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An Introduction to The Study of Collisions between Heavy Nuclei

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INTRODUCTION

My object is to discuss some of the physical problems currently being investigated concerning the collisions of nuclei, with kinetic energies comparable to what you will be able to achieve with your new accelerator. As we will see, this includes a wide range of phenomena, and our first task is to decide on a reasonable scheme for classifying the physically different processes that can occur. All these classification schemes start from the observation that the kinds of processes we will talk about are governed by small de-Broglie wave lengths—small compared to the dimensions of nuclei, or to the geometrical scale of the orbit. For example, the Sommerfeld parameter is defined by,

$$\eta = \frac{\text{distance of closest approach in a head-on Rutherford orbit}}{2 \times \text{de Broglie wave length}} \quad (1)$$

$$= \frac{Z_1 Z_2 e^2}{\hbar v} = .157 Z_1 Z_2 \sqrt{\frac{\text{projectile mass (amu)}}{E_{\text{lab}} (\text{MeV})}}$$

and has the value of 37 for 100 MeV oxygen incident on a dysprosium target. Similarly the ratio of the sum of the radii to the de-Broglie wavelength

$$\frac{R_1 + R_2}{\lambda} = \frac{1}{2} (R_1 + R_2)$$

is about 73 for this case. Thus the de-Broglie wavelength is small enough so that we can construct wave packets that localize the nuclei in regions small compared to their diameters, or to the scale of the orbit. This does not mean that we can ignore quantum mechanics, but it does mean that we can say that some phenomena are dominated by one region of configuration space, other phenomena by another region.

Fig 1a.

Fig 1b.

So we start with a schematic potential between colliding nuclei, as a function of their separation, and add in their electrostatic interaction. Of course this separation variable is not well defined when the nuclei overlap significantly. When we add V_C and V_N we can get two different kinds of pictures. For $r \gg R$ the force between the nuclei is always repulsive, but if the Coulomb repulsion isn't too strong there will be a region near the nuclear surface where the nuclear attraction overwhelms it, producing a net attraction. This leads to a

"pocket" in the combined potential, and an outer Coulomb barrier. If the nuclear potential is described by a Wood-Saxon form, the condition for this to occur is roughly

$$\frac{Z_1 Z_2 e^2}{(R_1 + R_2)^2} < \frac{V_0}{4a} \quad (2)$$

since $\frac{V_0}{4a}$ is the maximum attractive force associated with this potential.

On the other hand, if $\frac{Z_1 Z_2 e^2}{(R_1 + R_2)^2} \geq \frac{V_0}{4a}$, there is no region of attraction and the combined nuclear + coulomb potential looks like Fig. 1c with no pocket and no barrier. If there is a barrier and a pocket, we have the possibility of the approaching nuclei overcoming the barrier, and getting caught in the pocket, leading to a fusion process.

Fig. 1c

The potential curves drawn so far are adequate only for the discussion of head-on collisions, i.e. those with zero impact parameter. For finite impact parameter we must add the centrifugal potential, making a total effective radial potential of

$$V_{\text{total}} = V_N(r) + V_C(r) + \frac{l^2}{2mr^2} \quad (3)$$

Classically, the impact parameter b is related to the conserved angular momentum by $\hbar l = m v b$, with v the asymptotic relative velocity.

If you integrate the classical equations of motion for this problem, you find that the scattering angle Θ is given by

$$\Theta(E, l) = \pi - 2 \int_{r_{\min}}^{\infty} \frac{l \, dr}{m r^2 \sqrt{\frac{2}{m} \left(E - V_N(r) - V_C(r) - \frac{l^2}{2mr^2} \right)}} \quad (4)$$

where $r_{\min}$ is the distance of closest approach for the given energy E and impact parameter b . If the nuclear potential is absent, the integral yields the familiar Coulomb expression

$$\Theta(E, l) = 2 \arctan \left(\frac{z_1 z_2 e^2}{v l} \right) = 2 \arctan \frac{\eta}{l}, \quad (5)$$

which is sketched in Figure 4.

The scattering angle Θ as a function of l is referred to as the "deflection function". For the pure Coulomb case it is a monotonically decreasing function of l , but the situation is more interest-

ing when there is also an attractive potential strong enough to form a pocket . Then we get a situation such as that depicted in Figures 5 a, b, where there is seen to be a maximum in the deflection function, where the orbit comes close enough to feel a strong Coulomb repulsion, but not so close as to be pulled into the attractive well.

This maximum angle is called the rainbow angle, and the classical differential cross section

$$\frac{d\sigma}{d\Omega} = \frac{l}{m^2 v^2 \sin\Theta \left| \frac{d\Theta}{dl} \right|} \quad (6)$$

is singular there. The name rainbow comes from Descartes' explanation of the rainbow in terms of geometrical optics.

Fig. 6

Ray B corresponds to maximum deflection.

All incident rays in the neighborhood of B emerge near B, $\frac{d\sigma}{d\Omega}$ is singular at ray B.

Note that rays α and β emerge at the same angle in figure 5b, although they correspond to trajectories on different sides

of the nucleus. Ford and Wheeler pointed out that quantum mechanics requires us to associate a scattering amplitude with each path, and to add all the amplitudes that lead to a given scattering angle Θ . Thus we have the possibility of quantum mechanical interference phenomena even in situations

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where the deBroglie wavelength is small enough to allow us to talk in terms of classical orbits. When we discuss the angular distributions seen in particle transfer reactions we will see explicit examples of oscillations associated with interference between rays passing on opposite sides of the nucleus.

It is clear from this picture that large angular momenta (impact parameters) correspond to distant collisions, where we expect only elastic scattering, or perhaps Coulomb excitation. As l decreases we approach a regime in which the nuclei make a peripheral or grazing collision. Here we can expect some direct processes to occur, such as nuclear inelastic scattering, or the transfer of a few nucleons. For small impact parameter, and enough energy to overcome the Coulomb barrier, we get a sufficiently close collision to yield fusion of the projectile and target. This fusion can be followed by the evaporation of a few nucleons, with a large particle-stable "evaporation residue" remaining, or the fusion can be followed by fission into two roughly equal mass fragments. Notice that for sufficiently large values of $Z_1 Z_2$, where we get no pocket in the total potential, we can expect the transition between peripheral reactions and fusion to have quite different characteristics.

This transition region between peripheral and fusion reactions has yielded some of the most interesting new phenomena in heavy-ion physics. It turns out that there is a class of orbits such that the nuclei stay distant enough to maintain their own identities (except perhaps for the interchange of a few nucleons), yet they come close enough together, and stay together long enough, that their interaction converts a large fraction of their initial relative kinetic energy into internal excitation. These are called strongly damped collisions, or deep inelastic

collisions. Of course, the maximum damping occurs when the projectile and target fuse, because then the final relative kinetic energy is zero. If the target and projectile don't stick together, they will be driven apart by their Coulomb repulsion, and so the minimum final relative kinetic energy will be of the order of the Coulomb energy of touching spheres. This energy might be only a small fraction of the initial relative kinetic energy, the difference being converted into internal excitation energy of the fragments. We will see that in some circumstances, a large fraction of the reaction cross-section is associated with such events.

Reference to Fig. 5b suggests a correlation between the scattering angle and the extent of inelasticity. Since nuclei following orbit γ spend more time close together than nuclei following orbit α , we might expect more inelasticity associates with orbit γ . It also has a smaller scattering angle. This correlation between scattering angle and inelasticity was first observed by Wilczynski³, and was one of the clues that led to the interpretation of strongly damped collisions as being in the border between peripheral and fusion reactions.

Another way to relate the kind of physics occurring to the geometry and dynamics of the process, is to plot the partial reaction cross-section as a function of angular momentum. The expression for the differential cross-section for a projectile with intrinsic spin j_α and a target with intrinsic spin j_β , leading to a 2-particle final state with intrinsic spins j_γ, j_δ , can be written

$$\frac{d\sigma}{d\Omega}(j_\alpha j_\beta \rightarrow j_\gamma j_\delta, \theta) = \sum_{\substack{\lambda_\alpha \lambda_\beta \\ \lambda_\gamma \lambda_\delta}} \frac{1}{(2j_\alpha+1)(2j_\beta+1)} \frac{\lambda_{\alpha\beta}^2}{4} \left| \sum_J (2J+1) d_{\lambda_\alpha \lambda_\beta, \lambda_\gamma \lambda_\delta}^J(\theta) \left[S_{j_\alpha j_\beta \lambda_\alpha \lambda_\beta, j_\gamma j_\delta \lambda_\gamma \lambda_\delta}^J - \delta_{\alpha\gamma} \delta_{\beta\delta} \right] \right|^2 \quad (7)$$

Here $\lambda_\alpha, \lambda_\beta, \lambda_\gamma, \lambda_\delta$ are the helicities of the nuclei, and the $d_{\lambda_\alpha \lambda_\beta, \lambda_\gamma \lambda_\delta}^J(\theta)$ are

the usual R_3 representation matrix elements. Because of the spherical symmetry of the problem, flux is conserved for each partial wave angular momentum, which implies that each S^J forms a unitary matrix

$$S^J S^{J\dagger} = 1 \quad (8)$$

If we integrate over all final directions, we get the incoherent J-sum

$$\sigma(j_\alpha j_\beta \rightarrow j_\gamma j_\delta) = \pi \chi_{\alpha\beta}^2 \sum_{\lambda_\alpha \lambda_\beta \lambda_\gamma \lambda_\delta} \frac{(2J+1)}{(2j_\alpha+1)(2j_\beta+1)} \left| S_{j_\alpha j_\beta \lambda_\alpha \lambda_\beta, j_\gamma j_\delta \lambda_\gamma \lambda_\delta}^J - \delta_{\alpha\gamma} \delta_{\beta\delta} \right|^2 \quad (9)$$

To get the total reaction cross-section, we sum this over all the inelastic final channels

$$\begin{aligned} \sigma_r &= \pi \chi_{\alpha\beta}^2 \sum_J \frac{2J+1}{(2j_\alpha+1)(2j_\beta+1)} \sum_{\substack{j_\gamma j_\delta \\ \lambda_\alpha \lambda_\beta \lambda_\gamma \lambda_\delta}} \left| S_{j_\alpha j_\beta \lambda_\alpha \lambda_\beta, j_\gamma j_\delta \lambda_\gamma \lambda_\delta}^J \right|^2 = \sum_J \pi \chi_{\alpha\beta}^2 (2J+1) (\eta_{\alpha\beta}^J)^2 \\ &\equiv \sum_J \sigma_r^J \end{aligned}$$

where

$$(\eta_{\alpha\beta}^J)^2 \equiv 1 - \frac{1}{(2j_\alpha+1)(2j_\beta+1)} \sum_{\substack{\lambda_\alpha \lambda_\beta \\ \lambda_\gamma \lambda_\delta}} \left| S_{j_\alpha j_\beta \lambda_\alpha \lambda_\beta, j_\alpha j_\beta \lambda_\gamma \lambda_\delta}^J \right|^2 \quad (11)$$

We have used here the unitarity of S^J . All the S^J matrix elements involved in $(\eta_{\alpha\beta}^J)^2$ are associated with elastic scattering, but perhaps with change of helicity.

