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An experimental study of finite larmor radius effects

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**Lawrence
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An Experimental Study of Finite Larmor Radius Effects
in a Linear Z-Pinch

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ABSTRACT

Linear Z-pinches in Ar, Kr, Xe, N₂, and He are experimentally studied in regimes where strong finite Larmor radius effects could provide a significant stabilizing effect. Scaling arguments show that for deuterium such a pinch has an electron line density of order 2×10^{15} /cm. For higher Z plasmas a higher line density is allowed, the exact value of which depends on the average ion charge. The pinch is formed by puffing gas axially through the cathode towards the anode of an evacuated pinch chamber. When the gas reaches the anode, the pinch bank is fired. The pinch current rises in 2 to 3 μ sec to a maximum of 100 to 200 kA. The pinch bank capacitance is 900 μ F, and the external inductance is 100 nH. Additionally, the bank is fused to increase dI/dt . The primary diagnostics are a framing camera, a spatially resolved Mach-Zehnder interferometer, and X-ray absorption. It is found that the initially conically-shaped gas jet pinches to axis in a time proportional to $t \propto \rho_0^{-1/2}$, rather than the $\rho_0^{-1/4}$ scaling typical of the snowplow pinch. Observed linear pinch instabilities

have growth rates more than a factor of ten less than the MHD model. Instability types are found to scale with the ratio of the finite Larmor radius divided by the pinch radius, which requires determining the average ion charge for higher Z plasmas. The average ion charge is calculated by an approximate method for Ar plasmas. Least squares fits of Abel inverted electron densities show that the plasma has a Bennett density profile during the peak compression. It is found that FLR effects are important for high Z, low T pinches. For low Z pinches microinstabilities dominate and prevent enhanced confinement at low line densities. An anode voltage spike and associated X-ray burst is observed to coincide with a sausage-like instability. X-ray energies are measured to be of order e times the anode voltage. It is concluded that these instabilities are generated by localized, anomalously resistive regions, causing enhanced heating and subsequent rapid pinch expansion.

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Table of Contents

CHAPTER 1	Introduction	1
1.1.	Motivation	1
1.2.	Scaling	4
1.2.1.	Finite Larmor radius scaling	4
1.2.2.	Stability	10
1.3.	The experiment	11
	References for chapter 1	13
CHAPTER 2	Description of the Experiment	15
2.1.	The apparatus	16
2.1.1.	The capacitor bank and pinch circuitry	16
2.1.2.	The pinch chamber	20
2.1.3.	Pulsed gas valves	22
2.1.4.	The plasma gun	24
2.1.5.	Timing	27
2.2.	Diagnostics	30
2.2.1.	Framing cameras	30

2.2.2. Laser interferometers	40
2.2.3. X-ray measurements	44
2.2.4. Other measurements	46
References for chapter 2	48
 CHAPTER 3 The Z-pinch Formation	 51
3.1. Pinch formation	51
3.1.1. Gas flow into the chamber	51
3.1.2. Gas breakdown	58
3.1.3. Pinch formation	60
3.2. Examples of pinch formation and lifetimes	71
3.2.1. The gas-puff initiated Z-pinch	72
3.2.2. The plasma gun initiated Z-pinch	82
3.3. Deduced parameters	85
3.4. Electron density profiles	89
References for chapter 3	89
 CHAPTER 4 The Pinch Break-up and Observed Instabilities	 91
4.1. Description of the break-up processes	92
4.1.1. Scenario	92
4.1.2. Types of pinch instabilities	95
4.1.3. Other features	99
4.2. Measured parameters	102

4.2.1. X-ray absorption measurements	102
4.2.2. Growth rate and fluid velocity measurements	104
4.2.3. Instability type scaling with the FLR parameter	106
References for chapter 4	109
 CHAPTER 5 Discussion	 110
5.1. Linear growth rate comparisons	111
5.1.1. Analytical techniques	112
5.1.2. Skin current model growth rates	117
5.1.3. Diffuse current model growth rates	118
5.1.4. Finite Larmor radius effect on the growth rates	123
5.1.5. Comparisons of observed with calculated growth rates	128
5.2. The pinching dynamics and non-MHD effects	129
5.2.1. Pinching dynamics	130
5.2.2. The voltage spike and the sausage-like disruption ...	137
References for chapter 5	141
 CHAPTER 6 Summary	 145
6.1. Results	145
6.2. Further work	152
References for chapter 6	153

APPENDIX A Pulsed Gas Valve and Gas Flow Dynamics	154
A.1. Construction and operation of the gas valve	154
A.2. Gas dynamics	158
References for appendix A	167
APPENDIX B Laser Interferometry and Abel Inversion	169
B.1. General considerations	169
B.2. Abel inversion	171
References for appendix B	179
APPENDIX C Determination of the Effective Ion Charge	180
References for appendix C	188
APPENDIX D Electron Density Profiles	189

CHAPTER 1

Introduction

1.1. Motivation

The earliest efforts in controlled thermonuclear fusion were with the linear Z-pinch. (By convention, Z-pinch denotes the magnetic configuration in which a strong current flows in the z direction in a plasma inducing a magnetic field in the θ direction. Here z and θ refer to coordinates in cylindrical geometry. The self-induced magnetic field then squeezes, or *pinches* the plasma, thus confining it.) But these earlier experiments showed that the pinch was unstable to *sausage* and *kink* modes. In the *sausage* mode the plasma is similar

in shape to link sausage in that there are several islands of plasma linked by thin filaments. In the kink mode there is a gross motion of the plasma column off the axis of symmetry. This can be helical in structure and look much like a spring. The pinch instability was later predicted with a fluid theory using the energy principle by Kruskal and Schwarzschild,¹ and still later including a distributed current by Newcomb.² A related technique which also uses the fluid theory is to perturb the equilibrium fluid equations and solve for the instability growth rates. Kadomtsev³ summarizes this approach using a skin current model showing that a wavelength can always be found such that one of the modes is unstable. The energy principle shows that this is not quite as serious with distributed currents, and that with density gradients sufficiently small only the lowest order kink mode is unstable.⁴ Although more complicated, Z-pinch growth rate calculations using a diffuse current model have also been done by several researchers.^{5,6} They generally find that the growth rates with an axially stabilizing field are sensitive to the density profiles.

As a result of the theory, much effort has gone into altering the magnetic field configuration to stabilize the plasma. This usually involves adding an externally produced magnetic field roughly equal to, or greater in magnitude to the self-induced fields. Some examples are the tokamak and the stabilized pinch. The goal of the various approaches is to confine the plasma sufficiently long, and with a high enough particle density, that the fusion energy released exceeds the energy expended in confining the plasma. These requirements are

expressed as the Lawson criterion, which requires the product of the particle density and confinement time $n\tau$ be $\geq 10^{14}$ sec / cm³ for deuterium-tritium reactions and with a temperature ≥ 10 keV. The density in these configurations is limited by the maximum external magnetic field which can be produced to roughly 10^{14} / cm³. Therefore the confinement time must be nearly one second.

However, if an externally produced magnetic field is not used, one could consider a pinch with much higher magnetic fields and particle densities, and proportionally shorter confinement times. If the densities were made sufficiently large, then the required confinement time could be decreased to be less than the time for instabilities to develop. The fluid theory predicts instability growth times to be roughly equal to the ratio of the pinch radius to the ion sound velocity. This requires, however, densities of the order of solid densities and correspondingly large magnetic fields.⁷ Experimentally the observed growth times are found to be 10 to 100 times longer than those predicted by the fluid theory.⁸ This relaxes the density requirements somewhat, and allows the concept of a Z-pinch reactor to become feasible.

One mechanism that has been shown to have stabilizing effects with other configurations⁹ is the consideration of finite Larmor radius effects (hereafter denoted FLR). Hartman¹⁰ has proposed that these effects could be a stabilizing mechanism for the linear Z-pinch, provided the pinch is operated in regimes where these effects are

strong. In the fluid theory it is assumed that each ion Larmor radius is small compared to the dimensions of the plasma. When the ion Larmor radius becomes large the fluid theory breaks down. For nearly-fluid plasmas, these effects can be treated by adding viscous terms to the pressure tensor.¹¹ However, in regimes where these effects would be strong, the corrections also break down and need to be replaced with a particle theory.

Presently there exists no theory nor satisfactory numerical simulation which includes strong finite orbit effects in the linear Z-pinch. Nielsen, *et al*¹² have done three-dimensional particle simulations of the linear Z-pinch, but cannot investigate the high density regimes because of the need to follow many more particles than is possible with their code. Because of the complexity of the problem, the lack of a satisfactory theory, and the possibility of a significant stabilizing effect, it was decided to investigate experimentally these regimes.

1.2. *Scaling*

1.2.1. *Finite Larmor radius scaling*

Following the arguments of Hartman¹³, consider a Z-pinch where finite orbit effects would be important. This is a pinch which would

be, at most, only several ion orbits in diameter. For diffuse pinches the Larmor radius is a function of radius.

$$a_i = \frac{v_{\perp}}{\omega_{ci}} = \frac{2\pi r (m_i k T_i)^{1/2}}{Z_{av} e \mu_0 I(r)} \quad (1.1)$$

where Z_{av} is the average ion charge, and $I(r)$ is the integral of the current density.

$$I(r) = \int_0^r J_z(r') 2\pi r' dr' \quad (1.2)$$

Note that MKS units are used, as is the convention throughout the paper. However, when numbers are required, these will be expressed in cgs units, since these are commonly used units in experimental plasma physics.

With a uniform current density, the Larmor radius is proportional to $1/r$. Clearly, the Larmor radius becomes larger than the pinch radius as r approaches zero. To avoid this difficulty, find the average Larmor radius by integrating over the particle distribution.

$$\langle a_i \rangle = \frac{\int_0^{\infty} a_i(r) n(r) 2\pi r dr}{\int_0^{\infty} n(r) 2\pi r dr} \quad (1.3)$$

The number of ion orbits in the diameter is then the ratio of the pinch radius a to the average Larmor radius $\langle a_i \rangle$. Since this number is dependent on the current and density profiles, consider the results of three special cases as examples. Defining s as,

$$s = \frac{\mu_o I_o}{2\pi(m_i k T_i)^{1/2}} \quad (1.4)$$

Case 1. Uniform current and electron densities

$$\frac{a}{\langle a_i \rangle} = \frac{s}{2} \quad (1.5)$$

Case 2. Uniform current density and $n_e(r) = n_e(0)[1 - (r/a)^2]$

$$\frac{a}{\langle a_i \rangle} = \frac{3}{8} s \quad (1.6)$$

Case 3. Bennett profiles¹⁴ in current and electron densities.

$$\frac{a}{\langle a_i \rangle} = s \frac{b^{1/2} a}{\pi} \quad (1.7)$$

A Bennett density profile is given by

$$n(r) = \frac{n(0)}{(1 + b r^2)^2} \quad (1.8)$$

The parameter b describes the width of the Bennett profile, namely, $n(b^{-1/2}) = n(0)/4$. This profile arises from considering the electron drift velocity and the plasma temperature independent of r . If $b^{-1/2}$ is defined as the plasma radius a , then the number of ion orbits is s/π , which is very nearly the result for case 2. It is determined by the electron drift velocity v_{de} , the on-axis electron density $n(0)$, and the temperature, and equals

$$b = \frac{\mu_0}{8} e^2 v_{d0}^2 \frac{n(0)}{k(T_e + T_i)}. \quad (1.9)$$

The quantity s defined in eq. (1.4) can be shown to be the ratio of the pinch radius to the ion Larmor radius at the outer edge of the pinch. This is done by replacing r with a in eq. (1.1). Since this has appeared in the three examples given, it seems sensible to consider s as a scaling parameter relating to the number of ion orbits across the width of the pinch.

The temperature dependence of eq. (1.4) can be eliminated by considering a balance between the magnetic field and particle pressure. For arbitrary Z and T eq. (1.4) becomes,

$$s = \left[Z_{av} \left(1 + \frac{T_e}{T_i} Z_{av} \right) \frac{2e^2 \mu_0}{\pi m_i} \pi a^2 n_e \right]^{1/2}. \quad (1.10)$$

In cgs units, with $T_e = T_i$, the result for deuterium is,

$$s_D = \left(\frac{\pi n_e a^2}{1.6 \times 10^{15}} \right)^{1/2} \quad (1.11)$$

The subscript implies that the expression is evaluated for deuterium. For higher Z ions, and with unequal temperatures, s_D is modified by,

$$s = \left[\frac{Z_{av} \left(1 + Z_{av} \frac{T_e}{T_i} \right)}{\frac{m_i}{m_D}} \right]^{1/2} s_D \quad (1.12)$$

The quantity $\pi n_e a^2$ in eqs. (1.10) and (1.11) is the number of electrons per unit length, and is called the electron line density. Thus the number of ion orbits in a hydrogen or deuterium pinch scales only with the line density, and is independent of the ion temperature. For higher Z plasmas the temperature plays a more important role in that it determines the charge state Z_{av} . This complicates the analysis by requiring knowledge of both the temperature and electron density to determine the FLR scaling.

From these scaling arguments it can be seen that it is necessary to maintain a low line density to obtain a pinch where FLR effects are important. If the current and electron density profiles from case 3 are used, the scaling parameter s must equal approximately π to have a pinch containing one ion orbit. For deuterium, this dictates an electron line density of 1.6×10^{16} per cm. For higher Z plasmas this density scales approximately as $Z^{1/2}$, if fully ionized.

1.2.2. Stability

If, however, the line density is made too small, the electron drift velocity becomes large enough to exceed the threshold for the lower hybrid drift instability.¹⁵ Hartman¹⁶ has shown that when this happens there is a localized, rapid heating, and a subsequent expansion of the pinch column. That this threshold also scales with the line density and the FLR parameter s is shown by considering the ratio of the electron drift velocity v_{de} to the ion thermal velocity v_{ti} , and by using the same pressure balance condition used to derive eq. (1.10).

$$\begin{aligned} \frac{v_{de}}{v_{ti}} &= \frac{J_z}{n_e e} \left(\frac{m_i}{k T_i} \right)^{1/2} \\ &= \frac{4 Z_{av}^{1/2} \left(\frac{1}{Z_{av}} + \frac{T_e}{T_i} \right)}{s} \end{aligned} \quad (1.13)$$

Combining eqs. (1.12) and (1.13) gives a result related to the FLR parameter for deuterium.

$$\frac{v_{de}}{v_{ti}} = \frac{\frac{4}{Z_{av}^{1/2}} \left(\frac{1}{Z_{av}} + \frac{T_e}{T_i} \right)^{1/2} \left(2 \frac{m_D}{m_i} \right)^{1/2}}{s_D} \quad (1.14)$$

To avoid this instability it is necessary to require that this ratio of velocities be ≤ 1 . For deuterium plasmas, for example, s_D must be ≥ 8 . For an argon plasma, this requirement is not as stringent. The parameter s in this case must be only ≥ 0.3 .

These results suggest the possible existence of a stability window with a line density low enough to obtain strong FLR effects and yet still high enough to avoid the lower hybrid drift instability. Pinches formed with line densities below the lower limit would then exhibit very large microinstability growth rates, and those formed with line densities well above those needed for FLR considerations would exhibit the well-known MHD type instabilities and growth rates. Presumably, those formed with line densities within this window would be stable, or at least exhibit growth rates much slower than predicted by the MHD theory.

1.3. The Experiment

Considering the number of Z-pinch experiments which have been done previously, one would expect that FLR effects might have been observed. This is not the case, however, since earlier pinches operated at much higher line densities. They were usually initiated by breakdown along an insulator at the chamber wall. The current formed in a thin shell at the wall, which then collapsed radially, accruing

mass as it swept in. If the fill pressure of the chamber was lowered sufficiently that one might see FLR effects, it was impossible to induce electrical breakdown of the gas with the available electrode voltages. For these reasons, most of the previous work was done with line densities which did not permit observation of FLR effects.

This problem can be overcome by producing a localized high density region to allow gas breakdown, and by also controlling the extent of the region to insure a low line density. Low line density, and high local density can be achieved by puffing gas between the electrodes and along the axis of a previously evacuated pinch chamber. The line density is then controlled by the amount of gas puffed into the chamber. Another technique to achieve the same result is to direct a plasma stream from a plasma gun into the electrode gap. Both techniques have been tried, and will be discussed.

The goals of the experiment have been to; (1) set-up and operate a pinch in regimes where FLR effects might be important, (2) observe the instability types and measure their growth rates, and (3) determine what effect the line density has on the nature of the instabilities. Additionally, measured growth rates are compared with those predicted by the diffuse-current MHD theory, and with those estimated by a diffuse-current MHD theory which includes FLR corrections.

In chapter 2 the description of the experiment and diagnostics is given. The experimentally observed formation of the pinch is discussed in chapter 3. In chapter 4 the instability types are given and

categorized, and in chapter 5 the results are discussed, along with their implications. The results are summarized in chapter 6.

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CHAPTER 2

Description of the Experiment

To construct an experiment where finite Larmor radius effects are important it is necessary to maintain a low electron line density $\pi n_e a^2$ and yet maintain the number density n_e along the axis high enough that breakdown can be achieved. This is accomplished by using a pulsed gas valve to provide a localized puff of gas along the pinch axis. Thus, the local density is kept high, while the line density is kept low. In this chapter I will discuss how this is done experimentally, give the details of the experiment, and discuss the diagnostics.

2.1. The apparatus

2.1.1. The capacitor bank and pinch circuitry.

The finite Larmor radius Z-pinch experiment was designed to use the "green river" capacitor bank and control circuitry in building 431 at LLNL. This bank was constructed in the late 50's and consists of 120, $7.5 \mu\text{f}/20 \text{ kV}$ capacitors each connected to a common pad with ten RG-213 coaxial cables. The cables are about 10 meters long, are green in color, and appear to flow to the pad; hence the name "green river" bank. Each capacitor has its own fire and crowbar ignitron, all fired from a separate capacitor through a distribution panel. The total inductance of the system is 100 nH, with the time to current maximum being about $15 \mu\text{sec}$. Refer to the circuit diagram in fig. 2.1.

In order to increase the rate of current rise through the pinch, the first few microseconds of the bank current is shorted through a fuse placed in parallel with the pinch electrodes. The bank current is thereby allowed to rise slowly through the fuse until it blows, after which the current flows only through the pinch chamber. Additionally, a spark gap was located between the fuse and pinch anodes to prevent current flow through the pinch while the fuse is conducting.

Figure 2.1. Finite Larmor radius Z-pinch circuit.

The gas pressure in the spark gap region is adjusted to hold off the voltage as current is flowing through the fuse. When the fuse blows there is a large, inductive voltage spike at the fuse anode which then breaks down the gap and allows current flow to the pinch anode. A spark plug was also placed in the spark gap chamber region to assist in breakdown of the gap. It is connected to the bank pad through a 50 Ω resistor. The spark gap separation ranged from 3.8 to 5.5 mm. The separation varied because the gaps were gradually worn down with each shot and had to be replaced periodically. The gas fill is either N_2 or

operating pressures were found initially by hi-potting the spark gap to find the breakdown voltage as a function of pressure. Thereafter, the pressure was adjusted by trial and error: e.g., it is kept just low enough to breakdown the gap with the inductive voltage spike from the fuse.

The fuses consist simply of bare copper wires taped to the fuse terminals and imbedded in sand. See the diagram of the fuse terminals in fig. 2.3 for their location. Note that the assembly was extended below the pinch chamber to provide accessibility. The number and diameter of fuses varied for different bank charge voltages. At a 4 kV charge two 20 gauge wires were used. At other voltages, combinations of wires were used to maintain the same ratio of the wire cross-sectional area to bank energy.

This technique works very well and decreases the current rise time to 1 to 2 μ sec. Fig. 2.2 shows the effect of the fuse operation on the pinch current rise time. The upper trace is the total current flowing from the pad. The lower trace is the pinch current only. Note the factor of ten increase in the rate of current rise in the pinch current. The disadvantages of the system were that the bucket holding the sand around the fuses had to be dropped with each shot and the fuses replaced, and that the sand was pulverized in the process. While changing the fuses and sand between shots, a fine dust rose and then settled on the experiment, making it difficult to keep the optics and

Figure 2.2. The effect of the fuse on the pinch current rise time. The upper trace is the total current and the lower trace is the pinch current, both 100 kA / div. and 5 μ sec / div. The difference between the two is the current through the fuse.

the operator clean. But, in spite of these problems, the fusing was very reliable and allowed use of a slower bank.

The remainder of the circuit is completed by the pinch from the anode to the cathode. The plasma for the pinch comes from ionization of the gas which is puffed into the chamber through a hole in the cathode. Refer again to fig. 2.1. The electrode in the center of the gas flow channel serves either of two purposes: (1) It acts as a spark trigger to help form a conduction path, or (2) Without the 10 Ω resistor shown in fig. 2.1 it becomes an electrode of a Cheng type plasma gun.¹ The characteristics of the gun and gas valve are discussed in section 2.1.3. Some of the earlier shots were initiated

with this plasma gun. However, most of the discussion will be of plasmas where the gun only acts as a breakdown trigger.

2.1.2. The pinch chamber

Two different vacuum chambers and a variety of electrode styles were used in the experiment. The first chamber was 57 mm long, and allowed for a pinch length of 23 mm. A second chamber was 210 mm long, and is the chamber with which most of the shots were done, and which is

Figure 2.3. A cross-sectional view of the pinch chamber and fuse assembly. The long chamber with the adjustable length anode-cathode separation is shown. The length is changed by replacing the anode extension with one of a different length.

spark gap region remained the same.

The pinch chamber was constructed of brass as shown in fig. 2.3. The anode and cathodes are of copper. An earlier version used tantalum electrodes. However, the tantalum which was plated on the vacuum windows during the shot was difficult to remove. The copper electrodes plated the windows equally fast, but they were more easily cleaned with solvents. The window plating problem was also partially solved by using flange extensions to move the windows back about 30 cm.

The glass insulator which was placed around the throat of the anode prevented the current from shorting to the chamber wall, which was the return conductor. At the other end, because of its larger diameter, the anode was only 2.9 cm. from the wall. Breakdown did not occur however since the chamber was held at a vacuum until the gas was puffed in through the opening in the cathode. The vacuum was typically in the 10^{-5} Torr range, with the chamber being pumped by a liquid nitrogen cooled oil diffusion pump located roughly 4 m from the experiment. Typically, the capacitor bank was fired before the gas had time to fill the region between the anode and chamber wall. Breakdown to the wall occurred only after the pinch column broke up, at which time it could not effect pinch characteristics.

