as a function of the depth of penetration of the radiation below the surface.

After generation of the pairs the next question is that of the minority carrier charge collection. The deviation of the number of holes from equilibrium, Δp , in the top, n-type region is given by the diffusion equation

$$D_p \frac{d^2 \Delta p}{dx^2} - \frac{\Delta p}{\tau_p} + \frac{dN}{dx} = 0 \quad (2)$$

$D_p = \mu_p kT/q$ is the diffusion coefficient, μ_p is the mobility of the holes, T is the temperature, k is the Boltzmann constant, q is the electronic charge, and τ_p is the hole lifetime.

The optical pair generation rate, dN/dx , is

$$\frac{dN}{dx} = -N_0 \alpha \exp(-\alpha x)$$

where N_0 is the number of incident photons per unit time after correcting for reflection losses at the surface and in the dead region. α is the optical absorption coefficient. Both $N_0(\lambda)$ and $\alpha(\lambda)$ depend on the wavelength.

The boundary conditions are:

$$D_p \frac{d\Delta p}{dx} - S \Delta p = 0 \quad \text{at } x = t_d \quad (3a)$$

and

$$\Delta p = 0 \quad \text{at } x = t_1 \quad (3b)$$

Here S is the surface recombination velocity and t_d and t_1 are defined in Fig. 13.

This system of equations can be solved for Δp and used to determine the contribution to the photocurrent of the n-region, J_S^p , by

$$J_S^p = q D_p \left. \frac{d\Delta p}{dx} \right|_{x=t_1} \quad (4)$$

The second current to consider is that from the depletion region on the n-side of the cell, J_d^p . Because of the built in electric field we assumed that all the minority carriers generated in this region are collected; thus,

$$J_d^p = qN_o [\exp(-\alpha t_1) - \exp(-\alpha t_j)] \quad (5)$$

Similar expressions for the collection of electrons on the p-side J_d^n , and J_b^n can be made with the exception that in place of the boundary condition Eq. (3a) we will assume that

$$\Delta p = 0 \text{ at } x = d$$

This is equivalent to an infinite surface recombination velocity at the back contact.

The total current collected will be the sum of the contributions from the four different regions

$$J_t = J_s^p + J_d^p + J_d^n + J_b^n \quad (6)$$

The expressions for these currents can be found in Ref. 10. Other details of this calculation are presented in the Appendix.

If a unit flux of photons is incident then the charge collection efficiency can be calculated. Figures 16, 17 and 18 show the collection efficiency as a function of wavelength for three different junction depths with different dead layer thicknesses. A cell thickness of $100 \mu m$ was assumed. The contribution from each of the three volumes (p-, n-, and depletion-regions) that make up the solar cell is shown. In all practical cases the major contribution comes from the substrate or p-region.

Fig. 16. Charge collection efficiency as a function of wavelength for a junction depth of 0.1 μm , no dead layer, surface recombination velocity of 10^6 cm/sec and a 100 μm thick solar cell. The contribution from the n-region, depletion region, and p-region is shown. For an AM1 spectrum 80.8% of the maximum possible charge is collected.

Fig. 17. Same as Fig. 16 except the junction depth is $0.3 \mu\text{m}$ and the dead region is $0.1 \mu\text{m}$. For an AM1 spectrum 69.7% of the maximum possible charge is collected.

Fig. 18. Same as Fig. 16 and 17 except the junction depth is $0.5 \mu\text{m}$ and the dead region is $0.3 \mu\text{m}$. For an AM1 spectrum 60.4% of the maximum possible charge is collected.

To find the total charge generated current, I_{ℓ} , it is necessary to multiply the charge collection efficiency by the solar photon distribution and integrate over all wavelengths where $\alpha(\lambda) > 0$.

C. Electrical Characteristics of the Solar Cell

The last step is to relate the charge generated to an electrical output into a load resistance. To do this the current-voltage relationship for the solar diode was assumed to be

$$I = I_{\ell} - I_s [\exp(\beta V) - 1] - I_s' [\exp(\beta V/2) - 1] \quad (7)$$

where I is the current output of the diode (we have neglected any internal shunt resistances which is a good approximation for a well behaved diode) and V is the voltage across the diode. The diffusion recombination current, I_s , is

$$I_s = q n_i^2 \left[\frac{1}{N_A} \left(\frac{D_n}{\tau_n} \right)^{\frac{1}{2}} + \frac{1}{N_D} \left(\frac{D_p}{\tau_p} \right)^{\frac{1}{2}} \right] \quad (8)$$

and the depletion region recombination current, I_s' , is

$$I_s' = q n_i W / (\tau_p \tau_n)^{\frac{1}{2}} \quad (9)$$

where $\beta = q/kT$, n_i is the intrinsic carrier concentration, N_A is the acceptor concentration, N_D is the donor concentration in the p- and -n regions respectively. $W = t_2 - t_1$ is the width of the depletion region.

The power, P , delivered to a load resistor, R_L , will be

$$P = I^2 R_L \quad (10)$$

where

$$V = I (R_L + R_S) \quad (11)$$

A plot of P versus R_L will enable us to determine the maximum power for various conditions. R_S is the series resistance and is made up primarily of the resistance in the base of the solar cell and of the spreading resistance of the top or surface layer. Lindmayer and Allison⁴ showed that as a function of the number of metal grid lines collecting the current from the top surface, m , the series resistance is approximately

$$R_S = \rho_s / 8 m^2 \quad (12)$$

where $\rho_s = \rho_n / t_1$ is the surface resistivity. ρ_n is the bulk resistivity in the n region.

D. Numerical Calculations

We are now in a position to calculate the efficiency and electrical behavior of our solar cell. The Appendix gives a listing of the computer program used to carry out the calculation along with additional descriptions of the various assumptions made about the temperature and concentration dependences of parameters such as mobilities, minority carrier lifetimes, energy gap, effective masses etc.

Figure 19 shows the I-V curves for different numbers of grid lines per square. This is for a solar concentration of 5 at room temperature, 300°K. With 5 grid lines per square, which is the typical number used with commercial solar cells, a significant resistance and thus a loss in efficiency occurs. For a concentration of one little loss occurs. This is made especially evident in Fig. 20 where the efficiency, η , and open circuit voltage, V_{oc} , are shown as a function of concentration, R . V_{oc} increases steadily but for 5 grid lines where $R_S = 0.48\Omega$ in this particular example, η peaks at a concentration of 4 and then falls.

Fig. 19. Calculated current voltage characteristics for between 20 and 1 collector grid/ $\square$. The concentration was 5, the temperature $300^\circ K$ and the sheet resistivity $95.4 \Omega / \square$.

Fig. 20. The efficiency for $R_s = 0$ and $R_s = 0.48 \Omega$ (5 grid/□) as a function of concentration. The open circuit voltage is also shown.

Figure 21 shows the total power output versus concentration. When $R_S = 0$ a superlinear increase is obtained which can be fit quite well by an empirical expression of the form

$$P = P_0 R^n \quad (13)$$

where $n = 1.1$. This was true for a wide range of conditions and we will make use of this expression later during the cost analysis. The shape of the curve for 5 grid/ $\square$ is the same as that found experimentally (Fig. 7).

The behavior of the open circuit voltage and efficiency is shown as a function of the depth of the p-n junction in Fig. 22. For this calculation it was assumed that the dead layer was $0.2 \mu\text{m}$ less than the junction depth which is about what is expected for junctions formed by diffusion. The maximum in η versus t_j occurs since for very shallow junctions the sheet resistance is high reducing η and for deep junctions the dead region expands decreasing the total number of collected charge carriers. For a higher density of grid lines the better junction depth will be nearer the surface.

Another important parameter to examine is the temperature dependence of the efficiency and of the open circuit voltage. This is shown in Fig. 23. With increasing temperature the decrease, assuming a linear variation between 0 and 100°C , is about $0.5\%/^\circ\text{K}$. This is in good agreement with experimental predictions (see Section 1 and Fig. 7).

E. Non-Uniform Distribution

One further problem to be briefly addressed is the effect that non-uniform illumination of the solar cell will have on the performance. If the surface conductivity is high enough so the surface is a constant voltage, the solar cell current at any position across the surface, $I(y)$, can be written as

Fig. 21. Power output as a function of concentration for $R_s = 0$ and $R_s = 0.48 \Omega$ (5 grid/□) as a function of concentration.

Fig. 22. The efficiency for $R_S = 0$ and $R_S = 0.48 \Omega$ (5 lines/□) as a function of junction depth. The % of total charge collected is also shown. The concentration is 5.

Fig. 23. The open circuit voltage and efficiency as a function of temperature. The concentration was 5.

$$I(y) = I_{\ell} f(y) / L - I_s [\exp (\beta V) - 1] / L \quad (14)$$

where L is the length of the cell and $f(y)$ gives the distribution of radiation across the surface.

Integrating from 0 to L

$$I_T = \int_0^L I(y) dy = I_{\ell} \frac{\int_0^L f(y) dy}{L} - I_s [\exp (\beta V) - 1] \quad (15)$$

Note that if $\frac{1}{L} \int_0^L f(y) dy = 1$ even though $f(y)$ is very non-uniform I_T is not changed.

Thus we can conclude that as long as enough grid lines are present so the top is an equipotential surface, a non-uniform light distribution is not detrimental if the average intensity is unchanged.

III. ANALYTICAL STUDY OF COMPOUND PARABOLIC CONCENTRATORS

In this section, we present a theoretical analysis of the radiant intensity distribution at the exit aperture (or absorber) of a Compound Parabolic Concentrator (CPC). It has already been recognized that, for non-diffuse incident radiation on a CPC, the intensity distribution on the absorber will be highly non-uniform, especially when the incident angle approaches the maximum acceptance half-angle of the collector. With our current interest in concentrating sunlight on solar cells using CPC-s, it was desired to obtain a detailed understanding of this intensity distribution, its variation with incident angle, and the average distributions over daily, seasonal and yearly variations in the incident angle. The output of this analysis is expected to provide optimization criteria for the design of CPC-s for use with solar cells, as well as design criteria relating to the physical characteristics of the cells.

In the analysis presented below, we consider a cylindrical, trough-like CPC which is aligned in the east-west direction. Also, we consider the simplest coupling of the cell to the concentrator, which is to mount the solar cell flat on the bottom of the CPC (a single stage CPC is considered in the analysis). Part A of this section summarizes the well-known geometrical formulae for CPC-s which are used in the development of a ray-tracing program, and in the calculation of the intensity distribution on the cell. This program determines the solar flux which is incident at various angles on the cell as a function of the angle of incidence of the sunlight on the CPC, and the concentration, reflectivity and truncation ratio of the device. Following a brief review of the solar geometry, Part B of this section considers the average intensity distribution on the cell as a result of daily and seasonal variations in the angle of incidence. A comparison is made of various design collectors which give 3X, 5X and 10X concentrations, and optimization criteria are proposed for collector and solar cell design and selection. Finally, Part C considers the effects of temperature on the performance of solar cells in CPC-s.

A. Ray Tracing and Intensity Distribution Programs

1. Definitions and formulae

The coordinate system used to describe the CPC is shown in Fig. 24. The following parameters are used to specify a given collector under consideration:

Θ_m = maximum acceptance half-angle of the collector.

Tr = truncation ratio
= $\frac{\text{height of truncated collector (h')}}{\text{height of ideal collector (h)}}$

ρ = reflectivity of the collector surface.

d_2 = exit or absorber aperture width of the CPC

Other quantities in Fig. 24 are defined as follows:

d_1 (or d_1') = entrance or collector aperture width of the CPC (or truncated CPC)

Θ = angle between incident radiation and axis of CPC.

The concentration ratio of the CPC is given by the ratio of the collector to the absorber aperture widths which, for the ideal collector (i.e., Tr = 1), is

$$C = \frac{d_1}{d_2} = \frac{1}{\sin \Theta_m} \quad (16)$$

The height of an ideal CPC is given by

$$h = 1/2 (d_1 + d_2) \cot \Theta_m \quad (17)$$

The following relationships give the (X, Y) coordinates of the points F, A, B and C in Fig. 24:

Fig. 24. Coordinate system used for describing a CPC. The primed quantities refer to a truncated CPC.

$$\begin{aligned}
\begin{bmatrix} X_F \\ Y_F \end{bmatrix} &= \begin{bmatrix} 0 \\ 1/2 d_2 (1 + \sin \Theta_m) \end{bmatrix} \\
\begin{bmatrix} X_A \\ Y_A \end{bmatrix} &= \begin{bmatrix} d_2 \cos \Theta_m \\ 1/2 d_2 (1 - \sin \Theta_m) \end{bmatrix} \\
\begin{bmatrix} X_B \\ Y_B \end{bmatrix} &= \begin{bmatrix} (d_1 + d_2) \cos \Theta_m \\ Y_F + \frac{d_1 + d_2}{2 \sin \Theta_m} \cdot \cos 2\Theta_m \end{bmatrix} \\
\begin{bmatrix} X_C \\ Y_C \end{bmatrix} &= \begin{bmatrix} X_A \\ Y_B + \tan \Theta_m \cdot (X_B - X_A) \end{bmatrix}
\end{aligned} \tag{18}$$

The right side parabola (AB in Fig. 24) of the collector is described by the equation:

$$X^2 = 2 PY, \tag{19}$$

where

$$P = d_2 (1 + \sin \Theta_m). \tag{20}$$

The absorber (FA) and collector (CB) planes, respectively, are described by the expressions:

$$Y = (-\tan \Theta_m) X + 1/2 d_2 (1 + \sin \Theta_m) \tag{21}$$

and

$$Y = Y_B + \tan \Theta_m \cdot (X_B - X). \tag{22}$$

A ray which is incident at angle Θ and intersects the collector plane at (X_i, Y_i) is described by the equation:

$$Y = \tan \left(\frac{\pi}{2} - \Theta_m + \Theta \right) \cdot X + b_i, \tag{23}$$

where b_i is calculated from the condition that (X_i, Y_i) must also satisfy Eq. (22).

If the truncation ratio Tr is specified to be less than unity, the coordinates of the points B' and C' need to be calculated. The quantity $X_{B'}$ can be obtained from the larger root of the quadratic equation¹¹

$$h' = Tr \cdot h = X_{B'} \cdot \sin \Theta_m + \frac{X_{B'}^2 \cdot \cos \Theta_m}{2 d_2 (1 + \sin \Theta_m)} - 1/2 d_2 \cos \Theta_m (1 + \sin \Theta_m), \quad (24)$$

while $Y_{B'}$ is obtained from Eq. (19) with $X = X_{B'}$. The truncated collector width $C'B'$ is given by¹¹

$$d_1' = 2 X_{B'} \cdot \cos \Theta_m - \frac{X_{B'}^2 \cdot \sin \Theta_m}{d_2 (1 + \sin \Theta_m)} + d_2 (\sin \Theta_m - \cos^2 \Theta_m), \quad (25)$$

based on which the coordinates $(X_{C'}, Y_{C'})$ can be calculated.