For very large J , corresponding to large impact parameters, we only have elastic scattering. Then unitarity implies that $(\eta_{\alpha\beta}^J)^2$ defined in (11) equals zero. For small J , corresponding to small impact parameters, elastic scattering is an unlikely process and all the S^J matrix elements in (11) are nearly zero. Thus the $(\eta_{\alpha\beta}^J)^2$ factor in (11) goes from zero at large J to unity at small J , the transition occurring for J -values corresponding to impact parameters leading to peripheral collisions.

Fig. 7a

Fig. 7b

The amount of the reaction cross-section associated with a particular process is proportional to the corresponding area under the σ_r^J curve. Note that we have divided the fusion region into two parts. The fission barrier for the compound nucleus is lower if it has high angular momentum. Thus those compound nuclei that are formed in high-J events will be more likely to decay by fission than compound nuclei formed in low-J events.

Of course the dividing lines drawn on Fig. 7b must not be taken too seriously. There will generally be smooth transitions from one type of process to another. These transition regions contain much of the interesting physics. By studying the transition between fusion-evaporation and fusion-fission we can hope to learn about the cohesiveness of nuclear matter. The transitions at the two sides of the DIC region can teach us about energy dissipation and thermalization processes in nuclei.

As we pass from peripheral to deep-inelastic processes, the relative importance of single and multistep processes will change.

Fig. 7b illustrates the wide range of phenomena that are involved in the study of collisions between nuclei, even at the modest energies available from a Van der Graaf accelerator. From now our discussion will be organized according to the different regions of Fig. 7b .

Elastic Scattering.

Your new accelerator will give you beams of particles whose energy can be varied continuously up to a maximum of several hundred MeV, with an energy resolution of a few tens of keV. This is precisely what is needed for the study of elastic scattering. Clearly, unless you can resolve the ground state from the first excited state, you cannot be sure that the process you are studying is elastic scattering. At this time there are no cases where the data clearly resolves the elastic peak over an energy range as wide as you will have available, so this is a field where your laboratory can make a unique, and very positive, contribution.

Figure 8 illustrates the gross features expected for the angular distribution for elastic scattering from a strongly absorbing potential.

Figure 8. Schematic diagram illustrating the angular distribution of elastic scattering from a strongly absorbing potential.

We are generally not interested in the $\left[\text{cosec } \frac{\theta}{2} \right]^4$ angular dependence associated with the Coulomb repulsion, and thus the usual practice is to display the data as a ratio of the observed cross-section to the Rutherford scattering cross-section for scattering of point charges. Fig. 8 illustrates the situation in which the absorptive part of the internucleon potential has

approximately the same range as the real part. Then any orbits that correspond to nuclei coming close enough to feel the real nuclear potential will have a high probability of being absorbed out of the elastic channel. Thus

Figure 9. Elastic scattering of the $^{28}\text{Si} + ^{16}\text{O}$ system. The data are from Ref. 4, and the solid curve is an optical model calculation from Ref. 8.

we have pure Rutherford scattering right down to impact parameters where the elastic scattering is destroyed by absorption. Earlier studies of elastic scattering showed angular distributions rather similar to that shown in Figure 8, but they were confined largely to forward angles.

Figure 9 shows an angular distribution for the system $^{16}\text{O} + ^{28}\text{Si}$, where data have been obtained over the full angular range for θ_{cm} from 0° to 180° . The back-angle data have been obtained by study of the kinematically reversed reaction, namely ^{28}Si projectiles hitting an ^{16}O target. As illustrated in Figure 10, this results in higher final ^{16}O lab energy for events corresponding to 180° scattering in the center of mass system.

$$\begin{array}{ll}
 \text{^{16}O projectile} & \frac{12}{44} v_1 \leftarrow \text{^{16}O} \quad \text{^{28}Si} \longrightarrow \frac{32}{44} v_1 \quad v_1 = \sqrt{\frac{2 E_{\text{initial}} (\text{^{16}O})}{16}} \\
 \text{^{28}Si projectile} & \frac{12}{44} v_1 \longrightarrow \text{^{28}Si} \quad \text{^{16}O} \longrightarrow \frac{56}{44} v_1 \quad v_1 = \sqrt{\frac{2 E_{\text{initial}} (\text{^{28}Si})}{28}}
 \end{array}$$

Figure 10: Final kinematics of the $^{16}\text{O} + ^{28}\text{Si}$ reaction, when the scattering angle in the center of mass system is 180° . Note the ^{16}O comes out faster when ^{28}Si is the projectile than when ^{16}O is the projectile.

The oscillations of σ/σ_R about 1 shown in Fig. 9 for $\theta_{\text{cm}} < 50^\circ$ are reminiscent of the Fresnel diffraction pattern for scattering from a sharp edge. Frahn has shown that the long-range Coulomb potential ensures that the diffraction seen in heavy-ion reactions will be more like Fresnel diffraction than like Fraunhofer diffraction. The basic idea is illustra-

ted in Figure 11.

Figure 11: Grazing trajectories that have been bent by the Coulomb repulsion appear to have originated at a point A, quite near the nucleus.

The nucleons that follow a grazing trajectory appear to be emanating from point A quite close to the nucleus. Thus the closest optical analogy would be diffraction of light from a nearly source (Fresnel diffraction) rather than from an infinitely distant source (Fraunhofer diffraction).

There are three surprising features of the back-angle data:

- 1) for $\theta_{cm} > 120^\circ$ the ratio σ/σ_R shows strong regular oscillations
- 2) the envelope of these oscillations rises several orders of magnitude as θ_{cm} goes to 180° .
- 3) As the bombarding energy is changed, the 180° cross-section exhibits regular resonance-like changes (Fig. 12) with a gross structure of about 3 MeV. Fig. 12 also shows the 180° elastic scattering excitation function for the scattering of ^{12}C on ^{32}S . This also exhibits a 2-3 MeV gross-structure, but here there is also a fine structure of the order of several hundred keV.

If we apply equation (7) to the case of elastic scattering of

Figure 12. Some elastic scattering excitation functions.⁴

spinless particles, we get

$$\frac{d\sigma}{d\Omega}(\text{elastic}) = \frac{\pi^2}{4} \left| \sum_l (2l+1) (S_l^l - 1) P_l(\cos\theta) \right|^2. \quad (12)$$

Suppose the S^l are those that arise from a strongly absorbing potential, the sort that gives rise to the kind of angular distribution illustrated in Fig. 8. These S^l must be such that at small θ , where $P_l(\cos\theta) \approx 1$, we get constructive interference, but at $\theta \approx 180^\circ$, where $P_l(\cos\theta) \approx (-1)^l$ we get destructive interference. To increase the cross-section near $\theta = 180^\circ$, we must modify these strongly absorbing S^l in such a way that we destroy this back-angle destructive interference. One way to do this is to use "brute force" to modify some of the S^l produced by the strong-absorbing optical potential. A popular method is to use a Regge pole parameterization. For example, Braun-Munzinger et al. found they could get a good fit to their angular distribution at $E_{\text{cm}} = 35$ MeV by replacing their optical model nuclear S^l matrix elements by

$$S^l = S_{\text{optical model}}^l \left[1 + i \frac{.6}{l - 24.5 - 3i} \right]. \quad (13)$$

This changes the contribution to (12) of partial waves in the vicinity of $l = 24.5$. No specific physical mechanism is supplied to justify the extra term in (13), so this must be regarded as a parameterization of the data rather than an explanation.

A more natural way to account for the difference between Figs. 8 and 9 is to drop the assumption that the real and imaginary parts of the nuclear potential have the same radius. In particular, suppose we assume that the surface regions of the nuclei are fairly transparent. An example of such a "surface transparent" potential is shown in Figure 13. It is

Figure 13. The radial shape of the $^{28}\text{Si} + ^{16}\text{O}$ optical potentials used by Deinhart et al.⁸ Note that the real part of potential extends out beyond the imaginary part (surface transparency).

seen that this potential is strongly absorbing for $r < 5$ fm, but for $r > 6$ fm there is a region where grazing partial waves can be diffracted by the nuclear potential without being strongly absorbed. This extra diffraction can lead to the oscillations observed beyond $\theta_{cm} = 120^\circ$. The fit to the angular distribution provided by this potential is shown in Figure 9. Another ingredient of the calculated curve of Fig.9 is a small odd-even l -dependence of the potential -i.e. the depth of the potential is slightly different for even and odd l -values. If this feature were not included, one of the peaks in the angular region beyond 120° would be missing. This odd-even potential is somewhat controversial. Dehnhard et al. point out that if some of the events that we interpret as elastic scattering were actually exchange of a ^{12}C from one ^{16}O core to another, this would introduce an odd-even effect into the effective potential. Most people are rather skeptical, however, that transfer of an object as large as a ^{12}C can play a significant role.

The energy variation of the excitation function is much discussed, but its origin is still unclear. There is little hope of finding an energy-independent optical potential which would account for these data. Most optical model practitioners would be satisfied with potentials that varied smoothly as a function of energy, and gave a good fit to the elastic scattering angular distribution at every energy. It is still not clear that this is possible, since high-quality elastic scattering data are not available over a large energy range. This is a field that your laboratory would be able to make an almost unique contribution to.

It's perhaps unreasonable to expect too much of the optical model. Notice that Figure 9 implies that even though the $180^\circ \sigma/\sigma_R$ is larger than it is at 120° , it is still only a few percent. The imaginary part of the optical potential is only a very crude representation of the

actual processes that remove flux from the incident channel, and so it is surprising that it does so well in situations in which absorption is dominant. According to Bromley, we shouldn't look so critically at angular distributions at a single energy, or at excitation functions at one angle. Rather we should look at cross-section surfaces in an energy-angle plane, (Fig. 14), and not expect more of the optical model than a gross representation of the general topography.

Figure 12 indicates that there is resonance-like structure in the large-angle excitation function of the transfer reaction connecting the $^{16}\text{O} + ^{28}\text{Si}$ and $^{12}\text{C} + ^{32}\text{S}$ channels. Notice that there is no correlation between the locations of the maxima and minima for the two elastic processes and the transfer process. This indicates that these resonances are not manifestations of long-lived states in the ^{44}Ti compound system but rather are associated with some resonant effects in the entrance or exit channels. There is little structure in the forward angle excitation function of the transfer reaction in this case; however the α -transfer in the $^{24}\text{Mg}(^{16}\text{O}, ^{12}\text{C})^{28}\text{Si}$ shows resonance-like structure in the forward-angle excitation function. In general these structures are not seen when the nuclei involved cannot be assembled from an integral number of alpha-particles. The presence of additional valence nucleons evidently opens so many other channels that the resonance-like structure is washed out. All these facts presently form the pieces of a confusing but tantalizing puzzle. Many workers, especially at Tandem laboratories where high resolution is available, are adding pieces to the puzzle, but so far no over-all pattern has emerged.

Figure 14.

