MASTER

NOVEMBER 1977

UC-20g
PPPL-1397THEORY AND NUMERICAL SIMULATIONS
ON COLLISIONLESS DRIFT INSTABILI-
TIES IN THREE DIMENSIONS

BY

C. Z. CHENG AND H. OKUDA

PLASMA PHYSICS
LABORATORY

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

PRINCETON UNIVERSITY
PRINCETON, NEW JERSEY

This work was supported by Department of Energy Contract EY-76-C-02-3073. Reproduction, translation, publication, use and disposal, in whole or in part, by or for the United States Government is permitted.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Energy Research and Development Administration, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

Printed in the United States of America.

Available from
National Technical Information Service
U. S. Department of Commerce
5285 Port Royal Road
Springfield, Virginia 22151
Price: Printed Copy \$ * ; Microfiche \$3.00

<u>*Pages</u>	<u>NTIS Selling Price</u>
1-50	\$ 4.00
51-150	5.45
151-325	7.60
326-500	10.60
501-1000	13.60

Theory and Numerical Simulations on
Collisionless Drift Instabilities in Three Dimensions

C. Z. Cheng and H. Okuda

Plasma Physics Laboratory, Princeton University
Princeton, New Jersey 08540

PPPL-1397

November 1977

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Department of Energy, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

EB
DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

Theory and Numerical Simulations on Collisionless Drift Instabilities in Three Dimensions

C. Z. Cheng and H. Okuda

ABSTRACT

Nonlinear behavior of the collisionless drift instabilities and the resultant anomalous plasma diffusion have been studied by means of computer simulations and analytic theory. The simulation model used is a full three dimensional electrostatic model in a cylindrical geometry in an external magnetic field. Full dynamics is employed for the ion motion while the guiding center approximations are used for the motion of electrons which allows us to use rather realistic plasma parameters in the simulations. The results of simulations indicate that a strong turbulence develops through the nonlinear interaction of the drift instabilities which results in the formation of convective cells and anomalous particle diffusion. The broad frequency spectrum resembles to those observed recently in toroidal confinement devices. Analytic theory is developed based on the mode-coupling process to explain the generation of convective cells and strong plasma turbulence along with the estimate of the resultant particle diffusion.

I. INTRODUCTION

The importance of low-frequency microinstabilities in a magnetically confined plasma has long been recognized in relation with the research of controlled nuclear fusion [1]. Due to the inherent inhomogeneities of particle density and temperature associated with the confined plasmas, drift-type waves, whose characteristic frequency is the so-called diamagnetic drift frequency $\omega^* = (k_y c T_e) / (e B L_n)$, where $L_n = (d \ln n / dx)^{-1}$ is the density scale length, c , T_e , e , B and k_y are the speed of light, electron temperature, electronic charge, magnetic-field strength and the wavenumber in the direction of wave propagation, can be destabilized under a variety of plasma conditions. Depending on the collision frequency compared with the wave frequency, for example, the instabilities may be classified into collisional or collisionless. They may also be classified as electrostatic, electromagnetic, kinetic, or hydrodynamic depending on the nature of the instabilities.

While a great deal of work has been carried out on the linear stability of low-frequency drift-type instabilities in relation with toroidal confinement devices for controlled fusion [2,3], nonlinear behavior of these instabilities and the resultant anomalous diffusion of particles and energy have not been well understood until now. There are numerous nonlinear theories on drift instabilities and anomalous diffusion based on, for example, weak turbulence approximations [4], mode-coupling model [5,6,7] and strong

turbulence approximation [8]. These theoretical attempts are, however, either not very useful when a plasma is in strong turbulence or very approximate so that their validity has to be judged by comparing with experimental results or numerical simulations.

Recently, it has been demonstrated that particle simulation approach can be very useful for the understanding of drift wave turbulence and anomalous plasma diffusion [9,19,11]. Making use of guiding center drift approximations for the motion of electrons, both two-and-a-half (two space dimensions and three velocity dimensions) [12] and full three dimensional models [13] have been successfully constructed which can handle rather realistic plasma parameters. These models have been applied for the study of collisionless drift and trapped electron instabilities. Several nonlinear effects have been revealed which are inherent to low-frequency drift instabilities. In particular, it was found that the drift instabilities can generate low-frequency convective cells ($\omega \approx 0$, $k_{\parallel} \approx 0$) [14] which in turn enhance anomalous plasma diffusion [11]. The observed frequency spectrum is broad and peaks at $\omega \approx \omega^*$ for drift waves and $\omega \approx 0$ for convective cells which resembles with recent measurements of drift wave fluctuations in toroidal confinement devices [15,16,17].

It is the purpose of the present paper to report in some details the simulation results based on the full three dimensional simulation models [13] with and without magnetic shear. Analytic considerations are also given for the nonlinear excitation of

of convective cells via mode-coupling of drift waves. The estimate and the scaling of the observed particle diffusion will also be discussed and its implication to real experiments is suggested.

In Sec. II, the model is described along with the typical simulation parameters used. Results of the simulations are discussed in Sec. III and the nonlinear processes and the anomalous diffusion are compared with analytic theory in Sec. IV. Concluding remarks are given in Sec. V.

