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LA-UR--85-3416

DE86 000803

TITLE: SOURCE-JERK ANALYSIS USING A SEMI-EXPLICIT INVERSE KINETIC TECHNIQUE

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SUBMITTED TO: Workshop on Subcritical Reactivity Measurements
UNM
Albuquerque, New Mexico
August 26-29, 1985

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**SOURCE-JERK ANALYSIS USING A SEMI-EXPLICIT
INVERSE KINETIC TECHNIQUE**

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ABSTRACT

A method is proposed for measuring the effective reproduction factor, k , in subcritical systems. The method uses the transient response of a subcritical system to the sudden removal of an extraneous neutron source (i.e., a source jerk). The response is analyzed using an inverse kinetic technique that least-squares fits the exact analytical solution corresponding to a source-jerk transient as derived from the point-reactor model. It has been found that the technique can provide an accurate means of measuring k in systems that are close to critical (i.e., $0.95 < k < 1.0$). As a system becomes more subcritical (i.e., $k \ll 1.0$) spatial effects can introduce significant biases depending on the source and detector positions. However, methods are available that can correct for these biases and, hence, can allow measuring subcriticality in systems with k as low as 0.5.

INTRODUCTION

Recent interest in measuring k in subcritical systems for which $k \ll 1.0$ has spurred the development of several new techniques and has reincarnated an old, but seldom used technique, the "source jerk". Briefly, the source-jerk technique is as follows. A neutron source is placed into a subcritical system, and the neutron density is allowed to reach an equilibrium value proportional to the source strength and inversely proportional to the quantity $(1 - k)$. The source is then rapidly ejected (or jerked) from the system and the resultant transient observed. The reactivity of the system is then inferred from analysis of the transient data. In the past, there have been two analysis techniques: (1) the prompt-drop approximation and (2) the integral-flux method. A brief description of these two methods will be presented below followed by the description of a new method, "the semi-explicit inverse kinetic technique".

PROMPT-DROP APPROXIMATION METHOD

It is not known who originally derived the prompt-drop approximation, but the first reference to its use in measuring the reactivity of a subcritical system via the source-jerk technique was made by Jankowski et al. [1]. Using the nomenclature of Matrick [2], it can be shown that the reactivity of the system can be related to the sudden drop in power immediately following the source jerk via

$$\frac{\rho}{\beta} = - \left(\frac{n_0 - n_1}{n_1 - n_b} \right) \quad (1)$$

where ρ = reactivity [defined as $(k - 1)/k$], β = effective delayed neutron fraction, n_0 = initial equilibrium neutron density, n_1 = neutron density level obtained immediately after source jerk, and n_b = final equilibrium neutron density (i.e., background). In the derivation of Eq. 1, it was assumed that nonremovable neutron sources may be present in the system. These might include extraneous neutrons produced by spontaneous fissioning, photo-neutrons, etc. Note that the reactivity is measured in terms of "dollars" (i.e., ρ divided by β). This is an inherent characteristic of all reactivity measurements based upon the dynamic response of a reactor. The response is governed strictly by the ratio ρ/β , not ρ . It should also be noted that Eq. 1 is derived from the point-reactor model, and hence n_0 and n_1 are interpreted to be neutron densities proportional to the fundamental mode of the transient behavior of the neutron density.