MATHEMATICAL MODEL of the optical type, incorporating forces that act on two carbon-12 nuclei as they come together and then elastically scatter like colliding billiard balls, was used to generate a cross-section surface (*top*). The model parameters were adjusted until the calculated cross-section surface closely matched the measured cross-section surface (*bottom*).

Quasi-molecular states.

10

In 1960, Almqvist, Bromley and Kuehner studied reactions induced by bombarding ^{12}C with ^{12}C . They saw resonances in the yields of light particles (protons, neutrons, alphas, γ -rays) at energies at and below the Coulomb barrier ($E_{\text{cm}} = 5$ to 7 MeV). A few years later, it was shown that strong resonances at these energies also appeared in the elastic channel. Since then the studies have been extended to energies above the Coulomb barrier. The search for energy structure in the light-particle decays is complicated by the occurrence of statistical fluctuations in the yields. However, if the cross-sections are fluctuating statistically, you don't expect to find correlations between the yields of different channels. Such correlations were in fact found to exist, and revealed resonance-like structures with widths of several hundred keV.

Figure 15 shows cross-sections of 90° $^{12}\text{C} + ^{12}\text{C}$ elastic scattering, total reaction cross-section, inelastic scattering (the total $^{12}\text{C}(2^+ \rightarrow 0^+)$ yield) and the yield of the $3^-_1 0^+_1$ ^{16}O γ -ray, which is assumed to follow the formation of ^{20}Ne in an α -unbound state. When these cross-sections are plotted as function of E_{cm} , it is seen that there is a strong correlation between the occurrences of maxima and minima. Moreover, one can determine a gross-structure with a width of 2-3 MeV and an intermediate structure several hundred keV wide. Similarly correlated yields are also seen in the $^{12}\text{C}(2^+_1) + ^{12}\text{C}_{\text{g.s.}}$ and $^{12}\text{C}(2^+_1) + ^{12}\text{C}(2^+_1)$ channels.

A simple interpretation of these data is that the 2-3 MeV gross-structure corresponds to resonant states and that the few-hundred-keV intermediate structure corresponds to slightly more complicated (doorway) states of the same spin and parity as the gross-structure states. There is some evidence concerning the angular momenta of the intermediate structure

$^{12}\text{C} + ^{12}\text{C}$ INTERMEDIATE STRUCTURES

Figure 15. Comparison of some intermediate structure seen in the $^{12}\text{C} + ^{12}\text{C}$ system. 11

Figure 16. Experimentally observed resonant energy centroids vs $J(J+1)$ for reasonable assumptions about the assignments of the centroids.¹² The calculated energies are based on real nucleus-nucleus potentials.

states, from which we can infer the angular momenta of the enveloping gross-structure. This leads to the suggestion that the gross-structure states form a $K=0$ rotational band, which extends up to $J^\pi = 18^+$ at $E_{cm} \approx 36$ MeV. In fact a plot of the energy centroids of these states against the proposed $J(J+1)$ values leads to a very nearly straight line (Fig. 16).

There have been several attempts to calculate the locations and widths of the resonances of two ^{12}C nuclei moving relative to each other. Figure 16 shows the energies of resonances calculated with a real, rather shallow potential. An essential ingredient of these resonant states is reflection from the inner wall of the "pocket" of the effective internucleon potential. However, if the $^{12}\text{C} - ^{12}\text{C}$ potential is strongly absorbing inside this pocket, such reflection would be very strongly attenuated. Friedman and Goebel have shown that in circumstances such as this, with a barrier outside a region of strong absorption, a new type of single-particle resonance occurs, which can be described in terms of waves running around the top of the barrier. These barrier-top resonances are another possible explanation for the gross-structure illustrated in Figure 15.

It has also been argued that the gross-structure seen in the inelastic channels does not necessarily imply a compound nucleus mechanism, but that it can also arise if these are considered to be direct transitions from the initial to final channels. According to the Austern-Blair diffraction model, for each partial wave in the entrance channel there is

an energy interval (window) over which this partial wave is likely to make an appreciable contribution to a DWBA matrix element. Similarly for the exit channel. DWBA matrix elements can be expected to be large when the bombarding energies are such that the entrance and exit channel windows overlap, due attention being paid to the Q-value and angular momentum transfers. Phillips et al. ¹⁴ have shown that they can account for the energy variation of inelastic cross-sections using this model, which makes no reference to any intermediate quasi-molecular ^{24}Mg state. This explanation says the DWBA matrix elements will be large when the initial and final channels match in a certain way, which depends upon bombarding energy with a gross-structure variation of several MeV. There is no apparent explanation in this picture for the few-hundred keV intermediate structure referred to above. Thus if this intermediate structure does have a doorway-state like relationship to the gross-structure peaks, this would be an argument in favour of the quasi-molecular interpretation.

PERIPHERAL PARTICLE-TRANSFER REACTIONS.

Light ion particle transfer reactions have yielded an enormous amount of detailed nuclear structure information. For a wide range of energies and targets it has been found possible to analyze them in terms of direct reaction theory, using the distorted waves Born approximation (DWBA). The most common example is the (d,p) reaction, where one can write

$$\frac{d\sigma}{d\Omega}(\theta) = \sum_{\ell j} S_{\ell j} \sigma_{DWBA}^{\ell j}(\theta). \quad (14)$$

The spectroscopic factor $S_{\ell j}$ operator is the square of the matrix element of a neutron creation operator between the initial and final states

$$S_{\ell j} = \left| \langle \psi_m^{I_f} | [a_{\ell j}^{\dagger} \psi_m^{I_i}]^{I_f} \rangle \right|^2 \quad (15)$$

and contains the nuclear structure information. The shape of the angular distribution is given by the factors $\sigma_{DWBA}^{\ell j}(\theta)$, which depend mostly on the optical potentials that generate the optical wave functions used to describe the relative motion of the fragments in the initial and final channels. The simplest cases are those where the target is the ground state of an even-even nucleus ($I_i=0$), because then only one value of j ($j=I_f$) yields a non-zero value for $S_{\ell j}$, and then parity conservation picks out a unique value from the pair $\ell = j \pm 1/2$. Thus only one term contributes to the sum in (14). It is found that the angular dependence of $\sigma_{DWBA}^{\ell j}(\theta)$ is determined mostly by ℓ . Thus the experimental determination of the shape of the angular distribution gives information about the spins and parities of final states, and if we

have at our disposal a DWBA code to calculate $\sigma_{DWBA}^{\ell j}(\theta)$, we can use the absolute magnitude of the measured cross-section to infer the spectroscopic factor.

The quantities ℓ and j in (14) are referred to as the orbital and total angular momentum transfers. They describe the transfer of angular momentum between the relative motion of the fragments and their internal motion, just as the Q value specifies the transfer of energy between the relative and internal motions of the fragments.

The situation is similar for heavy-ion transfer reactions, such as the ($^{16}\text{O}, ^{15}\text{O}$) reaction, except that the angular dependence of $\sigma_{DWBA}^{\ell j}(\theta)$ is not so strongly determined by ℓ . To understand why this should be so, consider the amount of transverse momentum exchanged as a result of the collision. Some of this momentum is carried across by the transferred nucleon, and some of it is provided by the impulse due to the Coulomb field. We can make a simple estimate of the momentum transfer due to the Coulomb field by calculating the sideways Coulomb impulse as the target and projectile move past each other on a straight-line trajectory (Fig.

17). We find

$$\Delta p_{\perp} = \int_{t=-\infty}^{\infty} F_{\perp} dt \approx \int_{t=-\infty}^{\infty} \frac{z_1 z_2 e^2}{x^2 + b^2} \cdot \frac{b}{\sqrt{x^2 + b^2}} dt = \frac{z_1 z_2 e^2}{v} b \int_{x=-\infty}^{\infty} \frac{dx}{(x^2 + b^2)^{3/2}}$$

$$= 2 \frac{z_1 z_2 e^2}{vb} \quad (16)$$

Figure 17: The target and projectile moving past each other on straight-line trajectories

Now let us consider the amount of momentum carried across by the transferred nucleon, neglecting the effect of the Coulomb field. Let $\vec{p}_i$ be the momentum of the incoming projectile, let $\vec{p}_o$ be its momentum after the nucleon has been stripped from it, and $\vec{p}_t$ be the momentum of that stripped nucleon. Then momentum conservation implies (see Fig. 18)

$$\vec{p}_t = \vec{p}_i - \vec{p}_o, \quad p_t = \sqrt{p_i^2 + p_o^2 - 2 p_i p_o \cos \theta}$$

If p_i and p_o are not too different,

$$p_i \approx p_o \approx p, \quad p_t \approx p \sqrt{2 - 2 \cos \theta} \approx p \theta \text{ for } \theta < 1$$

Figure 18: Vector diagram showing the relation between the momenta of the incoming and outgoing projectiles, and the momenta of the transferred particle.

When these nucleons of linear momentum $\vec{p}_t$ strike a target nucleus of radius R , the amount of angular momentum they deliver has a maximum of about $p_t R$. Thus the angular momentum transferred to the target satisfies the inequality

$$\hbar l \leq p_t R, \quad \frac{\hbar l}{p R} \leq \theta \quad (17)$$

Comparison with (16) shows that

$$\begin{aligned}
 & \frac{\text{momentum transfer due to Coulomb repulsion}}{\text{momentum transfer due to particle transfer}} \approx \frac{2 \frac{z_1 z_2 e^2}{v b}}{\frac{\hbar \ell}{R}} \\
 & = 2 \frac{R}{b} \frac{\frac{z_1 z_2 e^2}{\hbar v}}{\ell} \approx \frac{\eta}{\ell}
 \end{aligned} \tag{18}$$

For 10 MeV deuterons bombarding ^{68}Z , η is about 2.1 and thus the momentum transferred by the neutron is comparable to that transferred by the Coulomb field. Equation (17) shows that the particle-transferred momentum is connected to the transferred angular momentum, and thus the angular distribution of the outgoing fragment should be related to the angular momentum transfer. However for 56 MeV ^{16}O bombarding ^{48}Ca , η is about 13.5. Here most of the transferred momentum is provided by the Coulomb impulse, which does not depend upon the angular momentum transfer. These features are illustrated by Figures 19 and 20. The (d,p) angular distributions are grouped together according to angular momentum transfer, and the family resemblance is very striking. It is also clear that the first maximum moves toward increasing angles as ℓ increases, in accord with (17). On the other hand $p_{3/2}$ and $f_{1/2}$ transfers in $^{48}\text{Ca}(^{16}\text{O}, ^{15}\text{N})^{49}\text{Sc}$ lead to quite similar angular distributions, although the angular momentum transfer differs by two units.

Table I shows an interesting summary of angle-integrated cross-sections for different processes observed when 56 MeV ^{16}O ions bombard a ^{48}Ca target. This table clearly indicates that the cross-sections decrease rapidly as more nucleons are transferred. Note also that two of the single-nucleon transfers have sizeable cross-sections, whereas two of them are very weak. This distinction can be understood in terms of the poor kinematic matching conditions in the latter case. To see this in more

Figure 19. Some Zn(d,p) angular momentum distributions classified according to angular momentum transfer.

