Seismic Design Study On
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Abstract

The best way to design a nuclear steam generating system to withstand seismic stresses is to establish a mathematical model, to which seismic loading can be applied, to determine the dynamic responses of the system. A practical approach for establishing such a model is to idealize the components in terms of beam elements which form the essential building blocks of the analytical model. The major shell structures of reactor vessels and steam generators are usually supported at a single elevation in the system, and are often simulated by cantilever beam elements. The use of a beam element to represent shell structure must be justified.

This paper presents the results of a parametric study of thin-walled cylindrical shells which are clamped at one end while the other end is free. By using 3-D plate/shell finite element analysis, the region of application, where the beam element can reasonably simulate the shell structure, has been defined as the combination of its geometrical parameters, e.g., the thickness/radius and length/radius. As an illustrative example, the 3-D finite element method has been used to investigate the seismic analysis of a cantilever cylindrical shell similar to the Clinch River LMFBR reactor vessel. The numerical values are compared with those produced by a beam element model. The results and conclusions obtained by this investigation can be applied to improve dynamic modeling techniques in general.

I. Introduction

The traditional engineering approach to determine the component structural response of a power plant under dynamic loading was to employ an equivalent static force method. With the increase in engineering knowledge and the improvement in analytical procedures in the dynamic field, the technique for seismic design of a nuclear steam system (NSS) has gradually shifted to actual dynamic analysis. This analysis involves analytical modeling and a method of calculation which may be based on either real-time integration or response spectra procedures. The choice of the method of calculation is predetermined by the specification of the seismic loading. However, the establishment of an analytical model, consistent with the actual hardware and amenable to numerical calculations, is subject only to each individual's ingenuity. In practice, major factors such as the structural geometry, the importance of the components in the system, personal experience, the applicable code and standards, and the manufacturing schedule, have either explicitly or implicitly controlled the course of action.

A typical engineering approach to the seismic design of an NSS component by using the finite element method is presented in this paper. Two major items affecting the validity of the solution have been considered during the preliminary modeling of the structure. First was the selection of a proper model that would preserve the essential dynamic characteristics yet be simple enough for mathematical formulation. Second was the accuracy and applicability of the calculation procedures, i.e., the computer program should be qualified for the selected model. Two finite element models, a beam element and a 3-D plate/shell element, simulating a typical liquid metal fast breeder reactor (LMFBR) vessel have been evaluated. The vessel is represented by a cantilever

cylindrical shell. A parametric study has been performed on such types of shells to define the region of applicability where a beam element can reasonably simulate the shell structure. The results can be used as guidelines for those who are developing a dynamic model of a nuclear steam generator system where the major shell structures are supported at a single elevation and can be modeled into a beam element. During the study, the model determined by 3-D plate/shell analysis of element Q-19 presented by Clough and Fellippa^[1]* has been evaluated and the computer program qualified for such application. A frequency comparison between the analytical results and the experimental data of a set of cantilever cylindrical shells^[2] is reported. Finally, the dynamic responses of the LMFBR reactor vessel, simulated by both the beam element and shell element, due to seismic loading (floor response spectra) are calculated. The numerical results have demonstrated that a beam model may produce misleading design information when the shell structure is oversimplified in the modeling process.

*Numbers in bracket refer to the entries of references.

II. Program Qualification

A classical thin cylindrical shell is modeled into an assemblage of the refined quadrilateral element (Q-19) representing both membrane and flexural stiffness properties. The assumptions and limitations of this Q-19 element have been discussed by Clough and Felippa in [1]. In brief, it is within the framework of the thin shell theory except that the normal derivatives of displacement functions are piece-wise continuous among the elements. However, the increase in number of elements will diminish such discrepancies. The shell model as shown in Figure 1 is fully restrained at the top end and is free of loading at the bottom. The shell has been modeled into various numbers of mesh sizes until the convergence of its numerical solutions (first five model frequencies) is achieved. It was found that with a mesh size of less than 15° circumferential and an aspect ratio no greater than 4/1 the computer solution produced reasonable accuracy. Based on this optimized mesh size, three particular cylindrical shells with known experimental data^[2] were investigated. The numerical values in terms of frequency factor (Δ) were plotted against the experimental data as shown in Figure 2. It is surprising to note that the major discrepancy (about 20% deviation) occurs at the lowest mode of vibration (circumferential wave number $n = 2$). But for higher modes of vibration ($n = 3$ and 4) the calculated frequencies and experimental data are in good agreement - within 10%. An independent analytical calculation^[6] has shown the same phenomenon. Normally, one would expect better agreement at the lowest mode of vibration than at higher modes. Without knowing the test fixture in detail, we could not offer a reasonable explanation. However, it has been experienced in the past^[3] that the end fixity could influence the frequencies of vibration especially with the small number of n . Overall, except $n = 2$, the computer results match the test data quite well.

