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stress analysis of the PM-2A reactor vessel

Contract No. AT[30-1]-2639
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Development Report
UC-81, Reactors, Power
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STRESS ANALYSIS OF THE PM-2A REACTOR VESSEL

By:
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Approved by:
M. H. Dixon, Project Engineer

Issued: May 14, 1962

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ABSTRACT

This report presents the stress analysis performed on the PM-2A reactor vessel and cover as part of the Program for Engineering Support and Development of Army Pressurized Water Reactor Power Plants. The maximum combined stress (51,200 psi) occurs in the studs after reaching steady state conditions. A fatigue analysis indicated that this stress could be safely applied 2500 times, and since the studs will never approach 2500 cycles from initial stud tightening to steady state conditions, they should not suffer any fatigue damage.

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LIST OF FIGURES

1.0 SUMMARY AND CONCLUSIONS

Calculated stresses in the PM-2A vessel were significant in three major areas: (1) the studs, (2) the nozzle penetrations, (3) the cover at the junction of the elliptical head and the transition section. The cover and flange stress calculations were for steady-state temperature distributions only. Figure 1 shows the location and magnitude of maximum stresses in the reactor vessel.

This analysis assumed the studs were initially tightened to the ASME Code allowable stress of 20,000 psi. Upon application of 1750 psi internal pressure, the tensile stress increased to 22,480 psi and 14,700 psi bending stress was added, for total stress of 37,180 psi. Under steady-state temperature conditions, the tensile stress decreased by 980 psi, but another 15,000 psi bending stress was added for total stress of 51,200 psi. The stud stress of 51,200 psi was shown to be safe from a fatigue standpoint, according to the Navy Code. Local yielding which takes place under the nut and tends to reduce bending stresses was ignored.

The nozzle penetrations were the areas of highest stress in the main body of the vessel. In the main coolant nozzles, the pressure stress was 34,100 psi at the inside corner of the longitudinal section. The maximum stress at the same location in the decay cooling nozzle was conservatively calculated at 38,000 psi. Secondary bending stresses at the main coolant nozzles due to piping loads were low, the highest being only 3400 psi. The maximum combined stress from pressure and piping loads was 36,320 psi which occurred at the upper inside corner of the longitudical section of the outlet nozzle. Thermal stresses in the nozzles were not included and will be computed as part of the Fiscal Year 1962 stress analysis program.

The highest stress calculated in the vessel, 62,575 psi, occurred in the cover at the junction of the elliptical head and transition piece, with the studs tightened to 20,000 psi and with internal pressure of 1750 psi applied. This condition existed only for several cycles during pressure tests in the early life of the reactor and cannot be repeated because of nil-ductility transition restrictions on pressure at low temperatures, which apply to the PM-2A reactor vessel. The calculated stress of 62,575 psi was considered safe because it was within Navy Code limits for 500 cycles of application. At operating conditions, the thermal stresses calculated from the temperature distributions were opposite in direction to the stud load-pressure stresses and tended to reduce them. As a result, the maximum principal stress in the cover at operating conditions was 26,675 psi.

When the transient temperature distributions are available, transient thermal stresses will be calculated and a fatigue analysis of the most highly stressed areas will be performed.

PM-2A VESSEL OUTLINE

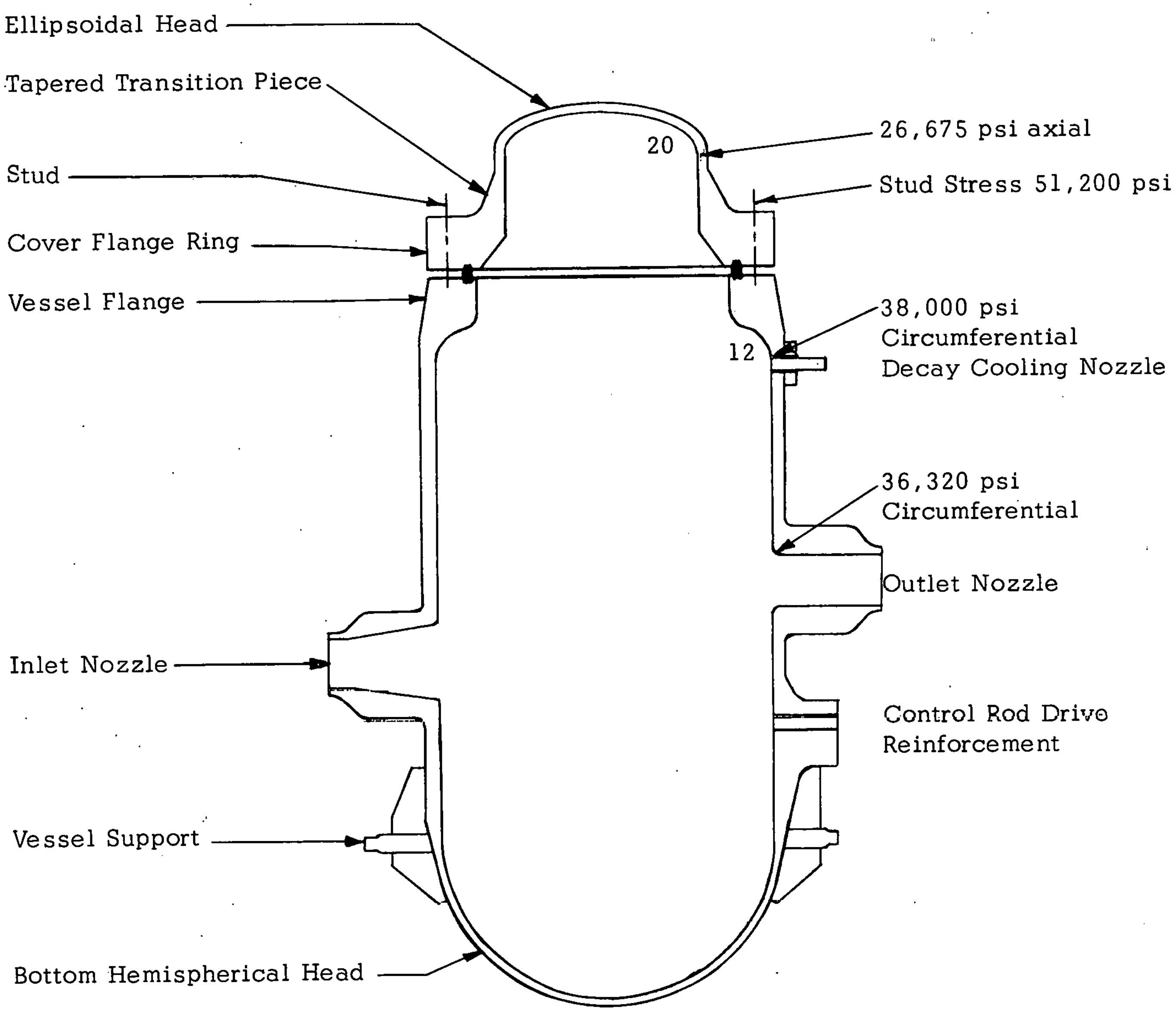

Figure 1. Maximum Combined Stresses at Steady State Operating Conditions

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2.0 INTRODUCTION

This report presents a stress analysis performed on the PM-2A reactor vessel under Item 6.9 of the Program Plan for Engineering Support and Development of Army Pressurized Water Reactor Power Plants.*

The PM-2A reactor vessel was designed for a maximum pressure of 2000 psi at 600°F. Actual operating conditions are 1750 psi and 518°F. The vessel is basically a 42-3/4 in. outside diameter cylindrical shell with a wall thickness of 2-3/8 in., flanged at the upper end, and closed at the bottom by a 1-1/4 in. thick hemispherical head which is welded to the cylindrical section. The top closure is a flanged 1-3/8 in. thick 2:1 elliptical head which is fastened to the vessel by 18 studs. An octagonal section Type 304 stainless steel ring gasket seals the opening. The cylindrical section of the vessel is fabricated of two A-350 Gr. LF-3 ring forgings with a welded overlay of Type 304 stainless steel applied to the inner surface to provide corrosion protection. Drawing F-41202-1-2 shows the vessel and cover configuration and dimensions.

The stresses presented herein were calculated for conditions of:

A. Initial stud tightening

B. Stud load plus 1750 psi internal pressure

C. Steady state temperature distributions which exist when the reactor is at operating temperature of 518°F.

Calculation of the transient thermal stresses will be completed under the Fiscal Year 1962 program and published in a later report.

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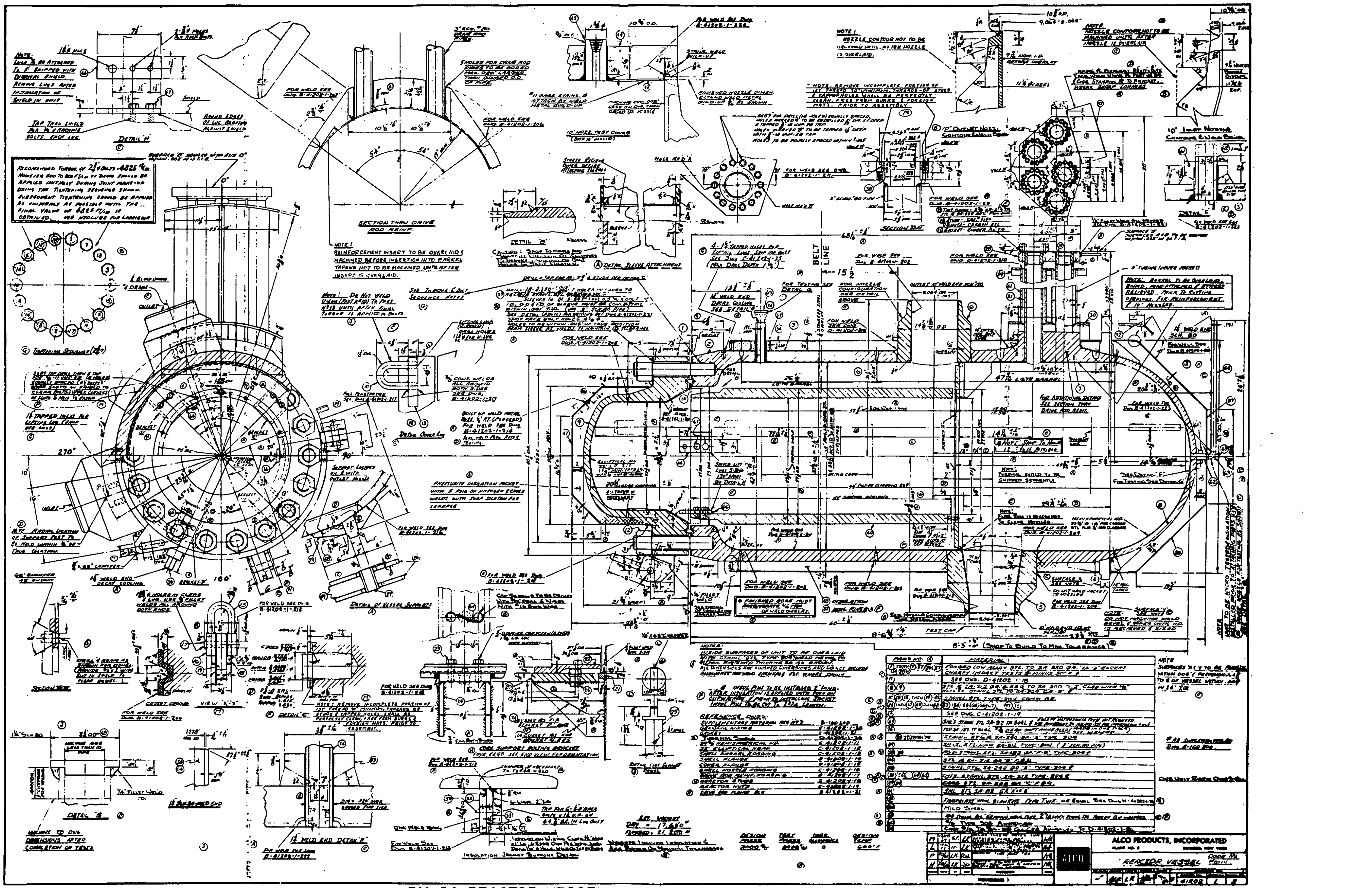
* AP Note 286 Addendum 1, Revision 1, May 1, 1961.



PM-2A REACTOR VESSEL

3.0 ANALYSIS OF VESSEL, EXCLUDING COVER AND FLANGE

3.1 MEMBRANE STRESSES DUE TO PRESSURE

Principal stress components at the inner and outer wall surfaces of the vessel were calculated from equations outlined in the Navy Code. The maximum pressure stress computed for the cylindrical vessel wall was 15,200 psi. It occurred at the inside surface in the circumferential direction. The membrane stress in the hemispherical head was 14,000 psi while the hoop stress at the edge of the hemispherical head was 14,750 psi compression.

Stress indices obtained from the Navy Code were used to calculate the increase in pressure stress at the main coolant nozzles. The maximum stress in both the inlet and outlet nozzles occurs at the inside corner of the longitudinal section where the stress is 34,100 psi circumferentially.

The highest pressure stress occurs at the decay heat cooling nozzle where the local circumferential stress is 38,000 psi at the inner surface. The nozzle penetration is a small externally reinforced hole in the vessel wall. The reinforcement is not of the type covered by the Navy Code; so stresses were conservatively calculated by ignoring the reinforcement and using the stress concentration factor for a small hole in a plate subjected to a 2:1 bi-axial tension.

The membrane stress at the control rod drive penetrations is 20,600 psi at the inside surface. Stress concentration factors used were obtained from the section of the Navy Code which covers openings in a reinforcing band in a vessel.

Figure 2 illustrates the location and magnitude of the highest membrane stresses in the vessel.

3.2 SECONDARY BENDING STRESSES AT BOTTOM HEMISPHERICAL HEAD

A discontinuity analysis was made at the region where the bottom hemispherical head joins the cylindrical section of the vessel.

Membrane deflections were calculated for the head and shell. Next, the discontinuity deflections and edge rotations for the head and shell were obtained in terms of the discontinuity bending moments and shear forces. Boundary conditions required that the edge rotation for the head and cylinder be equal and that the sum of the membrane deflection and discontinuity deflection be the same for both the head and the shell. After solving the equations of the junction, the maximum discontinuity bending moment in the cylindrical shell was found to occur 3.32 in. above the junction of the head and the cylindrical shell. The discontinuity stress produced in the shell at this point was an axial bending stress of 726 psi.

PM-2A VESSEL OUTLINE

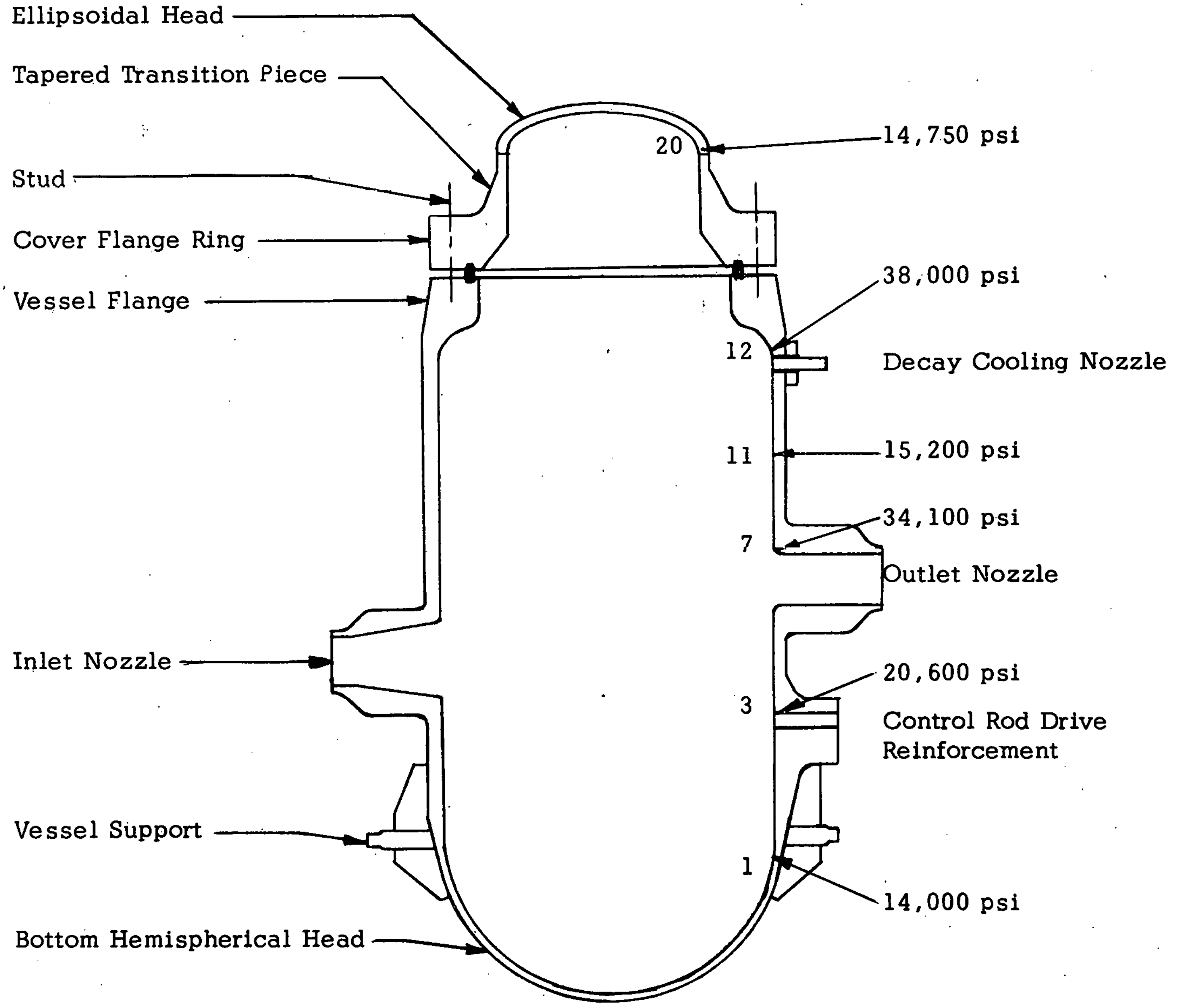


Figure 2. Membrane Stresses Due to Pressure

Since the discontinuity stresses at the point of maximum shear and moment are not expected to be large, only the discontinuity stress at the edge of the bottom head was calculated. In making the calculation, the edge of the head was approximated by a cylinder of the same radius and thickness. The maximum discontinuity stress at the edge of the head was a circumferential stress of 1882 psi at the outside surface.

Figure 3 shows the location and magnitude of the highest secondary discontinuity bending stresses at the lower head.

3.3 STRESSES DUE TO EXTERNAL LOADS

The external loads acting on the reactor vessel occur at the vessel supports and the main coolant nozzles.

At the vessel supports, external load stresses are due to the reactions required to support the total dead weight of the vessel and internals, including the primary water contained. The maximum stress at the supports was found to be an axial stress of 1200 psi.

The external load stresses in the vessel nozzles are due to the moments applied to the nozzles by the primary loop piping. These moments were previously calculated in the PM-2A Piping Design Analysis.[3]

In order to calculate these external load stresses, the method outlined in the Navy Code for obtaining stresses from radial loads and external moments acting on cylindrical shells was used.

The highest stress at the outlet nozzle is produced by the longitudinal piping moment which causes a circumferential stress of 2216 psi at the nozzle. At the inlet nozzle, the highest stress is produced by the circumferential piping moment which causes a circumferential stress of 3372 psi.

Figure 4 shows the magnitude and location of the maximum external load stresses.

3.4 THERMAL STRESS IN THE VESSEL WALL

The thermal stress in the vessel wall due to gamma heating at the mid-plane had been previously calculated to be 6830 psi,[4] based on a gamma heat generation rate of 1.089×10^5 Btu/ft^3-hr.

PM-2A VESSEL OUTLINE

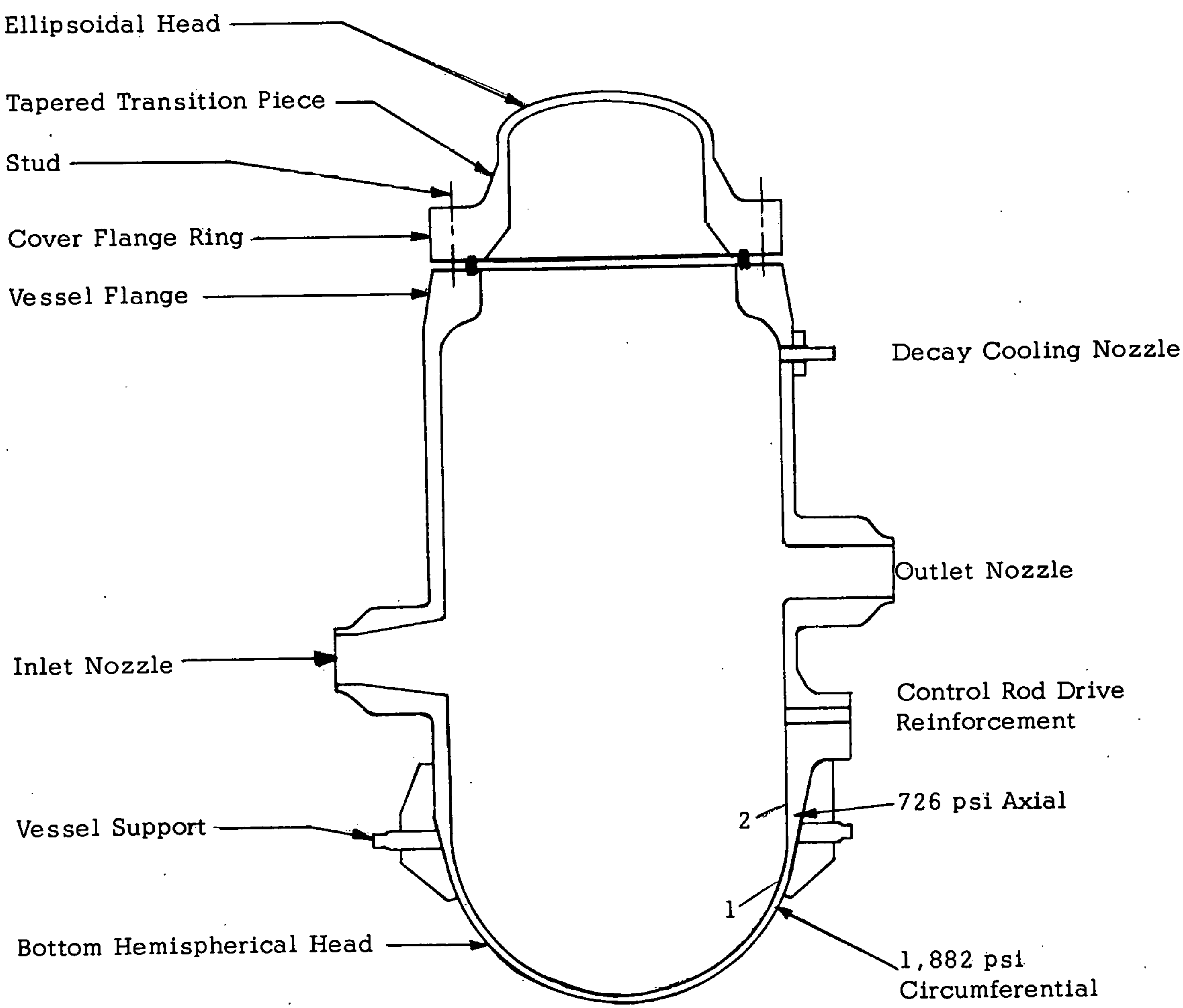


Figure 3. Secondary Discontinuity Bending Stresses

PM-2A VESSEL OUTLINE

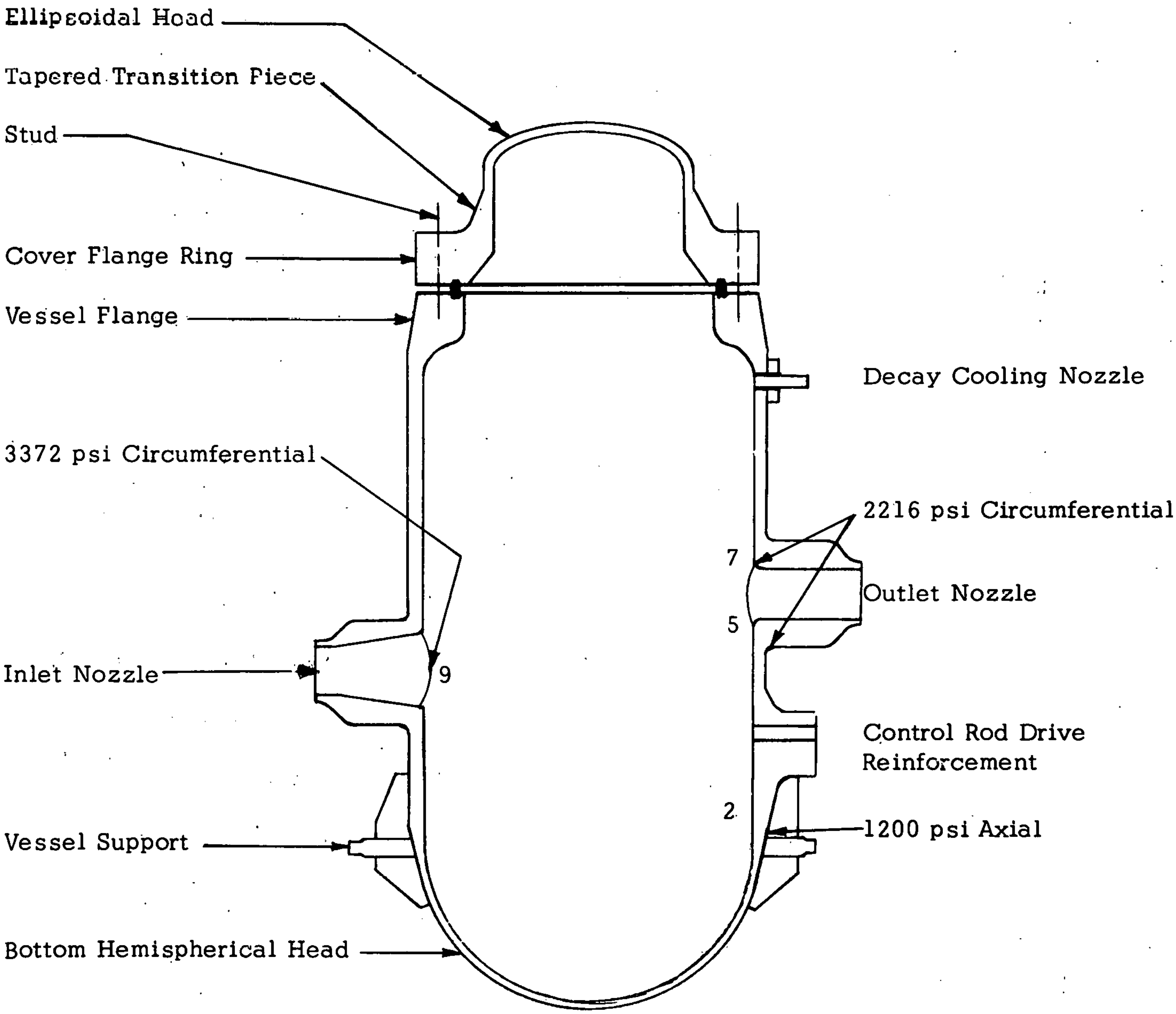


Figure 4. External Load Stresses

3.5 CLADDING STRESS

An analysis was made to determine the stresses in the vessel wall and cladding as a result of the following operations:

1. Application of the welded overlay
2. Stress relieving
3. Hydrostatic testing
4. Application of normal operating temperature and pressure.

This analysis was based on the assumption that the cladding is an elastic-perfectly plastic material and that interaction forces between the cladding and the base metal cause stresses which are uniformly distributed over the thickness of each.

After the cladding has been applied at an elevated temperature and cools to room temperature, it is at its yield strength of 30,000 psi in tension at 80°F. The stress which is induced in the vessel wall by the cladding is 3160 psi in compression and is due to the difference in thermal expansion between the cladding and the vessel wall.

Stress relieving the vessel at a temperature of 1150°F causes compression in the cladding, equal to its yield strength of 12,500 psi in compression at 1150°F. Upon completion of stress relieving, the cladding returns to a stress of 30,000 psi in tension. Assuming the stress to be uniformly distributed over the thickness, the stress induced in the vessel wall when stress relieved at 1150°F will be increased to 1315 psi in tension. After stress relieving, the stress in the vessel wall returns to its prior value of 3160 psi in compression.

During hydrostatic testing at an internal pressure of 3000 psi, the calculated increase in average vessel wall stress is 26,000 psi in tension. The cladding carries none of the load because it is already at its yield stress of 30,000 psi. It simply yields further at test pressure. The net tensile stress in the vessel wall during hydrostatic testing is 22,840 psi. Upon completion of hydrostatic testing, the stress in the cladding drops to 4030 psi in tension. This reduction of tensile stress in the cladding reduces the vessel wall stress to 424 psi in compression.

When the operating temperature reached 510°F, the change in cladding stress was calculated to be 31,000 psi in compression. Thus the resultant stress in the cladding is 26,970 psi in compression. The stress in the vessel wall due to the cladding stress is 2830 psi in tension.

The stresses due to an internal pressure of 1750 psi were calculated for the inner and outer surfaces of the cladding and the vessel wall. The calculated average stress in the cladding is 13,610 psi tension, but the stress in the cladding prior to applying internal pressure was 26,970 psi in compression. The resultant cladding stress then, is 13,360 psi in compression. The vessel wall has a stress increase of 1405 psi in tension due to the cladding to give a resultant stress of 14,925 psi tension.

4.0 REACTOR VESSEL COVER AND FLANGE ANALYSIS

The PM-2A reactor vessel cover is a flanged, carbon steel, elliptical head 1-3/8 in. thick with 1/8-in. stainless steel cladding on the inside surface. The elliptical head is formed with a short cylindrical skirt 2-1/4 in. long. This is welded to a tapered transition piece which is part of the cover flange. The cover flange is 7-3/8 in. high and 9-1/4 in. thick with a 30° taper at the bottom inside corner.

The vessel is a carbon steel cylindrical shell with 1/4 in. stainless steel cladding at the inside surface. The outside radius is 21-3/8 in. and the vessel wall thickness is 2-5/8 in. including the cladding. The vessel flange is 6-1/4 in. high and 7-3/8 in. thick with an inward taper at the top outside corner. It is joined to the vessel shell through a 4-1/4 in. fillet radius. The cover and vessel are shown in Dwg. F-41202-1-2. The reactor vessel studs are fabricated of AISI 403 material.

In order to calculate the stresses, the cover was divided into three pieces consisting of the elliptical head, a short cylindrical section representing the head skirt and tapered section, and the cover flange or ring. The upper end of the vessel was divided into two pieces consisting of the flange and the vessel wall cylinder. In order to obtain valid deflections, the thickness dimensions used included the cladding.

A discontinuity analysis was performed, and deflections, rotation and corresponding stresses for the various pieces were calculated for the following cases:

Case A - Stud load only
Case B - Stud load plus 1750 psi pressure
Case C - Steady state thermal condition

In each case, the cover ring and vessel flange were assumed to be connected by the octagonal gasket acting as a hinge.

4.1 CASE A - STUD LOAD ONLY, NO INTERNAL PRESSURE

The studs were assumed to be tightened to the ASME Code allowable stress of 20,000 psi which caused the cover ring to rotate about the gasket and move downward 0.0015 in. along the stud centerline. The vessel flange rotated about the gasket in the opposite direction, moving upward a distance of 0.0017 in. along the stud centerline. Thus, the total distance the cover ring and vessel flange deflected was the sum of the two or 0.0032 in. These flange positions were the reference from which changes in stud stress were computed for the other loading conditions. Stud bending stresses were not computed for Case A; but were included for Case B, which combines the effects of stud load and pressure, and for the most severe case, Case C.

Due to the absence of internal pressure and thermal distortions, the stresses for Case A were relatively low. The maximum stress developed in the cover occurred at the junction of the elliptical head and the tapered transition section. It is an axial bending stress of 11,050 psi. The other end of the transition developed a stress of 8380 psi at the point where it joined the cover flange. The maximum stress in the vessel occurred at the junction of the flange and the vessel wall. It was an axial bending stress of 13,800 psi. The shear stress developed in the gasket under this loading was negligible, only 150 psi.

Figure 5 shows the location and magnitude of the stresses for the case of stud load only.

4.2 CASE B - STUD LOAD PLUS 1750 PSI INTERNAL PRESSURE

Applying an internal pressure of 1750 psi after the studs have been tightened to a stress of 20,000 psi causes the cover ring to rotate about the gasket so that the ring moves downward a distance of 0.0012 in. along the stud centerline. The vessel flange rotates about the gasket causing it to move upward a distance of 0.0013 in. along the stud centerline. The total decrease in distance between flanges is 0.0025 in. Thus, the increase in stud length caused by the application of pressure is the difference between 0.0025 in. obtained above and 0.0032 in. obtained for Case A, or 0.0007 in. This increase in stud length results in an additional stud tension of 2480 psi. The rotation of the flanges also causes a bending stress in the studs conservatively calculated at 14,700 psi by ignoring local yielding under the nut face. The total stud stress for Case B, then, is 37,180 psi.

The maximum stress in the cover for the case of stud load plus pressure has increased to 62,575 psi and occurs in the same place as for Case A, at the junction of the elliptical head and the tapered transition piece.

The maximum vessel stress occurs at the junction of the vessel flange and wall where the axial stress is 24,620 psi in tension at the outside surface. The circumferential stress at this point was 16,900 psi.

The shear stress in the gasket has increased to 10,100 psi. Figure 6 shows the location of the above stresses.