The computer program described below utilizes the above equations in tracing the passage of rays through ideal or truncated CPC-s.

2. Description of computer program

A computer program has been developed to determine the distribution of radiation across the exit slit of a CPC as a function of the angle of incidence Θ . The program consists of two essential parts: (i) a ray tracing main program which examines the passage through the CPC of specular radiation (or "rays") incident on the collector plane; and (ii) a subroutine program which sums the rays reaching the absorber plane to give the intensity distribution. The computer program is appended to this report; in this section, we describe briefly the main operational features of the program.

The input to the computer program is a set of values Θ_m , Tr , d_2 and ρ . Initially, the various parameters, coordinates, etc., which characterize the CPC are calculated and printed.

The angle of incidence Θ (measured counter-clockwise from the axis of the CPC) is then specified; the program can deal with $\Theta < \text{or} > \Theta_m$ (in the case of a truncated CPC). In the trivial case where $\Theta = \Theta_m$, all incident rays are focussed at the point F in Fig. 24 (or the point A when $\Theta = -\Theta_m$). The collector plane CB (or C'B') is divided into N equi-spaced regions[†], and the coordinates (X_i, Y_i) at the center of each region are calculated. This gives the equation (see Eq. (23)) of N incident rays on the collector. For $\Theta < \Theta_m$, each of the N rays will do one of the following: (i) intersect FA directly; (ii) intersect the right parabola AB one or more times, then hit FA; or (iii) intersect the left parabola FC one or more times, then hit FA. For $\Theta > \Theta_m$, we need to consider only rays which either (i) intersect FC one or more times before hitting FA (this can occur in the case of a truncated CPC); or (ii) intersect FA directly. Rays which intersect AB in this case will never reach the absorber, but eventually exit through the collector plane. (Of course, when Θ is negative and $|\Theta| > \Theta_m$, the left and right parabolas are reversed in the above argument).

The program calculates the point of intersection and the angle of incidence of each ray on the absorber. The intensity which is associated with each incident ray on the absorber and the region of the absorber affected by each ray is determined by the following reasoning. At the collector plane, the incident ray contains unit solar intensity, and this intensity is uniform over

[†]In this way, we assume that the radiation incident on the collector plane is represented by N infinitely thin rays, each of which contains unit solar intensity. The present version of the program is limited to $N = 100$; the results reported below are, in fact, for $N = 100$.

an effective "width" $W_C = |CB|/N$ of the ray[†] (although the ray is treated as being infinitely thin in the ray tracing program). On a direct intersection with the absorber, a region of "width" $W_A = W_C$ is therefore illuminated by the ray; i.e., if the i th ray intersects directly with the absorber at a distance $d = d_i$ from the point F, then the region of the absorber from $d = d_i - W_C/2$ to $d = d_i + W_C/2$ receives light of unit solar intensity at every point. On an indirect intersection with the absorber (i.e., after one or more reflections from either parabola), the effective "width" of the ray on the absorber is

$$W_A = W_C \cdot \cos \Theta / \sin \Theta_{inc}, \quad (26)$$

where Θ_{inc} is the angle of incidence of the ray on the absorber (Θ_{inc} is measured counter-clockwise from the direction $\vec{FA}$ to the ray; on a direct hit, $\Theta_{inc} = \pi/2 + \Theta$, and the previous conditions obtain). Therefore, the region $d = d_i - W_A/2$ to $d = d_i + W_A/2$ on the absorber receives sunlight in this case; the intensity is uniform at every point and has the value $W_C/W_A = \sin \Theta_{inc} / \cos \Theta$ times unit solar intensity. Of course, when the reflectivity $\rho < 1$, the above intensity is reduced by a further factor ρ^n , where n is the number of reflections of this particular ray from either the left or the right parabola.

By properly specifying the output condition variables in the program, one can obtain: (i) a detailed printout of the ray tracing (intersections with parabola, reflected ray equations, etc.); (ii) a summary of the N rays intersecting the absorber (hit points, angles, intensities, number of reflections); or (iii) suppress output and calculate only the intensity distribution.

[†]For a truncated CPC, $W_C = |C'B'|/N$.

The subroutine which calculates the intensity distribution across the absorber sums the intensity contributions from each of the N rays at each of 100 equi-spaced intervals along the absorber. Of course, this discretizing process, as well as that in representing the incoming radiation by N rays, introduces errors into the analysis: for example, the "hit-regions" on the absorber of two adjacent rays on the collector plane can overlap. Also, some radiation is "lost" when the width W_A of a ray incident on the absorber is such that the "hit-region" extends beyond the points F or A . As shown below, this latter error is typically less than 5% insofar as the average intensity on the absorber is concerned. The intensity distribution is output in graphical (or histogram) form, as exemplified below.

3. Results

Figure 25 shows the intensity distributions across the absorber for 3X, 5X, 7.5X and 10X ideal CPC-s (i.e., $Tr = 1$ and $\rho = 1$) at normal incidence ($\Theta = 0^\circ$). At the top of each figure, the characteristics of the CPC are displayed (e_m , C , ρ , Tr , d_1 or d_1' , d_2 , h or h' , and Θ). The two columns on the left show the variation of intensity (in suns) with distance from the point F (in cm). We have used $d_2 = 2$ cm in the calculations. The intensity distribution is displayed as a histogram which is normalized with respect to the maximum in the distribution. For example, the maximum intensity in Fig. 25a is 7.669 suns at $d = 0.58$ cm or 1.44 cm. The average intensity in this case is 2.936 suns; the maximum and average values are also displayed in the figure. The distributions at normal incidence are, of course, symmetric, and they become more uniform as the concentration ratio and thus the height of the CPC increases. However, as Θ deviates from 0° or if the CPC is truncated, the latter concentrators also lead to highly non-uniform distributions, as shown below.

Fig. 25a
(3X-CPC)

44

45

Fig. 25d
(10X-CPC)

The symbols 1, 2, 3 and 4 in the figures are used to distinguish radiant energy which is incident at various angular ranges on the absorber. The symbol 1 denotes radiation incident at $80^\circ \leq \Theta_{\text{inc}} \leq 90^\circ$, the symbol 2 is for $50^\circ \leq \Theta_{\text{inc}} < 80^\circ$, the symbol 3 is for $20^\circ \leq \Theta_{\text{inc}} < 50^\circ$, while the symbol 4 denotes $0^\circ \leq \Theta_{\text{inc}} < 20^\circ$.[†] It is seen that only a small fraction of the radiation which reaches the absorber is at angles $< 20^\circ$. The division of the intensity distribution into such angular ranges is felt to be important in the eventual design and optimization of anti-reflection films for solar cells to be used with a given concentrator. See Fig. 14 to see how the reflectivity of Si varies with angle of incidence.

The error resulting from the discretized treatment of the incident radiation and the intensity distribution is apparent from the average intensity values in Fig. 25; these are typically a few percent lower than the concentration ratio of the CPC. Although it is a relatively easy task to rewrite the computer program to treat > 100 incident rays and/or intervals on the absorber, the present results were deemed to be sufficiently accurate for the purposes of this investigation. A further discussion of discretizing errors is given later in this section.

Effect of the angle of incidence

Figure 26 shows the variation of the intensity distribution with angle of incidence for the case of the 3X-CPC in Fig. 25a. As Θ approaches the maximum acceptance angle ($\Theta_m = 19.47^\circ$ in this case), the distribution becomes progressively more non-uniform, with an increasingly higher single peak approaching $d = 0$ (in Fig. 26d, the peak intensity is 66.76 suns at $d = 0.08$ cm).

[†] Θ_{inc} is taken to be the acute angle between the absorber plane and the incident ray.

see INTENSITY DISTRIBUTION ALONG ARC OF CIRCLES
 REFLECTIVITY = 0.9999, TRANSMISSION = 0.0001, LOSS = 0.0001, REFLECTIVITY = 0.9999, TRANSMISSION = 0.0001, LOSS = 0.0001

Fig. 26a. $\Theta = 5^\circ$
 Max. intensity = 9.98
 Ave. intensity = 2.96

see INTENSITY DISTRIBUTION ALONG ARC OF CIRCLES
 REFLECTIVITY = 0.9999, TRANSMISSION = 0.0001, LOSS = 0.0001, REFLECTIVITY = 0.9999, TRANSMISSION = 0.0001, LOSS = 0.0001

Fig. 26b. $\Theta = 10^\circ$
 Max. intensity = 13.25
 Ave. intensity = 2.98

For negative values of Θ , the peaks would, of course, shift in the opposite direction. For $\Theta > \Theta_m$, no radiation reaches the absorber of an ideal CPC. On the other hand, the effective maximum acceptance angle is $> \Theta_m$ for the case of a truncated CPC. Here radiation can reach a portion of the absorber directly, as well as by reflection from the parabola which is on the same side of the collector axis as the incident radiation (e.g., the left side parabola can reflect rays toward the absorber for positive Θ when $\Theta > \Theta_m$).

Effect of truncation

The significant effect of truncating a CPC and thereby increasing the angular acceptance of the device (albeit the average intensity is low when $\Theta > \Theta_m$) was discussed in the preceding paragraph. For a given angle of incidence, the effect of truncating a CPC is generally to reduce the intensity on both sides of the peak in the distribution. An example of this is given in Fig. 27 for the case of the 10X-CPC at normal incidence shown in Fig. 25d. In Fig. 27a, the CPC is truncated to 0.4 times its ideal height which reduces the concentration ratio to 8.54 (the average intensity of the distribution is somewhat lower because of the discretizing errors discussed earlier). By comparing Figs. 25d and 27a, we see that the regions most affected by truncation are the outer edges of the absorber. A further truncation of this CPC to 0.1 times its ideal height (concentration ratio ~ 5.01) again reduces the intensity on the outer edges of the absorber, as shown in Fig. 27b.

A somewhat disturbing result in Fig. 27 is that the absolute magnitude of the peak has increased from ~ 12.2 in Fig. 25d to ~ 15.0 in Fig. 27b. Since $\Theta = 0$ in both cases, this is simply another consequence of discretizing the incoming radiation into 100 rays. The collector width in the truncated CPC in Fig. 27b is about one-half that of the ideal CPC in Fig. 25d; therefore, one would have to take $N \approx 200$ in the ideal case to have the same incident

ray-width and hence the same accuracy. These results indicate that a further study of the effects of N on the intensity distribution would be a worthwhile undertaking.

Effect of reflectivity

The principal effect of setting $\rho < 1$ (which it must be for any real reflector material used in the construction of the CPC) is to reduce the intensity uniformly at every point on the absorber, without appreciably affecting the distribution itself. This conclusion holds for all angles of incidence of the radiation on the CPC. A case of normal incidence is considered in Figs. 28a and 28b which, by comparison to Fig. 25b where $\rho = 1$, examine the same 5X-CPC for $\rho = 0.9$ and $\rho = 0.8$, respectively. The distributions are essentially unchanged; the respective peak and average values are closely correlated when the $\rho = 1$ values are multiplied by a factor of about ρ^n , where n is the average number of reflections ($n = 1.30$ in this case).

B. Comparison of Various Design CPC-s

1. The solar geometry

It is not very meaningful, of course, to compare various design CPC-s only at some given, fixed angle of incidence of radiation. The angle of incidence Θ , which is also referred to as the solar elevation relative to the concentrator, undergoes daily variations which change from season to season. In order to understand the tracking requirements for the concentrator, as well as to form a useful basis for comparison of various CPC-s, we present in this section the relevant formulae which give Θ as a function of time based on the solar geometry.

From the recent analysis of Rabl,¹² we obtain the following relationship:

$$\tan (\Theta + \phi - s) = \frac{\tan \alpha}{\cos \omega t} , \quad (27)$$

Fig. 28. The effect on the intensity distribution of the reflectivity for a 5X-CPC at normal incidence.

where

- α = solar declination (i.e., the angular position of the sun at solar noon with respect to the plane of the equator; north positive)
- $\alpha = 23.45^\circ \cdot \sin \left[360^\circ \cdot \frac{284 + m}{365} \right]$
- m = day of the year
- ϕ = latitude (north positive)
- $w = 360^\circ/24$
- t = time after noon, in hours
- s = angle between the horizontal (i.e., tangent at the earth surface) and the plane of the east-west collector (positive when plane slopes toward the equator).

The angular quantities are shown in a schematic of the solar geometry in Fig. 29. The angle $\Theta_v = \Theta + \phi - s$ is also referred to as the solar elevation from the equatorial plane ($\Theta_v = \alpha$ at solar noon), while $\beta = \phi - s$ is the tilt of the collector normal with respect to the equatorial plane.

Figure 30 shows the variation of Θ_v with time as a function of the solar declination. The days corresponding to summer solstice ($\alpha = 23.45^\circ$), winter solstice ($\alpha = -23.45^\circ$), equinox ($\alpha = 0^\circ$), and midway between solstice and equinox ($\alpha = \pm 12^\circ$) are shown in the figure. The change in solar elevation, and therefore in the incident angle Θ , is greatest at solstice; the daily variation in Θ_v (and Θ) is seen to be symmetrical about solar noon. There is also a symmetry of the $\Theta_v - t$ curves with regard to changes in the seasons (i.e., $\pm \alpha$); however, it must be kept in mind that the total number of daylight hours is not symmetric, in general. The day length (in hours) is given by the formula¹³

$$T_d = \frac{2}{15} \cos^{-1} (-\tan \phi \cdot \tan \alpha) \quad (28)$$

Fig. 29. Schematic of the angles used in the analysis of the solar geometry.

Fig. 30. The variation of solar elevation with time as a function of solar declination.

For the latitude of Cambridge, Mass. ($\phi = 42.38^\circ$), the day lengths at summer and winter solstice are 15.1 and 8.9 hours, respectively.

The actual range of values over which Θ varies on a given day is, of course, dependent on the location of the concentrator (i.e., the latitude ϕ) and on the tilt angle s of the concentrator relative to the horizontal. For a fixed value of ϕ , the tilt angle can be varied to give $\Theta = 0$ at various times during the day; this flexibility allows the establishment of optimization criteria, as discussed in the next section. Finally, with the sign convention for Θ and solar geometry used here, the intensity distributions in Figs. 25 to 28 and in subsequent figures are to be regarded as being viewed in the west-to-east direction, i.e., the left-side ($d = 0$) of the absorber is the north-side of the cell in the concentrator.

2. Assumptions for comparison of CPC-s

A given concentration ratio CPC can be obtained by constructing a collector with the appropriate value of Θ_m (cf. Eq. (16)) or by truncating a device with a lower characteristic value of Θ_m . Table I lists some combinations of Θ_m and Tr values which give 3X, 5X, or 10X concentrations. The advantages of using truncated CPC-s are apparent from the increase in the effective acceptance angle and the decrease in collector height with truncation. The former leads to some relaxation of tracking requirements (at the expense of some loss in concentration), while the latter leads to a reduction in collector surface area and in device structure, and to an accompanying reduction in cost, as discussed elsewhere in this report. These various design CPC-s will now be examined with regard to their performance with solar cells for daily and seasonal variations in incident angle.