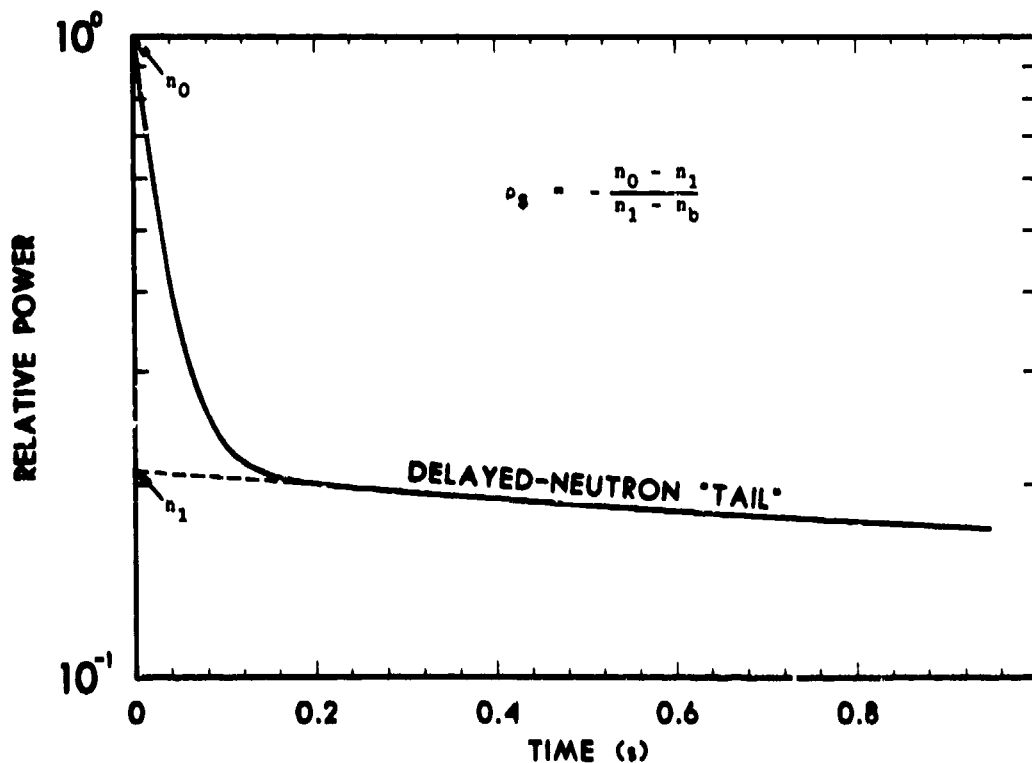


Figure 1
Neutron density as a function of time, illustrating
a source-jerk measurement of reactivity.

Although this method works, in principle, it has several shortcomings that have limited its use. First, and foremost, it is very susceptible to spatial effects produced by the rapid decay of higher flux harmonics present during the initial portion of the transient, the point in time at which n_1 is measured. Hence, a poorly positioned detector or source will yield a biased n_1 , and an "apparent" reactivity that can be significantly different from the true reactivity of the system. Second, for highly subcritical systems, n_1 will be several orders of magnitude lower than n_0 . The counting statistics at n_1 may become so low as to produce a large uncertainty in n_1 and a subsequently large uncertainty in the calculated ρ . And third, because the prompt-drop approximation assumes an instantaneous removal of the source, if the source is not removed in a time that is short compared to the shortest-lived delayed-neutron group, extrapolating back in time to find n_1 may become more tenuous, resulting again in a large uncertainty for ρ .

INTEGRAL-FLUX METHOD

A vast improvement to the prompt-drop approximation technique was introduced by Schmid.[3] Rather than just observe the initial behavior of the neutron density following the source jerk, the integral of the delayed-neutron tail following the source jerk is measured and related to the reactivity of the system. This relationship is derived as follows.

Immediately following the source jerk, it is assumed that the point-reactor equations for an arbitrary number of delayed-neutron groups is applicable. That is,

$$\frac{dn}{dt} = \left(\frac{\rho - \beta}{l}\right) n + \sum_1 \lambda_1 C_1 + q_b \quad , \text{ and} \quad (2)$$

$$\frac{dC_1}{dt} = \left(\frac{\beta_1}{l}\right) n - \lambda_1 C_1 \quad , \quad \text{for } i = 1, 2, 3, \dots, g \quad , \quad (3)$$

where n = neutron density, t = time, C_1 = delayed-neutron precursor density of i^{th} group, q_b = the final effective source strength, λ_1 = decay constant of i^{th} precursor group, β_1 = delayed-neutron yield of i^{th} precursor group, g = number of delayed-neutron groups, and l = neutron-generation time. Integrating Eqs. 2 and 3 from $t = 0$ to $t = \infty$ yields the following expression, which relates reactivity (in dollars) to the integral of the neutron density occurring during the decay of the delayed-neutron tail (Figure 2):

$$\frac{\rho}{\beta} = - \frac{(n_0 - n_b)}{\int (n - n_b) dt} \left[\frac{1}{\beta} \sum_1 \frac{\beta_1}{\lambda_1} + \frac{l}{\beta} \right] \quad (4)$$