4.3 CASE C - STEADY-STATE THERMAL CONDITION

Temperature distributions through the PM-2A reactor vessel and cover were determined by using the HOC Code. HOC is a steady-state heat conduction

PM-2A VESSEL OUTLINE

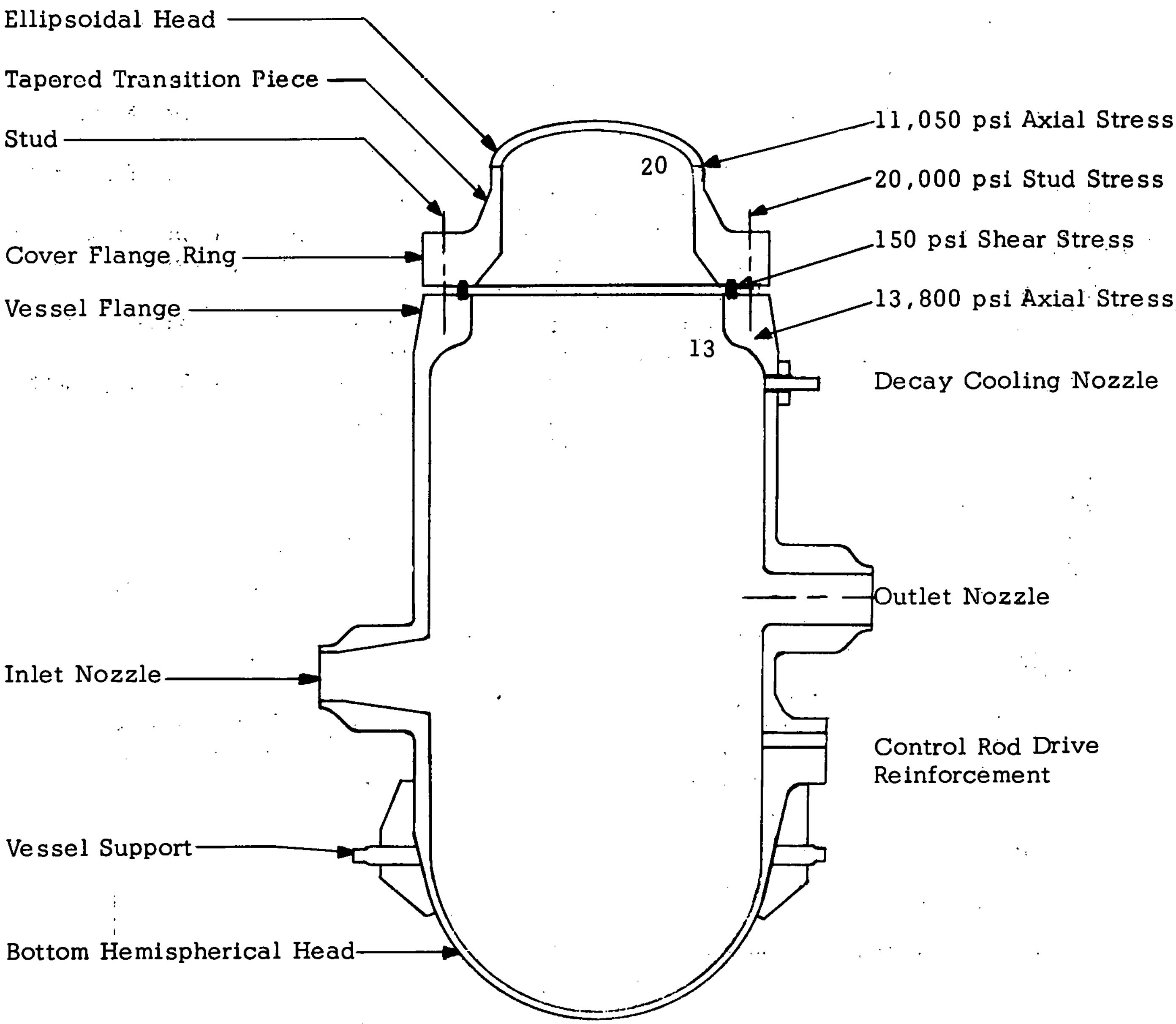


Figure 5. Case A - Stresses in the Cover and Vessel Flange Due to Stud Load Only

PM-2A VESSEL OUTLINE

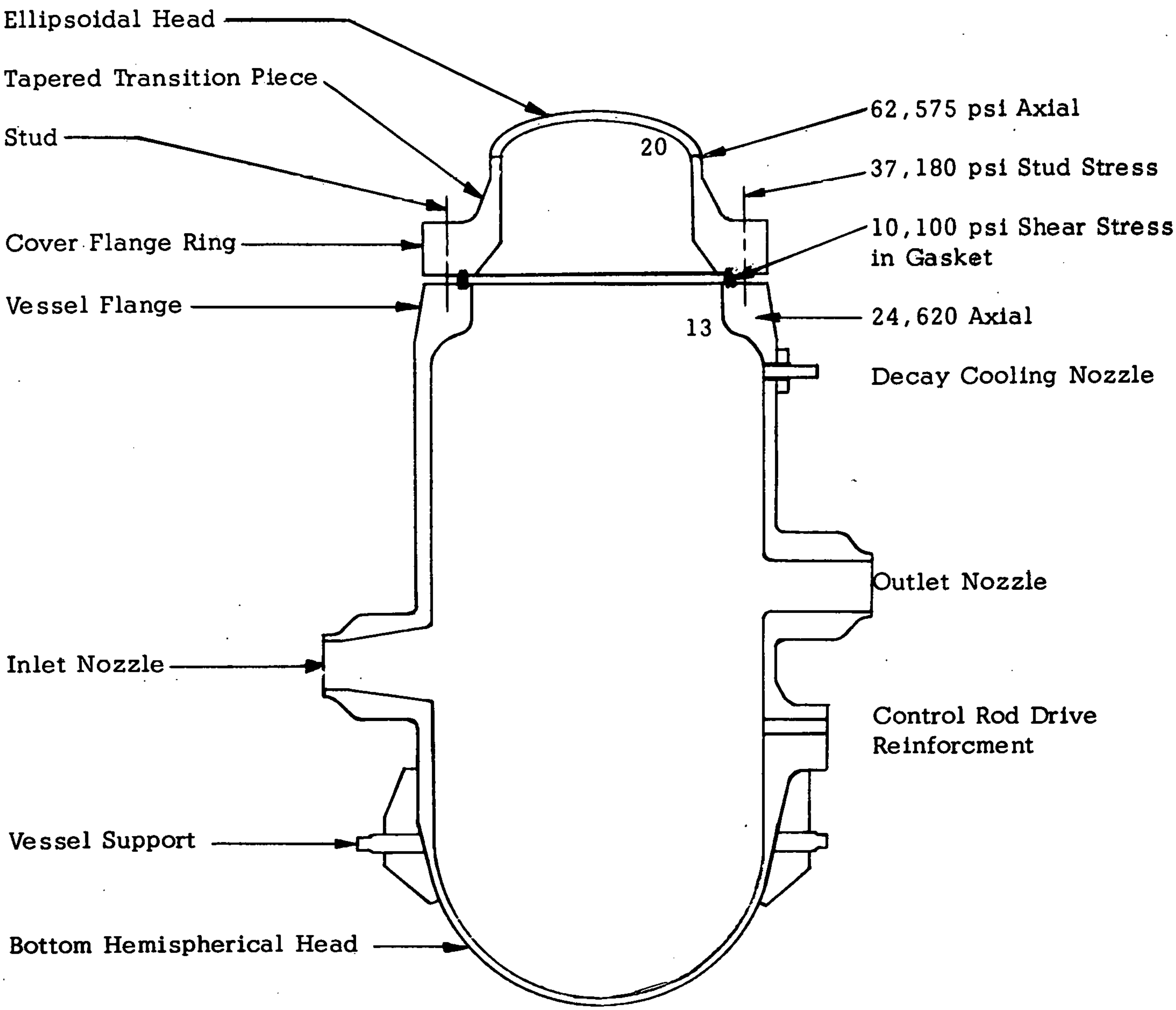


Figure 6. Case B - Stresses in the Cover and Vessel Flange Due to Stud Load Plus 1,750 psi Pressure

program which determines temperature distribution in two dimensions by the relaxation technique. The code can handle solids of irregular shape or with non-uniform heat generation that can be represented as a solid of revolution.

The bulk water temperature inside the reactor was assumed to be at an average temperature of 518°F and natural convection was assumed to be the mode of heat transfer. The air surrounding the vessel up to the vessel flange was assumed to be at 225°F. Boiling water was assumed to cover the vertical outside surface of the cover flange while steam at 212°F surrounded the cover dome insulation jacket and the studs and nuts. Details of the thermal analysis are presented in Appendix A.

The discontinuity analysis for the steady-state condition was based on free body thermal displacements and rotations calculated from the average temperature of each piece of the cover and flange.

For the steady-state condition, the cover flange and vessel flange both rotate about the gasket in such a way that they both move upward. The cover flange moves 0.0005 in. while the vessel flange moves 0.0033 in. both distances being measured along the vertical centerline of the stud. The net decrease in distance between them is 0.0028 in. However, there is also a difference in expansion between the stud and cover flange material, such that the stud is stretched 0.0025 in. The net change in stud length for both conditions is a decrease of 0.0003 in. This results in a decrease in stud tension of 980 psi. The rotation of the flanges also increases the bending stress by 15,000 psi and this stress raises the total stud stress to 51,200 psi.

The shear stress in the gasket under the thermal loads is increased by 21,600 psi to 31,700 psi.

The maximum steady-state thermal stress calculated for the cover occurs at the junction of the elliptical head and tapered transition section where the axial stress is 35,900 psi in compression. The combined effect of stud load, pressure, and steady-state thermal condition gives a total axial stress of 26,675 psi at this point.

In the vessel flange region, a maximum steady-state thermal stress of 17,900 psi axially occurs at the junction of the vessel flange and vessel wall. The maximum combined stress at this point is an axial stress of 19,620 psi in tension. Figure 7 shows the location of these stresses.

PM-2A VESSEL OUTLINE

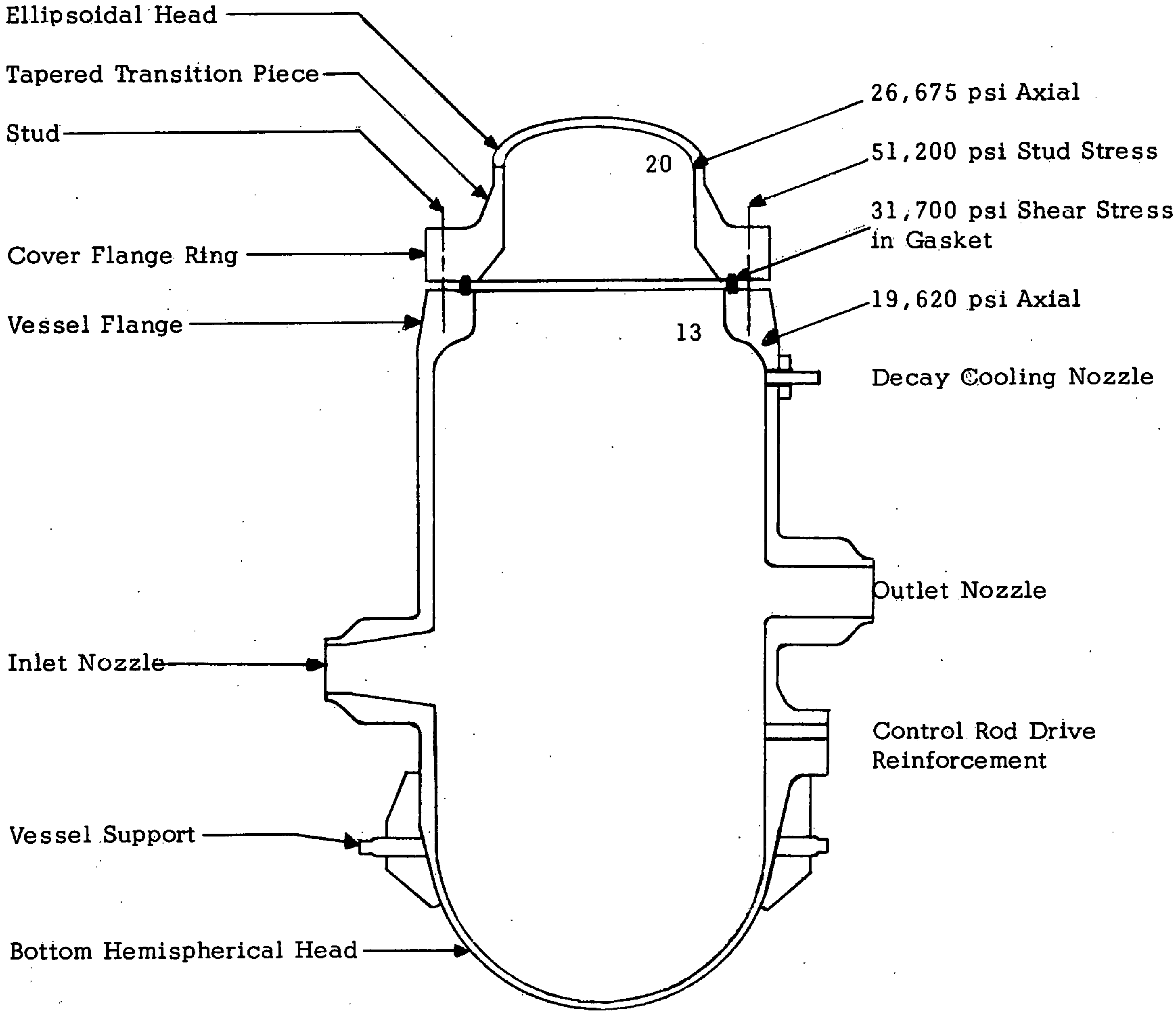


Figure 7. Case C - Stresses Due to the Combined Effect of Stud Load, 1750 psi Internal Pressure and Steady State Thermal Condition

5.0 EVALUATION OF MAXIMUM STRESSES

The combined stud stress after reaching steady-state temperatures is 51,200 psi, which includes 20,000 psi initial stud tension due to tightening, 1500 psi resultant tension after reaching steady-state temperature, 14,700 psi bending stress after applying internal pressure, and 15,000 psi bending stress after reaching steady-state temperature. Thus, the stress range for a cycle during which the vessel cover is tightened, internal pressure applied, and steady-state conditions are reached, is 51,200 psi. The alternating stress then is 25,600 psi which is considered safe from the fatigue standpoint.

The Navy Code does not list allowable values for the PM-2A stud material, AISI-403. However, there are values listed for AISI-410 (heat treated to 100,000 psi tensile strength) which has very similar physical and mechanical properties. The allowable value of alternating stress of this material is 25,000 psi for 2500 cycles of operation.

The calculated alternating stud stress of 25,600 psi was felt to be realistic even through two modifying factors which could not be evaluated numerically were not taken into account. Local yielding of material under the nut will reduce the bending stress in the stud. Stress concentrations at the root of the threads will increase it. Since these two factors oppose each other, the net effect on stud stress will be minimized.

The actual number of cylces from stud tightening to steady-state conditions will never approach 2500 times during a 20-year PM-2A lifetime. Therefore, the alternating stress of 25,600 psi which is in the Navy Code 2500 cycle range is considered to be safe.

The calculated stress of 62,575 psi in the cover, at the elliptical head and tapered transition section, after application of bolt load plus pressure, is high but well within Navy Code limits for 500 cycles. However, this condition occurs only during pressure testing of the primary system, and it is believed to have occurred only a few times during the original PM-2A installation. Operating procedures have been established since then which preclude pressure testing at low temperature in order to prevent brittle fracture. In view of this, a fatigue analysis was not performed at this time. When the transient stresses have been determined, a complete fatigue analysis of the worst point in the cover will be performed.

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6.0 MATERIALS DATA

The PM-2A vessel is fabricated of ASTM A-350 Grade LF-3 forgings. The base material is clad with Type 304 stainless steel overlay. Figures 8, 9, 10, and 11 present the tensile and physical properties for 304 stainless steel and A-350 Grade LF-3. The tensile properties for the base material are based on actual test data obtained from a PM-2A duplicate forging at testing temperatures of 1100°F, 600°F, and room temperature. Intermediate values were obtained by interpolation.

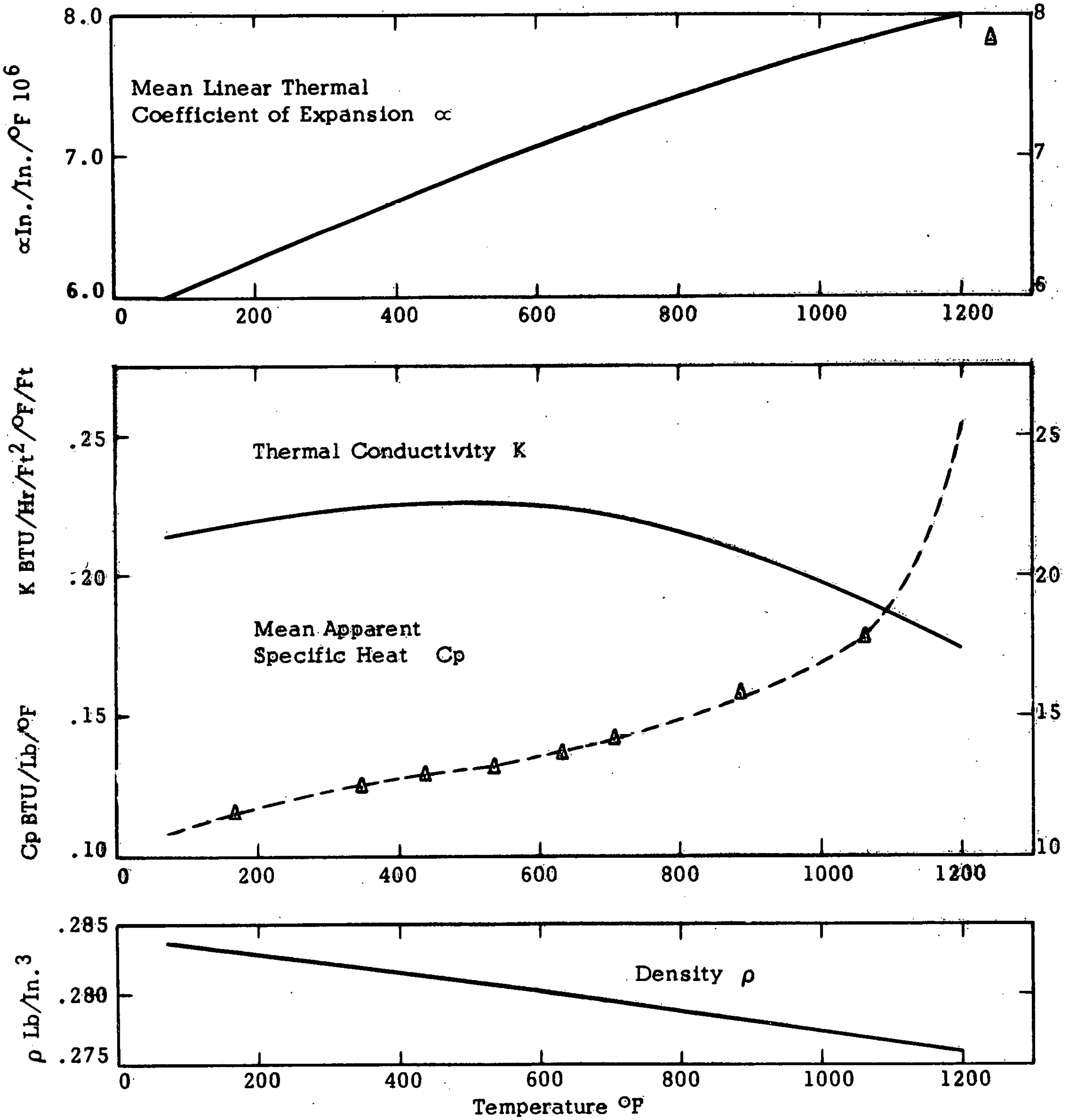

Figure 8. Physical Properties
A-350 Gr. LF-3

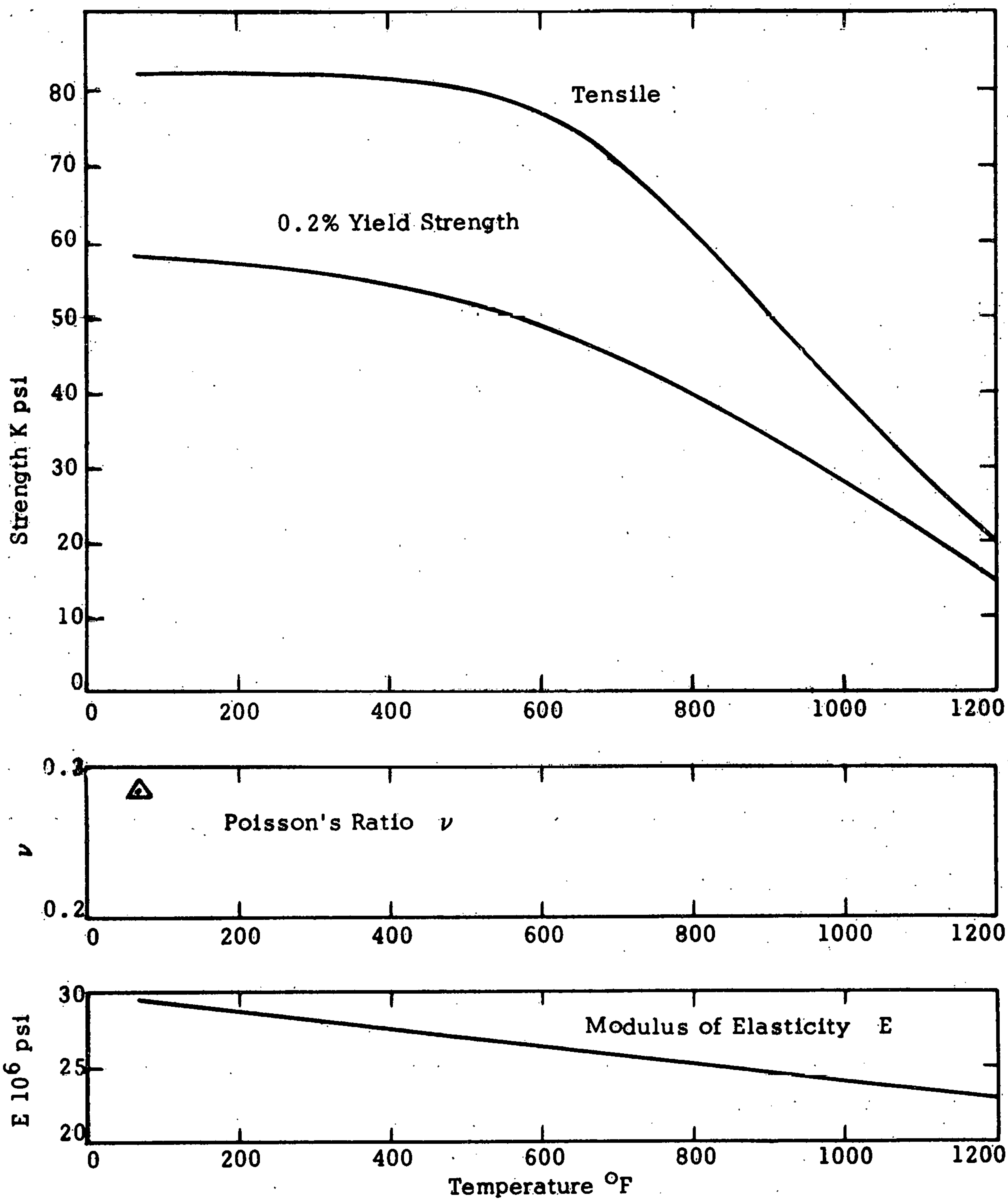

Figure 9. Tensile Properties
A-350 Gr. LF-3

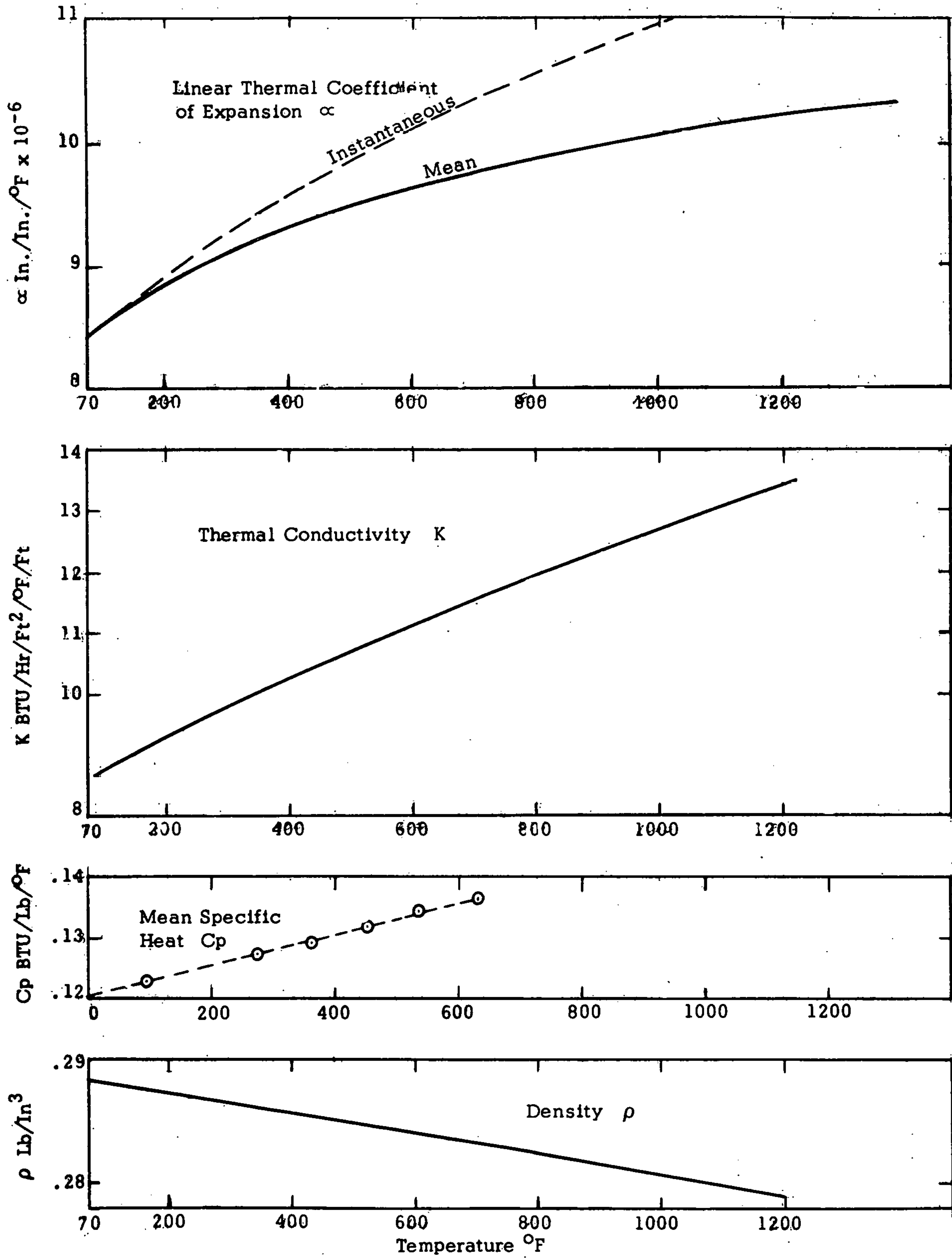

Figure 10. Physical Properties of Type 304 Stainless Steel

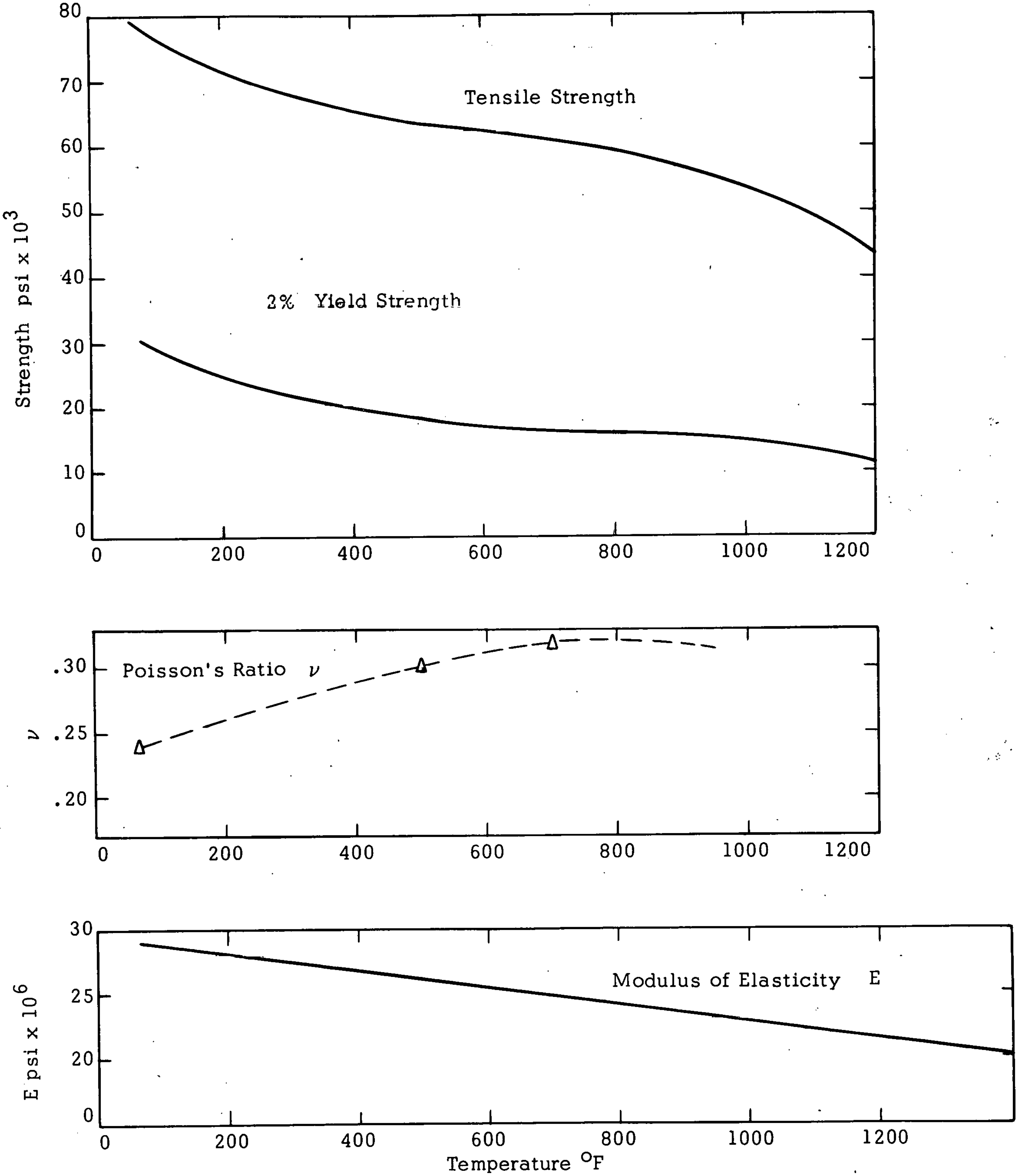

Figure 11. Tensile Properties of Type 304 Stainless Steel

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7.0 REFERENCES

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APPENDIX A - THERMAL ANALYSIS

A.1 STEADY STATE TEMPERATURE DISTRIBUTION ANALYSIS

A.1.1 Method of Analysis

In support of the thermal stress analysis of the PM-2A vessel and cover region, steady-state temperature distributions were calculated using the HOC Code.

The physical geometry of the vessel and cover were reduced to rectangular geometry, each region having uniform properties. Estimates of boundary temperatures and convective heat film transfer coefficients were made. The problem was run in r, z geometry.

For conservatism, the cross section analyzed passed through the center of the cover studs. In r, z geometry, this cross section is rotated to determine region volumes and integrated region temperatures. The heat flow paths between the bolts will reduce the severity of the problem.

A.1.2 Boundary Conditions

The bulk water temperature inside the vessel is at an average temperature of 518°F, and natural convection is employed as the mode of heat transfer to determine the heat transfer film coefficient. The air surrounding the vessel wall up to the top of the vessel flange is assumed to be at 225°F. Boiling water is assumed to cover the vertical outside surface of the cover flange. Steam at 212°F blankets the cover dome insulation jacket and the nuts and studs.

The following table summarizes the boundary conditions for this problem:

Location	Fluid	Fluid Temperature, °F	Heat Transfer Coefficient, Btu/hr-ft^2-°F
Vessel Cover	Steam	212	1
Nut	Steam	212	1
Cover Flange	Boiling Water	212	500
Vessel Wall	Air	225	0.45
Inside of Vessel	Water	518	340

The following table lists the materials and properties used in this analysis:

Part	Material	k, Btu/hr-ft^2-°F
Vessel Cover	A-350 Gr. LF-3	22.7
Cover Cladding	A-304	10.8
Cover Insulation	Fiberglass	0.04
Vessel	A-350 Gr. LF-3	22.5
Vessel Cladding	A-304	10.75
Vessel Insulation	Fiberglass	0.04
Nut	4130	24.9
Bolt	4130	24.8
Gasket	A-304	22.5

A.1.3 Results

Figure 12 shows a sketch of the vessel and cover, the outline of the various regions and the region average temperatures as calculated by the HOC Code.

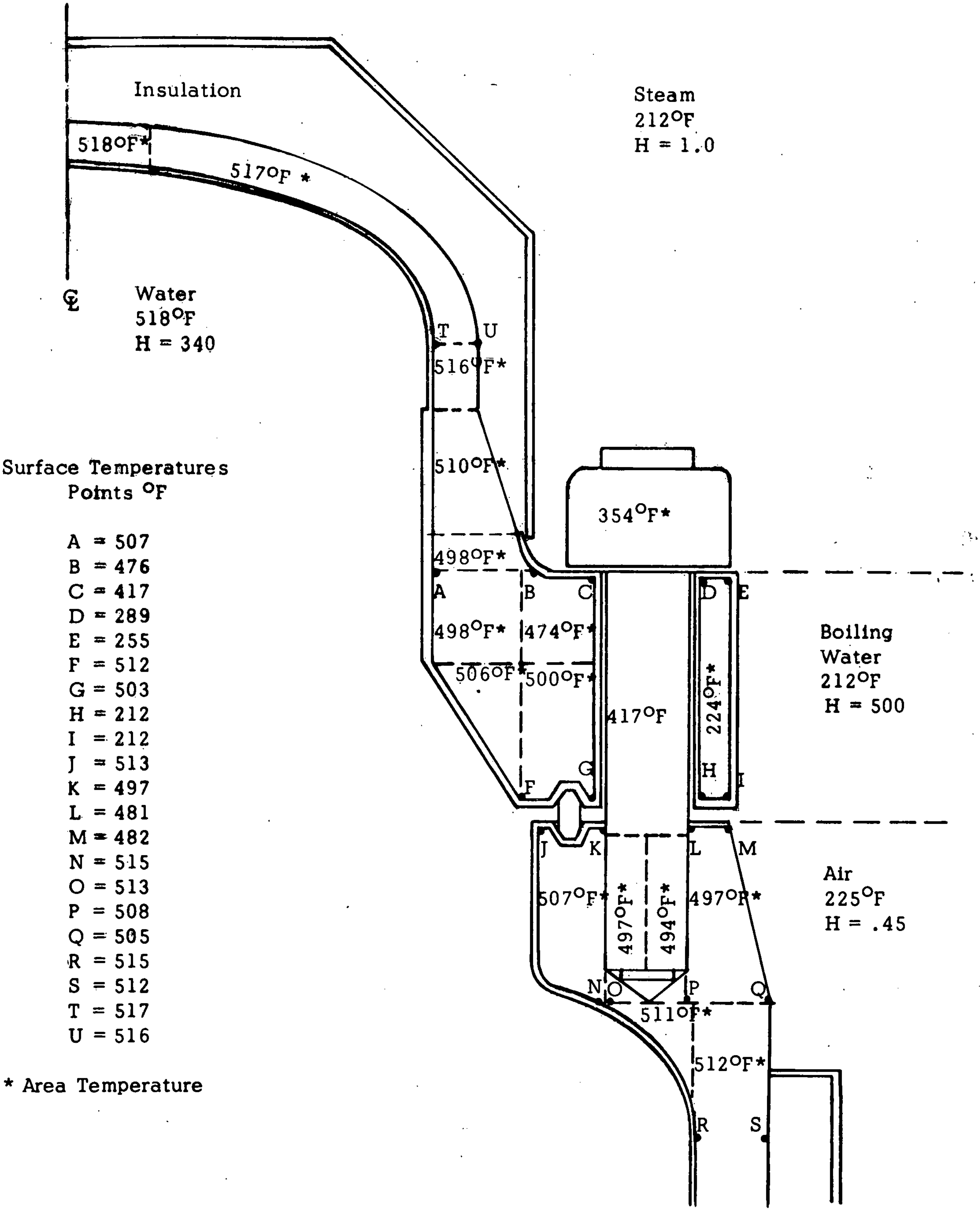

Figure 12. PM-2A Vessel Cover and Flange Steady State Temperature Distribution

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APPENDIX B - STRESS CALCULATIONS

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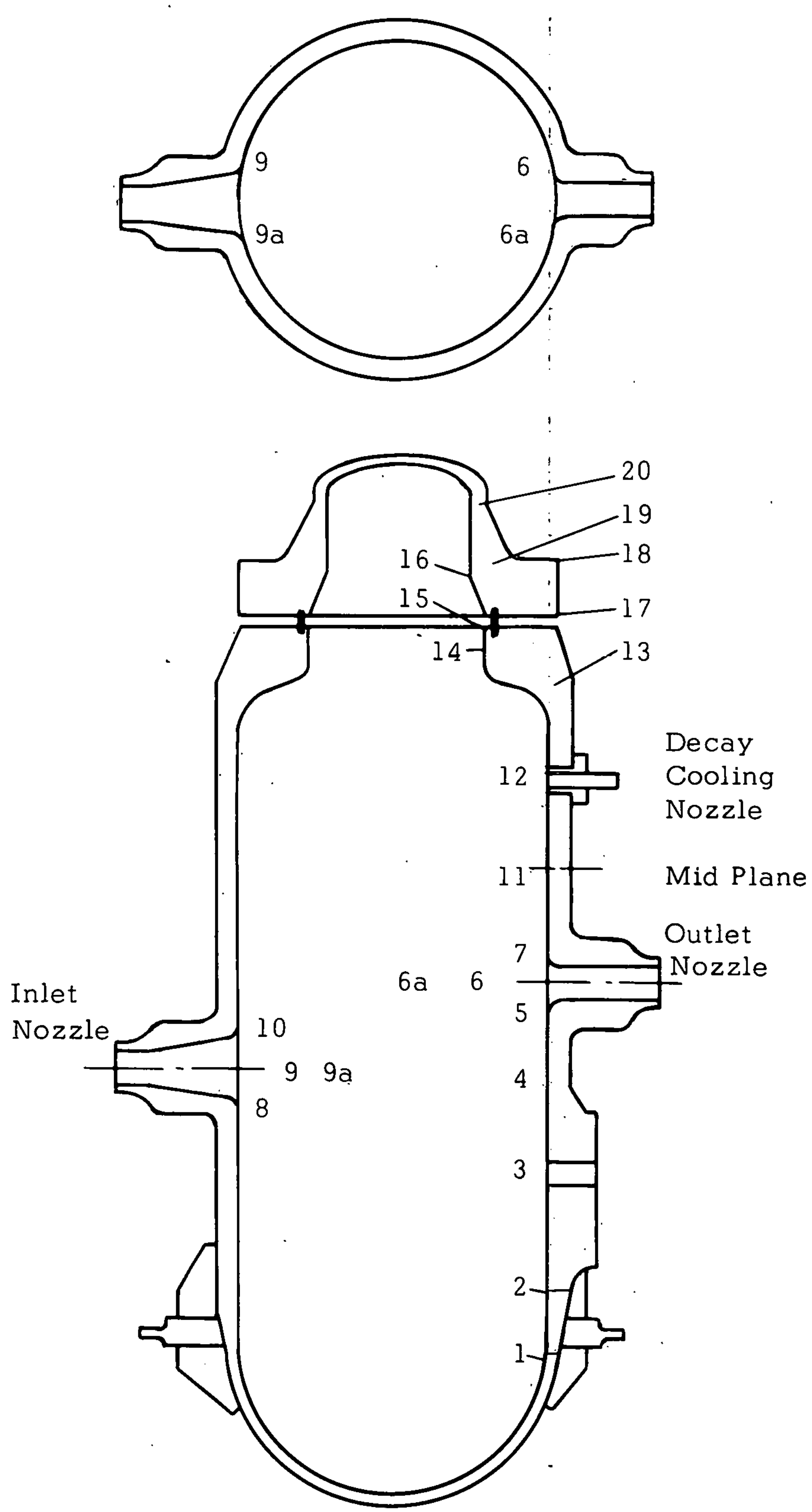
APPENDIX B - STRESS CALCULATIONS