We consider three specific days of the year: summer solstice ($\alpha = 23.45^\circ$), equinox ($\alpha = 0^\circ$), and $\alpha = 12^\circ$. Instead of the actual number of daylight hours (cf. Eq. (28)), we consider only the central eight hours (i.e.,

Table I. Properties of Various Design CPC-s which Give 3X, 5X or 10X Concentration

<u>Symbol</u>	<u>Conc. Ratio (C)</u>	<u>θ_m (deg)</u>	<u>Trunc. Ratio (Tr)</u>	<u>Height (cm)</u>	<u>Effective Max. Acceptance Angle (deg)</u>
a	3	19.47	1.0	11.31	19.47
b	3	15.0	0.30504	5.54	35.84
c	3	10.0	0.11069	4.24	43.31
d	3	7.5	0.05831	3.84	46.20
e	3	5.0	0.02460	3.51	48.75
f	5	11.54	1.0	29.39	11.54
g	5	10.0	0.43181	16.55	19.93
h	5	7.5	0.18907	12.44	25.75
k	5	5.0	0.07267	10.36	30.08
m	10	5.74	1.0	109.45	5.74
n	10	5.0	0.42807	61.03	10.22
p	10	2.5	0.06882	37.71	16.26

8:00 A.M. to 4:00 P.M.) of each day. We set the latitude $\phi = 42.38^\circ$ (Cambridge, Mass.), and allow the tilt angle s to be variable. For simplicity, we replace the continuous $\Theta_v - t$ (or $\Theta - t$) variation in Fig. 30 with a step-function given by the average Θ over successive 15 minute intervals. In this way, the total change in Θ is 16.0° for $\alpha = 23.45^\circ$ and 10.0° for $\alpha = 12^\circ$. Table II shows the extent of Θ -variation from 8:00 A.M. to 12:00 P.M. on these days as a function of the tilt angle. The variation from 12:00 P.M. to 4:00 P.M. is, of course, symmetrically opposite to that before noon. The choices for the tilt angle were such as to obtain $\Theta = 0$ (i.e., to "point" the collector) at noon, 11:00 A.M. and 1:00 P.M., etc. At equinox ($\alpha = 0$), Θ_v and therefore Θ is constant; the value of Θ is obtained on the basis of the tilt angle selected for that day.

In order to obtain the average intensity distribution on the absorber for the above angular variations, the computer program in Section A was modified to enable the summation and subsequent averaging of the distributions obtained for the Θ -values at 15 minute intervals. The angular distribution of the radiation on the absorber was ignored in the averaging procedure, and a reflectivity of one was assumed. The final output of the program is a histogram giving the average intensity distribution for the day and tilt angle under consideration.

3. Results

Figure 31 shows the intensity distribution for the 5X concentrators f, g, h and k in Table I. The distributions are for $\alpha = 23.45^\circ$ and a tilt angle to give $\Theta = 0$ at noon. The average intensity is seen to be highest for the ideal 5X-CPC (f), at least for this case where the average is taken over only the central 8 hours of the day. It is not apparent, however, that this conclusion should hold if the averaging were made over the full extent of the daylight hours, in view of the larger effective acceptance angles of the truncated

Table II. Extent of Variation of Incident Angle during Central Eight Hours on Selected Days of the Year as a Function of Collector Tilt Angle (for $\phi = 42.38^\circ$)

Declination, α (deg)	Tilt Angle, s (deg)	Θ Variation from 8:00 A.M. to 12:00 P.M. (deg)	Hours from noon at $\Theta = 0$
23.45 (summer solstice)	18.93	16.0 to 0.0	0
	18.20	15.3 to -0.7	1
	15.78	12.9 to -3.1	2
	10.85	7.9 to -8.1	3
	1.44	-1.5 to -17.5	4
12.0	30.38	10.0 to 0.0	0
	29.97	9.6 to -0.4	1
	28.59	8.2 to -1.8	2
	25.65	5.3 to -4.7	3
	19.35	-1.0 to -11.0	4

Fig. 31. Comparison of intensity distribution of 5X CPC-s for central 8 hours at summer solstice. Tilt angle is set to give $\Theta = 0$ at noon.

Fig. 31 (continued)

collectors (see Table I). The most uniform distribution is again for the ideal device in this case; this "advantage" may be overridden, however, by design considerations for the solar cells. For example, if the metal contact-grid covers (and, effectively, shields from radiation) a 0.4 cm wide portion on the south side of the cell, the average useful intensities for the collectors f and k (Figs. 31a and 31d, respectively) become ~ 3.4 and ~ 3.6 , respectively. The truncated collector is thus preferred in this case both from the point of view of economy and of performance.

Figure 32 examines the variation of the average intensity distribution with tilt angle for the case of collector f (cf. Fig. 31a). The optimum configuration, insofar as the average intensity or concentration is concerned, is seen to be the case where the collector points to the sun at 9:00 A.M. and 3:00 P.M. Here the extent of the Θ -variation (7.9° to -8.1°) is entirely within the acceptance angle of the CPC; the slight difference between the average intensity in Fig. 32c and the nominal concentration ratio of the device is caused by the discretizing errors discussed in Section A.

Such optimum tilt configurations exist for the truncated 5X devices and for the 3X and 10X concentrators as well. Below we compare the various design CPC-s on the basis of the average concentration during the selected days of the year.

Average concentration versus tilt angle

Figure 33 shows the average concentration for the 3X CPC-s in Table I as a function of the tilt angle. For each CPC, there is clearly an optimum value of the tilt angle which depends on the day of the year; $s \approx 25.5^\circ$, and $s \approx 11^\circ$ seem to give the highest average for $\alpha = 12^\circ$ and $\alpha = 23.45^\circ$, respectively, at least for the ideal and moderately truncated devices. Similar conclusions hold for the 5X and 10X CPC-s, as seen in Figs. 34 and 35, respectively. For CPC-s with the greater truncation (i.e., lower value of Tr), the

Fig. 32. Variation of the intensity distribution for the ideal 5X-CPC (collector f) with the tilt angle of the device on summer solstice.

Fig. 32c.
 Ave. intensity = 4.950
 Max. intensity = 7.729
 $\Theta = 0^\circ$ at 9:00 A.M.
 and 3:00 P.M.

Fig. 32d.
 Ave. intensity = 1.546
 Max. intensity = 2.668
 and 4:00 P.M.

Fig. 32 (continued)

Fig. 33. The dependence of average concentration on the tilt angle for the various design 3X CPC-s: (a) $\alpha = 12^\circ$; (b) $\alpha = 23.45^\circ$ (see Table I for collector data; $\Theta = 42.38^\circ$).

Fig. 34. The dependence of average concentration on the tilt angle for the various design 5X CPC-s: (a) $\alpha = 12^\circ$; (b) $\alpha = 23.45^\circ$ (see Table I for collector data; $\theta = 42.38^\circ$).

Fig. 35. The dependence of average concentration on the tilt angle for the various design 10X CPC-s: (a) $\alpha = 12^\circ$; (b) $\alpha = 23.45^\circ$ (see Table I for collector data; $\Theta = 42.38^\circ$).

peak in the curves is at higher values of the tilt angle (see collectors d and e in Fig. 33b, h and k in Fig. 34b, and p in Fig. 35a). The 10X collectors in Fig. 35b all perform better at the higher tilt angles.

Figure 36 compares the average concentration ratios of the collectors at equinox as a function of the angle of incidence (which is constant for that day) and therefore the tilt angle of the device. The actual intensity distributions are given by plots such as those shown in Section A.3 for a fixed angle of incidence.

Figures 33 to 36 give an indication of the tracking which is required in order to operate the CPC-s at their optimum level. Each curve, of course, gives only the central portion of the concentration-tilt angle variation where the optimum conditions occur; the curves are seen to be broader (hence less tracking required) for the lower concentration ratio devices. The curves at equinox (Fig. 36), for example, are symmetric about $\Theta = 0$; the ideal 3X, 5X and 10X devices will perform at their nominal average concentration as long as the tilt angle is within $\pm 19.47^\circ$, $\pm 11.54^\circ$ and $\pm 5.74^\circ$, respectively, of $s = 42.38^\circ$. This implies that adjustments are required even for the ideal 3X device; the average concentration of collector a in Fig. 33b decreases for $s \gtrsim 22.4$, while, on winter solstice, the same holds true for $s \lesssim 62.4^\circ$. This device thus requires 4 adjustments during the year in order to perform at the nominal concentration for at least 8 hours every day.

4. Discussion

The results of the preceding section suggest that the choice among various design CPC-s which yield some given concentration may be based on one of at least three optimization criteria. Firstly, one may select the device which gives the highest average intensity for the entire year. This was the basis used for comparison on selected days in the preceding section; with few exceptions (e.g., Fig. 35b), it was found that the ideal CPC gave the

Fig. 36. The variation of the average concentration with tilt angle (or with θ) at equinox for the various design 3X, 5X and 10X CPC-s. (See Table I for collector data; $\psi = 42.38^\circ$).

highest average concentration assuming that tracking was available to obtain the optimum tilt angle. This conclusion should also hold for other days of the year. It was also apparent, however, that, with tracking, some of the truncated devices could perform equally as well on this basis; for example, collectors a, b and c in Fig. 33, or f and g in Fig. 34 deliver the same average intensities for the days in question. On the other hand, if only limited tracking or no tracking were available, an extension of the present analysis could be used to obtain the design parameters (i.e., e_m and Tr) for some desired concentration ratio CPC which would give the highest average intensity for the entire year.

A second criterion would be to select the CPC which gives the most uniform intensity distribution when averaged over every day of the year. Of course, it is also desired to have the average intensity as close as possible to the nominally desired concentration. Based on our limited analysis thus far, it is not possible to reach any general conclusions with regard to the selection of a CPC based on this criterion. The problem is further complicated when the question of full, limited or no tracking is considered.

Contrary to the above condition, a third criterion might be to select the CPC which gives the most non-uniform average intensity distribution, while, at the same time, have a high average concentration ratio. This would aid in the design of the optimum contact-grid configuration for the solar cells, and, perhaps, in the design of other physical characteristics of the cell (e.g., junction depth or AR coating thickness). As exemplified in the preceding section, a truncated CPC with a lower average intensity than the ideal CPC can actually lead to a higher useful concentration ratio when the shadowing effects of the cell contacts are taken into account. The extent of tracking which is available is again an additional variable to be considered when CPC-s are compared subject to this criterion.

Either of the above three optimization criteria would also have to be examined in conjunction with economic considerations; this latter aspect of the use of CPC-s with solar cells is discussed in detail elsewhere in this report. Generally, it is expected that truncation will lead to a reduction and tracking to an increase in the system cost.

Finally, the present study clearly demonstrates that useful information is to be gained from further analytic work in this area. The study shows that optimum configurations of CPC-s and solar cells exist which can give some desired combination of economy and performance on selected days of the year. It is reasonable to project that such optimum conditions could also be determined for an entire year. We therefore expect that such analytic studies could aid in the selection and design of an entire concentrator scheme, including the choice of CPC parameters, the cell characteristics, and the tracking system.

C. Effect of Temperature in CPC on Solar Cell Performance

With increasing levels of solar flux the temperature of the solar cells will increase. Higher fluxes increase solar cell efficiency, provided they are properly designed to have a low series resistance, but higher temperatures decrease the efficiency. In Section II we found that the power output was approximately proportional to $R^{1.1}$ but the efficiency decreased by about $0.5\%/^{\circ}\text{C}$. For the purpose of the rest of this report we will empirically represent the efficiency, η , as

$$\eta = R^n \eta_o (1 - \beta T) \quad (29)$$

where $n = 0.1$, $\eta_o = 0.14$ and $\beta = 5 \times 10^{-3}/^{\circ}\text{C}$.

Information is available on the heat rejection of solar cells for different methods of cooling.¹⁴ For this analysis we will consider three types. 1) water or liquid cooling, 2) a finned strip and 3) a plane (flat) strip. The temperature gradient through the Si solar cell from the front to the back is negligible. For example, for a 0.025 cm (10 mil) thick cell with a concentration of $R = 10$, the temperature drop is only about 0.1°C . This is because the thermal conductivity of Si is quite high ($0.2 \text{ W/cm}/^\circ\text{C}$).⁸ Table III summarizes the thermal conductance values that have been estimated and that we will use for the different cooling configurations. In all cases the thermal resistance of the bond between the solar cell and the heat sink was taken to be $1.8 \text{ cm}^2/^\circ\text{C}/\text{W}$. In practice only for water cooling is such a resistance high enough to significantly reduce the heat flow.

Table III. The Effective Thermal Conductances for Various Types of Cooling¹⁴

Type of Cooling	Thermal Conductance (K_e) ($\text{W}/\text{cm}^2 - ^\circ\text{C}$)	
1. Liquid Flow	0.2	
2. Finned Heat Sink	0.03	} 220 cm/s wind
3. Plane Strip	0.006	

The total thermal heating of the solar cell, Q , assuming all the energy is deposited in the Si will be:

$$Q = P_o A_R (1 - \eta) \quad (30)$$

where P_o is the incident solar flux and A_R is the total area at the aperture of the CPC. η will in general be a function of concentration and temperature as specified in Eq. (29).

At equilibrium this thermal energy will be removed. Considering only heat rejection to a heat sink and neglecting radiation and direct convection, Q will be

$$Q = K_e A_{Si} (T - T_o) \quad (31)$$

where K_e is the effective thermal conductance, A_{Si} is the area of Si and T_o is the ambient temperature. Thus noting that $R = A_R / A_{Si}$ the temperature will be

$$T = T_o + P_o R (1 - \eta) / K_e \quad (32)$$

Using Eq. (29) and (32) we can calculate the temperature of the solar cell and the efficiency as a function of concentration for different values of the thermal conductance (Fig. 37 and 38 respectively). If we restrict ourselves to a concentration of 10 or less then it is obvious that a finned cooling strip ($K_e = 0.03 \text{ W/cm}^2/\text{°C}$) is satisfactory. A temperature rise of only 30°C with little change in efficiency occurs.

If we are interested in utilizing the rejected heat then liquid cooling becomes more desirable since the heat exchange medium need only be slightly cooler than the solar cell temperature. For example, if 100°C ambient temperature were desired ($R = 10$), with a liquid the solar cell could operate at 104°C where the efficiency is still 9.0% but with a finned strip and a gas the solar cell would operate at 131°C where $\eta = 6.7\%$. An additional disadvantage of gas as a heat transfer medium is the substantially higher power required to force the gas through the cooling channel compared to a liquid such as water with an equivalent cooling capacity.¹⁴

Fig. 37. Temperature of a silicon solar cell as a function of concentration for different types of cooling. $K_e = 0.005 \text{ W/cm}^2/^\circ\text{C}$ corresponds to a flat strip, 0.03 a finned heat sink in a slight breeze, 0.2 cooling by a circulating liquid, and 0.01 to some intermediate cooling. The ambient temperature was 25°C .