III. Parametric Studies

a. Open-Ended Shells

A series of computer runs have been performed with various combinations of two geometrical parameters, namely, the ratio of length to radius and the ratio of thickness to radius of the shell. The purpose of this study was to define the range of applicability where a shell sector can be approximately replaced by a beam element. The fundamental frequency of the finite element model was used as the criterion to compare with that of beam theory. The procedure is justified for this preliminary study since most seismic loadings are within the low frequency range and the structural responses shall be controlled by its fundamental mode. Results are plotted in Figure 3 for comparison with classical beam theory^[4] in terms of the frequency ratio with respect to the fundamental mode. The frequencies for the beam theory, taking into consideration the rotatory inertia and shear deformation^[5], are also calculated and plotted by a broken line shown in Figure 3. The horizontal line with the ratio value equal to unity shows the shell vibrates at the same frequency as the classical beam. This study has shown that classical beam theory accurately predicts fundamental frequencies for shells with longer length ($l/R \geq 20$) and is insensitive to the thickness. In shorter cylinders ($9 \leq l/R \leq 20$) the thickness of the shell moderately affects the frequency of vibration. However, in very short cylinders ($l/R < 8$), the shell no longer vibrates in the beam mode ($n = 1$); its vibration modes are controlled by circumferential bending energy ($n > 1$). Beam theory including the rotatory and shear effect predicts better solutions than the classical theory as long as the shell is vibrating at the swaying mode ($n = 1$). Neither theory can accurately predict the fundamental frequency of a shell vibrating at its circumferential

bending modes ($n > 1$). Information obtained from this study is applicable to cylinders with open ends, such as smokestacks.

b. Vessels With End Caps

When the analysis is applied to a storage tank or a pressure vessel, the parametric study shall be performed on cylindrical shells with end caps. One has to establish the analytical model simulating such a compound structure. The classical closed-form approach would be much too involved if not impossible to employ. Investigation centered on cylindrical shells with a flat end plate and used the finite element method. The thickness of the end plate is assumed to be the same as the vessel wall. Parametric studies on the various combinations of thickness/radius and length/radius ratios have been carried out. The numerical calculations plotted in the same format as for the open-ended shells are shown in Figure 4 to compare the frequency results between the plate/shell model and the Timoshenko beam model with a lumped mass on the tip. Results indicate improvement in the range of application for the beam element to represent the shell model. This is caused by the end plate which enhances the vessel's tendency to vibrate in the swaying mode ($n = 1$). For the swaying mode of vibration, the end plate contributes the inertia effect (added mass) to the vessel; and consequently, the fundamental frequency is reduced in comparison with that of the open-ended shell. It is also interesting to note, that as the vessel gets shorter and thinner, its fundamental frequency becomes the frequency due to vibration of the end plate.

The possibility of such plate modes of vibration should be evaluated when one is trying to simulate the actual vessel with a beam element. The discrepancies in terms of frequency ratios have been clearly indicated in Figure 4.