INDEX TO DATA SHEETS



PM-2A REACTOR
VESSEL OUTLINE

TABLE 1 - STRESS SUMMARY

Point	Location		Membrane Stress	Thermal Stresses: γ Heating	Thermal Stresses: Steady State	Ext. Load Stress	Discont. Stress	Clad Stress	Combined Stress (Steady State)
1	Inside	σ_1	14000	-	-	- 1080	- 1175	-	11745
		σ_2	14000	-	-	- 480	1178	-	14698
		σ_3	- 1750	-	-	0	0	-	- 1750
	Outside	σ_1	13100	-	-	1200	1175	-	15475
		σ_2	13100	-	-	893	1882	-	15872
		σ_3	0	-	-	0	0	-	0
2	Inside	σ_1	6720	-	-	1200	- 726	-	7194
		σ_2	15200	-	-	890	- 628	1405	16867
		σ_3	- 1750	-	-	0	0	-	- 1750
	Outside	σ_1	6720	-	-	- 1080	726	-	6366
		σ_2	13450	-	-	- 480	- 192	1405	14183
		σ_3	0	-	-	0	0	-	0
3	Incido	σ_1	- 967	-	-	-			967
		σ_2	20600	-	-	-	-	-	20600
		σ_3	- 967	-	-	-	-	-	- 967
	Outside	σ_1	3870	-	-	-	-	-	3870
		σ_2	6450	-	-	-	-	-	6450
		σ_3	0	-	-	-	-	-	0
4	Inside	σ_1	6720	-	-	-	-	-	6720
		σ_2	15200	-	-	-	-	1405	16605
		σ_3	- 1750	-	-	-	-	-	- 1750
	Outside	σ_1	6720	-	-	-	-	-	6720
		σ_2	13450	-	-	-	-	1405	14855
		σ_3	0	-	-	-	-	-	0
5	Inside	σ_1	- 2220	-	-	- 902	-	-	- 3122
		σ_2	34100	-	-	- 1804	-	-	32296
		σ_3	- 2220	-	-	0	-	-	- 2220
	Outside	σ_1	3710	-	-	1758	-	-	5468
		σ_2	3710	-	-	2216	-	-	5926
		σ_3	0	-	-	0	-	-	0
6	Inside	σ_1	-12600	-	-	641	-	-	13241
		σ_2	- 2220	-	-	989	-	-	- 1231
		σ_3	- 2220	-	-	0	-	-	- 2220
	Outside	σ_1	17100	-	-	- 353	-	-	16747
		σ_2	17100	-	-	- 817	-	-	16283
		σ_3	0	-	-	0	-	-	0
7	Inside	σ_1	- 2220	-	-	1758	-	-	- 462
		σ_2	34100	-	-	2216	-	-	36316
		σ_3	- 2220	-	-	0	-	-	- 2220
	Outside	σ_1	3710	-	-	- 902	-	-	2808
		σ_2	3710	-	-	- 1804	-	-	1906
		σ_3	0	-	-	0	-	-	0
8	Inside	σ_1	- 2220	-	-	261	-	-	- 1959
		σ_2	34100	-	-	224	-	-	34324
		σ_3	- 2220	-	-	0	-	-	- 2220
	Outside	σ_1	3710	-	-	- 159	-	-	3551
		σ_2	3710	-	-	- 44	-	-	3666
		σ_3	0	-	-	0	-	-	0
9	Inside	σ_1	12600	-	-	- 1006	-	-	11594
		σ_2	- 2220	-	-	- 2748	-	-	- 4968
		σ_3	- 2220	-	-	0	-	-	- 2220
	Outside	σ_1	17100	-	-	2306	-	-	19406
		σ_2	17100	-	-	3372	-	-	20472
		σ_3	0	-	-	0	-	-	0
10	Inside	σ_1	- 2220	-	-	- 159	-	-	- 2379
		σ_2	34100	-	-	- 44	-	-	34056
		σ_3	- 2220	-	-	0	-	-	2220
	Outside	σ_1	3710	-	-	261	-	-	3971
		σ_2	3710	-	-	224	-	-	3934
		σ_3	0	-	-	0	-	-	0
11 (Mid Plane)	Inside	σ_1	6720	-	-	-	-	-	6720
		σ_2	15200	6830	-	-	-	1405	23435
		σ_3	- 1750	-	-	-	-	-	- 1750
	Outside	σ_1	6720	-	-	-	-	-	6720
		σ_2	13450	6830	-	-	-	1405	21685
		σ_3	0	-	-	-	-	-	0
12	Inside	σ_1	6720	-	-	-	-	-	6720
		σ_2	38000	-	-	-	-	-	38000
		σ_3	- 1750	-	-	-	-	-	- 1750
	Outside	σ_1	6720	-	-	-	-	-	6720
		σ_2	33600	-	-	-	-	-	33600
		σ_3	0	-	-	-	-	-	0
13	Inside	σ_1	6720	-	- 8100	-	5000*	-	6180
		σ_2	15200	-	- 1000	-	- 5680*	-	2240
		σ_3	- 1750	-	0	-	0*	-	- 1750
	Outside	σ_1	6720	-	8100	-	- 5000*	-	19620
		σ_2	13450	-	3860	-	- 8680*	-	8230
		σ_3	0	-	0	-	0*	-	0
14	Inside	σ_1	0	-	-	-	-	-	0
		σ_2	4380	-	- 3830	-	3170*	-	- 2240
		σ_3	- 1750	-	-	-	-	-	- 1750
	Outside	σ_1	0	-	-	-	-	-	0
		σ_2	2648	-	-	-	-	-	2648
		σ_3	0	-	-	-	-	-	0
15	Inside	σ_1	0	-	-	-	-	-	0
		σ_2	4380	-	- 5700	-	- 643*	-	-12263
		σ_3	- 1750	-	-	-	-	-	- 1750
	Outside	σ_1	0	-	-	-	-	-	0
		σ_2	2648	-	-	-	-	-	2648
		σ_3	0	-	-	-	-	-	0
16	Inside	σ_1	0	-	-	-	-	-	0
		σ_2	3200	-	6070	-	4950*	-	20960
		σ_3	- 1750	-	-	-	-	-	- 1750
17	Outside	σ_1	0	-	-	-	-	-	0
		σ_2	1450	-	12900	-	-	-	5180
		σ_3	0	-	-	-	-	-	0
18	Outside	σ_1	0	-	-	-	-	-	0
		σ_2	1450	-	1930	-	2940*	-	5865
		σ_3	0	-	-	-	-	-	0
19	Inside	σ_1	3800	-	-13290	-	-11350*	-	8750
		σ_2	9360	-	9407	-	- 520*	-	2930
		σ_3	- 1750	-	0	-	0*	-	- 1750
	Outside	σ_1	3800	-	13290	-	11350*	-	- 1150
		σ_2	7610	-	1433	-	6280*	-	1800
		σ_3	0	-	0	-	0*	-	0
20	Inside	σ_1	7375	-	-24400	-	55200*	-	26675
		σ_2	-14750	-	-10485	-	21100*	-	- 7909
		σ_3	- 1750	-	0	-	0	-	- 1750
	Outside	σ_1	7375	-	24400	-	55200*	-	-11925
		σ_2	-14750	-	4155	-	- 2100*	-	- 9560
		σ_3	0	-	0	-	0*	-	0
6a	Inside	σ_1	12600	-	-	- 353	-	-	12247
		σ_2	- 2220	-	-	- 817	-	-	- 3037
		σ_3	- 2220	-	-	0	-	-	- 2220
	Outside	σ_1	17100	-	-	641	-	-	17741
		σ_2	17100	-	-	989	-	-	18089
		σ_3	0	-	-	0	-	-	0
9a	Inside	σ_1	12600	-	-	2306	-	-	+14906
		σ_2	- 2220	-	-	3372	-	-	11502
		σ_3	- 2220	-	-	0	-	-	- 2220
	Outside	σ_1	17100	-	-	-1006	-	-	16094
		σ_2	17100	-	-	-2748	-	-	14352
		σ_3	0	-	-	0	-	-	0

* Discontinuity stresses for cover and flange case B - bolt load plus pressure.

σ_1 Axial Stress σ_2 Circumferential Stress σ_3 Radial Stress

ALCO PRODUCTS INC.

BY B/R DATE 10/16/60 SUBJECT PM-2A STRESS ANALYSIS AE-90 TASK 6.9 SHEET NO. 1 OF 4
CHKD. BY [illegible] DATE
JOB NO. CONTR. No AT(30-1)-2639

MEMBRANE STRESSES – PRINCIPAL STRESS COMPONENTS
(REF. NAVY CODE SECT A.3.3)

① VESSEL SHELL $t = 2.375$

INSIDE

a) $\sigma_r = -P = \underline{-1750}$

b) $\sigma_R = P\left[\dfrac{Y^2+1}{Y^2-1}\right]$

OUTSIDE

$\sigma_r = \underline{0}$

$\sigma_R = \dfrac{2P}{Y^2-1}$

where Y = radius ratio = $\dfrac{R_i + t}{R_i} = \dfrac{19 + 2.375}{19} = 1.12$

$Y^2 = 1.26$

INSIDE

$\sigma_R = 1750\left[\dfrac{1.26+1}{1.26-1}\right] =$

✓ $\sigma_R = \underline{15,200 \text{ psi.}}$

c) $\sigma_a = \dfrac{P}{Y^2-1}$

$\sigma_a = \dfrac{1750}{1.26-1}$

✓ $\sigma_a = \underline{6720 \text{ psi.}}$

OUTSIDE

$\sigma_R = \dfrac{2(1750)}{1.26-1}$

✓ $\sigma_R = \underline{13,450 \text{ psi}}$

$\sigma_a = \dfrac{P}{Y^2-1}$

$\sigma_a = \dfrac{1750}{1.26-1}$

✓ $\sigma_a = \underline{6720 \text{ psi.}}$

σ_r corresponds to σ_3 on stress summary sheets
σ_R " " " σ_2 " " "
σ_a " " " σ_1 " " "

MEMBRANE STRESSES — PRINCIPAL STRESS COMPONENTS

(REF. NAVY CODE SECT A.3.3)

① VESSEL SHELL $t = 2.375$

INSIDE

a) $\sigma_r = -P = \underline{-1750}$

b) $\sigma_h = P\left[\dfrac{Y^2+1}{Y^2-1}\right]$

OUTSIDE

$\sigma_r = \underline{0}$

$\sigma_h = \dfrac{2P}{Y^2-1}$

where Y = radius ratio $= \dfrac{R_i + t}{R_i} = \dfrac{19 + 2.375}{19} = 1.12$

$Y^2 = 1.26$

INSIDE:

$\sigma_h = 1750\left[\dfrac{1.26+1}{1.26-1}\right] =$

✓ $\sigma_h = \underline{15,200 \text{ psi}}$

OUTSIDE:

$\sigma_h = \dfrac{2(1750)}{1.26-1}$

✓ $\sigma_h = \underline{13,450 \text{ psi}}$

c) INSIDE:

$\sigma_a = \dfrac{P}{Y^2-1}$

$\sigma_a = \dfrac{1750}{1.26-1}$

✓ $\sigma_a = \underline{6720 \text{ psi}}$

OUTSIDE:

$\sigma_a = \dfrac{P}{Y^2-1}$

$\sigma_a = \dfrac{1750}{1.26-1}$

✓ $\sigma_a = \underline{6720 \text{ psi}}$

σ_r corresponds to σ_3 on stress summary sheets
σ_h " " " σ_2 " " "
σ_a " " " σ_1 " " "

2. BOTTOM HEMISPHERICAL HEAD - PRINCIPAL STRESS COMPONENTS —

REF: NAVY CODE, SECT A.3.3 — SPHERE

INSIDE

$\sigma_3 = \sigma_r = -P$

$\sigma_a = \sigma_R = \frac{P}{2}\left[\frac{Y^3+2}{Y^3-1}\right]$

OUTSIDE

$\sigma_r = 0$

$\sigma_a = \sigma_R = \frac{3P}{2}\left[\frac{1}{Y^3-1}\right]$

$$Y = \frac{R_i + t}{R_i} = \frac{19.6875 + 1.25}{19.6875} = 1.062$$

$$Y^3 = 1.20$$

INSIDE

$\sigma_3 = \sigma_r = -1750$

$\sigma_a = \sigma_R = \frac{1750}{2}\left[\frac{1.20+2}{1.20-1}\right]$

$\sigma_a = \sigma_R = 14000 \text{ psi}$

OUTSIDE

$\sigma_r = 0$

$\sigma_a = \sigma_R = \frac{3(1750)}{2}\left[\frac{1}{1.20-1}\right]$

$\sigma_a = \sigma_h = 13100 \text{ psi}$

3. STRESS INDICES IN NOZZLES — numerical ratio of stresses in question to the calculated membrane stress intensity in the shell.

a) Per NAVY Code Fig. C.3-2 Pg. 144 stress indices are as follows —

	LONGITUDINAL SECTION		TRANSVERSE SECTION	
	INSIDE CORNER	OUTSIDE CORNER	INSIDE CORNER	OUTSIDE CORNER
σ_n	2.30	0.25	.85	1.15
σ_t	−0.15	0.25	−0.15	1.15
σ_r	−0.15	0	−0.15	0

b) MEMBRANE STRESS INTENSITY IN SHELL $S_M = \frac{P(2R_i + t)}{2t}$ NAVY CODE — SECT A.3.1(2) Pg. 52

$$S_M = \frac{1750(2(19) + 2.375)}{2(2.375)}$$

$R_i = 19''$
$t = 2.375''$

$$S_M = \frac{1750(40.375)}{4.750}$$

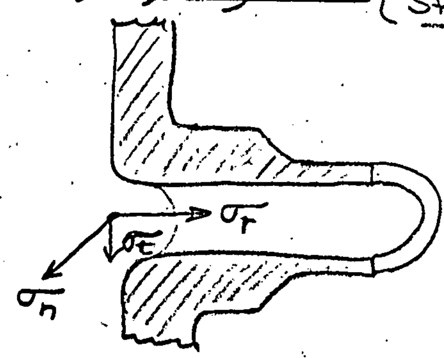
$$S_M = 14850 \text{ P.S.I.}$$

C. STRESSES IN NOZZLES

$$S_{NOZZLE} = \text{STRESS INDEX} \times S_M$$

LONGITUDINAL SECTION

PTS. 5, 7, 8, & 10 { STRESS SUMMARY SHEETS



By DEFINITION from NAVY CODE

σ_t is axial stress σ_1

σ_n is circumf. Stress σ_2

σ_r is radial STRESS σ_3

INSIDE CORNER

$\sigma_n = \sigma_2 = 2.30(14850) = \underline{\underline{34100 \text{ psi}}}$

$\sigma_t = \sigma_1 = -0.15(14850) = \underline{\underline{-2220 \text{ psi}}}$

$\sigma_r = \sigma_3 = -0.15(14850) = \underline{\underline{-2220 \text{ psi}}}$

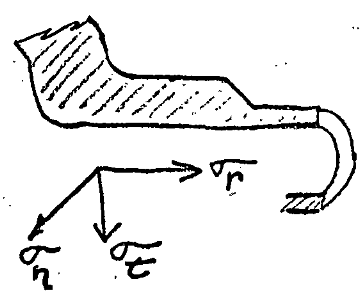
OUTSIDE CORNER

$\sigma_2 = .25(14850) = \underline{\underline{3710 \text{ psi}}}$

$\sigma_1 = .25(14850) = \underline{\underline{3710 \text{ psi}}}$

$\sigma_3 = 0(14850) = \underline{\underline{0}}$

TRANSVERSE SECTION – PTS. 6 & 9



INSIDE CORNER

$\sigma_n = \sigma_1 = .85(14850) = 12600 \text{ psi}$

$\sigma_t = \sigma_2 = -.15(14850) = -2220 \text{ psi}$

$\sigma_r = \sigma_3 = -.15(14850) = -2220 \text{ psi}$

OUTSIDE CORNER

$\sigma_1 = 1.15(14850) = 17100 \text{ psi}$

$\sigma_2 = 1.15(14850) = 17100 \text{ psi}$

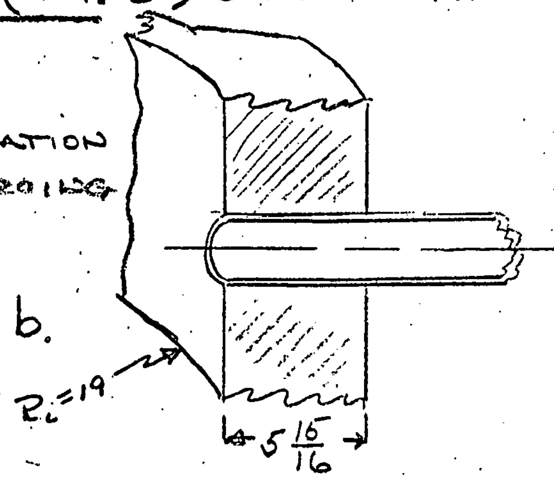
$\sigma_3 = 0(14850) = 0 \text{ psi}$

ALCO PRODUCTS INC.

BY: B.C.R. DATE: 12/13/61 SUBJECT: PM-2A STRESS ANALYSIS SHEET NO. 1 OF 2
CHKD. BY ______ DATE ______ JOB NO. ______

MEMBRANE STRESSES AT CONTROL ROD DRIVE PENETRATIONS

REINFORCING PAD (PT. 3) STRESS CHART



COMPENSATION FOR THE PENETRATION IS PROVIDED BY THE REINFORCING PAD.

REF. NAVY CODE FIG C.3-6b. AND PARA. C.3.4.

THE STRESS INDICES ARE

	LONGIT. SECTION		TRANSVERSE SECTION	
	INSIDE	OUTSIDE	INSIDE	OUTSIDE
σ_n	3.2	1.0	.85	1.5
σ_t	-.15	.6	-.15	1.5
σ_r	-.15	0	-.15	0

THE STRESS INDICES ARE NUMERICAL RATIO OF STRESSES IN QUESTION TO THE CALCULATED MEMBRANE STRESS INTENSITY IN THE SHELL (S_m). IN THIS CASE S_M IS CALCULATED, USING THE THICKNESS OF THE REINFORCING PAD.

$$S_M = \frac{P(2R_i + t)}{2t} = \frac{1750(2\times 19 + 5.95)}{2\times 5.95}$$

$S_M = 6450$ PSI

∴ THE STRESSES AT THE REINFORCING PAD

σ = STRESS INDEX × S_M

FOR THE LONGITUDINAL SECTION —

INSIDE	OUTSIDE
$\sigma_h = \sigma_2 = 3.2(6450) = 20600$ psi	$\sigma_2 = 1.0(6450) = 6450$ psi
$\sigma_t = \sigma_1 = -.15(6450) = -967$ psi	$\sigma_1 = .6(6450) = 3870$ psi
$\sigma_r = \sigma_3 = -.15(6450) = -967$ psi	$\sigma_3 = 0(6450) = 0$ psi

FOR TRANSVERSE SECTION —

INSIDE	OUTSIDE
$\sigma_h = \sigma_1 = .85(6450) = 5480$ psi	$\sigma_1 = 1.5(6450) = 9670$ psi
$\sigma_t = \sigma_2 = -.15(6450) = -967$ psi	$\sigma_2 = 1.5(6450) = 9670$ psi
$\sigma_r = \sigma_3 = -.15(6450) = -967$ psi	$\sigma_3 = 0 = 0$

ALCO PRODUCTS INC.

BY E.R. DATE 12/12/61 SUBJECT PM-2A STRESS ANALYSIS AE-90 TASK 6.9 SHEET NO. 1 OF 1

CHKD. BY ____ DATE ____ JOB NO. AT(30-1)-2639

MEMBRANE STRESS IN VESSEL WALL AT DECAY HEAT COOLING NOZZLE (PT 12)

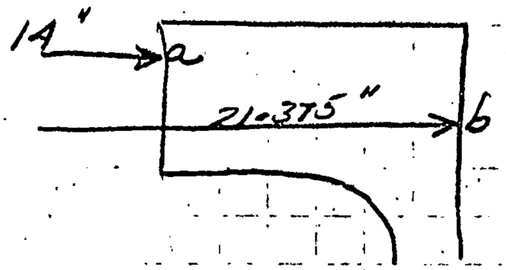
MEMBRANE STRESSES IN VESSEL WALL —

INSIDE	OUTSIDE
$\sigma_1 = 6720$ psi	$\sigma_1 = 6720$
$\sigma_2 = 15200$ psi	$\sigma_2 = 13450$
$\sigma_3 = -1750$ psi	$\sigma_3 = 0$

THE STRESS CONCENTRATION FACTOR FOR A SMALL HOLE IN A PLATE SUBJECT TO A BIAXIAL TENSION WHICH IS TWICE AS LARGE IN ONE DIRECTION AS THE OTHER (SUCH AS A PRESSURIZED CYLINDER) IS 2.5, IN THE DIRECTION OF THE LARGER FORCE, & 1.0 IN THE DIRECTION OF THE SMALLER ONE. IGNORING THE EXTERNAL REINFORCEMENT, THE STRESSES AT THE DECAY COOLING NOZZLE ARE

INSIDE	OUTSIDE
$\sigma_1 = 1.0(6720) = 6{,}720$ psi	$\sigma_1 = 1.0(6720) = 6{,}720$ psi
$\sigma_2 = 2.5(15200) = 38{,}000$ psi	$\sigma_2 = 2.5(13450) = 33{,}600$ psi
$\sigma_3 = -1750 = -1750$ psi	$\sigma_3 = 0 = 0$ psi

PRESSURE STRESS AT PTS 14 & 15 —



REF: FORMULAS FOR STRESS & STRAIN, ROARK

INSIDE

$\sigma_1 = 0 \text{ psi}$

$\sigma_2 = p\left(\dfrac{b^2+a^2}{b^2-a^2}\right)$

$\sigma_2 = 1750\,\dfrac{(21.375^2+14^2)}{21.375^2-14^2}$

$\sigma_2 = 4380 \text{ psi}$

$\sigma_3 = -1750 \text{ psi}$

OUTSIDE

$\sigma_1 = 0 \text{ psi}$

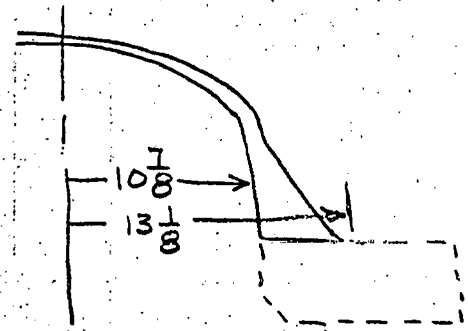
$\sigma_2 = p\,\dfrac{a^2}{b^2}\left(\dfrac{2b^2}{(b^2-a^2)}\right)$

$= 1750\left(\dfrac{196}{456}\right)\left(\dfrac{912}{260}\right)$

$\sigma_2 = 2648 \text{ psi}$

$\sigma_3 = 0 \text{ psi}$

PRESSURE STRESS AT BOTTOM of VESSEL COVER
TRANSITION PIECE
PT (19) of STRESS SUMMARY CHARTS
REF: NAVY CODE SECT A 3.3.



INSIDE SURFACE

$\sigma_3 = -P = \underline{-1750\text{ psi}}$

$\sigma_2 = P\left[\frac{Y^2+1}{Y^2-1}\right]$ where $Y^2 = \left[\frac{R_i+t}{R_i}\right]^2 = \left[\frac{10.875+2.250}{10.875}\right]^2$

$Y^2 = 1.46$

$\sigma_2 = 1750\left[\frac{1.46+1}{1.46-1}\right]$

$\sigma_2 = \underline{9360\text{ psi}}$

$\sigma_1 = \frac{P}{Y^2-1} = \frac{1750}{1.46-1} = \underline{3800\text{ psi}}$

OUTSIDE SURFACE

$\sigma_3 = 0$

$\sigma_2 = \frac{2P}{Y^2-1} = \frac{2(1750)}{1.46-1} = \underline{7610\text{ psi}}$

$\sigma_1 = \frac{P}{Y^2-1} = \underline{3800\text{ psi}}$

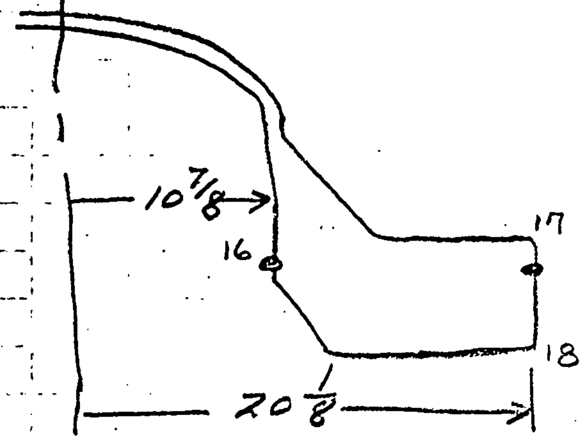
ALCO PRODUCTS INC.

BY ____ DATE ____ SUBJECT ____ SHEET NO. 3 OF 4

CHKD. BY ____ DATE ____ JOB NO. ____

PRESSURE STRESS AT VESSEL COVER FLANGE
(PT 16, 17 & 18)

REF: FORMULAS FOR STRESS & STRAIN, ROARK



INNER (PT 16)

$\sigma_1 = \underline{0 \text{ PSI}}$

$$\sigma_2 = P \frac{b^2 + a^2}{b^2 - a^2}$$

$$= 1750 \frac{(20.125^2 + 10.875^2)}{20.125^2 - 10.875^2}$$

$\sigma_2 = \underline{3200 \text{ PSI}}$

$\sigma_3 = -1750 \text{ PSI}$

OUTER (PTS 17 & 18)

$\sigma_1 = \underline{0 \text{ PSI}}$

$$\sigma_2 = P \frac{a^2}{b^2} \left(\frac{2b^2}{b^2 - a^2} \right)$$

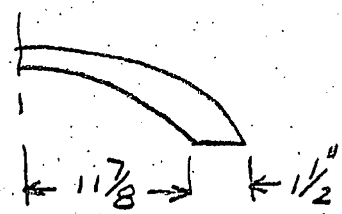
$$= 1750 \frac{(118)}{(403)} \frac{(806)}{(285)}$$

$\sigma_2 = \underline{1450 \text{ PSI}}$

$\sigma_3 = 0$

PRESSURE STRESS AT POINT 20 —

USING ELLIPSOIDAL HEAD FORMULA —



$$\sigma_R = \sigma_3 = -P = \underline{\underline{-1750 \text{ psi}}}$$

$$\sigma_h = \sigma_2 = -\frac{P}{t}\left(R_i + \frac{t}{2}\right)$$

$$= -\frac{1750}{1.5}(11.875 + .750) = \underline{\underline{-14750 \text{ psi}}}$$

$$\sigma_a = \sigma_1 = \frac{P}{2t}\left(R_i + \frac{t}{2}\right)$$

$$= \frac{1750}{2(1.5)}(11.875 + .750) = \underline{\underline{7375 \text{ psi}}}$$

ALCO PRODUCTS INC.

BY B.L.R. DATE 6/19/61 SUBJECT PM-2A STRESS ANALYSIS AE 90 TASK 6.9 SHEET NO. 1 OF 1
CHKD. BY ______ DATE ______ JOB NO. CONTR. NO AT(30-1) 2639

PRESSURE STRESS ~ (AT NOZZLE TAPER)

Ref. NAVY CODE SECT. A-3

The pressure STRESSES will be the same for both NOZZLES SINCE R_i & t are the same.

OUTSIDE SURFACE

a) $\sigma_r = -P = -1750$ psi

b) $\sigma_h = P\left[\frac{Y^2+1}{Y^2-1}\right]$

INSIDE SURFACE

a) $\sigma_r = 0$ psi

b) $\sigma_h = \frac{2P}{Y^2-1}$

where $Y^2 = \left(\frac{R_i+t}{R_i}\right)^2$

$= \left(\frac{4.5+.75}{4.5}\right)^2$

$Y^2 = 1.36$

$\sigma_h = 1750\left[\frac{1.36+1}{1.36-1}\right]$

$\sigma_h = 11480$ psi

$\sigma_h = \frac{2(1750)}{1.36-1}$

$\sigma_h = 9730$ psi

c) $\sigma_a = \frac{P}{Y^2-1}$

$= \frac{1750}{.36}$

$\sigma_a = 4860$ psi

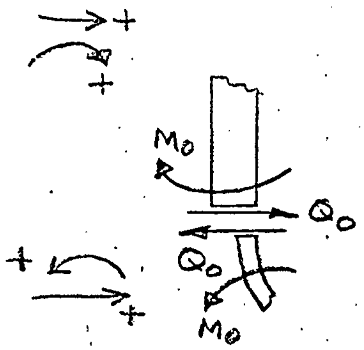
c) $\sigma_a = 4860$ psi

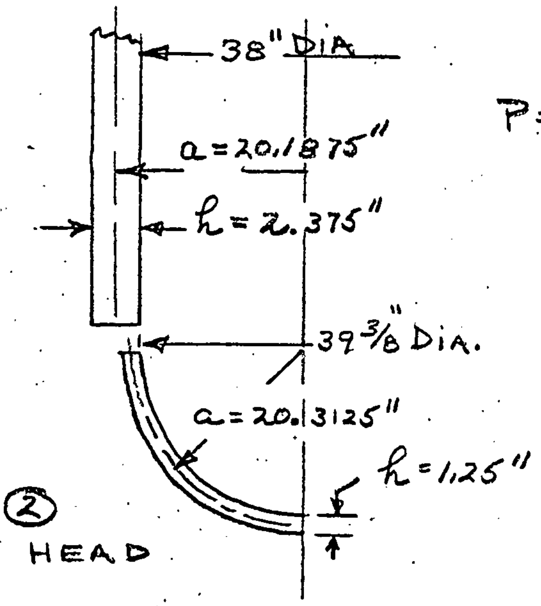
ALCO PRODUCTS INC.

BY BIR DATE 1/16/61 SUBJECT PM-2A STRESS ANALYSIS AE-90 TASK 6.9 SHEET NO. 1 OF 11

CHKD. BY [illegible] DATE 1/25/61 JOB NO. CONTR. NO. AT(30-1) 2639

DISCONTINUITY STRESSES AT BOTTOM SPHERICAL HEAD





P = 1750 PSI

$\delta_1 = +$ INWARD , $\omega_1 = +$ INWARD , $\theta_1 = +$ C.W.

$\delta_2 = +$ INWARD , $\omega_2 = +$ INWARD , $\theta_2 = +$ C.C.W.

① MEMBRANE DEFLECTION δ

SHELL

$$\delta_1 = -\frac{Pa_1^2}{Eh_1}\left(1 - \frac{\nu}{2}\right)$$

HEAD

$$\delta_2 = -\frac{Pa_2^2}{2Eh_2}(1-\nu)$$

② DISCONTINUITY DEFLECTION — ω

a) SHELL

$$\omega_1 = -\frac{1}{2\beta_1^3 D}\left[\beta_1 M_0 - Q_0\right]$$

b) HEAD

$$\omega_2 = -\frac{1}{2\beta_2^3 D_2}\left[\beta_2 M_0 + Q_0\right]$$

③ EDGE ROTATION —

a) SHELL

$$\theta_1 = \frac{dw_1}{dx} = \frac{1}{2\beta_1^2 D_1}\left[2\beta_1 M_0 - Q_0\right]$$

b) HEAD

$$\theta_2 = \frac{1}{2\beta_2^2 D_2}\left[2\beta_2 M_0 + Q_0\right]$$

④ BOUNDARY CONDITIONS —

$$w_1 + \delta_1 = w_2 + \delta_2 \qquad \theta_1 = -\theta_2$$

① MEMBRANE DEFLECTION

SHELL

$$\delta_1 = -\frac{Pa^2}{Eh}\left(1 - \frac{\nu}{2}\right)$$

$$\delta_1 = -\frac{1750(20,1875)^2}{(30\times10^6)(2.375)}\left(1 - \frac{.3}{2}\right)$$

$$\delta_1 = -\frac{1750(406)(.85)}{30\times10^6(2.375)}$$

✓ $\delta_1 = -.00848''$

HEAD

$$\delta_2 = -\frac{1750(20,3125)^2}{2(30\times10^6)(1.25)}(1-.3)$$

$$\delta_2 = -\frac{1750(412)(.7)}{2(30\times10^6)(1.25)}$$

✓ $\delta_2 = -.00672$

ALCO PRODUCTS INC.

BY B.J.R. DATE 1/16/61 SUBJECT PM-2A STRESS ANALYSIS SHEET NO. 3 OF 11

CHKD. BY ______ DATE ______ JOB NO. ______

(2) DISCONTINUITY DEFLECTION -

a) SHELL

$$\omega_1 = -\frac{1}{2\beta_1^3 D_1}\left[\beta_1 M_0 - Q_0\right]$$

$$\beta_1 = \frac{1.285}{\sqrt{a\,h}} = \frac{1.285}{\sqrt{20.1875 \times 2.375}}$$

$$\beta_1 = \frac{1.285}{6.9} = .181$$

$$D_1 = \frac{E h^3}{12(1-\nu^2)} = \frac{30\times10^6 (2.375)^3}{12(1-.09)}$$

$$D_1 = 36.8 \times 10^6$$

$$\omega_1 = -\frac{1}{2(.181)^3(36.8\times10^6)}\left[.181\, M_0 - Q_0\right]$$

$$\omega_1 = -\frac{.181 M_0}{2(.00593)(36.8\times10^6)} + \frac{Q_0}{2(.00593)(36.8\times10^6)}$$

$$\omega_1 = -.415\times10^{-6} M_0 + 2.29\times10^{-6} Q_0$$

$$\omega_1 \times 10^6 = -.415\, M_0 + 2.29\, Q_0$$

b) HEAD

$$\omega_2 = -\frac{1}{2\beta_2^3 D_2}\left[\beta_2 M_0 + Q_0\right]$$

$$\beta_2 = \frac{1.285}{\sqrt{a_2 h_2}} = \frac{1.285}{\sqrt{20.3125 \times 1.25}}$$

$$\beta_2 = \frac{1.285}{5.04} = .255$$

$$\beta_2^3 = .0166$$

$$D_2 = \frac{E h_2^3}{12(1-\nu^2)} = \frac{(30\times10^6)(1.25)^3}{12(1-.09)}$$

1.95 .91

$$D_2 = 5.36 \times 10^6$$

$$\omega_2 = -\frac{1}{2(.0166)(5.36\times10^6)}\left[.255 M_0 + Q_0\right]$$

$$\omega_2 = -\frac{.255\ M_0}{2(.0166)(5.36\times10^6)} - \frac{Q_0}{2(.0166)(5.36\times10^6)}$$

$$\omega_2 = -1.43\times10^{-6} M_0 - 5.6\times10^{-6} Q_0$$

$$\omega_2 \times 10^6 = -1.43\ M_0 - 5.6\ Q_0$$

(3) EDGE ROTATION —

a) SHELL

$\beta_1^2 = (.181)^2 = .0328$

$$\theta_1 = \frac{1}{2\beta_1^2 D_1}\left[2\beta_1 M_o - Q_o\right]$$

$D_1 = 36.8 \times 10^6$

$$\theta_1 = \frac{M_o}{\beta_1 D_1} - \frac{Q_o}{2\beta_1^2 D_1}$$

$$\theta_1 = \frac{M_o}{(.181)(36.8\times10^6)} - \frac{Q_o}{2(.0328)(36.8\times10^6)}$$

$$\theta_1 = .151 \times 10^{-6} M_o - .412 \times 10^{-6} Q_o$$

$$\theta_1 \times 10^6 = .151 M_o - .412 Q_o$$

b) HEAD

$$\theta_2 = \frac{1}{2\beta_2^2 D_2}\left[2\beta_2 M_o + Q_o\right]$$

$\beta_2^2 = (.255)^2 = .065$

$D_2 = 5.36 \times 10^6$

$$\theta_2 = \frac{M_o}{\beta_2 D_2} + \frac{Q_o}{2\beta_2^2 D_2}$$

$$\theta_2 = \frac{M_o}{.255(5.36\times10^6)} + \frac{Q_o}{2(.065)(5.36\times10^6)}$$

$$\theta_2 = .73 \times 10^{-6} M_o + 1.435 \times 10^{-6} Q_o$$

$$\theta_2 = .73 M_o + 1.435 Q_o$$

④ BOUNDARY CONDITION –

a) $\omega_1 + \delta_1 = \omega_2 + \delta_2$

$$-.415\,M_0 + 2.29\,Q_0 - 8480 = -1.43\,M_0 - 5.6\,Q_0 - 6720$$

✓ $1.015\,M_0 + 7.89\,Q_0 = 1760$

b) $\theta_1 = -\theta_2$

$$.151\,M_0 - .412\,Q_0 = -.73\,M_0 - 1.435\,Q_0$$

$$+1.023\,Q_0 = -.881\,M_0$$

$$Q_0 = -.86\,M_0$$

Substituting in a)

$$1.015\,M_0 + 7.89(-.86\,M_0) = 1760$$
$$1.015\,M_0 - 6.78\,M_0 = 1760$$
$$-5.76\,M_0 = 1760$$

$M_0 = -305$ in #/in

$Q_0 = -.86(-305) = 263$ #/in

✓ $M_0 = -305$ in #/in

✓ $Q_0 = 263$ #/in

⑤ DEFLECTION

SHELL

$$\omega_1 \times 10^6 = -.415\, M_0 + 2.29\, Q_0$$
$$= -.415(-305) + 2.29(263)$$
$$= +126.8 + 603$$
$$= 729.8$$
$$\omega_1 = .0007298''$$

HEAD

$$\omega_2 \times 10^6 = -1.43\, M_0 - 5.6\, Q_0$$
$$= -1.43(-305) - 5.6(263)$$
$$= 436 - 1475$$
$$= -1039$$
$$\omega_2 = -.001039''$$

⑥ SLOPE

SHELL

$$\theta_1 \times 10^6 = .151\, M_0 - .412\, Q_0$$
$$= .151(-305) - .412(263)$$
$$= -46 - 108.3$$
$$= -154.3$$
$$\theta_1 = -.000154$$

HEAD

$$\theta_2 \times 10^6 = .73\, M_0 + 1.435\, Q_0$$
$$= .73(-305) + 1.435(263)$$
$$= -223 + 378$$
$$= 155$$
$$\theta_2 = .000155$$

Solutions check $\theta_1 = -\theta_2$

$$\omega_1 + \delta_1 = \omega_2 + \delta_2$$

$-.00848$	$-.001039$
$+.00073$	$-.00672$
$-.00775 =$	$-.007759$

(7) DETERMINATION M_x & Q_x FOR SHELL

$$M_x = M_0 \varphi(\beta x) + \frac{Q_0 \zeta(\beta x)}{\beta} \qquad \beta = .181$$

Q_0 is opposite in direction to M_0 and will have negative sign

$$M_x = -305 \varphi(\beta x) - \frac{263 \zeta(\beta x)}{.181}$$

$$M_x = -305 \varphi(\beta x) - 1450 \zeta(\beta x)$$

$$Q_x = -\left[2 \beta M_0 \zeta(\beta x) - (-Q_0) \psi(\beta x)\right]$$

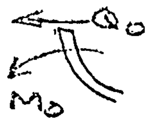
$$= -2(.181)(-305) \zeta(\beta x) - 263 \psi(\beta x)$$

$$Q_x = 110 \zeta(\beta x) - 263 \psi(\beta x)$$

Substitute VALUES FROM TABLE 45 Pg 394 "THEORY of PLATES & SHELLS" INTO EXPRESSIONS for Q_x and M_x for BOTH SHELL & SPHERICAL HEAD.