$T_0 = 25^\circ \text{C}$

Fig. 38. Solar cell conversion efficiency as a function of concentration. For the types of cooling schemes corresponding to various values at K_e , see the preceding figure or text.

IV. COST EFFECTIVENESS ANALYSIS

In photovoltaic systems the primary reason for considering concentration is to replace "expensive" Si with a "cheap" reflector thus decreasing the overall cost of the system. (For thermal systems the reason is also to produce higher quality heat, i.e., higher temperatures). The economics of doing this depend very critically upon the relative cost of the materials and methods of fabrication as well as the areas of reflectors and solar cell material involved.

A simple analysis has been made, which is presented in this section, for a single stage and a two stage CPC. Both are assumed to be concentrators for long, flat cells. The bottom section of the second type was assumed to be a medium with an index of refraction of 1.5. The analysis has focussed on the electrical output rather than the thermal heat that is generated.

A. Simple CPC

Figure 39 shows the collector area per unit length divided by the aperture per unit length as a function of concentration for various acceptance angle between 30° and 2.5° . For a given concentration the smaller the acceptance angle the smaller the collector area. The area increases very non-linearly as the concentration approaches its maximum value. For a system that does not continuously track the sun, the acceptance angle should be at least 5° to 6° which corresponds to a concentration of about 10 ($\Theta_{\max} = 5.7^\circ$). Such a system would have to be adjusted periodically, i.e., weekly or monthly, though. For a completely non-tracking system $\pm 23^\circ$ angle is required limiting the concentration to 2.6. Using a dielectric filled CPC with an index of refraction of 1.5 a concentration of 10 with an acceptance angle of 8.6° is possible.

For ease of construction the CPC should not be very tall. Figure 40 shows the height to width (aperture) ratio versus concentration again for several acceptance angles. For $\Theta_{\max} = 5^\circ$ and $R = 10$ the height to width ratio is 3 which is already slightly ungainly.

Fig. 39. The collector area per unit lengths divided by the aperture per unit length as a function of concentration for a CPC. Acceptance angles between 2.5 and 30° were used.

Fig. 40. The height divided by the width of the aperture as a function of concentration for a CPC.

B. Two Stage CPC

Similar calculations have been made for a two stage CPC. The bottom section was slightly truncated with an acceptance angle of 87.9° instead of 90° but this slight decrease in $\Theta_{\max}$ already reduces the length by about 25%. Referring to Fig. 14 which shows the reflectivity of Si, light incident at angles near 90° from the normal is almost completely reflected. Fig. 41 shows the total area versus concentration, and Fig. 42 the total height of both stages versus concentration. For low concentrations the 2-stage structure has a larger area than a single stage system but for concentrations above about 3, it has a smaller area and shorter overall length. For example when $R = 10$ and $\Theta = 5^\circ$ the two stage CPC has 55% the area and is only about 1/2 as tall as the simple CPC. Whether the decrease in material costs are enough to compensate for the increased complexity has not been analyzed yet.

C. Total System Cost

The total cost will be made up of a number of items including 1) the solar cells, 2) the reflector, 3) the supporting structure, 4) the tracking mechanism, and 5) the cooling mechanism. Table IV shows how these costs would be expected to vary with area. The solar cell cost, C_{Si} , and cooling cost, C_C , will be proportional to the area of Si, A_{Si} ; whereas, the support mechanisms costs, C_S , and tracking cost C_T , will probably vary with the area of the aperture A_R . The area of the reflector will be a complicated function that depends on the acceptance area, the concentration, and acceptance area but it is easily calculated numerically.

The total cost of a system, T , neglecting land, installation, and monetary considerations, will be the sum of these terms, i.e.,

$$T = A_{Si} [C_{Si} + C_R R_f + (C_S + C_T) R + C_C]$$

Fig. 41. The collector area per unit length divided by the aperture per unit length as a function of concentration for a two stage CPC. The bottom stage is filled with a dielectric of index of refraction of 1.5.

Fig. 42. The total height divided by the width of the aperture as a function of concentration for a two stage CPC .

Table IV. Costs per Unit Area

Solar Cell	$C_{Si} A_{Si}$
Reflector	$C_R A_R f(R, \theta_{max})$
Support	$C_s A_R$
Tracking	$C_T A_R$
Cooling	$C_C A_{Si}$

Since the peak electric power generated, E , is

$$E = A_{Si} R \eta (R, T) P_o$$

the installed cost, K , per unit power will be

$$K = \left[\frac{C_{Si} + C_C}{R} + C_s + C_T + C_R f(R, \theta_{max}) \right] / \eta$$

This expression is the materials cost of a system in terms of dollars per kilowatt of peak power. Our next task is to assign reasonable values to the various components in this expression.

The most critical, since it is the highest price item, is that of the solar cell. Spectrolab in a cost analysis¹⁵ projects that in 1977 Si solar cells can be manufactured for \$256/m² (\$23.81/ft², 2.56¢/cm² or \$2.15/W at AM1).^{*} This includes the solar cell encapsulant and mounting on an extruded

^{*}In the rest of this report we will use units of \$/ft with (¢/cm²) in parenthesis.

aluminum I-beam to form a complete array. Presumably without much increase in cost the I-beam could be modified to be used as a finned heat sink, thus we will lump the cooling cost with the Si cost. This cost projection was made assuming conventional crystal growing and fabrication technology.

Table V is a list of various materials that might be used in a solar concentration system and some of their characteristics. The density, relative tensile strength and specific stiffness are important from the construction point of view. For example with a specific stiffness of 100 a completely self supporting structure can be made but for a value of 10 or less additional support is necessary. The cost data is based on that in 1974.

Figure 43 is a plot of the cost versus concentration and was calculated assuming that $C_{Si} = \$23/\text{ft}^2$ ($2.5\text{¢}/\text{cm}^2$) and $C_R = \$0.046/\text{ft}^2$ ($0.005\text{¢}/\text{cm}^2$). This corresponds to the use of 0.010 in. PVC, Mylar or nylon as the reflector surface. Also shown is the result of using a reflector cost of ten times as much, i.e., $C_R = \$0.46/\text{ft}^2$ ($0.05\text{¢}/\text{cm}^2$) which is equivalent to 0.010 in. stainless steel, to 0.010 in. TFE fluorocarbons, or to 0.060 in. of aluminum. An acceptance angle of 5° was assumed to calculate $f(R, \Theta)$. The support structure and tracking costs were assumed to be $\$0.46/\text{ft}^2$ ($0.05\text{¢}/\text{cm}^2$) and a value of $K = 0.03 \text{ W}/\text{cm}^2 \cdot ^\circ\text{C}$ appropriate to a finned heat sink was used.

At a concentration of 1 the cost is $\$2090/\text{kW}$ but drops rapidly to $\$225/\text{kW}$ at $R = 11$ for the first case and to $\$323/\text{kW}$ at $R = 9$ for the second case. Any errors in estimating C_S and C_T will not change the position of the minimum but will only shift the curves vertically up and down. In the case just considered these costs are approximately $\$40/\text{kW}$.

Spectrolab projects¹⁵ that without changing the basic procedures of using conventional processes but with a reduction in the Si prices to $\$6/\text{kg}$ and modest improvements in fabrication, the array manufacturing cost can be reduced almost 50% to $\$12/\text{ft}^2$ ($1.3\text{¢}/\text{cm}^2$). With only this change, holding everything else fixed, Fig. 44 was calculated. Now the minimum costs are $\$111/\text{kW}$ at $R = 10.5$ and $\$221/\text{kW}$ at $R = 8$ for the two different reflector costs.

Table V. Properties and Costs of Materials for CPC

	Specific Gravity	Density (lb./in. ³)	Tensile Strength per unit density (in.)	Specific Stiffness Young's Modulus/Density (in.) $\times 10^{-6}$	1974 Costs		
					Cost per Weight (\$/lb)	0.010" Thick (\$/ft ²)	0.25 mm Thick (¢/cm ²)
Stainless Steel	7.90	0.286	483	100	1.20	.49	0.053
Carbon Steel	7.83	0.283	502	106	0.12	.049	0.0054
Aluminum - 890 Alloy	2.90	0.104	890	105	0.50	.075	0.0081
Aluminum - 290 Cast	2.90	0.104	290	-	0.50	.075	0.0081
Glass	2.55	0.092	-	129	0.30	.040	0.0043
Epoxy (Cast)	1.12 - 2.4	0.040 - 0.086	386	85	4.00	.43	0.046
TFE Fluorocarbons	2.4	0.087	51	1	3.45	.43	0.046
TFE Film	2.15	0.078	51	1	4.00	.45	0.048
PVC	1.55	0.058	209	.07	0.55	.046	0.0050
Polyester (Mylar)	1.39	0.050	171	16	0.64	.046	0.0050
Nylon 6 Film	1.12	0.040	258	8	0.76	.044	0.0047
Polystyrene Film	1.07	0.038	250	18	0.14	.0077	0.00083
Polyethylene	.94	0.034	152	1	0.20	.0098	0.0011
Polypropylene	.90	0.032	-	6	0.24	.011	0.0012

Cost to metallize with Al or Ag is \$0.08/ft².

Fig. 43. The cost in \$/kW of a CPC system versus concentration for silicon solar costs of \$23/ft. Reflector costs of 0.05 ¢/cm² (\$0.46/ft²) and 0.005 ¢/cm² (\$0.046/ft²) were assumed. An acceptance of 5° which gives a maximum concentration of 11.5 was used. $K_c = 0.03 \text{ W/cm}^2/\text{°C}$.

Fig. 44. The cost in \$/kW of a CPC system versus concentration. The same as the preceding figure except silicon costs of \$12/ft² were assumed.

As the cell costs decrease relative to the other costs the optimum concentration occurs at lower values. This is more clearly demonstrated in Fig. 45 where different ratios of $\gamma = C_R/C_{Si}$ have been assumed. Here $C_{Si} = \$10/\text{ft}^2$, $C_S + C_T = \$0.10/\text{ft}^2$. Curves for γ at 0.01, 0.03, and 0.1 are shown. In this figure $\Theta_{\max} = 10^\circ$.

The effectiveness of the cooling can have a drastic effect as shown in Fig. 46 for K_e of 0.006, 0.03, and $0.2 \text{ W/cm}^2\text{-}^\circ\text{C}$. Because the efficiency drops rapidly with increasing temperature, the cost rises since the area necessary to generate a fixed amount of power increases drastically for inefficient cooling.

D. Two Stage CPC

Figures 47 and 48 were calculated for a two stage CPC with a dielectric with an index of refraction of 1.5 in the bottom stage. Exactly the same assumptions as used for the simple collector (Figs. 43 and 44) were made. Note now that the maximum concentration for the 5° acceptance angle is 17.2 instead of 11.5. The minimum cost is roughly 20% lower for the two versus one stage CPC, but no consideration of any increased fabrication difficulties or the cost of the dielectric have been made. Overall though it appears that the two stage system may offer some cost as well as operational advantages.

E. Present Cost Advantage of CPC

A very direct demonstration of the reduction possibly with a CPC is to cost an actual CPC even though it was designed without any consideration of low cost. The ribbed collector is shown in Fig. 49. Based on January 1975 material prices the breakdown for a one foot length CPC section that has a concentration of 5 and is designed for a 1.05 in. solar cell is as follows:

Fig. 45. The cost in \$/kW of a CPC system versus concentration. γ is the ratio of reflector costs per unit area to Si solar cell costs. An acceptance angle of 10° and $K_e = 0.003 \text{ W/cm}^2/\text{ }^\circ\text{C}$ were used.

Fig. 46. The cost in \$/kW of a CPC system versus concentration for different types of cooling. See the text for types of cooling corresponding to different values of K_E . The acceptance angle is 5° .

Fig. 47. The cost in \$/kW of a two stage CPC system with same assumptions as used for the simple collector.

Fig. 48. The cost in \$/kW of a two stage CPC system with Si costs of \$13/ft². All other assumptions were the same as in the preceding figure.

Fig. 49. Drawing of a times 5 CPC used to calculate a cost breakdown.

1. Al Ribs = $(13.77 \text{ in.}^3) (0.104 \text{ lb/cm}^3) (\$0.78/\text{lb}) = \$1.12.$
2. Extruded Al Heat Sink = $(0.62 \text{ lb/ft}) (\$0.90/\text{lb}) (1 \text{ ft}) = \$0.56.$
3. Polished Al Reflector, 25 mils thick, 85% reflectivity = $(1.67 \text{ ft}^2) (\$0.292/\text{ft}^2) = \$0.49.$
4. Glass Cover, 1/8 in. thick = $(7.88 \text{ in.}^3) (0.090 \text{ lb/in.}^3) (\$0.30/\text{lb}) = \$0.21.$
5. Exterior plywood sheet, 5/8 in. thick = $(0.44 \text{ ft}^2) (\$0.25/\text{ft}^2) = \$0.11.$
6. Extruded Al Rib Supports = $(1 \text{ ft}) (0.153 \text{ lb/ft}) (\$0.90/\text{lb}) = \$0.14.$
7. Adhesives, Coatings and Fasteners (estimated) = $\$0.50.$

Material Cost = $\$3.13$ for 0.44 ft^2 projected area. Labor cost (10 minutes of assembly labor at $\$4/\text{hr}$) = $\$0.67$. Total Cost = $\$8.69/\text{ft}^2$ of CPC.

In early 1975 solar cells could be purchased at about $\$20/\text{W}$ or $\$223/\text{ft}^2$ for a 12% cell.¹⁵ Thus with concentration

$$K = \frac{\$223}{(5)(0.85)} + \$8.69 = \$61.16/\text{ft}^2 = \$5.49/\text{W}$$

Without concentration the additional cost besides the solar cells will be for a glass cover, plywood sheet, hardware and assembly:

$$K = \$223 + \$2.53 = \$225.53/\text{ft}^2 = \$20.23/\text{W}$$

Thus even today a cost reduction of almost four can be obtained just by the use of a simple CPC concentrator.

V. SUMMARY

An investigation of the compound parabolic collector (CPC) for use with photovoltaic solar cells has been carried out. Using a heliostat and neutral density filters, the characteristics of state-of-the-art commercial silicon solar cells were studied for concentrations between one and ten suns. The primary conclusion reached was that as presently manufactured the series resistance is too high by a factor of about 5 for optimum performance at concentrations above 2 or 3. Increasing the number of collector grids per unit length by two or more will eliminate the problem. Typical antireflection coating thicknesses (800 - 1200 Å) and the depth of the p-n junction (0.25 - 0.35 μm) were satisfactory. Testing in a nominal times eight CPC confirmed these findings. Measurements made versus temperature found a decrease in efficiency of 0.4%/°C independent of solar concentration.