2.1.3. Pulsed gas valves

Two different pulsed gas valves were used. They were (1) a Berkeley puffatron valve,² which was used on the initial sequence of shots, and (2) a fast gas valve modeled after the one used by Cheng³ with his deflagration gun. The puffatron was used because it was fast, it was available, being left over from a previous experiment, and because it is capable of puffing into a region known amounts of gas. The Cheng style fast gas valve was built and used with later shots to reduce the timing jitter, and to increase the amount of gas puffed into the chamber.

The puffatron gas valve consists of a steel rod which is held on an O-ring seal by springs, and a solenoidal coil which drives the rod off the seal when pulsed with a high current. The seal encloses a small plenum which had a volume of .84 cm³ and had a static fill pressure that ranged anywhere from 100 Torr to several atmospheres. The gas valve rod assembly is 25 mm in diameter and 80 cm long, with the solenoid driving coil 100 mm in diameter, mounted at the end of the rod. The gas valve was mounted in the pinch cathode flange in a similar position to the gas valve shown in fig. 2.3.

The puffatron valve opens in 160 μ sec and has a jitter time of as much as $\pm 50 \mu$ sec. The current for the solenoid was supplied by a 2.2 μ F capacitor charged to 11 kV, which was fired and crow-barred with ignitrons. The current rise time was 15 μ sec and the peak current about 10 kA.

The second puff valve, which was built and used with most of the shots, was modified from Cheng's design to allow use with the existing

Figure 2.4. Cross-sectional view of the Z-pinch cathode showing the plasma gun and pulsed gas valve.

geometry in our Z-pinch electrode. Fig. 2.4 shows this gas valve and how it was positioned at the throat of the plasma gun on the cathode flange. The seal is made by either a small copper disk or coil wound bobbin which is held against a rubber O-ring by ambient plenum pressure. Either the disk or the bobbin serves as a secondary coil of a transformer, and is shot off the seal as the primary coil is pulsed, thus opening the valve. When the disk reaches the stop, it rebounds with the same speed. Measurements of the stop position show that the

valve closes and does not reopen. A small spring was also inserted behind the disk to keep it on the seal while pumping the plenum. It was weak enough, however, to not affect the opening and closing times. A more detailed discussion of the construction and gas dynamics of the valve is given in appendix A.

The total quantity of gas entering the pinch chamber, as a function of time, is given in fig. 2.5. The derivation of these results is in appendix A. The vertical coordinate is the fraction of gas from the plenum which enters the chamber. The total number of particles is obtained by multiplying by a constant, which is 3.3×10^{20} for the Cheng style valve, and 1.5×10^{19} for the Berkeley valve, and by the number of atmospheres of plenum fill pressure. The origin of the horizontal scale is the time the gas valve is fired. The valve opening time is measured to be $40 \mu\text{sec}$. The delay in gas arrival is the sound transit time from the gas valve opening to the cathode plane.

2.1.4. *The plasma gun*

As mentioned at the end of chapter 1, rather than puffing gas into the region between the electrodes, it is also possible to direct a plasma stream across the electrode gap to allow breakdown but still maintain a low line density. The primary disadvantage of using the gun initiation was that because of the jitter in the gun fire time it was not possible to fuse the bank current to obtain shorter rise times in the pinch current. The gun used was a small version of Chengs

Figure 2.5. Total quantity of gas admitted to the pinch chamber from the fast gas valves as a function of time, as calculated from the gas dynamics. Each curve is a set of values for stop settings of 1/2, 1, 1 1/2, and 2 1/2 turns, at .907 mm/turn. The highest curve is the output of the Berkeley puffatron valve.

deflagration gun.⁴ The gun is of coaxial design and consists of a cylindrical barrel held at ground potential, and an on-axis electrode which is held at a high voltage. A fast gas valve puffs gas into the throat of the gun, which then breaks down in a uniform discharge. The resulting current ionizes the gas, creating a low temperature plasma.

and produces an azimuthal magnetic field which provides the energy to launch a deflagration wave from the gun barrel. The magnetic field serves the same purpose as the chemical heat of combustion in a rocket engine, which drives a deflagration wave out the exhaust nozzle. This mode of operation differs from that of the Marshall gun,⁴ which is of similar construction, in that with the Marshall gun there is a localized current sheet behind the gas which acts as a magnetic piston to push the neutral gas and plasma out of the gun barrel. Thus, the Marshall gun mode is much like a chemical detonation, where neutral gas is driven and heated by a shock wave. With the deflagration gun, the plasma thermal energy remains low, whereas with the Marshall gun part of the magnetic energy goes into shock heating. The mode in which either gun operates depends on whether the voltage is applied to the center electrode before or after the gas arrives. If the gas arrives after the voltage is applied, the gun operates in the deflagration mode; if the gas arrives before the voltage, the gun operates in the Marshall or snowplow mode, as it is called.

The construction of the gun is shown in fig. 2.4. An earlier version was slightly longer, but with the same diameters. The major difference was that the center electrode was bent and brought out through the wall near the throat of the valve. This asymmetry was removed with the design shown. The gun current was supplied by charging a $2.2 \mu\text{F}$ capacitor, normally to 8 kV, as shown in fig. 2.1, except that the 10Ω resistor was shorted. For the later shots, the resistor was used to prevent large currents and let the gun only serve

as a spark trigger. The current had a quarter period of $3 \mu\text{sec}$, and a peak current of 10 kA. Since the gun fire circuit was not crow-barred, the current rang through zero for several periods. During the first half period the gun appeared to operate in a deflagration mode, but thereafter presumably in a snowplow mode, as has been observed by Cheng. Framing camera photographs of the gun plasma are given in the next chapter in fig. 3.4.

Attempts were made to crow-bar the gun current to keep the gun operating in the deflagration mode longer. These were not successful because of the gas valve jitter. It was possible to pre-fire the gun fire ignitron and keep it conducting with a $1 \text{ k}\Omega$ keep-alive resistor, but it was difficult to time the crowbar trigger to coincide with the current maximum. Perhaps a multipulse trigger would have solved this problem, but was not tried.

2.1.5. Timing

The timing of the experiment was accomplished by use of a number of pulse distribution and variable-delay pulse amplifiers. The timing and sequence of events for the two modes of operation are shown in fig. 2.6. Fig. 2.6(a) shows a diagram of the timing for the fused-bank mode where the plasma gun is used only as a pre-ionizer. In this mode of operation it was necessary to trigger the oscilloscopes and diagnostics on a precursor because of the jitter in the pinch current start. The pinch current Rogowski loop proved to be a satisfactory

(b)

(c)

Figure 2.6. The timing and sequence of events for the FLR experiment. Timing for the fused-bank, no plasma gun mode is shown in (a). The vertical lines represent pulse distribution amplifiers, and the horizontal lines represent pulse delays. The arrowheads on the delay lines mean that the delay is variable. The dots represent ignitron firing chassis. Parts (b) and (c) show the sequence of events, with (c) being an expanded time scale. Times are in μsec .

precursor and was applied to the input of a marker generator with the input voltage threshold set high enough to reject noise signals. The marker output was then used to trigger the scopes and diagnostics. In figs. 2.6(b) and (c) the sequence of events for the fused-bank mode is shown. The sequence of events for the gun mode of operation is not shown, but it is similar. The main difference is that the pinch bank and gun ignitrons were fired shortly before the gas arrived.

The equipment used for the timing was at least fifteen years old and of vacuum tube design. It didn't have the low jitter times and shorter trigger delays of some of the newer designs, but it was fairly reliable. The timing control racks consisted of a coaxial cable patch panel, and the various delay and amplifier chassis. The variable delay chassis⁶ was dual channel, and had a minimum delay of $1.2 \mu\text{sec}$ and a maximum of 100 msec. Jitter was $\pm 0.1 \mu\text{sec}$ in the fastest range. The pulse distribution amplifiers⁷ had a fanout of four, and had a pulse delay of order $0.1 \mu\text{sec}$. The ignitrons were fired with a dual-ignitron-firing chassis,⁸ or a kip-tube chassis.⁹ The delay for the first was about $1 \mu\text{sec}$ and for the kip chassis less than $0.1 \mu\text{sec}$. For events requiring ignitron firing after short delays, the kip-tube chassis was used. The marker generators¹⁰ were dual channel, had minimum delay (less than 50 nsec), had a 2000 volt output pulse, and had a variable pulse length which was determined by the length of an attached coaxial cable. The minimum pulse width was 10 nsec, and the maximum was limited only by the amount of line used. These generators

were found to be very useful for camera and scope triggers, and for fiducial markers.

2.2 Diagnostics

A number of diagnostics were used on the experiment. The goals were to first observe the pinch structure and evolution, and then to measure the plasma parameters: the most important being the current and voltage, and then the plasma density, and density profiles, and finally, the plasma temperature. Several attempts were made to measure plasma temperature, but were unsuccessful. For this reason, plasma temperatures are inferred from the equilibrium conditions and the observed fluid velocities. Table 2.1 lists the diagnostics from which data were received. The UV monochromator is in parentheses since it was tried at the very end of the experiment, and there was not enough time available to get it working properly. The remainder of the chapter will be a discussion of each of these diagnostics.

2.2.1. Framing cameras

The use of a framing camera enabled observing the formation and decay of the pinch, and allowed measuring fluid velocities of the plasma. Framing cameras were used rather than streak cameras to allow 2D observation of the plasma at discrete times. The streak mode would have given better information on time behavior and growth rates, but

Table 2.1. Diagnostics for the finite Larmor radius Z-pinch experiment.

Type	Measured
1. Framing camera	Three 10 ns frames for each shot
2. Laser interferometer	Electron density
a. He-Ne	
b. CO ₂	
c. Argon-ion	
3. X-ray scintillator/PM tube	Hard X-rays from pinch break-up
4. Voltage probes	Voltage vs time
a. V _{bank}	
b. V _{anode}	
5. Rogowski loops	Current vs time
a. Bank current	
b. Pinch current	
c. Plasma gun current	
6. Light pipe/PM tube	Total visible output vs. time
7. Gas arrival probe	Gas arrival timing
(8. UV monochromator)	

would have lost in ease of interpretation since one spatial dimension is necessarily sacrificed to allow for a time scan. Two framing cameras were used, although not concurrently, for the entire set of shots. They were; (1) the TRW model 1D image-converter camera, and (2) the Imacon model 700 image converter camera, made by Hadland Photonics Limited. The TRW camera is commonly known as the STL camera, referring

to Space Technology Laboratories, the original manufacturer. Both cameras have streak capabilities, but were not used in that mode.

The STL camera provided the most useful set of information, and was used with almost all of the shots. The most commonly used sweep voltage plug-in was the 6B, which provided three 17 x 25 mm frames per shot. Exposure times could be set to 5, 10, or 20 nsec, and the delay between frames could be independently set to 50, 100, 200, or 500 nsec for each frame. The photocathode has an S-11 spectral response so that the images are an integral of the visible spectrum. The system gain is 2x, and final image resolution is 8 line pairs/mm. The gate on/gate off light extinction ratio is 10^6 , which was sufficient to block out increased electrode light after the experiment.

Care had to be taken to align the image position properly, since the cathode gating grid left two shadows running lengthwise in the final image. It was necessary to adjust the camera position such that the grid wires did not block out the desired image. Since the grid wires are not on a focal plane, they behaved as if not entirely opaque. The gain in their shadow was reduced, but not entirely to zero. This effect was observed in several shots where the plasma had an extended size.

The STL camera was mounted as close as possible to a side window of the vacuum chamber to allow use of as much light as possible. While in the focus mode, the image size was adjusted to make the pinch electrodes just visible inside either side of the image frame. When

the laser interferometer was added it was necessary to raise the camera to the top window and view through a mirror. For these shots the original f2 objective lens was replaced with a 100 mm / f4.5 lens to maintain the same image size. There was, however, more than sufficient light with the argon plasmas. The light intensity was usually reduced with a combination of neutral density filters, and by closing down the lens aperture. (With the hydrogen plasmas there was still sufficient light, except that the electrodes were usually brighter than the plasma. Efforts to mask out the electrodes to look only at the plasma were not entirely successful.)

The image on the anode phosphor was focused by a f1.2 transfer lens onto a film plane. Polaroid type 57 film, which has a speed of 3000 ASA, was used for most of the shots. Where higher dynamic range was desired, and to provide the capability to be scanned, Kodak Royal X pan film was also used. It has a 1250 ASA speed, which can be doubled at development time, provided the dynamic range can be sacrificed. For protection, all the Polaroid prints were coated with a spray laquer.

The Imacon camera was used with the 2×10^7 frames/sec plug-in. It provided eight to sixteen frames separated 50 nsec with 10 nsec exposures. The image size is 16mm x 18mm with eight frames and 8mm x 18mm with sixteen frames. The image resolution is 5 line pairs/mm, and the system gain is 2x. The image was recorded on either Polaroid type 47 or type 410 film.

The main difficulty with the Imacon was its inability to effectively block out light after the scan. The aperture stops of the objective lens and the neutral density filters had to be carefully adjusted to prevent a bright blur over the entire image plane. Even then, the film had a background fog which reduced the available dynamic range, and resolution. For these reasons the camera was only used on a small number of shots.

In order to understand the relationship of the framing camera photographs to the plasma density and temperature, the emitted light intensity for an argon plasma as a function of temperature was calculated. At lower temperatures, spectral line radiation and recombination radiation are important, and at higher temperatures electron-ion bremsstrahlung is important. All three of these mechanisms are dependent on the ionization state, or average charge Z_{ave} of the ion, which was calculated as a function of temperature by the technique discussed in appendix C.

The spectral line intensity was determined by summing the intensities of individual lines. Expressing in units of radiated power per unit volume, the intensity is

$$P_i = \sum_j h\nu_j^{k,l} A_j^{k,l} n^k, \quad (2.1)$$

where $h\nu_j^{k,l}$ is the photon energy released in the transition from the

k^{th} to the l^{th} excited level, $A_{j,l}^{k,l}$ is the transition probability (units sec^{-1}), and n_j^k the density of ions which have an electron in the k^{th} excited level. The subscript j denotes the particular line. Since there are several hundred argon lines in the visible, only the strongest lines, and only those in a wavelength range of 0.4 to 0.5 μm were included. To be consistent, the bremsstrahlung and recombination radiation were also limited to this range.

The population of the excited levels is related to the density of the ground state by the Boltzmann formula.

$$n^k = n^l \frac{g_k}{g_l} \frac{e^{-E_k/kT_e}}{e^{-E_l/kT_e}} \quad (2.2)$$

Here g_k and g_l are the degeneracies, and E_k and E_l are the energies of the levels k and l . The population of the ground levels of each ionization state are approximated by gaussian distributions with the average charge as an independent variable. Such profiles closely match those calculated by House.¹¹ The density of the j^{th} ionization state is then

$$n_j(Z_{ave}) = n_i \exp(-a(Z_j - Z_{ave})^2), \quad (2.3)$$

where n_i is the total number density of ions and neutrals, and Z_{ave} is a continuous function of temperature, and is calculated by the technique shown in appendix C. The constant a is chosen such that

$n_i(z_i + 1/2) = 1/2$. Note that the definition of the subscript i has been changed here to indicate the i^{th} ionization state. The values of g , E , A , and the wavelength, are tabulated by Wiese, *et al*¹² for neutral and singly ionized atoms. For doubly and more highly ionized ions the transition probabilities and degeneracies of the visible lines are not available. However, for doubly ionized argon they can be estimated by comparing the relative intensities given by Striganov and Sventitskii¹³ with those in the ultraviolet for which these parameters are known.

The frequency dependent expressions for the bremsstrahlung and the recombination radiation are given by Cooper¹⁴ in the hydrogenic ion approximation. That is, the ionization potential of an ion with a charge Ze is $Z^2 E_H$, and the energy of the excited level k is $Z^2 E_H(1 - 1/k^2)$, where E_H is the ionization potential of hydrogen. The intensity of the bremsstrahlung radiation is

$$P_b(\nu) = 1.70 \times 10^{-40} n_i n_e Z^2 \left(\frac{E_H}{T_e} \right)^{1/2} \exp\left(-\frac{h\nu}{kT_e}\right). \quad (2.4)$$

The units are ergs/(sec cm³) for the intensity, with T_e and E_H given in eV. A similar expression for the recombination radiation is

$$P_r(\nu) = c n_e n_i Z^4 \left(\frac{E_H}{T_e} \right)^{3/2} \exp\left(-\frac{h\nu}{kT_e}\right) \sum_{q \geq \left(\frac{Z^2 E_H}{h\nu} \right)^{1/2}}^{\infty} -\frac{1}{q^3} \exp\left(\frac{Z^2 E_H}{q^2 T_e}\right), \quad (2.5)$$

where c is a constant equal to 3.40×10^{-40} . The units are the same as those in the previous equation.

The intensities for argon from these three mechanisms were then integrated over the frequencies corresponding to the 0.4 to 0.5 μm wavelength range used for the line radiation, and were plotted, as is shown in fig. 2.7. At low temperatures, the line radiation dominates, and at higher temperatures the bremsstrahlung does. The line intensity falls rapidly after 6 eV because the strongest lines are in the ultraviolet and x-ray spectrum for the more highly ionized ions. The dip in the total light is probably not as pronounced in a real plasma as is shown in the graph. Line intensities for higher level transitions, which are known to exist, are not included since data for them is not available. (Striganov and Sventitskii,¹³ for example, list sixteen visible lines for triply ionized argon for which no data other than the wavelength is given.)

The other interesting feature is that the bremsstrahlung radiation falls off gradually with increasing temperature, despite the sharply increasing average ion charge. However it is necessary to remember that the camera detects only the visible radiation. The total

Figure 2.7. Calculated visible radiation of an argon plasma in a 0.4 to 0.5 μm wavelength window, as a function of temperature, for $n = 10^{18} \text{ cm}^{-3}$. The curves are marked B for bremsstrahlung, L for line radiation, and R for recombination radiation. The unmarked curve is the sum.

bremsstrahlung radiation increases with increasing temperature and ion charge as $Z^2 T_e^{1/2}$, but the maximum is shifted to the ultraviolet sufficiently that the intensity drops in the visible. This effect is shown by a plot in Cooper's paper.¹⁵

Thus, to conclude the discussion of the plasma radiation, the light emitted from an argon plasma rises sharply to a maximum at about 3 eV, and then stays relatively constant (probably to within a factor of five) as higher temperatures are attained, given a fixed particle density. For fixed temperatures, the intensity varies linearly with the density at lower temperatures, provided the excited states are in local thermodynamic equilibrium (LTE), and as the square of the density at higher temperatures, as can be seen in eqs. (2.1) and (2.4). If the plasma is not in LTE at low temperatures, then the line intensity is also proportional to the square of the density. If the density is sufficiently low the excitation process cannot keep up with the radiative decay. Thus, a dark region on the framing camera image must mean that either the electron temperature is below 2 eV, or that the density has decreased. It will not indicate a region of higher temperature, however.

Before concluding the discussion of the framing cameras, it is necessary to mention a system that was useful in analyzing the framing camera photographs, and that is the PDS photodensitometer which is operated by the Technical Photography group at LLNL. This system has the capability of digitizing an x-y scan with up to 13 million points, with a resolution of anywhere from 4 to 200 μm . This data is read onto a magnetic tape, which may be used to transfer the scan to the laboratory's computer system, where software is available for processing the data. Routines are available from Trimble,¹⁶ which can do contour plots, 3D plots, alter the scan data, rotate, invert, or

Fourier transform and filter the scans. The only drawback to the system is that the densitometer can only scan transparencies. For this reason only the photographs on the Royal X pan film were scanned.

2.2.2. Laser Interferometers

Measurements of the Z-pinch electron density and density profile were made with a spatially resolved Mach-Zehnder laser interferometer. A diagram of the set-up is shown in fig. 2.8. Rather than use a pulsed laser, a CW (continuous wave) laser was used. Time resolution and light gain was obtained by gating a micro-channel plate intensifier with a 10 nsec pulse. The gated image was recorded by a Silicon Intensifier Target (SIT) tube TV camera and displayed on a TV monitor. The image was permanently recorded by photographing the monitor image. The use of short gate times is possible because of the high gain of the intensifier and SIT tube combination; it being 10^4 times more sensitive than the standard vidicon.

The construction of the system was relatively simple since mechanical vibration is non-existent for the short exposure times used. The beam splitters and mirrors were 75 mm diameter and flat to $\lambda/10$. The vacuum windows were not optically flat because of the susceptibility to damage. However, they were selected to introduce the least amount of distortion. The plasma light was filtered out by spatial filtering, and by using narrow-band-transmission interference filters.

Figure 2.8. Diagram of the Mach-Zehnder laser interferometer used to measure electron densities and density profiles. The TV camera consists of a micro channel plate intensifier and SIT tube. The intensifier is gated by a 10 nsec pulse.

The advantages of the system are that the alignment is easier with a CW laser, and that data recording with Polaroid film is much less complicated than exposing and developing the film plates used in pulsed interferometry. There are, however, several disadvantages with this system. First, it is not possible to use double pulse holographic

techniques. Therefore, it is necessary to use optically flat elements within the interferometer. Unfortunately, the vacuum windows are coated with and damaged by electrode debris with each shot, making frequent replacement necessary to obtain good interferograms. Second, the resolution of the final image is also limited by the resolution of the intensifier/SIT tube/TV monitor combination. The nominal resolution of the micro channel plate is 10 line pairs/mm. This resolution would be acceptable except that the beam size had to be reduced 5x to maintain sufficient intensity, and to obtain the proper image size on the monitor. Thus, the resolution in the scale of the pinch is only 2 line pairs/mm. The poor resolution could be corrected by using a smaller beam diameter through the pinch, but nevertheless it is still a limitation. Third, for many of the higher density shots, the plasma light within the wavelength band of the filter was as bright as the laser illumination.