(17) DETERMINATION M_x & Q_x

a) <u>HEAD</u> $\beta = .255$



$$M_x = -D\frac{d^2w}{dx^2} = -D \times \frac{-1}{2\beta D}\left[2\beta M_0\varphi(\beta x) + 2Q_0\zeta(\beta x)\right]$$

$$M_x = M_0\,\varphi(\beta x) + Q_0\frac{\zeta(\beta x)}{\beta}$$

$$M_x = -305\,\varphi(\beta x) + \frac{263\,\zeta(\beta x)}{.255}$$

$$M_x = -305\,\varphi(\beta x) + 1030\,\zeta(\beta x)$$

$$Q_x = -D\frac{d^3w}{dx^3} = -D \times \frac{1}{D}\left[2\beta M_0\,\zeta(\beta x) - Q_0\,\psi(\beta x)\right]$$

$$Q_x = -\left[2\beta M_0\,\zeta(\beta x) - Q_0\,\psi(\beta x)\right]$$

$$Q_x = -2(.255)(-305)\,\zeta(\beta x) + 263\,\psi(\beta x)$$

$$Q_x = 155\,\zeta(\beta x) + 263\,\psi(\beta x)$$

22

				HEAD		SHELL	
βx	$\varphi(\beta x)$	$\psi(\beta x)$	$\zeta(\beta x)$	M_x	Q_x	M_x	Q_x
0	1.0	1.0	0	−305	* 263	−305	−263
.3	.9267	.488	.2189	−57	162.5	−601	−104.3
.6	.7628	.143	.3099	88	85.6	−682 *	−3.6
.9	.5712	−.0657	.3185	155	32.1	−637	+52.3
1.2	.3899	−.1716	.2807	171	−1.6	−535	+76
1.5	.2384	−.2086	.2226	156.4	−20.5	−395	+79.5
1.8	.1234	−.1985	.1610	128.4	−27	−270	69.9

SPHERICAL HEAD —

Max. mom. $M_x = -305$ in#/in
$Q_x = 263$ #/in
occurs at edge where $w = -1039 \times 10^{-6}$

CYLINDRICAL SHELL —

Max mom. $M_x = -682$ in#/in
$Q_x = -3.6$ #/in

Max Moment occurs at $\beta x = .6$

$$x = \frac{.6}{.181} = 3.32''$$

Deflection at M_x

$$w_x \times 10^6 = -.415\, M_x + 2.29\, Q_x$$
$$= -.415(-682) + 2.29(-3.6)$$
$$= +283 - 8.28$$
$$= 274.7$$

$w_x = .0002747$ in.

STRESS AT MAX BENDING MOMENT

CYLINDRICAL SHELL

a) AXIAL

$$\sigma_1 = \frac{ap}{2h} \pm \frac{6M_x}{h^2}$$

$$= \frac{(20.1875)(1750)}{2(2.375)} \pm \frac{6(-682)}{(2.375)^2}$$

$$= 7430 \pm 726 \quad \begin{matrix} + \text{ OUTSIDE} \\ - \text{ INSIDE} \end{matrix}$$

$$\sigma_1 \begin{cases} 8156 \text{ psi} & \text{OUTSIDE} \\ 6704 \text{ psi} & \text{INSIDE} \end{cases}$$

b) CIRCUMFERENTIAL

$$\sigma_2 = \frac{ap}{h} - \frac{Ew_x}{a} \pm \frac{6\nu M_x}{h^2}$$

$$= 14860 - \frac{(30\times10^6)(274.7\times10^{-6})}{20.1875} \pm \frac{6(.3)(-682)}{(2.375)^2}$$

$$= 14860 - 410 \pm 218$$

$$= 14860 \begin{matrix} -192 \text{ psi} & \text{OUTSIDE} \\ -628 \text{ psi} & \text{INSIDE} \end{matrix}$$

$$\sigma_2 = \begin{cases} 14668 \text{ psi} & \text{OUTSIDE} \\ 14232 \text{ psi} & \text{INSIDE} \end{cases}$$

STRESS AT MAX BENDING MOMENT

SPHERICAL HEAD

occurs at edge
$M_x = -305$
$\omega = .001039$

a) Axial

$$\sigma_1 = \frac{ap}{2h} \pm \frac{6M_x}{h^2}$$

$$= \frac{20.3125(1750)}{2(1.25)} \pm \frac{6(-305)}{(1.25)^2}$$

$$= 14200 \left[\pm 1175 \begin{matrix} \text{OUTSIDE Surface} \\ \text{INSIDE Surface} \end{matrix} \right]$$ DISC STRESS

$$\sigma_1 = \begin{cases} 15,375 \text{ psi} & \text{OUTSIDE} \\ 13,025 \text{ psi} & \text{INSIDE} \end{cases}$$

b) Circumferential

$$\sigma_2 = \frac{ap}{2h} - \frac{E\omega}{a} \pm \frac{6\nu M}{h^2}$$

$$= 14200 - \frac{(30\times10^6)(.1039\times10^{-6})}{20.3125} \pm \frac{6(.3)(-305)}{1.25^2}$$

$$= 14200 + -1530 \pm 352 \left. \begin{matrix} - \text{INSIDE} \\ + \text{OUTSIDE} \end{matrix} \right]$$

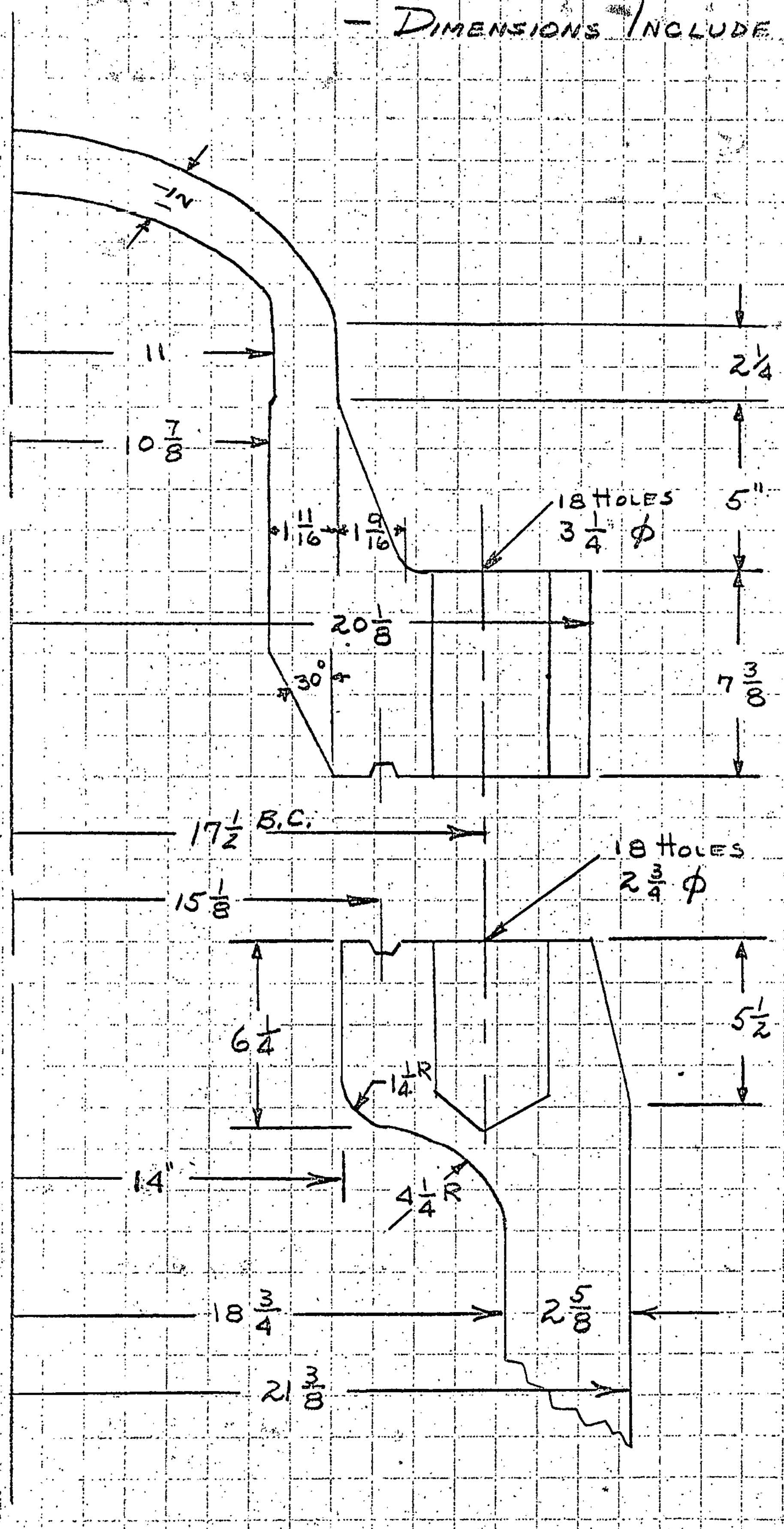
$$= 14200 \begin{matrix} +1882 \text{ psi} & \text{OUTSIDE} \\ +1178 \text{ psi} & \text{INSIDE} \end{matrix}$$

$$\sigma_2 = \begin{cases} 16,082 \text{ psi} & \text{OUTSIDE} \\ 15,378 \text{ psi} & \text{INSIDE} \end{cases}$$

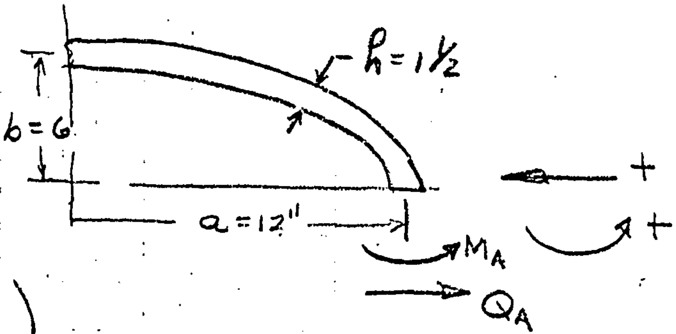
ALCO PRODUCTS INC.

BY B/R DATE 7/3/61 SUBJECT PM-2A COVER STRESS ANALYSIS TASK 619 SHEET NO. OF
CHKD. BY DATE JOB NO.

— DIMENSIONS INCLUDE CLADDING —



1 ELLIPTICAL HEAD (2:1)



DEFLECTION DUE TO PRESSURE

$$\delta_A = -\frac{Pa^2}{Eh}\left(1 - \frac{a^2}{2b^2} - \frac{\nu}{2}\right)$$

DISCONTINUITY DEFLECTION

$$\delta_A = -\frac{2\beta b^2}{Eh}\left(\beta \frac{b^2}{a^2} M_A + Q_A\right)$$

a) DEFLECTION FOR BOLT LOAD + PRESSURE

$$\Delta R_A = -\frac{Pa^2}{Eh}\left(1 - \frac{a^2}{2b^2} - \frac{\nu}{2}\right) - \frac{2\beta b^2}{Eh}\left(\beta \frac{b^2}{a^2} M_A + Q_A\right)$$

FOR BOLT LOAD ONLY   CASE "A"   P = 0

$$\beta = \frac{1.285}{\sqrt{a h}} = \frac{1.285}{\sqrt{12 \times 1.5}} = .304$$

$$\Delta R_A = -\frac{\overset{495}{2(.304)(36)}}{29.5 \times 10^6 (1.5)}\left(.304\left(\frac{36}{144}\right) M_A + Q_A\right)$$

$$\Delta R_A = -.0376 \times 10^{-6} M_A - .495 \times 10^{-6} Q_A$$

$$10^6 \Delta R_A = -.0376 M_A - .495 Q_A \qquad (1A)$$

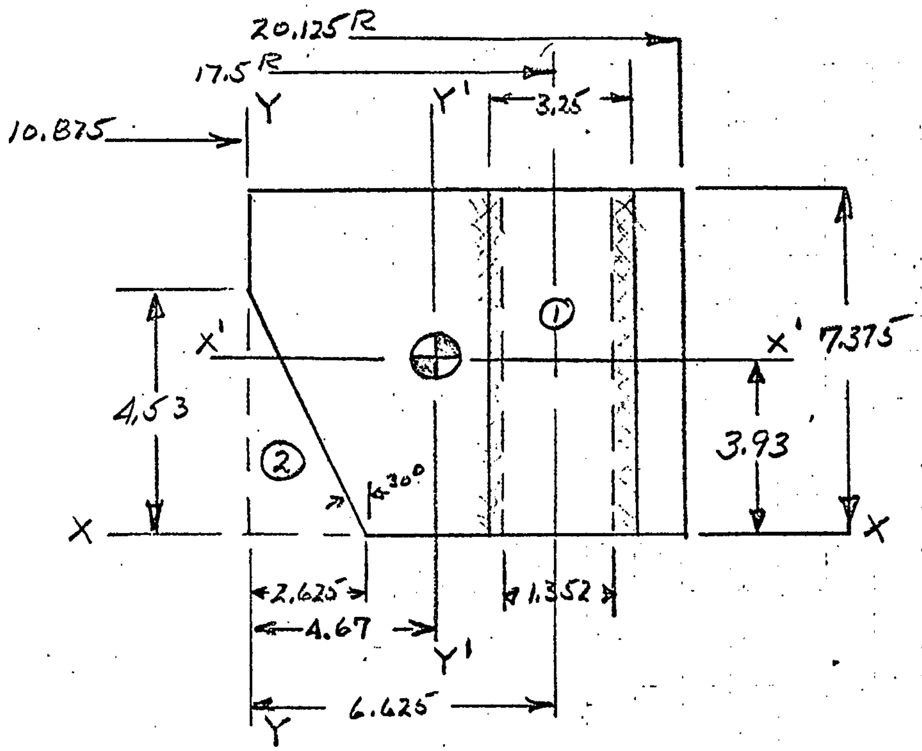
b) ROTATION

$$\theta_A = \frac{2\beta^2 b^2}{Eh} \frac{b^2}{a^2}\left(2\beta \frac{b^2}{a^2} M_A + Q_A\right)$$

$$10^{-6}\theta_A = .0056 M_A + .0376 Q_A \qquad (2A)$$

COVER FLANGE —

CORRECTION FACTOR FOR STUD HOLES IN RING —



$$C = \frac{\text{VOL of HOLES}}{\text{TOTAL VOL of RING CONTAINING HOLES}}$$

$$= \frac{\pi/4\,(3.25)^2\,18 \times 7.375}{\pi\,\big((19.125)^2 - 15.875^2\big)\,7.375}$$

$$C = .416$$

CORRECTED HOLE DIA. $= 3.25 \times .416 = 1.352$

LOCATION OF CENTROIDAL AXIS $Y'-Y'$

$$A_{\substack{\text{ACTUAL}\\\text{AREA}}}\, X' = A_T X_T - \left[A_1 x_1 + A_2 x_2 \right]$$

$$52.5\, X' = 68.4 \times \frac{7.375}{2} - \left[9.98 \times \frac{7.375}{2} + 5.95 \times \frac{4.53}{3} \right]$$

$$x' = \frac{206.2}{52.5} = 3.93$$

ALCO PRODUCTS INC.

BY ______ DATE ______ SUBJECT PM-2A Cover Stress Analysis SHEET NO. 3 OF 25

CHKD. BY ______ DATE ______ JOB NO. ______

LOCATION of CENTROIDAL AXIS YY'

$$A_{\substack{ACT\\AREA}}\, y' = A_{TOT}\, y_{o_{TOT}} - a_1 y_1 - a_2 y_2$$

$$= (7.375)(9.250) \times 4.625 - 7.375(1.352)\,6.625 - \frac{(4.53)(2.625)}{2}(.875)$$

$$= 316 - 66 - 5.2 = 245$$

$$y' = \frac{245}{68.4 - 9.98 - 5.95} = \frac{245}{52.5}$$

$$y' = 4.67''$$

Moment of Inertia About X-X'

$$I_{XX} = \frac{9.25(7.375)^3}{3} - \frac{1.352\,(7.375)^3}{3} - \frac{2.625(4.53)^3}{12}$$

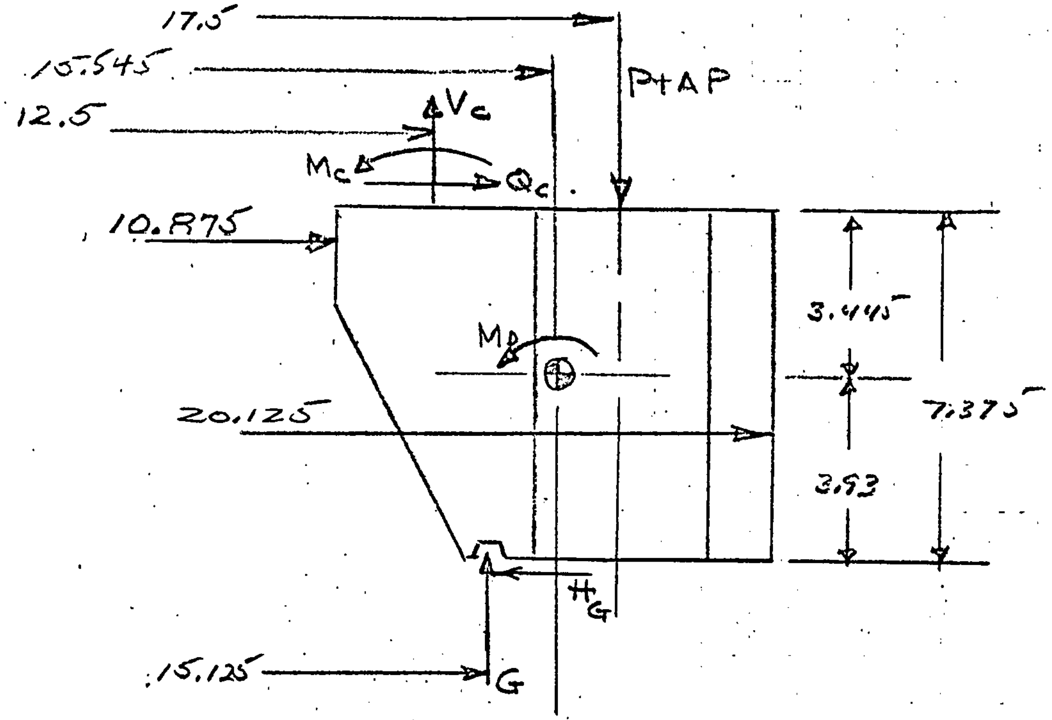
$$I_{XX} = 1049 \text{ in}^4$$

Moment of Inertia About Centroidal Axis X'X'

$$I_{X'X'} = I_{YY} - 52.5(3.93)^2$$

$$= 1049 - 811$$

$$I_{X'X'} = 238 \text{ in}^4$$



ROTATION Θ_D —

Rotation due to a moment applied at inner face of flange (Ref: "Stresses & Deformations of flanged Shells" paper by Horvay & Clausen)

$$\Theta_D = \frac{Ma^2}{EI}\left[1 + \left(\frac{1}{2}+\mu\right)\frac{w}{R} + \frac{1+\mu}{2}\,\frac{w^2}{aR}\right]$$

a = inner rad of flange
μ = .3
w = flange width 9.25"
R = mean Rad. of flange
M = moment applied at inner face
I = Moment of Inertia about Centroidal axis

$$\Theta_D = \frac{15.545}{10.875}\,\frac{M_D\,(10.875)^2}{29.5\times10^6\,(238)}\left[1 + .8\,\frac{(9.25)}{15.2} + \frac{1.3\,(9.25)^2}{2\,(10.875)(15.2)}\right]$$

$$10^6\,\Theta_D = .044\ M_D$$

Sum of Moments about D

$$15.545 M_D = 15.125 H_G 4.15 - 17.5 (P + \Delta P)(17.5 - 15.545)$$
$$- 15.125 G_T (15.545 - 15.125) - 12.5 Q_c 3.445$$
$$+ 12.5 M_c$$

$$M_D = -4.04 H_G - 12.2 (P + \Delta P) - .409 G - 2.77 Q_c + .804 M_c$$

For Bolt Load Only Case "A"

$$G = 1.158 P \quad \Delta P = 0$$
$$P = 16450$$

$$M_D = -4.04 H_G - 2.77 Q_c + .804 M_c - 40,110 \qquad (2A)$$

Subst. in θ_D where

$$10^6 \theta_D = .044 \; M_D$$

$$10^6 \theta_D = -.18 H_G - .12 \; Q_c + .035 \; M_c - 1765 \qquad (3A)$$

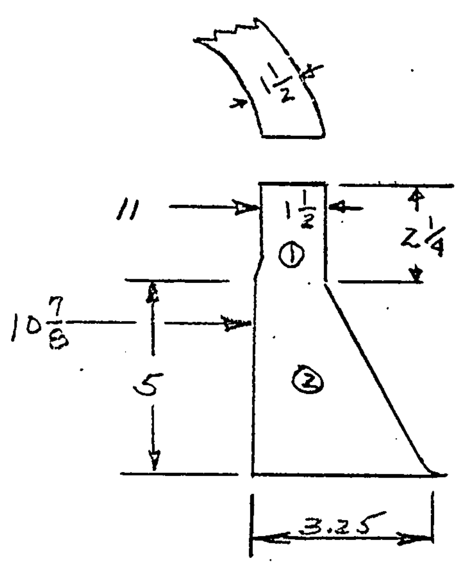
DEFLECTION ΔR_I

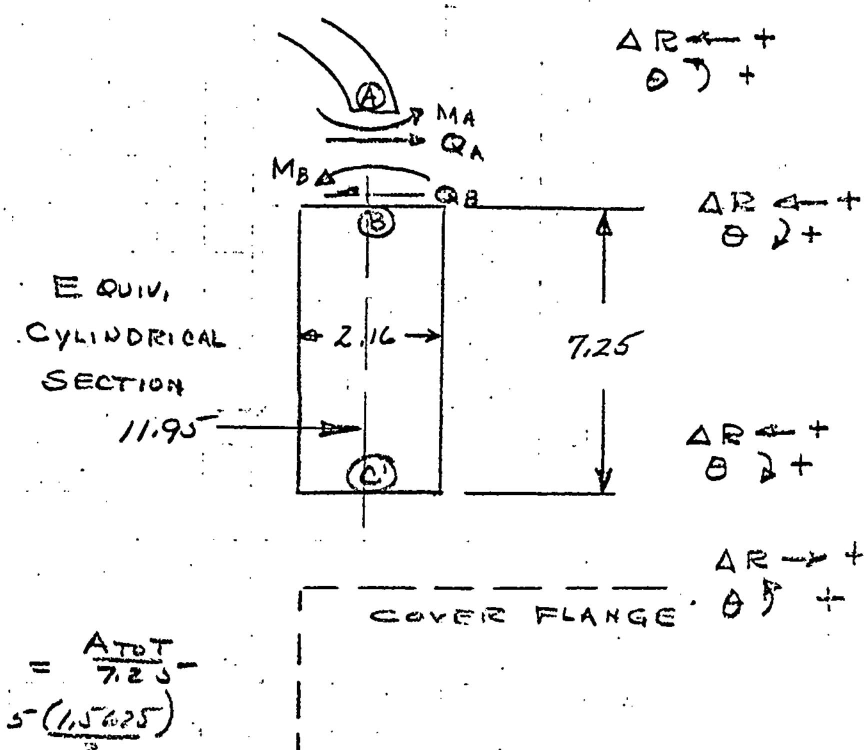
$$\Delta R_I = \left[\frac{12.5}{10.875} Q_c - \frac{15.125}{10.875} H_G\right] \frac{aR}{EA} \left[1 + \frac{uW}{R} + \frac{W^2}{4R^2}\right]$$

$$= \left[1.15 Q_c - 1.39 H_G\right] \frac{10.875 (15.545)}{29.5 \times 10^6 (5.25)} \left[1 + \frac{.3 (9.25)}{15.545} + \frac{9.25^2}{4 (15.545)^2}\right]$$

$$10^6 \Delta R_I = .159 \; Q_c - .192 \; H_G \qquad (4A)$$

SHORT CYLINDRICAL SECTION & TAPER SECTION —





EQUIV. CYL. SECT THKNS = $\frac{A_{TOT}}{7.25}$

$$t = \frac{1.5 \times 2.25 + 5(1.6875) + 5\left(\frac{1.5625}{2}\right)}{7.25}$$

$t =$ 2.16"

Ref: "Theory of Plates & Shells"(6) Timoshenko p. 392, 393

Basic Equation of Cylinder is

$$\frac{d^4 w}{dx^4} + 4\beta^4 w = \frac{Z}{D}$$

Gen'l Solution

$$w = e^{\beta x}(C_1 \cos\beta x + C_2 \sin\beta x) + e^{-\beta x}(C_3 \cos\beta x + C_4 \sin\beta x) + f(x)$$

Equations for slope, moment, & shear obtained by taking successive derivitives

Expression for deflection, slope, moment & shear

$$w = (e^{2\beta x}C_1 + C_3)\theta(\beta x) + (e^{2\beta x}C_2 + C_4)\zeta(\beta x)$$

$$\frac{dw}{dx} = \beta\left[(e^{2\beta x}C_1 + C_4)\psi(\beta x) + (e^{2\beta x}C_2 - C_3)\phi(\beta x)\right]$$

$$M_x = -D\left(\frac{d^2w}{dx^2}\right) = -D2\beta^2\left[(-e^{2\beta x}C_1 + C_3)\zeta(\beta x) + (e^{2\beta x}C_2 - C_4)\theta(\beta x)\right]$$

$$Q_x = -D\frac{d^3w}{dx^3} = -D2\beta^3\left[(-e^{2\beta x}C_1 + C_4)\phi(\beta x) + (e^{2\beta x}C_2 + C_3)\psi(\beta x)\right]$$

At Point B where $x = 0$

$$\beta = \frac{1.285}{\sqrt{a \times h}} = \frac{1.285}{\sqrt{11.95 \times 2.16}} = .253 \qquad 2\beta^2 = .1296 \qquad 2\beta^3 = .0322$$

$$D = \frac{Eh^3}{12(1-\nu^2)} = 27.2 \times 10^6 \qquad e^{2\beta x} = 1$$

From Tables "Theory of Plates & Shells"

$$\theta(\beta x) = 1 \qquad \zeta(\beta x) = 0$$
$$\psi(\beta x) = 1 \qquad \phi(\beta x) = 1$$

Substituting these values in above equation

$$\Delta R_B = C_1 + C_3 \qquad (5A)$$

$$\theta_B = .253\,(C_1 + C_4 + C_2 - C_3) \qquad (6A)$$

$$M_B = -3.52\,(C_2 - C_4) \qquad (7A)$$

$$Q_B = -.875\,(C_4 - C_1 + C_2 + C_3) \qquad (8A)$$

At Point C' where $x = 7.25$

$\beta = .253$ $\qquad e^{\beta x} = 6.25$

$2\beta^2 = .1296$

$2\beta^3 = .0322$ $\qquad e^{2\beta x} = e^{3.66} \doteq 39$

From tables Timoshenko for $\beta x = \frac{1.285}{\sqrt{a \times h}} \times 7.25$

$\beta x = 1.83$

$\theta(\beta x) = -.0408$

$\psi(\beta x) = -.1959$

$\zeta(\beta x) = .1551$

$\phi(\beta x) = .1143$

Substituting into equations for w, θ, M & Q

$$\Delta R_{c'} = -1.595 C_1 - .0408 C_3 + 6.05 C_2 + .1551 C_4 \qquad (9A)$$

$$\theta_{c'} = -1.93 C_1 - .0495 C_4 + 1.13 C_2 - .0289 C_3 \qquad (10A)$$

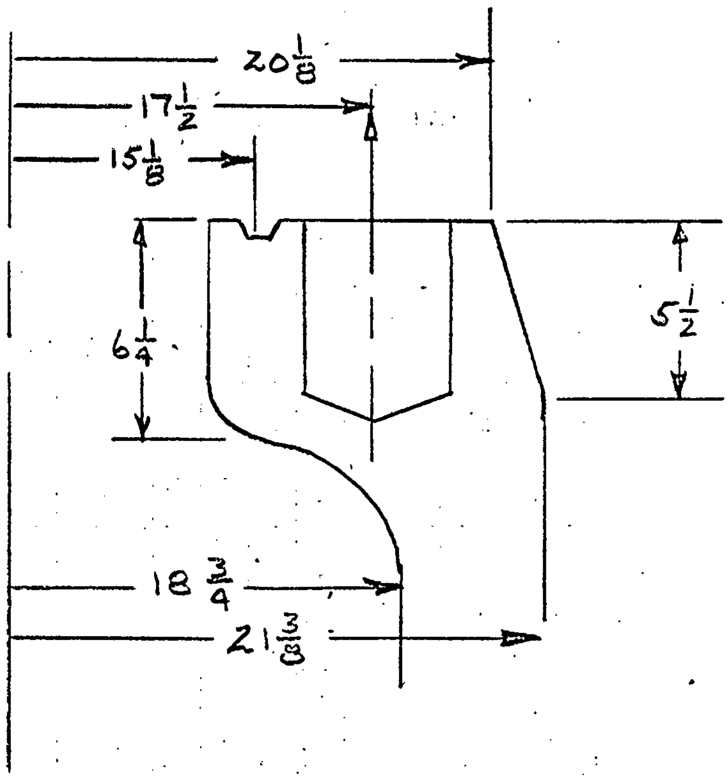
$$M_{c'} = 21.3 C_1 - .546 C_3 + 5.61 C_2 - .144 C_4 \qquad (11A)$$

$$Q_{c'} = 3.90 C_1 - .10 C_4 + 6.66 C_2 + .171 C_3 \qquad (12A)$$

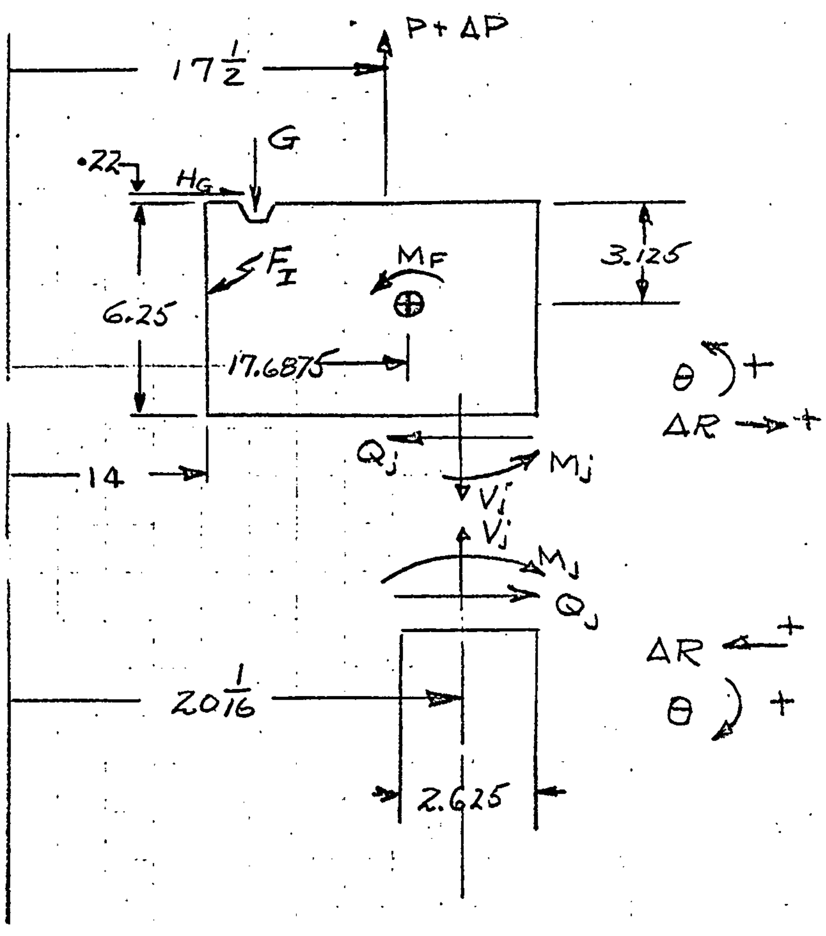
ALCO PRODUCTS INC.

BY BJR DATE
CHKD. BY DATE
SUBJECT PM-2A COVER STRESS ANALYSIS TASK 6.9
SHEET NO. 9 OF 75
JOB NO.

VESSEL FLANGE



LOWER FLANGE IS ASSUMED TO BE RIGID. THE FLANGE ROTATES BUT DOES NOT BEND & IS ASSUMED TO ROTATE ABOUT THE GASKET, POINT G



FOR SIMPLIFICATION, THE VESSEL FLANGE IS CONSIDERED AS SHOWN & EQUATIONS OBTAINED FOR LOWER FLANGE MUST BE CORRECTED TO ACTUAL CONDITION SHOWN ABOVE.

CORRECTION FACTOR FOR LOWER FLANGE —

TOTAL VOL OF RING (simplified SKETCH) $= \pi(21.375^2 - 14^2)6.25 = 5090 \text{ in}^3$

Comparing actual flange to simplified, the volume of metal removed by bolt holes and outside & inside Corners, and added by 4¼" fillet, must be Considered.

Vol. of bolt holes $= 18\frac{\pi}{4}(2.75)^2(5.4375) = -582 \text{ in}^3$ Removed

Vol. of taper $= \frac{5.5 \times 1.25}{2} \times 2\pi(20.95) = -453 \text{ in}^3$ Removed

Vol. of inside Corner $= \left(1.25^2 - \frac{\pi}{4}(1.25)^2\right)2\pi(14.4) = -30.3 \text{ in}^3$ Removed

-1065.3 in^3 Removed

Vol. of fillet $= \left(4.25^2 - \frac{\pi(4.25)^2}{4}\right)2\pi(17.3) = 424 \text{ in}^3$ Added

NET VOLUME REMOVED from Simplified SKETCH to GIVE ACTUAL -641.3 in^3

CORRECTION FACTOR $= \dfrac{5090}{5090 - 641} = \underline{1.143}$

THE ACTUAL FLANGE HAS LESS METAL THAN THE SIMPLIFIED SKETCH, thereby GIVING GREATER DEFLECTION & ROTATION. THE EQUATIONS obtained for the simplified flange must be multiplied by this CORRECTION FACTOR.