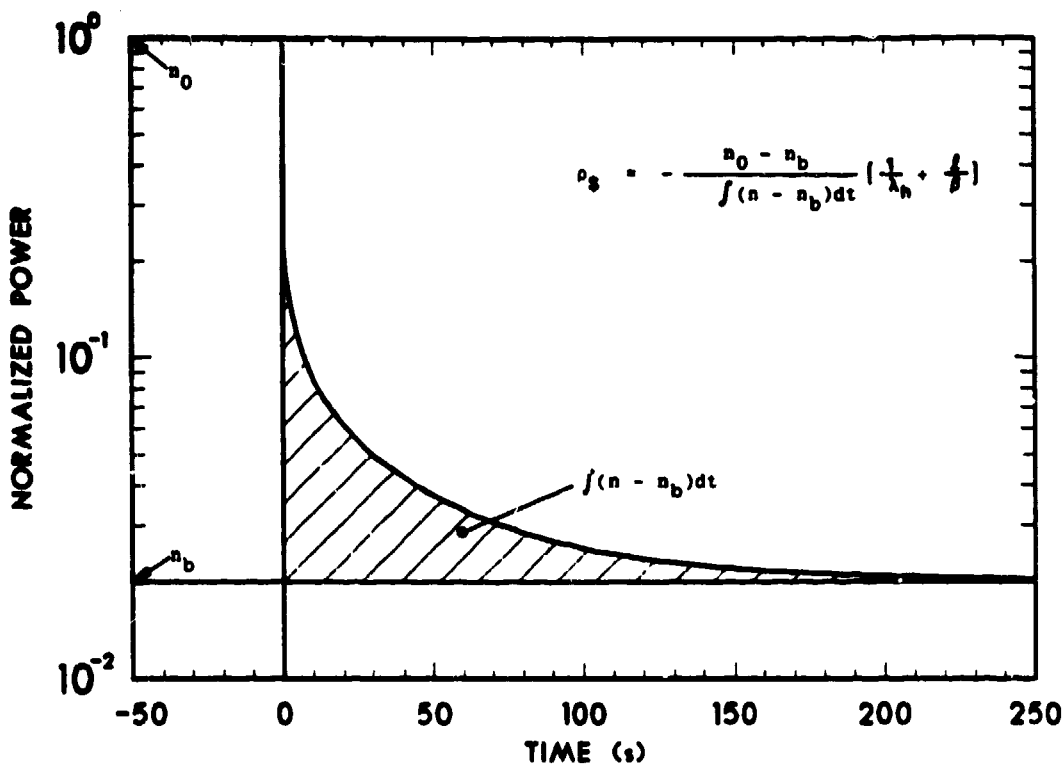


Figure 2
Integral of the neutron density used in source-jerk analysis.

The first term in the bracket on the right-hand side of Eq. 4 is recognized as the weighted harmonic-mean decay constant for the delayed-neutron precursors. That is,

$$\frac{1}{\lambda_h} = \frac{1}{\beta} \sum_1 \frac{\beta_1}{\lambda_1} \quad (5)$$

It is obvious that λ_h is a function of both the effective relative yields and the decay constants of each delayed-neutron group for the reactor system upon which the source jerk is performed. Therefore, some additional knowledge of the reactor system is required in order to perform a reactivity measurement using this technique. Using the delayed-neutron parameters measured by Keepin, Wiggert, and Zeigler, typical values of λ_h are shown in the Table.[4]

TABLE
 DELAYED-NEUTRON HARMONIC MEAN DECAY CONSTANT

Energy	Fuel	$\lambda_h(s^{-1})$
Thermal	^{235}U	0.0767
Thermal	^{239}Pu	0.0648
Thermal	^{233}U	0.0543
1.45 MeV	^{235}U	0.0784
1.58	^{239}Pu	0.0683
1.45	^{233}U	0.0559

Because neutron generation times range from 10^{-3} to 10^{-8} for most reactors, the second term in the bracket on the right-hand side of Eq. 4 is usually negligible in comparison to $1/\lambda_h$. Contingent upon this condition being satisfied, Eq. 4 can be simplified to

$$\frac{\rho}{\beta} = - \frac{(n_0 - n_b)}{\lambda_h \int (n - n_b) dt} \quad (6)$$

Although this method represents an improvement over the prompt-drop approximation, in some respects it still suffers. First, additional information about the reactor is required. The effective delayed-neutron parameters (i.e., λ_1 and β_1) must be known in order to evaluate λ_h . Although the vast majority of reactor systems normally encountered can be well characterized by the values shown in the Table, the requirement of knowing the delayed-neutron parameters does preclude using an integral-flux technique on "black-box" type systems. This is in contrast to the prompt-drop method. In that formulation, the only information necessary to measure the dollars subcritical is the initial equilibrium power level and the power level immediately following the source jerk.