II. MODEL

The model used in the simulations is a three dimensional cylinder with square cross section as sketched in Fig. 1. The plasma is surrounded by a conducting wall in x and y directions while it is periodic in z . Particles reaching the wall are elastically reflected back to the system. The imposed external magnetic field is given by

$$\vec{B}(r) = B_0 \hat{z} + B_p(r) \hat{\theta} \quad (1)$$

where $B_p(r)/B_0 = 2\pi r/qL_z$ is the poloidal field, q is the safety factor and is assumed to take the form of $q = q_0(1 + \lambda r^2/a^2)$ [18]. (r, θ) is the polar coordinate in the (x, y) plane, L_z is the length of the model in z , a is the plasma radius ($a = L_x/2$), and q_0 and λ are numerical constants, typically

$q_0 = 1.1$ and $\lambda = 2 \sim 4$. The shear length is given by

$$L_s = \left(\frac{2\pi r}{L_z q^2} \frac{dq}{dr} \right)^{-1}.$$

Full Lorentz force is used to move the ions in the simulations while the guiding center drift approximations are used for the electrons eliminating the fast time scale associated with the electron gyration. This approximation allows us to use simulation parameters which may be not too far from the realistic ones.

Poisson's equation is solved by first expanding in z direction,

$$\Phi(x, y, z) = \sum_{n=-N}^N \phi_n(x, y) \exp(i2\pi n/L_z)$$

and then using the fast Fourier transform technique to solve $\phi_n(x, y)$ on a two-dimensional spatial grid [13]. This hybrid approach greatly saves the computer memory for the simulations of three dimensional plasmas compared with the conventional method of using three dimensional spatial grid and allows us to carry out simulations of large plasma volume. N characterizes the number of Fourier modes kept in the model and is determined from analytic theory and by experience. For typical laboratory plasmas in which $L_z/a \approx 10 \sim 20$, $N = 5 \sim 10$ is chosen. Short-wavelength modes in z (large N) do not grow at all or grow a little compared with much more unstable long-wavelength modes for all the simulations reported here. This is commonly observed in laboratory

experiments [19] and is due to the fact that the short-wavelength modes are subject to strong ion-Landau damping.

Typical simulation parameters are the following: a 64×64 spatial grid in (x,y) plane, $L_z/L_x = 10$ or 20 , $m_i/m_e = 100$ or 400 , $T_e/T_i = 4$, $\Omega_e/\omega_{pe} = 5$ (Ω_e , ω_{pe} are the electron gyro and plasma frequencies), $L_x/\rho_i = 32$ (ρ_i is the thermal ion gyroradius). The initial density is assumed to be a gaussian $n(r) = n_0 \exp(-\alpha r^2/a^2)$ which gives the diamagnetic drift frequency ω^* constant in r initially. $\alpha = 2$ or 4 .

Initial temperature is assumed flat. The shear length is varied such that $L_s/L_n = \infty$ (shearless), 50 , 25 , and 10 . In terms of physical plasma dimensions, the simulation plasma is smaller than a large confinement device such as a tokamak but it may be larger than, for example, a conventional Q device; and one can simulate quite a few interesting phenomena using the present model.

One of the advantages of the simulation plasmas is that one can obtain almost any kind of diagnostics without disturbing a plasma with desired accuracy. Temporal evaluation of each unstable Fourier mode, mode structure in space, frequency spectrum, particle distribution functions and phase-space distributions are some of the available diagnostics. In particular, precise measurements may be carried out for the transport of plasmas across magnetic field which may be used to construct a nonlinear theory for anomalous plasma diffusion. This is one of the final goals of these numerical simulations. We believe that the results reported

here from the large scale three dimensional simulations have never been obtained before, and they demonstrate the simulation approach for the plasma confinement in large fusion devices is very important. While the present model is only electrostatic and in a cylindrical geometry, more realistic models including electromagnetic interactions [20] and using a toroidal geometry [13] are being used to investigate finite β and toroidal effects on plasma transport.

III. RESULTS OF THE SIMULATIONS

A. Drift Waves and Convective Cells

Before discussing the simulation results, let us briefly review the basic properties of drift waves and convective cells. This will help understand and identify the simulation results discussed later.

The dispersion relation of electrostatic, collisionless drift instabilities may be given by, in a local approximation [1],

$$1 + \tau + \frac{\omega - \omega^*}{k_{\parallel} v_e} Z\left(\frac{\omega}{k_{\parallel} v_e}\right) + \frac{\tau\omega + \omega^*}{k_{\parallel} v_i} \Gamma Z\left(\frac{\omega}{k_{\parallel} v_i}\right) = 0 \quad (2)$$

where $\tau = T_e/T_i$, $b = k_{\perp}^2 \rho_i^2$, $\Gamma = I_0(b) \exp(-b)$, I_0 is the modified Bessel function and Z is the plasma dispersion

function [21]. For $v_i \ll \omega/k_{\parallel} \ll v_e$, Eq. (2) may be solved to find

$$\omega_k/\omega^* = \Gamma/(1 + \tau - \tau\Gamma)$$

and

(3)

$$\gamma_k/\omega^* = (\pi)^{1/2} \omega^* \Gamma(1 - \Gamma)(1 + \tau)/[(k_{\parallel} v_e)(1 + \tau - \tau\Gamma)^3] .$$

The maximum growth, however, takes place when $\omega \approx k_{\parallel} v_e$ and $k_y \rho_s \approx 1$ ($\rho_s = \rho_i \tau^{1/2}$) for which Eq. (2) may be solved numerically. A typical numerical solution of Eq. (2) is shown in Fig. 2 for $m_i/m_e = 1800$ with the other parameters given earlier. Similar curves are found for $m_i/m_e = 100$ and 400 which are used for the simulations. In Fig. 2, we note the frequency of the drift waves increases with $k_{\perp} \rho_s$ and reaches the maximum at around $k_{\perp} \rho_s \approx 1$ and then decreases as $k_{\perp} \rho_s$ exceeds unity. The maximum growth takes place also at around $k_{\perp} \rho_s \approx 1$. Note the rapid decrease of the growth rate with the increase $k_{\parallel}$ because of the onset of ion-Landau damping. For this reason, drift waves become unstable only in a highly elongated system.