A theoretical model of a Si solar cell was developed and should be useful for maximizing the performance under conditions found in a CPC. The excellent agreement between experiment and theory obtained, lends confidence in the model. Among the functional dependences studied were the temperature, donor and acceptor concentrations, junction depths, surface recombination velocities, solar concentrations, and minority carrier lifetimes.

A comprehensive calculation of the energy distribution across the exit aperture for ideal and truncated CPC's was made. Input parameters examined were the concentration, acceptance angle, degree of truncation and angle of incidence of the incoming radiation. In addition to the total energy, the angular distribution of energy across the exit slit was determined. Energy distributions averaged over an entire day for different angles of inclination of the collector were calculated to determine the best placement of the collector grids and the optimum collector for any given situation.

The effect solar flux has on the temperature and thus efficiency of the solar cell was studied. For concentrations the order of ten or less cooling with a finned heat sink is satisfactory.

A cost effectiveness analysis was made for a CPC photovoltaics system. A substantial reduction in power generation costs appears to be quite definite. A rather broad minimum of the cost occurs for concentrations in the range of 6 to 10. Using projected 1977 costs for Si solar cell arrays ($\$23.81/\text{ft}^2$), we estimate a manufacturing cost of $\$323/\text{kW}$ at a concentration of 9 versus $\$2150/\text{kW}$ without concentration. Even without any technical advances it should easily be possible to reduce the manufacturing costs today by a factor of 4 by using a CPC.

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APPENDIX A - COMPUTER PROGRAMS

A. Solar Cell Characteristics

Figure A1 is a program listing of the computer program used to calculate the properties of a Si solar cell. Two subroutines, one of which calculates the current generated and charge collection efficiency of the p-n junction by an AM1 solar spectrum, LIGHT, and the second which generates the I-V characteristics, VI, are listed in Figs. A2 and A3 respectively.

In addition to the discussion in Section II, the following assumptions and procedures have been used:

1. The donor level is 0.044 eV below the conduction band and the acceptor level is 0.045 eV above the valence band. These levels are characteristic of phosphorus and boron respectively.¹⁷

2. The intrinsic carrier concentration is given by¹⁸

$$n_i^2 = N_V N_C \exp(-E_g/kT)$$

where

$$N_V = 2 (2\pi m_n^* kT/h^2)^{3/2} \quad \text{and}$$

$$N_C = 2 (2\pi m_p^* kT/h^2)^{3/2}$$

2. The effective masses of the electrons, m_n^* , and holes, m_p^* , are given by:¹⁹

$$m_n^*/m_o = 1.062 + 3.67 \times 10^{-4} T$$

$$m_p^*/m_o = 0.5902 + 6.17 \times 10^{-4} T$$

where T is in °K.

```

C
C PROGRAM TO CALCULATE THE EFFICIENCY OF A SOLAR CELL.
C CASE OF A SI SOLAR CELL WITH A DEAD LAYER, N DIFFUSION, P RIBBON
C AND UNDER CONCENTRATION.
C
COMMON LP, LN, DP, DN, WIDN, WIDP, S, JD, DO, DEAD
IMPLICIT REAL(A-Z)
INTEGER I
ETDO=.044
ETA=.045
DO=1.E-2
NDS=2.51007E19
Q=1.602E-19
EP=1.07E-12
TO=300.
5 CONTINUE
TYPE 76
TYPE 10
10 FORMAT(/ ' INPUT T, NA, ND, CON, S, JD AND TAU. ' $)
ACCEPT 20,T,NA,ND,CON,S,JD,TAUPO
20 FORMAT(7G)
IF(T.EQ.0.0) STOP
TAUNO=TAUPO
DEAD=JD-2.E-5
IF(DEAD.LT.0.0) DEAD=0.0
PIN=0.1*CON
C FROM NBS REPORT 400-4.
MUNO=65.+1265./(1.+(ND/8.5E16)**.72)
MUPO=47.7+447.3/(1.+(NA/1.9E17)**.76)
MN=1.062+T*3.67E-4
MP=0.5902+T*6.17E-4
EG=1.1388-(T-195.)*2.67E-4
MUP=MUPO*(TO/T)**1.5
MUN=MUNO*(TO/T)**1.5
ETD=EG-ETDO
NV=NDS*(T/TO*MP)**1.5
NC=NDS*(T/TO*MN)**1.5
KT=T/11605.6
C ASSUME EXPRESSION FOR MINORITY CARRIER MOBILITIES ARE THE SAME AS FOR
C MAJORITY BUT WITH A CHANGE OF THE TYPE OF IMPURITY LEVELS.
DN=(65.+1265./(1.+(NA/8.5E16))**.72)*(TO/T)**1.5*KT
DP=(47.7+447.3/(1.+(ND/1.9E17))**.76)*(TO/T)**1.5*KT
TYPE 21
21 FORMAT(/ ' HOLE MOBILITY ELECTRON MOBILITY HOLE DIFF
1 COEFF ELEC DIFF COEFF ')
TYPE 22
22 FORMAT(' CM2/V.SEC CM2/V.SEC CM2/SEC
1 CM2/SEC ')
TYPE 23,MUP,MUN,DP,DN
23 FORMAT(4F17.3)
EFD=ETD+KT*ALOG((SQRT(1.+4.*ND/NC*EXP((EG-ETD)/KT))-1.)/2.)
NDP=ND/(1.+EXP((EFD-ETD)/KT))
EFA=ETA-KT*ALOG((SQRT(1.+4.*NA/NV*EXP(ETA/KT))-1.)/2.)
NAM=NA/(1.+EXP((ETA-EFA)/KT))
RHON=1./NDP/MUN/Q
RHOP=1./NAM/MUP/Q
NVC=SQRT(NV)*SQRT(NC)
NI=NVC*EXP(-EG/KT/2.)
NI2=NI**2
VBI=EFD-EFA

```

Fig. A-1. Computer program used to calculate properties of a Si solar cell under concentration.

```

WID=SQRT(2.*EP/Q*(1./NAM+1./NDP)*VBI)
WIDN=WID*NAM/(NDP+NAM)
WIDP=WID-WIDN
IF(WIDN.GT.JD) WIDN=JD
TYPE 74,WID
TYPE 19,WIDN,WID,DEAD
19 FORMAT(/ ' DEPLETION WIDTH ON N-SIDE IS '1PE8.3,' AND ON P-SIDE
1 IS '1PE8.3,'. DEAD REGION IS '1PE8.3,' CM.' )
RSUR=RHON/JD
TYPE 26,RSUR
26 FORMAT(/ ' SURFACE RESISTANCE IS '1PE8.3,' OHMS/SQUARE.' )
C LIFETIMES LIMITED BY AUGER RECOMBINATION
TAUP=TAUPO/(1.+TAUPO*NDP*1.7E-31*NDP)
TAUN=TAUNO/(1.+TAUNO*NAM*1.2E-31*NAM)
LP=SQRT(DP*TAUP)
LN=SQRT(DN*TAUN)
TYPE 24,LP,LN
24 FORMAT(/ ' LP IN N-REGION IS '1PE8.3,' AND
1 LN IN P-REGION IS '1PE8.3,' CM.' )
TYPE 25,NA,ND
25 FORMAT(/ 24H NUMBER OF ACCEPTORS IS 1PE9.3, 25H AND NUMBER OF
1 DONORS IS 1PE9.3, 6H CM-3. )
C TYPE 27,ETDO,ETA
C 27 FORMAT(/ 16H DONOR LEVEL IS F5.3, 23H AND ACCEPTOR LEVEL
C 1 IS F5.3, 4H EV. )
TYPE 31,S
31 FORMAT(/ ' SURFACE RECOMBINATION VELOCITY IS '1PE8.3,' CM/SEC.' )
IS=Q*NI2*(LN/TAUN/NAM+LP/TAUP/NDP)
CALL LIGHT(IL,CON)
IL=IL*(1.87-.783219*EG)
C VOC=KT*ALOG(IL/IS+1.)
C BVOC=VOC/KT
C DEL=0.
C DO 30 I=1,10
C 30 DEL=ALOG(1.+BVOC-DEL)
C BVMP=BVOC-DEL
C VMP=BVMP*KT
C IMP=IS*BVMP*EXP(BVMP)
C ETAA=IMP*VMP/PIN
ISP=Q*WID*SQRT(NI2/TAUP/TAUN)
RR=ISP/IS
VOCP=KT*2.*ALOG(SQRT(1.+IL/IS+RR+RR**2/4.))-RR/2.)
BVOCP=VOCP/KT
RDR=RR*EXP(-BVOCP/2.)
DEN=1.+RDR
TYPE 34,JD,DO
34 FORMAT(/ ' JUNCTION DEPTH IS '1PE10.3,' AND CELL THICKNESS IS '
1 1PE10.3,' CM.' )
TYPE 35,RDR
35 FORMAT(/ ' RATIO OF DIFF/RECOM CURRENT IS '1PE10.3 )
DEL=0.1*BVOCP
DO 40 I=1,10
BV=BVOCP-DEL
40 DEL=ALOG((1.+BV+RDR*(EXP(DEL/2.))*(1.+BV/2.))/DEN)
BVMPP=BVOCP-DEL
VMPP=BVMPP*KT
IMPP=ABS(IS*(EXP(BVMPP)-1.)+ISP*(EXP(BVMPP/2.))-1.)-IL)
ETAP=IMPP*VMPP/PIN
TYPE 50
50 FORMAT(/ ' ENERGY GAP N FERMI LEVEL P FERMI LEVEL BUILT-I
1N-VOLTAGE ' )

```



```

TYPE 52,EG,EFD,EFA,VBI
52 FORMAT( 4F15.4)
TYPE 54
54 FORMAT(/ ' IONIZED DONORS IONIZED ACCEP INTRIN CARR CON
1 V B DEN STATES C B DEN STATES' )
TYPE 56,NDP,NAM,NI,NV,NC
56 FORMAT(1P5E16.3)
TYPE 58,TAUP,TAUN
58 FORMAT(/ 18 H HOLE LIFETIME IS 1PE10.3, 28H SEC. ELECTRON
1 LIFETIME IS 1PE10.3, 6H SEC. )
TYPE 59,RHON,RHOP
59 FORMAT(/26H RESISTIVITY ON N SIDE IS 1PE10.3, 18H AND ON P SIDE IS
1 1PE10.3 , 8H OHM-CM. )
TYPE 60,IL,PIN,T
60 FORMAT(/ 5H IL= F7.5, 14H A/CM2. PIN= F7.4, 15H WATT/CM2. T=
1 F7.2, 8H DEG K. )
C TYPE 62
C 62 FORMAT(/// ' FOR AN IDEAL JUNCTION ' )
C TYPE 63
C 63 FORMAT( ' -----
C 1-----' )
C TYPE 64
64 FORMAT(/ ' OPEN CIRCUIT VOLTAGE MAX POWER VOLTAGE REVERSE SAT
1CURRENT MAX POWER CURRENT ' )
C TYPE 66
66 FORMAT( ' (VOLTS) (VOLTS) (A/CM2)
1 (A/CM2) ' )
C TYPE 68,VOC,VMP,IS,IMP
68 FORMAT( F14.5,4X,F18.5,2X,2E18.4)
C TYPE 70,ETAA
C TYPE 76
76 FORMAT(///// )
70 FORMAT(/ ' EFFICIENCY WITH ZERO SERIES RESISTANCE IS ' F6.4 )
C TYPE 72
C 72 FORMAT(/ ' FOR CASE OF RECOMBINATION
C 1 AND DIFFUSION CURRENT . ' )
TYPE 63
74 FORMAT(/ ' DEPLETION WIDTH IS ', 1PE10.3, ' CM.' )
TYPE 64
TYPE 66
TYPE 68,VOC,VMPP,ISP,IMPP
TYPE 70,ETAP
C TYPE 76
ANS=0.0
TYPE 75
75 FORMAT(/ ' PLOT I-V CURVE? ' $ )
ACCEPT 20, ANS
IF(ANS.EQ.1.) CALL VI(KT,RSUR,RR,IL,IS,PIN)
GO TO 5
END

```

Fig. A-1 (continued)

C
C CALCULATION OF LIGHT GENERATED CURRENT AND CHARGE COLLECTION
C EFFICIENCY OF A SI SOLAR CELL.
C

```

SUBROUTINE LIGHT(IL,CON)
DIMENSION AL(25),PH(25)
COMMON LP, LN, DP, DN, L1, L2, S, T, D, DEAD
IMPLICIT REAL(A-Z)
INTEGER N
PH(5)=0.
PH(6)=.0604
PH(7)=1.444
PH(8)=1.912
PH(9)=3.397
PH(10)=3.674
PH(11)=3.737
PH(12)=3.955
PH(13)=4.122
PH(14)=4.122
PH(15)=3.963
PH(16)=3.019
PH(17)=2.395
PH(18)=3.08
PH(19)=.526
PH(20)=2.818
PH(21)=2.853
PH(22)=2.657
AL(5)=1.E6
AL(6)=5.E5
AL(7)=1.8E5
AL(8)=6.5E4
AL(9)=2.5E4
AL(10)=1.1E4
AL(11)=6400.
AL(12)=4700.
AL(13)=4000.
AL(14)=2800.
AL(15)=1700.
AL(16)=1000.
AL(17)=550.
AL(18)=290.
AL(19)=160.
AL(20)=80.
AL(21)=30.
AL(22)=1.
DO 5 N=5,22
5 PH(N)=PH(N)*1.E17
TP=T-L1
TP1=TP/LP
AA=S*LP/DP
TYPE 16
16 FORMAT(/ ' CHARGE COLLECTION' )
TYPE 15
15 FORMAT(' WAVELENGTH JT JSP JDRP JBM JDRN' )
JTOT=0.
DO 20 N=5,22
LAM=N*0.05
A=AL(N)
ALP=A*LP
ATT=EXP(-A*DEAD)

```

Fig. A-2. Subroutine to calculate current generated and charge collection efficiency of a p-n junction.

```

TH=TANH(TP1)
DEN=ALP**2-1.
JSP=ALP**2/DEN*((DP+S/A)/(DP+S*LP)*EXP(-TP1)
1 -EXP(-A*TP))+ALP*(TH+AA)/(1.+AA*TH)*(ALP*(DP+S/A)/(DP+S*LP)*
2 EXP(-TP1)-EXP(-A*TP))/DEN
JDRP=EXP(-TP*A)-EXP(-A*T)
EXAD=0.
IF(A*D.LT.82.) EXAD=EXP(-A*D)
JBN=EXP(-A*(T+L2))*(A/(A+1./LN)-A*(EXP(-D/LN)-EXAD)/
1 (A**2-1./LN**2)/SINH(D/LN))
JDRN=EXP(-A*T)*(1.-EXP(-A*L2))
JSP=JSP*ATT
JDRP=JDRP*ATT
JBN=JBN*ATT
JDRN=JDRN*ATT
JT=JSP+JDRP+JBN+JDRN
JTOT=JT*PH(N)*0.05+JTOT
20 TYPE 10, LAM, JT, JSP, JDRP, JBN, JDRN
10 FORMAT(6F9.5)
FRAC=JTOT/2.254E17
IL=JTOT*1.6E-19*CON
TYPE 25, IL
25 FORMAT(/ ' CURRENT GENERATED AT 300 DEG K IS ',F8.5,' A/CM2' )
TYPE 30,FRAC
30 FORMAT(' WHICH IS ',F8.5,' OF POSSIBLE.'/)
END