Fig. 20. The solid line for the $f_{7/2}$ state and the dashed line for the $p_{3/2}$ state correspond to DWBA fits using elastic scattering optical-model parameters. The solid line for the $p_{3/2}$ state is the fit obtained after modifying the parameters in the outgoing channel.

TABLE I. Cross sections observed for representative low-lying levels populated in the reaction channels measured.

Nucleons transferred	Reaction	ΔN	State populated	Q value (MeV)	σ (mb)
0	$(^{16}\text{O}, ^{16}\text{O}')$	0	$^{42}\text{Ca}(2^+)$	-3.83	6.04
			$^{42}\text{Ca}(3^-)$	-4.51	5.67
1	$\left\{ \begin{array}{l} (^{16}\text{O}, ^{15}\text{N}) \\ (^{16}\text{O}, ^{17}\text{O}) \\ (^{16}\text{O}, ^{15}\text{O}) \\ (^{16}\text{O}, ^{17}\text{F}) \end{array} \right.$	$\left\{ \begin{array}{l} -1p \\ +1n \\ -1n \\ +1p \end{array} \right.$	$\left\{ \begin{array}{l} ^{49}\text{Sc}(\frac{7}{2}^-)_{\text{g.s.}} \\ ^{47}\text{Ca}(\frac{7}{2}^-)_{\text{g.s.}} \\ ^{48}\text{Ca}(\frac{3}{2}^-)_{\text{g.s.}} \\ ^{47}\text{K}(\frac{3}{2}^+)_{\text{g.s.}} \end{array} \right.$	$\left\{ \begin{array}{l} -2.50 \\ -5.80 \\ -10.52 \\ -14.66 \end{array} \right.$	$\left\{ \begin{array}{l} 10.29 \\ 10.88 \\ \leq 0.03 \\ \leq 0.03 \end{array} \right.$
2	$\left\{ \begin{array}{l} (^{16}\text{O}, ^{14}\text{C}) \\ (^{16}\text{O}, ^{14}\text{N}) \\ (^{16}\text{O}, ^{18}\text{O}) \\ (^{16}\text{O}, ^{16}\text{N}) \end{array} \right.$	$\left\{ \begin{array}{l} -2p \\ -1p-1n \\ +2n \\ +1p-1n \end{array} \right.$	$\left\{ \begin{array}{l} ^{50}\text{Ti}(0^+)_{\text{g.s.}} \\ ^{50}\text{Ti}(2^+) \\ ^{50}\text{Sc}(5^+)_{\text{g.s.}} \\ ^{46}\text{Ca}(0^+)_{\text{g.s.}} \\ ^{48}\text{Sc}(6^+, 5^+, 4^+)_{\text{g.s.}}^a \end{array} \right.$	$\left\{ \begin{array}{l} -0.54 \\ -2.09 \\ -6.85 \\ -5.03 \\ -10.13 \end{array} \right.$	$\left\{ \begin{array}{l} 0.370 \\ 0.480 \\ 0.147 \\ 0.190 \\ 0.168 \end{array} \right.$
3	$\left\{ \begin{array}{l} (^{16}\text{O}, ^{13}\text{B}) \\ (^{16}\text{O}, ^{13}\text{C}) \\ (^{16}\text{O}, ^{19}\text{O}) \\ (^{16}\text{O}, ^{15}\text{C}) \end{array} \right.$	$\left\{ \begin{array}{l} -3p \\ -2p-1n \\ +3n \\ -2p+1n \end{array} \right.$	$\left\{ \begin{array}{l} ^{51}\text{V}(\frac{7}{2}^-)_{\text{g.s.}} \\ ^{51}\text{Ti}(\frac{3}{2}^-)_{\text{g.s.}} \\ ^{45}\text{Ca}(\frac{7}{2}^-)_{\text{g.s.}} \\ ^{49}\text{Ti}(\frac{7}{2}^-)_{\text{g.s.}} \\ ^{49}\text{Ti}(\frac{5}{2}^-, \frac{7}{2}^-) \end{array} \right.$	$\left\{ \begin{array}{l} -13.32 \\ -2.34 \\ -11.48 \\ 10.27 \\ -12.53 \end{array} \right.$	$\left\{ \begin{array}{l} \leq 0.007 \\ 0.042 \\ \leq 0.003 \\ 0.003 \\ 0.019 \end{array} \right.$
4	$\left\{ \begin{array}{l} (^{16}\text{O}, ^{12}\text{C}) \\ (^{16}\text{O}, ^{12}\text{B}) \end{array} \right.$	$\left\{ \begin{array}{l} -2p-2n \\ -3p-1n \end{array} \right.$	$\left\{ \begin{array}{l} ^{52}\text{Ti}(0^+)_{\text{g.s.}} \\ ^{52}\text{Ti}(2^+) \\ ^{53}\text{V}(1^+, 5^+)_{\text{g.s.}}^a \end{array} \right.$	$\left\{ \begin{array}{l} +0.51 \\ -0.54 \\ -10.89 \end{array} \right.$	$\left\{ \begin{array}{l} \leq 0.002 \\ \leq 0.012 \\ < 0.002 \end{array} \right.$
5	$\{ (^{16}\text{O}, ^{11}\text{B})$	$-3p-2n$	$^{53}\text{V}(\frac{3}{2}^-, \frac{5}{2}^-, \frac{7}{2}^-)_{\text{g.s.}}^a$	-5.44	< 0.004

^a Unresolved ground-state groups.

detail, consider a Coulomb trajectory. The distance of closest approach is related to the reduced mass m , the center-of-mass energy E , and the relative angular momentum ℓ by

$$r_{\min} = \frac{z_1 z_2 e^2}{2E} + \sqrt{\left(\frac{z_1 z_2 e^2}{2E}\right)^2 + \frac{\ell^2}{2mE}}$$

Our picture of particle transfer as a peripheral process suggests that appreciable cross-sections will occur only when $r_{\min}$ for the entrance and exit channels are comparable to the corresponding sums of fragment radii, that is when

$$\begin{aligned} \ell_{\text{graz}}^{(\text{incident})} &= \sqrt{2m \left[E(R_1 + R_2)^2 - (R_1 + R_2) z_1 z_2 e^2 \right]} \\ \ell_{\text{graz}}^{(\text{final})} &= \sqrt{2m' \left[E'(R'_1 + R'_2)^2 - (R'_1 + R'_2) z'_1 z'_2 e^2 \right]} \end{aligned} \quad (19)$$

where the unprimed quantities refer to the entrance channel and the primed quantities to the exit channel. Table II gives these numbers for the entrance channel, and for the exit channels corresponding to the four 1-particle transfer cases listed in Table I.

Channel	$\ell_{\text{gr/h}}$	$\Delta \ell_{\text{gr/h}}$	$\ell_{\text{transfer}}/\hbar$
$^{16}\text{O}(0^+) + ^{48}\text{Ca}(0^+)$	20.2	0	0
$^{15}\text{N}(\frac{1}{2}^-) + ^{49}\text{Sc}(\frac{7}{2}^-)$	19.5	.7	3,4
$^{17}\text{O}(\frac{5}{2}^+) + ^{47}\text{Ca}(\frac{7}{2}^-)$	15.1	5.1	1,2,3,4,5,6
$^{15}\text{O}(\frac{1}{2}^-) + ^{49}\text{Ca}(\frac{3}{2}^-)$	6.7	13.5	2,3
$^{17}\text{F}(\frac{5}{2}^+) + ^{47}\text{K}(\quad)$	—	>20.2	1,2,3,4

Table II: Grazing angular momenta for the elastic and several particle-transfer channels when 56 MeV ^{16}O ions bombard a ^{40}Ca target. ¹⁷
The last column gives the angular momentum transfers consistent with the spins of the final fragments.

The absence of an $\ell_{\text{gr}}/\hbar$ entry in the $^{17}\text{F}+^{47}\text{K}$ case implies that there is no ℓ -value which will allow a grazing collision in this case. It is seen from Table II that in the transitions to ^{15}N and ^{49}Sc , and to ^{17}O and ^{47}Ca , the change in grazing angular momentum is consistent with the angular momentum that can be supplied to the internal motion. These cases lead to relatively large cross-sections. However in the transitions to $^{15}\text{O}+^{49}\text{Ca}$ and $^{17}\text{F}+^{47}\text{K}$, one cannot obtain the required large change in grazing orbital angular momentum from the change in internal angular momentum. Alternatively, the angular momentum and energy transfers required in these last two cases are not consistent with grazing trajectories in both entrance and exit channels. Such reactions are said to suffer from angular momentum mismatch, and Table I shows that their cross-sections are very low.

Next we discuss the shape of the angular distribution expected for particle transfer between heavy ions. We get the qualitative features right with considerable simplification of the formulae if we set all the helicities equal to zero. Then (7) applied to nucleon transfer yields

$$\frac{d\sigma}{d\Omega} = \frac{1}{4 k_{\alpha\beta}^2} \left| \sum_J (2J+1) S^J P_J(\cos\theta) \right|^2 \quad (20)$$

¹⁸
Following Strutinsky, let us sketch in Figure 21 a reasonable dependence of $|S^J|$ on J .

Figure 21: Schematic dependence of $|S^J|$ on J for peripheral particle transfer and strong absorption.

We expect $|S^J|$ to peak at J_0 , close to the grazing angular momentum and to fall off for large J because then the nuclei don't get close enough to exchange nucleons, and for small J because the associated small impact parameters would lead to more complicated non-peripheral processes. A convenient explicit form is

$$\sqrt{2J+1} S^J \approx e^{-\frac{(J-J_0)^2}{\Gamma^2} + i\Theta J} \quad (21)$$

The parameters J_0, Γ, Θ could be chosen[†] to yield a good fit to a DWBA calculation. Since the important J values will be quite large, we can use the asymptotic formula

$$P_J(\cos \theta) \xrightarrow{\theta > \frac{1}{J}} \frac{\sin([J + \frac{1}{2}]\theta + \frac{\pi}{4})}{\sqrt{\frac{\pi}{2}(J + \frac{1}{2}) \sin \theta}}$$

If these expressions are inserted into (20) and the J sum is approximated by an integral, we find

[†] For pure Coulomb scattering, Θ would be the scattering angle associated with a classical orbit with angular momentum J_0 .

$$\frac{d\sigma}{d\Omega} = \frac{p^2}{4k_{\alpha p}^2 \sin\theta} e^{-\frac{\Gamma^2}{2}(\theta-\Theta)^2} \left[1 + e^{-2\Gamma^2\theta\Theta} - 2e^{-\Gamma^2\theta\Theta} \cos\left([2J_0+1]\theta + \frac{\pi}{2}\right) \right] \quad (22)$$

The first term in (22) is approximately a Gaussian function of θ centered around the parameter Θ , with a width of about $2/\Gamma$. The second term is usually unimportant, and produces only a slight skewing of the peak. The third term introduces wiggles in the angular distribution with period $\Delta\theta \approx \pi/J_0$. The $e^{-\Gamma^2\theta\Theta}$ factor implies that these wiggles are stronger on the low θ side of the Gaussian than on the high θ side. Thus we expect an angular distribution something like that shown in Figure 22 (compare Fig. 20)

Figure 22: Sketch of $d\sigma/d\Omega$ as given by (22).