DEFLECTION OF LOWER FLANGE AT F_I —

$$\Delta R_{F_I} = \frac{Pa}{E}\left(\frac{b^2+a^2}{b^2-a^2}+\nu\right) - \frac{Pa}{E}\left(\frac{2b^2}{b^2-a^2}\right)$$

$$= \frac{15.125}{14}\left(\frac{H_G}{6.25}\right)\left(\frac{14}{29.5\times10^6}\right)\left(\frac{21.375^2+14^2}{21.375^2-14^2}+.3\right)$$

$$- \frac{20.0625}{21.375}\left(\frac{Q_j}{6.25}\right)\frac{14}{29.5\times10^6}\left(\frac{2\,(21.375)^2}{21.375^2-14^2}\right)$$

$$10^6\Delta R_{F_I} = .229\,H_G - .251\,Q_j$$

APPLYING CORRECTION FACTOR $\times 1.143$

$$10^6\Delta R_{F_I} = .262\,H_G - .285\,Q_j \qquad (13A)$$

DEFLECTION OF LOWER FLANGE AT J —

$$\Delta R_{F_j} = \frac{Pb}{E}\left(\frac{2a^2}{b^2-a^2}\right) - \frac{Pb}{E}\left(\frac{a^2+b^2}{b^2-a^2}-\nu\right)$$

$$= \frac{15.125}{14}\;\frac{H_G}{6.25}\;\frac{21.375}{29.5\times10^6}\left(\frac{2(14)^2}{21.375^2-14^2}\right)$$

$$- \frac{20.0625}{21.375}\;\frac{Q_j}{6.25}\;\frac{20.0625}{29.5\times10^6}\left(\frac{14^2+21.375^2}{21.375^2-14^2}-.3\right)$$

$$10^6\Delta R_{F_j} = (.189\,H_G - .224\,Q_j)\times 1.143 \text{ CORR. FACTOR}$$

$$10^6\Delta R_{F_j} = .216\,H_G - .257\,Q_j \qquad (14A)$$

DEFLECTION of CYLINDRICAL SHELL AT J

$$\Delta R_j = -\frac{1}{2\beta^3 D}(\beta M_j + Q_j)$$

$$\beta^3 = \left[\frac{1.285}{\sqrt{20.06 \times 2.625}}\right]^3 \qquad D = \frac{E h^3}{12(1-\nu^2)}$$

$$\beta^3 = .00555 \qquad D = 48.9 \times 10^6$$

$$\Delta R_j = -\frac{1}{2(.00555)(48.9 \times 10^6)}(.177 M_j + Q_j)$$

$$10^6 \Delta R_j = -.326 M_j - 1.84 Q_j$$

ROTATION of CYLINDRICAL SHELL AT J

$$\theta_j = \frac{1}{2\beta^2 D}(2\beta M_j + Q_j)$$

$$\theta_j = \frac{1}{2(.177)^2(48.9 \times 10^6)}(2(.177)M_j + Q_j)$$

$$10^6 \theta_j = .1158 M_j + .327 Q_j \qquad (15A)$$

SUM of MOMENTS about F — ↶+

$$17.6875 M_F = G(17.6875 - 15.125)(15.125) - H_G(3.345)(15.125)$$
$$-17.5(P+\Delta P)(17.6875 - 17.5) - 20.06\,Q_j\,3.125 + 20.06 M_j$$

$$M_F = 2.195 G - 2.86 H_G - .186(P+\Delta P) - 3.55 Q_j + 1.135 M_j$$

For Bolt Load Only Case "A"

$$15.125\,G = 17.5\,P$$
$$G = 1.158\,P$$

$\Delta P = 0$ and P = Load which produces ASME Code Allowable Stress in Studs

$S_{ALL} = 20\,000$ P.S.I. for SA-193 Gr. B6 AISI 403 Stud Mat'l

$$\text{Stud Area} = \frac{\pi}{4}(2.531)^2\,18 = 90.6\ in^2$$

$$P = \frac{90.6 \times 20000}{\pi \times 35} = 16450\ \#/in$$

Subst. Values of G & P

$$M_F = -2.86\,H_G - 3.55\,Q_j + 1.135\,M_j + 38640 \qquad (16A)$$

Rotation of Lower Flange θ_F —

$$\theta_F = \frac{12\,M_F\,a}{E\,R^3 \ln a/c} = \frac{12\,M_F\,17.6875}{29.5\times10^6(6.25)^3 \ln \frac{21.375}{14}}$$

$$10^6\,\theta_F = .0697\,M_F$$

Subst. expression for M_F in θ_F

$$10^6\,\theta_F = -.199\,H_G - .2475\,Q_j + .0791\,M_j + 2700$$

Applying Correction Factor

$$10^6\,\theta_F = -.228\,H_G - .283\,Q_j + .0905\,M_j + 3090 \qquad (17A)$$

SUMMARY of EQUATIONS

HEAD —

$$10^6 \Delta R_A = -.0376\, M_A - .495\, Q_A \qquad (1)$$

$$10^6 \theta_A = .0056\, M_A + .0376\, Q_A \qquad (2)$$

Top of Cyl. Sect —

$$10^6 \Delta R_B = C_1 + C_3 \qquad (3)$$

$$10^6 \theta_B = .253\,(C_1 + C_4 + C_2 - C_3) \qquad (4)$$

$$10^6 M_B = -3.52\, C_2 + 3.52\, C_4 \qquad (5)$$

$$10^6 Q_B = -.875\, C_4 + .876\, C_1 - .875\, C_2 - .875\, C_3 \qquad (6)$$

Bottom of Cyl. Sect —

$$10^6 \Delta R_{C'} = -1.595\, C_1 - .0408\, C_3 + 6.05\, C_2 + .1551\, C_4 \qquad (7)$$

$$10^6 \theta_{C'} = -1.93\, C_1 - .0495\, C_4 + 1.13\, C_2 - .0289\, C_3 \qquad (8)$$

$$10^6 M_{C'} = 21.3\, C_1 - .546\, C_3 + 5.61\, C_2 - .144\, C_4 \qquad (9)$$

$$10^6 Q_{C'} = 3.90\, C_1 - .10\, C_4 + 6.66\, C_2 + .171\, C_3 \qquad (10)$$

Upper Flange —

$$10^6 \theta_D = -.18\, H_G - .12\, Q_C + .035\, M_C - 1765 \qquad (11)$$

$$10^6 \Delta R_I = .159\, Q_C - .192\, H_G \qquad (12)$$

Lower Flange

$$10^6 \Delta R_{FI} = .262\, H_G - .285\, Q_J \qquad (13)$$

$$10^6 \Delta R_{FJ} = .216\, H_G - .257\, Q_J \qquad (14)$$

$$10^6 \theta_F = -.228\, H_G - .283\, Q_J + .0905\, M_J + 3090 \qquad (15)$$

Vessel Cylindrical Section

$$10^6 \Delta R_J = -.326\, M_J - 1.84\, Q_J \qquad (16)$$

$$10^6 \theta_J = .1158\, M_J + .327\, Q_J \qquad (17)$$

BOUNDARY CONDITIONS —

19) $\Delta R_A = \Delta R_B$

20) $M_A = M_B$

21) $Q_A = -Q_B$

22) $\theta_A = -\theta_B$

(23) $-\Delta R_{c'} = \Delta R_c = \Delta R_I - 3.445\,\theta_D$

(24) $-\theta_{c'} = \theta_c = \theta_D$

(25) $\Delta R_G = \Delta R_I + 4.15\,\theta_D = \Delta R_{F_I} - 3.345\,\theta_F$

(26) $-\Delta R_J = \Delta R_{F_J} + 3.125\,\theta_F$

(27) $\theta_F = -\theta_J$

(28) $-M_{c'} = M_c$

(29) $Q_{c'} = Q_c$

Substituting (1) & (3) into 19 above

$$\Delta R_A = \Delta R_B$$

$$-.0376\,M_A - .495\,Q_A - C_1 - C_3 = 0 \qquad (30)$$

Substituting (2) & (4) into 22

$$\theta_A = -\theta_B$$

$$.0056\,M_A + .0376\,Q_A + .253\,C_1 + .253\,C_4 + .253\,C_2 - .253\,C_3 = 0 \qquad (31)$$

Substituting (6) into 21

$$Q_A = -Q_B$$

$$Q_A - .875\,C_4 + .875\,C_1 - .875\,C_2 - .875\,C_3 = 0 \qquad (32)$$

Subst. (7)(12) & (11) into 23

$$-\Delta R_C' = \Delta R_C = \Delta R_I - 3.445\,\theta_D$$

$$1.595 C_1 + .0408 C_3 - 6.05 C_2 - .1551 C_4 - .428 H_G - .569 Q_C + .12 M_C = 6080 \qquad (34)$$

Subst (8) & (11) into 24

$$-\theta_C = \theta_D$$

$$1.93 C_1 + .0495 C_4 - 1.13 C_2 + .0289 C_3 + .178 H_G + .122 Q_C - .035 M_C = 1765 \qquad (35)$$

Subst (12)(13)(15)(11) into 25

$$\Delta R_G = \Delta R_I + 4.15\,\theta_D = \Delta R_{FI} - 3.345\,\theta_F$$

$$-.339 Q_C - 1.963 H_G - .609 Q_J + .1453 M_C + .303 M_J' = -3030 \qquad (36)$$

Subst (16)(14) & (15) into 26

$$-\Delta R_J = \Delta R_{FJ} + 3.125\,\theta_F$$

$$.043 M_J + 2.98 Q_J + .497 H_G = 9660 \qquad (37)$$

Subst (15) & (17) into 27

$$\theta_F = -\theta_J$$

$$-.228 H_G + .044 Q_J + .2063 M_J = -3090 \qquad (38)$$

Subst equat. (9) into 28

$$-M_c' = M_c$$

$$-M_c - 21.3\,C_1 + .546\,C_3 - 5.61\,C_2 + .144\,C_4 = 0 \qquad (39)$$

Subst equat. (10) into 29

$$Q_c' = Q_c$$

$$Q_c - 3.90\,C_1 + .10\,C_4 - 6.66\,C_2 - .171\,C_3 = 0 \qquad (40)$$

Subst equat (5) into 20

$$M_A = M_B$$

$$M_A + 3.52\,C_2 - 3.52\,C_4 = 0 \qquad (41)$$

Thus solving eqn's 39 thru 41 for 11 unknowns — yields —

$C_1 = -580$

$C_2 = -404$

$C_3 = 9$

$C_4 = 773$

$Q_A = 839$

$M_A = 4142$

$Q_C = -5030$

$M_C = 14743$

$H_G = -114$

$M_J = -15848$

$Q_J = 3489$

Substituting above values to check boundary conditions, The values for deflection & Rotation obtained are tabulated on the following page —

CHECK OF BOUNDARY CONDITIONS —

1. $\Delta R_A = \Delta R_B$

$$-.0376\big(\overset{-156}{\underset{M_A}{4142}}\big) - .495\big(\overset{-415}{\underset{Q_A}{839}}\big) = \underset{C_1}{-580} + \underset{C_3}{9}$$

$$-571 = -571$$

2. $M_A = M_B$

$$4142 = -3.52\big(\overset{1420}{\underset{C_2}{-404}}\big) + 3.52\big(\overset{2720}{\underset{C_4}{773}}\big)$$

$$4142 = 4140$$

3. $Q_A = -Q_B$

$$839 = -\Big[-.875\big(\overset{-677}{\underset{C_4}{773}}\big) + .875\big(\overset{-507}{\underset{C_1}{-580}}\big) - .875\big(\overset{+354}{\underset{C_2}{-404}}\big) - .875\big(\overset{-7.9}{\underset{C_3}{9}}\big)$$

$$839 = 838$$

4. $\theta_A = -\theta_B$

$$.0056\big(\overset{23.2}{\underset{M_A}{4142}}\big) + .0376\big(\overset{31.6}{\underset{Q_A}{839}}\big) = -.253\big(\overset{-220}{\underset{C_1}{-580} \underset{+C_4}{+773} \underset{+C_2}{-404} \underset{-C_3}{-9}}\big)$$

$$54.8 = -55.7$$

5. $$\theta_D = -.18\big(\overset{+20.5}{\underset{H_G}{-114}}\big) - .12\big(\overset{+603}{\underset{Q_C}{-5030}}\big) + .085\big(\overset{516}{\underset{M_C}{14743}}\big) - 1765$$

$$\theta_D = -625$$

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BY.............DATE......... SUBJECT .... SHEET NO. 20 OF 25
CHKD. BY.......DATE......... JOB NO.

6. $-\Delta R_{c'} = \Delta R_c = \Delta R_I - 3.445\,\theta_D$

$$-\Big[\underbrace{\overset{+925}{-1.595\big(\underset{C_1}{-580}\big)} \overset{-.0368}{-.0408\big(\underset{C_3}{9}\big)} \overset{-2440}{+6.05\big(\underset{C_2}{-404}\big)} \overset{120}{+.1551\big(\underset{C_4}{773}\big)}}_{\Delta R_{c'}}\Big] =$$

$$\underbrace{\overset{-800}{.159\big(\underset{Q_C}{-5030}\big)} \overset{21.9}{-.192\big(\underset{H_G}{-114}\big)}}_{\Delta R_I = -778} \overset{+2160}{-3.445\big(\underset{\theta_D}{-625}\big)}$$

$$1395 = 1382$$

7. $-\theta_{c'} = \theta_c = \theta_D$

$$-\Big[\overset{1120}{-1.93\big(\underset{C_1}{-580}\big)} \overset{-38}{-.0495\big(\underset{C_4}{773}\big)} \overset{-457}{+1.13\big(\underset{C_2}{-404}\big)} -.0289\big(\underset{C_3}{9}\big)\Big] = \underset{\theta_D}{-625}$$

$$-625 = -625$$

8. $\theta_F = -\theta_J$

$$\overset{+26}{-.228\big(\underset{H_G}{-114}\big)} \overset{-987}{-.283\big(\underset{Q_J}{3489}\big)} \overset{-1435}{+.0905\big(\underset{M_J}{-15848}\big)} + 3090 =$$

$$-\Big(\overset{-1835}{.1158\big(\underset{M_J}{-15848}\big)} \overset{+1140}{+.327\big(\underset{Q_J}{3489}\big)}\Big)$$

$$+694 = +695$$

9. $\Delta R_G = \Delta R_I + 4.15\,\theta_D = \Delta R_{FI} - 3.345\,\theta_F$

$$\overset{-800}{.159\big(\underset{Q_C}{-5030}\big)} \overset{+21.9}{-.192\big(\underset{H_G}{-114}\big)} \overset{-2595}{+4.15\big(\underset{\theta_D}{-625}\big)} =$$

$$\underbrace{\overset{-29.9}{.262\big(\underset{H_G}{-114}\big)} \overset{-1000}{-.287\big(\underset{Q_J}{3489}\big)}}_{\Delta R_{FI} = -1030} \overset{-2330}{-3.345\big(\underset{\theta_F}{694}\big)}$$

$$-3373 = -3360$$

10. $M_{C'} = -M_C$

$$\overset{-12350}{21.3(\underset{C_1}{-580})} - \overset{-4.6}{.546(\underset{C_3}{9})} + \overset{-2260}{5.61(\underset{C_2}{-404})} - \overset{-111}{.144(\underset{C_4}{773})} = -M_C$$

$$14726 = -(-14743)$$

11. $Q_{C'} = Q_C$

$$\overset{-2260}{3.90(\underset{C_1}{-580})} - \overset{-77.3}{.10(\underset{C_4}{773})} + \overset{-2700}{6.66(\underset{C_2}{-404})} + \overset{+1.54}{.171(\underset{C_3}{9})} = -5030$$

$$-5035 = -5030$$

12. $\Delta R_J = \overset{+5160}{-.326(\underset{M_J}{-15848})} - \overset{-6420}{1.84(\underset{Q_J}{3489})}$

$$\Delta R_J = -1260$$

13. $-\Delta R_J = \Delta R_{F_J} + 3.125\,\theta_F$

$$-(-1260) = \overset{-24.6}{.216(\underset{H_G}{-114})} - \overset{-896}{.257(\underset{Q_J}{3489})} + \overset{2170}{3.125(695)}$$

$$1260 = 1249$$

CASE A — STUD LOAD ONLY
SUMMARY of RESULTS

— RESULTS —

$Q_A = -Q_B = 839$

$M_A = M_B = 4142$

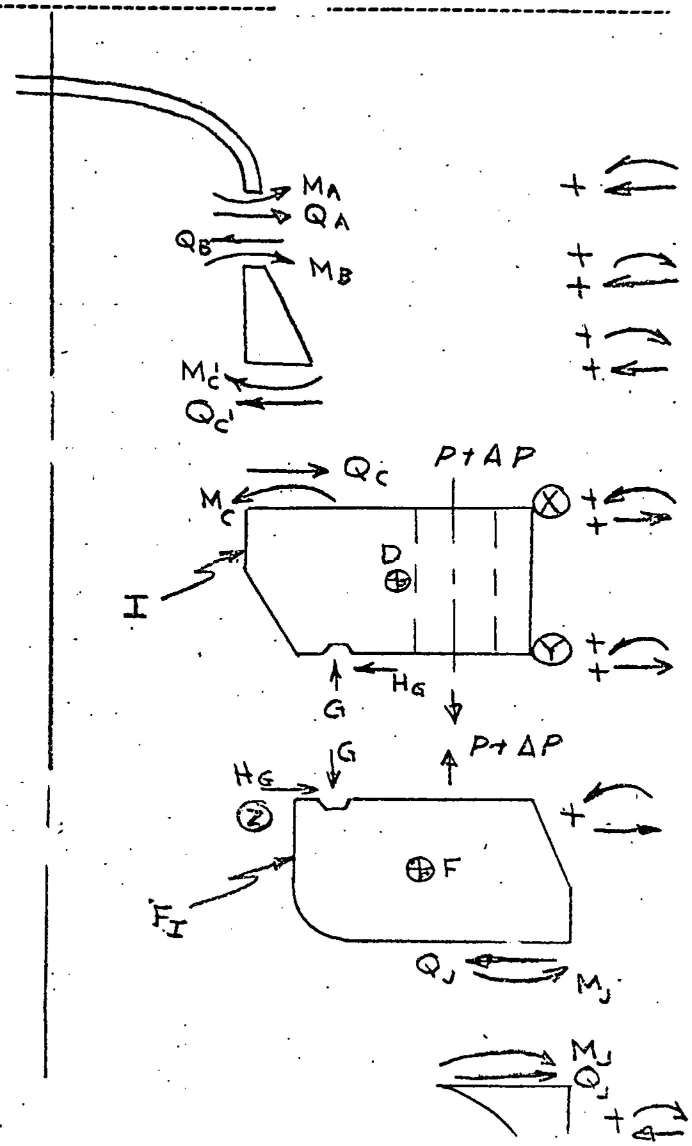
$Q_C = Q_{C'} = -5030$

$M_C = -M_{C'} = 14743$

$H_G = -114$

$M_J = -15848$

$Q_J = 3489$



LOCATION	DEFLECTION ΔR (10^{-6} in)	ROTATION θ (10^{-6} RAD)	STRESS (PSI) AXIAL σ_1	CIRCUMF σ_2	RADIAL σ_3	SHEAR
A (20)	−571	55	−11050 OUTER +11050 INNER	−1905 4715	0	
C' (19)	1382	−625	+8380 OUTER −8380 INNER	5770 750	0	
I (16)	−778	−625	—	−2120	—	
G (14)	−3377	—	—	—	—	152
F_I	−1030	695		−2170		
J (13)	−1260	−695	+13800 OUTER −13800 INNER	5590 −2290	0	
X (18)	1725	−625	—	2530	—	
Y (17)	−2884	−625	—	−4230	—	
Z (15)	−3200	695	—	−6750	—	

STRESSES —

PT. A —

AXIAL: $\sigma_1 = \pm \frac{6 M_A}{h^2} = \pm \frac{6(4142)}{(1.5)^2} = 11{,}050$ PSI — OUTSIDE + INSIDE

CIRCUMF: $\sigma_2 = -\frac{\Delta R_A}{R_A} \times E \pm \nu \frac{6 M_A}{h^2}$

$$= -\underbrace{\frac{(-571)(29.5)}{12}}_{+1405} \pm \underbrace{.3(11060)}_{3310}$$

$\sigma_2 =$ − 1905 PSI OUTSIDE
+ 4715 PSI INSIDE

PT. C′ —

AXIAL: $\sigma_1 = \pm \frac{6 M_C}{h^2} = \pm \frac{6(14743)}{(3.25)^2} = 8{,}380$ PSI − INSIDE + OUTSIDE

CIRCUMF: $\sigma_2 = \frac{\Delta R_C}{R_C} \times E \pm \nu \frac{6 M_C}{h^2}$

$$= -\underbrace{\frac{(1382)(29.5)}{12.5}}_{3260} \pm \underbrace{.3(8380)}_{2510}$$

= 750 PSI INSIDE
5,770 PSI OUTSIDE

PT I —

CIRCUMF: $\sigma_2 = \frac{\Delta R_I}{R_I} \times E = \frac{-778}{10.875} \times 29.5 = -2{,}120$ PSI

PT G —

SHEAR STRESS AT GASKET $= \frac{H_G}{.75} = \frac{114}{.75} = 152$ PSI

PT F_I –

CIRCUMF: $\sigma_2 = \frac{\Delta R_{F_I}}{R_{F_I}} \times E = -\frac{1030}{14} \times 29.5 = -2170$ PSI

PT J –

AXIAL: $\sigma_1 = \pm\frac{6 M_J}{h^2} = \pm\frac{6(-15848)}{(2.625)^2} = 13800$ PSI + OUTSIDE − INSIDE

Circumf: $\sigma_2 = \frac{Pa}{h} - \frac{E\omega}{a} \pm \frac{\nu 6 M_J}{h^2}$ $P=0$ for Case A

$= 0 + \left[-\frac{(29.5)(-1260)}{20.0625} \pm .3(13800)\right]$

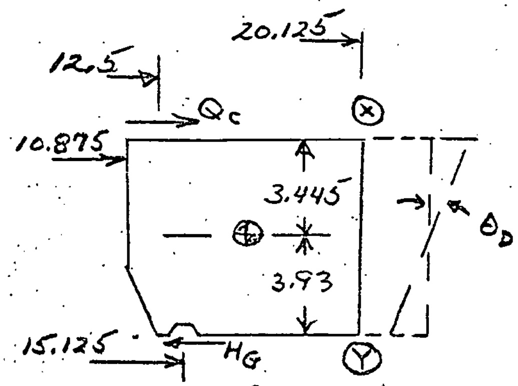
$= 0 + [+1850 \pm 4140]$,

$\sigma_2 = 0 + \left[\begin{array}{l} -2290 \text{ PSI INSIDE} \\ +5590 \text{ PSI OUTSIDE} \end{array}\right]$

RADIAL: $\sigma_3 = 0$

PT X –

DISCONTINUITY DEFLECTION AT OUTER SURFACE



$\Delta R_o = P\frac{b}{E}\left(\frac{2a^2}{b^2-a^2}\right)$

$10^6 \Delta R_o = \left(\frac{12.5}{10.875} Q_c - \frac{15.125}{10.875} H_G\right)\frac{20.125}{(7.375)(29.5)}\left(\frac{2(10.875)^2}{20.125^2 - 10.875^2}\right)$

$= (1.15\ Q_c - 1.39\ H_G)(.0762)$

$= .0875\ Q_c - .106\ H_G$

$10^6 \Delta R_o = .0875(-5030) - .106(-114) = -.428$

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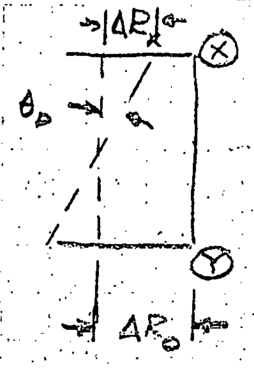


$$\Delta R_x = \Delta R_o - 3.445\,\theta_D$$

$$= -.428 - 3.445(-.625) \quad {+2153}$$

$$\Delta R_x = +.1725$$

STRESS AT X

$$\sigma_2 = \frac{\Delta R_x}{R_x} \times E = \frac{.1725}{20.125}(29.5) = 2530 \text{ PSI}$$

PT Y –

$$\Delta R_y = \Delta R_o + 3.93\,\theta_D$$

$$= -.428 + 3.93(-.625) = -2884 \quad {-2456}$$

STRESS AT Y

$$\sigma_2 = \frac{\Delta R_y}{R_y} \times E = \frac{-.2884}{20.125}(29.5) = -4230 \text{ PSI}$$

PT Z –

$$\Delta R_z = \Delta R_{F_z} - 3.125\,\theta_F$$

$$\Delta R_z = -.1030 - 3.125(.695) = -.3200 \quad {-2170}$$

STRESS AT Z –

$$\sigma_2 = \frac{\Delta R_z}{R_z} \times E = \frac{-.3200}{14}(29.5) = -6750 \text{ PSI}$$

CASE "B" — STUD LOAD PLUS PRESSURE

Stud Load at 20000 psi/bolt $P = 16450$ #/in

$$\text{Gasket Load} = \frac{17.5}{15.125} \times 16450 = 19000 \text{ \#/in}$$

WITH INTERNAL PRESSURE APPLIED, THE LOAD ON THE STUD = $P + \Delta P$ WHERE ΔP = CHANGE IN STUD TENSION DUE TO ADDED PRESSURE LOAD & UNLOADING OF GASKET.

$$17.5(P + \Delta P) = (B + G)\,15.125$$

B = BLOWOFF LOAD
G = LOAD ON GASKET WITH PRESSURE

$$B = \frac{1750\,\pi(15.125)^2}{2\pi\,15.125} = 13230 \text{ \#/in}$$

RELATIVE STIFFNESS OF GASKET & STUDS —

$$\delta_s = \frac{P l_s}{A_s E_s} \qquad \delta_g = \frac{G l_g}{A_g E_g}$$

$$\frac{\delta_s}{\delta_g} = \frac{2\pi\,17.5}{18}\frac{P l_s}{A_s E_s} \times \frac{A_g E_g}{\frac{2\pi\,15.125}{18}(1.157P)\,l_g}$$

$$= \frac{A_g E_g\, l_s}{A_s E_s\, l_g}$$

$$= \frac{7.813 \times 3.96}{5.03 \times .4375} \times \frac{26}{28} = 13.05$$

Change in stud deflection = change in gasket deflection

$$\Delta \delta_s = \Delta \delta_g$$

$$\frac{2\pi(15.125)(16.125)}{18}\frac{\left(\frac{17.5}{15.125}B + \frac{16.125}{17.5}G - P\right)l_s}{A_s E_s}$$

$$= \frac{2\pi(15.125)}{18}\frac{\left(\frac{17.5}{15.125}P - G\right)l_g}{A_g E_g}$$

Solving for $G = 6800$

$$P + \Delta P = \frac{B + G}{1.158} = \frac{13230 + 6800}{1.158} = 17300 \text{ \#/in}$$

ELLIPTICAL HEAD —

DEFLECTION DUE TO PRESSURE

$$\delta R_A = -\frac{Pa^2}{ER}\left(1 - \frac{a^2}{2b^2} - \frac{\nu}{2}\right)$$

$$= -\frac{1750(144)}{(29.5\times10^6)(1.5)}\left(1 - \frac{144}{2(36)} - .15\right)$$

$$10^6\,\delta R_A = 6550$$

Eqn 1A Becomes

$$10^6\,\Delta R_A = -.0376\,M_A - .495\,Q_A + 6550 \quad (1B)$$

LOWER FLANGE —

$$\delta \ell_{F_I} = \frac{Pa}{E}\left[\frac{b^2+a^2}{b^2-a^2} - \nu\left(\frac{a^2}{b^2-a^2}-1\right)\right]$$

$$= \frac{1750(14)}{29.5\times10^6}\left[\frac{21.375^2+14^2}{21.375^2-14^2} - .3\left(\frac{14^2}{21.375^2-14^2}-1\right)\right]$$

$$10^6\,\delta R_{F_I} = 2145$$

APPLYING CORRECTION FACTOR OF 1.143, EQN 3A BECOMES

$$10^6\,\Delta \ell_{F_I} = .262\,H_G - .285\,Q_J + 2450 \qquad (3B)$$

$$\delta R_{F_J} = \frac{Pb}{E}\left[\frac{a^2}{b^2-a^2}(2-\nu)\right]$$

$$= \frac{1750(21.375)}{29.5\times10^6}\left[\frac{(14)^2}{21.375^2-14^2}(2-.3)\right]$$

$$10^6\,\delta R_{F_J} = 1630$$

APPLYING CORRECTION FACTOR of 1.143 EQN 4 A BECOMES

$$10^6\,\Delta R_{F_J} = .216\,H_G - .257\,Q_J + 1865 \qquad (4B)$$

$$\delta R_J = -\frac{Pa^2}{ER}\left(1-\frac{\nu}{2}\right)$$

$$= -\frac{1750(20.0625)^2}{29.5\times10^6(2.625)}(1-.15)$$

$$= -7735\times10^{-6}$$

Eqn 5A becomes

$$10^6\,\Delta R_J = -.326\,M_J - .184\,Q_J - 7735 \qquad (5B)$$

SUM OF MOMENTS ABOUT F —

$$17.6875\, M_F = 15.125\overset{G}{(6800)}\,2.625 - 3.345\, H_G\, 15.125 - 17.5\overset{P+\Delta P}{(17300)}(.1875)$$

$$- \frac{1750\,\pi(18.75)^2(2.375)\overset{V_J}{(20.0625)}}{2\pi(20.0625)}$$

$$- \frac{1750\,\pi(18.75^2 - 15.125^2)}{2\pi(16.9375)^2} \times .75 \times 16.9375$$

$$+ 20.06\, M_J' - 20.06\, Q_J'(3.125)$$

$$M_F = -.286\, H_G - 3.55\, Q_J' + 1.135\, M_J' - 33800$$

FROM Pg 6
$$10^6\, \theta_F = .044\, M_F$$

$$\theta_F = -.0199\, H_G - .247\, Q_J + .0791\, M_J - 2360$$

APPLYING CORRECTION FACTOR 1.143

$$10^6\, \theta_F = -.228\, H_G - .283\, Q_J + .0905\, M_J - 2740 \qquad (6B)$$

COVER FLANGE

$$\delta R_I = \frac{1750\,(10.875)}{29.5}\left[\frac{20.125^2 + 10.875^2}{20.125^2 - 10.875^2} - .3\left(\frac{10.875^2}{20.125^2 - 10.875^2} - 1\right)\right]$$

$$10^6\, \delta R_I = 1290$$

eqn 10A becomes

$$10^6\, \Delta R_I = .159\, Q_c - .192\, H_G + 1290 \qquad (7B)$$

Sum of Moments about D –

$$15.545\, M_D = -15.125\, H_G\, 4.15 - 17.5\overset{P+\Delta P}{(7300)}\,1.766 - 15.125\overset{G}{(6800)}(.610$$

$$- 12.5\, Q_C(3.445) + 12.5\, M_C - \frac{1750\,\pi(10.875)^2(12.5)(3.235)}{2\pi(12.5)}$$

$$15.545\, M_D = -62.8\, H_G - 534000 - 62700 - 43.1\, Q_C + 12.5\, M_C$$

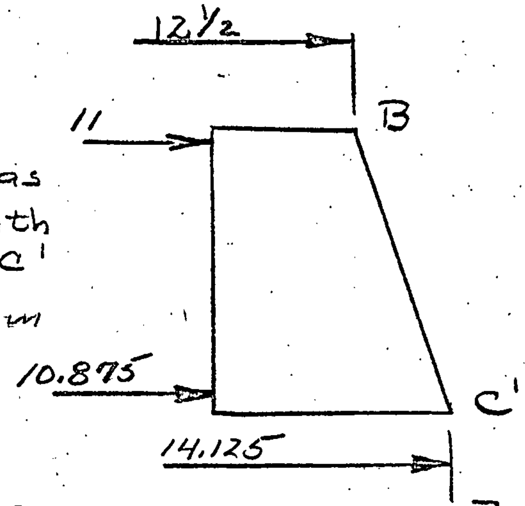
$$- 334000$$

$$M_D = -4.04\, H_G - 2.77\, Q_C + .80\, M_C - 77000$$

From Pg 9

$$10^6\,\theta_D = .044\, M_D$$

$$10^6\,\theta_D = -.178\, H_G - .122\, Q_C + .035\, M_C - 3390 \quad (8B)$$



CYLINDRICAL SECTION –

To obtain deflections due to PRESSURE AT B & C' The Tapered section is Treated as a cylinder in each case with thickness as shown at B & C'

Then using expressions from Roark "FORMULAS FOR STRESS & STRAIN, the following is obtained.

$$\delta R_B = -\frac{1750(11)}{29.5\times10^6}\left[\frac{12.5^2+11^2}{12.5^2-11^2} - .3\left(\frac{11^2}{12.5^2-11^2}-1\right)\right]$$

$$10^6\,\delta R_B = -4680$$

EQN, 11 A NOW BECOMES

$$10^6\,\Delta R_B = C_1 + C_3 - 4680 \quad\text{———— (9B)}$$

$$\delta R_{c'} = -\frac{1750(10.875)}{29.5}\left[\frac{14.125^2+10.875^2}{14.125^2-10.875^2} - .3\left(\frac{10.875^2}{14.125^2-10.875^2}-1\right)\right]$$

$$10\,\delta R_{c'} = -2440$$

EQN. 15 A NOW BECOMES

$$10^6\,\Delta R_{c'} = -1.595C_1 - .0408\,C_3 + 6.05\,C_2 + .1551\,C_4 - 2440 \quad\text{— (10B)}$$

Summary of Equat's. —

ELLIPT. HD: $10^6 \Delta R_A = -.0376\, M_A - .495\, Q_A + 6550$

$10^6\, \theta_A = .0056\, M_A + .0376\, Q_A$

CYL SECT (TOP): $10^6 \Delta R_B = C_1 + C_3 - 4680$

$10^6\, \theta_B = .253\, C_1 + .253\, C_4 + .253\, C_2 - .253\, C_3$

$10^6\, M_B = -3.52\, C_2 + 3.52\, C_4$

$10^6\, Q_B = -.875\, C_4 + .875\, C_1 - .875\, C_2 - .875\, C_3$

CYL SECT (BOTTOM): $10^6\, \Delta R_C' = -1.595\, C_1 - .0408\, C_3 + 6.05\, C_2 + .1551\, C_4 - 2440$

$10^6\, \theta_C' = -1.93\, C_1 - .0495\, C_4 + 1.13\, C_2 - .0289\, C_3$

$10^6\, M_C' = 21.3\, C_1 - .546\, C_3 + 5.61\, C_2 - .144\, C_4$

$10^6\, Q_C' = 3.90\, C_1 - .10\, C_4 + 6.66\, C_2 + .171\, C_3$

UPPER FLANGE: $10^6\, \theta_D = -.18\, H_G - .12\, Q_C + .035\, M_C - 3390$

$10^6\, \Delta R_I = .159\, Q_C - .192\, H_G + 1290$

LOWER FLANGE: $10^6\, \Delta R_{F_I} = .262\, H_G - .285\, Q_j + 2450$

$10^6\, \Delta R_{F_J} = .216\, H_G - .257\, Q_j + 1865$

$10^6\, \theta_F = -.228\, H_G - .283\, Q_J + .0905\, M_j - 2740$

VESSEL CYL: $10^6\, \Delta R_J = -.326\, M_J - 1.84\, Q_J - 7735$

$10^6\, \theta_J = .0115\, M_J + .327\, Q_J$

COMBINING ABOVE EQUAT'S. WITH BOUNDARY CONDITIONS ON PG. 14 (which also hold for CASE "B"), 11 EQUAT'S. WITH 11 UNKNOWNS ARE OBTAINED.