```

```

C
C SUBROUTINE TO CALCULATE THE IV CURVE FOR THE SOLAR CELL INCLUDING
C SERIES RESISTANCE
C
      SUBROUTINE VI(KT,RSUR,RR,IL,IS,PIN)
      IMPLICIT REAL(A-Z)
      INTEGER NGRID,N
      5  TYPE 10
      10 FORMAT(/' NUMBER OF GRID LINES PER SQUARE? ' $ )
      ACCEPT 20, NGRID
      20 FORMAT(G)
      RS=RSUR/NGRID**2/8.
      TYPE 30,RS
      30 FORMAT(/' SERIES RESISTANCE IS ',1PE8.3,' OHM.' /)
      TYPE 40
      40 FORMAT(/' CURRENT VOLTAGE POWER EFF. ')
      TYPE 41
      41 FORMAT(' MA/CM2 V MW/CM2 ')
      DEL=0.1*IL
      P=-PIN
      I=-DEL
      R2=RR/2.
      DO 50 N=1,11
      I=I+DEL
      V=2.*KT*ALOG(SQRT((1.+R2)**2+(IL-I)/IS)-R2)
      VL=V-I*RS
      PN=VL*I
      IF(PN.LT.P) GO TO 80
      P=PN
      EFF=P/PIN
      50 TYPE 60,I,VL,P,EFF
      60 FORMAT(4F10.5)
      80 I=I-2.*DEL
      DEL=DEL*0.2
      DO 85 N=1,40
      IF(P.LT.0.0) GO TO 90
      I=I+DEL
      V=2.*KT*ALOG(SQRT((1.+R2)**2+(IL-I)/IS)-R2)
      VL=V-I*RS
      P=VL*I
      EFF=P/PIN
      85 TYPE 60, I,VL,P,EFF
      90 ANS=0.
      TYPE 70
      70 FORMAT(/' DIFFERNT NUMBER OF GRID LINES? ' $)
      ACCEPT 20,ANS
      IF(ANS.EQ.1.) GO TO 5
      RETURN
      END

```

Fig. A-3. Subroutine to calculate I-V characteristics.

4. The room temperature (300°K) mobility of the holes, μ_p , and electrons μ_n when they are majority carriers are:¹⁹

$$\mu_p = 47.7 + 447.3 / [1 + (N_A / 1.9 \times 10^{17})^{0.76}]$$

$$\mu_n = 65 + 1265 / [1 + (N_D / 8 \times 10^{17})^{0.72}]$$

The mobility when holes and electrons are minority carriers will be the same expression except N_A and N_D are interchanged.

5. The mobilities for both holes and electrons will vary with temperature $T^{-3/2}$.

6. The temperature dependence of the energy gap, E_g , will be approximated by:¹⁹

$$E_g = 1.1108 - (T-300) 2.67 \times 10^{-4}$$

7. The width of the depletion region, W , is given by:¹⁸

$$W = \left[\frac{2\epsilon}{q} V_{BI} \left(\frac{1}{N_A^-} + \frac{1}{N_D^+} \right) \right]^{1/2}$$

where the built-in voltage V_{BI} is given by the difference in the Fermi level between the p and n sides of the junction. N_A^- and N_D^+ are the number of ionized acceptors and donors respectively. The dielectric constant, ϵ , is 1.07×10^{-12} f/cm.⁸ The depleted width on the n side, W_n , and on the p side, W_p , are related by:¹⁵

$$W_n N_D^+ = W_p N_A^-$$

8. The minority carrier lifetimes are limited by Auger recombination²⁰ so that

$$\frac{1}{\tau_p} = \frac{1}{\tau_{p_0}} + 1.7 \times 10^{-31} N_D^2$$

$$\frac{1}{\tau_n} = \frac{1}{\tau_{n_0}} + 1.2 \times 10^{-31} N_A^2$$

τ_{p_0} and τ_{n_0} are the lifetimes for low doping levels and can be specified. For most examples we assumed $\tau_{p_0} = \tau_{n_0}$.

9. The optical absorption data was taken from Dash and Newman⁵ and the spectral irradiance data from the RCA Optical Handbook.⁷ The light generated current was obtained by multiplying the current generated for one sun by the concentration and a factor linearly proportional to the energy gap i.e.,

$$I_l = I_l(E_g(300^\circ K)) (1.87 - .7832 E_g)$$

Table A1 shows the input parameters that were used if they were not otherwise specified.

B. CPC Geometry and Parameters

The program listed in Fig. A4 is used to calculate the coordinates of the reflector of a CPC if the acceptance angle, input aperture and output width are specified. The height and area of the collector are also determined.

Table AI

Temperature, T	300°K
Number of Acceptors, N_A	$1 \times 10^{16} \text{ cm}^{-3}$
Number of Donors, N_D	$2 \times 10^{20} \text{ cm}^{-3}$
Concentration, R	5 suns
Surface Recombination Velocity, S	$1 \times 10^6 \text{ cm/s}$
Junction Depth, t_j	0.3 μm
Hole-Electron Lifetime, $\tau_{p_0} = \tau_{n_0}$	2 μs

C CALCULATION OF SHAPE OF CPC COLLECTOR FOR A FLAT RIBBON

```

C
C
C
      IMPLICIT REAL(A-Z)
      INTEGER N
10  CONTINUE
      TYPE 20
20  FORMAT(/// ' INPUT TH, D1, AND D2. ' $)
      ACCEPT 30,THD,D1,D2
30  FORMAT(3G)
      IF(THD.EQ.0.0) STOP
C D1=ENTRANCE WIDTH
C D2=EXIT WIDTH
C TH=MAXIMUM COLLECTION ANGLE
      THR=THD*3.141592/180.
      S=0.
      XOLD=D2
      YOLD=0.0
      SITH=SIN(THR)
      D1P=D2/SITH
      IF(D1P.LT.D1) GO TO 40
      X0=(D1+D2)/2.
      X1=(D1P+D2)/2.
      L=X1/SITH*COS(THR)
      RMAX=X1/SITH
      RMAXP=RMAX*(D1-D2)/(D1P-D2)
      CON=D1/D2
      TYPE 35,THD,CON
35  FORMAT(// ' ACCEPTANCE ANGLE IS ',F6.2, ' DEG AND CONCENTRATION
      1 IS ',F8.4 )
C      TYPE 80
C      80  FORMAT(/ '          HEIGHT          WIDTH          XPLUS
C      1          XMINUS '/')
      DELR=(RMAXP-D2)/50.
      IF(ABS(DELR).LT.0.0001*D1) DELR=(RMAX-D2)/100.
      AA=2.*X1*SITH
      DO 60 N=0,300
      R=D2+DELR*N
      PH=ACOS(AA/R-1.)
      DE=PH+THR-1.570796
      XP=R*COS(DE)
      X=XP
      XM=-XP+D2
      W=XP-XM
      Y=R*SIN(DE)
      DELS=SQRT((X-XOLD)**2+(Y-YOLD)**2)
      S=S+DELS
      TYPE 70,Y,W,XP,XM
70  FORMAT(4F15.4)
      IF(W.GE.D1-0.00001) GO TO 90
      YOLD=Y
      XOLD=X
60  CONTINUE
90  DELX=X0-XOLD

```

Fig. A-4. Program used to calculate coordinates and other properties of a CPC.


```

DELY=(Y-YOLD)*DELX/(X-XOLD)
S=S-DELS+SQRT(DELX**2+DELY**2)
XS=2.*S
TYPE 72,N
72 FORMAT(/' N= ',I3 )
XL=YOLD+DELY
TYPE 100,XL,XS
100 FORMAT(/ 21H COLLECTOR LENGTH IS F8.4, 19H AND TOTAL AREA
1 IS F10.4 /)
XLP=L
TRUN=XL/XLP
CONP=D1P/D2
TYPE 120,XLP,CONP
120 FORMAT(' UNTRUNCATED LENGTH IS ',F8.3,' CM AND CONCENTRATION
1 IS 'F8.4 ')
TYPE 130,TRUN
130 FORMAT(/ ' LENGTH IS TRUNCATED BY A FACTOR OF 'F8.4 /)
AP=XS/D1
TYPE 110,AP
110 FORMAT( 31H RATIO OF AREA TO APERTURE IS F10.4 //)
GO TO 10
40 TYPE 50
50 FORMAT(// ' NO SOLUTION EXISTS. ' //)
GO TO 10
END

```

APPENDIX B - RAY TRACING AND INTENSITY DISTRIBUTION PROGRAM

The details of the following computer programs were described in Section III.A of this report. The input and output variables of the program are defined in the various "comment" statements in the program.

C PROGRAM TO CALCULATE INTENSITY DISTRIBUTION ALONG ABSORBER
 C IN IDEAL CPC-COLLECTOR AS FUNCTION OF ANGLE (THETA) INCIDENT
 C RADIATION MAKES WITH OPTIC AXIS OF CPC.
 C TRUNCATED CPC CAN ALSO BE TREATED BY PROGRAM.
 C THIS PROGRAM IS A "RAY-TRACER"; SUBROUTINE PROGRAM
 C PLOTS INTENSITY DISTRIBUTION ALONG ABSORBER
 C PROGRAM CAN ALSO AVERAGE DISTRIBUTION OVER RANGE OF INCIDENT ANGLES

COMMON ANGH(100),WH(100),DIST(100),HITINT(100)

COMMON ABB(100),D(100)

COMMON REFL,TRUNC,THETAM,CONC,D1,D2,HEIGHT,ACCONC,D1TR

COMMON NRIGHT(100),NLEFT(100),ONE,TWO,THREE,FOUR,BLANK,IOUT,N

COMMON IAVE,PLUS,INEG,IHIST(100,100)

REAL MA,IA,MC,IC,MIN,IIN,MR,IR

INTEGER ONE,TWO,THREE,FOUR,BLANK,PLUS

DIMENSION XIN(100),YIN(100),XH(100),YH(100),STORE(100)

C READ PRINT/OUT CONDITION: IOUT=0 IF NO DETAILED OUTPUT IS REQUIRED

C IPUNCH=0 IF NO CARD OUTPUT IS REQUIRED.

C READ IN HISTOGRAM OUTPUT SYMBOLS.

C IF IAVE=0, HISTOGRAM OF EACH THETA IS GIVEN, WITHOUT AVERAGING;

C IF IAVE=1, AVERAGE HISTOGRAM IS CALCULATED FOR A GIVEN SET OF THETA

C IF IAVE=2, EACH HISTOGRAM AND AVERAGE IS PRINTED.

READ(5,82) IOUT,IPUNCH,IAVE,ONE,TWO,THREE,FOUR,BLANK,PLUS

82 FORMAT(3I1,6A1)

C SPECIFY THETA-MAX(IN DEGS) AND ABSORBER APERTURE D2 (CM OR INS)

C SPECIFY REFLECTIVITY AND TRUNCATION RATIO

2 READ(5,80) THETAM,D2,REFL,TRUNC

80 FORMAT(4F10.5)

PI=3.14159265

C CALCULATE AND PRINT COLLECTOR PARAMETERS FOR IDEAL CPC

THETAD=THETAM*PI/180.0

CONC=1.0/SIN(THETAD)

D1=CONC*D2

HEIGHT=(D1+D2)*0.5/TAN(THETAD)

WRITE(6,90) THETAM,CONC,D1,D2,HEIGHT,REFL,TRUNC

90 FORMAT('1','***** PARAMETER VALUES FOR IDEAL CPC COLLECTOR *****'///

1' THETA-MAX (DEGS) =',F10.5//' CONCENTRATION =',F10.4//' D1 =',F10

2.4,' D2 =',F10.4,' HEIGHT =',F10.4//' REFLECTIVITY =',F10.3,

3' TRUNCATION RATIO =',F10.5//')

IF(IPUNCH.EQ.0) GO TO 320

WRITE(7,998) THETAM,CONC,D1,D2,HEIGHT

998 FORMAT(F10.6,F10.4,3F15.6)

320 CONTINUE

C CALCULATE AND PRINT COORDINATES OF POINTS F, A, B, AND C.

SM=SIN(THETAD)

CM=COS(THETAD)

TM=TAN(THETAD)

S2M=SIN(2.0*THETAD)

C2M=COS(2.0*THETAD)

C

XF=0.0

YF=D2*(1.0 + SM)*0.5

XA=D2*CM

YA=YF - D2*SM

```

XB=(D1+D2)*S2M/2.0/SM
YB=YF + (D1+D2)*C2M/2.0/SM
XC=XA
YC=-TM*XA + YB + TM*XB

```

```

WRITE(6,91) XF,XA,XB,XC

```

```

91 FORMAT('0','X-COORDINATES OF POINTS F,A,B,C =',4F10.6///)

```

```

WRITE(6,92) YF,YA,YB,YC

```

```

92 FORMAT('0','Y-COORDINATES OF POINTS F,A,B,C =',4F10.6///)

```

```

IF(IPUNCH.EQ.0) GO TO 321

```

```

WRITE(7,997) XF,XA,XB,XC

```

```

WRITE(7,997) YF,YA,YB,YC

```

```

997 FORMAT(4E15.6)

```

```

321 CONTINUE

```

```

C CALCULATE EQUATIONS OF PARABOLA, COLLECTOR AND ABSORBER
C PLANES, AND PRINT PARAMETERS.