The origin of the oscillations at small θ is the interference between waves scattered from opposite sides of the nucleus, as in a 2-slit interference pattern. In fact the period of such oscillations should be

$$\Delta\theta = \frac{\lambda}{2(R_1+R_2)} = \frac{2\pi\lambda}{2(R_1+R_2)} = \frac{\pi}{k(R_1+R_2)} = \frac{\pi}{J_0}$$

which agrees with the period calculated by the Strutinsky method.

The differential cross-sections calculated with modern one-particle transfer programs generally do a good job of accounting for the shapes of the observed angular distributions, and one can even understand the absolute cross-sections using the spectroscopic factors that were extracted from analyses of light-ion reactions. One anomalous situation is the $\ell=1$ transfer that occurs in a few cases such as $^{40}\text{Ca}(^{13}\text{C}, ^{14}\text{N})^{39}\text{K}_{3/2^+}$. Figure 23 shows that the absolute magnitude of the cross-section is calculated cor-

Figure 23. Differential cross-sections for some transfer reactions induced by 68 MeV incident ^{13}C .

rectly, as is the period of the oscillations. However the calculated curve is clearly out of phase with the data. On the other hand, the calculations for $^{40}\text{Ca}(^{13}\text{C}, ^{12}\text{C})^{41}\text{Ca}_{3/2}^+$ fit the data in magnitude, period and phase. The origin of the problem of these few anomalous cases is as yet unclear.

Amongst the possible explanations are spin-orbit contributions to the optical potentials, ^{29/} and multistep effects such as inelastic excitation of the ^{40}Ca to its first 3^- state before transfer. ^{21/} Both these effects can shift the phase in the right direction, but when applied in reasonable amounts ^{22/} they don't shift it enough. The possibility of inelastic excitation before or after the channel is of course always present. It can be included by extending the DWBA to make it CCBA (coupled-channels Born approximation). However adding more than one or two channels leads to a prohibitively large calculation. There is some reason to hope that as the bombarding energy is increased and the transfer process takes place on a shorter time scale, the relative importance of inelastic excitation 2-step processes will decrease.

We have to consider another kind of multistep process when we study multi-particle transfer reactions. For example, when the reaction $^{206}\text{Pb}(^{18}\text{O}, ^{16}\text{O})^{208}\text{Pb}$ occurs, can it be treated in first-order Born approximation as a single-step process, or must we assume that the reaction proceeds through states of an intermediate $^{207}\text{Pb}+^{17}\text{O}$ system? Let us use the usual notation

and write the exact matrix element schematically as

$$M = \left\langle \chi_{\text{Eb}}^{(-)} \psi_f(A, n_1, n_2) \phi_f(b) \left| V(b, n_1) + V(b, n_2) \right| \psi^+(A, b, n_1, n_2) \right\rangle \quad (23)$$

$$= 2 \cdot \langle \chi^{(-)}(r_{bb}) \psi_f(A, n_1, n_2) \phi_f(b) | V(b, n_1) | \bar{\psi}^+(A, b, n_1, n_2) \rangle.$$

Here $\bar{\psi}^+$ is the exact wave function, corresponding to A+a as the entrance channel. The second equality follows because both ψ_f and $\bar{\psi}^+$ are antisymmetric with respect to the interchange of n_1 and n_2 . Since we don't know what $\bar{\psi}^+$ is, we use the Born approximation and replace it by the entrance channel wave function only

$$\bar{\psi}^+(A, b, n_1, n_2) \approx \chi^{(+)}(r_{Aa}) \psi_i(A) \phi_i(b, n_1, n_2) \quad (24)$$

Now we have to consider the question: how good is the approximation (24) in the region of configuration space that is important in the evaluation of M in (23)?

We have most trust that (24) is reasonably good in the region of configuration space that looks most like the entrance channel, that is where n_1 and n_2 are quite near b. The presence of $V(b, n)$ in (23) tells us that we only need $\bar{\psi}^+$ in the region where n_1 is near b. But what about n_2 ? There is nothing in (23) that tells up that the important region of configuration space is where n_2 is near b. In fact $\psi_f(A, n_1, n_2)$ in the bra in (23) is probably largest when n_2 is near A. There is no reason to believe that the entrance channel wave function on the right-hand side of (24) gives a good representation of the exact wave function when n_2 is near A. Therefore, if our object is to find an approximation to $\bar{\psi}^+$ that is adequate for the evaluation of M, (24) is probably not sufficient. We need to add to the right-hand side of (24) something that improves the description of $\bar{\psi}^+$ when n_2 is near A. If we use the Schrodinger equation to find this addition, we are calculating something like the one-neutron transfer of n_2 to an intermediate complex $(A+n_2)+(b+n_1)$, and then (23) is equivalent to the one-particle transfer of n_1 from this complex to the final state B+b.

This problem does not arise in the evaluation of the matrix element needed in one-particle transfer

$$\langle \chi_f^{(-)}(r_{bb}) \psi_f(A, n_1) \phi_f(b) | V(b, n_1) | \Psi^+(A, b, n_1) \rangle$$

Here we do not need to worry about n_2 , and the presence of $V(b, n_1)$ is sufficient to guarantee that we only need to know Ψ^+ in the region of configuration space where the elastic channel gives a good representation of Ψ^+ .

If the interaction between n_1 and n_2 , is assumed to be strong compared with the interaction between either of them and A, then the 2-step term is small compared with the one-step term. Conversely, if the n_1 - n_2 interaction is relatively weak, the two neutrons transfer independently and then the two-step term dominates. So far, all calculations have shown that in realistic situations the two terms are comparable. The above discussion suggests that the one-step approximation may also be inadequate for three and four particle transfer, although the tight binding of the 2-proton-2-neutron system might increase the validity of this approximation in α -transfer reactions.

One reason why the problem of relative importance of single-step and multistep processes is still unsolved after many years of experience with these reactions is that nobody has found a good way to use experimental data to distinguish them. Calculated angular distributions are quite similar for the two processes. It might be that they have different dependence on bombarding energy. Of course when you try to do a Born approximation calculation to study the variation of a cross-section

over a wide energy range, you must worry about the energy dependence of the optical wave functions. Therefore it would be especially valuable to have examples where the reaction angular distribution, as well as the elastic scattering for the entrance and exit channels, is measured over the kind of energy range that will be accessible to you with your new accelerator.

FUSION.

We now discuss the distribution of the reaction cross-section σ_r between fusion and deep-inelastic scattering. We will see that this distribution is based on a competition between nuclear, Coulomb, and centrifugal forces.

In (10) we gave an expression for the total reaction cross-section in terms of elastic scattering S-matrix elements. Suppose the Coulomb repulsion is weak enough so that for $J=0$ we have a "pocket" and an outer Coulomb barrier, of height E_B

Figure 24: Schematic representation of a situation in which the $J=0$ potential has a pocket. E_B is the height of the $J=0$ barrier, which occurs at a radius R_B .

To get a first estimate for σ_r , we assume that

$$\begin{aligned} \left[\eta_{\alpha\beta}^J \right]^2 &= 1 \quad \text{for } J \leq J_{cr} \\ &= 0 \quad \text{for } J > J_{cr}. \end{aligned} \tag{25}$$

J_{cr} is taken to be the largest J value for which the outer bump in the total effective potential is lower than E , the center-of-mass energy in

the entrance channel. In the case illustrated in Figure 24, $J_{cr} \approx 20$. The idea is that for $J \leq J_{cr}$, the two fragments are free to come close enough together for reactions to occur, whereas for $J > J_{cr}$ we get reflection at the outer edge of the barrier. The simplest estimate of J_{cr} is obtained by neglecting the fact that the position of the outer barrier varies with J , so we write

$$E = E_B + \frac{\hbar^2 J_{cr} (J_{cr} + 1)}{2m R_B^2}$$

Then the expression for the total reaction cross-section is

$$\sigma_r = \pi \chi_{\alpha\beta}^2 \sum_{J=0}^{\infty} (2J+1) (\eta_{\alpha\beta})^2 = \frac{\pi}{k_{\alpha\beta}^2} \sum_{J=0}^{J_{cr}} (2J+1) = \frac{\pi}{k_{\alpha\beta}^2} (J_{cr} + 1)^2$$

$$\sigma_r \approx \frac{\pi}{k_{\alpha\beta}^2} \frac{2m R_B^2}{\hbar^2} [E - E_B] = \pi R_B^2 \left[1 - \frac{E_B}{E} \right]. \quad (26)$$

The simple dependence of σ_r on E predicted by (26) can be tested by plotting ²³ the total reaction cross-section as a function of $\frac{1}{E}$. The slope and intercept of the straight line obtained in this way should yield the parameters R_B and E_B . However, since (26) neglects the dependence of R on J , it should probably be regarded as essentially qualitative, and should not be relied on to give precise numerical information.

An important empirical observation is that for low energies and for targets and projectiles that are not too heavy, almost all the reaction cross-section goes into fusion, that is into compound nucleus formation. As the energy is increased, or as more highly charged nuclei are used, the fusion cross-section falls below the reaction cross-section, the difference going into the deep-inelastic collision process. This can be understood by referring to Figure 25.

The sketch shows the effective potentials for three partial waves.

V_A , V_B , V_C are the heights of the potentials at the outer barriers, and V_a , V_b , V_c are their heights at a certain radius R_{crit} . Notice that the energy differences between V_a , V_b and V_c are greater than those between V_A , V_B and V_C . This is because the centrifugal potentials $\hbar^2/2m \ell(\ell+1)/r^2$ differ more at small r than at large r .

Following Glas and Mosel, we make the following two assumptions:

- a) We get a nuclear reaction whenever the kinetic energy is greater than the height of the outer bump.
- b) We get nuclear fusion whenever the trajectory of the relative motion can reach the critical distance R_{crit} .

At low energy, in the vicinity of E_A , the outer bump V_A is higher than the inner crossing V_a . Thus any trajectory that can pass over the outer bump can reach R_{crit} . At energy E_A all reactions lead to fusion, $\sigma_{fus} = \sigma_r$.

At higher energy, in the vicinity of E_C , the outer bump V_C is lower than the inner crossing V_c . Thus there is a range of energies between V_C and V_c where the trajectories with angular momentum ℓ_c contribute to reactions (since they get over the outer bump), but not to fusion (since they don't reach R_{crit}). At energy E_C some reactions do not lead to fusion, so that $\sigma_{fus} < \sigma_r$.

The intermediate potential labelled ℓ_B is such that $V_b = V_B$. For energies below E_B any ℓ -value that can contribute to reactions also contributes to fusion. For energies above E_B , there are reaction that do not lead to fusion.

Figure 25. The potentials in the vicinity of R_{crit} and the outer barrier, for three values of angular momentum.