— EQUATIONS —

$-.0376 M_A - .495 Q_A - C_1 - C_3 = -11230$ — (1)

$.0056 M_A + .0376 Q_A + .253 C_1 + .253 C_4 + .253 C_2 - .253 C_3 = 0$ — (2)

$1.595 C_1 + .0408 C_3 - 6.05 C_2 - .1551 C_4 - .428 H_G - .569 Q_c + .12 M_c = 10530$ (3)

$Q_A - .875 C_4 + .875 C_1 - .875 C_2 - .875 C_3 = 0$ — (4)

$1.93 C_1 + .0495 C_4 - 1.13 C_2 + .0298 C_3 + .18 H_G + .12 Q_c - .035 M_c = -3390$ (5)

$-.339 Q_c - 1.963 H_G - .659 Q_j + .145 M_c + .303 M_j = 24420$ — (6)

$.043 M_j + 2.98 Q_j + .497 H_G = -14430$ — (7)

$-.228 H_G + .044 Q_j + .2063 M_j = 2740$ — (8)

$M_A + 3.52 C_2 - 3.52 C_4 = 0$ — (9)

$-M_c - 21.3 C_1 + .546 C_3 - 5.61 C_2 + .144 C_4 = 0$ — (10)

$Q_c + 3.90 C_1 - .10 C_4 + 6.66 C_2 + 1.71 C_3 = 0$ — (11)

MACHINE RESULTS of ABOVE EQN'S

$C_1 = -.779$	$M_A = 20733$
$C_2 = -493$	$Q_c = -5816$
$C_3 = 6118$	$M_c = 23482$
$C_4 = 5397$	$H_G = -7595$
$Q_A = 10326$	$M_J = 5668$

$Q_J = -3657$

CHECK OF BOUNDARY CONDITIONS —

1) $\Delta R_A = \Delta R_B$

$$-.0376(\overset{-780}{\underset{M_A}{20733}}) - .495(\overset{-5110}{\underset{Q_A}{10326}}) + 6550 =$$

$$-\underset{C_1}{779} + \underset{C_3}{6118} - 4680$$

$$+659 = 659$$

2) $M_A = M_B$ -

$$20733 = -3.52(\overset{-5890}{\underset{C_2}{-493} - \underset{C_4}{5397}})$$

$$20733 = 20730$$

3) $Q_A = -Q_B$

$$10326 = +.875^{.11801}(\underset{C_4}{5397} - \underset{-C_1}{(-779)} \underset{+C_2}{- 493} \underset{+C_3}{+ 6118})$$

$$10326 = +10320$$

4) $\theta_A = -\theta_B$

$$.0056(\overset{116}{\underset{M_A}{20733}}) + .0376(\overset{389}{\underset{Q_A}{10326}}) = -.253(\overset{-1993}{\underset{C_1}{-779} + \underset{C_4}{5397} \underset{C_2}{-493} \underset{-C_3}{-6118}})$$

$$505 = 503$$

5) $\theta_D = -.18(\underset{H_G}{-7595}) - .12(\underset{Q_c}{-5816}) + .035(\underset{M_c}{23483}) - 3390$

$$= +1367 + 698 + 822 - 3390$$

$$\theta_D = -503 \checkmark$$

6) $-\Delta R_{C'} = \Delta R_c = \Delta R_I - 3.445\,\theta_D$

$-\Big[-1.595(\overset{+1245}{\underset{C_1}{-779}}) - .0408(\overset{-250}{\underset{C_3}{6118}}) + 6.05(\overset{-2980}{\underset{C_2}{-493}}) + .1551(\overset{+836}{\underset{C_4}{5397}})$

$-2440\Big] = .159(\overset{-924}{\underset{\Theta_C}{-5816}}) - .192(\overset{+1460}{\underset{H_G}{-7595}}) + 1290$

$-3.445(\overset{+1733}{\underset{\Theta_D}{-503}})$

$[-\Delta R_{c'} = +3589] = [\Delta R_I = 1826] + 1733$

$3589 = 3559$

7) $-\theta_c' = \theta_D$

$-\Big[-1.93(\overset{+1503}{\underset{C_1}{-779}}) - .0495(\overset{-267}{\underset{C_4}{5397}}) + 1.13(\overset{-557}{\underset{C_2}{-493}}) - .0289(\overset{-177}{\underset{C_3}{6118}})\Big]$

$= -502$

$-503. = -503$

8) $\Delta R_G = \Delta R_I + 4.15\,\theta_D = \Delta R_{F_I} - 3.345\,\theta_F$

where $\theta_F = -.228(\overset{1730}{\underset{H_G}{-7595}}) - .283(\overset{+1035}{\underset{Q_J}{-3657}}) + .0905(\overset{+514}{\underset{M_J}{5668}})$

-2740

$\theta_F = 540$

$\Delta R_{F_I} = .262(\overset{-1990}{\underset{H_G}{-7595}}) - .285(\overset{+1045}{\underset{Q_J}{-3657}}) + 2450$

$\Delta R_{F_I} = 1505$

Subs in $\Delta R_I + 4.15\,\Theta_D = \Delta R_{F_I} - 3.345\,\Theta_F$

$$\underset{-2095}{1826 + 4.15(-503)} = \underset{-1800}{1505 - 3.345(540)}$$

$$-269 = -295$$

9) $-\Delta R_J = \Delta R_{F_J} + 3.125\,\Theta_F$

$$-[\underset{-1848}{-.326(\underset{M_J}{5668})} - \underset{+6729}{1.84(\underset{Q_J}{-3657})} - 7735] =$$

$$\underset{-1640}{.216(\underset{H_G}{-7595})} - \underset{+940}{.257(\underset{Q_J}{-3657})} + 1865 + \underset{+1685}{3.125(\underset{\Theta_F}{539})}$$

$$[-\Delta R_J = 2854] = [\Delta R_{F_J} = 1165] + 1688$$

$$2854 = 2853$$

10) $M_C = -M_{C'}$

$$23483 = -[\underset{-16600}{21.3(\underset{C_1}{-779})} - \underset{-3340}{.546(\underset{C_3}{6118})} + \underset{-2770}{5.61(\underset{C_2}{-493})} - \underset{-775}{.144(\underset{C_4}{5397})}]$$

$$23483 = 23485$$

11) $Q_C = Q_{C'}$

$$-5816 = \underset{-3040}{3.90(\underset{C_1}{-779})} - \underset{-540}{.10(\underset{C_4}{5397})} + \underset{-3290}{6.66(\underset{C_2}{-493})} + \underset{1050}{.171(\underset{C_3}{6118})}$$

$$-5816 = -5820$$

12. $\theta_J = -\theta_F$

$$.1158 (\underbrace{5668}_{M_J}) + .327 (\underbrace{-3657}_{\theta_J}) = -539$$

$$656 - 1195 = -539$$

$$-539 = -539$$

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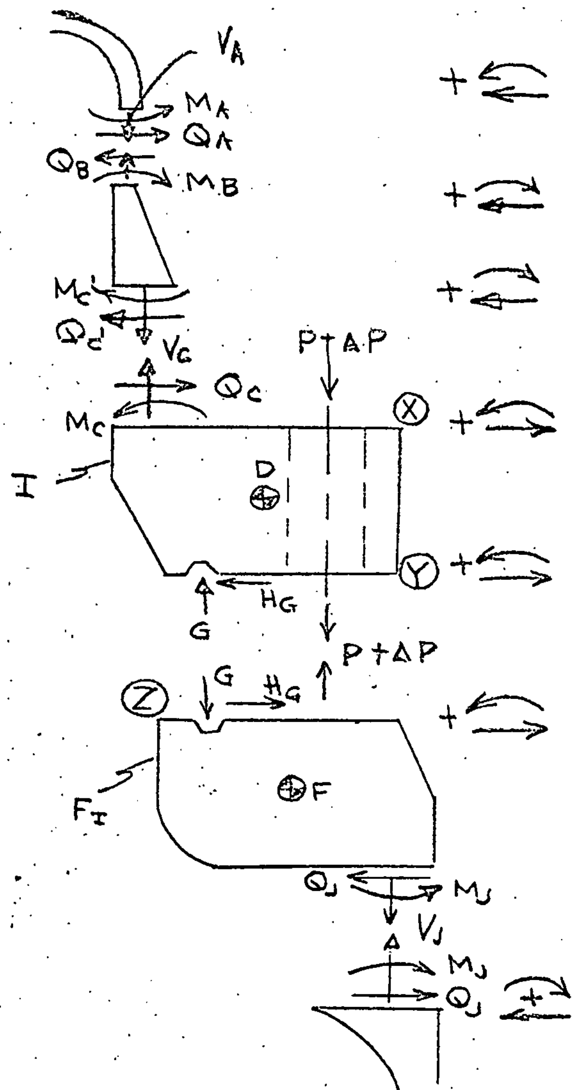
— SUMMARY —

CASE B - STUD LOAD PLUS PRESSURE



RESULTS

$Q_A = Q_B = 10{,}326$

$M_A = M_B = 20{,}733$

$Q_C' = Q_C = -5{,}816$

$M_C = -M_C' = 23{,}483$

$H_G = -7{,}595$

$M_J = 5{,}668$

$Q_J = -3{,}657$

LOCATION	DEFLECTION ΔR 10^6 IN	ROTATION θ 10^6 RAD	STRESS (PSI) AXIAL σ_1	CIRCUMF σ_2	RADIAL σ_3	SHEAR
A (20)	659	505	−55200 OUTER +55200 INNER	−2,100 +2,100	0	
C' (19)	−3589	503	+11350 OUTER −11350 INNER	+6280 −520	0	
I (16)	1826	−503	—	4950		
G	−234	—	—	—	—	−10100
J (13)	−2854	−540	−5000 OUTER +5000 INNER	−8680 −5680	0	
X (18)	1998	−503	—	2940	—	
Y (17)	−1715	−503	—	−2520	—	
Z (15)	−304	540	—	−643	—	
F_I (14)	1505	540	—	3170	—	

STRESSES FOR CASE B8

A) AXIAL:

$$\sigma_1 = \pm \frac{6M_A}{h^2} = \frac{6(20733)}{(1.5)^2} = 55200 \text{ psi} \begin{matrix} -\text{OUTER} \\ +\text{INNER} \end{matrix}$$

CIRCUMF:

$$\sigma_2 = -\frac{Pa}{h} - \frac{Ew}{a} \pm \frac{\nu 6 M_A}{h^2} \quad \text{where } w = \Delta R_{A\,DISC}$$

$$w = \Delta R_{A\,DISC} = \Delta R_{A_{TOT}} - \delta_{RA}$$

$$= 659 - 6550$$

$$= -5891$$

$$= -\frac{(1750)(12)}{1.5} - \frac{29.5(-5891)}{12} \pm .3(55200)$$

$$= -14000 + 14500 \pm 16600$$

$$= -14000 + \begin{bmatrix} +21{,}100 \text{ psi} & \text{INNER} \\ -2100 \text{ psi} & \text{OUTER} \end{bmatrix}$$

C') AXIAL:

$$\sigma_1 = \pm \frac{6 M_{c'}}{h^2} = \pm \frac{6(23483)}{(3.52)^2} = 11350 \text{ psi} \begin{matrix} +\text{OUTER} \\ -\text{INNER} \end{matrix}$$

CIRCUMF:

$$\sigma_2 = \frac{Pa}{h} - \frac{Ew}{a} \pm \frac{\nu 6 M_c}{h^2} \quad \text{where } w = \Delta R_{c'_{DISC}}$$

$$\Delta R_{c'_{DISC}} = \Delta R_{TOT} - \delta_{c'}$$

$$= -3589 - (-2440)$$

$$= -1149$$

$$\sigma_2 = \frac{(1750)(11.75)}{1.5} - \frac{29.5(-1149)}{11.75} \pm .3(11350)$$

$$\sigma_2 = 13700 + \left[2880 \pm 3400 \right]$$

$$\sigma_2 = 13700 + \begin{bmatrix} -520 \text{ psi} & \text{INNER} \\ +6280 \text{ psi} & \text{OUTER} \end{bmatrix}$$

G) SHEAR STRESS IN GASKET $= \frac{H_G}{.75} = \frac{7595}{.75} =$ 10100 PSI

F_I) CIRCUMF:

$$\sigma_2 = \frac{\Delta R_{F_I}}{R_{F_I}} \times E = \frac{1505}{14} \times 29.5 = 3170 \text{ PSI}$$

J) AXIAL:

$$\sigma_1 = \pm \frac{6M_J}{h^2} = \pm \frac{6(5666)}{(2.625)^2} = 5000 \text{ PSI} \quad \begin{matrix} -\text{OUTER} \\ +\text{INNER} \end{matrix}$$

Circumf:

$$\sigma_2 = \frac{Pa}{h} - \frac{Ew}{a} \pm \frac{\nu 6M_J}{h^2}$$

where w = discontinuity deflection ΔR_J

$$\Delta R_{J_{TOT}} = \Delta R_{J_{DISC.}} + \delta_{R_J}$$

$$-2854 + 7735 = \Delta R_{J_{DISC}} = 4881$$

$$\sigma_2 = \frac{(1750)(20.0625)}{2.625} - \frac{(29.5)(4881)}{20.0625} \pm .3(5000)$$

$$\sigma_2 = 13300 + [-7180 \pm 1500$$

$$\sigma_2 = 13300 + \begin{bmatrix} -8680 \text{ psi outer} \\ -5680 \text{ psi inner} \end{bmatrix}$$

RADIAL:

$$\sigma_3 = 0$$

I) CIRCUMF:

$$\sigma_2 = \frac{\Delta R_I}{R_I} \times E = \frac{1826}{10.875} \times 29.5 = 4950 \text{ PSI}$$

PT X —

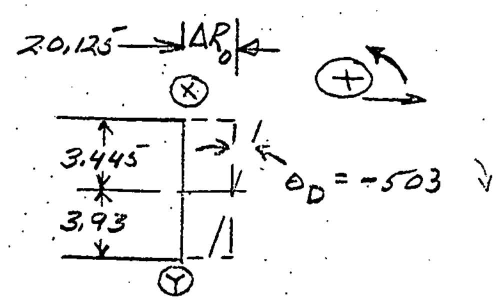
DISCONTINUITY DEFLECTION of OUTSIDE SURFACE ΔR_o

$10^6 \Delta R_o = .0875\, Q_c - .102\, H_G$ (from SHT. 24 CASE A)

$-510 \qquad\qquad +775$

$= .0875(-5816) - .102(-7595)$

$10^6 \Delta R_o = 265$



$10^6 \Delta R_x = \Delta R_o - 3.445\, \theta_D$

$+1733$

$= 265 - 3.445(-503)$

$10^6 \Delta R_x = 1998$

STRESS AT X

$\sigma_z = \frac{\Delta R_x}{R_x} \times E = \frac{1998}{20.125} \times 29.5 = \underline{\underline{2940 \text{ PSI}}}$

PT Y —

$10^6 \Delta R_y = \Delta R_o + 3.93\, \theta_D$

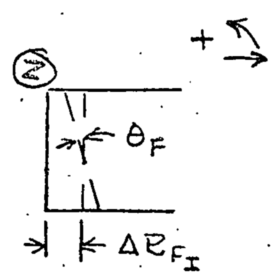
$+1910$

$= 265 - 3.93(-503)$

$10^6 \Delta R_y = -1715$

STRESS AT Y

$\sigma_z = \frac{\Delta R_y}{R_y} \times E = \frac{-1645}{20.125} \times 29.5 = \underline{\underline{-2520}} \text{ PSI}$



PT Z —

$10^6 \Delta R_z = \Delta R_{F_I} - 3.345\, \theta_F$

-1810

$= 1505 - 3.345(540) = -305$

STRESS AT Z

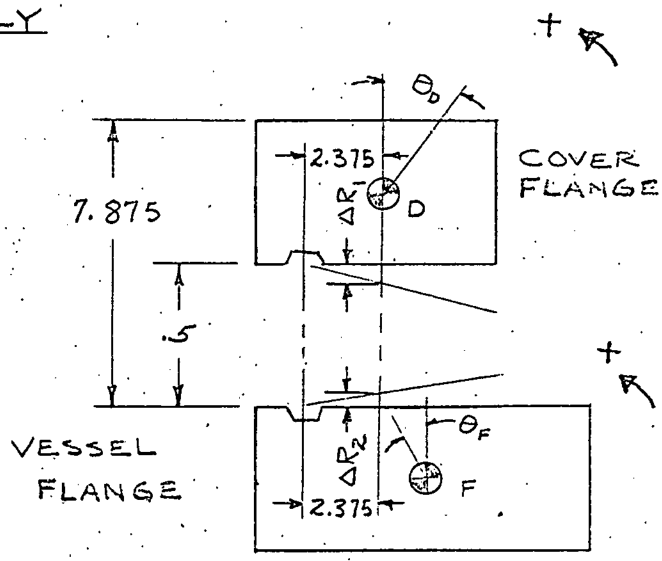
$\sigma_z = \frac{\Delta R_z}{R_z} \times E = \frac{-305}{14} \times 29.5 = \underline{\underline{-643 \text{ PSI}}}$

DEFLECTION OF TOP & BOTTOM FLANGES ALONG CENTERLINE OF BOLT HOLE

CASE A - BOLT LOAD ONLY

$10^6 \Delta R_1 = 2.375\ \theta_D$
$= 2.375\ (.625)$

$\downarrow \Delta R_1 = 1485 \times 10^{-6}$ in



$10^6 \Delta R_2 = 2.375\ \theta_F$
$= 2.375\ (695)$

$\uparrow \Delta R_2 = 1650 \times 10^{-6}$ in

Total decrease in distance between flanges =
$1485 + 1650 = 3135 \times 10^{-6}$ in

CASE B - BOLT LOAD + PRESSURE

$10^6 \Delta R_1 = 2.375\ \theta_D$
$= 2.375\ (.503)$

$\downarrow \Delta R_1 = 1193 \times 10^{-6}$ in

$10^6 \Delta R_2 = 2.375\ (\theta_F)$
$= 2.375\ (540)$

$\uparrow \Delta R_2 = 1280 \times 10^{-6}$ in

Total decrease in distance between flanges =
$1193 + 1280 = 2473 \times 10^{-6}$ in

INCREASE in distance between flanges due to pressure alone $= 3135 - 2475 = 662 \times 10^{-6}$ in

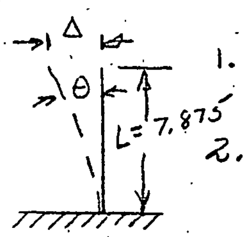
$\therefore$ Stud stresses are increased

$$S = \frac{660 \times 10^{-6}}{7.875}(29.5 \times 10^{6}) = 2480 \text{ psi}$$

STRESSES IN STUD –

	TENSILE
CASE A:	20,000 psi
CASE B:	20,000 + 2480 = 22480 psi Tensile

Bending Stress for Case B



1. RESULTANT ROTATION of STUD ℄ = 540 + 503 = 1043

2. HORIZONTAL DEFLECTION of STUD
$$\Delta = L \times \theta = (7.875)(1043) = 8240 \times 10^{-6} \text{ in.}$$

3. FOR CANTILEVER BEAM
$$\Delta = \frac{WL^3}{3EI} \quad \& \quad \text{MAX. MOM} = -WL \; ; \quad I = \frac{\pi r^4}{4} ; \; r = 1.25$$
$$\frac{\Delta \, 3EI}{L^2} = WL = M$$

$$\frac{(8240)(3)(29.5)(\pi)(1.25)^4}{4(7.875)^2} = M_{MAX} = 22500 \text{ in}\#$$

4. $$S_{Bending} = \frac{Mc}{I} = \frac{(22500)(1.25)(4)}{\pi(1.25)^4} = 14700 \text{ PSI}$$

TOTAL STRESS for CASE B $= 22480 + 14700 = \underline{\underline{37180}}$ PSI

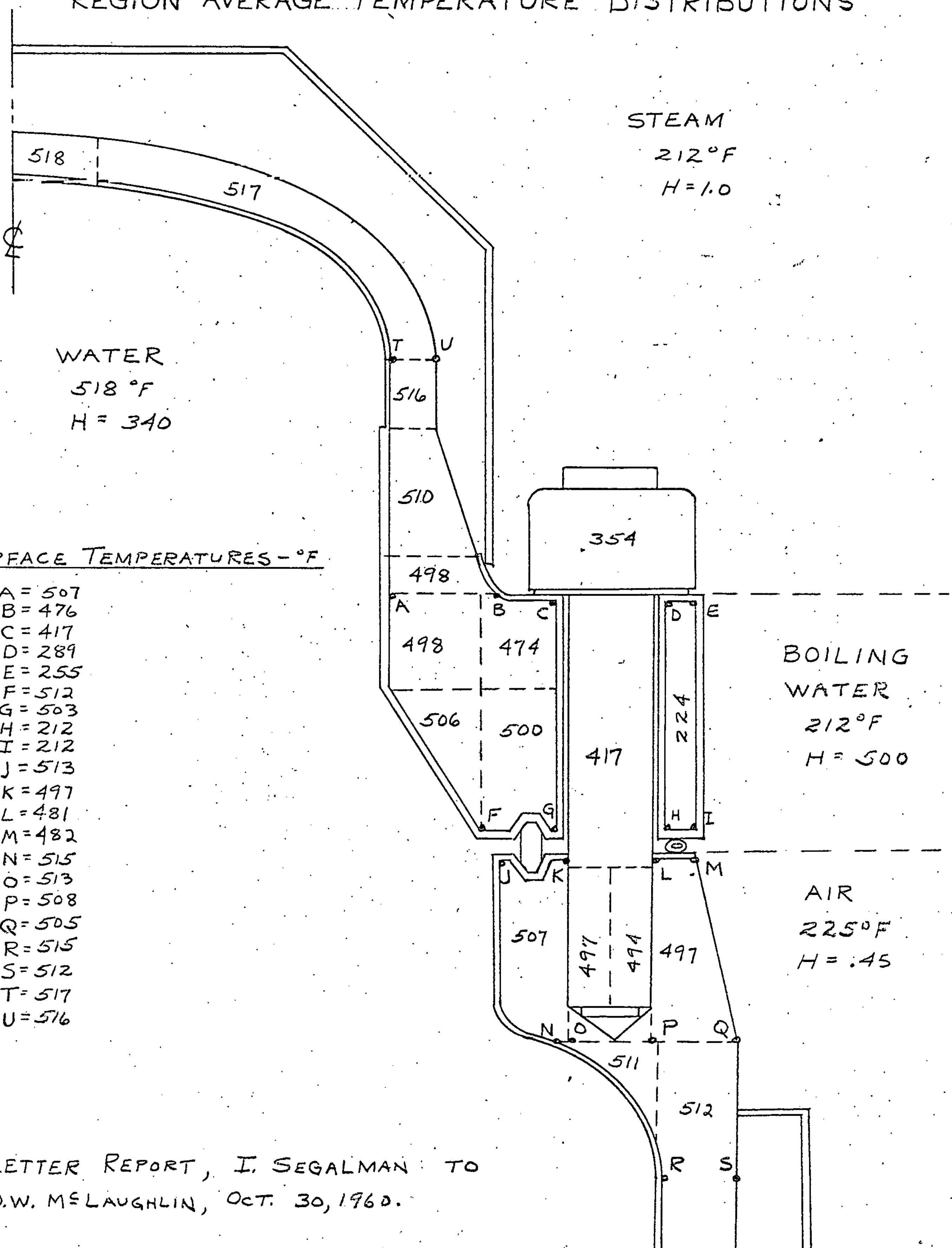
ALCO PRODUCTS INC.

BY RAC DATE 11/6/61 SUBJECT PM-2A VESSEL COVER AND FLANGE SHEET NO. 1 OF 25

CHKD. BY ______ DATE ______ JOB NO. ______

THERMAL STRESS ANALYSIS - STEADY STATE

REGION AVERAGE TEMPERATURE DISTRIBUTIONS*



SURFACE TEMPERATURES - °F

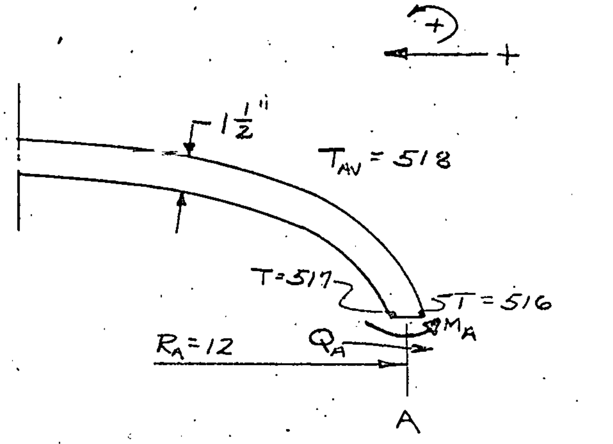
A = 507
B = 476
C = 417
D = 289
E = 255
F = 512
G = 503
H = 212
I = 212
J = 513
K = 497
L = 481
M = 482
N = 515
O = 513
P = 508
Q = 505
R = 515
S = 512
T = 517
U = 516

* LETTER REPORT, I. SEGALMAN TO D.W. McLAUGHLIN, OCT. 30, 1960.

COVER -

Deflection at A due to temperature distribution:

SIGN CONVENTION



$$-\Delta R_A = \alpha_{518}\, \Delta T\, R_A$$

$$= 6.91 \times 10^{-6}\,(518-70)\,12$$

$$10^6 \Delta R_A = 6.91\,(448)\,12 = -37{,}200$$

Including Previous Analysis:

$$10^6 \Delta R_A = -.0376\, M_A - .495\, Q_A - 37{,}200 \qquad (1)$$

Rotation at A due to temperature distribution:

$$M_{TH} = \frac{\Delta T}{2}\,\alpha\,\frac{E}{1-\nu}\,\frac{t^2}{6}$$

Theory of Plates & Shells
— Timoshenko

This equation is the equivalent bending moment at edge A which would produce the same effect as the temperature difference at the edge.

ΔT = Temperature difference across thickness at edge.

E & α are taken at the average temperature at the edge.

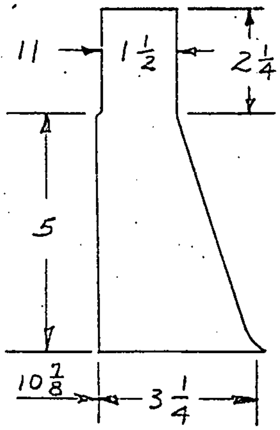
$$M_{TH} = \frac{1}{2}\,\alpha_{518}\,\frac{27 \times 10^6}{.7}\left[\frac{(1.5)^2}{6}\right] = \frac{1}{2}(6.91)(38.6)(.375) = 50$$

Including Previous Analysis:

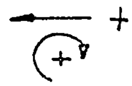
$$10^6 \theta_A = .0376\, Q_A + .0056\,(M_A + 50)$$

$$10^6 \theta_A = .0376\, Q_A + .0056\, M_A + .28 \qquad (2)$$

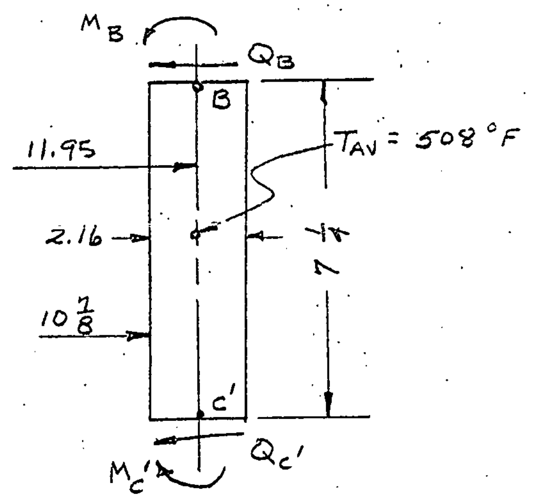
EQUIVALENT CYLINDRICAL SECTION -



SIGN CONV. @ B



SIGN CONV. @ C′



TRANSITION BETWEEN COVER & COVER FLANGE

EQUIVALENT CYLINDRICAL SECTION

Deflection at B:

$$-\Delta R_B = \alpha_{516}\,\Delta T\,R_B = 6.91\times10^{-6}\,(516-70)\,(11.95)$$

$$10^6\Delta R_B = -36{,}800$$

Including Previous Analysis:

$$10^6\Delta R_B = c_1 + c_3 - 36{,}800 \qquad (3)$$

Rotation at B:

Average upper surface temp (at pts. T & U) = 516 °F

Average lower surface temp (at pts. A & B) = 491 °F

$$-\Delta R_{UPPER} = \alpha_{516}\,\Delta T\,R_B = 6.91\times10^{-6}\,(516-70)\,11.95$$

$$10^6\Delta R_U = -36{,}800$$

$-\Delta R_{LOWER} = \alpha_{491}\ \Delta T\ R_{c'} = 6.88 \times 10^{-6}(491-70)(11.95)$

$10^6 \Delta R_L = -34{,}600$

$$10^6\ \theta_B = \frac{\Delta R_{LOWER} - \Delta R_{UPPER}}{7.25} = \frac{-34{,}600 + 36{,}800}{7.25} = 304$$

Including Previous Analysis:

$$10^6 \theta_B = .253(c_1 + c_2 + c_4 - c_3) + 304 \qquad (4)$$

<u>Deflection at c'</u>

$-\Delta R_{c'} = \alpha_{498}\ \Delta T\ R_{c'} = 6.89 \times 10^{-6}(498-70)\ 11.95$

$10^6 \Delta R_{c'} = -35{,}200$

Including Previous Analysis:

$$10^6 \Delta R_{c'} = -1.59\, c_1 - .0408\, c_3 + 6.05\, c_2 + .1551\, c_4 - 35{,}200 \qquad (5)$$

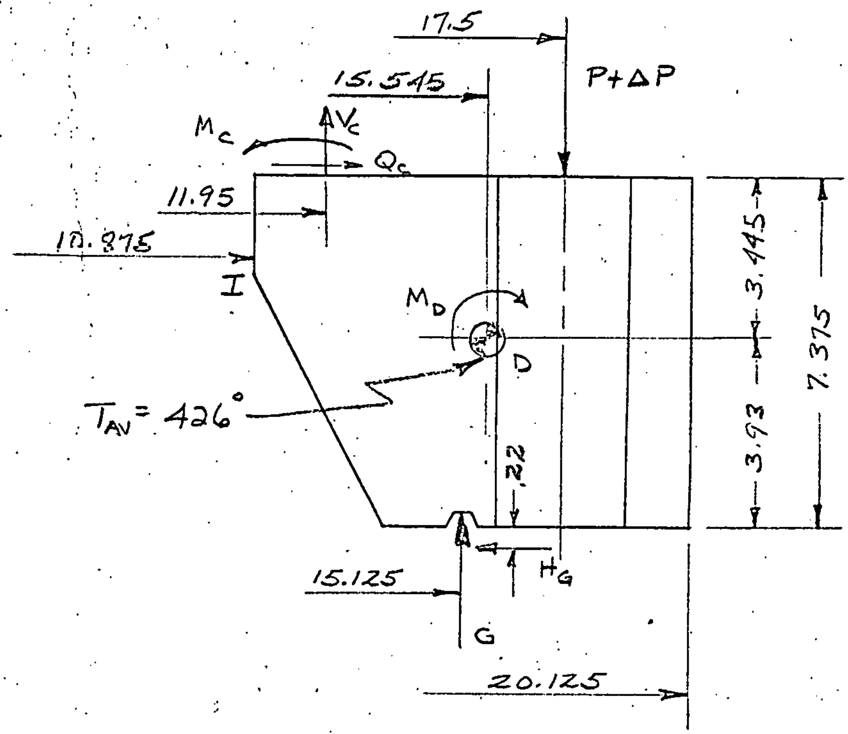
<u>Rotation at C'</u>:

$\theta_{c'} = \theta_B$

$\therefore$ Including Previous Analysis:

$$10^6 \theta_{c'} = -1.93\, c_1 - .0495\, c_4 + 1.13\, c_2 - .0289\, c_3 + 304 \qquad (6)$$

COVER FLANGE



Deflection at I :

$$\Delta R = \alpha_{426} \Delta T R_c$$

$$= 6.72(426-70)(10.875)$$

$$10^6 \Delta R_c = 26000$$

Average Upper Surface Temperature = 395°F

Average Lower Surface Temperature = 380°F

SIGN CONV.

\+ →

(+)

Including Previous Analysis :

$$10^6 \Delta R_I = .159\, Q_c - .192\, H_G + 26{,}000 \quad (7)$$

Rotation at C ($\theta_c = \theta_D$):

$$\Delta R_{UPPER} = \alpha_{395}(395-70)(15.545) = 33{,}800$$

$$\Delta R_{LOWER} = \alpha_{380}(380-70)(15.545) = 32{,}000$$

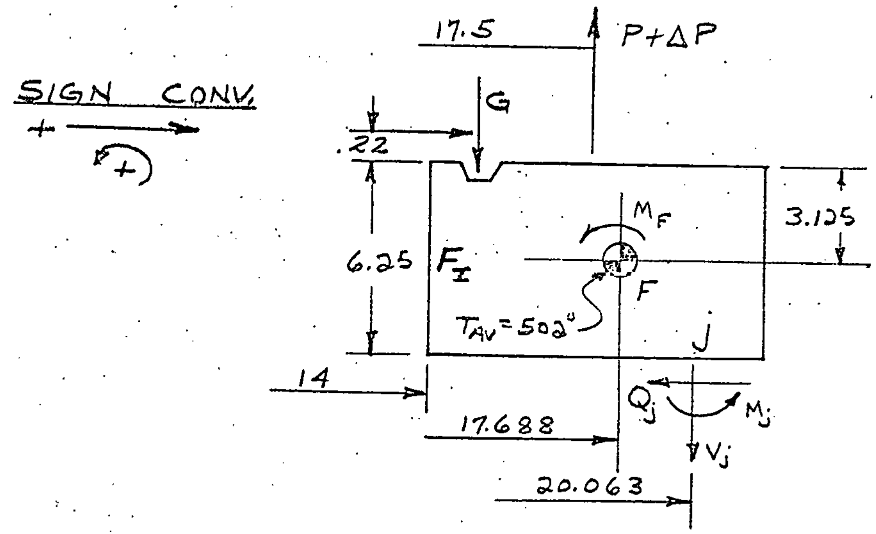
$$10^6 \theta_D = \frac{\Delta R_L - \Delta R_U}{7.375} = \frac{32{,}000 - 33{,}800}{7.375} = \frac{1800}{7.375} = -244$$

Including Previous Analysis :

$$10^6 \theta_D = .044\, M_D - 244$$

$$10^6 \theta_D = -.18\, H_G - .12\, Q_c + .035\, M_c - 244 \quad (8)$$

VESSEL FLANGE



Deflection at I:

$\Delta R_{F_I} = \alpha_{502} \Delta T R_I$

$10^6 \Delta R_{F_I} = 6.89(502-70)\,14$

$10^6 \Delta R_{F_I} = 41{,}700$

Including Previous Analysis:

$10^6 \Delta R_{F_I} = .262\, H_G - .285\, Q_j + 41{,}700$ —— (9)

Deflection at j:

$\Delta R_{F_j} = \alpha_{502} \Delta T R_j = 6.89 \times 10^{-6} (502-70)\, 20.063$

$10^6 \Delta R_{F_j} = 59{,}600$

Including Previous Analysis:

$10^6 \Delta R_{F_j} = .216\, H_G - .257\, Q_j + 59{,}600$ —— (10)

Average temperature of upper surface of Vessel flange = 492°F

Average temperature of lower surface of Vessel flange = 509°F

Rotation at F:

$\Delta R_{UPPER} = \alpha_{492} \Delta T R_F = 6.88 \times 10^{-6} (492-70)(17.688)$

$10^6 \Delta R_U = 51{,}350$

$\Delta R_{LOWER} = \alpha_{509} \Delta T R_F = 6.9 \times 10^{-6} (509-70)(17.688)$

$10^6 \Delta R_L = 53{,}600$

$$10^6\theta_F = \frac{\Delta R_L - \Delta R_U}{6.25} = \frac{53600 - 51350}{6.25} = \frac{2250}{6.25} = 360$$

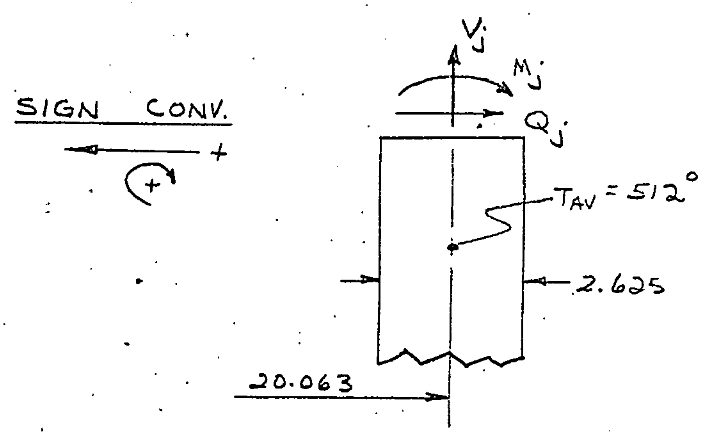
Including Previous Analysis:

$10^6\theta_F = .0697\,M_F + 360$ or

$10^6\theta_F = -.228\,H_G - .283\,Q_j + .0905\,M_j + 360$ ———— ———— (11)

Vessel Wall

Deflection at j:



$-\Delta R_j = \alpha_{512}\,\Delta T\,R_j$

$= 6.91\times10^{-6}(512-70)\,20.063$

$10^6\Delta R_j = -61,100$

Including Previous Analysis:

$10^6\Delta R_j = -.326\,M_j - 1.84\,Q_j - 61,100$ ———— ———— ———— ———— (12)

Rotation at j:

$M_{TH} = \frac{\Delta T}{2}\,\alpha_{507}\,\frac{E}{1-\nu}\,\frac{t^2}{6}$ (see sheet 2 for reference)

$M_{TH} = \frac{3}{2}\,(6.91\times10^{-6})\,\frac{27\times10^6}{.7}\,\frac{(2.625)^2}{6}$

$M_{TH} = 460$

Including Previous Analysis:

$10^6\theta_j = .1158\,(M_j + 460) + .327\,Q_j$ or

$10^6\theta_j = .1158\,M_j + .327\,Q_j + 53$ ———— ———— ———— ———— (13)

SUMMARY OF EQUATIONS

COVER

$10^6 \Delta R_A = -.0376\, M_A - .495\, Q_A - 37{,}200$ (1)

$10^6\, \theta_A = .0376\, Q_A + .0056\, M_A + .28$ (2)

EQUIVALENT CYLINDRICAL SECTION

TOP

$10^6 \Delta R_B = C_1 + C_3 - 36{,}800$ (3)

$10^6 \theta_B = .253\,(C_1 + C_2 + C_4 - C_3) + 304$ (4)

$M_B = -3.52\, C_2 + 3.52\, C_4$ (14)

$Q_B = -.875\, C_4 + .875\, C_1 - .875\, C_2 - .875\, C_3$ (15)

BOTTOM

$10^6 \Delta R_{C'} = -1.595\, C_1 - .0408\, C_3 + 6.05\, C_2 + .1551\, C_4 - 35{,}200$ (5)

$10^6 \theta_{C'} = -1.93\, C_1 - .0495\, C_4 + 1.13\, C_2 - .0289\, C_3 + 304$ (6)

$M_{C'} = 21.3\, C_1 - .546\, C_3 + 5.61\, C_2 - .149\, C_4$ (16)

$Q_{C'} = 3.90\, C_1 - .10\, C_4 + 6.66\, C_2 + .171\, C_3$ (17)

COVER FLANGE

$10^6 \theta_D = -.18\, H_G - .12\, Q_c + .035\, M_c - 244$ (8)

$10^6 \Delta R_I = .159\, Q_c - .192\, H_G + 26{,}000$ (7)

VESSEL FLANGE

$10^6 \Delta R_{F_I} = .262\, H_G - .285\, Q_j + 41{,}700$ (9)

$10^6 \Delta R_{F_j} = .216\, H_G - .257\, Q_j + 59{,}600$ (10)

$10^6 \theta_F = -.228\, H_G - .283\, Q_j + .0905\, M_j + 360$ (11)

VESSEL WALL

$10^6 \Delta R_j = -.326\, M_j - 1.84\, Q_j - 61{,}100$ (12)

$10^6\, \theta_j = .1158\, M_j + .327\, Q_j + 53.$ (13)

BOUNDARY CONDITIONS

$\Delta R_A = \Delta R_B$ (18)

$M_A = M_B$ (19)

$Q_A = -Q_B$ (20)

$\theta_A = -\theta_B$ (21)

$-\Delta R_{c'} = \Delta R_c - 3.445\,\theta_D$ (22)

$-\theta_{c'} = \theta_c = \theta_D$ (23)

$\Delta R_G = \Delta R_I + 4.15\,\theta_D = \Delta R_{F_I} - 3.345\,\theta_F$ (24)