```

```

C PARABOLA CAB IS:  $X^2 = 2.0 * P * Y$ 
 $P = D2 * (1.0 + SM)$ 

```

```

C ABSORBER PLANE FA IS:  $Y = MA * X + IA$ 
 $MA = -TM$ 

```

```

 $IA = YF$ 

```

```

C COLLECTOR PLANE CB IS:  $Y = MC * X + IC$ 

```

```

 $MC = -TM$ 

```

```

 $IC = YB + TM * XB$ 

```

```

WRITE(6,93) P,MA,IA,MC,IC

```

```

93 FORMAT('0','PARAMETERS P, MA, IA, MC, IC =',5E15.6///)

```

```

C CALCULATE TRUNCATED CPC PARAMETERS UNLESS TRUNC = 1.0

```

```

IF(TRUNC.GE.0.999) GO TO 510

```

```

HEIGHT=HEIGHT*TRUNC

```

```

SMP1=SM + 1.0

```

```

 $A = CM / (2.0 * D2 * SMP1)$ 

```

```

 $B = SM$ 

```

```

 $C = -(D2 * CM * SMP1 / 2.0 + HEIGHT)$ 

```

```

 $XB = (-B + \sqrt{B^2 - 4.0 * A * C}) / (2.0 * A)$ 

```

```

 $YB = XB * XB / 2.0 / P$ 

```

```

 $IC = YB + TM * XB$ 

```

```

 $D1TR = 2.0 * XB * CM - XB * XB * SM / D2 / SMP1 + D2 * (SM - CM * CM)$ 

```

```

 $YC = YB + D1TR * SM$ 

```

```

 $XC = (IC - YC) / TM$ 

```

```

 $ACCONC = D1TR / D2$ 

```

```

 $SLOPE = \text{ABS}((YC - YA) / (XC - XA))$ 

```

```

 $ACCANG = \text{ATAN}(SLOPE)$ 

```

```

 $ACCANG = ACCANG - \text{THERAD}$ 

```

```

 $ACCANG = \pi / 2.0 - ACCANG$ 

```

```

 $ACANGD = ACCANG * 180.0 / \pi$ 

```

```

WRITE(6,520) ACCONC,D1TR,HEIGHT,ACANGD

```

```

520 FORMAT('0','**** TRUNCATED CPC PARAMETERS ****'//,
  'ACTUAL CONC =',
  1F10.5,' TRUNC. COLL. WIDTH =',F10.5,' HEIGHT =',F10.5,
  2' MAX. ACC. ANGLE =',F10.4//)

```

```

WRITE(6,91) XF,XA,XB,XC

```

```

WRITE(6,92) YF,YA,YB,YC

```



```

WRITE(6,93) P,MA,IA,MC,IC
510 CONTINUE
IF(IAVE.NE.1) GO TO 1520
WRITE(6,1521)
1521 FORMAT('1','*** SUMMARY OF THETA VALUES EXAMINED IN AVERAGE ***'//
1//)
1520 CONTINUE
C INITIALIZE ENERGY IN SUM OF DISTRIBUTIONS
C NTHETA COUNTS NUMBER OF THETA VALUES IN AVERAGING
DO 1525 I=1,100
1525 STORE(I)=0.0
NTHETA=0
C READ IN ANGLE OF INCIDENCE OF RADIATION MEASURED FROM OPTIC AXIS
C OF CPC. NOTE: THETA (IN DEGS) MUST BE LESS THAN THETA-MAX
C PRESENT VERSION OF PROGRAM CAN ALSO DEAL WITH THETA .GT. THETA-MAX
C ILAST=1 IF THIS IS LAST THETA FOR THIS CPC
C ILAST=2 IF THIS IS LAST CASE EXAMINED
C INEG=1 IF THETA IS NEGATIVE (IT MUST BE POSITIVE WHEN READ IN)
1 READ(5,81) THETA,ILAST,INEG
81 FORMAT(F10.6,2I1)
NTHETA=NTHETA + 1
TRAD=THETA*PI/180.0
IF(IAVE.NE.1) GO TO 1620
WRITE(6,1621) THETA,INEG
1621 FORMAT('0','THETA =',F10.4,' IS IT NEGATIVE?',I5)
1620 CONTINUE
C SET DESIRED NO. OF INTERVALS IN COLLECTOR PLANE (MAXIMUM = 100)
N=100
XN=FLOAT(N)
C FIND "WIDTH" OF EACH INCIDENT RAY AND PRINT INFO FOR THIS RUN.
DIFFX=XB - XC
DIFFY=YB - YC
DXSQ=DIFFX*DIFFX
DYSQ=DIFFY*DIFFY
WIDTH=SQRT(DXSQ + DYSQ)/XN
IF(ROUT.EQ.0) GO TO 299
WRITE(6,941) THETA,N,WIDTH
941 FORMAT('1','ANGLE OF INCIDENCE, THETA (DEGS) =',F10.4/' NO. OF I
INTERVALS IN COLLECTOR PLANE =',I5,' INTERVAL WIDTH =',E15.6)
299 CONTINUE
IF(IPUNCH.EQ.0) GO TO 322
WRITE(7,996) THETA,N,WIDTH
996 FORMAT(F10.3,I5,F15.6)
322 CONTINUE
C CALCULATE (X,Y) COORDS. OF POINTS IN COLLECTOR PLANE THRU
C WHICH INCIDENT RAY PASSES
DELX=(XB - XC)/XN
DO 10 I=1,N
IF(I.EQ.1) GO TO 1010
XIN(I)=XIN(I-1) + DELX
GO TO 1011
1010 XIN(I)=XC + DELX/2.0
1011 YIN(I)=MC*XIN(I) + IC

```

10 CONTINUE

C START DO LOOP TO EXAMINE EACH OF N INCIDENT RAYS ON COLLECTOR PLANE

DO 200 I=1,N

C COUNT REFLECTIONS FROM LEFT AND RIGHT PARABOLAS

NREFLT=0

NREFRT=0

C CALCULATE EQUATION OF I-TH INCIDENT RAY: $Y=MIN*X + IIN$ $MIN=TAN(PI/2.0 - THERAD + TRAD)$ $IIN=YIN(I) - MIN*XIN(I)$

C CALCULATE INTERSECTION WITH ABSORBER PLANE

 $XAIN=(IIN - IA)/(MA - MIN)$ $YAIN=MA*XAIN + IA$

IF(IOUT.EQ.0) GO TO 301

WRITE(6,94) I,XIN(I),YIN(I),MIN,IIN,XAIN,YAIN

94 FORMAT('0','INCIDENT RAY NO. ',I5,' COLLECTOR PLANE COORDS.=',

1 2E15.6// 'SLOPE, INTERCEPT =',2E15.6,' INTERSECTION WITH ABS

SORBER PLANE =',2E15.6)

301 CONTINUE

C IF RAY INTERSECTS TO LEFT OF F, NEED TO TREAT MIRROR IMAGE;

C IF RAY INTERSECTS TO RIGHT OF A, NEED TO REFLECT FROM PARABOLA;

C IF RAY INTERSECTS BETWEEN F AND A, IT IS A DIRECT HIT ON ABSORBER.

IF(XAIN.GT.(XA+1.0E-4)) GO TO 40

IF(XAIN.LT.(XF-1.0E-4)) GO TO 30

C

IF(XAIN.GE.0.0) GO TO 49

XFIT=XF

YHIT=YF

GO TO 149

49 CONTINUE

IF(XAIN.GE.XA) GO TO 249

XHIT=XAIN

YHIT=YAIN

GO TO 149

249 XHIT=XA

YHIT=YA

149 CONTINUE

THHIT=PI/2.0 + TRAD

THHITD=THHIT*180.0/PI

WHIT=WIDTH

GO TO 50

40 CONTINUE

C FIND INTERSECTION WITH PARABOLA AND SLOPE AT THAT POINT.

C SET INITIAL VALUE OF THHIT (EACH INTERSECTION WITH PARABOLA REDUCES

C THIS VALUE BY 2*BETA, WHERE BETA IS ANGLE BETWEEN RAY AND

C TANGENT TO PARABOLA.

C IF THETA.GT.THETA-MAX, THIS RAY DOESN'T HIT ABSORBER

IF(THETA.GT.THETAM) GO TO 145

THHIT=PI/2.0 + TRAD

45 CONTINUE

NREFRT=NREFRT + 1

A=1.0

B=-2.0*P*MIN

C=-2.0*P*IIN

XR=(-B - SQRT(B*B - 4.0*A*C))/(2.0*A)

YR=XR*XR/2.0/P

DYDXR=XR/P

BETA=ATAN(MIN) - ATAN(DYDXR)

BETAD=BETA*180.0/PI

C CALCULATE EQUATION OF REFLECTED RAY: $Y = MR * X + IR$

GAMMA=ATAN(DYDXR) - BETA

MR=TAN(GAMMA)

IP=YR - MR*XR

IF(IOUT.EQ.0) GO TO 302

WRITE(6,900) XR,YR,BETAD

900 FORMAT('0','INTERSECTION WITH PARABOLA AT',2E15.6,' BETA =',

1 F10.4,' REFLECTED RAY PARAMETERS ARE:')

302 CONTINUE

C FIND INTERSECTION WITH ABSORBER PLANE AND PRINT.

XAIN=(IR - IA)/(MA - MR)

YAIN=MA*XAIN + IA

IF(IOUT.EQ.0) GO TO 303

WRITE(6,901) MR,IP,XAIN,YAIN

901 FORMAT('0','SLOPE, INTERCEPT =',2E15.6,' INTERSECTION WITH ABSC

ORBER PLANE =',2E15.6)

303 CONTINUE

C CHECK IF HIT ABSORBER OR REFLECT AGAIN

IF(XAIN.GT.XA) GO TO 60

IF(XAIN.LT.(XF-1.0E-4)) GO TO 660

C

XHIT=XAIN

YHIT=YAIN

GO TO 661

660 XHIT=XF

YHIT=YF

661 CONTINUE

THHIT=THHIT - 2.0*BETA

THHITD=THHIT*180.0/PI

WHIT=WIDTH*CCS(THHITD)/SIN(THHIT)

GO TO 70

60 CONTINUE

THHIT=THHIT - 2.0*BETA

MIN=MR

MIN=IR

GO TO 45

C

C THIS RAY DOESN'T HIT ABSORBER; SET EQUAL TO DUMMY VALUES

145 CONTINUE

XHIT=XAIN

YHIT=YAIN

THHIT=0.0

THHITD=0.0

WHIT=0.0

GO TO 170

C

30 CONTINUE

IF(IOUT.EQ.0) GO TO 304

```
WRITE(6,95)
95 FORMAT('0','INCIDENT RAY REFLECTS FROM LEFT SIDE PARABOLA, CONSIDER
IR MIRROR IMAGE APCUT OPTIC AXIS')
304 CONTINUE
C CALCULATE COORDS. AND EQUATION OF MIRROR RAY
XIMG=XIN(N-I+1)
YIMG=YIN(N-I+1)
MIN=TAN(PI/2.0 - THRAD - TRAD)
IIN=YIMG - MIN*XIMG
IF(IOUT.EQ.0) GO TO 305
WRITE(6,951) I,XIMG,YIMG,MIN,IIN
951 FORMAT('0','INCIDENT RAY NO. ',I5,' IMAGE COORDS ON COLLECTOR P
ILANE =',2F15.6// ' SLOPE, INTERCEPT =',2E15.6)
305 CONTINUE
C FIND INTERSECTION WITH PARABOLA AND SLOPE AT THAT POINT
THHIT=PI/2.0 - TPAD
46 CONTINUE
NREFLT=NREFLT + 1
A=1.0
B=-2.0*P*MIN
C=-2.0*P*IIN
XR=(-B - SQRT(B*B - 4.0*A*C))/(2.0*A)
YR=YR*XR/2.0/P
DYDXR=XR/P
BETA=ATAN(MIN) - ATAN(DYDXR)
BETAD=BETA*180.0/PI
C CALCULATE EQUATION OF REFLECTED RAY: Y = MR*X + IR
GAMMA=ATAN(DYDXR) - BETA
MR=TAN(GAMMA)
IR=YR - MR*XR
IF(IOUT.EQ.0) GO TO 306
WRITE(6,900) XR,YR,BETAD
306 CONTINUE
C FIND INTERSECTION WITH ABSORBER PLANE AND PRINT
XAIN=(IR - IA)/(MA - MR)
YAIN=MA*XAIN + IA
IF(IOUT.EQ.0) GO TO 307
WRITE(6,901) MR,IR,XAIN,YAIN
307 CONTINUE
C CHECK IF HIT ABSORBER OR REFLECT AGAIN
IF(XAIN.GT.XA) GO TO 61
XHIT=XAIN
YHIT=YAIN
THHIT=THHIT - 2.0*BETA
THHITD=THHIT*180.0/PI
WHIT=WIDTH*CCS(TRAD)/SIN(THHIT)
GO TO 71
61 CONTINUE
THHIT=THHIT - 2.0*BETA
MIN=MR
IIN=IR
GO TO 46
71 CONTINUE
```


IF(IOUT.EQ.0) GO TO 308

WRITE(6,902)

902 FORMAT('0','MIRROR RAY REFLECTED FROM PARABOLA HITS ABSORBER')

WRITE(6,97) XHIT,YHIT,THHITD,WHIT

308 CONTINUE

C REFLECT BACK ABOUT OPTIC AXIS

XDIFF=XHIT - XA

YDIFF=YHIT - YA

XDSC=XDIFF*XDIFF

YDSC=YDIFF*YDIFF

AH=SQRT(XDSC + YDSC)

XHIT=XF + AH*CM

YHIT=YF - AH*SM

THHIT=PI - THHIT

THHITD=THHIT*180.0/PI

IF(IOUT.EQ.0) GO TO 309

WRITE(6,903)

903 FORMAT('0','ACTUAL RAY HITS ABSORBER AS FOLLOWS:')

WRITE(6,97) XHIT,YHIT,THHITD,WHIT

309 CONTINUE

GO TO 20

C

50 CONTINUE

IF(IOUT.EQ.0) GO TO 310

WRITE(6,96)

96 FORMAT('0','INCIDENT RAY HITS DIRECTLY ON ABSORBER PLANE')

WRITE(6,97) XHIT,YHIT,THHITD,WHIT

97 FORMAT('0','XHIT =',F15.6,' YHIT =',F15.6,' INC. ANGLE =',

1 F10.3,' INC. WIDTH =',F15.6/////)

310 CONTINUE

GO TO 20

C

170 CONTINUE

IF(IOUT.EQ.0) GO TO 313

WRITE(6,196)

196 FORMAT('0','INCIDENT RAY REFLECTS OUT OF PARABOLA SINCE THETA .GT.

1 THETA-MAX')

WRITE(6,97) XHIT,YHIT,THHITD,WHIT

313 CONTINUE

GO TO 20

C

70 CONTINUE

IF(IOUT.EQ.0) GO TO 311

WRITE(6,98)

98 FORMAT('0','RAY REFLECTED FROM PARABOLA HITS ABSORBER')

WRITE(6,97) XHIT,YHIT,THHITD,WHIT

311 CONTINUE

20 CONTINUE

XH(I)=XHIT

YH(I)=YHIT

ANGH(I)=THHITD

WH(I)=WHIT

NRIGHT(I)=NREFRT

NLEFT(I)=NREFLT

200 CONTINUE

C SUMMARIZE ABSORBER DATA FOR THIS VALUE OF THETA

IF(IAVF.EQ.1) GO TO 1942

WRITE(6,941) THETA,N,WIDTH

WRITE(6,942)

942 FORMAT('0', 'RAY NO. X-HIT Y-HIT DISTANCE
1 ANGLE HIT-WIDTH INTENSITY RIGHT AND LEFT REFLS.')

1942 CONTINUE

C CALCULATE HIT-POINT DISTANCES FROM ABSORBER, AND HIT- INTENSITY

DO 230 I=1,N

IF(WH(I).LT.1.0E-5) GO TO 231

XDIFF=XH(I) - XF

YDIFF=YH(I) - YF

XDSC=XDIFF*XDIFF

YDSC=YDIFF*YDIFF

DIST(I)=SQRT(XDSC + YDSC)

IP=NLEFT(I) + NRIGHT(I)

HITINT(I)=(WIDTH/WH(I))*REFL*IP

GO TO 230

231 DIST(I)=0.0

HITINT(I)=0.0

230 CONTINUE

IF(IAVE.EQ.1) GO TO 1210

DO 210 I=1,N

WRITE(6,943) I,XH(I),YH(I),DIST(I),ANGH(I),WH(I),HITINT(I),

1 NRIGHT(I),NLEFT(I)

943 FORMAT(' ',15,5X,2F15.6,F10.5,F10.3,2F10.5,5X,13,6X,13)

IF(IPUNCH.EQ.0) GO TO 312

WRITE(7,999) THETA,I,XH(I),YH(I),ANGH(I),WH(I),NRIGHT(I),NLEFT(I)

999 FORMAT(F10.2,15,2F15.6,F10.3,5F15.6,215)

312 CONTINUE

210 CONTINUE

1210 CONTINUE

C CALCULATE AVERAGE NO. OF REFLECTIONS AND AVERAGE POWER/CONCENTRATION
C AT THE ABSORBER

NSUM=0

PWRSUM=0.0

DO 220 I=1,N

IF(HITINT(I).LT.1.0E-5) GO TO 220

NP=NRIGHT(I) + NLEFT(I)

NSUM=NSUM + NP

PWRSUM=PWRSUM + REFL*NP

220 CONTINUE

XSUM=NSUM

AVEREF=XSUM/XN

AVEPWR=PWRSUM/XN

WRITE(6,701) AVEREF,AVEPWR

701 FORMAT('0', 'AVERAGE NUMBER OF REFLECTIONS =',F10.3,' AVERAGE INT
ENSITY/CONCENTRATION AT ABSORBER =',F10.4)

C

C INSERT ABSORBER ENERGY HISTOGRAM HERE

C

CALL ABSORB(THETA).