IV. Seismic Responses

A preliminary seismic analysis has been applied to a typical IMFBR reactor vessel to determine its maximum responses. The calculation procedure is based on the seismic response spectra^[7] combined with the normal mode method. Two simplified analytical models (beam model and shell model) were established to simulate the reactor vessel of the Clinch River Breeder Reactor Plant (CRBRP). For illustrative purposes, the internal structures were omitted in this model. The actual structure (shown in Figure 5) is represented by a simple cylindrical shell fixed at the top and an end plate attached at the bottom end. The vessel is 310 cm. in radius, 1727 cm. in length, and 7.9 cm. in wall thickness. The total weight of the structure including its internals and sodium is 1,794,210 kg. which is assumed to be uniformly distributed over the vessel. The beam model was composed of 40 sectors with the end-plate mass lumped at its tip. It was treated as a cantilever beam taking into consideration the transverse shear and rotatory inertia. The calculated modal frequencies are listed in Table I. The shell model consisted of 241 nodes and 264 finite elements; ten layers in the axial direction, 24 elements (Q-19) in the circumferential direction for each layer. The end plate was divided into 24 triangular plate elements. The quadrilateral element has an aspect ratio of about 2:1. Its calculated frequencies are also listed in Table I for comparison. This shows that the beam model does not have the capacity of predicting the system fundamental frequency. The modes having identical frequencies correspond to the modes of vibration in the X- and Y- directions because the structure is symmetric with respect to both axes.

As an illustrative example, let us consider a horizontal seismic loading. The CRBRP OBE seismic response spectrum in the horizontal direction shown in Figure 6 was applied to both models. In the shell model, the first modal frequency was due to the transverse vibration of the end plate.

However, the horizontal seismic loading did not strongly excite this mode of vibration. Structural response was dominated by the second and third modal participation, corresponding to the beam bending modes. In such cases, one would expect the maximum stress results of the shell and beam models to be the same. Table II shows the maximum principal stress determined by the shell model is about 3% higher than that of the beam model. This is due to the fact that the biaxial stresses have been generated in the plate/shell analysis while the beam model considers only the longitudinal bending stresses. Although the difference is small, it points out the possibility that the beam model may be less conservative. This demonstrates the necessity of the preliminary parameter study on the selection of a proper analytical model.

V. Conclusions

This paper presents a typical design procedure for the seismic analysis of cantilever cylindrical shells using a finite element program. A qualification evaluation provides the validity of the analytical method used in the Finite Element Stress Analysis Program (FESAP). The parameter studies have generated the information defining the range of application for both open- and close-ended cylinders which can be modeled by beam elements. Results have been reported in terms of dimensionless parameters and plotted in curves for general use. One can reference these curves as a guideline to establish a proper dynamic analytical model. The application need not be limited to seismic design. It can also be useful to check against flow-induced vibration where low frequency excitation may be expected. The numerical solution example applied on the reactor vessel of the CRBRP has demonstrated that a beam model may produce less conservative design information when the physical structure is oversimplified in the modeling process. This further justifies preliminary study on the choice of a proper model.

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TABLE I - MODAL FREQUENCIES (HZ)

MODE NO.	BEAM MODEL	SHELL MODEL
1	5.324	4.720
2	5.324	5.308
3	20.95	5.308
4	20.95	7.255
5	23.67	7.255

TABLE II - MAXIMUM RESPONSE SRSS STRESSES (MPA)
(OBE Seismic Response Spectrum, Horizontal, CRBRP RV)

LOCATION	Beam Element	Shell Model	
	MEMBRANE	MEMBRANE	BENDING & MEMBRANE
Fixed	93.75	92.6	96.6
Middle Section	38.0	38.2	38.5
Bottom End	0.6 "	4.98	5.30

FIGURE 1

Finite Element Shell Model
and Vibration Modes

$N=3$

$N=2$

FIGURE 2 - Comparisons, Finite Element Solution vs. Experimental Data

FIGURE 3 - Frequency Comparison, Beam Theories vs. Finite Element Shell Model

FIGURE 4 - Frequency Ratio, Shell Model/Beam Model

FIGURE 5 — REACTOR VESSEL, CLINCH RIVER BREEDER REACTOR PLANT

ACCELERATION - G

FIGURE 6 - OBE HORIZONTAL ACCELERATION
RESPONSE SPECTRUM AT R. V.
SUPPORTS (2% DAMPING)

2.5

2.05

2.0

1.5

1.0

0.5

0

FREQUENCY - HZ

Fig 6 OBE HORIZONTAL RESPONSE
SPECTRUM AT R.V. SUPPORTS
(2% DAMPING)