$-\Delta R_j = \Delta R_{F_j} + 3.125\,\theta_F$ (25)

$\theta_F = -\theta_j$ (26)

$-M_{c'} = M_c$ (27)

$Q_{c'} = Q_c$ (28)

Subst. 1 & 3 into 18 – gives

$$c_1 + c_3 + .0376\,M_A + .495\,Q_A = -400 \quad (29)$$

Subst. 2 & 4 into 21 – gives

$$.253\,(c_1 + c_2 + c_4 - c_3) + .0376\,Q_A + .0056\,M_A = -304 \quad (30)$$

Subst. 15 into 20 – gives

$$Q_A - .875\,c_4 + .875\,c_1 - .875\,c_2 - .875\,c_3 = 0 \quad (31)$$

Subst. 5, 7, & 8 into 22 – gives

$$1.595\,c_1 + .0408\,c_3 - 6.05\,c_2 - .1551\,c_4 - .569\,Q_c + .428\,H_G - .121\,M_c = -8359 \quad (32)$$

Subst. 6 & 8 into 23 – gives

$$1.93\,c_1 + .0495\,c_4 - 1.13\,c_2 + .0289\,c_3 + .18\,H_G + .12\,Q_c - .035\,M_c = 60 \quad (33)$$

Subst. 7, 8, 9 & 11 into 24 – gives

$$-.339\,Q_c - 1.963\,H_G + .1453\,M_c - .659\,Q_j + .303\,M_j = 15508 \quad (34)$$

Subst. 10, 11 & 12 into 25 – gives

$$.043\,M_j + 2.98\,Q_j + .196\,H_G = -375 \quad (35)$$

Subst. 11 & 13 into 26 – gives

$$-.228\,H_G + .044\,Q_j + .2063\,M_j = 413 \quad (36)$$

Subst. 16 into 27 – gives

$$M_c - 21.3\,C_1 + .546\,C_3 - 5.61\,C_2 + .144\,C_4 = 0 \quad (37)$$

Subst. 17 into 28 – gives

$$Q_c - 3.9\,C_1 + .10\,C_4 - 6.66\,C_2 - .171\,C_3 = 0 \quad (38)$$

Subst. 14 into 19

$$M_A + 3.52\,C_2 - 3.52\,C_4 = 0 \quad (39)$$

Solving the above 11 equations (29 thru 39) yields :

$Q_A = -2,100$

$M_A = -13,481$

$Q_C = 12,059$

$H_G = -16,249$

$M_C = -28,713$

$Q_J = 2,876$

$M_J = -20,574$

$C_1 = 1,026$

$C_2 = 1,167$

$C_3 = 121$

$C_4 = -2,663$

CHECK OF BOUNDARY CONDITIONS CASE C

1. $\Delta R_A = \Delta R_B$

$$-.0376(\overset{507}{\underset{M_A}{-13481}}) - .495(\overset{1040}{\underset{Q_A}{-2100}}) - 37200 = \underset{C_1}{1026}$$

$$+ \underset{C_3}{121} \quad -36800$$

$$-35653 = -35653$$

$\Delta R_{A_{DISC}} = +1547$

$\Delta R_{B_{DISC}} = +1147$

2. $M_A = M_B$

$$-13481 = -3.52(\overset{-4108}{\underset{C_2}{1167}}) + 3.52(\overset{-9374}{\underset{C_4}{-2663}})$$

$$-13481 = -13482$$

3. $Q_A = -Q_B$

$$= -[-.875(\overset{+2330}{\underset{C_4}{-2663}}) + .875(\overset{+898}{\underset{C_1}{1026}}) - .875(\overset{-1021}{\underset{C_2}{1167}})$$

$$-.875(\overset{-106}{\underset{C_3}{121}})]$$

$$-2100 = -2101$$

4. $-\Delta R_{C'} = \Delta R_I - 3.445\,\theta_D$

$$\theta_D = -.18(\overset{+2925}{\underset{H_G}{-16249}}) - .12(\overset{-1447}{\underset{Q_C}{12059}}) + .035(\overset{-1005}{\underset{M_C}{-28713}}) - 244$$

$\theta_{D_{TOT}} = 229$ $\quad\theta_{D_{DISC}} = +473$

$$\Delta R_I = .159(\overset{1917}{\underset{Q_C}{12059}}) - .192(\overset{+3120}{\underset{H_G}{-16249}}) + 26000$$

$\Delta R_{I_{TOT}} = 31037$

$\Delta R_{I_{DISC}} = +5037$

$$-\Delta R_{c'} = -\Big[\overset{-1636}{-1.595\,(\underset{C_1}{1026})} \overset{-5}{-.0408\,(\underset{C_3}{121})} \overset{7060}{+6.05\,(\underset{C_2}{1167})}$$

$$\overset{-413}{+.1551\,(\underset{C_4}{2663})} - 35200\Big]$$

$$-\Delta R_{c'_{TOT}} = +30194 \qquad \Delta R_{c'_{DISC}} = +5006$$

$$\underset{-\Delta R_{c'}}{30194} = \underset{\Delta R_I}{31037} - 3.445\,(\underset{\theta_D}{229})$$

$$30194 = 30248$$

5. $-\theta_{c'} = \theta_D$

$$-\Big[\overset{-1980}{-1.93\,(\underset{C_1}{1026})} \overset{+132}{-.0495\,(\underset{C_4}{-2663})} \overset{1319}{+1.13\,(\underset{C_2}{1167})}$$

$$\overset{-3}{-.0289\,(\underset{C_3}{121})} + 304 = \underset{\theta_{D_{TOT}}}{229}$$

$$228 = 229$$

6. $\Delta R_G = \Delta R_I + 4.15\,\theta_D = \Delta R_{F_I} - 3.345\,\theta_F$

$$31037 + \overset{950}{4.15\,(\underset{\theta_D}{229})} = \overset{-4257}{.262\,(\underset{H_G}{-16249})} \overset{-820}{-.285\,(\underset{Q_J}{2876})} + 41700$$

$$\overset{-4646}{-3.345\,(\underset{\theta_F}{1389})}$$

$$31987 = 31977$$

where $\Delta R_{F_{I_{DISC}}} = -4257 - 820 = -5077$

$\Delta R_{F_{I_{TOT}}} = +36623$

7) $-\Delta R_J = \Delta R_{F_J} + 3.125\,\theta_F$

$$\theta_F = -.228\underbrace{\overset{+3705}{(-16249)}}_{H_G} - .283\underbrace{\overset{-814}{(2876)}}_{Q_J} + .0905\underbrace{\overset{-1862}{(-20574)}}_{M_J}$$

$+360$

$\theta_{F_{TOT}} = 1389$; $\theta_{F_{DISC}} = +1029$

$$-\Delta R_J = -\left[-.326\underbrace{\overset{+6707}{(-20574)}}_{M_J} - 1.84\underbrace{\overset{-5292}{(2876)}}_{Q_J} - 61100\right]$$

$-\Delta R_{J_{TOT}} = +59685$ $\quad \Delta R_{J_{DISC}} = 6707 - 5292 = +1415$

$$\Delta R_{F_J} = .216\underbrace{\overset{-3510}{(-16249)}}_{H_G} - .257\underbrace{\overset{-739}{(2876)}}_{Q_J} + 59600$$

$\Delta R_{F_J} = -4249 + 59600$

$\Delta R_{F_{J\,TOT}} = 55351$ $\quad \Delta R_{F_{J\,DISC}} = -4249$

$$\underset{-\Delta R_J}{59685} = \underset{\Delta R_{F_J}}{55351} + \overset{4341}{3.125\underset{\theta_F}{(1389)}}$$

$59685 = 59692$

8) $\theta_F = -\theta_J$

$$1389 = -\left[.1158\,(\overset{-2382}{-\underset{A_R}{20574}}) + .327\,(\overset{940}{\underset{\theta_J}{2876}}) + 53\right]$$

$$1389 = -\left[-1389\right]$$

$$1389 = 1389 \qquad \theta_{J_{DISC}} = -1442$$

9) $M_C = -M_{C'}$

$$-28713 = -\left[21.3\,(\overset{21854}{\underset{C_1}{1026}}) - .546\,(\overset{-66}{\underset{C_3}{121}}) + 5.61\,(\overset{6547}{\underset{C_2}{1167}}) - .144\,(\overset{383}{-\underset{C_4}{2663}})\right]$$

$$-28713 = -28718$$

10) $Q_{C'} = Q_C$

$$3.90\,(\overset{4001}{\underset{C_1}{1026}}) - .10\,(\overset{+266}{-\underset{C_4}{2663}}) + 6.66\,(\overset{+7772}{\underset{C_2}{1167}}) + .171\,(\overset{+.21}{\underset{C_3}{121}}) = 12059$$

$$12060 = 12059$$

11) $M_A = M_B$

$$-13481 = -3.52\,(\overset{-4108}{\underset{C_2}{1167}}) + 3.52\,(\overset{-9373}{-\underset{C_4}{2663}})$$

$$-13481 = -13481$$

12) $\theta_A = -\theta_B = -(.253(C_1 + C_2 + C_4 - C_3) + 304)$

$$= -(.253(1026 + 1167 - 2663 - 121) + 304)$$

-591

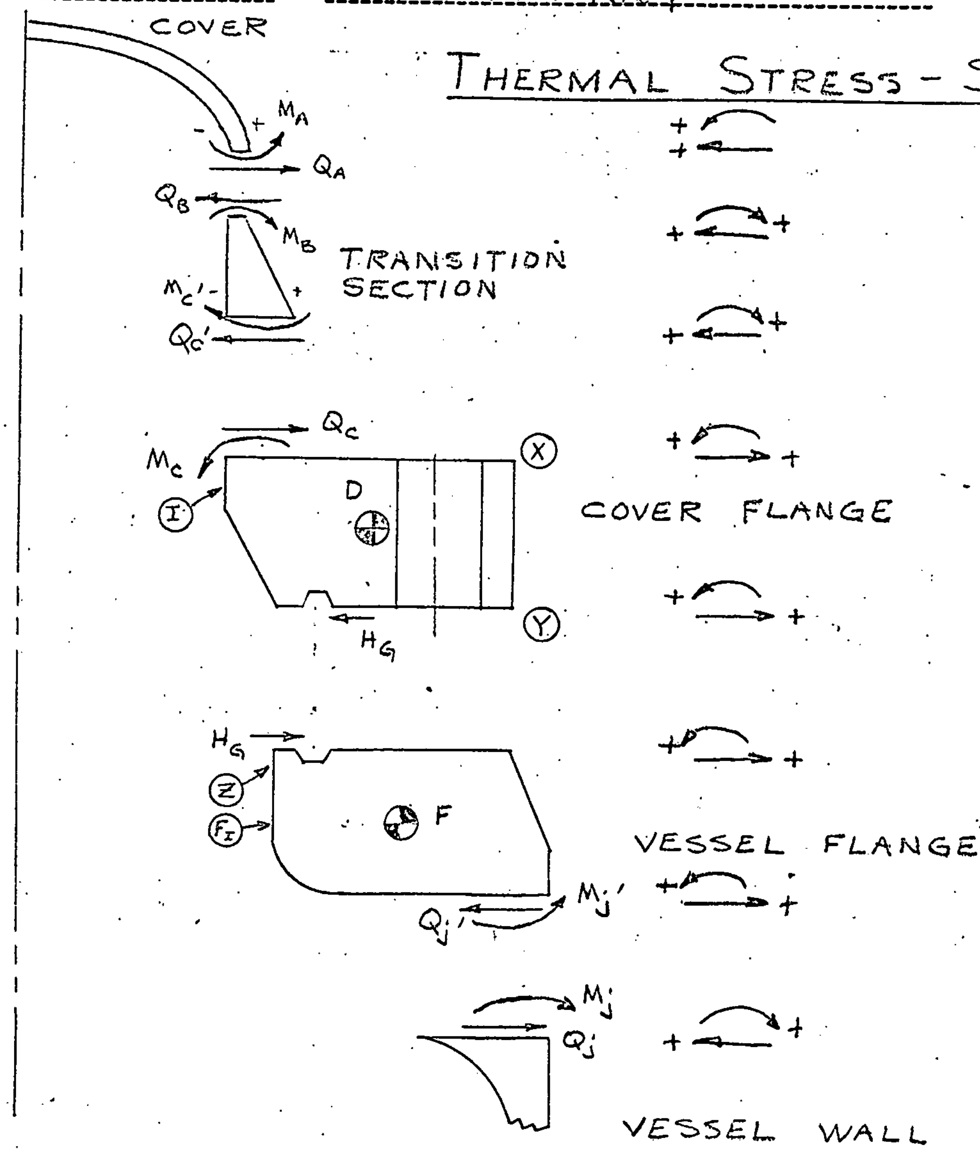
$$= -(-149 + 304) = -155$$

ALCO PRODUCTS INC.

BY RAC DATE 11/8/61 SUBJECT PM-2A STRESS ANALYSIS - AE 90 TASK 6.9 SUMMARY SHEET NO. 16 OF 7–

CHKD. BY ______ DATE ______ JOB NO. ______

THERMAL STRESS - STEADY STATE



LOCATION	STRESS CHART	TOTAL DEFLECTION ΔR (10^{-6} IN.)	TOTAL ROTATION θ (10^{-6} RAD)	STRESSES (PSI) CIRCUMF. (σ_2)	AXIAL (σ_1)	RADIAL (σ_3)	SHEAR
A	20	−35,653	−155	− 14250 INNER + 7290 OUTER	−35900 +35900	0	—
C'	19	−30,194	−229	− 5910 INNER − 15690 OUTER	+16300 −16300	0	—
I	16	31,037	229	12750	—	—	—
G		31,987	—	—	—	—	21,600
F_I	14	36623	1389	−9790	—	—	—
J	13	−59,685	−1389	− 7280 INNER + 3460 OUTER	−17900 +17900	0	—
X	18	49178	229	1475	—	—	—
Y	17	52668	229	6250	—	—	—
Z	15	33403	−1389	− 16000	—	—	—

STRESSES

THE DEFLECTIONS DUE TO DISCONTINUITY FORCES ONLY ARE USED TO FIND CIRCUMFERENTIAL STRESSES —

1. PT. A.

AXIAL $\sigma_1 = \pm \frac{6M_A}{h^2} = \pm 6 \frac{(-13481)}{(1.5)^2} = \begin{array}{l} -35900 \text{ psi INNER} \\ +35900 \text{ psi OUTER} \end{array}$

CIRCUMF: $\sigma_2 = -\frac{\Delta R_{A\,DISC}}{R_A} \times E \pm \frac{\nu 6 M_A}{h^2}$

where $\Delta R_{A_{DISC}} = +.1547$ SHT. 12

$\sigma_2 = -\frac{.1547}{12} \times 27 \pm .3(35900)$

$\sigma_2 = -3480 \pm 10770$

$\sigma_2 = \begin{array}{l} -14250 \text{ psi INNER} \\ +7290 \text{ psi OUTER} \end{array}$

RADIAL: $\sigma_3 = 0$

2. PT C'

AXIAL: $\sigma_1 = \pm \frac{6M_{C'}}{h^2} = \pm 6 \frac{(28713.)}{(3.25)^2} = \begin{array}{l} -16300 \text{ psi OUTER} \\ +16300 \text{ psi INNER} \end{array}$

CIRCUMF: $\sigma_2 = -\frac{\Delta R_{C'DISC}}{R_C} \times E_{498} \pm \frac{\nu 6 M_C}{h^2}$

where $\Delta R_{C'_{DISC}} = +.5006$ SHT. 13

$\sigma_2 = -\frac{.5006}{12.6} \times 27 \pm .3(16300)$

$\sigma_2 = -10,800 \pm 4170 = \begin{array}{l} -15690 \text{ psi OUTER} \\ -5900 \text{ psi INNER} \end{array}$

RADIAL: $\sigma_3 = 0$

3. $P_T G$

SHEAR STRESS IN GASKET $= \frac{H_G}{.75} = \frac{-16249}{.75} = 21600$ PSI

4. $P_T J$

$\sigma_1 = \pm \frac{6 M_J}{h^2} = \pm \frac{6(-20574)}{(2.625)^2} = $ − 17900 PSI INNER, + 17900 PSI OUTER

$\sigma_2 = -\frac{\Delta R_{J\,DISC}}{R_J} \times E_{512} \pm \frac{\nu\, 6 M_J}{h^2}$

where $\Delta R_{J_{DISC}} = +1415$

$\sigma_2 = -\frac{1415}{20.063} \times 27 \pm .3(17900)$

$\sigma_2 = -1910 \pm 5370 = $ − 7280 PSI INNER, + 3460 PSI OUTER

$\sigma_3 = 0$

5. $P_T I$

$\sigma_1 = 0$

$\sigma_2 = \frac{\Delta R_{I\,DISC}}{R_I} \times E_{426}$ where $\Delta R_{I_{DISC}} = +.5037$

$\sigma_2 = \frac{.5037}{10.875} \times 27.5$

$\sigma_2 = +12750$ PSI

6. Pt F_I

$$\sigma_2 = \frac{\Delta R_{F_I\,DISC}}{R_{F_I}} \times E_{SO2} = \left(\frac{-5077}{14}\right)(27)$$

$$\sigma_2 = -\underline{9790}\ \text{psi}$$

7. Pts. X Y & Z

ΔR_o = DEFLECTION of outside SURFACE AT X & Y

$$10^6 \Delta R_o = .0875\, Q_c - .102\, H_G \quad (\text{SHT. 24 CASE A})$$

$$= .0875\overset{1053}{(12059)} - .102\overset{+1655}{(-16249)}$$

$$10^6 \Delta R_o = +2708$$

$$\Delta R_X = \Delta R_o - 3{,}445\, \theta_{D\,DISC}$$

$$= 2708 - \overset{-1630}{3{,}445(473)}$$

$$\Delta R_X = \underline{1078}$$

$$\Delta R_Y = \Delta R_o + 3.93\, \theta_{D\,DISC}$$

$$= 2708 + \overset{1860}{3.93(473)}$$

$$\Delta R_Y = \underline{4568}$$

$$\Delta R_Z = \Delta R_{F_I\,DISC} - 3.125\, \theta_{F\,DISC}$$

$$= -5077 - \overset{-3220}{3.125(1029)}$$

$$\Delta R_Z = -\underline{8297}$$

STRESS AT X

$$\sigma_z = \frac{\Delta R_x}{R_x} \times E_{426} = \frac{1078}{20{,}125} \times 27.5 = \underline{\underline{1{,}475 \text{ psi}}}$$

STRESS AT Y

$$\sigma_z = \frac{\Delta R_Y}{R_Y} \times E_{426} = \frac{4568}{20{,}125} \times 27.5 = \underline{\underline{6250 \text{ psi}}}$$

STRESS AT Z

$$\sigma_z = \frac{\Delta R_Z}{R_Z} \times E_{502} = -\frac{8297}{14} \times 27 = \underline{\underline{-16{,}000 \text{ psi}}}$$

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TOTAL DEFLECTIONS AT CORNERS of FLANGES —

PT X

$$10^6 \Delta R_{X_{TOT}} = \Delta R_{o_{DISC}} - 3.445\, \theta_{D_{DISC}} + \Delta R_{X_{TH}}$$

$$= 1078 + \alpha_{426} \Delta T R_X$$

$$= 1078 + \overbrace{6.72(426-70)(20{,}125)}^{48100}$$

$$10^6 \Delta R_{X_{TOT}} = \underline{49178}$$

PT Y

$$10^6 \Delta R_{Y_{TOT}} = \Delta R_{o_{DISC}} + 3.93\, \theta_{D_{DISC}} + \Delta R_{X_{Th}}$$

$$= 4568 + \alpha_{380} \Delta T R_Y$$

$$= 4568 + \overbrace{6.72(426-70)(20.125)}^{48100}$$

$$10^6 \Delta R_{Y_{TOT}} = \underline{52668}$$

PT Z

$$10^6 \Delta R_{Z_{TOT}} = \Delta R_{F_{I_{DISC}}} - 3.125\, \theta_{F_{DISC}} + \Delta R_{Z_{Th}}$$

$$= -8297 + \alpha_{502} \Delta T R_Z$$

$$= -8297 + \overbrace{6.9(502-70)(14)}^{41700}$$

$$10^6 \Delta R_{Z_{TOT}} = \underline{33{,}403}$$

STUD STRESS

DEFLECTION OF FLANGE SURFACES —

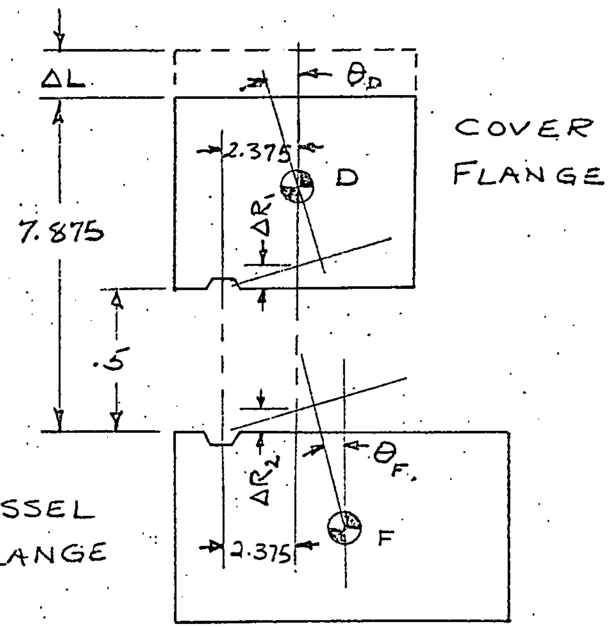
$$10^6 \Delta R_1 = 2.375\ \theta_{D\,TOT.} = 2.375(229)$$

$$\uparrow 10^6 \Delta R_1 = 544$$

$$10^6 \Delta R_2 = 2.375\ \theta_{F\,TOT} = 2.375(1389)$$

$$\uparrow 10^6 \Delta R_2 = 3300$$

DECREASE in distance between flange surface along ℄ of stud $= 544 - 3300 = -2756 \times 10^{-6}$



Note: θ_D & θ_F are total rotations due to discontinuity forces and thermal expansion.

THERMAL EXPANSION OF COVER FLANGE & STUD -

$$\Delta L_{\substack{COVER\\FLANGE}} = \alpha_{426}\,\Delta T\,L_{\substack{COVER\\FLANGE}} = 6.72\times10^{-6}(426-70)(7.375)$$

$$\Delta L_{C.F.} = 17{,}640 \times 10^{-6} \text{ in}$$

$$\Delta L_{STUD} = \alpha_{417}\,\Delta T\,L_{STUD} = 5.55(417-70)(7.875) = 15{,}170\times10^{-6} \text{ IN.}$$

The difference in thermal expansion between the cover flange & stud is

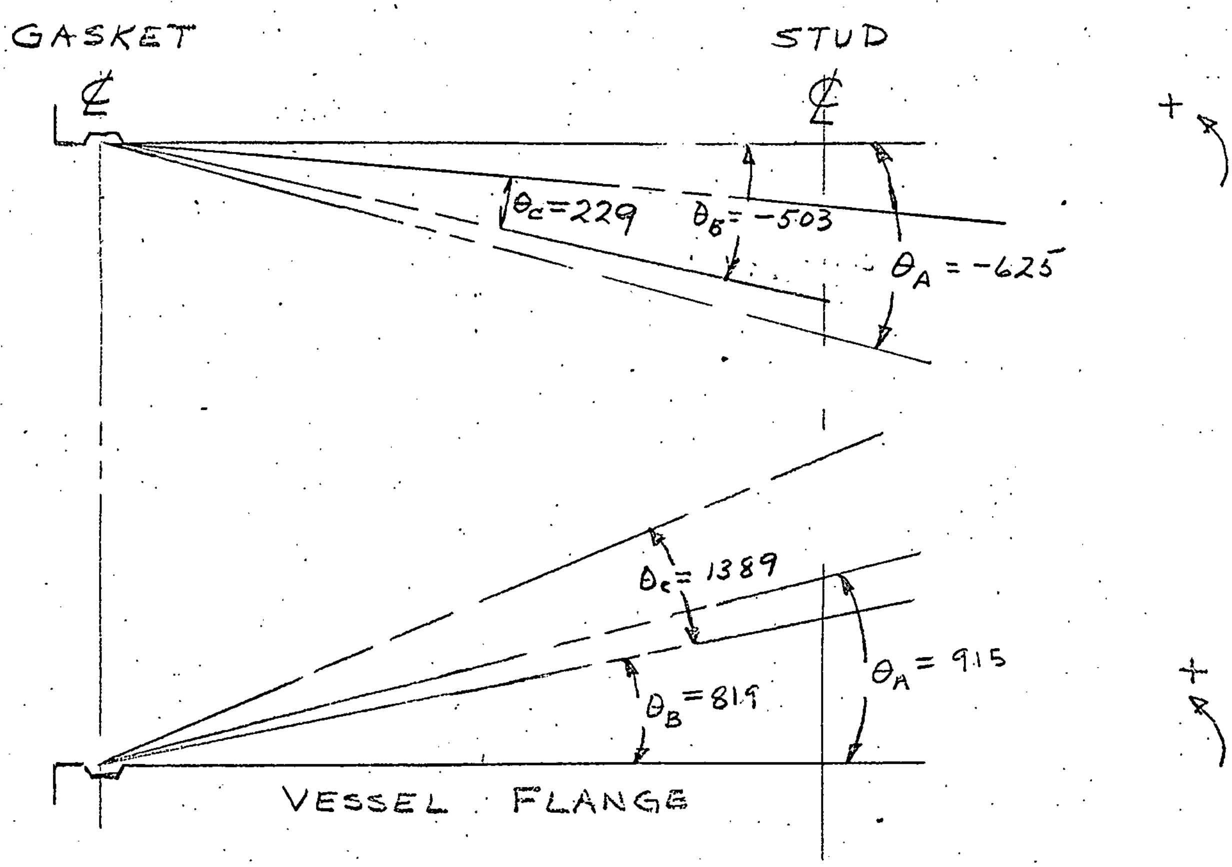
$$\begin{array}{r} 17{,}640 \\ -15{,}170 \\ \hline 2{,}470 \times 10^{-6} \text{ in} \end{array}$$

However, the distance between flange faces has decreased 2759×10^{-6} in

∴ Net RELAXATION in stud length is $2470 - 2756 = -286 \times 10^{-6}$ in

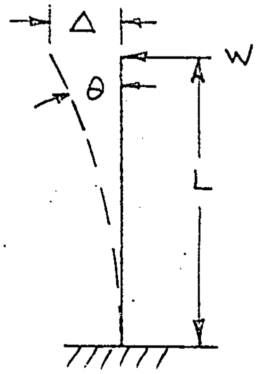
$$\text{stud stress} = \frac{\Delta L}{L} \times E = \frac{-286}{7.875} \times 27 = -980 \text{ psi}$$

BENDING STRESS IN STUD – (Subscripts A, B & C refer to cases A, B & C)



Resultant Rotation of stud ℄ for Case C ONLY $= 1389 - 229$

$= 1160 \times 10^{-6}$



Horizontal Deflection of the stud:

$$\Delta = L\theta = 7.875\,(1160 \times 10^{-6})$$

$$\Delta = 9140 \times 10^{-6}$$

For a cantilever beam -

$$\Delta = \frac{WL^3}{3EI} \qquad \text{where: } I = \frac{\pi}{4} r^4, \; r = 1.25$$

$$M_{max} = -WL$$

$$\therefore\ M_{max} = \frac{3EI\Delta}{L^2} = \frac{9140 \times 10^{-6}\,(27 \times 10^6)(3)(.785)(1.25)^4}{(7.875)^2}$$

$$M_{max} = 22{,}900 \quad \text{in-lbs}$$

$$S = \pm \frac{Mc}{I} \qquad \text{where } c = r$$

$$S = \frac{22900\,(1.25)}{.785\,(1.25)^4} = \frac{28{,}600}{1.915} = 15{,}000 \quad \text{psi}$$

STUD STRESS SUMMARY

CASE	ΔL STUD (IN.)	INCREASE IN STUD STRESS (PSI)
A	—	20,000
B (TENSION)	660×10^{-6}	2,480
(BENDING)		14 700
C (TENSION)	-286×10^{-6}	- 980
(BENDING)		15 000
Max. Total (case B & C plus bending)	—	51,200

BENDING STRESSES IN VESSEL SHELL DUE TO PIPING LOAD ON NOZZLES -

(These are listed as EXTERNAL LOADS ON the stress summary charts)

1. The INLET & OUTLET NOZZLE OUTSIDE DIAMETER ARE DIFFERENT INLET = 17.5"
OUTLET = 14.875"

PER NAVY Code P. 67, the factors β & γ ARE -

$$\beta = \frac{0.875\, r_o}{a} \qquad \gamma = a/t$$

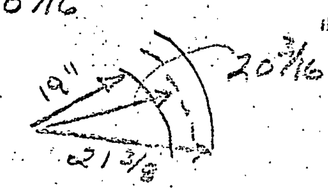
where r_o = outer radius of NOZZLE

$r_o = \frac{17.5}{2} = 8.75''$ INLET

$r_o = \frac{14.875}{2} = 7.4375''$ OUTLET

a = radius of middle surface of shell = $20\frac{3}{16}''$

t = shell thickness = 2.375"



INLET NOZZLE	OUTLET NOZZLE
$\beta = \frac{0.875(8.75)}{20.1875}$	$\beta = \frac{0.875(7.4375)}{20.1875}$
$\beta = .381$	$\beta = .324$
$\gamma = \frac{20.1875}{2.375}$	$\gamma = \frac{20.1875}{2.375}$
$\gamma = 8.5$	$\gamma = 8.5$

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CHKD. BY [illegible] DATE 12/20/60 JOB NO.

2. The following factors are obtained from NAVY Code from the figure numbers Listed—

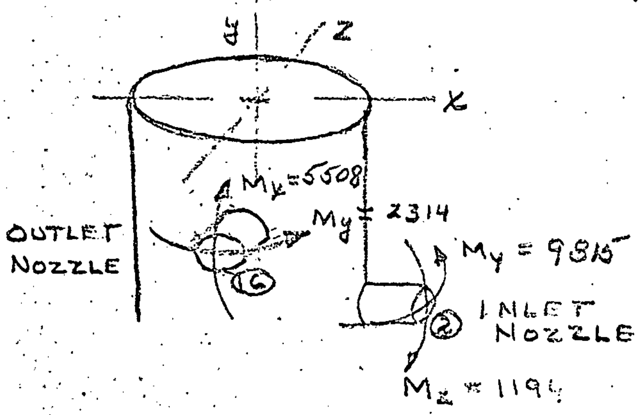
LONGITUDINAL MOMENT

FIG. NO.	FACTOR	INLET NOZZLE $\begin{cases}\beta = .38 \\ \gamma = 8.5\end{cases}$	OUTLET NOZZLE $\begin{cases}\beta = .324 \\ \gamma = 8.5\end{cases}$
A.5-6	$\frac{M_x}{M/a\beta}$	.047	.055
A.5-7	$\frac{M_\phi}{M/a\beta}$	.03	.034
A.5-8	$\frac{N_x}{M/a^2\beta}$	.48	.46
A.5-9	$\frac{N_\phi}{M/a^2\beta}$	.85	.90

CIRCUMFERENTIAL MOMENT

FIG. NO.	FACTOR	INLET NOZZLE	OUTLET NOZZLE
A.5-10	$\frac{M_x}{M/a\beta}$	.045	.049
A.5-11	$\frac{M_\phi}{M/a\beta}$	.083	.089
A.5-1	$\frac{N_x}{M/a^2\beta}$	.75	.60
A.5-13	$\frac{N_\phi}{M/a^2\beta}$	.36	.36

3. MOMENTS ACTING ON VESSEL NOZZLE - REF. PM-2A PRIMARY PIPING STRESS ANALYSIS AP NOTE 204. - NORMAL OPERATING CONDITIONS Pg. 27



INLET NOZZLE

LONGITUDINAL MOMENT = 1194 ft #
CIRCUMF. " " = 9815 ft #

OUTLET NOZZLE

Longitudinal MOMENT = 5508 ft #
CIRCUMF. " " " = 2314 ft #

4. FOR THE INLET NOZZLE -

LONGITUDINAL MOMENT

a) $\dfrac{M_x}{M/a\beta} \times \dfrac{6M}{a\beta t^2} = \dfrac{6M_x}{t^2}$

$a\beta t^2 = (20.1875)(.38)(2.1375)^2 = 43.2$

$.047 \times \dfrac{6M}{43.2} = \dfrac{6M_x}{t^2}$

$.00652 M =$

$.00652\,(1194 \times 12)$

$$\boxed{93.5 = \dfrac{6M_x}{t^2}}$$

b) $\dfrac{M_\phi}{M/a\beta} \times \dfrac{6M}{a\beta t^2} = \dfrac{6M_\phi}{t^2}$

$.03 \times \dfrac{6M}{43.2} =$

$.00418 M =$

$.00418\,(1194 \times 12) = \dfrac{6M_\phi}{t}$

$$\boxed{59.6 = \dfrac{6M_\phi}{t^2}}$$

c) $\frac{N_x}{M/a^2\beta} \times \frac{M}{ta^2\beta} = \frac{N_x}{t}$ $\qquad ta^2\beta = 2.375(20.1875)^2(.38)$
$= 367$

$.48 \times \frac{M}{367} =$

$.00131\ M =$

$.00131\ (1194 \times 12) = \frac{N_x}{t}$

$$\boxed{18.8 = \frac{N_x}{t}}$$

d) $\frac{N_\phi}{M/a^2\beta} \times \frac{M}{ta^2\beta} = \frac{N_\phi}{t}$

$.85 \times \frac{M}{367} =$

$.00232\ M =$

$.00232(1194 \times 12) = \frac{N_\phi}{t}$

$$\boxed{33.2 = \frac{N_\phi}{t}}$$

CIRCUMFERENTIAL MOMENT

a) $\frac{M_x}{M/a\beta} \times \frac{6M}{a\beta t^2} = \frac{6M_x}{t^2}$

$.045 \times \frac{6M}{43.2} =$

$.00625\ M =$

$.00625(9815 \times 12) = \frac{6M_x}{t^2}$

$$\boxed{736.1 = \frac{6M_x}{t^2}}$$

b) $\frac{M\phi}{M/a\beta} \times \frac{6M}{a\beta t^2} = \frac{6M\phi}{t^2}$

$.083 \times \frac{6M}{43.2} =$

$.01155 \; M =$

$.01155(9815 \times 12)$

$$\boxed{1360 = \frac{6M\phi}{t^2}}$$

c) $\frac{N_x}{M/a^2\beta} \times \frac{M}{ta^2\beta} = \frac{N_x}{t}$

$.75 \times \frac{M}{367} =$

$.00204 M =$

$.00204(9815 \times 12)$

$$\boxed{241 = \frac{N_x}{t}}$$

d) $\frac{N\phi}{M/a^2\beta} \times \frac{M}{ta^2\beta} = \frac{N\phi}{t}$

$.36 \times \frac{M}{367} = \frac{N\phi}{t}$

$.00098 \; M =$

$.00098(9815 \times 12)$

$$\boxed{115.5 = \frac{N\phi}{t}}$$

5. FOR THE OUTLET NOZZLE —

LONGITUDINAL MOMENT

a) $\frac{M_x}{M/a\beta} \times \frac{6M}{a\beta t^2} = \frac{6M_x}{t^2}$ $\quad a\beta t^2 = (20.1875)(.324)(2.375)^2 = 36.9$

$.055 \times \frac{6M}{36.9} =$

$.00895\ M =$

$.00895(5508 \times 12)$

$$\boxed{591 = \frac{6M_x}{t^2}}$$

b) $\frac{M_\phi}{M/a\beta} \times \frac{6M}{a\beta t^2} = \frac{6M_\phi}{t^2}$

$.083 \times \frac{6}{36.9} M =$

$.0135\ M =$

$.0135(5508 \times 12)$

$$\boxed{892 = \frac{6M_\phi}{t^2}}$$

c) $\frac{N_x}{M/a^2\beta} \times \frac{M}{ta^2\beta} = \frac{N_x}{t}$ $\quad ta^2\beta = (2.375)(20.1875)^2(.324) = 312$

$.75 \times \frac{M}{312} =$

$.0024\ M =$

$.0024(5508 \times 12)$

$$\boxed{158.5 = \frac{N_x}{t}}$$

d) $\frac{N\phi}{M/a^2\beta} \times \frac{M}{t a^2 \beta} = \frac{N\phi}{t}$

$.36 \times \frac{M}{312} =$

$.00115\ M =$

$.00115(5308 \times 12)$

$$\boxed{76.2 = \frac{N\phi}{t}}$$

CIRCUMFERENTIAL MOMENT

a) $\frac{M_x}{M/a\beta} \times \frac{6M}{a\beta t^2} = \frac{6M_x}{t^2}$ $a\beta t^2 = 36.9$

$.049 \times \frac{6M}{36.9} =$

$.00796\ M =$

$.00796(2314 \times 12)$

$$\boxed{221 = \frac{6M_x}{t^2}}$$

b) $\frac{M\phi}{M/a\beta} \times \frac{6M}{a\beta t^2} = \frac{6M\phi}{t^2}$

$.089 \times \frac{6M}{36.9} =$

$.0145\ M =$

$.0145(2314 \times 12)$

$$\boxed{402 = \frac{6M\phi}{t^2}}$$

c) $\dfrac{N_k}{M/a^2\beta} \times \dfrac{M}{ta^2\beta} = \dfrac{N_k}{t}$ $ta^2\beta = 312$