A 60 mW CW He-Ne laser, the first laser used, gave a number of good density profiles. With this laser, it was necessary to use 50 nsec exposure times because of the low power. There was also trouble with too much plasma light, and with a lack of sensitivity at lower line densities, and with lower Z materials. The fringe shift N of an interferometer can be shown to be proportional to the wavelength λ of the laser, times the line integral of the electron density,

$$N(y) = \frac{\lambda}{2.3 \times 10^{13} \text{ cm}^{-2}} \int_0^{L(y)} n_e(x, y) dx \quad (2.6)$$

where x and y describe the coordinate position, and $L(y)$ is the thickness of the plasma.

With the motivation of 17x greater sensitivity, and with the hope of improving the resolution of the interferogram, it was decided to use a pulsed TEA CO_2 laser at the $10.6 \mu\text{m}$ wave length. The system was built with a 1 J, 70 nsec laser and with the same interferometer configuration, except that NaCl and Ge optics were used. The detection system was also replaced by films sensitive to the $10.6 \mu\text{m}$ radiation.¹⁷⁻¹⁹ However, in opting to use the system, the negative index of refraction of the plasma was ignored. The plasma acted like a diverging cylindrical lens, refracting the beam enough that it was not detectable at the film plane. It was not possible to correct for the divergence since the index of refraction is a function of the electron density, which varies with both time and position.

For these reasons, a CW argon-ion laser operated at 514.5 nm was substituted for the CO_2 laser for the last four weeks of operation. The argon-ion laser was used because of its higher CW power, and because of availability. Its major drawback is that with argon, the plasma has optical emission lines at every wavelength the laser does. Therefore, even with the higher power level of the argon laser, it was

not possible to filter out the plasma light. The problem was less severe when using other gases.

Density profiles were obtained from Abel inversions of the interferograms. Appendix B contains a discussion of this technique and the scaling and limitations of the laser interferogram measurements.

2.2.3. X-ray measurements

Measurements of x-ray emission with energies ≥ 1 keV were made with a scintillator/photomultiplier tube combination. Attempts were also made to obtain x-ray pinhole photographs during the last week of the experiment, but these were unsuccessful.

The first measurements were with a NaI(Tl) scintillator. This crystal has a $1/e$ decay time of 250 nsec. For later shots a plastic scintillator was used to decrease the decay time to 2 nsec, and hence increase the frequency response. Both scintillators were located at the chamber wall and behind a thin (.0006") Al foil.

The photomultiplier, an RCA 6810A, was mounted directly behind the scintillator. It has an S-11 photo-response, and operated with a negative 2000 volt cathode potential while the anode was held at ground potential. The last four dynode stages were connected by capacitors of sufficient size that the photomultiplier could sustain 1 volt across 50 Ω for 0.2 μ sec without any linearity loss.

Energy measurements were made by determining the absorption through Al foils of different thickness. These foils were placed in the vacuum and in front of the scintillator. Neutral density filters were sandwiched in between the scintillator and photomultiplier to prevent saturation. There was some fluctuation ($\sim 10\%$) in the signal height for the different shots, however. This required averaging over several shots for each foil thickness. The data points were plotted on a logarithmic scale, and fitted to a straight line, the slope of which determined the mass absorption coefficient. These coefficients were then compared with absorption coefficients given by Osher²⁰ to obtain x-ray energy.

By fitting the data to a straight line it is implicitly assumed that the x-rays are mono-energetic. If the relative intensity is not logarithmically linear in the absorber thickness, but rather has a positive 2nd derivative, then one must conclude that a spectrum of energies is being observed. Then it is necessary to follow the analysis of Jahoda, *et al*,²¹ by integrating over the energy spectrum, while weighting the integrand with the absorption per unit energy. The energy spectrum is determined from bremsstrahlung radiation using a Maxwell distribution of electron velocities. The distribution is Maxwellian if the electron self-collision time is smaller than the experimental time scale, which it is by a factor of 10^3 for this experiment. With this technique, the total absorption, as a function of absorber thickness, can be calculated for a given electron temperature.

2.2.4. Other measurements

The bank current, the pinch current, and the gun current were measured with Rogowski loops. These were hand-constructed by stripping back and removing the outer conductor of RG-58 coaxial cable approximately 40 cm, removing 20 cm of dielectric from the center conductor, winding the center conductor back on itself, and then soldering it to the outer conductor. The end was then wound into one turn, and placed around the conductor to be measured.

The Rogowski coil output, which is proportional to the derivative of the current, was integrated with a passive RC network to provide a signal proportional to the current. The capacitor was chosen to be large enough that the integrator remained linear for times of interest. The loops were calibrated by comparing their output with a coil of known dimensions and turns.

Pinch anode and fuse voltages were measured with resistive voltage dividers each of which consisted of a string of 10 $500 \Omega / 2$ watt carbon resistors, yielding a voltage reduction of 100:1. The resistors in the divider were small enough that stray capacitance did not affect the frequency response. The resolution of the anode voltage was 10 nsec or better.

The total light output as a function of time was monitored with a light pipe connected to an RCA 1P21 photomultiplier tube, which has an S-4 photo-response. No efforts were made to collimate the light into

the end of the light pipe; it was simply pointed in the general direction of the pinch.

The gas arrival time was measured in the throat of the gas gun approximately 5 mm from the cathode plane. A ceramic insulated probe, which was bare at the end, was inserted perpendicular to the axis of the opening and biased it with a small capacitor charged to 2 kV. When the gas arrived there was an electrical breakdown from the probe to the wall, which discharged the capacitor. The signal was the voltage drop across a 50 Ω resistor in series with the capacitor. The capacitor was chosen to give a 10 μ sec pulse width, and a charge resistor was chosen large enough that the capacitor could not recharge until after the event. This signal enabled adjusting the bank timing to coincide with the gas arrival.

Attempts were also made to look at near-UV line spectra with a McPherson 1/3 m. f2 monochromator, with the hope of obtaining electron temperature measurements. The monochromator was located beneath the pinch chamber, with the pinch imaged lengthwise along the entrance slit. It was converted to a narrow band spectrograph by removing the exit slit and focusing the entrance slit image on a film plane. UV grade quartz optics were used throughout to allow use down to 185 nm wavelength. Both Polaroid type 57 and Kodak Royal X pan films were used and were found to be sensitive down to 250 nm. Calibration was done with a Hg arc lamp. The goal was to do several shots in the spectrograph mode and find a higher ionization-level spectral line

which did not have any close neighbors, and to then re-insert the slit and on subsequent shots observe the time evolution of the line intensity with a photomultiplier. However, in looking for lines, all that could be observed was a continuum. For this reason, this diagnostic was not pursued.

References for chapter 2

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CHAPTER 3

The Z-pinch Formation

3.1. Pinch formation

3.1.1. Gas flow into the chamber

In the last chapter, and in appendix A, the gas valve and its output as a function of time are discussed, however, the distribution of the gas as it enters the chamber is ignored. In this section the gas distribution, the breakdown process, and formation of the pinch is discussed. A small number of shots were initiated with a deflagration

plasma gun. A discussion of the formation of these types of pinches is deferred to the next section, however.

As the gas valve opens, the gas travels through the various orifices and intermediate regions at sonic velocities, as is discussed in appendix A. (See figs. 2.3 and 2.4 for the geometry.) This behavior is a result of the theory of flow through nozzles, where it is found that the flow is sonic if the pressure drop across the nozzle is greater than a factor of two, and that a sonic flow through the constriction of the nozzle becomes supersonic at the diverging end.^{1,2} Similarly, with the Z-pinch geometry, as the gas enters the vacuum region between the pinch electrodes, it expands and becomes supersonic.

The phenomenon of a steady-state free-jet expansion into a vacuum has been studied extensively in the context of supersonic wind-tunnels, molecular beam sources, and rocket exhaust plumes in the upper atmosphere. The flow of gas from the gas valve plenum to the pinch chamber is not a steady state process. However, many properties of the flow can be deduced by considering the steady-state results, after which non-steady state effects will be examined.

The general features of the jet, and the density of the core region as a function of the distance from the orifice are shown in fig. 3.1. An isentropic part of the flow appears to pass from the opening to the chamber through a barrel, the sides of which are the shock wave formed by the coalescence of compression waves from the jet boundary.⁴ These waves originate at the boundary as reflections of the

Figure 3.1. Free jet expansion into the vacuum chamber. (a) Geometry of the jet. (b) Density and pressure variation of the core region from the Owen-Thornhill theory.³

initial expansion fan. The bottom of the barrel is the Mach intersection of the lateral shock waves, and is called the Mach disk, or the Riemann wave. The strength of the shocks is related to the back pressure of the pinch chamber. In the limit of a perfect vacuum the shocks cease to exist. As the back pressure approaches the jet pressure, the shock waves become stronger.

The isentropic core was first described theoretically by Owen and Thornhill³ who used a numerical method of characteristics solution. Their results were verified experimentally by many others. A

particularly exhaustive study was done by Love, *et al.*⁵ who both experimentally and theoretically studied such aspects of the problem as the initial angle of the jet, the location of the shock boundaries, and the size and location of the Mach disk. These parameters were found to vary mainly with the mach number at the orifice, and the ratio of the orifice pressure to the background pressure. The papers by Sherman,⁴ and Ashkenas and Sherman⁶ give experimental data on the density of the shock regions, as well as to verify many of the predictions of Love, *et al.*⁵

Considering the flow through the orifices in the plasma gun anode (see fig. 2.4) as sonic, it is possible to determine the flow parameters at the cathode plane by using mass continuity, the Euler equation, and the isentropic equation of state. Thus, it is found that at the entrance to the chamber the jet has a Mach number of 4, and that the ratio of the jet pressure to the chamber pressure is 10^4 to 10^5 , based on a 10^{-4} Torr chamber vacuum. The mean free path for collisions at the orifice is 3×10^{-3} cm, with a one atmosphere plenum pressure. Thus the Knudsen number, which is the ratio of the mean free path to the jet orifice diameter, in this case, is $\sim 2 \times 10^{-3}$ at the cathode, and $< 2 \times 10^{-1}$ at the anode. For Knudsen numbers $\ll 1$, a gas is collisional, and a continuum theory is valid.

As the gas leaves the cathode plane it expands into a fan similar to that shown in fig. 3.1(a). Its initial angle is $65 \pm 10^\circ$ from the axis. The distance to the Mach disk is ~ 60 times the diameter of the

jet orifice d , which is well behind the anode. The density and pressure on-axis fall off as shown in fig. 3.1(b). For distances x/d greater than 2, the pressure drops as $(x/d)^{-2}$, where x is measured from a point slightly downstream from the orifice. Because of the adiabatic equation of state, the density falls off as $(x/d)^{-2/\gamma}$, where γ is the ratio of the specific heat capacities. Thus, for argon, the density falls off as $(x/d)^{-6/5}$. In the radial direction, the density and pressure vary as $\cos^2\theta$, where θ is measured from the axis about a pivot at the cathode plane. This variation from the axis is valid up to the point where the local pressure equals the background pressure. At that point there is a shock, and a higher density region. For small back pressures, the shock region is well away from the axis, has a low density, and therefore need not be considered. Fig. 3.2 shows a contour plot of this scaling in the Z-pinch geometry. The off-axis scaling is only valid for $x/d \geq 1$, since the orifice geometry has a much larger effect there. The axial variation does correspond to the Owen-Thornhill calculations, however. The contours for $x/d < 1$ are not calculated values, but are estimated by smoothing from the orifice density to the density in the expanded jet. The general features of the jet are apparent, however. The shock region would then be located at the uniform density contour where the vacuum pressure equals the jet pressure.

A good experimental example of this barrel shock structure is shown in the interferogram in fig. 3.12. The photograph was taken after the pinch current had started, for a shot with a higher back

Figure 3.2. (a) Contour and (b) 3D plots of the density profiles from the Owen-Thornhill theory. The units are orifice diameters. The cathode is on the left, and the anode is on the right.

pressure. Because of a pinhole used in the optics, the image has a Schlieren effect, i.e., regions of large gradients in the plasma density are seen as dark lines running across the image.

Up to this point the position of the anode has been ignored. Being located only 4.2 orifice diameters downstream, it certainly affects the flow. The question is what happens to the gas after it strikes the anode. If the flow at the anode is molecular, i.e., if it has a high Knudsen number, then the gas particles would rebound from the anode at all angles, and increase the backpressure at all positions between the electrodes. The Knudsen number is low, however. Thus the gas is collisional, and puddles against the electrode, escaping in a

more radial direction, and not affecting the flow upstream more than several mean free paths away. This effect is verified numerically by Bird⁷ in considering the time evolution a freely expanding jet impacting on a wall. He does find, however, that at later times there is a reflected shock which moves toward the orifice, radically changing the character of the jet. Typically, the pinch bank was fired before this structure could develop further upstream, however.

Two other effects are related to the non-steady state gas distribution. (1) Because of the short time between when the gas enters the chamber, and when the pinch bank is fired, the radial variation of the gas cannot attain the $\cos^2\theta$ distribution. Rather, the gas distribution can only extend a distance corresponding to the local flow velocity, times the time from when the gas first enters the chamber. The density is then expected to be zero past one of the constant density profiles shown in fig. 3.2(a). The position of the limiting profile depends on the time of the bank firing, however. (2) Because the pressure in the orifice is a steeply varying function of time, the axial variation of the pressure is necessarily steeper than the $(x/d)^{-2}$ variation since the pressure of the gas entering the chamber at later times is higher. Fig. A.5 of appendix A shows the pressure in the intermediate chamber to rise a factor of ten in 150 μsec , stay level for about 100 μsec , and then fall off almost as rapidly as it rises. The pressure at the orifice is expected to have the same variation. The magnitude of this effect is of course also dependent on the timing of the bank fire.

3.1.2. Gas breakdown

The breakdown characteristics and the Paschen curves for argon, and the noble gases are well-known and available.⁸ The breakdown voltages for the noble gases are much lower than those of other gases. The minimum breakdown voltage for both argon and krypton is ~ 230 volts, with a Pd (pressure times gap distance) of 1 Torr-cm. For our geometry this is a number density of $7 \times 10^{15} \text{ cm}^{-3}$. At both lower and higher pressures the breakdown voltage is much higher. With 4 kV across the pinch electrodes, which was also the most commonly used bank charge voltage, the Paschen curve shows an upper limit of $2 \times 10^{18} \text{ cm}^{-3}$, and a lower limit of $1 \times 10^{15} \text{ cm}^{-3}$ on the gas density. That is, breakdown is possible at any density between 10^{15} to 10^{18} cm^{-3} . The anode voltage is not instantaneously 4 kV, but rises at a rate of $40 \text{ kV}/\mu\text{sec}$. Therefore, as the anode voltage rises, the range of densities over which breakdown may occur, increases from a small range about the Paschen minimum, to a range spanning over three orders of magnitude.

From the gas valve calculations, using a one atmosphere plenum fill pressure, and from the curve in fig. 3.1, the on-axis density is estimated to be 5 to $10 \times 10^{15} \text{ cm}^{-3}$ at the anode. The value is of course very dependent on the timing. The path of breakdown is not observed to be perpendicular to the electrodes, but rather follows the gas distribution. Shots with the camera timed early show the initial current to be fan-, or barrel-shaped, with the distribution very

similar to that shown in figs. 3.1 and 3.2. An example of one of these shots is shown in fig. 3.3. There is also evidence that the initial current path is at the edge of the pinch. The light emission in the darker fan-shaped regions, as is seen in the framing camera images, is the brightest on the edges. This effect is seen in figs. 3.10, 3.11, 3.12, and 3.14 of the next section. The interferogram and Abel-inverted density profile of fig. 3.12 show the density to be mainly in a small region near the edge, as would be expected in a

Figure 3.3. Framing camera photograph of an early stage of the pinch formation. The pinch current is 40 kA, and the electrode separation is 23 mm.

snow-plow type collapse of the pinch.

3.1.3. Pinch formation

After the gas breakdown, the plasma column contracts towards the axis, often forming a straight, long-lasting column. The time for the plasma to collapse, the pinch time t_p , was found to vary with the gas valve plenum pressure, and the delay to bank fire. The results of measurements from a number of shots are shown in fig. 3.4. The time to pinch was measured from the onset of the pinch current to the time of the voltage spike and current dip, which usually corresponded to the time that the entire column had pinched. That the pinch time should vary with the delay to bank fire is reasonable, since for longer delays the gas is allowed to expand a longer time in a radial direction between the electrodes, and hence has further to go when the pinching begins. The effect also depends on the time behavior of the gas density. How the gas density and initial radius are related to the pinch time is discussed in the following paragraphs.

The pinch time can be predicted by the well-known snowplow model,^{9,10} given the initial radius and mass density of the plasma, and the current as a function of time. Consider a force balance between an infinitesimally thin current carrying shell and the magnetic field pressure.

$$\frac{d}{dt} \left(m \frac{dr}{dt} \right) = - \frac{B^2}{2\mu_0} 2\pi r \quad (3.1)$$

Figure 3.4. Measured time to pinch t_p versus the pulsed gas valve fill pressure, for three separate bank-fire delay times. The lines show the scaling with the density. The solid line is $t_p \propto \rho_o^{1/2}$, and the dashed line is $t_p \propto \rho_o^{1/4}$.

function of r .

$$m(r) = \rho_0 \pi (R_0^2 - r^2) \quad (3.2)$$

The magnetic field B is also a function of r and t .

$$B(r, t) = \frac{\mu_0 I(t)}{2\pi r} \quad (3.3)$$

For these last three equations r is the pinch radius, R_0 the initial radius, I the pinch current in amperes, ρ_0 the initial mass density, and m the mass per unit length. For the following equations, all units will be cgs (with I still in amps). Letting $y = r/R_0$, and including the above expressions for m and B , eq. (3.1) becomes,

$$\frac{d}{dt} \left[(1-y^2) \frac{dy}{dt} \right] = - \frac{I^2}{\pi \rho_0 10^2 R_0^4} \left(\frac{1}{y} \right). \quad (3.4)$$

The solution to this equation depends on the form of the current $I(t)$. Consider three different current profiles:

1. $I = s_0 t$, where s_0 is a constant. A numerical solution of eq. (3.4) with a linearly increasing current is found by Anderson, et al.,⁹ who define a dimensionless parameter τ

$$\tau = \left(\frac{\pi 10^2 \rho_o R_o^4}{s_o^2} \right)^{-1/4} t. \quad (3.5)$$

With $y(0) = 1$, and $dy(0)/d\tau = 0$, the normalized radius y is found to reach one-eighth of its initial radius for $\tau = 1.43$. The Anderson paper defines the pinch time t_p as the time for y to reach $1/8$, which was a commonly observed final radius for the snow-plow pinch. The choice of the final radius is somewhat arbitrary, but not critical, because of the high sheath velocity near the final stages of the collapse. Choosing $y = 0$, for example, gives a value for the pinch time that is only 3 % longer. Since the final value of the radius does not greatly affect the time to pinch, the convention that the plasma has pinched when $y = 1/8$ is adopted. Thus, the time to pinch with a linearly increasing current is

$$t_p^l = 1.43 \left(\frac{\pi 10^2 \rho_o R_o^4}{s_o^2} \right)^{1/4}. \quad (3.6)$$

The superscript l denotes a linearly increasing current. The pinch time then scales as $\rho_o^{1/4}$, $s_o^{-1/2}$, and is linear in R_o .

2. $I = I_o$, where I_o is a constant. For a current which rises to a constant value in a time much shorter than the pinch time, the pinch dynamics can be modeled by letting the current I in eq. (3.4) be a constant I_o . Define a new dimensionless time τ

$$\tau = \left(\frac{I_o}{(\pi 10^2 \rho_o)^{1/2} R_o^2} \right) t. \quad (3.7)$$

Now write eq. (3.4) in terms of τ , expanding the differentials.

$$(1 - y^2) \frac{d^2 y}{d\tau^2} - 2y^2 \left(\frac{dy}{d\tau} \right)^2 + \frac{1}{y} = 0 \quad (3.8)$$

An analytic solution to this differential equation is given by Kamke,¹¹ who expresses the solution in terms of an integral. Eq. (3.8) then reduces to the following integral, which must be evaluated numerically.

$$\tau = \int_y^1 \frac{(1 - y^2) dy}{\sqrt{y^2 - 2 \ln y - 1}} \quad (3.9)$$

For a lower limit $y = 1/8$, the dimensionless pinch time is $\tau = .873$. This result is verified by both a numerical integration of eq. (3.9), and a numerical solution to eq. (3.4) with a constant current. In dimensioned variables, the time to pinch is

$$t_p^c = .873 \frac{(\pi 10^2 \rho_o)^{1/2} R_o^2}{I_o} \quad (3.10)$$

The superscript c denotes a constant current. Thus, the time to pinch with a constant current scales as $\rho^{1/2} R_o^2$ and I_o^{-1} .

3. $I = s_0 t$, $t \leq t_1$, and $I = s_1 t$, $t > t_1$. With the fused-bank arrangement, the pinch current was found to rise quickly, and then level off to either a constant, or a slowly increasing current. If the rate of rise of the second slope is small, the time to pinch could scale either as in case (1), or case (2), depending on what fraction of the pinch time is spent in the linearly increasing current regime. With the current as an arbitrary function of time, it is not possible to write eq. (3.4) in terms of a dimensionless time, rather it is necessary to numerically solve the dimensioned equation. This was done with a 2nd order Runge-Kutta scheme.¹² The results of one particular run, with the slope s_1 equal to zero, are shown in fig. 3.5.

Although an analytic scaling of the pinch time was not found for the current that rises to a maximum, and levels off, it was noted from the numerical solution to eq. (3.4) for the case $s_1 = 0$, that with a fixed maximum current I_0 and $s_0 = I_0/t_1$, the time to pinch t_p varied approximately as

$$t_p = a + b t_1. \quad (3.11)$$

The parameters a and b are constants, and t_1 is constrained to be less than or equal to the time for the plasma to pinch on the linear portion of the current profile. Otherwise, the solution is given by eq. (3.6). For $t_1 = 0$, the pinch time is given by eq. (3.10), which requires that $a = t_p^c$. The constant b is determined by noting that $t_p = t_1$ as t_1 is

Figure 3.5. Numerical solution to eq. (3.4) with a current that rises to a maximum, and then levels-off to a constant. The dotted line is the solution if the current continues to rise linearly, rather than leveling-off.

increased to the point where the current levels off just as the plasma pinches. Using eq. (3.6) with $s = I_0/t_f$, the pinch time is

$$t_p = t_1 = 1.43 \left(\frac{\pi 10^2 \rho_0 R_0^4}{I_0^2} \right)^{1/4} t_1^{1/2}. \quad (3.12)$$

This equation is solved for t_1 , and combined with eq. (3.10), noting that the various parameters cancel. Thus, t_1 is found to equal $2.34 t_p^c$ at this point, which makes $b = .573$. Writing out the coefficients, the pinch time is

$$\begin{aligned} t_p &= t_p^c + .573 t_1, \quad t_1 \leq 2.34 t_p^c \\ &= t_p^l, \quad t_1 > 2.34 t_p^c \end{aligned} \quad (3.13)$$

where t_p^c and t_p^l are given by eqs. (3.10) and (3.6) respectively.