To be more quantitative, we should try to do better than the sharp cut-off limit of (25). We really need elastic channel S-matrix elements. It is reasonable to try to mimic the elastic channel of the real nucleus by the elastic channel of the optical model. If the S-matrix elements calculated from the optical model give the correct angular distribution when substituted into (9), they will probably not do too badly in (26). Because of the strong absorption in the pockets of Figure 24, the optical wave functions there will be mostly incoming waves. Thus instead of solving the full optical model Schrodinger equation, with the boundary conditions that the wave function vanishes at the origin like r^l , we can solve a problem in which we have incoming and outgoing waves on the outside of the barrier, and purely incoming waves inside the barrier. The fraction of the flux that gets past the barrier is the transmission coefficient T_l . It was shown by Hill and Wheeler that if the barrier can be represented by an inverted parabola,

$$V(r) = -\frac{1}{2} m \omega^2 r^2$$

then the transmission coefficient is given exactly by

$$T = \frac{1}{1 + \exp\left(2\pi \frac{a-E}{\hbar\omega}\right)} \quad (28)$$

The usual application of the WKB method to this barrier penetration problem gives the same formula, without the 1 in the denominator.

Glas and Mosel thus approximate the fusion cross-section by

$$\sigma_{\text{fus}} = \pi \lambda_{\alpha\beta}^2 \sum_{l=0}^{l_{\text{crit}}} \frac{2l+1}{1 + \exp\left[2\pi \left(V_B + \frac{\hbar^2 l(l+1)}{2J_B} - E\right) / \hbar\omega\right]} \quad (29)$$

Here V_B is the height of the $l=0$ outer barrier, J_B is an effective moment of inertia close to mR^2 , and $\hbar\omega$ is chosen to fit the curvature of the top of the barrier. For $E=V_A$ in Figure 25, the upper limit l_{crit} is taken to be l_a (at which $T_l = 1/2$), whereas for $E=V_C$ in Figure 25 the upper limit is taken to be l_c . This model has been very successful in accounting for the energy dependence of the total fusion cross-section. Vaz and Alexander²⁶ have correlated a large amount of fusion data using (29), with the barrier heights V_B calculated from a modified proximity potential. Figure 26 shows some of their results.

Note that the above argument to explain why $\sigma_{\text{fus}} < \sigma_r$ at high energies is based upon the use of entrance channel effects to cut down the contributions to σ_{fus} from high angular momentum partial waves. There have also been suggestions that the restriction on high partial waves is due to a limitation on the amount of angular momentum that the compound nucleus can absorb.²⁷ However cases are where the compound nucleus is formed at energy and angular momentum above the yrast line, and yet $\sigma_{\text{fus}} < \sigma_r$. It is now generally believed that the high partial wave limitations come from entrance channel effects.

Figure 26. Comparison of experiment with calculation²⁶ of fusion cross-sections for some light systems.

Recent results of heavy-ion reaction theories suggest that at high energies it is also difficult for very low partial waves to lead to fusion. ^{28,29} The projectile and target bounce off or pass through each other when they make a head-on collision. Figure 27 shows a comparison of the experimental excitation function for $^{16}\text{O}+^{16}\text{O}$ σ_{fus} with the TDHF predictions, ²⁸ and with the prediction of (29) using the optical model to obtain the barrier penetrability. The reason that the TDHF result is so much lower than the optical model result at $E_{\text{lab}} = 120$ MeV is the above-mentioned exclusion of low-partial waves at this high energy. Thus in this case the experimental data seem to support the idea of a low-angular-momentum barrier.

Figure 28 shows an interesting feature of some fusion data measured by the Argonne group. ³⁰ Note the oscillatory structure in the $^{12}\text{C}+^{12}\text{C}$ fusion excitation function. A similar effect is seen in the $^{16}\text{O}+^{16}\text{O}$ (Fig. 27) and $^{16}\text{O}+^{12}\text{C}$ fusion excitation functions. Notice that the presence of an extra particle (in ^{13}C) or hole (in ^{15}N) damps the oscillations considerably. Optical models with weak absorption in the surface predict total reaction cross-sections which exhibit this oscillatory structure. Their absence when there are valence particles or holes might be due to the lower surface transparency caused by the extra channels that are available.

The dependence of fusion cross-sections on nuclear deformation is illustrated in Figure 29, taken from the work of Stokstad et al. ³¹ They bombarded $^{148,150,152,154}\text{Sm}$ targets with ^{16}O ions, and collected the Yb nuclei that resulted from their fusion. The different Yb isotopes were identified by their x-rays. At 75 MeV lab energy, σ_{fus} is about the same for ^{148}Sm and ^{154}Sm targets, but at 60 MeV they differ by a factor of 20. The lower part of Figure 29 plots the ratio of the ^{154}Sm and ^{148}Sm cross-

Fig. 27 Comparison between TDHF and experimental fusion cross sections for $^{16}\text{O} + ^{16}\text{O}$. The total reaction cross section given by the optical model is also shown.

Figure 28. Total fusion cross-section excitation functions³⁰ for the $^{12}\text{C} + ^{12}\text{C}$, $^{13}\text{C} + ^{12}\text{C}$, $^{14}\text{N} + ^{12}\text{C}$ and $^{15}\text{N} + ^{12}\text{C}$ systems. The solid and open symbols represent cross-sections established from full angular distributions and single-angle measurements, respectively.

Fig. 29. a) Values³¹ of σ_{fus} for $^{160} + ^{148}, ^{150}, ^{152}, ^{154}\text{Sm}$ from $E_{lab} = 60-75$ MeV. The full curve connecting the ^{148}Sm data points is the result of a static calculation. The other curves are there to guide the eye. The arrow denotes the location in the lab system of the fusion barrier for $^{160} + ^{148}\text{Sm}$.
 b) The ratio of σ_{fus} for ^{154}Sm and ^{148}Sm . The full curves are the results of static calculations.

sections. Also presented there is the result of a prediction, based upon (29), but using transmission coefficients appropriate to a static non-spherical potential. The $\beta_\lambda=0$ curve results from only the difference in radii of ^{148}Sm and ^{154}Sm , and the difference between the bombarding energies used. The effect of non-zero deformation for ^{154}Sm is shown by the other solid curve. The deformation parameters used were taken from other experimental data. It is seen that this calculation correctly predicts the shape of the fusion ratios as a function of energy, but it overestimates the effect of the deformation. This may be because the calculation is a static one, that is the orientation of the ^{154}Sm does not change as the ^{16}O comes to fuse with it.

It is clear from the foregoing discussion that high quality fusion data over a wide energy range and for a variety of target projectile systems is needed before we can be confident that we have a clear picture of the physics of the fusion process.

DEEP INELASTIC COLLISIONS.

The name "deep inelastic collision" is perhaps a bit misleading, since it seems to imply a deep penetration of one collision partner by the other. Actually, it refers to a peripheral process with contact between the nuclei limited to their surface regions. We have in mind a picture in which the nuclei maintain this contact for a brief time, during which:

- 1) they move around each other through an angle which is related to the deflection angle θ ,
- 2) they exchange a few neutrons and protons, and
- 3) energy and angular momenta are converted from the relative motion of the nuclei into their internal motion.

Most of the discussion of deep-inelastic collisions has been organized around the study of the relationship between these three aspects of the interaction between the nuclei.

Figure 30 shows angular distributions of some of the light reaction products when ^{232}Th is bombarded by 288 MeV ^{40}Ar . Plotted vertically is the differential cross-section for slices of the final energy spectrum. When K nuclei come out with Q near zero, we get an angular distribution peaked a little below 60° . As Q becomes more negative, the peak shifts to smaller angles, until by the time we reach $Q = -120$ MeV, the angular distribution is forward-peaked. Similar behavior is shown by the angular distributions of the outgoing Cl and Al nuclei. Wilczynski showed that it is convenient to display this kind of information using a map of contours of constant double-differential cross-section, in an energy-angle plane. Figure 31 shows a so-called Wilczynski diagram for the reaction $^{232}\text{Th}(^{40}\text{Ar}, K)$ at $E_{\text{lab}} = 388$ MeV. The surface depicted here shows two ridges, one mov-

Figure 30. Angular distributions³² of isotopes of Al, Cl and K from the reaction $^{232}\text{Th} + ^{40}\text{Ar}$ ($E_{\text{lab}}=288$ MeV) as a function of energy damping.

Fig. 31. Contour map of the differential cross section $(d^2\sigma/dE \cdot d\theta)_{CM}$ (in $\mu b/MeV \cdot rad$) in the reaction $^{232}Th(^{40}Ar, K)$ at $E_{lab} = 388 MeV$, as a function of the scattering angle θ_{CM} and the C.M. energy of the detected K ions.

Pictorial description of the proposed model for transfer reactions at high angular momenta.

Illustration of possible experiments which could prove the idea on nuclear orbiting in heavy-ion reactions

Figure 32, From Ref. 3.

ing steeply from upper-right down to lower left, and the other moving almost horizontally across the bottom of the diagram. The steep ridge summarizes what is illustrated in Fig.30. As the Q value becomes more negative and so the outgoing ions have less energy, the peak in the angular distribution moves towards smaller angles, and eventually we get a forward-peaked distribution. This was interpreted by Wilczynski in terms of the orbiting picture shown in Fig.32. Smaller deflection angle means that the orbit produces closer contact between the nuclei (where they feel the attractive part of the potential more strongly) and this closer contact results in more conversion from relative to internal energy. The lower ridge, representing strongly damped events, was believed by Wilczynski to result from collisions where the nuclei have orbited to negative deflection angles.

If the interaction between the orbiting nuclei is something like that between two interlocking gears, then the separating nuclei will be spinning as indicated in Figure 32. Notice that orbits 1 and 2 emerge at the same angle, but are spinning in opposite directions. Wilczynski thus suggested that there should be a correlation between polarization and location in the Wilczynski diagram. ³³ Bond has pointed out that this argument is too primitive. Using the example of 1-nucleon transfer, which can be calculated in the DWBA, he showed that the relative polarizations of 1 and 2 depend upon many factors, including the reaction Q-value.

Wilczynski's analysis strongly suggests that as energy loss increases the reaction time increases, resulting in broader and more forward-peaked angular distributions.

The contours in the Wilczynski diagram correspond to constant values of $d^2\sigma/dE d\theta$ rather than to $d^2\sigma/dE d\Omega$. Differential cross-sections

with respect to θ are useful when we are interested in comparing with a situation in which the two fragments make many revolutions about each other before they separate. Figure 33 symbolizes a situation in which an orbit approaches a target in one direction, but as a result of random processes that go on during the many revolutions about the target, all angles of emission in the orbital plane are equally probable. Thus in this situation $\frac{d\sigma}{d\theta}$ would be constant. To get the usual differential cross-section

Figure 33: An incoming particle is scattered with equal probability in all directions in the plane.

$d\sigma/d\Omega$, we have to average this figure over all azimuthal planes containing the incident particle, which yields $\frac{d\sigma}{d\Omega} \sim \frac{1}{\sin\theta}$. Thus departure of $d\sigma/d\theta$ or $\sin\theta \frac{d\sigma}{d\Omega}$ from constancy is a measure of the memory the binary system has of its direction of formation.