Assuming that the additional information requirement is not a constraint, the integral method is still based upon the point-reactor model and, as such, requires that the measured neutron densities n and n_0 must be proportional to the fundamental mode of decay throughout the entire transient if Eq. 6 is to yield the correct answer. This can only be accomplished by the judicious choice of both source and detector position within the reactor system, particularly if k is well below 1.0. As with the prompt-drop method, a poorly positioned source or detector will yield an "apparent" reactivity that can be significantly biased from the true reactivity.

Several methods have been proposed to convert from an apparent (or spatially-dependent) reactivity to the true reactivity of the system. In general, there have been two approaches. The first approach is to place the source and detector(s) in a location where the first harmonic is nulled out, thereby allowing n and n_0 to be nearly proportional to the fundamental mode. However, this method presumes that this node position is known. For asymmetric systems, this presumption may be questionable. The second approach has been to assume that the true reactivity is related to an apparent reactivity via

$$\left(\frac{\rho}{\beta}\right)_t = f(\vec{r}) \left(\frac{\rho}{\beta}\right)_{\vec{r}} \quad , \quad (7)$$

where the correction factor f is a spatially-dependent quantity that must be determined from either a direct measurement or a calculation of both the neutron and adjoint fluxes. To date, the adequacy of Eq. 7 has been shown to be successful for subcritical systems with k of 0.95 or higher.[5-11]

Although Eq. 7 corrects for the problem that arises from spatial effects, another potential problem plagues this method of analysis. If the source is not removed instantaneously, the integral of the neutron density may be altered significantly during the source-removal "ramp". For systems that are highly subcritical, the contribution to the integral during the source-removal ramp may be as large or larger than the contribution from the entire delayed-neutron tail. This, obviously, will result in an erroneous calculation of reactivity. To circumvent this potential problem, a new method for analyzing source-jerk data is proposed as follows.

INVERSE-KINETIC TECHNIQUE

Rather than integrate Eqs. 1 and 2, it can be shown that the solution to that system of differential equations for a source-jerk transient in a system subcritical by ρ/β corresponds to

$$\frac{n - n_b}{n_0 - n_b} = \sum_j \frac{(\rho/\beta) e^{\omega_j(t-t_0)}}{\omega_j \left[\frac{\lambda}{\beta} + \sum_1 \frac{\lambda_1 \beta_1}{\beta(\omega_j + \lambda_1)^2} \right]} \quad , \quad (8)$$

where ω_j equals the j^{th} root of the inhour equation, \sum_1 represents the sum from $i = 1$ to $i = g$, and \sum_j represents the sum from $j = 1$ to $j = g + 1$. Equation 8 is valid for $t > t_0$, where t_0 represents an arbitrary time shift. For $t < t_0$, the reactor is at its steady-state value of n_0 .

Equation 8 describes the time-dependent behavior of the fundamental mode neutron density. It is noted that the right-hand side of Eq. 8 is a function of the following parameters: ρ/β , β_1/β , λ_1 , and l/β . The roots, ω_1 , are known quantities that can be determined from the inhour equation once the above parameters are specified. As with the integral-flux method, this requires some additional information about the system upon which the source jerk is to be performed. If it is assumed that the delayed-neutron parameters are known, then the right-hand side of Eq. 8 becomes strictly a function of reactivity (in terms of dollars). Hence, given a power history produced by a source jerk in a well defined system, it is possible to determine the reactivity of the system by successively iterating on ρ/β until the power history predicted by Eq. 8 matches the observed power history produced by the source jerk. In order to perform this iteration, a nonlinear least-squares fitting code has been adapted for this purpose.[12] An example of a source-jerk transient analysis using this technique is shown in Figure 3.