Note that eq. (3.13) has been constructed to be equal to the solutions of cases (1) and (2) at either limit. For values of t_1 between the limits, the equation was checked against the numerical solution and found to be about 3 % low at mid-range, with a smaller error toward the boundaries. This error is well within those introduced by the approximations of the snow-plow model. A plot of the pinch time calculated by the snow-plow model versus t_1 for several runs, is shown in fig. 3.6, superimposed on a plot of eq. (3.13).

Upon writing out the expressions for t_p^c and t_p^l , eq. (3.13) becomes

Figure 3.6. Pinch time versus the time the current levels off, in units of t_p^c . The circles are the numerical solutions to eq. (3.4), and the solid line is a plot of eq. (3.13).

$$\begin{aligned}
 t_p &= .873 \frac{(\pi 10^2 \rho_o)^{1/2} R_o^2}{I_o} + .573 t_1, \quad t \leq 2.04 \frac{(\pi 10^2 \rho_o)^{1/2} R_o^2}{I_o} \\
 &= 1.43 \left(\frac{\pi 10^2 \rho_o}{s_o^2} \right)^{1/4}, \quad t_1 > 2.04 \frac{(\pi 10^2 \rho_o)^{1/2} R_o^2}{I_o}
 \end{aligned}
 \tag{3.14}$$

Perhaps one of the most interesting features of eq. (3.14) is that it is independent of the scaling of the linearly rising current solution, but rather depends on the constant current scaling. Thus, the pinch time for these pinches would scale as $\rho_o^{1/2}$, R_o^2 , and I_o^{-1} , as does case (2).

For a fixed R_o and I_o , the pinch current varies as

$$t_p = a \rho_o^{1/2} + b, \quad (3.15)$$

where a and b are constants equal to

$$a = 8.73 \frac{\sqrt{\pi} R_o^2}{I_o} \quad (3.16)$$

$$b = .573 t_f,$$

which is exactly the scaling observed in fig. 3.4.

The main difficulty encountered in applying eq. (3.15) is that the initial radius R_o is not known, except by estimates from the initial gas distribution, and by accounting for the time delay to the bank fire, assuming the gas jet radius expands at the sound velocity. The mass density is determined by using the plot in fig. 2.5, the estimated radius, and the known pinch length. Estimates of this sort were done with several shots, and the calculated pinch times were found to be within 10 % of the measured values.

The pinch time is also experimentally observed to vary linearly

Figure 3.7. Measured time to pinch versus bank fire delay time (time in μsec), for several plenum fill pressures.

with the bank-fire delay time t , as is seen in fig. 3.7. The radius R_0 of the gas puff is proportional to t , and, since the total gas quantity between the electrodes remains constant at later times (see fig. 2.5), the density ρ_0 varies as R_0^{-2} , or t^{-2} . Thus, $t_p \propto \rho_0^{1/2} R_0^2 \propto t$, from eq. (3.14), which is consistent with observations.

Two aspects of the axial variation of the pinch should also be discussed. First, the framing camera photographs show that the pinch collapses to the axis first near the cathode, and last at the anode. If the current flows mostly in the skin, as is indicated by the interferogram in fig. 3.12 and by the light emission from the fan-shaped regions in the framing camera photographs, then the pinch velocity is not entirely radial but has an axial component directed toward the anode. The magnitude of the axial velocity is then the sine of the angle α which the fan-shaped region makes the z-axis just prior to peak compression, times the maximum pinch velocity. For α equal to 25° , which is the angle observed in fig. 3.8 of the next section, the axial velocity is 0.4 times the pinch velocity. Second, since the mass density and gas jet radius vary with the axial position z , the pinch time is also expected to vary with z . From sec. 3.1.1 the density was found to fall off as z^{-1} , and the radius to be approximately linear with z . Thus, the time to pinch scales as $z^{3/4}$ for the scaling eq. (3.6), and as $z^{3/2}$ for eq. (3.10). The experimentally observed scaling of the pinch time in z was found to vary from shot to shot, but was generally of the form $t_p(z) \propto a + b z$, where a and b are constants.

3.2. Examples of Pinch Formation and Lifetimes

Two modes of initializing linear z-pinches were used: (1) the fused bank, no gun mode, and (2) the plasma gun initiated mode. Both of these modes are described in chapter 2. The following examples

show example shots from each of these types, discussing the parameter variations. These included use of low and high Z gases, short and long pinch lengths (23 mm vs 53 mm), and use of various densities. In each of the shots studied, the pinch was observed to have an instability which eventually destroyed the pinch. These instabilities are examined

Table 3.1. Description of the of shots shown in this chapter

Figure	Description	Gas	Length (mm)	Plasma Gun	Fused
3.8, 3.9	low density gas-puff, short	Ar	31		x
3.10	low density gas-puff, long	Ar	53		x
3.11	high density gas-puff, long	Ar	53		x
3.12	high density, high Z, long	Kr	53		x
3.13	low Z gas-puff, long	He	53		x
3.14	low Z, high current, long	N ₂	53		x
3.15	plasma gun initiated, short	D ₂	23	x	

in chapter 4.

3.2.1. The gas-puff initiated Z-pinch

Framing camera photographs of a gas-puff initiated Z-pinch are shown in figs. 3.8 and 3.9. The photograph is consistent with the diagram of the chamber in fig. 2.3. That is, the cathode is on the right and the anode on the left. This convention is also used with all

Table 3.2. Measured parameters for the shots shown as examples in this section. These are the values measured at peak compression.

Figure	Radius (mm)	V _{bank} (kV)	I _{pinch} (kA)	τ_{conf} (μ s)	τ_{pinch} (μ s)	n _e (cm ⁻³)
3.8. 3.9	2.0	4.0	100	0.6	1.3	1.5 x 10 ¹⁸
3.10	2.0	4.0	90	1.0	2.0	5 x 10 ^{19b}
3.11	3.5	5.5	210	0.6	7.5	3 x 10 ^{19b}
3.12	2.0	4.0	120	0.5	6.0	2 x 10 ¹⁹
3.13	2.7	4.0	140	< 0.1	3.5	1.3 x 10 ¹⁸
3.14	2.7	5.5	220	0.4	6.5	2 x 10 ¹⁹
3.15	1.0	8.0 ^a	120	0.2	3.0	6 x 10 ¹⁹

^aAlthough the voltage was higher for these shots, the bank energy was lower since a 112 μ F capacitor bank was used. The other shots were done with the 900 μ F capacitor bank.

^bThese values are estimates based on gas valve settings, and the Saha equation.

other framing camera photographs. Note also that fig. 3.8, as well as others in this chapter, is constructed of a sequence of several shots, varying only the camera timing. A comparison of the parameters of these shots with the others in this section is given tables 3.1, 3.2, and 3.3. In table 3.1 the types of shots observed are listed. Table 3.2 gives a comparison of the directly measured parameters from these

Figure 3.8. Typical life history of a lower density argon Z-pinch from framing camera photographs. The time in μsec from the pinch current start is given to the right of each frame. The electrode separation is 31 mm.

shots, and table 3.3 compares the deduced parameters. Figure 3.8 shows the typical evolution of the gas puff, fused-bank operation. Argon was used instead of deuterium to make the pinch easier to photograph. The gas comes out of the plasma gun orifice in the form of a conically-shaped jet, is ionized, and starts to pinch, as shown in the first frame. (For orientation, note the shadow of the tip of the gun electrode.) Subsequent frames show that the conically-shaped plasma then pinches to a straight column, starting from the cathode end. It then breaks up in a sausage-like instability. Other shots of the same configuration, but with different densities, show the same pinching characteristics, but different break-up mechanisms. Some pinches are

(c)

Figure 3.9. Density measurement for the type of pinch shown in fig. 3.8. (a) Framing camera photographs, with a pinch length of 31 mm. (b) Interferogram from a Mach-Zehnder interferometer operated at 632.8 nm. The position of the arrow represents the relative time of exposure in μsec . (c) Abel inversions at the indicated axial positions.

observed to remain as a straight column up to $0.6 \mu\text{sec}$. The observed instability scaling with density is discussed in chapter 4, however.

Fig. 3.9 shows the electron density measurement for a gas-puff initiated z-pinch. The Abel inversion of the cross-sections in (b) are given in (c), where the position of the letters on the plots represent the calculated densities as a function of r . The solid lines are a least-squares fit of the points to a third order polynomial with a zero slope at the pinch axis. The line density is found to vary from 6×10^{16} to $1.9 \times 10^{17} \text{ cm}^{-1}$ along the axis of the pinch. This density implies a Bennett¹³ temperature of 200 to 490 eV along the axis. Other deduced parameters are discussed in the next section.

Fig. 3.10 shows the evolution of a gas-puff initiated, fused-bank pinch with the electrode separation increased to 53 mm. There are no direct measurements of the electron density for this set of shots, but the electron line density is estimated from the gas valve settings, from the results of appendix A, and the ionization calculations of appendix C to be $6 \times 10^{18} \text{ cm}^{-1}$. The technique by which this is done is explained in sec. 3.3.

Fig. 3.10(b) shows the current and voltage versus t for the shot. The current is observed to start with the large voltage spike shown on the lower trace. This voltage is measured at the capacitor bank side of the spark gap, and shows the inductive effect of the fuse blowing. As the current across the gap and the pinch rises, the voltage quickly drops, and remains low until the pinch-column is disrupted. The second

Figure 3.10. Evolution of argon, gas-puff initiated Z-pinch with an electrode separation of 53 mm. (a) Framing camera photographs of two shots differing only in camera timing. The time in μsec from pinch initiation is shown to the right of each frame. (b) The upper trace is the pinch current at 100 kA/div., and the lower trace is the anode voltage at 4.0 kV/div. The horizontal scale is 5.0 $\mu\text{sec}/\text{div}$.

voltage spike occurs at this time, and the current is observed to dip, but does not go to zero, because of the inductance of the system. Thereafter, a pinch often reforms, but usually with a lower inductance path. These restrikes were not studied extensively, however.

Hard x-ray signals were also observed to coincide with the voltage spikes. For those shots which had several restrikes and breaks, additional x-ray bursts were observed which also coincided with the later inductive voltage spikes. The energy of these bursts was

measured by absorption methods, and the results are discussed in the

Figure 3.11. Evolution of a long, higher density argon Z-pinch. (a) Framing camera photograph of one shot with an electrode separation of 53 mm. Time shown in μsec from pinch current start. (b) The upper trace is the pinch current, 100 kA/div., and the lower trace is the anode voltage, 4.0 kV/div. The horizontal scale is 5 $\mu\text{sec}/\text{div}$.

next chapter.

Figure 3.11 shows an example of a long, high density argon pinch, for which the pinch current at peak compression is 210 kA. Density measurements were not obtained for this and similar shots because of the excessive light from the plasma. Estimates from the gas valve settings indicated an ion density of $3 \times 10^{18} \text{ cm}^{-3}$. These estimates are made by use of fig. 2.5, using the plenum fill pressure and the bank fire timing to obtain the total quantity of gas puffed into the chamber. The line density is estimated by dividing this quantity by the pinch length, and the local density is estimated by assuming the

Figure 3.12. Evolution of a krypton pinch. (a) Framing camera photograph. (b) The upper trace is the pinch current, 100 kA/div., and the lower trace is the anode voltage, 4.0 kV/div. The horizontal scale is 5 μ sec. (c) Interferogram during pinch formation. (d) Abel inversion of the density profile.

camera image. Use of the Bennett relation, and the ionization calculations indicate a temperature of only 3 eV.

Figure 3.12 shows an example of krypton shot, with an interferogram at the early stages of pinching, and its Abel inversion. The measured electron density is $1.2 \times 10^{18} \text{ cm}^{-3}$, whereas using the gas valve calculations the ion density is estimated to be $5 \times 10^{17} \text{ cm}^{-3}$. The difference between the two could very well be attributed to the plasma being doubly ionized. From these measurements, the electron density at peak compression is estimated to be $2 \times 10^{19} \text{ cm}^{-3}$. It was not possible to get density measurements at peak compression, presumably because of the higher temperature, which leads to a higher ionization state, resulting in more electrons, and a much higher fringe shift.

Another interesting feature of this class of shots is that during the compression the region of highest density is at the edge of the plasma. This is seen clearly in the interferogram and Abel inversion, and also in the framing camera photographs. In the darker regions of the pinch the optical emission is strongest near the edge, and weaker in the center. An Abel inversion of the light, which is also proportional to the density for line radiation, would also show much higher densities at the edge. This type of density profile is very similar to the infinitesimally thin plasma shell that is considered with the snow-plow model.

Figure 3.13. Example of a helium Z-pinch. (a) Framing camera photographs with the time of the frame given in μsec . (b) The upper trace is pinch current, 100 kA/div., and the lower trace is the anode voltage, 4.0 kV/div. The horizontal scale is 5 $\mu\text{sec}/\text{div}$. (c) Interferogram at 7.0 μsec . (d) Abel inversions at the

Fig. 3.13 shows an example of a helium pinch. The measured electron density is $1.3 \times 10^{18} \text{ cm}^{-3}$. The time of the frames shown is after the first pinch and break-up, so that the plasma seen is likely a restrike, and contains electrode material. The helium and hydrogen pinches were very difficult to obtain, and often came apart on one end before the other end could pinch.

Figure 3.14, the last gas-puff example, is of a nitrogen pinch. The ion line density, determined from the gas valve settings, and by comparing with electron density measurements of similar shots, is

Figure 3.14. Example of a nitrogen pinch. (a) Framing camera photograph. (b) Pinch current, 100 kA/div., and anode voltage, 4.0 kV/div. The horizontal scale is 5 μsec/div.

$1 \times 10^{18} \text{ cm}^{-1}$.

3.2.2. The plasma gun initiated Z-pinch

An example of a deuterium, plasma-gun initiated Z-pinch is shown

Figure 3.15. Evolution of a deflagration gun initiated deuterium Z-pinch, assembled from framing camera photographs of four shots with only the camera timing varying between shots. Time in μsec from start of pinch current is given to the right of each frame. The electrode separation is 23 mm.

in fig. 3.15. For this set of shots, the anode had a conically-shaped protrusion, the outline of which is best seen in the frames of 0.0 and 1.6 μsec . This configuration was used to insure that the pinch remained on the central axis to enable photographing. I. was later found that the pinch could also be prevented from wandering by dishing the anode by 4° toward the center.

The first frame of fig. 3.15 shows the plasma as it is shot from the deflagration gun. At $t = 0.0 \mu\text{sec}$ the pinch current begins. At $t = 0.6 \mu\text{sec}$ the current is still rising almost linearly, and has

reached approximately 80 kA. At $t = 0.6 \mu\text{sec}$ a straight pinch channel is seen, which then develops a dark region near the anode at $t \sim 0.8 \mu\text{sec}$. The dark region then propagates axially to the opposite electrode. At $1.4 \mu\text{sec}$ no light is seen at all. The frames at 1.4 and $1.6 \mu\text{sec}$ show that anode has become much brighter after the pinch light goes out. The light pipe/photomultiplier combination also shows the light to increase almost a factor of ten at this time. One possible explanation is that high-Z electrode material is being ionized at these later times, greatly increasing the line radiation.

No electron density measurements were made with the gun-initiated shots, therefore it is not possible to reliably use the Bennett relation to estimate electron and ion temperatures. However, from the framing camera photographs in fig. 3.15, it is possible to estimate the ion density by observing the velocity of the propagating dark region in the photographs. Any disruption of the magnetic field must flow at the Alfvén velocity, which is a function of the magnetic field strength and the ion density. Equating the observed velocity to the Alfvén velocity, and using $B = 260 \text{ kG}$ (obtained from the current and pinch radius), the ion density is found to be $6 \times 10^{18} \text{ cm}^{-3}$. The electron line density is 1.9×10^{17} , which means that $s = 11$, by eq. (1.5). Based on this density, and considering the pinch to be fully-ionized, the Bennett temperature is found to be 120 eV.

The time the column remains straight without breaking apart, is estimated from the frames at 0.6 and $0.8 \mu\text{sec}$ to be less than 200 nsec;

perhaps only 100 nsec. This gives a respectable $n\tau$ of 4×10^{12} . (The ion temperature is much too low to consider fusion, however.)

The FLR scaling s is determined to be 22 by using eq. (1.5) and the estimated electron line density.

3.3 Deduced Parameters

In table 3.3 several deduced parameters are shown for the example shots shown in the last section. How these parameters were determined is discussed in the following paragraphs.

The electron density n_e and the electron line density N_e ($= \pi n_e a^2$) were most reliably determined from laser interferometry and subsequent Abel inversion. Then by using the Bennett relation and the results of the average ion charge calculations, it is possible to determine the average ion charge Z_{ave} , and the temperature T ($T_e = T_i$) by an iterative scheme. The Bennett relation is

$$I^2 = 200 N_e \left(1 + \frac{1}{Z_{ave}} \right) kT, \quad (3.17)$$

where the units are I in amperes, N_e in cm^{-1} , and kT in ergs. A value of Z_{ave} is guessed, and then eq. (3.17) is solved for the temperature. This temperature is then used with the plots in appendix C to find a

Table 3.3. deduced parameters for examples shown in section 3.2.

Figure	Z_{av}	T (eV)	s	$a/\langle a_i \rangle$	v_{de}/v_{ti}	T_{eq} (ns)
3.8, 3.9	16	200	36	11	.007	180
3.10	1	8 ^b	14	4.4	.0008	1.7
3.11	1	3	43	14	.0006	1.0
3.12	7	11	54	17	.002	1.7
3.13	2	140	24	7.6	.006	100
3.14	1	76 ^b	9.4	3.0	.004	100
3.15	1	120 ^b	11	3.5	.7	20

^a From the Bennett relation with $T = T_e = T_i$.

^b Measured from flow velocities

new guess for the average charge. This process can be repeated several times until a consistent T and $Z_{av,e}$ are found. The details of the average ion charge calculations are given in appendix C.

For various reasons, it was not possible to obtain electron density measurements for many of the shots. For these cases the ion density can be estimated from the gas valve settings, and the ion temperature inferred from wavefront velocities as the pinch breaks apart. It is then possible to use the Bennett relation and Saha equation to find a consistent T and $Z_{av,e}$, remembering that $N_e =$

$Z_{ave} N_i$. It was often found that the inferred ion temperature was too high for a consistent result, however. Therefore, many of the deduced parameters for these shots can only be estimates.

So far, it has been assumed that $T_e = T_i$. The equilibration between electrons and ions is discussed by Spitzer,¹⁴ who shows the time behavior of the temperature as

$$\frac{dT_i}{dt} = \frac{T_e - T_i}{\tau_{eq}} \quad (3.18)$$

The quantity τ_{eq} is the equipartition time, and is defined as

$$\tau_{eq} = \frac{3m_f k^{3/2}}{8(2\pi)^{1/2} n_f Z_f^2 e^4 \ln \Lambda} \left(\frac{T}{m} + \frac{T_f}{M_f} \right)^{3/2}, \quad (3.19)$$

where the subscript f is used to differentiate between field and test particles. For ions heating in a field of electrons eq. (3.19) becomes

$$\tau_{eq} = 3.18 \times 10^8 \frac{A T_e^{3/2}}{Z n_e \ln \Lambda}, \quad (3.20)$$

where T_e is in electron volts and n_e is in cm^{-3} , and $A = m_i/m_e$. The Coulomb logarithm term $\ln \Lambda$ is given by Book¹⁵ as

$$\begin{aligned} \ln \Lambda &= 23 - \ln \left(\frac{n_e^{1/2} Z}{T_e^{3/2}} \right), \quad T_e \leq 10 \text{ eV} \\ &= 24 - \ln \left(\frac{n_e^{1/2}}{T_e} \right), \quad T_e > 10 \text{ eV} \end{aligned} \quad (3.21)$$

The value of the FLR parameter s is determined by eqs.(1.5) and (1.6). For fully ionized plasmas, s varies as $Z^{1/2}$ for higher Z plasmas. However, if the plasma has a low enough temperature that it is only singly ionized, then s scales as $A^{-1/2}$, the inverse square root of the mass ratio. A related parameter, the average number of ion orbits in the pinch diameter $a/\langle a_i \rangle$, is taken as s/π , as is discussed in chapter 1. The scaling of the plasma stability as a function of s is presented in the next chapter.

The quantity v_{de}/v_{ti} , the ratio of the electron drift velocity to the ion thermal velocity, is determined by using eq. (1.8). If this ratio becomes ≥ 1 , the plasma becomes subject to micro-instabilities.

3.4. Electron density profiles

Electron density profile measurements were made of a number of shots by doing Abel inversions of the interferograms, and fitting the points to curves using a least squares fit. These profiles are shown in appendix D. It is found that at peak compression the profiles fit

best to a Bennett profile, rather than a gaussian or quadratic profile. All three of these profiles are described in the appendix.

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CHAPTER 4

The Pinch Break-up and Observed Instabilities

The intent of this chapter is to describe the phenomena, and present the measurements related to the break-up of the Z-pinch, leaving the analysis of these observations and measurements to the next chapter. In section 4.1 the break-up is characterized, and examples of the most common instabilities shown. In section 4.2 the measured parameters are given. Section 4.2.1 contains the x-ray energy measurements. In section 4.2.2 the observed break-up velocities, and linear growth rates are given. Finally, in section 4.2.3 the scaling

of the instability type with the FLR parameter s is presented, which is probably one of the most interesting results of the experiment.