When a magnetic shear is introduced, the drift waves are expected to localize near the rational surfaces. The energy input from the destabilizing inverse electron-Landau damping is now balanced with the convective energy loss in which the wave carries energy away from the rational surface to the point where the ion-Landau damping absorbs wave energy [22]. When the shear is strong enough, the convective loss of energy dominates over the energy

input from the inverse electron-Landau damping. This occurs at the shear length of

$$L_s/L_n \lesssim (m_i/m_e)^{1/3} \quad (4)$$

When the shear is weaker than this value, the growth rate in the slab geometry for $\tau = 1$ is given by [22]

$$\frac{\gamma_k}{\omega^*} = \left(\frac{2}{2-\Gamma} \right) \left(\sqrt{\frac{\pi}{2}} \frac{1-\Gamma}{2-\Gamma} \left| \frac{\omega_k}{k_{||} v_e} \right| - \frac{X_L}{L_s} \frac{k v_i}{\omega^*} (2n+1) \right) \quad (5)$$

with $n = 0, 1, 2, \dots$ and $X_L = \rho_i (-\partial \ln \Gamma / \partial b)^{1/2}$. In the presence of shear, nonlinear effects such as anomalous diffusion are expected to reduce because of the localization of unstable wave packet which limits the available free energy of expansion.

In many experimental situations, however, the simple shear stabilization criterion, Eq. (4), may not be applicable. In a toroidal system, for example, one-dimensional analysis which assumes each Fourier mode is independent, is not true any more due to the toroidal effects and one has to deal with the two-dimensional eigenvalue problem [23]. Furthermore, nonuniform magnetic shear along the particle orbit [24] or the double valued q profile associated with the hollow current discharges [25] can cause significant wave reflection from the rational surfaces and the shear stabilization will be completely defeated.

It is well known by now that there are other kinds of low-frequency oscillations in a magnetic field called convective

cells [14]. The frequency of convective cells is much lower than the drift frequency and the anomalous plasma diffusion due to thermally excited convective cells has been studied extensively [26]. In thermal equilibrium, the convective cells are purely damped modes with the dispersion relation given by [14]

$$\omega = \frac{-ik_{\perp}^2 D_{\perp}}{1 + \Omega_i^2 / \omega_{pi}^2} (k_{\parallel} = 0) \quad (6)$$

where $D_{\perp}$ is the cross field particle diffusion coefficient. The anomalous diffusion caused by the convective cells has been observed in multipoles and the scaling of diffusion coefficient confirmed [27,28]. These observations also confirmed that the non-linear damping is much more important than the collisional damping even in thermal equilibrium [29].

When the inhomogeneity of plasma density is introduced, the low-frequency dielectric function including both the ion flute mode and convective cells may be written as, for $\omega \ll \Omega_i$, [25]

$$\epsilon = 1 + \frac{\omega_{pi}^2}{\Omega_i^2} \frac{\omega - \omega_i^*}{\omega} \frac{\omega - ik_{\perp}^2 D_{\perp}}{\omega} (k_{\parallel} = 0) \quad (7)$$

which gives the convective cells $\omega = -ik_{\perp}^2 D_{\perp} / (1 + \Omega_i^2 / \omega_{pi}^2)$ and ion flute mode destabilized by the ion diffusion

$$\omega = \frac{\omega_i^*}{1 + \Omega_i^2 / \omega_{pi}^2} + i \frac{k_{\perp}^2 D_{\perp}}{1 + \Omega_i^2 / \omega_{pi}^2}$$

When the temperature gradient is considered instead of the density gradient, it is found that the convective mode now becomes

a purely growing mode [26]. These instabilities have been checked by the two-dimensional simulations and anomalous plasma diffusion across magnetic field has been observed.

While the drift waves and convective cells may be both unstable in a confined plasma, it appears that the drift waves have much larger growth rate in general and can become unstable in much wider region of phase space. However, later we will show that strong couplings can take place for an unstable plasma between drift waves and convective cells when the drift waves grow to finite amplitudes. At that stage, frequency spectrum becomes broad as seen recently in toroidal experiments. As a result of nonlinear generation of convective cells, a strong turbulence with anomalous diffusion develops.

B. Universal Instability Without Shear (Case 1)

In this first example of numerical simulations, we shall discuss the case where there is no poloidal magnetic field ($B_p = 0$) and henceforth there is no shear. This case will correspond to a conventional linear machine such as a Q device. Here, $m_i/m_e = 100$, $L_z/L_x = 20$, $L_x/\rho_i = 32$, $\alpha = 4$, and $\Omega_e/\omega_{pe} = 5$ were used. The characteristics of the linear dispersion relation is very much like that shown in Fig. 2 where $n = \pm 1$ ($k_z = 2\pi n/L_z$) modes are by far the most unstable modes. In this case $N = 5$ was chosen but the modes with $n = \pm 4$ or ± 5 grew very little as shown later even at the time of saturation.