Performing an analysis of a source-jerk transient using this technique improves the experiment in two ways. First, the result becomes only weakly dependent on the source-removal ramp time. This occurs primarily because the relative power, at times several seconds into the transient, is essentially identical to the relative power corresponding to a true step removal of the source. Hence, for source-removal ramp times of 1 second or less, the

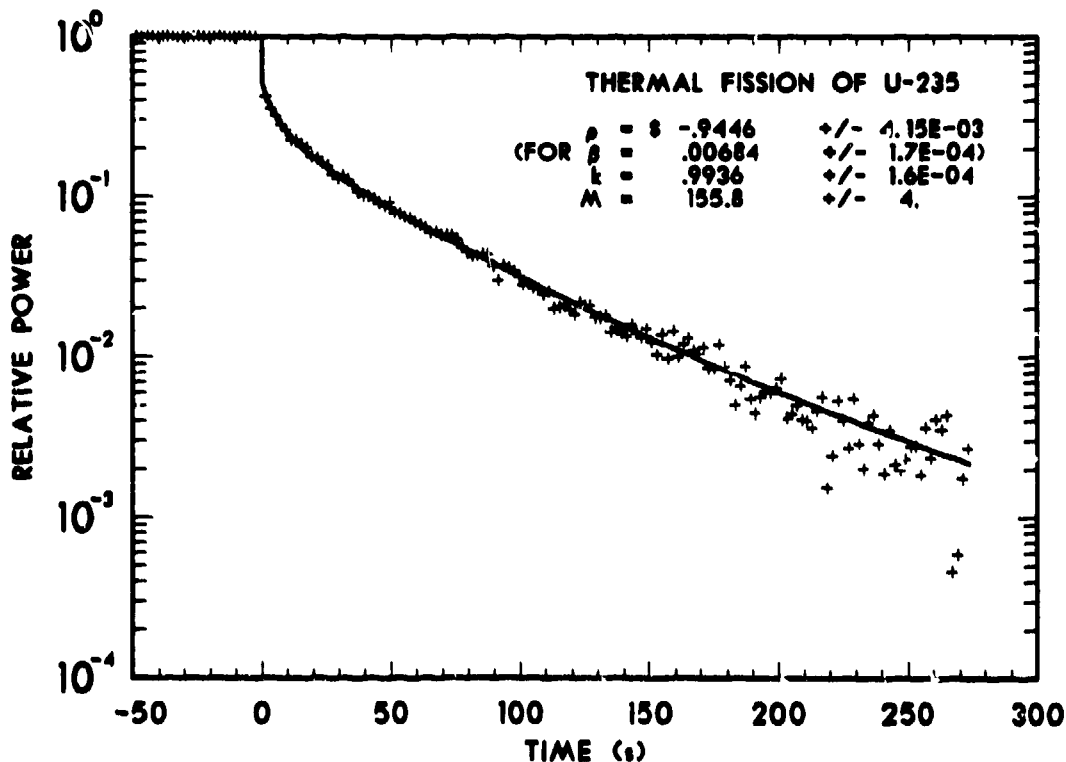


Figure 3
Example of analysis of source-jerk transient using the inverse-kinetic method.

least-squares fit of the data following the source ramp will yield the same answer as if it were a true step change in source strength. Second, by analyzing the transient behavior from beginning to end, it is easily ascertained if the delayed-neutron tail is well correlated by the choice of the delayed-neutron parameters assumed for that system by visually comparing the data against the least-squares fit.

On the other hand, the inverse-kinetic method still suffers from the problems that plague both the prompt-drop method and the integral-flux method. Namely, spatial effects can significantly bias the reactivity measurement.

CONCLUSIONS

Measuring reactivity of a subcritical system by way of a source jerk is an easy and viable technique. It requires very little electronic equipment and can be performed in less than 30 minutes. Depending on the method chosen to analyze the transient data (i.e., the prompt-drop, the integral-flux, or the inverse-kinetic method), the results can be very accurate in the range of $k > 0.95$ and can be reasonably accurate in the range of $0.5 < k < 0.95$.

The only serious limitation to the use of the source-jerk technique arises primarily from biases introduced by spatial effects in far subcritical systems. However, with proper choice of source and detector position, these biases can be reduced to within acceptable limits.

Currently, work is in progress at Los Alamos to quantify the biases that are created by spatial effects in relatively simple systems. Once this work is completed, more difficult (and perhaps more practical) systems will be studied in hopes of defining a regime in which k can be measured adequately using the source-jerk technique.

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