The way the total nuclear charge is re-distributed between the nuclei during the collision is illustrated in Figure 34, where the angular distributions of various Z-products of the reaction $^{197}_{79}\text{Au} + ^{86}_{36}\text{Kr}$ (620 MeV) are presented. Notice that when the outgoing light projectile has a Z near that of the incident projectile ($34 \leq Z \leq 38$), the angular dis-

Fig. 34. Angular distributions as a function of fragment atomic number for the $^{197}\text{Au} + ^{86}\text{Kr}$ reaction at a bombarding energy of 620 MeV.

tribution shows a strong grazing peak. However, as the charge of the outgoing light projectile becomes increasingly different from the incident projectile charge, the angular distributions become increasingly forward peaked. These observations are consistent with Wilczynski's (energy loss)-(scattering angle)-time correlation, since they show that large amounts of charge are exchanged only when the projectile and target are together for a long time.

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This relationship is also exhibited by Figure 35, where it is seen that large charge transfers only occur when there has been appreciable energy damping, and therefore long interaction time. For fragments with Z near the projectile Z there is an undamped peak, plus a tail corresponding to damped events. In Figure 36 the same data are plotted in a way that shows that as energy damping increases, the width of the Z -distribution of the outgoing light fragment increases. This monotonic relation between energy damping and the width of the Z -distribution is plotted in Figure 37, where data for several reactions are summarized. The conclusion drawn from this evidence is that energy damping and charge transfer take place on comparable time scales. This is the basis for various diffusion models of deep-inelastic collisions, which focus on the exchange of nucleons during the entire time of contact.

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Figure 38 illustrates that there is a similar correlation between the width of the mass distribution of the light outgoing fragment and energy damping. The colliding nuclei here are ^{86}Kr and ^{166}Er , with $E_{\text{cm}} = 400$ MeV. Note the remarkable agreement in the shapes of the corresponding Z and A distributions. The white circles of the 230 MeV Z distribution are scaled from the 228 MeV points of the A distribution. This similarity indicates that the exchange of neutrons and protons is highly correlated on all times scales.

Fig. 35. Plot of $d^2\sigma/dZ d(TKE)$ as a function of the total kinetic energy for different elements produced in the $^{209}\text{Bi} + ^{136}\text{Xe}$ reaction. Each bin contains three elements. The ordinate is a logarithmic scale. The arrows indicate Coulomb energies for touching spheres

Fig. 36. Plot of the differential cross section $d^2\sigma/dE dZ$ as a function of atomic number for various final kinetic-energy bins of 50-MeV width measured for the $^{209}\text{Bi} + ^{136}\text{Xe}$ reaction. The solid lines are Gaussian fits to the data

Fig. 37. Total kinetic energy loss in MeV as a function of the variance; of the proton distribution of the projectile-like fragment.

Fig.38. Upper portion: Angle-integrated fragment-primary-mass distributions at four $E_{K \text{ tot}}$ intervals (312, 270, 228, and 186 MeV) as functions of the primary mass of the light fragment, A_L , measured for the reaction ^{86}Kr with ^{166}Er at $E_{\text{c.m.}} = 396$ MeV. The cross sections are averaged over intervals of 8 amu in mass and 42 MeV c.m. in $E_{K \text{ tot}}$. Lower portion: Angle-integrated fragment-charge distributions at four $E_{K \text{ tot}}$ intervals (310, 270, 230, and 190 MeV) as functions of the atomic number of the light fragment, Z_L , measured for the reaction ^{86}Kr with ^{166}Er at $E_{\text{c.m.}} = 405$ MeV. The cross sections are averaged over intervals of 1 charge unit and 20 MeV (c.m.) in $E_{K \text{ tot}}$. The center values of the $E_{K \text{ tot}}$ intervals are indicated. The open circles near the fragment-charge distribution at $E_{K \text{ tot}} = 230$ MeV (lower portion) represent cross section obtained by scaling the fragment-mass distribution at $E_{K \text{ tot}} = 228$ MeV (upper portion).

Another aspect of mass equilibration is illustrated in Figure 39, where contour plots of constant $d^3\sigma/d\Omega dA dE$ are plotted in a mass-energy plane.³⁸ The two entrance channels ($^{64}\text{Ni} + ^{40}\text{Ca}$ and $^{58}\text{Ni} + ^{40}\text{Ar}$) are quite different. ^{64}Ni is neutron rich while ^{58}Ni is neutron poor, and viceversa for ^{40}Ca and ^{40}Ar . However, the N/Z ratios are similar (1.17 and 1.13). Reference to Figure 39 shows that for the strongly energy-damped events, which took place on a long time scale, the A-distributions of the out-going K-isotopes are very similar. In other words these events have had the opportunity to forget the differences of their incident channels. However the undamped or quasi-elastic events have retained strong memories of the incident channels: the N-distributions are peaked at N=20 and N=22, exactly as for the corresponding elastic channels. Thus we see from the last two figures that the neutrons are also equilibrated on the same time scale as the energy. But there is also evidence that the Z-distributions equilibrates faster.³⁹ Figure 40 shows the widths of the Z-distribution for different masses produced in the reaction $^{86}\text{Kr} + ^{98}\text{Mo}$ with $E_{\text{lab}} = 430 \text{ MeV}$. For energy loss $\leq 30 \text{ MeV}$, the widths increase with energy loss, but then they remain constant up to the largest energy losses of about 80 MeV. Thus the change in N/Z was confined to the early stages of the reaction (about the first third).

The relative oscillation of protons and neutrons during the

Contour plot of mass against energy for K isotopes produced in two different reactions involving very different total N/Z ratios in the entrance channel, ($^{62}\text{Ni} + ^{40}\text{Ca}$) and ($^{58}\text{Ni} + ^{40}\text{Ar}$), at an impact energy of 230 MeV. The mass distribution for DRC events is the same, while it differs strongly for the quasi-elastic processes

Figure 40. Widths³⁹ of the Z-distributions for several masses, plotted as a function of the total kinetic energy. The lines are drawn to guide the eye.

time that the collision partners are in contact has been discussed in terms of an isovector giant dipole excitation in the collision complex. ⁴⁰

The problem of the distribution between the two fragments of the angular distribution extracted from the orbital motion is a difficult one, which has been addressed only in the last few years. The angular momenta of the departing fragments have been determined by observation of γ and light-particle emission, and by observation of the angular distribution of fission fragments. An interesting example of the latter approach is a recent paper by Harrach et al., ⁴¹ who bombarded ^{58}Ni and ^{90}Zr targets with beams of ^{208}Pb and ^{238}U . In these cases, the projectile-like fragment was the heavy one, and it was projected forward into a cone of $45\text{--}50^\circ$ opening angle. It's fission was then observed. Figure 41 shows the distribution of fission events as functions of the relative speed of the fragments. The narrow peaks clearly demonstrate the existence of an intermediate fissioning system of well-defined energy. Moreover, these distributions were independent of the direction of the velocity of the fissioning nucleus relative to the incident beam, and to the Q-value of the reaction. This implies that the fission took place far away from the target-like fragments.

Figure 42 shows the angular distributions of the fission fragments. The abscissa, θ , is the angle between the direction of the fission fragments and the normal to the reaction plane of the previous deep-inelastic collision. Note that the most probable value of θ is 90° , which means that the fission fragments tend to remain in the reaction plane. The $\theta=90^\circ/\theta=0^\circ$ ratio for ^{208}Pb is 25. The smaller ratio of 3 for ^{238}U is due to the fact that in this case, fission can occur for quasi-elastic reactions, where little angular momentum

Fig. 41. Distribution of the vector difference $|\vec{v}_1 - \vec{v}_2|$ of the fission-fragment laboratory velocities, integrated over all other observables.

Fig. 42. Overall fission-fragment angular correlations. The polar distributions (upper part) are integrated over all azimuth angles ϕ_{cm}^F . The in-plane distributions (lower part) represent a cut close to the equator. The rms width (center part) is evaluated from the individual polar distributions for each ϕ_{cm}^F .

Fig. 43. Fission coincidence yield, fission probabilities (ratio of ternaries to total), rms angles of the polar distributions (integrated over ϕ_{cm}^F), and a lower limit for the average oriented part $\langle I_2 \rangle$ of the heavy-fragment spin (assuming complete alignment).

is transferred. The r.m.s. widths of the θ distributions are also shown, and the rather flat in-plane azimuthal angle distributions, which indicate that all directions in the reaction plane are approximately equally favored by the fission fragments.

Harrach et al. use these observations to try to infer the distributions of the angular momenta and angular momentum projections of the fissioning product of the deep-inelastic collision. Their conclusion about $\langle I_z \rangle$ ($\hat{z}$ is the direction normal to the reaction plane) is shown in Figure 43. A noteworthy feature in the U case is the rise in $\langle I_z \rangle$ with increasing energy loss. This is consistent with the diffusion model picture of greater mass-transfer probability for greater energy loss. It is interesting, however, that this rise is followed by a plateau which persists up to the largest energy losses. This feature points to a more complicated relation between energy conversion and angular momentum than the simplest considerations would imply. The occurrence of the plateau is consistent with the results of a calculation by Broglia et al. based on a model of heavy-ion collisions in terms of elementary modes of excitation.

Theories of fusion and deep-inelastic collisions.

The attempts to understand the properties of nuclear collisions in the fusion or deep-inelastic collision region can be divided into two classes:

- 1) attempts that focus on a few explicit degrees of freedom and try to follow their evolution in time,
- 2) attempts that treat the sharing of energy, angular momentum neutrons and protons, as statistical processes.

The first class has produced more detailed predictions, and thus we will consider them here more fully. We will describe two examples, the time-dependent Hartree-Fock (TDHF) method, and the coherent-state model (CSM).

TDHF is a mean-field theory, that is the force experienced by each nucleon is due only to the instantaneous collective field to which all the nucleons contribute. It can be regarded as a generalization of the independent-particle model which we have used for many years to describe bound and nearly bound nuclear states. The justification for using the independent-particle model at low energies is that the presence of the degenerate Fermi sea prevents most of the nucleon-nucleon scatterings that would occur in free space. Thus the mean free path of a nucleon in a nucleus is comparable to the dimensions of the nucleus. Although the bombarding energy of the heavy-ion may be several hundreds of MeV, when this energy is distributed among all the nucleons in the system, the excitation energy per nucleon is only 1 or 2 MeV. Of course there may be local "hot-spots" where a few nucleons are excited to energies well above the Fermi energy, but most of the nucleons in the collision complex will still be dominated by the Pauli principle and an almost filled Fermi sea.

Thus long mean-free-paths and mean-field theories also are applicable in this situation.