Figure 3 shows the time development of the most unstable $n = \pm 1$ modes for different poloidal wavelengths $k_\theta = m/r$. Here, the electric field energy of the (m,n) mode integrated over the plasma radius is shown. All the modes grow at about the growth rate of $\gamma/\Omega_i = 0.04 \sim 0.06$ in good agreement with linear theory. After growing about two orders of magnitude, the drift wave amplitude saturates and gradually decreases thereafter. This is partly because the fluctuation energy of drift instabilities is dissipated to excite low-frequency convective cells and partly because the anomalous diffusion flattened the density gradient as shown later.

The radial mode structure associated with one of the most unstable modes, $(m,n) = (5,1)$, is shown in Fig. 4. After reaching the maximum amplitude of $|e\phi_{mn}(r)/T_e| \approx 0.1$ at $r/a \approx 0.75$, the mode amplitude decreases toward its initial noise level. Essentially, the mode extends the entire plasma column, but the peak amplitude tends to locate towards the plasma edge and the fluctuation level is small near the center of the plasma.

Time evolution of the fluctuation spectra with respect to m and n are plotted in Fig. 5. Here, the electric field energy integrated over the plasma radius and summed over m or n are shown for different time steps. The shape of the fluctuation spectrum with respect to m resembles to that of the linear growth rate shown in Fig. 2. It peaks at around $k_\theta \rho_s \approx 1$ and decreases for both large and small m . In fact, the spectrum shown here is in close resemblance with the measurement in a

tokamak device [15]. At the time of saturation, however, the spectrum spreads toward small m modes ($m = 1, 2, 3$). This is because of the excitation of convective cells which tend to coalesce into larger cells.

Evolution of the spectrum with respect to n indicates a very interesting characteristics. At early times, the spectrum peaks at $n = 1$ since they are the most unstable modes. As the fluctuations grow to large amplitude, $n = 0$ modes ($k_{||} = 0$) grow rapidly and takes over the other modes at later times. Note these $n = 0$ modes are not drift modes. Investigation of the frequency spectrum reveals that these modes are essentially convective cells whose frequency is essentially zero. Note also the higher n modes grow little throughout the simulations and their amplitude is much smaller in agreement with laboratory experiments [19].

Frequency spectrum for drift and convective cells measured at $r/a = 0.5$ are shown in Fig. 6 for $m = 1$ and $m = 3$ modes. For $(m, n) = (3, 1)$ mode which is one of the most unstable drift waves, the observed frequency is close to ω^* while the $(m, n) = (3, 0)$ mode is found at zero frequency. The amplitude of the $(3, 1)$ mode is much larger than the $(3, 0)$ mode. For $(1, 1)$ and $(1, 0)$ modes, on the other hand, the convective cells, $(1, 0)$ mode, are much stronger. In fact, it is larger than $(3, 1)$ mode more than a factor of two. This is because the convective cells tends to coalesce to larger cells comparable to the plasma size in the absence of magnetic shear which can cause large scale

particle diffusion.

Figure 7 shows the evolution of convective cells ($n = 0$) for several different m modes. All the modes grow at about the same growth rate which is almost a factor of two larger than the growth rate of drift waves shown in Fig. 3. When the drift instabilities saturate, convective cells also stop growing. We believe these observations provide evidence for the fact that the convective cells are nonlinearly excited as the result of mode-coupling of the drift instabilities. We note the saturation level of the convective cells is slightly higher than but close to that of the drift waves. Furthermore, large amplitude convective cells persist even after the instabilities saturated. This is because the convective cells exist even in a uniform plasma and they do not require presence of density gradient in contrast to drift waves.

Radial mode structure for the largest convective cells, $(1,0)$ mode, are shown in Fig. 8. Note the peak amplitude is $e\phi_k/T_e = 0.2$ and the mode extends the entire plasma column peaking at $r/a \approx 0.5$. Particle diffusion is strongly enhanced in the presence of large convective cells.

Let us now look at the diffusion of particles in velocity space. Figure 9 shows the development of the electron velocity distribution along the magnetic field. Due to the inverse electron-Landau damping which converts thermal energy to drift wave fluctuations, steepening of the distribution at the wave phase velocity takes place. While the steepening increases the Landau damping

on the drift waves, it is not generally strong enough to cause the total stabilization when the plasma is far away from the marginal stability [30]. The change of the slope for strongly unstable modes with $\omega/k_z \approx v_e$ is small as seen in Fig. 9, whereas the steepening may stabilize weakly unstable modes with $\omega/k_z \ll v_e$ since the increase of Landau damping is significant for such modes.

Figure 10 indicates the evolution of electron and ion temperatures (mean kinetic energy) which are set to be uniform initially. The electron temperature decreases at $r/a \gtrsim 0.5$ as the instability develops because of the inverse Landau damping for $t \lesssim 1000 \omega_{pe}$ and at later times significant increase of electron temperature is observed for $r/a \lesssim 0.5$. This increase of temperature is believed to be due to the fact that the particle diffusion for low-energy electrons, $v_z \lesssim \omega/k_z$, is much larger than that for high-energy electrons. As a result of particle diffusion, hot electrons remain at the center of a plasma while the cold electrons move toward the edge creating a strong temperature gradient. Under such conditions, energy diffusion is smaller than the particle diffusion. More will be mentioned in Sec. IV. These observations are similar to the previous simulation results based on a two-and-a-half-dimensional model [30].

Ion-perpendicular energy $T_{i\perp}$ increases almost by a factor of 2 across the entire plasma column. This increase of kinetic energy is associated with the $c\mathbf{E} \times \mathbf{B}/B^2$ drift velocity which gives rise to the increase of ion transverse kinetic energy of $(\omega_{pi}^2/\Omega_i^2) \sum_k |E_k|^2/8\pi$ where $|E_k|^2/8\pi$ is the electric field energy

associated with drift waves, convective cells and ambipolar field.