4.1 Description of the break-up processes

4.1.1 Scenario

The formation of the pinch is amply described and shown in the last chapter. However, it was also apparent, especially in figures 3.8 and 3.10, that the initially straight pinch column is abruptly broken. Shortly before the time of the disruption, a dip in the pinch current, and an anode voltage spike are observed. The magnitude of the voltage spike is often two to three times the bank charge voltage. It is always seen, and the plasma column may or may not exhibit a disruption at that time. In figs. 3.8 and 3.10 a disruption associated with the voltage spike is seen. Fig. 4.1 (a) shows, on the other hand, a pinch which remains for more than $1 \mu\text{sec}$ after the voltage spike, which coincides with the first frame. It is also interesting to note that the sausage-like instabilities of the first frame have been obliterated by the next frame.

The half-width of the voltage spike was observed to be $\sim 100 \text{ nsec}$ for those pinches which had only one break. For those with multiple breaks, a more jagged, and longer lasting voltage spike was seen.

Figure 4.1. (a) Framing camera photographs (b) X-ray signal and anode voltage, 2 kA/div. The time scale is 5 $\mu\text{sec}/\text{div}$. (c) pinch current, 100 kA/div., and anode voltage, 2 kA/div. The time scale is 5 $\mu\text{sec}/\text{div}$. (d) Anode voltage and pinch current, same scales, but at 1 $\mu\text{sec}/\text{div}$. The three pulses on the pinch current trace show the timing of the camera frames.

contributed to the anode voltage. Often, the plasma was observed to form sausage like islands of light and dark intensities, much like those seen starting to form in fig. 4.2.

Also observed, and coinciding with the anode voltage spike, was a hard x-ray signal. Figure 4.1(b) shows the x-ray signal from a NaI (Tl) crystal-photomultiplier combination. The leading edge of the x-ray burst coincides with the voltage spike, to within the resolution of the oscilloscope traces. The decay time of the pulse is due to the slow decay of the detector. Later shots using a plastic scintillator show that the x-ray pulse duration is approximately equal to the voltage spike width. The energy of the x-rays, measured by absorption methods, is found to be ~ 10 keV, whereas the height of the voltage spike was about 11 kV. These measurements are given in section 4.2.1. The source of the X-rays was not established, since attempts to obtain X-ray pinhole photographs were not successful.

The eventual break-up of the pinch was very dependent on the initial parameters, i.e., the bank-fire time, and the pulsed gas valve fill pressure and stop adjustments. With some pinches, a sausage-like disruption would develop before the entire length of the pinch was formed. With others, the pinch could be observed to remain straight for more than $1 \mu\text{sec}$ before either breaking up, or slowly expanding. The various type of instabilities which were observed to cause these disruptions are discussed in the next section.

Events occurring after the pinch broke up were not studied extensively. The pinch current remained high, with the current presumably finding alternative paths. However, on many occasions the plasma was observed to repinch, and a second, and sometimes third X-ray burst were seen. Damage marks indicate that much of the later current was from the anode edge to the wall, thus shorting-out the cathode. Furthermore, the framing camera was seldom able to pick up images at these late times, even though the total light was much higher.

4.1.2. Types of pinch instabilities

By far the most commonly observed deformation was the sausage-like, or $m = 0$ mode instability. Clear examples of these have already been seen in figs. 3.8 and 4.1. Other examples, which show an abrupt disruption which propagates down the length of the pinch, also appear to be sausage-like. An example of this type is seen in fig. 3.10. In figure 4.2 another such example is shown, where close examination of the framing camera image in (a) shows brightness variations possibly corresponding to a sausage-like structure. However, when the pinch does break up, it does not break up into droplets, but rather comes apart between two of the "links" of the sausage. The link-sausage structure is seen more clearly in the contour plots in (b).

The location of the break is most often seen to originate at the

(a)

(b)

Figure 4.2. (a) Framing camera photograph of a shot with a sausage-like disruption. (b) Contour plot of a photo-densitometer scan of the framing camera image. The electrode gap is 53 mm, and the frame separation 200 nsec. The cathode is on the top, and time goes from left to right.

shots, the reverse is also seen, although it is not as common. Interferograms of the dark region between the sausages also show that the electron density is much less than that of the brighter regions of

region, or it is rapidly heated and thus expands. Often, an apparently lower density filament appears to remain in the darker region. The only evidence for the core region is from the framing camera photographs, for which the image intensity is proportional to the square of the electron density. The interferograms do not show this effect. Examples of shots showing the propagating expansion region, or

Figure 4.3. Framing camera photographs of sausage-like instabilities which propagate across the pinch. (a) The instability can be seen as an expanding expansion plume, in this shot. (b) A faint, lower density core appears to have formed in the lower density region of the pinch. (c) The pinch appears to be rapidly expanding in a radial direction.

Another instability that is seen with the higher line density shots is the long wave-length kink, or $m = 1$ mode. This instability results in gross motions which distort the pinch column, causing it eventually break apart. Examples of two shots with the kink mode are

(a)

(b)

Figure 4.4. Two higher density shots showing the development of the kink instability. The time between frames is 200 nsec, and the electrode gap is 53 n for both shots.

shown in fig. 4.4. Higher order modes ($m > 1$) have not been observed.

At very low densities a very wispy, short-lived pinch is observed. Framing camera photographs show that the pinch breaks apart near the cathode before the plasma nearer the anode can pinch. Then later, the plasma near the anode pinches, but that nearer the cathode has expanded to a large, dark plume. At the other end of the scale, i.e., with high density, fat and cool pinches, the pinch column undergoes a non-uniform radial expansion, with the pinch looking much like a fat, fuzzy caterpillar. This fuzziness, or spindle structure as it is sometimes called, has been observed in many previous experiments, and is considered to be the non-linear MHD $m = 0$ mode.¹ Examples of both of these types of pinches are shown in fig. 4.5.

4.1.3 Other features

Two other interesting features were noted related to the instabilities. The first is that often, in the case of pinches which remain straight after the voltage spike, an oscillation on the anode voltage was observed. The oscillation is seen in fig. 4.6, which is the anode voltage trace for such a shot. This oscillation in the anode voltage was not seen with all the shots, however. Since the current is only slowly varying in this time scale, the voltage variation must be due to rapid variations of the pinch inductance. The inductance is proportional to $\ln(R/a)$, where R is the chamber radius. Thus it appears that the pinch is oscillating about a small radius.

Figure 4.5. Framing camera photographs of (a) low density, wispy pinch, which pinches and blows apart on one side before it can pinch on the other, and (b), a high line density pinch which expands radially, and non-uniformly along its length. The frame spacing is 500 nsec, and the electrode gap 53 mm for both shots.

The other interesting feature was observed when the pinch chamber was disassembled. The anode, which is copper, was found to have a 6 mm diameter and 2 mm deep spherical depression, which was dug by the pinch. It was located directly in the center of the anode, on the

Figure 4.6. Anode voltage, 4 kV/div., and pinch current, 100 kA/div. for a shot where the pinch radius oscillates, producing a periodic variation in the anode voltage just prior to the voltage spike. The horizontal scale is 1.0 μ sec/div.

chamber axis. This anode was used for nearly 400 shots, with pinch currents commonly ranging from 100 to 200 kA, and up to over 500 kA with a few of the later shots. The cathode, on the other hand, showed no such damage.

4.2. Measured parameters

4.2.1. X-ray absorption measurements

The energy of the burst of x-rays associated with the voltage spike was measured by an x-ray absorption technique, as described in chapter 2. Aluminum foils of varying thickness were placed in front of the SPM detector, and the maximum intensity per shot measured. Fig. 4.7 shows such plots for three separate configurations of the pinch and detector. Neutral density filters were placed between the scintillator and the photomultiplier to keep the detector from saturating. The data points shown in the figure are averages over several shots, with those points where the detector was near saturation rejected. The last point on curve *B* is probably too high because of the stray light getting into the detector, and therefore was not used in the determination of the x-ray energy.

The mass attenuation coefficient μ/ρ was determined from fig. 4.7 by the technique discussed in chapter 2, and was used to determine the x-ray energy with the plot given by Osher.² These results are summarized in table 4.1. It is interesting to note that the x-ray energy is much greater than the plasma temperature, and corresponds closely to the anode spike voltage V_s .

Figure 4.7. X-ray intensity versus Al foil thickness, for three separate configurations of the pinch and SPM detector. The lines connecting points are a least squares fit for those curves with more

Table 4.1. Summary of data from figure 4.7. The quantity μp is the most attenuation coefficient; E is the x-ray energy, and V_2 is the voltage of the anode spike.

Sequence	μp (cm ² /g)	E (keV)	V_2 (kV)
A	44 ± 4	$8.0 \pm .2$	9 ± 1
B	31 ± 2	$9.8 \pm .2$	11 ± 1
C	98 ± 20	$2.8 \pm .7$	8 ± 1

4.2.2. Growth rate and fluid velocity measurements

With the sausage-like instability, linear growth rates were not measurable. Rather, the pinches remained straight, then abruptly broke at one location. Pinches which clearly had a link-sausage structure were not seen to grow linearly, but rather, either re-formed to straight columns, or abruptly grew dark between two of the sausages. The observed, and apparently profile-stabilized $m = 0$ modes might have originated from a non-linear coupling to internal kink modes, as is suggested by Kadomtsev.² Neglecting FLR and wall effects, the internal modes are shown to be always unstable, and hence could easily cause other modes to appear.

The axial velocity of the break-up of the pinch column is found to range from .8 to 8×10^5 cm/sec for the argon pinches with a 4 kV bank charge voltage. The velocities are mildly density dependent, with the lowest densities having the largest break-up velocities. These velocities are an order of magnitude greater than the axial component of the snowplow sheet during the pinch compression. Hence, it does not appear that they are due simply to plasma streaming. They are of the same order of magnitude as the Alfvén velocity, however, where v_A is determined using densities from the gas valve output calculations.

The radial expansion of the plasma in the dark region was very quick, and difficult to determine because as the plasma expands the light emission decreases to the point that it is no longer visible. However, lower limits on the expansion velocity can be made from framing camera photographs of the break-up. These indicate that the lower bound on the radial velocity at the break is about 1×10^7 cm/sec. This velocity is then somewhat larger than the Alfvén velocity, suggesting that some other mechanism is the source of the rapid expansions.

Kink instabilities were not generally observed, except for those shots where the density was high enough. When they were observed, the growth rates were found to be linear. Thus, it seems sensible to characterize them in terms of dimensionless parameters used in the linearized MHD theory. Thus consider ka , the wave number ($k = 2\pi/\lambda$) times the pinch radius, and $\gamma a/v_A$, the measured growth rate, times a

and divided by the Alfvén velocity. The Alfvén velocity is calculated by using the magnetic field at the plasma edge, and using the gas valve

Table 4.2. Measured growth rates for the kink ($m = 1$) instability. The first three quantities are measured from the framing camera photographs. The Alfvén velocity is calculated at the plasma radius, and by using the density determined from the graph in fig. 2.5.

Shot no.	k (cm^{-1})	ka	γ (10^6 sec^{-1})	v_A (10^7 cm/sec)	$\gamma a/v_A$
108	9.5	2.9	1.2	4.0	.009
292	4.6	1.4	1.6	1.3	.1
296	1.2	.36	2.2	1.3	.04
303	.59	.18	2.4	1.3	.05
593	1.8	.54	2.4	4.6	.3
672	3.5	1.0	4.4	3.9	.06

dynamics to determine the mass density. These measurements are compared with the theoretical values in the next chapter.

4.2.3. Instability type scaling with the FLR parameter.

A quick scan through the shots reveals that the type of instability, and the length of time the pinch remains straight, is very dependent on the amount of gas puffed into the chamber, which is proportional to the line density. Very low density shots, as is shown

densities more typical sausage instabilities are observed. With yet higher gas densities, straight pinches which last several microseconds are seen. At higher densities, the classical kink instabilities are seen, and at the highest gas densities studied, fat, low temperature pinches are observed to expand in a radial direction, but at different velocities along the axis.

Since these effects are also dependent on the temperature, it is important to use a scaling which includes both temperature and density,

Figure 4.8. A comparison of the types of instabilities observed, arranged in ascending order of the quantity of neutral gas puffed into the chamber. Refer to table 4.3 for the parameters.

as well as the average ion charge. This suggests scaling the instability types with the FLR parameter s , as is defined in eq. (1.5), which is related to the strength of the FLR corrections to the MHD

Table 4.3. Description and parameters for the pinch instabilities seen in fig. 4.8.

	Description	Ion line density (cm ⁻¹)	Z_{av}	s	$a/\langle a_l \rangle$
A	wispy, does not completely pinch	$< 3 \times 10^{16}$	8	8.2	2.6
B	sausage like	5×10^{16}	12	16	5
C	straight, slowly expands	2×10^{17} ^a	14	45	14
D	kink	4×10^{18}	2	27	9
E	non-linear sausage	1×10^{19}	3	47	15

^aThis value comes from the measured electron density, using the Bennett relation to obtain the temperature, which is used to find Z_{av} , and hence the line density. The gas valve dynamics give the value 1×10^{18} .

theory. These results are shown in fig. 4.8, which shows the variation of the instability type versus s.

Unfortunately, density measurements were only made with a few of these types. Therefore it is not possible to accurately know the line densities of all of these pinches. The primary means of determining these values thus rests on the PGV dynamics, which at best are only very approximate calculations. But, in spite of the uncertainty of

these numbers, the general scaling of the instabilities is very obvious.

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CHAPTER 5

Discussion

In this chapter the results of the measurements and observations are discussed. Section 5.1 contains a discussion of the growth rate calculations, using both the skin current and diffuse current models. Finite orbit effects on the growth rates are then discussed, and measured and calculated growth rates are compared. In section 5.2 possible causes for the voltage spike and x-ray burst are discussed. These include enhanced resistivity effects, inductive effects of filamentation, radiation collapse, and electromagnetic effects.

5.1. Linear growth rate comparisons

Comparison of experimentally observed growth rates with those predicted by the linearized, ideal MHD theory is very risky, primarily for two reasons. (1) The experimentally observed deformations are generally large amplitude, thereby violating the initial premise of the linearized theory that the perturbations are small. Thus, to make a reliable comparison, one would have to use growth rates predicted by a non-linear theory. (2) It is found from the linearized theory that the predicted growth rates are strongly dependent on which equilibrium profiles in pressure and magnetic field are used.^{3,13} Therefore, it is necessary to know both the pressure and current variation with r to provide a valid comparison with even the linear theory.

In spite of these difficulties, it would be useful to know what growth rates could be expected from the linearized, MHD theory, both with and without the FLR damping effects, and with arbitrary equilibrium profiles. A comparison could then be made with experimentally observed growth rates, provided they are small amplitude, and it is checked that the growths are not saturated; i.e., that the perturbed quantities grow as $\exp(\gamma t)$. Furthermore, equilibrium profiles, which are needed for the theory, could be determined from the Abel inversions of the interferometer data. It is

thereby hoped that some conclusions on the magnitude of the FLR effects might be made.

Therefore, to prepare for this comparison, the next several paragraphs are devoted to a discussion of the relevant MHD theory. Since a non-linear theory does not exist, the discussion is limited to small amplitude (linear) calculations.

5.1.1. Analytical techniques

Calculations which determine the growth rates of an unstable, equilibrium plasma are usually based on procedures formally established in a paper by Bernstein, *et al.*¹ The approach is to first consider small variations from an equilibrium solution to the ideal (meaning infinite conductivity), magnetohydrodynamic (MHD) equations, keeping only first order terms in the variation. The variations of the various parameters are written in terms of a vector ξ , which is a measure of the displacement of the location of a fluid element from its equilibrium position. Thus, the location $\mathbf{r}$ of a fluid element at time t is

$$\mathbf{r} = \mathbf{r}_0 + \xi.$$

The ideal MHD equations are thus written in the form

$$\rho_o \frac{\partial^2 \xi}{\partial t^2} = \mathbf{F}(\xi) , \quad (5.1)$$

where

$$\begin{aligned} \mathbf{F}(\xi) = & \nabla_o [\gamma \rho_o \nabla \cdot \xi + (\xi \cdot \nabla_o) \rho_o] + \frac{1}{\mu_o} [\mathbf{J}_o \times \mathbf{a} - \mathbf{B}_o \times (\nabla_o \times \mathbf{a})] \\ & + [\nabla_o \cdot (\rho_o \xi)] \nabla_o \phi_o . \end{aligned} \quad (5.2)$$

The subscripts on the operators imply that they operate on the zeroeth order quantities only. The vector $\mathbf{a}$ is the first order variation of the magnetic field and equals

$$\mathbf{a} = \nabla_o \times (\xi \times \mathbf{B}_o) . \quad (5.3)$$

Note that the form of $\mathbf{F}$ depends on which equilibrium, and which coordinate system is used.

All the first order quantities are now written as

$$Q(r, \theta, z, t) = Q(r) \exp(i\omega t + im\theta + ikz),$$

and eq. (5.1) becomes the eigenvalue equation

$$-\omega^2 \rho_0 \xi = F(\xi, \omega) . \quad (5.4)$$

The associated boundary conditions are that $r\xi_r = 0$ at $r = 0$, and $\xi_r = 0$ at $r = b$, the location of the wall, if the plasma extends to the wall. This boundary condition restricts the solution to the internal modes of the pinch, since the plasma boundary is then fixed to the wall. For a pinch surrounded by a vacuum region, the boundary conditions are more complicated. The first requirement is that the zeroeth and first order pressures be continuous across plasma-vacuum interface. This results in the following two conditions.²

$$p_0 + \frac{B_{0i}^2}{2\mu_0} = \frac{B_{0e}^2}{2\mu_0} \quad (5.5)$$

and,

$$-\gamma p_0 \nabla \cdot \xi + \frac{1}{\mu_0} \mathbf{B}_{0i} \cdot \mathbf{B}_{1i} = \frac{1}{\mu_0} \mathbf{B}_{0e} \cdot \mathbf{B}_{1e} + \frac{\xi_r}{2\mu_0} \left(\frac{\partial B_{0e}^2}{\partial r} - \frac{\partial B_{0i}^2}{\partial r} \right) . \quad (5.6)$$

One further boundary condition, which is a result of the requirement that the tangential component of the electric field be continuous across the boundary, is that

$$\mathbf{B} \cdot \mathbf{B}_{1e} = \mathbf{B} \cdot [\nabla \times (\xi \times \mathbf{B}_{0e})] \quad (5.7)$$

Note, that in eqs. (5.5)–(5.7), the subscripts 0 and 1 refer to the zeroth and first order quantities, respectively, and the subscripts i and e refer to the quantities interior and exterior to the plasma. The explicit form of eq. (5.4) is too long to write out here, but can be found in a paper by Goedbloed and Hagebeuk.³ There it is written in cylindrical coordinates, includes a longitudinal magnetic field, and allows arbitrary profiles for the zero order pressure and magnetic field, provided they satisfy the equilibrium condition

$$\nabla \left(p + \frac{\mathbf{B}^2}{2\mu_0} \right) = \frac{(\mathbf{B} \cdot \nabla) \mathbf{B}}{\mu_0}. \quad (5.8)$$

Eq. (5.4) has been solved for the growth rates for various configurations by several early researchers.^{4–11} Kruskal and Schwarzschild⁴ solved for $m = 0$ growth rates of a linear pinch with a skin current, having no B_z nor wall-stabilization. Tayler⁵ extended the model to a diffuse current, uniform density pinch, for which the pressure varies as $p = p_0(1 - (r/a)^2)$. For this particular equilibrium eq. (5.4) can be analytically reduced to a transcendental equation in terms of the boundary values, from which the maximum growth rate can be obtained. These results are discussed in a following subsection.

Taylor later extended the work to include wall effects and a stabilizing B_z .⁶ These efforts were paralleled by several Russian researchers. Trubnikov⁷ considered a uniform current distribution, and no axial field, to find approximate solutions to $m = 0$ growth rates for a limited number of wavelengths. Shafranov⁸ added wall and axial magnetic field effects to a skin current model, and later to a uniform density, diffuse current model,⁹ doing an analysis very similar to that of Taylor. A good review of the growth rate calculations, and general stability principles is given in an article by Kadomtsev.¹⁰

The diffuse current form of eq. (5.4) was first written in general form by Hain and Lust,¹¹ who numerically calculated $m = 1$ and higher mode growth rates using a Bennett profile. Their analysis includes an axial magnetic field, and considers a vacuum region outside the plasma which extends to infinity. Their calculations, however, only consider ka in the range 0.1 to 1.0, and with $B_z \neq 0$. Their results are therefore not suited for the present work. After the paper on the diffuse pinch stability theory by Newcomb¹² growth rate calculations of diffuse pinches stopped. The subject was however reopened ten years later by Friedberg¹³ to determine the effects on the growth rate of diffuse versus skin current distributions. His analysis is done with an incompressible ($\nabla \cdot \xi = 0$) approximation, with tokamak ordering ($B_z \gg B_\theta$), and with the plasma extending to the wall. Therefore, the results do not apply to the present experiment, but they do show a strong dependence on the current profile, with the growth rate varying as much as a factor of ten between the skin current and

the diffuse profiles. Goedbloed and Hagebeuk³ extended the analysis by considering the plasma to be compressible, but they also kept the same ordering in B_z/B_θ .

Thus, it is apparent, that to compare experimentally measured growth rates with diffuse growth rate calculations for anything but a uniform density, it is necessary to first numerically calculate the growth rates from eq. (5.4), using the appropriate boundary conditions. This will not be done, however. Rather, the approach will be to compare observed growth rates with those predicted by the uniform particle current density model of Tayler. And as is also done in his paper, estimates of the variation of the growth rates with non-uniform particle density profiles will be made.

5.1.2. Skin current model growth rates

In order to understand the general features of the growth rates, consider first a skin current model. Growth rates calculated by this model, with $B_z = 0$, are given by Kadomtsev.¹⁴

$$\frac{\omega^2}{v_A^2} = -ka \frac{I_m'(ka)}{I_m(ka)} \left(m^2 \mathcal{K}_m(ka, kb) + 1 \right) \quad (5.9)$$

Here k is the wavenumber, a is the pinch radius, b is the wall radius, v_A is the Alfvén velocity, and I_m is the modified Bessel function of the first kind, and of order m . Similarly, $\mathcal{K}_m$ is a function of Bessel

$$\mathcal{K}_m(ka, kb) = \frac{K_m(ka) kb I_m'(kb) + I_m(ka) kb K_m'(kb)}{ka K_m'(ka) kb I_m'(kb) + ka I_m'(ka) kb K_m'(kb)} . \quad (5.10)$$

For large wall radii, eq. (5.10) becomes

$$\mathcal{K}_m(ka, kb) = \frac{K_m(ka)}{ka K_m'(ka)}, \quad \text{for } b \gg a. \quad (5.11)$$

And for large values of ka , $\mathcal{K}_m \approx 1$.