The TDHF approximates the total wave function of the system by a single Slater determinant, whose single-particle states satisfy the following set of coupled non-linear differential equations

$$i\hbar \frac{\partial \psi_j(\vec{r}, t)}{\partial t} = - \frac{\hbar^2}{2m} \nabla^2 \psi_j(\vec{r}, t) + \int d^3r \left[\int d^3s \sum_k \psi_k^*(\vec{s}, t) U(\vec{r}, s) \psi_k(\vec{s}, t) \right] \psi_j(\vec{r}, t) \\ - \int d^3r \left[\int d^3s \sum_k \psi_k^*(\vec{s}, t) U(\vec{r}, s) \psi_j(\vec{s}, t) \psi_k(\vec{r}, t) \right]. \quad (30)$$

Here v is taken to be an interaction whose parameters are adjusted to yield good results for nuclear ground states. The time-independent Hartree-Fock equations can be recovered from (30) by setting

$$\psi_j(\vec{r}, t) = e^{-\frac{i}{\hbar} \epsilon_j t} \phi_j(\vec{r}), \quad (31)$$

and the RPA can be derived by considering small-amplitude time-independent oscillations about the solution of this time-independent problem. Thus an attractive feature of the TDHF approach is that it provides a fully microscopic, fully quantum-mechanical extension of methods that are known to work well near the ground state.

The principal limitation of the method is the requirement that the wave function be approximated by a single Slater determinant. Thus the TDHA cannot be expected to provide reliable predictions about quantities depending upon many-body correlations, or correlations involving small internucleon separations. Fortunately, the kinds of properties measured in heavy-ion experiments (for example average distributions of neutrons and protons between the fragments, or kinetic energy loss) which can be expres-

sed in terms of one-body operators can probably be reliably calculated with a single determinant.

The integration of the set (30) begins at an initial time with a determinant that describes two non-overlapping nuclei moving towards each other along a Coulomb orbit with specified impact parameter. The time integration is performed numerically. The outcome of the collision is interpreted in terms of quantities calculated with the final wave function. Figure 44 shows contour plots of density integrated over the normal to the scattering plane for an $^{16}\text{O} + ^{16}\text{O}$ collision at $E_{\text{lab}} = 105 \text{ MeV}$ and $L = 13\hbar$. The unit of time is 10^{-22} sec. This is an event that resulted in a fusion of the two nuclei after a rather complicated series of damped oscillations and rotations. On the other hand, Figure 45 shows a collision with smaller impact parameter that did not fuse. The outgoing fragments are strongly deformed, and the kinetic energy has been relaxed down to the Coulomb barrier. This is an example of the absence of fusion for low partial waves, referred to on page 47.

It is convenient to discuss time evolution in terms of orbits. To extract a relative separation coordinate $\vec{R}(t)$ from the calculated wave functions, the following integral was calculated

$$\vec{R}(t) = \hat{n}(t) \int d^3x \rho(\vec{x}, t) |\hat{n} \cdot \vec{x}| / A \quad (32)$$

Here $\hat{n}(t)$ is a unit vector directed along the principal axis (with large-

Fig. 44.

Contour plots of the density integrated over the normal to the scattering plane for an $^{16}\text{O} + ^{16}\text{O}$ collision at $E_{\text{Lab}} = 105 \text{ MeV}$ and $L = 13 \hbar$. Time is in units of 10^{-22} s .

Fig. 45., Similar to Fig. II.1 for $L = 5 \hbar$.

st eigenvalue) of the mass-quadrupole tensor, and $\rho(\vec{x}, t)$ is the calculated instantaneous density. Some of the trajectories followed by this $\vec{R}(t)$ are illustrated in Figure 46. The large L trajectories behave in the manner expected for nuclei moving under the influence of a combined nuclear + Coulomb potential. Fusion occurs for intermediate L, but not for $L \leq 11.5$.

Another example of the kind of detail TDHF can provide is shown in Figure 47. Various properties of the orbits of the $^{84}\text{Kr} + ^{209}\text{Bi}$ ($E_{\text{lab}} = 600 \text{ MeV}$) system are plotted as functions of angular momentum. Notice that for the most strongly-damped collisions, the final kinetic energy is lower than the Coulomb barrier. This is a consequence of the deformations of the two fragments at the moment they separate. The contact time τ is defined as the period during which the minimum density on the line between the fragment centers exceeds half the central nuclear density. Note that the upward fluctuation in θ_{cm} at about $L=180$ corresponds to a downward fluctuation in contact time.

The coherent-state model has evolved in the past few years into a rather complex description of the deep-inelastic process. Like the TDHF, it integrates a time-differential equation to determine the time evolution of the system along a selected set of orbits. However, it does not deal with the individual nucleon degrees of freedom. Rather, it works with a variable $\vec{r}(t)$ defining the separation of the centers of the fragments, and a set of parameters $\alpha_{\lambda\mu}^i(t)$ which define, in the usual way, the shapes of the surfaces of the two nuclei ($i=1,2$). These surfaces interact via a proximity potential, which is a function of the minimum

Fig. 46. Trajectories of the fragment separation vector $\vec{R}(t)$ for $^{16}\text{O} + ^{16}\text{O}$ at $E_{\text{lab}} = 105 \text{ MeV}$. The curves are labeled by the initial orbital angular momentum.

Fig. 47. Quantitative summary of TDHF results for Kr-induced collisions. Plotted as functions of the initial orbital angular momentum are: θ_{CM} , the final CM scattering angle; TKE_{CM} , the final CM total kinetic energy of the fragments; τ , the contact time; ΔA , the net mass change of the light fragment; Γ_A , the full width at half maximum of the mass distribution; and P_{Kr} , the percentage of Kr orbitals remaining in the light fragment after the collision. Also shown on the θ_{CM} graph are: θ_{COUL} , the Rutherford deflection function; $l_{1/2}$, the angular momentum where the optical-model transmission coefficients fall to 1/2; and θ_{P} , the peak angle in the experimental strongly damped angular distribution. The coulomb barriers indicated on the TKE_{CM} graphs correspond to point charges separated by 14.28 fm, the experimental strong-absorption radius for $^{84}\text{Kr} + ^{209}\text{Bi}$ at $E_{\text{Lab}} = 600 \text{ MeV}$.

Figure 48: The proximity interaction potential energy is a function of the minimum separation, S , between the surfaces and of the curvatures at the points a, b .

separation distances between the surfaces, and of the curvatures of the surfaces at the points at opposite ends of this minimum distance line. For simplicity, Broglia et al approximate this quantity s by the distance s' between the surfaces along the line connecting the centers of the nuclei.

If the nuclei were isolated, the $\alpha_{\lambda\mu}^i(t)$ would undergo independent damped harmonic oscillations. The restoring force and mass parameters are chosen to give the correct energy and summed oscillator strength for the corresponding collective mode. The damping constant is chosen to give a good description of the width of the distribution of this collective mode in the real nucleus. These parameters, determined for low energy nuclear states, are assumed to remain constant throughout the collision. During the collision, the surface-surface interaction potential couples the relative vector $\vec{r}(t)$ with the $\alpha_{\lambda\mu}^i(t)$ and thus can result in the transfer of energy and angular momentum between the relative motion and the collective oscillations. Since this oscillation is damped, some of this energy is continually being bled off into more complicated degrees of

freedom, from which it cannot return to the relative motion. This is one of the three sources of energy damping in the CSM. A second source is the energy that is contained in the collective oscillations after the fragments have separated. These two effects account for about half the energy removed from the relative motion.

The remaining half of the energy dissipated is related to the flow of nucleons between the surfaces when they are near each other. This is described phenomenologically in terms of a mass flux, which is driven by the mass asymmetry, and a friction force which is related to the momentum transported by the transferred nucleons. These quantities are incorporated into the description using arguments similar to those that go into the derivation of the proximity force.

The gradient of the interaction potential with respect to the relative vector $\vec{r}$ gives part of the force between the fragments. The other part is due to friction. This total force is used in Newton's 2nd law, at each point of the orbit, to obtain the instantaneous acceleration. Similarly, the interaction potential drives the damped harmonic oscillations of the collective modes. The entire set of coupled equations is integrated simultaneously to yield $\vec{r}(t)$ and the $\alpha_{\lambda\mu}^i(t)$.

The coupling between the surfaces is non-central. One consequence of this is the production of accelerations normal to the instantaneous plane of the orbit. Thus the orbit will not remain plane. In most of the calculations done so far, these out-of-plane accelerations are ignored.

The main assumption of this model is that there are two kinds of coupling between the relative and internal motion:

Figure 49. Development of a collision of 610 MeV ^{44}Kr on Pb, according to the coherent-state model. The impact parameter is 8 Fm. The unit of time is 10^{-21} sec.

Figure 50. Deflection angle (a) and final kinetic energy (b) as functions of impact parameter⁴⁴. The significance of impact parameters P_1, P_2, P_3, P_4 are discussed in the text.

- 1) A coupling that proceeds via excitation of collective shape oscillations. This is treated explicitly.
- 2) A coupling associated with flux of nucleons between the fragments. This is treated phenomenologically.

Earlier versions of this theory incorporated only the first variety of coupling.

4.4

An example of the development of a collision in this model is shown in Figure 4.9, for the collision of 610 MeV Kr on Pb with an impact parameter of 8 fm ($L=282h$). The pictures show the development of the collision at intervals of $.12 \times 10^{-21}$ sec.

In figure 50 are plotted the deflection angle and final kinetic energy E_f as functions of impact parameter b . For $b > \rho_4$ we have peripheral, elastic, quasi-elastic and particle transfer. These curves are interrupted between ρ_3 and ρ_4 . For the corresponding impact parameters, the orbits showed a second turning point—that is r decreased, increased and then started to decrease again.

Figure 51: The occurrence of a second turning point is taken as an indication that fusion has occurred.

Although the orbit was able to surmount the outer barrier on its way in, dissipation in the pocket has lowered its kinetic energy enough so that

it gets caught on the way out (Fig.51). However, while the kinetic energy is decreasing in the pocket, the relative angular momentum is decreasing as well, so it is possible that in spite of the energy dissipation, the slowed down fragments will be able to overcome the outer barrier and to separate. Broglia et al find that this can occur for certain values of the initial angular momentum. This corresponds to the region of impact parameter between ρ_2 and ρ_3 in Figure 50. It is the CSM explanation for the non-occurrence of fusion at low impact parameters. ²⁷✓

Recently, a comparison has been made of CSM and TDHF calculations for the same system. The results were quite similar for all the quantities calculated (nuclear shapes, total energy loss, orbit geometry, etc.) Since these models start from quite different assumptions, the similarity of their predictions seems to imply that both accomplish their objective of accounting for the gross features of the time development of nuclei in collision. ⁴⁵✓

The statistical descriptions of deep-inelastic collisions take the viewpoint that during the time the nuclei are in contact, a very large number of degrees of freedom are excited. We really don't need to know, or want to know, their individual time dependence, since experiment only determines a few average properties. In the spirit of statistical mechanics, we can calculate these averages as ensemble averages, with the characteristics of the ensemble governed by our intuition and experience in other areas of statistical mechanics. These ensemble averages replace the detailed microscopic dynamics by an average dynamics. They provide a conceptual framework for the introduction of macroscopic properties like diffusivity or friction, and they also offer the hope of accounting for

gross-features of the reactions using only rather general principles, without having first to calculate details of orbits which are subsequently thrown away in an averaging process. I think we should not regard the statistical and explicit theories of deep-inelastic collisions as rivals. They attempt to accomplish different things, and we need them both if we are to have a complete an understanding of these reactions.

02

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