Ion-parallel temperature $T_{i||}$ hardly changes during the course of simulations because the ion-Landau damping is very weak.

Anomalous particle diffusion is shown in Fig. 11 in which the evolution of ion and electron densities is shown. Initially both the ions and electrons diffuse more or less together, however, ions are always found to diffuse slightly faster than the electrons which results in the build-up of ambipolar potential of the order of the ion thermal energy. The fact that the ions diffuse slightly faster than the electrons appears to be because the electrons behave more adiabatically than the ions whose nonlinear behavior is the dominant mechanism for the generation of convective cells and plasma turbulence. This is in contrast to the marginally stable drift waves in which it is known that the diffusion of ions and electrons is the same [4].

The mechanism of particle diffusion may be elucidated by observing the diffusion of a set of test particles in time. Figure 12 shows the diffusion of the guiding center location for a set of ions sampled initially to form a ring at $r/a = 0.5$. At $t = 800 \omega_{pe}^{-1}$, the ions respond to $cE_{\theta} \times B/B^2$ drift where E_{θ} is the electric field associated with the drift waves. Excitation of several different azimuthal modes may be seen. As the ions move in and out radially, radial electric fields are built-up since there is a slight phase lag between the motions of electrons and ions. Radial electric fields in turn move particles in azimuthal direction thereby forming a closed pattern

of plasma convections as seen in Fig. 12(c). One can estimate the number of such cells for one drift mode is of the order of m where m is the azimuthal mode number. If there are several unstable drift modes, then the number of the cells are so many that the nonlinear interaction of these cells leads to strong plasma turbulence, resulting in the formation of convective cells. As will be shown in Sec. IV, the nonlinear excitation of convective cells becomes important even when the amplitude of the drift waves is small.

Let us compare the observed plasma diffusion with the theoretical estimate. From Fig. 11, one can estimate the overall particle diffusion coefficient is

$$D_{\perp} = \frac{\langle (\Delta r)^2 \rangle}{2t} \approx 1.6 \times 10^{-2} \rho_i^2 \omega_{pe}$$

which greatly exceeds the classical diffusion due to Coulomb collisions. More relevant estimate would be the diffusion due to convective cells given by [14],

$$D_{\perp} = \frac{c}{B} \left(\frac{T_{eff} \ln(L_x/\rho_i)}{(1 + \omega_{pi}^2/\Omega_i^2) L_z} \right)^{1/2} \approx 1.5 \times 10^{-2} \rho_i^2 \omega_{pe}$$

which is close to the measurement. Here, T_{eff} is the effective temperature defined from the level of electric field fluctuations of the convective cells determined approximately from Fig. 7 [28]. Note the thermal level fluctuations for the convective cells are given by [14]

$$V \frac{(E^2)_k}{8\pi} = \frac{T_i}{2} \frac{1}{1 + \omega_{pi}^2 / \Omega_i^2}$$

and the saturated level of convective cells is about 400 times larger than the thermal noise level ($T_{\text{eff}}/T_i \approx 400$). Here V is the plasma volume, $V = L_x L_y L_z$.

If one estimates the conventional formula of diffusion $D_{\perp} \approx \gamma_k / k_{\perp}^2$, then we find this number is less than $10^{-2} \rho_i^2 \omega_{pe}$ which is smaller than the observation. While this value is not too far numerically from the observation, it is clear that this estimate is very approximate and phenomenological, and certainly does not consider the generation of convective cells at the later stage of instabilities when the drift waves are nonlinearly saturated so that $\gamma_k \approx 0$ for them.

While the observed particle diffusion is mainly due to the growing drift instabilities for the early stage, presumably due to quasilinear effects, it is mainly due to the convective cells at the later stage when the drift instabilities saturate by nonlinear effects ($t \gtrsim 10^3 \omega_{pe}^{-1}$).

In order to confirm the diffusion process in more detail, another simulation has been carried out with the same parameters except that the convective cells ($n = 0$ modes) are removed from the system artificially. This is done by deleting the electric field components with $n = 0$ in the equation of motions. Particle diffusion for such a run is shown in Fig. 13 (dotted line) in comparison with the previous case. We see at early times ($t \lesssim 10^3 \omega_{pe}^{-1}$), diffusion for both cases is about the same

although the full code calculation shows slightly larger diffusion. At later times, however, the diffusion without convective cells stops completely before the density gradient is wiped out. At that time, drift instabilities have saturated by nonlinear effects and the diffusion becomes much smaller. This observation confirms that the diffusion at the later stage when the drift instabilities reach the marginal stability is dominantly due to convective cells. It is important to emphasize that the convective cells can cause large diffusion in a steady state even after the saturation of drift instabilities due to the nonoscillatory feature of the mode. In laboratory experiments for plasma confinement, a steady state is established by balancing the diffusion with the particle ionization and recycling. Under such conditions, drift waves may be in marginal stability and it is likely that the convective cells play an important role for plasma diffusion.

Finally, we have run the same example by increasing the number of modes in z to $N = 10$ and found the essential physics remains the same. Higher n modes did not grow as expected and the small n modes played a crucial role for plasma turbulence.