C

C

INSERT AVERAGING ROUTINE HERE

C

IF(IAVE.EQ.0) GO TO 2010

DO 2020 I=1,N

IF(INEQ.EQ.1) GO TO 2030

STORE(I)=STORE(I) + ABR(I)

GO TO 2020

2030 STORE(I)=STORE(I) + ABB(N-I+1)

2020 CONTINUE

IF(!!LAST.EQ.0) GO TO 1

C CALCULATE HISTOGRAM FOR AVERAGE ENERGY

XNTH=FLOAT(NTHETA)

DO 2040 I=1,N

2040 STORE(I)=STORE(I)/XNTH

STMAX=STORE(1)

NMAX=1

DO 2050 I=2,N

IF(STORE(I).GT.STMAX) GO TO 2060

GO TO 2050

2060 STMAX=STORE(I)

NMAX=I

2050 CONTINUE

DO 2070 I=1,N

JMAX=STORE(I)*100/STMAX + 0.5

IF(JMAX.GT.100) JMAX=100

IF(JMAX.EQ.0) GO TO 2180

DO 2080 J=1,JMAX

2080 IHIST(I,J)=PLUS

2180 CONTINUE

JMAXP1=JMAX + 1

IF(JMAXP1.GT.100) GO TO 2070

DO 2090 J=JMAXP1,100

2090 IHIST(I,J)=BLANK

2070 CONTINUE

IF(TRUNC.GE.0.999) GO TO 805

THM=THETAM

DC2=D2

HT=HEIGHT

CNC=ACCCNC

DC1=D1TR

GO TO 810

805 CONTINUE

THM=THETAM

DC2=D2

HT=HEIGHT

CNC=CONC

DC1=D1

810 CONTINUE

WRITE(6,2091)

2091 FORMAT('1','*** AVERAGE INTENSITY DISTRIBUTION ALONG ABSORBER OF C
1PC *** AVERAGE IS FOR FOLLOWING CONDITIONS:')

```
WRITE(6,2092) THM,CNC,REFL,TRUNC
2092 FORMAT(' ', 'THETA-MAX(DEGS) =',F10.4,' CONCENTRATION =',F10.4,
1 ' REFLECTIVITY =',F10.3,' TRUNCATION =',F10.5)
WRITE(6,2093) DC1,DC2,HT
2093 FORMAT(' ', 'COLL. WIDTH =',F10.4,' ABS. WIDTH =',F10.3,' HEIGH
1T =',F10.4,' THETA RANGE =')
WRITE(6,2094)
2094 FORMAT('0', ' DISTANCE INTENSITY 1 2
1 3 4 5 6 7 8 9
2 10')
DO 2100 I=1,N
WRITE(6,2095) D(I),STORE(I),(IHIST(I,J),J=1,100)
2095 FORMAT(' ',F10.4,4X,F10.3,4X,100A1)
2100 CONTINUE
C CALCULATE AVERAGE OF DISTRIBUTION
SUMINT=0.0
DO 2110 I=1,N
2110 SUMINT=SUMINT + STORE(I)
AVEINT=SUMINT/XN
WRITE(6,2096) STMAX,NMAX,AVEINT
2096 FORMAT(' ', 'MAXIMUM INTENSITY CORRESPONDS TO',F10.5,' AT NMAX = '
1,15,' AVERAGE INTENSITY ON ABSORBER =',F10.4)
210 CONTINUE
IF(I LAST.EQ.2) GO TO 55
IF(I LAST.EQ.1) GO TO 2
GO TO 1
55 STOP
END
```


SUBROUTINE ABSORB(THETA)

C PROGRAM TO PLOT INTENSITY DISTRIBUTION ALONG ABSORBER OF

C IDEAL OR TRUNCATED CPC COLLECTOR..

COMMON ANGH(100),WH(100),DIST(100),RATINT(100)

COMMON ABB(100),D(100)

COMMON REFL,TRUNC,THETAM,CONC,D1,D2,HEIGHT,ACCONC,D1TR

COMMON NRIGHT(100),NLEFT(100),ONE,TWO,THREE,FOUR,BLANK,IOUT,N

COMMON IAVE,PLUS,INEG,THIST(100,100)

INTEGER ONE,TWO,THREE,FOUR,BLANK,PLUS

DIMENSION DMIN(100),DMAX(100)

DIMENSION ABS1(100),ABS2(100),ABS3(100),ABS4(100)

XN=FLOAT(N)

C CALCULATE EXTENT OF EACH HIT REGION ON ABSORBER

C AND ACUTE ANGLE OF EACH HIT

DO 20 I=1,N

DMIN(I)=DIST(I) - WH(I)/2.0

DMAX(I)=DIST(I) + WH(I)/2.0

IF(DMIN(I).LT.0.0) DMIN(I)=0.0

IF(DMAX(I).GT.D2) DMAX(I)=D2

IF(ANGH(I).GT.90.0) ANGH(I)=180.0 - ANGH(I)

20 CONTINUE

C INITIALIZE INTENSITY ON ABSORBER (N INTERVALS, 100 MAXIMUM)

DO 30 I=1,N

ABB(I)=0.0

ABS1(I)=0.0

ABS2(I)=0.0

ABS3(I)=0.0

ABS4(I)=0.0

30 CONTINUE

C SET INTERVAL POINTS ON ABSORBER

DABS=D2/XN

DO 390 I=1,N

IF(I.EQ.1) GO TO 391

D(I)=D(I-1) + DABS

GO TO 390

391 D(I)=DABS

390 CONTINUE

C EXAMINE EACH OF THE N RAYS; ADD CONTRIBUTIONS INTO

C APPROPRIATE BOXES ON ABSORBER.

DO 400 I=1,N

IMN=DMIN(I)/DABS + 1

IMX=DMAX(I)/DABS + 1

IF(IMX.GT.N) IMX=N

C PRINTOUT IS DETAILED UNLESS IOUT=0

IF(IOUT.EQ.0) GO TO 401

WRITE(6,900) I,DMIN(I),DIST(I),DMAX(I),ANGH(I),RATINT(I),IMN,IMX

900 FORMAT('0','RAY NO =',I5,' DMIN,DIST,DMAX =',3F10.6,' ANGLE =',
1,F10.3,' INTENSITY =',F10.5,' BOX NOS HIT =',2I5)

401 CONTINUE

DLOW=DMIN(I)

DO 410 J=IMN,IMX

IF(J.EQ.IMX) GO TO 420

DHIGH=D(J)

```

GO TO 430
420 DHIGH=DMAX(I)
430 CONTINUE
FRACT=(DHIGH - DLOW)/DABS
SUNINT=RATINT(I)*FRACT
IF(ANGH(I).LT.80.0) GO TO 42
ABS1(J)=ABS1(J) + SUNINT
GO TO 39
42 IF(ANGH(I).LT.50.0) GO TO 43
ABS2(J)=ABS2(J) + SUNINT
GO TO 39
43 IF(ANGH(I).LT.20.0) GO TO 44
ABS3(J)=ABS3(J) + SUNINT
GO TO 39
44 ABS4(J)=ABS4(J) + SUNINT
39 ABB(J)=ABB(J) + SUNINT
C PRINTOUT UNLESS ICUT=0
IF(ICUT.EQ.0) GO TO 402
WRITE(6,901) J,DLOW,DHIGH,FRACT,SUNINT,ABB(J),ABS1(J),ABS2(J),
1 ABS3(J),ABS4(J)
901 FORMAT(' ',I3,' PCX LIMITS =',2F10.5,' FRACT,SUNINT =',2F10.5/
1 ABB,ABS1,ABS2,ABS3,ABS4 =',5F10.5)
402 CONTINUE
DLOW=D(J)
410 CONTINUE
400 CONTINUE
C PPINT-OUT INTENSITY DISTRIBUTION ALONG ABSORBER
C IF IAVE=1, HISTOGRAM IS NOT PRINTED
IF(IAVE.EQ.1) GO TO 1194
WRITE(6,94) THETA,INEG
94 FORMAT('1','**** INTENSITY DISTRIBUTION ALONG ABSORBER ****'///
1 THETA =',F10.4,' POSITIVE(0) OR NEGATIVE(1) ?',I5///' DISTANC
2F 80 TO 90 50 TO 80 20 TO 50 0
3TC 20 TOTAL'///)
DO 50 I=1,N
WRITE(6,95) D(I),ABS1(I),ABS2(I),ABS3(I),ABS4(I),ABB(I)
95 FORMAT(' ',F10.3,5F20.4)
50 CONTINUE
1194 CONTINUE
C
C ADD HISTOGRAM OUTPUT HERE
C
ABBMAX=ABB(1)
NMAX=1
DO 600 I=2,N
IF(ABB(I).GT.ABBMAX) GO TO 601
GO TO 600
601 ABBMAX=ABB(I)
NMAX=I
600 CONTINUE
IF(IAVE.EQ.1) GO TO 1195
DO 60 I=1,N
IF(ABBMAX.LT.0.0001) GO TO 169

```

```
IM4=ABS4(I)*100/ABRMAX + 0.5
IF(IM4.GT.100) IM4=100
IM3=ABS3(I)*100/ABRMAX + 0.5
IM2=IM3 + IM4
IF(IM3.GT.100) IM3=100
IM2=ABS2(I)*100/ABRMAX + 0.5
IM2=IM2 + IM3
IF(IM2.GT.100) IM2=100
IM1=ABS1(I)*100/ABRMAX + 0.5
IM1=IM1 + IM2
IF(IM1.GT.100) IM1=100
IF(IM4.EQ.0) GO TO 61
DO 62 J=1,IM4
62 IHIST(I,J)=FCUR
61 IF(IM4.EQ.100) GO TO 60
IMP=IM4 + 1
IF(IM3.EQ.IM4) GO TO 63
DO 64 J=IMP,IM3
64 IHIST(I,J)=THREE
63 IF(IM3.EQ.100) GO TO 60
IMP=IM3 + 1
IF(IM2.EQ.IM3) GO TO 65
DO 66 J=IMP,IM2
66 IHIST(I,J)=TWO
65 IF(IM2.EQ.100) GO TO 60
IMP=IM2 + 1
IF(IM1.EQ.IM2) GO TO 67
DO 68 J=IMP,IM1
68 IHIST(I,J)=ONE
67 IF(IM1.EQ.100) GO TO 60
IMP=IM1 + 1
GO TO 269
169 IMP=1
269 CONTINUE
DO 69 J=IMP,100
69 IHIST(I,J)=BLANK
60 CONTINUE
C PRINT OUT HISTOGRAM
IF(TRUNC.GE.0.999) GO TO 805
THM=THETAM
DC2=D2
HT=HEIGHT
CNC=ACCONC
DC1=D1TR
GO TO 810
805 CONTINUE
THM=THETAM
DC2=D2
HT=HEIGHT
CNC=CONC
DC1=D1
810 CONTINUE
WRITE(6,8000)
```

```
8000 FORMAT('1','*** INTENSITY DISTRIBUTION ALONG ABSORBER OF CPC ***')
      WRITE(6,8001) THM,CNC,REFL,TRUNC
8001 FORMAT(' ','THETA-MAX(DEGS) =',F10.4,' CONCENTRATION =',F10.4,
1 ' REFLECTIVITY =',F10.3,' TRUNCATION =',F10.5)
      WRITE(6,8002) DC1,DC2,HT,THETA
8002 FORMAT(' ','COLL.WIDTH =',F10.4,' ABS.WIDTH =',F10.3,' HEIGHT
1 =',F10.4,' INCIDENT THETA =',F10.4)
      WRITE(6,801)
801  FORMAT('0',' DISTANCE          INTENSITY          1          2
1 3          4          5          6          7          8          9
2 10')
      DO 70 I=1,N
      IF(INEG.EQ.1) GO TO 1802
      WRITE(6,802) D(I),ABB(I),(IHIST(I,J),J=1,100)
802  FORMAT(' ',F10.4,4X,F10.3,4X,100A1)
      GO TO 70
1802 IROW=N-I+1
      WRITE(6,802) D(I),ABB(IROW),(IHIST(IROW,J),J=1,100)
70  CONTINUE
1195 CONTINUE
C  CALCULATE AVERAGE INTENSITY ON ABSORBER
      SUMINT=0.0
      DO 75 I=1,N
      SUMINT=SUMINT + ABB(I)
75  CONTINUE
      AVEINT=SUMINT/XN
      IF(INEG.EQ.1) NMAX=101 - NMAX
      WRITE(6,803) ABBMAX,NMAX,AVEINT
803  FORMAT(' ','MAXIMUM INTENSITY CORRESPONDS TO',F10.5,' AT NMAX ='
1,15,' AVERAGE INTENSITY ON ABSORBER =',F10.4)
      RETURN
      END
```