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MASTER

SEQUENTIAL INTERNAL SUPERSYMMETRY

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SEQUENTIAL INTERNAL SUPERSYMMETRY

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ABSTRACT

The supergroups $SU(2/1)$, $SU(5/1)$ and $SU(5=k/1)$ provide fitting classifications, unifying weak-electromagnetic, color, and sequential flavor respectively.

 $SU(2/1)$: ELECTROWEAK DYNAMICS

Experience with old-fashioned $SU(3)$ has shown that a good classification group should be studied, even if neither its dynamics nor the quantum-statistics of its representations are understood at the time. It has recently been suggested^{1,2} that the simple supergroup $SU(2/1) \supset SU(2) \times U(1)$, applied as an internal supergauge, provide a highly-restricted Salam-Weinberg model.

We have proved elsewhere³ that the standard-model matrices, when applied to leptons and quarks sequentially, obey a condition of supertracelessness when left and right chiral fields are assigned to different gradings. This is because $SU(2)$ is traceless, and acts only within the left-chiral sector; and because electric charges Q have to match in both sectors. The $U(1)$ is then a supertraceless matrix U , since it is a linear combination, $Q = I_3 + \frac{1}{2}U$.

The defining representation is $(\nu_L^0, e_L^-, [e_R^-]_L)$ with the quantum numbers $(-\frac{1}{2}, \frac{1}{2})$ in the SNR notation⁴ $(\frac{1}{2}u_{mid}, i_{3max})$ characterizing the eigenvalues of the highest I_3 state. To commute with the Lorentz group, this is a left-chiral spin-half multiplet. The boson $[e_R^-]_L$ has the internal quantum numbers of e_R^- . Several suggestions have been made with respect to the "wrong" statistics and chirality

- a) adding dimensions to space-time², preferably chiral-fermionic⁵
- b) treating the entire system as a classification with no statistics implications. For example, a study of anomaly restrictions relating constituents and their composites⁶ has shown that the $SU(n/m)$ groups fulfill these conditions
- c) we have shown⁷ that the renormalization-ghosts include an effective field, with "wrong" statistics, for every matter field in the theory. Our $[]$ would be the parity-transforms of these ghosts. One of us (Y.N.) may have a preference for this interpretation, but the results of this article do not depend on that choice, except where stated.

Note that one might be tempted to forego the complications of a supergroup, and replace it by $SU(3)$ with $(\nu_L^0, e_L^-, (\overline{e_R^-})_L^\dagger)$ and $|\Delta\ell|=2$ generators and currents, and the same $\theta_W=30^\circ$. However, $SU(2/1)$ scores highly in providing us with the 4-dimensional fundamental representation^{4,8} $(u_{L3}^{2/3}, d_{L3}^{-1/3}/[d_{R3}^{-1/3}]_L, [u_{R3}^{2/3}]_L)$, $(1/6, 1/2)$ in SNR notation. Indeed, the 4 representation is characterized⁹ by

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$b = \frac{1}{2} u_{\text{mid}} + \frac{1}{2}$; $b = 2/3, 1/3$ respectively for quarks and antiquarks, and $b = 0, 1$ respectively for leptons and antileptons. In the system of representations of $SU(n/m)$ that we use⁹, integer values for b imply reducibility. $SU(2/1)$ indeed predicts the decoupling of ν_R^0 due to the integer charges of the leptons!

$SU(2/1)$ also predicts the $I = \frac{1}{2}$, $U = \frac{1}{3}$ assignments of the Goldstone-Higgs multiplet^{1,2}. In the ghost interpretation these scalar fields make a perfect 8, together with the Faddeev-Popov ghosts of $SU(2) \times U(1)$. Indeed, this restriction could be taken as a definition of the theory. In that picture, the ϕ^4 coupling is determined and the mass of the Higgs field ~ 250 GeV. Treating the full $SU(2/1)$ as a local gauge makes the basic set of leptons or quarks have masses of the order of the W mass. This looked surprising at the time but it is precisely the picture of "technicolor" or "hypercolor" etc.¹⁰.

Local $SU(2/1)$ has one main difficulty, relating to the Killing metric $g_{AB} = \text{str}(M_A M_B)$ which is non-positive-definite for the 8th direction $g_{UU} = \text{str}(M_U)^2 < 0$. This precludes the straightforward construction of a second-quantized theory. Either we break the symmetry (by using the metric of $SU(3)$, with a certain justification in the ghost picture⁸) or we treat $SU(2/1)/SU(2) \times U(1)$ globally only (this also cancels the heavy fermion masses).

$SU(5/1)$: COLOR ADDED

In $SU(5/1)$, taking for $I_3 = \text{diag}((\frac{1}{2}, -\frac{1}{2}, 0, 0, 0/0))$ there are two choices of U . Previous authors^{11,12} adopted $U \subset SU(5) \subset SU(5/1)$ i.e. the Georgi-Glashow choice

$$\frac{1}{2}N = \text{diag } \frac{1}{2}((1, 1, -2/3, -2/3, -2/3 / 0)) \quad (1)$$

but this clashes with $SU(2/1) \subset SU(5/1)$. We have selected⁹

$$\frac{1}{2}U = \text{diag } \frac{1}{2}((0, 0, 1/3, 1/3, 1/3 / 1)) \quad (2)$$

which have $SU(5/1) \supset SU(3)_{\text{color}} \times SU(2/1)$. The $b=2$ representation is 32 dimensional and contains precisely two generations (or a generation and its ghosts). It is reducible, and one may select ^{11,12} 16 or 31 (dropping the ν_R).

$SU(5+k/1)$: SERIALITY ADDED

The representations of $SU(n/m)$ have recently been described^{9,11,13}. They follow the same Clifford-algebra pattern

$$| "0" > \otimes I | "0" > \otimes X | "0" > \otimes Y | "0" > \otimes \dots$$

which we used in exploring extended supergravity and discovering the $N=8$ limit. Here too, we have a restriction on the range of U going from $+2$ to -2 , with $\pm 4/3, \pm 1, \pm 2/3, \pm 1/3$ as the only allowed intermediate values. We have proved¹⁴ that these values restrict the color group uniquely to $SU(3)$, or no color at all as an alternative.

Our representations $b = \frac{4+k}{2}$, with $u_{\text{max}} = 2$, for

$$\frac{1}{2}U = \text{diag } \frac{1}{2}((\frac{-k}{4+k}, \frac{-k}{4+k}, (\frac{4}{4+k}) k \text{ times}, (\frac{4-2k}{3(4+k)}) 3 \text{ times}, \frac{4}{4+k})) \quad (3)$$

$$I_3 = \text{diag}((\frac{1}{2}, -\frac{1}{2}, 0 (k+3) \text{ times} / 0)) \quad (4)$$

are 2^{5+k} dimensional. They contain 2^{k+1} generations, or 2^k generations plus their ghosts, depending upon the interpretation (see discussion of ϵ in ref⁸). Thus $k=2$ (or $k=3$) predicts 8 generations. We have provided the exact construction elsewhere^{9,3}. Half of the generations have inverted physical chiralities.

Note that for $k \geq 1$, we now have $g_{UU} > 0$, so that a quantized local supergauge theory becomes possible.

The defining representation describes the primitive constituents of the theory. For $k=2$ they have charges,

$$Q = \text{diag} \left((1/3, -2/3, 1/3, 1/3, 0, 0, 0, 1/3) \right) \quad (5)$$

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