The growth rate is defined as $\gamma = \pm i\omega$. Normalized to v_A/a , it is plotted as a function of ka in fig. (5.1) for $m = 0$, and $m = 1$ modes. Several features are readily apparent. (1) The larger growth rates occur for the smallest wavelengths, (2) the wall has no stabilizing effect on the $m = 0$ mode, and (3) the wall effects are stronger for long wavelengths and negligible for short wavelengths. Plots for higher order modes show that the growth rate is much higher for long wavelengths, despite the stronger wall stabilization. Therefore, from the skin-current model, with no B_z , it appears that short wavelength MHD instabilities will dominate for $m = 0$ and $m = 1$, and that higher order modes are also unstable.

5.1.3. Diffuse current model growth rates.

If the current is not restricted to flow on the surface of the pinch, the predicted growth rates vary substantially from those predicted by the skin current model. For a pinch of radius a with a

SKIN CURRENT MODEL, $m = 0$

SKIN CURRENT MODEL, $m = 1$

Figure (5.1). Growth rate versus ka for (a) $m = 0$, and (b) $m = 1$, for a wall radius to pinch radius ratio of 5, 10, 20, and 50.

uniform density ρ_0 , and a pressure $p_0(1 - (r/a)^2)$, an equilibrium is obtained with a uniform current density J_0 , and a magnetic field $B_\theta(r) = B_{\theta 0} r/a$, with $p_0 = B_{\theta 0}^2/\mu_0$. With this equilibrium eq. (5.4) can be reduced to Bessel's equation, with an incompressible fluid approximation. By applying the boundary conditions a transcendental equation for the normalized growth rate Ω^2 ($\Omega^2 = \gamma^2 a^2 / v_A^2$) can be found. This is given by Tayler⁵ as

$$\left(\frac{\varphi}{\chi}\right)_{r=a} = - \frac{m^2 + \Omega^2}{m^2 + \Omega^2} \frac{\rho_1/\rho_0}{\rho_1/\rho_0} ka \frac{K'_m(ka)}{K_m(ka)}. \quad (5.12)$$

With $\beta = 2/(\Omega^2 + m^2)$, and ρ_1 is the density outside the pinch. The quantities φ and χ are given in terms of Bessel functions.

$$\chi = C \frac{J_m[(m^2\beta^2 - 1)^{1/2} ka]}{ka} \quad (5.13)$$

$$\varphi = C \frac{m^2\beta J_m[(m^2\beta^2 - 1)^{1/2} ka]}{(m^2\beta^2 - 1) ka} + \frac{J'_m[(m^2\beta^2 - 1)^{1/2} ka]}{(m^2\beta^2 - 1)^{1/2}}.$$

Eq. (5.12) is then solved numerically for the largest growth rate. The results for the $m = 1$ mode are shown in fig. (5.2). The important differences between the skin current and diffuse current models are that (1) the growth rate does not approach infinity as $k \rightarrow \infty$, and (2)

DIFFUSE CURRENT MODEL, $m = 1$

Figure (5.2). Growth rate versus ka for the $m = 1$ mode of a diffuse pinch with uniform particle and current densities.

in the previous analysis. If included, it could be expected that they would have the largest effect for long wavelengths, as is seen with the skin current mode.

Closer examination of eqs. (5.12) and (5.13) show that no positive solutions of Ω^2 are allowed for $m > 1$, making this particular diffuse current equilibrium much more stable than the skin current model (see discussion by Taylor.⁵) For $m = 0$, eq. (5.12) becomes indeterminate for an incompressible fluid, and reduces to an equation similar to Bessel's equation for a compressible fluid. With the ratio of specific heat equal to one, the difference amounts to adding an additional term $(4\nu_A^2/\omega^2 a^4)r^4 \xi_r$. Presumably, this equation would need to be solved numerically. Trubnikov⁷ solved for approximate values of

that with a uniform current density and temperature, but with the particle density varying as $n = n_0(1 - (r/a)^2)$. He found that for $ka \ll 1$, $\Omega^2 \approx 2\sqrt{3} ka$, and for $ka \gg 1$, $\Omega^2 \approx 4/3 ka$. His numerical check shows that the formula for the lower limit is generally applicable up to $ka = 1$.

For a pinch with an arbitrary equilibrium profile, eq. (5.4) must be solved numerically for each particular equilibrium. Solutions for growth rates of other interesting compressible equilibria have not been found in the literature. However, Tayler⁵ does estimate an upper bound for the growth rates for a more realistic density and pressure profile, i.e., one with a uniform temperature. The equilibrium is described by

$$\begin{aligned}
 \rho &= \rho_0 \left(1 - (r/a)^2 \right) \\
 p &= p_0 \left(1 - (r/a)^2 \right) \\
 B &= B_{\theta 0} (r/a) \hat{\theta} \\
 J &= j_0 \hat{z}, \quad r \leq a, \\
 &= 0, \quad r > a
 \end{aligned}
 \tag{5.14}$$

where outside the pinch the density and pressure are uniform, and are given by

$$\begin{aligned} \rho_1 &= \rho_0 \left(1 - (a/b)^2 \right) \\ \rho_1 &= \rho_0 \left(1 - (a/b)^2 \right). \end{aligned} \quad (5.15)$$

Again, a and b are the pinch and wall radii, respectively. If η is defined by $(a/b)^2 = 1 - \eta$, then the maximum possible growth rate is found to be

$$\Omega^2 = 3/\eta. \quad (5.16)$$

Thus, it can be seen that as the plasma extends to the wall, and the background density becomes small, the maximum growth rate becomes much larger than that of the uniform density pinch. For this reason, the growth rates shown in fig. (5.2) will be considered as the minimum growth rate which can be expected from the MHD theory.

5.1.4. Finite Larmor radius effect on the growth rate

If the line density of a pinch is small enough, then finite Larmor radius effects can become an important stabilizing mechanism, as is discussed in a number of papers.¹⁵⁻¹⁹ However, none of these or other papers consider FLR effects in the linear, no B_z , z-pinch. The effect is caused by an $E \times B$ drift, where the ions experience a slightly different average electric field because of their larger orbits, than do the electrons. Thus, a charge separation current is

built up parallel to the z axis with a perturbed electric field in a radial direction. The $E \times B$ drift for the ions is

$$\mathbf{v} = (1 + a_a^2/4 \nabla^2) \frac{\mathbf{E} \times \mathbf{B}}{B^2}, \quad (5.17)$$

where ∇^2 operates on $\mathbf{E}$, and a_a is either the electron or ion cyclotron radius. For electrons the second term is negligibly small and is therefore omitted. If the ion orbits are the same order of magnitude as the plasma radius, then the following current results.

$$\begin{aligned} J_{FLR} &= n_i e \mathbf{v}_{di} - n_e e \mathbf{v}_{de} \\ &= \frac{1}{4} n e a_i^2 \nabla^2 \frac{\mathbf{E} \times \mathbf{B}}{B^2} \end{aligned} \quad (5.18)$$

In order to estimate a condition for stability, this current is then set equal to the currents which drive the instability, which are the curvature and the ∇B drifts. This is done by Rosenbluth, et al.¹⁵ to arrive at a stability condition $(ka_i)^2 \approx \Omega_{MHD}/\Omega_i$, where Ω_i is the ion gyrofrequency, with $\Omega_i = v_{\perp i}/a_i$. This result can be further expanded by noting that the MHD growth rate Ω_{MHD} has a maximum value v_A/a , as shown in the last section. Thus, rewriting this relation in terms of s and ka , the following result is obtained,

$$\begin{aligned}
 (ka)^2 &= \frac{s}{\pi} \frac{v_A}{v_{\perp j}} \\
 &= \frac{s}{\pi} \sqrt{2(1 + Z_{av})} ,
 \end{aligned}
 \tag{5.19}$$

where the second line is obtained by again using the Bennett equilibrium condition. Solving for s , it is found that the pinch is stable for large ka if

$$s \leq \frac{\pi(ka)^2}{\sqrt{2(1 + Z_{av})}} .
 \tag{5.20}$$

With a deuterium pinch which is fully ionized, for example, s must equal 1.6 for $ka = 1$, and only 160 for $ka = 10$. For longer wavelengths eq. (5.20) is no longer valid since $\Omega_{MHD} \approx kv_A(\ln(1/ka))^{1/2}$, which goes to zero for small ka . Using this value for Ω_{MHD} , eq. (5.20) becomes

$$s \leq \frac{\pi}{\sqrt{2(1 + Z_{av})}} \frac{ka}{\sqrt{\ln(1/ka)}}
 \tag{5.21}$$

Eq. (5.21) thus requires that s be .1 for $ka = .1$, and .007 for $ka = .01$. Even these values are probably excessively strict since the wall stabilization effects, which are important for small ka , have not

least the short wavelength instabilities can be stabilized for $a_1 \approx a$, and that longer wavelength instabilities may also be stabilized.

These are heuristic estimates, of course, based on the notion that the perturbed field quantities are small. However, this assumption was violated several times in arriving at eq. (5.20). This equation can only be meant to provide the general scaling features of FLR stabilization.

No attempts are made to calculate the diffuse current model MHD growth rates. However, it is possible to place bounds on the growth rates by considering viscous effects. It has been shown by Roberts and Taylor¹⁶ that the FLR effects can be included in the MHD equations by writing them as viscous, off-diagonal terms in the pressure tensor. This adds one further complication, which makes a numerical solution most certainly necessary. It has been shown by Tayler⁵, however, that viscous terms added to the MHD equations decrease the growth rate to zero for small values of the wavelength λ (or for large values of ka). Furthermore, the wavelength of maximum growth rate, λ_M , can be estimated as a function of a collisional mean free path. For a magnetically confined plasma the collisional mean free path is the Larmor radius. Tayler's expression is

$$\frac{\lambda_M}{a} = \left(\frac{l}{a} \right)^{2/3}, \quad (5.22)$$

and letting $l = a_1$, eq. (5.22) becomes

$$k_M a = \frac{\pi}{3} s^{2/3}, \quad (5.23)$$

where $k_M = 2\pi/\lambda_M$. Growth rates at wavenumbers less than k_M remain unchanged, while those at greater k are strongly damped. Note that upon comparing eq. (5.23) with fig. (5.2) it can be seen that the growth rate goes to zero as s becomes small, which verifies the conclusions based on eq. (5.20).

Use of eq. (5.23) then provides a technique for estimating the maximum growth rate expected with FLR effects. One first determines the value s by eq. (1.6), determines the value of ka which gives the maximum growth rate by using eq. (5.23), and then matches $k_M a$ with either fig. 5.1 or fig. 5.2 to obtain the growth rate.

Several other papers should be mentioned before continuing. The first is by Stringer¹⁸, who calculates analytic solutions for the growth rates near the $k \cdot B = 0$ surfaces of a cylindrical screw-pinch. He also provides a technique for determining the growth rate with FLR effects, given the MHD growth rate, provided, of course, that $a_1 \ll a$. Because of the included B_z , his results do not apply to this work. Trubnikov and Zhdanov¹⁹ consider an anisotropic pressure tensor which includes FLR effects in determining the growth rate of an $m = 0$ linear pinch (no B_z) with the approximation that $ka \gg 1$. Their result is

$$\omega = -\frac{1}{2} (ka_i)^2 \omega_{ci} + i \sqrt{|kg_{eff}| - (ka_i)^4 \omega_{ci}^2 / 4}, \quad ka \gg 1, \quad (5.24)$$

where ω_{ci} is the ion Larmor frequency, and $g_{eff} = (2p/rp) \approx v_{ti}^2/r$. This equation is of limited value in the present experiment since ka is observed to be ≈ 1 .

From these considerations several general conclusions can be drawn. (1) As shown by eq. (5.23), short wavelength growths are most strongly damped by FLR effects. (2) Those modes with the finer structure, i.e., the higher order m modes will be more strongly stabilized. (3) If FLR stabilization can occur, it must be for $a \sim a_i$.

5.1.5. Comparisons of observed with calculated linear growth rates

As mentioned in the last chapter, growth rates were difficult to measure for the $m = 0$ mode. It was only possible to place lower bounds on the growth rates. With the Trubnikov model,⁷ for $ka \approx 1$, which was the most commonly observed wavelength, $\gamma a/v_A = 1.9$. Experimentally, two separate $m = 0$ growth rates were observed for each shot; one several orders of magnitude less than, and later one much greater than the predicted value. For the shots with low s , the pinch stayed quiescent, or slowly growing, for up to $1 \mu\text{sec}$, before abruptly breaking in a sausage-like instability. This sausage-like instability was observed to grow at a rate much faster than that predicted by the

For the $m = 1$ mode, a comparison can be made between the data in table 4.2 and fig. (5.2). Because of the relatively few shots that exhibited the kink instability, the point scatter is large. However, several general features can be deduced from such a plot as is seen in fig. (5.3); (1) The experimentally observed rates are roughly a factor of ten less than those predicted by the theory, and (2) No kinks are observed for $ka > 3$. This last statement is possibly not entirely correct. It is possible for the kink mode to non-linearly couple to the sausage mode so that a short wavelength internal kink might evolve into an external sausage instability.²⁰ It was not possible to resolve this effect from the framing camera images because of the finer structure at the shorter wavelengths.

5.2. *The pinching dynamics and non-MHD effects*

Recently, there has been much discussion in the literature of additional effects neglected by Anderson, et al²¹ and Leontovich and Osovetz²² on the dynamics of the pinch compression. These additions enable the theory to better explain the final states of the pinch, as well as to predict the abrupt disruptions observed in almost all pinches. Since these ideas have direct bearing on the observed dynamics and instabilities, they will be discussed in this section in the context of this experiment.

Figure 5.3. Normalized $m = 1$ growth rate versus ka . The solid line is the result of the Tayler theory, and the points come from table 4.2.

5.2.1. Pinching dynamics

The snow-plow model described in chapter 1 is derived by considering the forces on a current carrying sheet that is accelerated inwards by its self-generated magnetic field, accruing mass as it sweeps in. The particle pressure of the gas is neglected, and therefore it is not possible to predict the final radius, or subsequent behavior of the pinch. The derivation of the Leonotovich and Osovetz equation²² goes one step further and includes the particle pressure. Thus, the model is able to predict a final radius and temperature, a

subsequent expansion, and a recompression. The snow-plow model is limited, however, in that it considers the plasma to reside in a thin, hollow shell. It is therefore not possible to correctly model the mixing and heating that takes place at peak compression. This problem is corrected by considering a shock-model, as done by Allen.²³ Potter²⁴ improved on the shock model with a slug model, which includes the effects of a cylindrical strong shock preceding a piston of plasma (the slug) which collapses to the axis.

However, even these improved techniques do not include many effects which become important in the later stages of the compression. Axial streaming of plasma during the compression can lead to removal of aggregated material, thereby allowing much higher compression than in a symmetric Z-pinch.^{24,25} This effect, which is important in the plasma focus, also pertains to this experiment because of the conical distribution of gas prior to pinching. As the pinch current is increased, there is an axial component of the compression directed towards the anode, as is discussed in chapter 3. This effect can also lead to enhanced heating, as is noted by Vikhrev and Korzhavin,²⁵ where localized, low-density regions with higher electron drift velocities are established. If the density becomes low enough, there is an anomalously high resistivity, and rapid heating and expansion.

Any mechanism which reduces the line density locally could be a cause of increased resistivity. Thus, for example, it is also possible

region, could also cause an onset of anomalous resistivity, and increased heating. For the incompressible case, $\nabla \cdot \mathbf{v} = 0$ gives an axial flow of

$$v_z = \frac{i}{kr} \frac{\partial}{\partial r} (rv_r) . \quad (5.25)$$

For $m \neq 0$, there is also an azimuthal term $-mv_\theta/kr$, which decreases the axial flow. Thus, this effect could be expected to be strongest for $m = 0$ modes, and perhaps this is a reason that this mode is the most commonly observed disruptive mode, despite the fact that the ideal MHD theory shows that it can be stabilized by a sufficiently diffuse profile.

There are a number of micro-instability interactions which will lead to enhanced resistivity of the pinch, and therefore must be included in a dynamical theory of the later compression stages. Liewer and Krall²⁷ discuss three types: the Buneman, the ion acoustic, and the lower hybrid drift instabilities. Vikhrev and Korzhavin²⁵ list these and four additional types, conveniently compared in a table, which can affect the pinch dynamics. Each becomes important when the electron drift velocity exceeds a specific threshold for that instability. The lower-hybrid drift instability becomes important when $v_{de} > v_{fi}$, where v_{fi} is the ion thermal velocity. The ion-acoustic instability, which also has a low threshold, turns on when the drift

restriction probably eliminates this instability for consideration in the earlier stages of the pinch dynamics since $T_i \gg T_e$ during the snow-plow portion of the compression. The Buneman instability is also probably of lesser importance since it requires that $v_{de} > v_{te}$. The only other micro-instability that has a low turn-on threshold is the electron-acoustic instability, which has an identical threshold, and gives an identical resistivity as that of the lower hybrid drift instability. Therefore, it is sufficient to consider only the lower hybrid drift instability for $v_{de} \approx v_{ti}$.

As was shown in the derivation of eq. (1.8), the criteria that the drift velocity exceed the ion thermal velocity for the lower hybrid drift instability can be written in terms of the FLR parameter s , which in turn is written in terms of a line density $N = \pi n a^2$ by the use of the Bennett relation. Similarly, the thresholds for the other instabilities can also be written in terms of line densities. These are also tabulated by Vikhrev and Korzhavin²⁵ for deuterium. For higher Z gases the line density thresholds are complicated somewhat by their dependence on the average ion charge, which is temperature dependent. Thus, for argon, it was shown that the onset of the lower-hybrid-drift instability is relaxed somewhat from that of deuterium. The magnitude of the anomalous resistivity caused by the micro-instabilities can be appreciably larger than that of the classical coulomb resistivity. The anomalous resistivity η is written in terms of the classical resistivity η_C as

$$\eta = \eta_C (1 + \nu_{eff} \tau_{ei}) , \quad (5.26)$$

where ν_{eff} is the effective collision frequency, and τ_{ei} is the classical electron-ion collision time. The values of ν_{eff} are given by Vikhrev and Korzhavin,²⁵ where for the lower hybrid drift instability it equals $(v_{de}/v_{ti})^2 \omega_{LH}$, with the lower hybrid frequency $\omega_{LH} = \sqrt{\omega_{ce}\omega_{ci}}$. The product $\nu_{eff} \tau_{ei}$ is typically of order 10^4 to 10^8 , depending on the temperature and line density.

In order to demonstrate what effect anomalous resistivity has on the expansion of the pinch, consider an equilibrium pinch of radius a that suddenly becomes anomalously resistive. The external PdV work goes into resistive heating of the pinch and radiative losses. Thus,

$$PdV = P_{\Omega} - P_R$$

$$\frac{\mu_o}{8\pi^2 a^2} \frac{d}{dt} (\pi a^2) = \frac{I^2 \eta}{\pi a^2} . \quad (5.27)$$

The radiative term is dropped because it is much smaller than the anomalous heating term. The pressure balance relation was also used to express the particle pressure of the first line of the equation in terms of a magnetic pressure. Simplifying, the following is obtained.

$$\frac{da}{dt} = \frac{4\eta}{\mu_o a} \quad (5.28)$$

For a pinch of radius 0.5 cm, and with the resistivity $\eta = 10^4 \eta_c$, which is low in many instances, the expansion rate is

$$\begin{aligned} \frac{da}{dt} &= 37 \text{ mm} / \text{ns}, \quad T = 20 \text{ eV} \\ &= 0.1 \text{ mm} / \text{ns}, \quad T = 200 \text{ eV} . \end{aligned} \quad (5.29)$$

Thus, it can be seen that the expansion of the pinch by anomalous heating is indeed very fast.

One other physical effect has been found to be very important in more correctly describing the dynamics of a pinch, is consideration of the radiation losses, as alluded to in eq. (5.27). Shearer²⁸ has shown that if the radiation losses of the plasma exceed the ohmic heating, then the pinch cools and collapses to a high density filament. This effect is also discussed by Vikhrev²⁶ who indicates that the pinch can collapse only until the density becomes high enough that it radiates as a black-body radiator. The condition that the radiative cooling rate exceeds the ohmic heating rate is expressed in terms of the Pease-Braginskii current I_{PB} ,^{29,30} namely that

$$\frac{1}{a} \frac{da}{dt} = \frac{3}{4} \frac{1}{\tau_R} \left[\left(\frac{I_{PB}}{I} \right)^2 - 1 \right], \quad (5.30)$$

where the radiation time constant τ_R is defined as

$$\tau_R = \frac{3NkT}{P_R} = \frac{3}{4} \frac{I^2}{P_R}. \quad (5.31)$$

Eq. (5.30) was derived from eq. (5.27) using the Bennett equilibrium.

With Coulomb resistivity, the Pease-Braginskii current is of order 1.5 MA for deuterium, and 110 kA for argon, being also dependent on the density profile. If, however, the resistivity is not classical, but rather anomalous, the Pease-Braginskii current becomes very high, and the pinch expands, which is consistent with eq. (5.29).

Thus, a complete description of the dynamics of the z-pinch must include these three effects; axial flow, anomalous resistivity, and radiation, if it is to successfully explain the complex phenomena often experimentally observed. Recent 1D numerical calculations with LASNEX³¹ have been done by Nielsen,³² and show that an enhanced resistivity prevents the pinch from attaining the small radii found when just radiation and Coulomb resistivity are considered. Vikhrev and Korzhavin²⁵ modeled the plasma focus and found the rapid heating and expansion, and subsequent recompression with the anomalous resistivity playing a large role in the dynamics. They have explained

many of the effects observed with the plasma focus, but they also do not consider radiation effects.