C. Universal Instability With Shear (Case 2,3,&4)

In the absence of magnetic shear, we have seen the drift instabilities grow to large amplitude resulting in strong plasma turbulence and formation of convective cells. The anomalous

particle diffusion is very large which completely wiped out the initial density gradient of a plasma. In this section, we shall study the effects of magnetic shear on drift instabilities and resultant anomalous diffusion. As stated previously, shear is introduced in the model by imposing a poloidal magnetic field given by

$$B_p(r)/B_0 = \frac{2\pi r}{q_0(1 + \lambda r^2/a^2)L_z}.$$

q_0 and λ are numerical constants with $q_0 \approx 1$ and $\lambda = 2 \sim 4$. This choice of safety factor q corresponds to an axial current profile $j_z \sim (1 + \lambda r^2/a^2)^{-2}$. In the presence of shear, drift waves are expected to localize near the rational surfaces at $r = r_s$ where r_s 's are found from $\mathbf{k} \cdot \mathbf{B} = 0$. For a perturbation of the form $\exp(im\theta + inz2\pi/L_z)$, there are more rational surfaces as m, n become larger. For a cylindrical plasma model, each Fourier mode localized near the specific rational surface is expected to be independent in the linear theory. Making use of the results from the one-dimensional slab geometry [22], the localization width around a rational surface is estimated to be $X_T/\rho_s = (L_s/L_n)^{1/2}$ and the distance at which the ion-Landau damping absorbs the wave energy is $X_i/\rho_s = (L_s/L_n)(T_e/T_i)^{1/2}$. For a system with weak shear, L_s/L_n may be larger than 50, for example, and it is conceivable that the ion-Landau damping point is outside of the plasma radius even in a real experimental devices. Under such conditions, the so-called out going energy boundary conditions

may not be appropriate because of the reflection of waves near the plasma edge.

When there are multiple rational surfaces in the system, it becomes important if each surface overlaps with each other, i.e., if the distance between rational surface is smaller than X_T . If overlapping takes place, then it is conceivable that, for example, particles can move across rational surfaces one by one, thereby diffusing through the entire plasma radius. The distance between two neighboring surfaces of m and $m + 1$ will be given by $\Delta r_{m+1,n} \approx [nq'(r_m)]^{-1}$ or

$$\frac{\Delta r_{m+1,n}}{a} \approx \frac{a}{r_m} \frac{1}{2q_0 \lambda n}$$

which becomes progressively small for large n and it is likely in large toroidal experiments, that several rational surfaces tend to overlap with each other.

In the following, we shall describe the results of simulations in the presence of weak shear, $L_s/L_n = 50$ (Case 2), intermediate shear, $L_s/L_n = 25$ (Case 3) and finally strong shear, $L_s/L_n = 10$ (Case 4) where L_s is the approximate shear length averaged over r .

We shall briefly describe the case in which a weak shear of $L_s/L_n = 50$ is introduced in the system (Case 2). The strength of shear is much too weak for the stabilization of drift waves as shown in Eq. (3) and only a small reduction of the growth rate is expected to occur as shown in Eq. (4). Here $q_0 = 1.1$,

$\lambda = 2.5$, $\alpha = 4$, and $m_i/m_e = 100$ are used.

Figure 14 shows the time development of the electric field energy for drift waves ($n = \pm 1$) and convective cells ($n = 0$). The growth rate of the drift modes are only slightly reduced from the shearless case and overall characteristics are very much the same as the shearless case. After saturating at about the same amplitude, the amplitude of drift waves tend to decrease while the amplitude of convective cells stay large until the density is completely flattened. The convective cells grow at about the same growth rate but the onset of the growth is substantially delayed. This was observed in the shearless case and is one of the evidence of nonlinear excitation due to drift instabilities. Note that the largest amplitude convective cells ($m = 1$) attains a very large amplitude. The measurements of frequency spectrum indicate the presence of drift and convective modes similar to Fig. 6.

Figure 15 shows the mode structure of a drift wave (m, n) = (4,1) and convective cells (1,0) for different times. Clearly no localization is observable for both modes and their behaviors are essentially the same to the shearless case. The convective cells reach much larger amplitude than the drift mode and extend over the entire plasma radius in spite of the presence of shear. It will be shown in Sec. IV that the convective cells may be excited regardless of the strength of shear as long as drift waves are unstable. This is so because of the beating of two drift waves is responsible for the nonlinear excitation of convective cells.

As far as the particle diffusion is concerned, the observed diffusion is just about the same to the shearless case and the initial density gradient is completely wiped out at about the same time as shown in Fig. 11. We therefore conclude the effects of shear is essentially negligible for the present case (Case 2).

Then the shear is raised to $L_s/L_n = 25$ with $q_0 = 1$, $\lambda = 3$, $L_z = 1280$, $\Omega_e/\omega_{pe} = 10$, $\alpha = 2$ and $m_i/m_e = 400$ for this case (Case 3). Note that the mass ratio is increased and the density gradient is decreased. The shear length for this case is about the same to many laboratory experiments in toroidal confinement devices. We observe the drift instabilities become less violent, however, the generated turbulence is still strong enough to eventually flatten the initial density gradient completely.

Figure 16 indicates the evolution of plasma diffusion with time. While the time scales are expanded compared with the previous cases, the initial density is completely wiped out by $t \approx 4000 \omega_{pe}^{-1}$. Note again the ions diffuse faster relative to the electrons so that the plasma column charges up negative at the center as seen in all the other cases. Diffusion coefficient for this case is only two times smaller than the previous cases.