$.60 \times \dfrac{M}{312} =$

$.001925\, M =$

$.001925(2314 \times 12)$

$$\boxed{53.4 = \frac{N_k}{t}}$$

d) $\dfrac{N_\phi}{M/a^2\beta} \times \dfrac{M}{ta^2\beta} = \dfrac{N_\phi}{t}$

$.36 \times \dfrac{M}{312} =$

$.00115\, M =$

$.00115(2314 \times 12)$

$$\boxed{320 = \frac{N_\phi}{t}}$$

6. STRESS CONCENTRATION FACTORS –
REF. NAVY CODE, FIG A.7-1 PG 92

$\dfrac{\text{WELD FILLET RADIUS}}{\text{SHELL THICKNESS}} = \dfrac{r}{t}$ where $r = 3/8" = .375"$
DWG B-41202-1-208

$= \dfrac{.375}{2.375}$

$= .158$

FROM CURVE – $K_M = 2.25$ ~ BENDING

$K_M = 2.7$ ~ MEMBRANE

7. DETERMINATION of STRESSES –

a) INLET NOZZLE

LONGITUDINAL

$$\sigma_x = K_N \frac{N_x}{t} \pm K_M \frac{6M_x}{t^2}$$

$$= 2.7(18.8) \pm 2.25(93.5)$$

$$50.7 \pm 210$$

$$\boxed{\sigma_x = \begin{cases} 260.7 \\ -159.3 \end{cases}}$$

$$\sigma_\phi = K_N \frac{N_\phi}{t} \pm K_M \frac{6M_\phi}{t^2}$$

$$= 2.7(33.2) \pm 2.25(59.6)$$

$$= 89.6 \pm 134$$

$$\boxed{\sigma_\phi = \begin{cases} 223.6 \\ -44.4 \end{cases}}$$

CIRCUMFERENTIAL

$$\sigma_x = 2.7(241) \pm 2.25(736)$$

$$= 650 \pm 1656$$

$$\boxed{\sigma_x = \begin{cases} 2306 \\ -1006 \end{cases}}$$

$$\sigma_\phi = 2.7(110.5) \pm 2.25(1360)$$

$$312 \pm 3060$$

$$\boxed{\sigma_\phi = \begin{cases} 3372 \\ -2748 \end{cases}}$$

b) OUTLET NOZZLE

$$\sigma_x = K_N \frac{N_x}{t} \pm K_M \frac{6M_x}{t^2}$$

$$= 2.7(158.5) \pm 2.25(591)$$

$$428 \pm 1330$$

$$\boxed{\sigma_x = \begin{cases} 1758 \\ -902 \end{cases}}$$

$$\sigma_\phi = K_N \frac{N_\phi}{t} \pm K_M \frac{6M_\phi}{t^2}$$

$$= 2.7(76.2) \pm 2.25(892)$$

$$206 \pm 2010$$

$$\boxed{\sigma_\phi = \begin{cases} 2216 \\ -1804 \end{cases}}$$

$$\sigma_x = 2.7(53.4) \pm 2.25(221)$$

$$= 144 \pm 497$$

$$\boxed{\sigma_x = \begin{cases} 641 \\ -353 \end{cases}}$$

$$\sigma_\phi = 2.7(32) \pm 2.25(402)$$

$$865 \pm 903$$

$$\boxed{\sigma_\phi = \begin{cases} 989 \\ -817 \end{cases}}$$

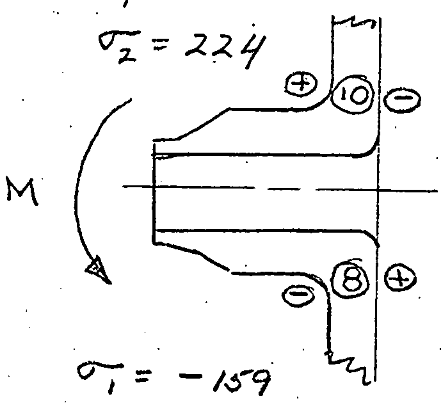
SUMMARY

– INLET NOZZLE –

LONGITUDINAL

OUTSIDE
$\sigma_1 = 261$
$\sigma_2 = 224$

INSIDE
$\sigma_1 = -159$
$\sigma_2 = -44$



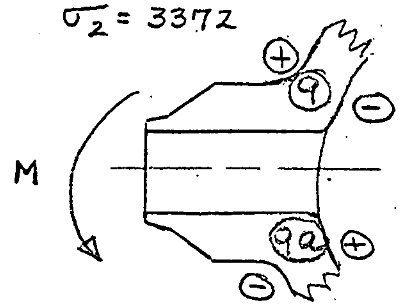
$\sigma_1 = -159$
$\sigma_2 = -44$

$\sigma_1 = 261$
$\sigma_2 = 224$

CIRCUMFERENTIAL

OUTSIDE
$\sigma_1 = 2306$
$\sigma_2 = 3372$

INSIDE
$\sigma_1 = -1006$
$\sigma_2 = -2748$



$\sigma_1 = -1006$
$\sigma_2 = -2748$

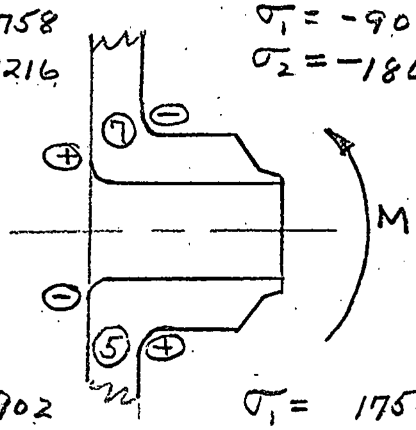
$\sigma_1 = 2306$
$\sigma_2 = 3372$

– OUTLET NOZZLE –

LONGITUDINAL

INSIDE
$\sigma_1 = 1758$
$\sigma_2 = 2216$

OUTSIDE
$\sigma_1 = -902$
$\sigma_2 = -1804$



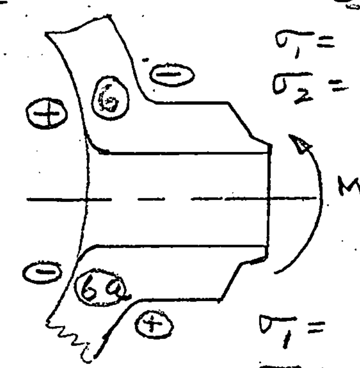
$\sigma_1 = -902$
$\sigma_2 = -1804$

$\sigma_1 = 1758$
$\sigma_2 = 2216$

CIRCUMFERENTIAL

INSIDE
$\sigma_1 = 641$
$\sigma_2 = 989$

OUTSIDE
$\sigma_1 = -353$
$\sigma_2 = -817$



$\sigma_1 = -353$
$\sigma_2 = -817$

$\sigma_1 = 641$
$\sigma_2 = 989$

ALCO PRODUCTS INC.

BY B.R. DATE 12/22/60 — SUBJECT PM-2A STRESS ANALYSIS - BENDING STRESS AT VESSEL SUPPORTS — SHEET NO. 1 OF 11

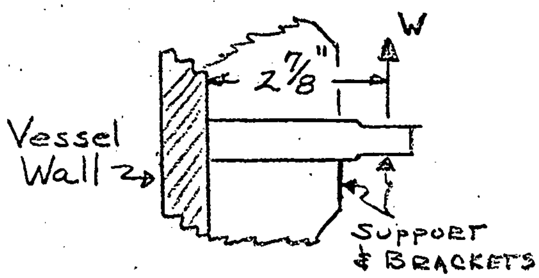
CHKD. BY R.W. DATE 1/25/61 — JOB NO. CONTR. NO. AT(30-1) 2639

(VESSEL SHELL)

BENDING STRESS AT VESSEL SUPPORTS —
(Ref. Dwg F-41202-1-2)

TOTAL WEIGHT OF FLOODED VESSEL including core structure, and elements, etc. } 21,500 lbs.

ASSUME THE 4 SUPPORTS to be at the STRAIGHT PORTION of THE VESSEL CYLINDER.

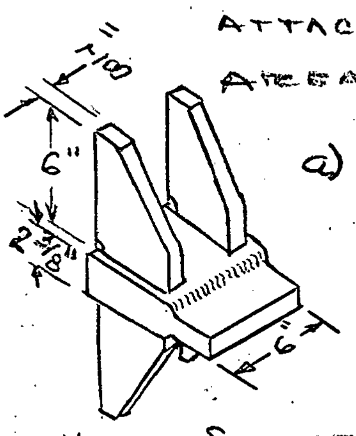


Moment Arm = 2 7/8" { REF. DWGS: R-9-46-1044, C-9-46-2146

$$\text{Moment at each Support} = \frac{21500}{4} \times 2.875$$

$$M = 15500 \text{ in.\#}$$

1. ASSUME A LONGITUDINAL MOMENT ONLY, ACTING ON AN ATTACHMENT TO THE VESSEL which has a circular AREA EQUIVALENT TO THE SUPPORT —



VESSEL SUPPORT

a) AREA of SUPPORT in Contact with Vessel WALL —

$$2.375" \times 6" = 14.25 \text{ in}^2$$
$$4(6" \times .875) = \underline{21.0} \text{ in}^2$$
$$35.25 \text{ in}^2$$

b) RADIUS of CIRCLE of EQUIVALENT AREA —

$$A = \pi r_o^2$$

$$r_o = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{35.25}{\pi}}$$

$$r_o = 3.35$$

2. Per Navy Code

$$\left\{ \begin{array}{l} a = 20\tfrac{3}{16}'' \\ t = 2.375'' \\ r_0 = 3.35'' \end{array} \right.$$

a) $\beta = \dfrac{0.875\, r_0}{a}$; $\gamma = \dfrac{a}{t}$

$\beta = \dfrac{0.875(3.35)}{20.1875}$ $\gamma = \dfrac{20.1875}{2.375}$

$\underline{\beta = .146}$ $\underline{\gamma = 8.5}$

b)

Fig A.5-6 Pg. 72 ~ $\dfrac{M_x}{M/a\beta} = .09$

Fig A.5-7 Pg. 73 ~ $\dfrac{M_\phi}{M/a\beta} = .054$

Fig A.5-8 Pg. 74 ~ $\dfrac{N_x}{M/a^2\beta} = .20$

Fig A.5-9 Pg. 75 ~ $\dfrac{N_\phi}{M/a^2\beta} = .70$

c) $\dfrac{M_x}{M/a\beta} \times \dfrac{6M}{a\beta t^2} = \dfrac{6M_x}{t^2}$

$.09 \times \dfrac{6(15500)}{16.6} = \dfrac{6M_x}{t^2}$

$.09 \times 5600 = \dfrac{6M_x}{t^2}$

$$\boxed{504 = \frac{6M_x}{t^2}}$$

$$\left\{ \begin{array}{l} a\beta t^2 = (20.1875)(.146)(2.375)^2 \\ \quad = 16.6 \\ M = 15{,}500 \text{ in\# from sheet 1} \end{array} \right.$$

d) $\dfrac{M_\phi}{M/a\beta} \times \dfrac{6M}{a\beta t^2} = \dfrac{6M_\phi}{t^2}$

$.054 \times 5600 = \dfrac{6M_\phi}{t^2}$

$$\boxed{303 = \dfrac{6M_\phi}{t^2}}$$

e) $\dfrac{N_x}{M/a^2\beta} \times \dfrac{M}{ta^2\beta} = \dfrac{N_x}{t}$

$ta^2\beta = 2.375(20.1875)^2(.146) = 141$

$.2 \times \dfrac{15500}{141} = \dfrac{N_x}{t}$

$.2 \times 110 = \dfrac{N_x}{t}$

$$\boxed{22 = \dfrac{N_x}{t}}$$

f) $\dfrac{N_\phi}{M/a^2\beta} \times \dfrac{M}{ta^2\beta} = \dfrac{N_\phi}{t}$

$.70 \times 110 =$

$$\boxed{77.0 = \dfrac{N_\phi}{t}}$$

3. DETERMINATION OF STRESSES –

$$\sigma_i = K_N \frac{N_i}{t} \pm K_M \frac{6M_i}{t^2}$$

a) STRESS CONCENTRATION FACTORS K_N & K_M
REF. NAVY CODE FIG A7-1 PG 92

$\dfrac{r}{t} = \dfrac{\text{fillet radius}}{\text{Shell thickness}} = \dfrac{.375}{2.375} = .158$

from curve $K_M = 2.26$ ~ BENDING STRESS

$K_N = 2.7$ ~ MEMBRANE STRESS

b) DETERMINATION of STRESSES —

$$\sigma_x = K_N \frac{N_x}{t} \pm K_m \frac{6M_x}{t^2} \qquad \sigma_\phi = K_N \frac{N_\phi}{t} \pm \frac{6M_\phi}{t^2}$$

$$\sigma_x = 2.7(22) \pm 2.26(504) \qquad = 2.7(77) \pm 2.26(303)$$

$$\sigma_x = 59.5 \pm 1139 \qquad = 208 \pm 685$$

$$\sigma_x = \begin{cases} 1198.5 \text{ PSI} \\ -1079.5 \text{ PSI} \end{cases} \qquad \sigma_\phi = \begin{cases} 893 \text{ PSI} \\ -477 \text{ PSI} \end{cases}$$

SUMMARY

		PT. 1	PT 2
Longitudinal — (σ_1)			
σ_1 =	-1079.5 psi	INSIDE	OUTSIDE
σ_1 =	1198.5 psi	OUTSIDE	INSIDE
Circumferential — (σ_2)			
σ_2 =	-477 psi	INSIDE	OUTSIDE
σ_2 =	893 psi	OUTSIDE	INSIDE

BY VB/R DATE 1/23/61	SUBJECT PM-2A STRESS ANALYSIS - VESSEL & CLADDING STRESS	SHEET NO. 1 OF 12
CHKD. BY ____ DATE ____	AE 90 TASK 6.9	JOB NO. CONTR NO AT(30-1) 2639

PM-2A VESSEL DATA

PREHEAT TEMPERATURE WELDING - 250°F

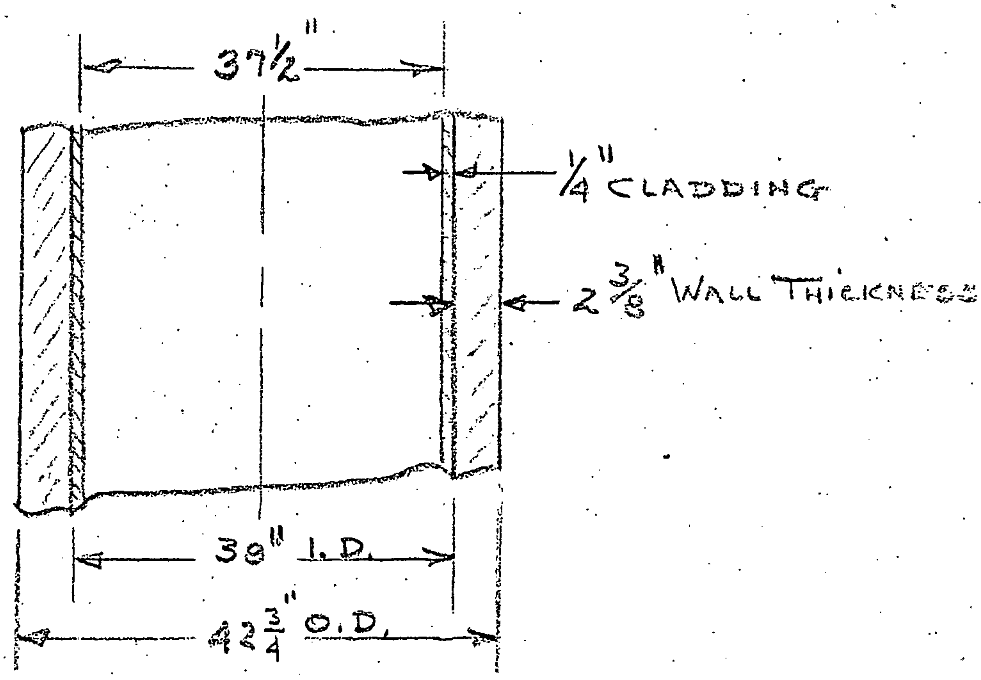
STRESS RELIEVING TEMPERATURE - 1150°F

HYDROSTATIC TEST PRESSURE - 3000 PSI

OPERATING TEMPERATURE (AV) - 510°F

VESSEL MATERIAL - SA 350 Gr LF-3

CLADDING MATERIAL - 304 S.S.

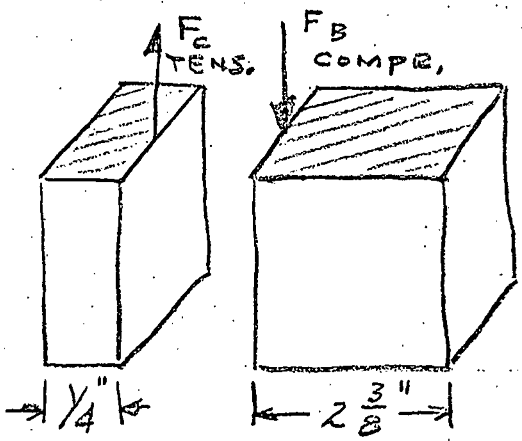


1. STRESS IN CLADDING DUE TO WELDING —

THE WELDED OVERLAY IS APPLIED TO THE VESSEL AT THE MELTING TEMPERATURE OF THE STAINLESS STEEL AND AS THE CLADDING COOLS to ROOM TEMPERATURE, IT IS STRESSED TO ITS YIELD STRENGTH IN TENSION AT 80°F IS 30000 psi.

$\therefore S_c$ AFTER WELDING = 30000 PSI

2. STRESS IN BASE MATERIAL DUE TO WELDING —



ASSUME UNIFORMLY DISTRIBUTED LOAD ACROSS CLADDING & BASE MATERIAL

THE EFFECT OF THE DIFFERENCE IN ELASTIC MODULUS between CLAD AND BASE METAL IS SMALL & IS IGNORED HERE. THE AVERAGE OF THE TWO MODULI WILL BE USED IN THE CALCULATIONS WHERE REQUIRED

$F_c = -F_b$

$S_c A_c = -S_b A_b$

for a unit width

$A_c = t_c$

$A_b = t_b$

Then $S_c t_c = -S_b t_b$

$$S_b = -S_c \frac{t_c}{t_b} = -\frac{30000(.25)}{2.375}$$

$S_b = -3160$ PSI COMPRESSION

2, STRESS INDUCED DURING STRESS RELEIVING

a) CLADDING STRESS

The change in cladding temperature required to change the cladding stress from the cold tensile yield strength to the hot compressive yield stress is found as follows:

$$\Delta S_c = 30000 + 12500 = 42500 \text{ psi}$$

$$\Delta T = \frac{\Delta S_c}{E_{AVG}(\alpha_c - \alpha_b)} \qquad \alpha_c = 10.2 \times 10^{-6}$$

$$= \frac{42500}{22(10.2 - 7.95)} \qquad \alpha_b = 7.95 \times 10^{-6}$$

$$E_{AVG} = 22 \times 10^6 \text{ psi}$$

$$\Delta T = 860^\circ F$$

SINCE THE TEMPERATUR INCREASE IS GREATER THAN 860° DURING STRESS RELIEVING, THE CLADDING will YIELD IN COMPRESSION.

THEREFORE THE STRESS RELIEVING WILL CAUSE COMPRESSION IN THE CLADDING WHICH WILL BE THE YIELD STRENGTH of the CLADDING AT THE STRESS RELIEVING TEMPERATURE

STRESS RELIEVING TEMP = 1150° F

YIELD STRESS @ 1150° = 12500 PSI

AFTER STRESS RELIEVING THE CLADDING RETURNS TO ITS PREVIOUS STRESS 30000 PSI IN TENSION

b) BASE MATERIAL STRESS AT STRESS RELIEVING TEMP OF 1150°F

$$S_b = -S_c \frac{t_c}{t_b}$$

$$S_b = -(-12500)\left(\frac{.25}{2.375}\right)$$

$$S_b = +1315 \text{ psi Tension}$$

THE BASE MATERIAL WILL RETURN TO THE ORIGINAL STRESS OF -3160 PSI IN COMPRESSION AFTER STRESS RELIEVING IS COMPLETED & ROOM TEMPERATURE IS REACHED,

3. EFFECT OF HYDROSTATIC TESTING - TEST PRESSURE = 3000 PSI

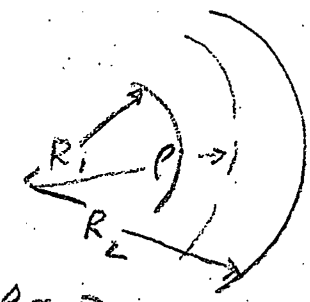
THE HYDROSTATIC TESTING WILL CAUSE TENSILE STRESS IN THE VESSEL WALL, AND SINCE THE CLADDING IS ALREADY AT ITS TENSILE YIELD STRENGTH (30,000 PSI). IT IS ASSUMED THE CLADDING YIELDS FURTHER AND DOES NOT CARRY ANY OF THE TEST PRESSURE.

ALCO PRODUCTS INC.

BY________ DATE________ SUBJECT________ SHEET NO. 5 OF 15
CHKD. BY________ DATE________ JOB NO.________

FOR THICK WALL CYLINDER

$$S_T = \frac{P R_1^2 (R_2^2 + R_1^2)}{\rho^2 (R_2^2 - R_1^2)}$$



STRESS AT INNER SURFACE WHERE $\rho = R_1$

$$S_T = \frac{P \cancel{R_1^2} (R_2^2 + R_1^2)}{\cancel{R_1^2} (R_2^2 - R_1^2)}$$

$R_1 = 19$
$R_2 = 21.375$

$$S_T = \frac{3000 \, (21.375^2 + 19^2)}{21.375^2 - 19^2}$$

$$S_T = \frac{3000 \, (455 + 361)}{455 - 361}$$

$$S_T = 26000 \text{ psi}$$

a) CLADDING STRESS DUE TO HYDROSTATIC TEST PRESSURE

① UNIT STRAIN of VESSEL WALL AT TEST PRESSURE

$$\epsilon = \frac{S_T}{E_B}$$

$$\epsilon = \frac{26 \times 10^3}{29 \times 10^6}$$

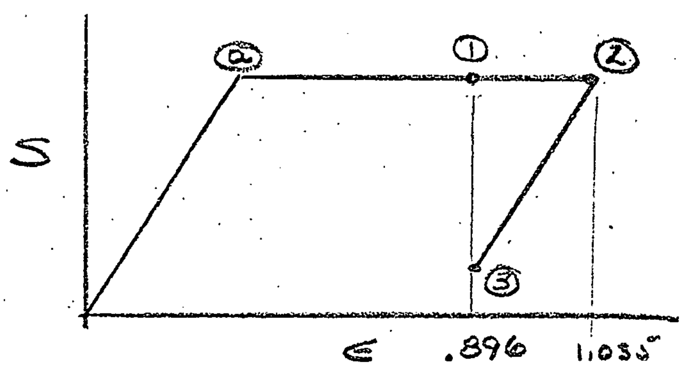
$$\epsilon = .896 \times 10^{-3}$$

2. UNIT STRAIN OF CLADDING ALSO WILL BE 0.896×10^{-3}
SINCE CLADDING AND VESSEL WALL ARE ONE PIECE.

3. UNIT STRAIN OF CLADDING AT YIELD POINT (COLD) $S = 30000$ PSI

$$\epsilon = \frac{S}{E} = \frac{30 \times 10^{3}}{29 \times 10^{6}}$$

$$\epsilon = 1.035 \times 10^{-3}$$



① CLADDING BEFORE HYDRO. TEST
② " " " DURING " "
③ " " " AFTER " "

$$E = \frac{S_a}{\epsilon_a} = \frac{S_3}{\epsilon_a - \epsilon_{1-2}}$$

$$29 \times 10^{6} = \frac{S_3}{(1.035 - .896) \times 10^{-3}}$$

$$S_3 = 29(1.035 - .896) \times 10^{3}$$

$$S_3 = 4030 \text{ PSI TENSION}$$

RESULTANT STRESS IN CLADDING AFTER HYDRO. TEST

b) BASE MATERIAL STRESS DUE TO HYDROSTATIC TEST PRESSURE

THE STRESS AT THE INNER SURFACE OF THE BASE MATERIAL IS 26000 psi (TENSION), but prior to application of test pressure IT WAS −3160 psi (compression)

∴ Resultant stress in BASE MATERIAL DURING HYDRO TEST

$$\begin{array}{r} 26000 \text{ psi} \\ - \underline{3160} \\ 22840 \text{ psi (Tension)} \end{array}$$

Upon completion of Hydro the base material would return to its original stress of −3160 psi compression, but the reduction of cladding stress to +4030 psi causes reduction in compression

∴ RESULTANT STRESS IN BASE MATERIAL AFTER HYDRO TEST

$$S_b = -S_c \frac{t_c}{t_b}$$

$$S_b = -4030 \frac{(.25)}{2.375}$$

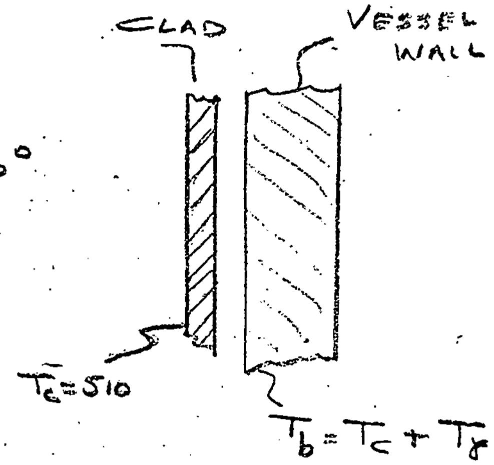
$$S_b = -424 \text{ psi compression}$$

④ STRESS DUE TO GOING TO OPERATING TEMP. —

a) CLADDING STRESS DUE TO NORMAL OPERATING TEMPERATURE

AVERAGE COOLANT TEMP. = 510°F

ASSUME TEMP INCREASE IN VESSEL WALL DUE TO GAMMA HEATING $T_\gamma = 20°$



HEATING TO OPERATING TEMPERATURE WILL TEND TO REVERSE THE TENSILE STRESS IN CLADDING

$$S_c = \frac{(\alpha_c \Delta T_c - \alpha_b \Delta T_b) E_c}{(1-u)\left[1 + \frac{E_c t_c}{E_b t_b}\right]}$$

$\Delta T_c = 510 - 80 = 430°F$
$T_r = 20°F$
$\alpha_c = 9.5 \times 10^{-6}$ @ 510
$\alpha_b = 6.95$ @ 530
$E_c = 25 \times 10^6$ @ 510
$E_B = 27 \times 10^6$ @ 530
$t_c = .25''$
$t_b = 2.375''$

$\Delta T_b = \Delta T_c + T_r$

Subst. ΔT_b in above & solve in terms of T_r.

$S_c = S_{yield\ @510°} = 19000$ P.S.I.

$$S_c = -\frac{\left[\alpha_c \Delta T_c - \alpha_b (\Delta T_c + T_r)\right] E_c}{(1-\mu)\left[1 + \frac{E_c t_c}{E_b t_b}\right]}$$

$$S_c = -\frac{\left[9.5\times10^{-6}(430) - 6.95\times10^{-6}(430 + T_r)\right] 25\times10^{6}}{1-.3\left[1 + \frac{25\times10^{6}(.25)}{27\times10^{6}(2.375)}\right]}$$

$$S_c = -\frac{\left[4080 - 2990 - 6.95\, T_r\right] 25}{.7\left[1 + .0974\right]}$$

$$S_c = -\frac{\left[1090 - 6.95\, T_r\right] 25}{.7(1.0974)}$$

$$S_c = -(1090 - 6.95\, T_r)\, 32.6$$

$$S_c = -226(157 - T_r)$$

Assuming $T_r = 20°$

$$S_c = -226(157 - 20)$$

$S_c = -31000$ psi compression

The cladding has a stress of + 3050 psi in Tension prior to operating temp, The resultant stress in cladding after reaching operating Temp. is

−31000	psi	compression
+ 4030	psi	Tension
− 26970	psi	compression

ALCO PRODUCTS INC.

BY B.J.R. DATE 1/23/61 SUBJECT ______ SHEET NO. 10 OF 13
CHKD. BY ______ DATE ______ JOB NO. ______

b) STRESSES IN VESSEL WALL DUE TO GOING TO OPERATING TEMPERATURE

$$S_b = -S_c \frac{t_c}{t_b}$$

$$= -(-26970)\frac{(.25)}{2.375}$$

$$S_b = +2830 \quad \text{Tension}$$

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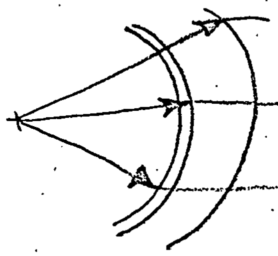
BY B/R DATE 11/28/66 SUBJECT ________ SHEET NO. 11 OF 13
CHKD. BY ______ DATE ______ JOB NO. ______

⑤ CLADDING STRESS & VESSEL WALL STRESS DUE TO NORMAL OPERATING PRESSURE (1750 psi)



$R_1^2 = (18.75'')^2 = 352$

$R_2^2 = (21.375)^2 = 455$

$$S_{T_{INSIDE}} = \frac{PR_1^2(R_2^2+\rho^2)}{\rho^2(R_2^2-R_1^2)}$$ at inside $\rho = R_1$

$$S_{T_{INSIDE}} = \frac{P\cancel{R_1^2}(R_2^2+R_1^2)}{\cancel{R_1^2}(R_2^2-R_1^2)}$$

$$S_{T_{INSIDE}} = \frac{P(R_2^2+R_1^2)}{R_2^2-R_1^2}$$

$$= \frac{1750\overbrace{(455+352)}^{807}}{\underbrace{455-352}_{103}}$$

$S_{T_{INSIDE}} = $ 13700 psi

$$S_{T_{OUTSIDE}} = \frac{PR_1^2(R_2^2+\rho^2)}{\rho^2(R_2^2-R_1^2)}$$ $\rho = R_2$

$$= \frac{PR_1^2(2R_2^2)}{R_2^2(R_2^2-R_1^2)}$$

$$= \frac{2PR_1^2}{R_2^2-R_1^2} = \frac{2(1750)(352)}{103}$$

$S_{t_{OUTSIDE}} = $ 12000 psi

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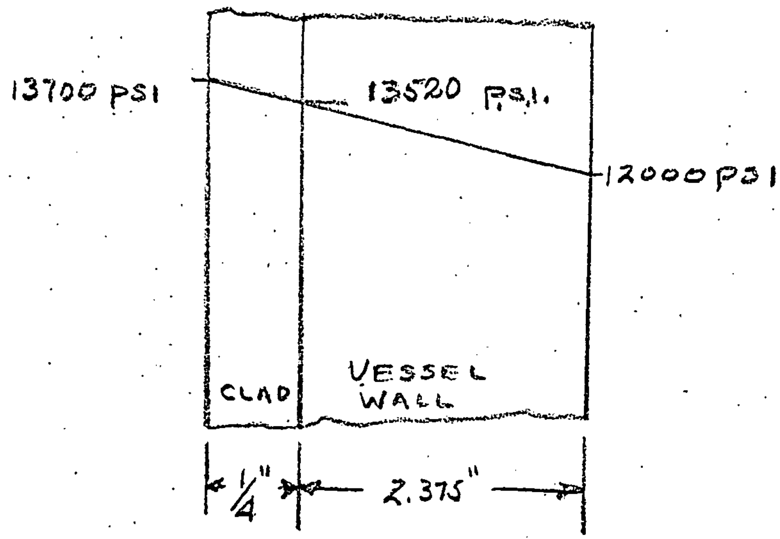
BY A J K DATE 1/23/60 SUBJECT ____ SHEET NO. 12 OF 13
CHKD. BY ____ DATE ____ JOB NO. ____



STRESS at outer surface of cladding

$$1700 \times \frac{.25}{2.375} = 179$$

$$\begin{array}{r} 13700 \\ -\ \underline{179} \\ 13521 \text{ PSI} \end{array}$$

CLADDING STRESS $= \dfrac{13700 + 13521}{2} = +13610$ PSI

The resultant stress in cladding after reaching operating pressure $\left.\right\}$ $\begin{array}{r} -26970 \text{ PSI} \\ \underline{+13610} \\ -13360 \text{ PSI} \end{array}$

VESSEL WALL STRESS –

$$S_b = -S_c \frac{t_c}{t_b} = -(-13360)\frac{.25}{2.375} = 1405 \text{ PSI}$$

$$S_b = 1405 + 13620 = 14925 \text{ PSI}$$

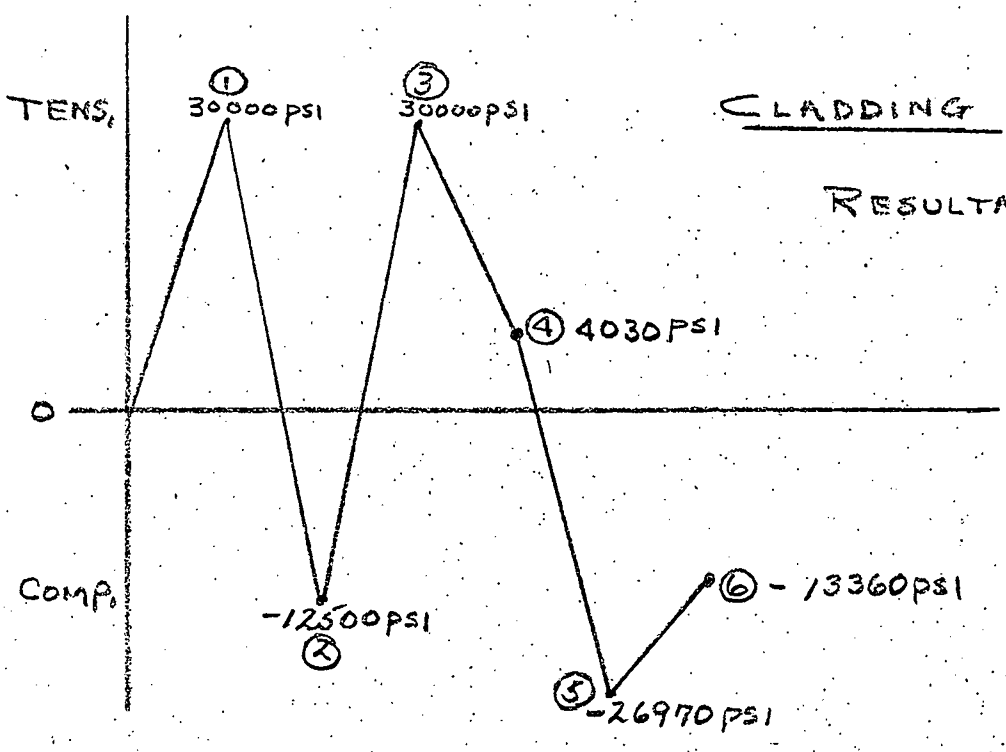
ALCO PRODUCTS INC.

BY________ DATE________ SUBJECT________ SHEET NO. 13 OF 13
CHKD. BY________ DATE________ JOB NO.________

SUMMARY OF STRESSES

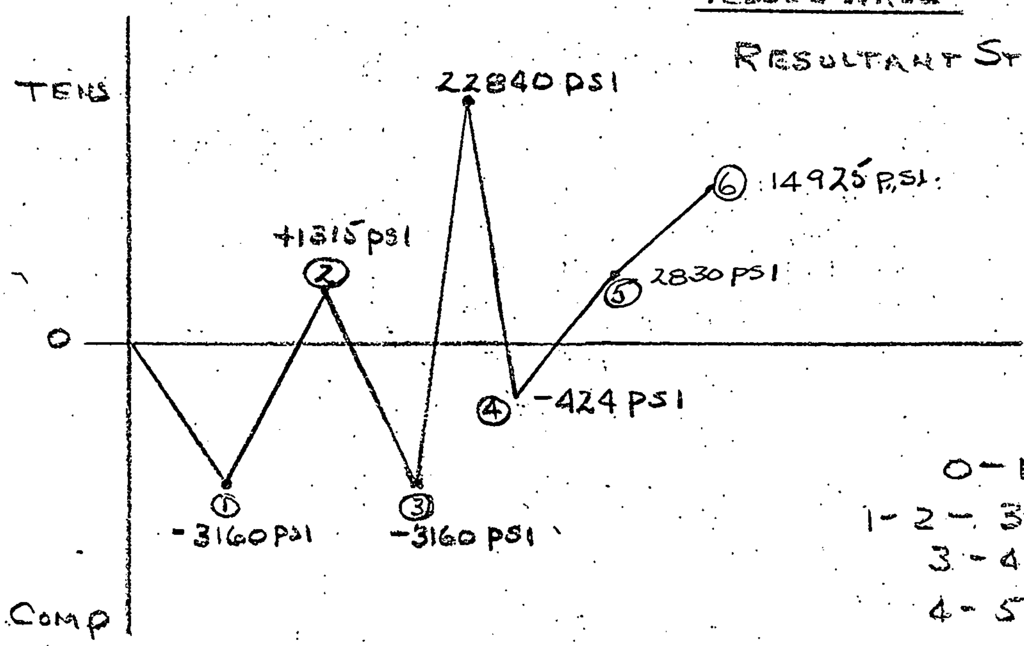
CLADDING

RESULTANT STRESS = −13,360 PSI



VESSEL WALL −

RESULTANT STRESS = +1405 PSI



0 − 1	WELDING
1 − 2 − 3	STRESS RELIEVING
3 − 4	HYDRO TEST
4 − 5	OPER. TEMP
5 − 6	OPER. PRESS