5.2.2. *The voltage spike and the sausage-like disruption*

Voltage spikes, dips in the pinch current, x-ray bursts, and, for deuterium, neutron emission, have long been observed in pinches and pinch-like devices. It has long been known that the neutrons generated are not thermonuclear in origin,²¹ but have directed energies and are therefore not useful for controlled fusion. There have been many proposed explanations, which can fit into three broad categories; (1) Inductive effects due to the narrowing of the pinch column, (2) Anomalous resistive effects, and (3) Electromagnetic effects. The intent of this section is to discuss each of these in the context of the present experiment.

The induced voltage across the pinch electrodes has three possible sources;

$$\begin{aligned} \mathcal{E} &= \frac{d}{dt}(Li) + iR \\ &= L \frac{di}{dt} + i \frac{dL}{dt} + iR . \end{aligned} \tag{5.32}$$

The inductance L per unit length of the pinch is $L = \mu_0/2\pi \ln(b/a)$, where a and b are the pinch and wall radii, respectively. Neglecting

the resistive effects for the moment, and noting that $a = a(t)$ and that di/dt is small, the induced voltage is

$$\mathcal{E} = i \frac{dL}{dt} = i \frac{\mu_0 l}{2\pi a} \frac{b}{u} \frac{dy}{dt} . \quad (5.33)$$

The velocity dy/dt is the pinch radius collapse velocity, normalized to the initial wall radius, and l is the pinch length. From fig. (3.5) this velocity is found to be of order 1.0×10^6 cm/sec. Thus, to produce a voltage spike of 16 kV, which was often seen, with a current of 100 kA, and a radius of 0.3 cm. requires that the filament radius be 0.1 mm. Filaments of this size, which would be produced by a radiative collapse, were not observed with the framing camera, nor with the interferometer. Furthermore, the calculations of chap. 2 indicate that the plasma filament should still be visible, if it is present. It is therefore very doubtful that filamentation was the cause of the voltage spikes in this experiment. Those filaments which do sometimes remain, such as the one seen in fig. 4.2, appear to be less dense, perhaps being the remainder after the outer layers have been thrown off.

Bernard, *et al.*,³³ were the first to suggest that anomalous resistivity might be the cause of the voltage spikes, and disruptions in the plasma focus, and other z-pinch like devices. As discussed in the last section, axial flows induced by non-axially uniform configurations (i.e., the plasma focus, or this experiment) or those

axial flows arising from an $m = 0$ instability, can cause regions of lower line density, where the local electron drift velocity exceeds the ion thermal velocity, making the plasma unstable to a lower hybrid drift instability. The result is that the plasma quickly heats and expands to a lower density, and emits less visible light, since the bremsstrahlung is proportional to the square of the density.

Consider now eq. (5.32) with the resistive term dominating. Using the same parameters as used with the inductive case, R need only be $.16 \Omega$ to produce the 16 kV voltage spike. Thus, $\eta/\eta_c = 30$ for $T = 20$ eV, and 10^3 for $T = 200$ eV, considering only a portion of the pinch length to be anomalously resistive. Both of these values are approximately a factor of ten less than the resistivity estimated for the lower hybrid drift instability, but they do indicate that anomalous effects could very well be the cause of the voltage spike.

One additional observation helps substantiate the concept that axial flows lead to reduced line density and localized micro-instabilities. Namely, the break-up of the pinch was observed to often form near the gas valve opening in the cathode, where the gas distribution transitions from a cylindrically shaped to a conically shaped jet. Nearest the cathode the pinching is primarily radial. Further away there is an axial component directed away from the cathode. Thus, in the transition region between the two there will then be a region of reduced density, i.e., one where more gas is flowing out than coming in. Therefore, any rapid heating could be

expected to start in this region. This phenomenon was seen with most of the pinches which broke up in a sausage-like instability. See, for example, figs. 3.8, 3.10, 4.2, 4.3, and 4.5.

The third possible explanation for the voltage spike, and break-up of the pinch is that it is associated with the displacement current, which is not included in the ideal MHD equations. The addition of the displacement current then allows electromagnetic effects. Trubnikov and Zhdanov³⁴ write the cylindrical wave equation, and find a solution for a sudden break in an infinitesimally thin wire with current J_0 as

$$E_z(r,t) = \frac{2 J_0}{c (c^2 t^2 - r^2)^{1/2}} . \quad (5.34)$$

This solution has the properties of an initially very strong axial electric field, going to infinity at $r = ct$. Thus very strong electric fields are present which accelerate electrons and ions across the magnetic field (assuming $E \gg B$). X-ray bursts and neutron yields are then associated with the interaction of these accelerated particles with the plasma on either side of the discontinuity.

Pickels³⁵ suggests that an electro-magnetic effect causes B -field detachment from the current sheet and penetration of the plasma, resulting in strong displacement fields, as in eq. (5.34). This hypothesis is based on clues from the LLNL, and the Gribkov, *et al*³⁶ plasma focus experiments which show a tearing of the current sheet

before peak compression. This mechanism could possibly also explain why the voltage spike does not always coincide with the peak compression. This theory has yet to be worked out in detail, however.

It was not possible to verify either of these two theories with the present experiment, and therefore no conclusions are made concerning their validity. It is obvious, however, that both approaches can include anomalous resistivity effects, and are possibly even required to explain the sudden current disruption used to produce the wave of eq. (5.34).

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CHAPTER 6

Summary

6.1. Results

The goal of the experiment was to observe FLR effects in the linear z-pinch. The experiment was successful to a large extent in that the scaling of the instability types was clearly observed to be dependent on the electron line density of the pinch, a parameter which is directly related to the strength of the FLR effects. No theory was constructed which includes these effects because of the difficulties involved in using a fluid theory when the ion orbits are large.

In the first chapter the Finite Larmor radius scaling was developed. It was found, by using the pressure balance condition, that the number of pinch orbits in the radius is proportional to the square root of the line density and independent of the temperature if the plasma is completely ionized. For higher Z plasmas, which are usually not completely ionized, there is a mild dependence on the average effective charge of the ion. This Z_{eff} dependence implies that low temperature, high Z plasmas can have strong FLR effects at much higher line densities than hydrogen.

It was also found in the first chapter, that to avoid regimes where micrainstabilities exist it is important to keep the electron line density high enough that the electron drift velocity not exceed the ion thermal velocity. This restriction places severe limitations on the amount of FLR stabilization that could be achieved, but suggests that there might exist a stability window between the conflicting requirements of these two mechanisms. For higher Z materials it was found that the line density threshold is relaxed somewhat.

The experiment, which differed significantly from earlier pinch devices, is described in the second chapter. The apparatus consisted of an evacuated pinch chamber with gas puffed on-axis through a hole in the cathode. The pinch bank was fired before the gas had time to reach the walls. Therefore the dynamics of the pinch were somewhat different from the previous snowplow pinches where the pinch current starts at the wall. With this technique it was possible to maintain the line

density low enough that FLR effects are important, and yet have a local density high enough that the gas could breakdown.

Spatial electron density measurements were made of a number of the shots with a CW laser, time resolved with a gated intensifier and TV camera. A small code was developed to Abel invert the interferometer data, and to do least-squares fits to various density profiles of the computed radial densities. However, since density measurements were not available for all shots, it was necessary to provide a means of scaling the shots by density from the control settings of the experiment. This was done by calculating the pulsed gas valve output as a function of time, so that a knowledge of the timing and plenum fill pressures allowed determining the pinch density.

The other primary diagnostic of the experiment was the STL framing camera. To understand how the plasma brightness in the visible spectrum varied with density and temperature, the relative brightness of the line radiation, the recombination, and bremsstrahlung radiation was calculated. This calculation was complicated by the need to use and argon plasma to provide sufficient light for the framing camera, and therefore required calculation of the average ion charge. The average ion charge, which is a function of temperature, was determined with an approximate technique (good to 10 %) that is primarily saha-like at lower ionization levels, and corona-like at the higher levels. From these calculations the brightness of the plasma was found to stay relatively constant over a large range of temperatures. At low

temperatures line radiation was found to dominate, and as higher ionization levels are attained, the bremsstrahlung dominates. The recombination radiation was not found to be significant. Furthermore, the intensity of the line radiation is found to scale as n_i and the bremsstrahlung as $n_i n_e$ for a given temperature.

In the third chapter the density variation of the gas jet between the electrodes is estimated from supersonic jet studies to be $(x/d)^{-6/5}$, along the axis, and to vary radially out to 45° as $\cos^2\theta$, where θ is measured about the orifice. The breakdown of the gas was observed to occur at the outer edges of the gas, thus causing the initial plasma to have cone-shaped distribution, with the apex of the cone at the cathode. As the pinch current rises, the plasma is observed to pinch to a straight column, reaching peak compression at the cathode end. This non-uniform radial compression then has an axial component directed towards the anode, the magnitude of which can be as large as 0.4 times that of the radial velocity, depending on the angle of the compressing plasma near the axis.

The time to pinch, and the velocity of the imploding shock wave are calculated with the well-known snowplow model, both with linearly increasing and constants currents. For the constant current case the time to pinch is found to be proportional to $\rho_0^{-1/2}$, rather than the more usual $\rho_0^{-1/4}$ scaling found for the linearly increasing current. A numerical solution was also done with a current which rose linearly, and then leveled off to a constant current before pinching. For this

case, a scaling law was found that relates the time to pinch to that of the constant current case. Furthermore, the scaling of the time to pinch for this equation, i.e., $t_p \propto \rho^{-1/2}$, was also experimentally observed.

The formation and life of the gas-puff, and the gun initiated pinches are also shown from framing camera images. Several gases were used, although the most success in obtaining straight, slowly growing pinches was with the higher Z gases. No straight, long lasting hydrogen pinches were observed. Abel inversions of the interferograms showed that the plasma density resides primarily in a shock region during the compression phase, but becomes a Bennett profile at peak compression. That is,

$$n(r) = \frac{n_0}{(1 + (r/a)^2)^2} \quad (6.1)$$

where n_0 and a are constants.

With almost every shot, near peak compression, a large voltage spike (2 to 3 times the charge voltage) appeared across the pinch electrodes. The pinch column was observed to go dark at this time in a small region located usually near the cathode. Coinciding with the voltage spike a hard X-ray spike was also always observed. The X-ray energies were measured by absorption techniques to be of order θ times

The pinch usually appeared to break in sausage-like ($m = 0$) disruption, with a growth rate at least an order of magnitude higher than those predicted by the MHD theory. Often, however, the pinches would remain straight for up to $1 \mu\text{sec}$ before suddenly disrupting. Thus, it is concluded that the instability is not ideal MHD, but must include effects neglected in the MHD theory.

The $m = 1$ external kink mode was also observed, although it was not seen nearly as often as the sausage-like. The observed growth rates were found to have growth rates approximately a factor of ten less than those predicted by the diffuse MHD theory with a quadratic pressure profile.

The type of instability observed was found to vary with the electron line density, which is proportional to s , the number of ion orbits in the pinch diameter. For low densities, pinches were observed which expanded at one end of the pinch column before reaching maximum compression at the other. At higher densities, the more usual sausage-like instabilities were observed. At still higher densities pinches were observed that remained straight for up to $1 \mu\text{sec}$, before slowly expanding. At the highest densities studied, the more typical kink and sausage instabilities were observed. It is concluded that at the lowest line densities the microinstabilities dominate the development of the pinch, whereas at the higher line densities, MHD instabilities dominate. For the high Z plasmas a stability window

possibly exists between the two regimes, which is non-existent for the low Z plasmas.

Growth rates for the $m = 0$, and the $m = 1$ modes were calculated and plotted for the skin current model, and for a diffuse current, quadratic pressure profile model. The skin current model, which contains no B_z , predicts growth rates which differ significantly from those usually discussed with the screw pinch, in that as $ka \rightarrow 0$, the growth rate $\rightarrow 0$, whereas for the screw pinch it remains finite. The effect of adding a diffuse current is to limit the growth rate at large ka to $\gamma a/v_A = 1$, whereas with the skin current model $\gamma a/v_A \rightarrow \infty$.

When considering FLR effects by adding viscous terms to the MHD equations, it is found that the short wavelength growth rates are strongly reduced, and that the wavelength of the maximum growth rate, written in terms of the wavenumber, is $k_M a = \pi/3 s^{2/3}$. For wavenumbers less than this value, the growth rates remain unchanged.

Finally, it was concluded that anomalous resistivity effects probably play the largest role in causing the large voltage spikes, and associated X-ray bursts that are observed in this experiment. Purely inductive effects caused by simple radiative cooling and compression of the plasma, are simply not observed. Rather, the pinch is found to expand to a lower density, larger diameter pinch, characteristic of a pinch which is quickly heated by anomalously resistive heating.

6.2. Further work

Probably because of the known instability of the linear z-pinch, there have been very few calculations done on linear growth rates of the diffuse pinch with anything more than a quadratic profile, for which a transcendental equation for the dispersion relation can be obtained analytically. An important extension to this work would be a numerical solution to the linearized MHD equations for the growth rates using a Bennett profile, allowing viscous effects, and preferably not requiring incompressibility. Furthermore, since most pinches do not extend to the wall, solutions to the free-boundary pinch should be found, in spite of the difficulties with boundary conditions at the plasma-vacuum interface. FLR effects could be added by considering them as viscous effects. Or, they could be included in the MHD equations by starting with the Vlasov equation and not neglecting the FLR terms, as is commonly done in obtaining the ideal MHD equations. A similar approach is done by Kerbel¹ to calculate the magnitude of the FLR effects for the diffuse-current screw pinch with the ordering that $B_z \gg B_\theta$.

With these proposed calculations, it would be hoped a more precise dividing line between FLR stabilization, and microinstabilities could be established.

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APPENDIX A

Pulsed Gas Valve Construction and Gas Flow Dynamics

This appendix contains a discussion of the Cheng style gas valve construction and operation, and of the gas dynamic calculations used in preparing fig. 2.5 in chapter 2.

A.1. Construction and operation of the gas valve

The construction of the gas valve is shown in fig. 2.4 of chapter 2. The body and base of the gas valve are constructed of Lexan to electrically insulate the plasma gun, and to provide mechanical strength. Several valve bodies which were made of plexiglass cracked during use.

The primary coil had 5 1/2 turns and was constructed from a 95 mm wide, 0.5 mm thick copper strip. The leads are B & S no. 8 gage copper wires with the ends pounded flat and silver-soldered at right angles to

Figure A.1. Construction of the primary coil.

the copper strip. The leads and connecting strip were covered with overlapping mylar tape to prevent electrical breakdown. The coil was wound on a form, was covered, and filled with epoxy. In the process of winding both leads were bent back 180° and spaced 180° apart on the circumference. After the epoxy hardened, the coil and valve base were epoxied into the gas valve body. The leads coming out the top were

electrically insulated from the cover plate by 1/4" D polyflo vinyl tubing.

The secondary coil can be either a coil-wound bobbin, or a copper disk, as shown in fig. 2.4 of chapter 2. The copper disk was found to be satisfactory because its driving force was more than sufficient, and because there was some concern about jitter caused by the movement of the windings with a coil-wound bobbin. Since the magnetic field diffusion time through copper is relatively slow the need to slot the valve stop to reduce its eddy currents was eliminated.

The primary coil current was supplied by two $7.5 \mu\text{F} / 20\text{kV}$ capacitors normally charged from 10 to 16 kV. Fire and crowbar ignitrons were used so that the current rose to a maximum in $10 \mu\text{sec}$ and decayed in $100 \mu\text{sec}$. The peak current ranged from 10 to 16 kA. The capacitors were located about 8 m from the gas valve. For this reason the current profile was determined primarily by the external inductance of the connecting transmission line.

The jitter in the valve opening time was measured with a gas breakdown probe located 3 cm downstream from the valve opening and found to be $\pm 5 \mu\text{sec}$. Some of this jitter might be attributed to turbulence and breakdown characteristics of the gas. Therefore one could expect smaller jitter times than measured from the breakdown probe.

The delay in the gas valve opening and open time was measured by shining a He-Ne laser on the disk/O-ring seal, and by using a photomultiplier tube to measure the transmitted light as the valve opens. Figure A.2 shows an oscilloscope trace of the light output and the primary current as a function of time with a 12 kV capacitor

Figure A.2. Oscilloscope traces of the gas valve stop position and primary coil current. The upper trace is the photomultiplier output of a laser beam directed across the valve at the O-ring seal. The lower trace is the current at 8 kA/div. Both horizontal scales are 100 μ sec/div.

charge. The opening delay was found to be 40 μ sec, and was only slightly less at higher voltages. The light output indicates that the valve disk is bounced off the stop, and returns to the seal rather

quickly. The length of time the valve was open was found to be independent of the plenum pressure below about eight atmospheres. At higher pressures the gas provided an additional restoring force which more quickly closed the valve. The time was found to be very dependent on the stop position, however. At a 12 kV capacitor charge the disk velocity was measured at 1.1×10^3 cm/sec. The amount of time the valve remains open is the time for the disk to reach the stop and bounce back.

The total gas valve output was measured with argon at various plenum pressures and stop settings, for an early version with a somewhat different primary coil construction. The output ranged from .2 to 20 Atm-cm³ (5×10^{18} to 5×10^{20} particles.) The results of these measurements are given in fig. A.3. The decrease in output at higher pressures is apparently due to a higher restoring force on the disk which more quickly closes the valve.

A.2. Gas Dynamics

The gas output as a function of time can be estimated from rate equations. Consider the geometry of fig. 2.4, which shows the gas valve and gun assembly. As the valve opens, the gas enters an intermediate chamber, building up pressure, and escapes into the pinch chamber through holes around the gun electrode. The quantity of gas

Figure A.3. Total gas valve output as a function of plenum pressure for various stop settings.

which passes the cathode plane is thus dependent on the pressure differences between three chambers. Therefore, consider a simplified model of this geometry as shown in fig. A.4. The quantities P_0 , P_1 , and P_2 are the pressures, as a function of time, in the plenum, the intermediate volume, and the pinch chamber, respectively. The volumes V_0 and V_1 are 13.3 cm and .72 cm. The area A_1 is .20 cm². The area $A_0(t)$ varies with time as

Figure A.4. Model of pulsed gas valve and pinch chamber used in gas flow calculations.

$$\begin{aligned}
 A_0(t) &= 0, \quad t < t_d \\
 &= \frac{A_{max}}{w}(t - t_d), \quad t_d \leq t < t_d + w \\
 &= \frac{A_{max}}{w}(t_d + 2w - t), \quad t_d + t < 2w + t_d \\
 &= 0, \quad t > 2w
 \end{aligned}
 \tag{A.1}$$

The quantity t_d is the delay time in the gas valve opening, and w is time for the valve disk to reach the stop. It is a function of the valve stop position, being the number of turns of the stop multiplied by a constant and divided by the disk velocity. With a 12 kV capacitor

geometrical factor equal to .0907 cm/turn. A_{max} is also a function of the stop position, and is the number of turns multiplied by a constant, .274 cm²/turn.

The gas quantities in each chamber, as a function of time, are found by considering sonic flow between the chambers. The sonic flow assumption is allowed if the pressure drop between successive chambers is more than a factor of two.¹ Only for the largest valve openings, and for only very short durations is this condition not met. For an ideal gas, the sound velocity is $\sqrt{\gamma kT/M}$ and thus varies only with the temperature. The flow through a restricted orifice is a Joule-Thompson process, which conserves enthalpy, and hence requires, with the case of an ideal gas, that the temperature on either side of the orifice be the same.² Thus, it is sufficient to consider the sound velocity to be a constant. For argon, at room temperature, it is 3.19×10^4 cm/sec.

The rate of change in the quantity of gas in the plenum is proportional to the difference in chamber pressures. (The pressure drop due to turbulence in the orifice is insignificant because of the finite dimensions, and is therefore neglected.)

$$\frac{d}{dt}(P_0 V_0) = -v_s A_0 (P_0 - P_1) \quad (A.2)$$

The rate of change of the gas quantity in the intermediate chamber is proportional to the gas flow in minus the gas flow out. Initially,

sound transit time l_1/v_s after the flow starts on the right end. Further, the time dependence of the pressure at the left end must also be displaced by the transit time. For convenience, define $t_1 = l_1/v_s$, the transit time across the intermediate volume, and $t_2 = l_2/v_s$, the transit time from the chamber opening to the cathode plane.

$$\begin{aligned} \frac{d}{dt}(P_1 V_1) &= v_s A_0 (P_0 - P_1), \quad t < t_1 \\ &= v_s A_0 (P_0 - P_1) - v_s A_1 P_1 (t - t_1), \quad t \geq t_1 \end{aligned} \quad (A.3)$$

Since the chamber volume is so much larger than the plenum, its pressure rise is very small, and the effect on the flow rate is negligible. The time rate of change of the quantity Q of gas entering the chamber ($Q = P_2 V_2$) is also displaced by the transit time. At the opening to the chamber it is

$$\begin{aligned} \frac{dQ}{dt} &= 0, \quad t < t_1 \\ &= v_s A_1 P_1 (t - t_1), \quad t \geq t_1. \end{aligned} \quad (A.4)$$

Integrate eq. A.4 to obtain the total quantity which enters the chamber.

$$Q(t) = \int_{t_1}^t v_s A_1 P_1(\tau - t_1) d\tau$$

(A.5)

$$= \int_0^{t - t_1} v_s A_1 P_1(\tau) d\tau, \quad t \geq t_1$$

The quantity of gas that has passed the cathode plane at time t is the value of eq. A.5 displaced by the transit time t_2 .

$$Q(t) = v_s A_1 \int_0^{t - (t_1 + t_2)} P_1(\tau) d\tau, \quad t \geq t_1 + t_2 \quad (A.6)$$

The solution thus requires the knowledge of the time behavior of $P_1(t)$, which must be obtained from eqs. A.2 and A.3. These equations were solved numerically with a 4th order Runge-Kutta scheme³ with the initial conditions that $P_0(0)$ is a given constant, and that $P_1(0)$ and $Q(t)$ are zero. The results for several stop settings are shown in fig. A.5. Fig. 2.5 shows the same results, expanded with a semi-logarithmic scale, for the total quantity of gas $Q(t)$.