Mode structure at different times is shown in Fig. 17 for $(m,n) = (1,0)$ (a) and $(1,1)$ (b). It is clear that mode $(1,0)$ is much larger than $(1,1)$ mode. Note $(1,0)$ mode consists of both drift and convective modes as shown later since $k_{||}$ is no longer zero because of strong shear. Indeed, using $k_{||} = k_{\theta} B_p/B$, for $n = 0$,

$$\frac{\omega^*}{k_{||} v_e} = \frac{\alpha}{\pi} \left(\frac{\rho_i}{a} \right)^2 \left(\frac{T_e}{T_i} \right) \frac{\Omega_i L_z}{v_e} q(r)$$

which is smaller than 1 for $r/a \leq 1$ so that the drift instability may develop for $n = 0$ modes.

Figure 18 shows the frequency spectrum for the mode (1,0). In addition to a peak at $\omega \approx \omega_e^*$, one more additional peak may be seen near $\omega \approx 0$ and we believe it is the convective cells. Another indirect evidence for the generation of large scale convective cells is the particle diffusion shown in Fig. 16. We have seen previously, in the absence of shear, that even very strong drift instabilities do not wipe out the density gradient completely unless the convective cells are generated (Fig. 13). The fact that the density gradient is wiped out for the present case gives an additional support for the generation of convective cells.

Time evolutions of several m modes with $n = 0$ are shown in Fig. 19. All the modes shown have about the same growth rate until saturation due to the flattening of the initial density gradient. Note the (1,0) mode attains the largest amplitude near saturation which causes large scale convective diffusion. Note that there is no rational surface present for these modes. There is no contradiction here, however, because the existence of drift instabilities requires a finite $k_{||}$ such that $v_i \ll \omega/k_{||} \lesssim v_e$ and the localization width is estimated to be $x_T/\rho_s \approx (L_s/L_n)^{1/2} = 5$ which is a sizable fraction compared with the plasma radius for this case. The ion resonance point at which $\omega/k_{||} \approx v_i$ is

is satisfied lies outside the plasma radius and one may wonder the conventional shear stabilization criterion may not be applicable here. Let us note, however, that the time required for a convective wave packet to reach the wall and reflect back to the central region will be estimated by [30]

$$t = 2 \int_0^a \frac{dx}{|v_g|} = \frac{\sigma a^2}{4\omega} \approx 2 \times 10^4 \omega_{pe}^{-1}$$

for $k_\theta \rho_i \approx 0.5$ which is much longer than the total simulation time and therefore the reflection of wave packet from the wall may be neglected.

Formation of large scale convective cells can be clearly seen in Fig. 20 where the orbits of the test electrons (a-c) and ions (d-f) are plotted. The way ions diffuse is not much different from the shearless case as shown in Fig. 12 since the ion motion along the field lines is slow. Electrons diffuse more or less the same manner as the ions but they tend to spread out more rapidly because of the rapid motion along the field lines.

Finally the strength of shear is raised to $L_s/L_n = 10$ ($L_z = 640$) (Case 4) with the other parameters the same to Case 3. Note the strength of shear for this case is close to the marginal stability criterion and the drift instabilities are found indeed very weak.

Figure 21 shows particle diffusion for this case. In contrast to all the previous cases, only small diffusion takes place and the bulk of the initial density profile is maintained as the

instabilities saturate at very small amplitude. The density gradient in a steady state is about 15% less than the initial gradient and it appears that the drift instabilities reached marginal stability due to the quasilinear diffusion in density. Investigation of the frequency spectrum reveals that the drift waves remain at small amplitude and no convective cells are found. This is consistent with the density profile in steady state since the convective cells, if generated, would have caused much larger diffusion even at small amplitude.

The measured overall particle diffusion coefficient is plotted with respect to L_s/L_n in Fig. 22. When the shear is weak, $L_s/L_n \gtrsim 50$, diffusion is very large and its numerical value is close to the convective cell diffusion. For $L_s/L_n = 25$, typical for laboratory experiments, diffusion is reduced by a factor of two. A very strong shear reduced the diffusion by a factor of ten for the initial unstable phase and when the drift instabilities saturate by quasilinear diffusion in real space, anomalous particle diffusion stops completely. This happens at $L_s/L_n \approx 8$ close to the theoretical stability criterion given by Eq. (3). It appears these simulation results are in reasonable agreement with the recent measurements in toroidal confinement experiments [15,16,17]. In particular, the presence of strong plasma turbulence and the broad frequency spectrum observed both in the simulations and experiments appear to confirm the possibility of anomalous particle diffusion due to drift instabilities and the convective cells.

IV. DISCUSSIONS AND THEORETICAL INTERPRETATIONS

We have seen, in the previous section, that the drift instabilities are very strong and do not stop until the density gradient is wiped out. In the presence of moderate strength of shear, anomalous diffusion is found to decrease, however, it does not stop until complete stabilization of drift instabilities are achieved. This requires a rather strong shear and it is likely that the plasmas in toroidal devices remain unstable in laboratory experiments. It was also found that, in addition to drift instabilities, convective cells are responsible for the observed particle diffusion. These convective cells are excited nonlinearly by the mode-coupling process to be discussed later [7].

In addition to the excitation of convective cells, several other nonlinear processes have been found which are responsible for the saturation of drift instabilities. Some of them are the quasilinear steepening of electron parallel velocity distribution which enhances the Landau damping, generation of electron temperature gradient in the direction of density gradient, and the particle diffusion in real space due to drift waves and convective cells. We now discuss the consequences of each nonlinear process.