The numerical solver was compared with an analytical solution for the case that A_0 is constant, and t_d , t_1 , and t_2 are zero. The two

Figure A.5. The numerical solution of $P_0(t)$ and $P_1(t)$ normalized to the initial plenum pressure $P_0(0)$, and $Q(t)$, normalized to the initial quantity in the plenum for $1/2$, 1 , $1\ 1/2$, and $2\ 1/2$ turns of the valve stop. The family of curves in the upper portion of the frame is $P_0(t)$. Those lines in the lower right-hand corner are $Q(t)$. The remaining family of solid lines is $P_1(t)$. The dotted lines are $A_0(t)$, the plenum opening areas, in units of cm^2 .

sufficient for this model.

It is worth noting, that for times after about 500 μsec , the quantity of gas in the chamber does not increase appreciably. This indicates that measurements of gas output, as shown in fig. A.3, would also give good estimates for the quantity in the chamber at this time. Comparing fig. A.5 with fig. A.3 at 1 Atm shows a disagreement by almost a factor of ten, however. As stated on page 159, the data displayed in fig. A.3 is from an earlier version with a different coil construction. It was replaced with a newer version because its primary coil was shorting out as it was pulsed. This would have led to a smaller driving force, and hence smaller disk velocity, which would have left the valve open longer. For that valve it is therefore not valid to consider the total output equivalent to the gas present at the time the pinch bank is fired. Rather, it is necessary to consider the same problem, but with a lower disk velocity. Fig. A.6 shows the results of the same analysis, but with a disk velocity arbitrarily set to one-fourth of that used for generating fig. A.5. From this plot it can clearly be seen that the quantity of gas in the chamber at 500 μsec is approximately equivalent for the two valves. However, the asymptotes are very different, showing that the total gas output measurement does not provide sufficient information to know the quantity in the chamber at the bank fire time. The reason the total quantities differ with the two cases is that one valve is open longer than the other.

Figure A.6. The numerical solution of $P_0(t)$, $P_1(t)$, and $Q(t)$ for the case that the disk velocity is one-fourth that used for fig. A.5. Refer to fig. A.5 for a description of the families of curves.

The output of the Berkeley puffatron valve can also be modeled with this technique by adjusting the various orifice areas and plenum volumes. The steel rod which makes the seal is connected to the

spring, and driven off by pulsing the solenoidal coil. The motion of the rod was not measured, but it can be estimated by considering the magnetic flux of the coil supplying an impulse over a period of $\sim 100 \mu\text{sec}$. Doing this, the rod velocity is estimated to be $3 \times 10^3 \text{ cm/sec}$. The restoring force of the spring is considered to have a much smaller effect on the rod velocity. Thus, for the time scale of the experiment, the valve is considered to remain open. With these estimates, and the measured geometry of the valve, the gas output as a function of time is calculated as shown in fig. A.7. As is seen in the figure, the motion of the rod after $600 \mu\text{sec}$ would not have an appreciable effect since most of the gas has left the plenum prior to this time.

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Figure A.7. The numerical solution for normalized values of $P_0(t)$, $P_1(t)$, and $Q(t)$ for the Berkeley puffatron. Refer to fig. A.5 for a description of the curves.

APPENDIX B

Laser Interferometry and Abel Inversion

B.1. General considerations

Use of an interferometer provides an aesthetically pleasing approach to making electron density measurements of a plasma. The probing radiation, be it ultraviolet, visible, infra-red, or microwave, need not be of sufficient strength to affect the plasma. The technique also does not require that the results depend on a calibration since the number of fringe shifts depends only on the frequency of the probing radiation, and size and density of the plasma. The techniques of microwave and laser interferometry are discussed by a number of authors.¹⁻³ Jahoda⁴ has given a number techniques for spatially resolved laser interferometry and holographic interferometry.

The number of fringe shifts $N(\gamma)$ in a Mach-Zehnder interferometer is the difference between the path lengths of the two legs of the interferometer, divided by the wavelength λ . The path length in each leg is adjusted such that there is no fringe shift without the plasma. When the plasma is present, the index of refraction ζ of the experimental leg differs from that of the reference leg, and results in a fringe shift.

$$N(\gamma) = \int_0^{L(\gamma)} (1 - \zeta(x, \gamma)) \frac{dx}{\lambda} \quad (B.1)$$

The index of refraction of a plasma, for frequencies much greater than the electron cyclotron frequency (which it is by several orders of magnitude for the lasers used), is

$$\zeta = \left(1 - \frac{\omega_p^2}{\omega^2} \right)^{1/2}. \quad (B.2)$$

The electron plasma frequency ω_p is a function of the electron density n_e and mass m_e , and the electron charge e . In mks units

$$\omega_p = \left(\frac{n_e e^2}{\epsilon_0 m_e} \right)^{1/2}. \quad (B.3)$$

For $\omega \gg \omega_p$, expand eq. (B.2) in a binomial series, keeping only the

first term, and substitute into eq. (B.1).

$$N(\gamma) = A(\lambda) \int_0^{L(\gamma)} n_e(x, \gamma) dx \quad (B.4)$$

where $A(\lambda)$ contains all the constants, and in mks units equals

$$A = \frac{\mu_0 e^2}{8\pi^2} \lambda. \quad (B.5)$$

With cgs units eq. (B.4) is identical to eq. (2.1) of chapter 2.

Values of A are given for He-Ne, argon-ion, and CO_2 lasers in Table B.1. Table B.1 shows the validity of the small ω_p/ω approximation for a density of $10^{18} / \text{cm}^3$. A similar parameter, the electron cut-off density, is also shown. For electron densities above this value the index of refraction is imaginary, and the laser beam is reflected.

B.2. Abel Inversion

The geometry of the experiment was such that it was necessary to look across the pinch, rather than to look down the axis. If it can be assumed that the plasma consists of a number of concentric shells, each

Table B.1. Comparison of parameters for argon-ion, He-Ne, and CO₂ lasers. The quantity ω_p/ω is calculated for an electron density of $10^{18} / \text{cm}^3$. The parameter A is defined in eq. (B.4).

Laser	Wavelength (μm)	Cut-off density (electrons/ cm^3)	ω_p/ω	A (cm^2)
Argon-ion	0.5145	4.2×10^{21}	.002	2.31×10^{-18}
He-Ne	0.6328	2.8×10^{21}	.003	2.84×10^{-18}
CO ₂	10.6	9.9×10^{18}	.05	4.75×10^{-17}

the fringe shift measured at several positions perpendicular to the axis. This technique is called an Abel inversion and is discussed by several authors.^{5,6}

The inversion can be easily derived. Consider the geometry of fig. B.1 with a collimated laser beam propagating in the x-direction. Consider m concentric shells of outer radius r_i , and m measured frequency shifts at positions y_k along the y-axis, where the indices i and k range from 1 to m . For the outer shell, the fringe shift is a result of the density in the $k = m$ shell only. For beam paths intersecting more shells, the fringe shift is the sum of contributions of several shells. Since the measured fringe shift is an average over an increment Δy , calculate the average contribution from each shell

Figure B.1. Geometry used in the Abel inversion. The plasma consists of m concentric cylindrical shells of uniform density $n(r_i)$.

segment within the increment. The contribution from the cross-hatched segment in fig. B.1 is

$$N_i(y_k) = \frac{A}{\Delta y} \int_{y_k - \Delta y/2}^{y_k + \Delta y/2} dy \int_{\sqrt{r_{i-1}^2 - y^2}}^{\sqrt{r_i^2 - y^2}} n(r_i) dx. \quad (B.6)$$

The contributions from all the segments in the beam path are summed to obtain the total fringe shift at y_k . Since the sum is symmetrical about the y -axis, only the contributions of the 1st quadrant are used,

$$\begin{aligned}
 N(y_k) &= 2 \sum_{i=k}^m N_i(y_k) \\
 &= 2 \frac{A}{\Delta y} \sum_{i=k}^m \int_{y_k - \Delta y/2}^{y_k + \Delta y/2} dy \int_{\sqrt{r_{i-1}^2 - y^2}}^{\sqrt{r_i^2 - y^2}} n(r_i) dx.
 \end{aligned}
 \tag{B.7}$$

Note also, that because of symmetry, $N(-y_k) = N(y_k)$. Hence, only positive values of y_k need be considered.

The integral can be simplified by noting that the electron density is constant on each shell, thus allowing it to be factored out of the integral. The integrations over x and y can then be performed. Doing first the x integration, and defining a geometrical factor a_{ik} , it is found that

$$\begin{aligned}
 N(y_k) &= \frac{A}{\Delta y} \sum_{i=k}^m n(r_i) \int_{y_k - \Delta y/2}^{y_k + \Delta y/2} dy \left(\sqrt{r_i^2 - y^2} - \sqrt{r_{i-1}^2 - y^2} \right) \\
 &= \sum_{i=k}^m a_{ik} n(r_i),
 \end{aligned}
 \tag{B.8}$$

where a_{ik} is defined as,

$$a_{ik} = 2 \frac{A}{\Delta y} \int_{y_k - \Delta y/2}^{y_k + \Delta y/2} dy \left(\sqrt{r_i^2 - y^2} - \sqrt{r_{i-1}^2 - y^2} \right), \quad i \neq k. \quad (B.9)$$

For the case $i = k$, the second term in the parentheses in eqs. (B.8) and (B.9) is zero since the y -axis is the lower limit of integration for the integral over x .

Note that the limits of the remaining integral $y_k + \Delta y/2$ and $y_k - \Delta y/2$, are equal to the shell radii r_k and r_{k-1} , respectively. Integrating for $i = k$, it is found that

$$\begin{aligned} a_{kk} = \frac{A}{\Delta y} & \left(-r_{k-1} \sqrt{r_k^2 - r_{k-1}^2} \right. \\ & \left. + \frac{\pi}{2} r_k^2 - r_k^2 \sin^{-1} \frac{r_{k-1}}{r_k} \right) \end{aligned} \quad (B.10)$$

For $i > k$, the coefficient is

$$\begin{aligned} a_{ik} = \frac{A}{\Delta y} & \left[r_k \left(\sqrt{r_i^2 - r_k^2} - \sqrt{r_{i-1}^2 - r_k^2} \right) \right. \\ & - r_{k-1} \left(\sqrt{r_i^2 - r_{k-1}^2} - \sqrt{r_{i-1}^2 - r_{k-1}^2} \right) \\ & + r_i^2 \left(\sin^{-1} \frac{r_k}{r_i} - \sin^{-1} \frac{r_{k-1}}{r_i} \right) \\ & \left. + r_{i-1}^2 \left(\sin^{-1} \frac{r_{k-1}}{r_{i-1}} - \sin^{-1} \frac{r_k}{r_{i-1}} \right) \right], \quad i > k. \end{aligned} \quad (B.11)$$

The solution to this set of equations is readily seen upon writing out the sum of eq. (B.8).

$$\begin{aligned}
 N(y_m) &= a_{mm} n(r_m) \\
 N(y_{m-1}) &= a_{m-1,m-1} n(r_{m-1}) + a_{m-1,m} n(r_m) \\
 &\quad \cdot \quad \cdot \\
 &\quad \cdot \quad \cdot \\
 &\quad \cdot \quad \cdot \\
 N(y_1) &= a_{1,1} n(r_1) + a_{1,2} n(r_2) + \dots + a_{1,m} n(r_m)
 \end{aligned} \tag{B.12}$$

Since the geometrical factors a_{ik} are known, it is a simple process of back-substitution to determine the shell densities, given the fringe shift as a function of position. That is, the density of the $i = m$ shell is determined by solving the first line of eq. (B.12). This value is substituted into the second line, which is solved for the $m-1$ shell density, which is then substituted into the next equation. This process continues until all the $n_e(r_i)$ are determined.

The electron line density $\mathcal{N}$ of the pinch is also easily determined with the approach. It is the integral of the electron density over the cross-sectional area of the pinch.

$$\mathcal{N} = \int dx \int dy n_e(x,y) \tag{B.13}$$

The limits of integration are such that the entire pinch cross-section

is included in the integral. This integral, however, is identical to the sum and integral of eq. (B.7) for an area of thickness Δy . Therefore, sum the contributions from each Δy , and substitute in $\mathcal{N}$ for the integral.

$$\begin{aligned}
 2 \sum_{k=1}^m N(y_k) &= 4 \frac{A}{\Delta y} \sum_{k=1}^m \sum_{i=k}^m \int_{y_k - \Delta y/2}^{y_k + \Delta y/2} dy \int_{\sqrt{r_i^2 - y^2}}^{\sqrt{r_i^2 - y^2}} n(r_i) dx \\
 &= \frac{A}{\Delta y} \mathcal{N}
 \end{aligned}
 \tag{B.14}$$

The factor of two appears because the sum over k only includes the positive values of y . Equation (B.14) is solved for the line density $\mathcal{N}$:

$$\mathcal{N} = 2 \frac{\Delta y}{A} \sum_{k=1}^m N(y_k)
 \tag{B.15}$$

An algorithm was written to compute the shell densities and the line density. The resulting points were fit to a 5th or lower order polynomial which was constrained to have a zero slope on the axis. Test cases were run to verify that it worked correctly, and to see how sensitive the density measurements are to variations in precision of

the fringe shift measurements. The results of several runs of varying

Table B.2. Density measurement errors from the Abel inversion routine. Shell densities and line densities determined by the inversion routine are compared with the input for a uniform density test case. The standard deviation of the density for each shell, and the error in the line density is shown to vary with the precision of the fringe shift used, and with the number of shells.

significant figures in input	% standard deviation in shell density with			% error in line density with		
	3	5	10	3	5	10
	shells	shells	shells	shells	shells	shells
4	.02	.03	.03	.01	< .01	< .01
3	.07	.17	.29	.02	.03	< .01
2	1.49	2.84	2.30	.12	.12	.03
1	6.12	17.8	27.9	5.36	4.36	.12

precision are given in table B.2. The data shows that increasing the number of shells increases the standard deviation in the shell densities, but also decreases the error in the line density. Therefore, one needs to decide which measurement is most important in choosing the number of shells for the Abel inversion.

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APPENDIX C

Determination of the Effective Ion Charge

In using the electron density measurements to determine the Bennett temperature, it is necessary to first determine the average charge per ion Z_{ave} . The technique used to find Z_{ave} depends on which model is used for the ionization, radiation, recombination, and collision processes. For a plasma in thermodynamic equilibrium, i.e., one that behaves as a black-body radiator, the Saha-Eggert equations are used.^{1,2} Most laboratory plasmas are not black-body radiators in that they do not reabsorb most of the emitted radiation, and they are thus termed *optically thin* plasmas. If collision processes still play a dominant role, the plasma can be in local thermodynamic equilibrium (LTE). Thus the populations of the bound electron states of an ion or atom can obey a Boltzmann distribution despite the radiation losses. For this class of plasmas the Saha-Eggert equations are also valid, provided the electron density is sufficiently high. McWhirter³ gives a

criterion for LTE which places a lower bound on the electron density.

$$n_e \geq 1.7 \times 10^{14} T_e^{1/2} \phi(p,q)^3 \text{ cm}^{-3} \quad (C.1)$$

T_e is the electron temperature, and $\phi(p,q)$ is the excitation potential of level p from level q . Both are in units of eV.

The maximum value of ϕ is the potential from the ground state to the first excited state, for a given ionization stage. As higher temperatures are considered, the atom is more highly ionized, and the potential to the first excited state much larger. Therefore, despite the only mild *explicit* dependence on the temperature, eq. (C.1) is strongly dependent on the electron temperature because of the cubic dependence on ϕ , which is a strong function of T_e . An argon plasma, for example, with $T_e \geq 100$ eV, must have an electron density greater than 10^{21} cm^{-3} to be in LTE. Thus, at higher temperatures, and with lower electron densities, it is necessary to consider other models to determine the effective charge.

With lower densities the ionization levels become less dependent on the electron density, but strongly dependent on various radiative processes. One model which considers some of these processes is the *corona* model, which is discussed in detail by McWhirter.⁴ House⁵ has used this model to calculate the ionization levels of the elements H through Fe, for temperatures ranging from $\log_{10} T(\text{eV}) = -0.5$ to 3.0.

⁴ R. McWhirter, *Electron-Atom Collisions*, Butterworths, London, 1962, p. 100.

to be populated. His results, for lower temperatures, are generally valid for $n_e \leq 10^{15} \text{ cm}^{-3}$.

Thus, it is obvious, that a model which would correctly predict the effective charge as a function of temperature must include LTE effects for low temperatures, and corona like effects at higher temperatures. This has been done by Hyman⁶ using a technique proposed by Raizer⁷ to obtain an approximate solution to the Saha equations. Since this method is used to obtain figs. C.1 - C.6, I will explain his approach.

The standard corona-like equations expressing the degree of ionization are

$$\frac{n_{z+1}}{n_z} = \frac{S(T_e)}{\alpha(T_e, n_e)} . \quad (C.2)$$

the quantities n_z and n_{z+1} are the fraction of ions in the z and $z+1$ charge states, respectively. $S(T_e)$ is the normal collisional ionization coefficient, and $\alpha(T_e, n_e)$ is the recombination coefficient, which can be written as the sum of radiative and three-body collisional terms.

$$\alpha(T_e, n_e) = \alpha^r(T_e) + n_e \beta(T_e) \quad (C.3)$$

The superscript r designates the radiative term, and β is the collisional term. For plasmas in LTE, $n_e \beta \gg \alpha^r$, and eq. (C.2) becomes

the Saha equation. In the opposite limit, it becomes the corona equation.

Eq. (C.2) can be easily solved, following the method of Raizer,⁷ by considering n_z and χ_z , the ionization potential of the z^{th} ion, to be continuous functions of the charge z . Therefore,

$$n(z+1) = n(z) + \frac{dn}{dz} (\Delta z = 1). \quad (C.4)$$

Substituting eq. (C.4) into eq. (C.2) gives

$$1 + \frac{1}{n} \frac{dn}{dz} = \frac{S}{\alpha}. \quad (C.5)$$

For a given electron temperature and density, only a relatively few ionization states are populated, and $n(z)$ is a sharply peaked function. Therefore, look for a solution of eq. (C.5) with $dn/dz = 0$. Doing this, eq. (C.5) becomes

$$S(T_e, \bar{\chi}) = \alpha(T_e, n_e, \bar{\chi}), \quad (C.6)$$

where $\bar{\chi}$ is an effective ionization potential at the peak. Generally, $\bar{\chi}$ will not equal the ionization potential of any of the ionization states, but rather will lie between the values of two consecutive states. The effective charge is then determined by using a table of

corresponding to $\bar{\chi}$. However, the interpolated value (call it z^*) is not quite the average charge. Rather, as is seen by eqs. (C.6) and (C.2), it is the value for which $n_{z^*+1} = n_{z^*}$. The average charge is thus the interpolated value plus 1/2, $Z_{av} = z^* + 1/2$.

Specific expressions for a and b are taken from McWhirter.⁴ Thus, eq. (C.7) reduces to the transcendental equation

$$e\bar{\chi}/T_e \left(a \bar{\chi}^{1/4} + \frac{b n_e}{T_e^{3/4}} \right) = T_e^{3/4}, \quad (C.7)$$

where a and b are constants equal to 7.82×10^{-9} and 1.65×10^{-22} , respectively. The units of $\bar{\chi}$ and T_e are in eV, and n_e is in cm^{-3} .

Eq. (C.7) is solved for $\bar{\chi}$ by an interval-halving technique, to produce a plot of $\bar{\chi}$ vs. T_e , as is shown in fig. C.1 for an electron density $1.0 \times 10^{21} \text{ cm}^{-3}$. This solution agrees with fig. 1 of Hyman's paper. More sophisticated root finders, which use the gradient of the function, do not work. The gradients away from the solution are so large that the methods do not converge to the solution in a reasonable number of iterations.

The average ion charge $Z_{av}(T_e)$ is determined by interpolation using the ionization potentials given by Lotz.⁸ Plots of Z_{av} vs. T_e for electron densities $10^{16} - 10^{20} \text{ cm}^{-3}$ are shown in figs. C.2 - C.6. The values are found to agree to within 10% with the calculations of House⁵ at high temperatures, and, for lower temperatures, to within 10%

Figure C.1. Solution to eq. (C.7). Plot of T_e versus $\bar{X}$ with $n_e = 1 \times 10^{21} \text{ cm}^{-3}$.

Figure C.3. Plot of $Z_{\text{effective}}$ vs. T_e for argon with $n_e = 1 \times 10^{17} \text{ cm}^{-3}$.

Figure C.4. Plot of $Z_{\text{effective}}$ vs. T_e for argon with $n_e = 1 \times 10^{18} \text{ cm}^{-3}$.

Figure C.5. Plot of Z_{ave} vs. T_e for argon with $n_e = 1 \times 10^{19} \text{ cm}^{-3}$.

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APPENDIX D

Electron Density Profiles

In calculating growth rates with the diffuse current, ideal MHD theory, it is necessary to know the pressure and current profiles. If the pinch is considered to have a uniform temperature across its radius, then the pressure is linearly proportional to the density. The density profiles are determined, in turn, from Abel inverting the laser interferograms. In this appendix, several example interferograms of shots with roughly the same parameters as those of figures 3.8 and 3.9 are given. The Abel inverted densities are connected by a least squares fit to (1) a quadratic profile, $n = n_0 [1 - (r/a)^2]$; and (2) to a Bennett profile, $n = n_0 / [1 + (r/a)^2]^2$. These least squares fits differ from those in the rest of the paper in that they are parameterized by only two constants. (The other Abel inversions are generally fit to a power series with more than two terms.) As is obvious from the following figures, the Bennett profile provides a much

is a gaussian profile, $n = n_0 \exp[-(r/a)^2]$. It generally fit the density points better than the quadratic, but not as well as the Bennett profiles.

Figure D.1. Electron density profiles of shot 1113, comparing least squares fits of the Abel inverted densities to quadratic and Bennett profiles.

Figure D.2. Electron density profiles of shot 1115, comparing least squares fits of the Abel inverted densities to quadratic and Bennett profiles.

Figure D.3. Electron density profiles of shot 2115, comparing least squares fits

Figure D.4. Electron density profiles of shot 3115, comparing least squares fits of the Abel inverted densities to quadratic and Bennett profiles.

Figure D.5. Electron density profiles of shot 6115, comparing least squares fits of the Abel inverted densities to quadratic and Bennett profiles.