A. Velocity Space Steepening

Let us first discuss the effect of the steepening of the electron distribution although this is not the dominant mechanism responsible for the saturation for the case where the drift

instabilities are far away from the marginal stability. This effect has been considered analytically [4] and was also observed previously in the 2-1/2D simulations [30]. Suppose the steepening of electron distribution leads the system to marginal stability. The growth rate of a drift wave near marginal stability is given by [4],

$$\frac{\gamma}{\omega} = - \frac{\pi}{2|k_{\parallel}|} \left(\frac{T_e}{m_e} \right) \left(k_{\parallel} \frac{\partial}{\partial v_{\parallel}} + \frac{k_y}{\Omega_i} \frac{\partial}{\partial x} \right) f_e(v_{\parallel}, x) \quad (8)$$

where the first term on the right hand side is the stabilizing Landau damping and the second term is the destabilizing density gradient term. For strongly unstable drift instabilities studied here $[\omega/(k_{\parallel} v_e) \approx 1]$, the second term is several times larger than the first term. This means that in order to achieve the stability by steepening the distribution alone, the slope of the electron distribution must increase by a factor of four or five at $\omega/k_{\parallel} v_e \sim 1$. We see, however, the steepening is small, less than a factor of 2, as seen in Fig. 9 and stability is in fact achieved by reducing the second term of Eq. (8), namely, by anomalous particle diffusion. This is because the fluctuation level would be too high if the steepening took place all the way near $\omega/k_z \sim v_e$ without particle diffusion. Note, however, the steepening of the velocity distribution for low energy electrons, $|v_{\parallel}| \ll v_e$ is very large as shown in Fig. 9 and it is likely that the weakly unstable drift waves with $|\omega/k_{\parallel} v_e| \ll 1$, may be stabilized by this process.

B. Plasma Heating and Generation of Temperature Gradient

While it is generally believed that the low-frequency drift instabilities ($\omega \ll \Omega_i$) cause anomalous particle diffusion, we have also seen that considerable temperature diffusion and heating take place as shown in Fig. 10. The initial uniform temperatures of ions and electrons become nonuniform as a result of drift instabilities. Let us emphasize here that temperature includes both the particle random thermal motion and coherent motion associated with the waves. In the presence of strong turbulence, we believe the distinction between random and coherent motions is not clear. Furthermore, the coherent motion is eventually randomized by either classical or anomalous collision processes giving rise to true temperature. Electron parallel temperature $T_{e\parallel}(r)$ decreases near the plasma edge and increases near the center. Ion perpendicular temperature increases more or less uniformly across the plasma radius. Ion parallel temperature does not change much since the ion-Landau damping is weak.

Electron temperature $T_{e\parallel}(r)$ decreases at $r/a > 0.5$ as the drift waves become large amplitude for the shearless case. By the time of saturation, $t \lesssim 1600\omega_{pe}^{-1}$, the initial flat temperature has undergone a drastic change and has a strong gradient. This change is caused partly by the instability itself which extracts electron thermal energy and converts it to the wave energy. This process is dominant for early times ($t < 800\omega_{pe}^{-1}$) when there is no significant particle diffusion. Electron cooling at $r/a > 0.5$

is believed due to this process. The reason that the electrons in the outer region loose more energy is simply because there are less number of electrons there so that each electron looses more energy to support the wave growth.

When the instabilities fully develop at later times, heating of electrons for $r/a \lesssim 0.5$ becomes significant in addition to the further cooling for $r/a \gtrsim 0.5$ creating a strong temperature gradient. This heating or cooling is only superficial and happened as a result of particle diffusion due to drift instabilities. Since the electron diffusion is primarily resonant, it is much larger for the low-energy electrons, $|v_{\parallel}| \lesssim \omega/k_{\parallel}$, compared with that for high energy electrons, $|v_{\parallel}| \gtrsim \omega/k_{\parallel}$, so that the low-energy particles diffuse out preferentially through the $c\tilde{E} \times \tilde{B}/B^2$ drift. This would result in the increase in electron temperature for $r/a \lesssim 0.5$ and decrease for $r/a \gtrsim 0.5$. The reason why the high-energy electrons diffuse less is because they are primarily adiabatic and the Doppler-shifted frequency for each particle $\omega' = |\omega - k_{\parallel} v_{\parallel}| \approx |k_{\parallel} v_{\parallel}| \gg |\omega|$ and therefore the excursion associated with $c\tilde{E} \times \tilde{B}/B^2$ drift is small resulting in a relatively small diffusion.

To confirm the strong dependence of electron diffusion on particle energy, velocity distributions are measured locally. Figure 23 shows the electron velocity distributions for electrons located at $r/a \lesssim 0.5$ (a) and at $r/a \gtrsim 0.5$ (b) for the shearless case (Case 1). It is clearly seen that for the region of $r/a \lesssim 0.5$, low-energy electrons are lost and they diffuse to the region of

$r/a \gtrsim 0.5$, increasing the distribution for low-energy electrons. It is also clear that the change of the populations of high-energy electrons is much smaller.

Our conclusion drawn from these observations is that the energy diffusion is smaller than the particle diffusion as far as the collisionless drift instabilities are concerned. In fact the change of nT_e is considerably smaller than the change of n or T_e itself.

The heating of the ion perpendicular kinetic energy is associated with the increase of the wave energy which accompanies the ion kinetic energy associated with the $c\tilde{E} \times \tilde{B}/B^2$ drift. The wave energy, W_k , per k mode may be estimated using a small signal energy theorem [31].

$$W_k = \frac{|E_k|^2}{8\pi} \omega \frac{\partial \epsilon}{\partial \omega} = \frac{|E_k|^2}{8\pi} \left(1 + \frac{1}{k^2 \lambda_e^2} + \frac{1-\Gamma}{k^2 \lambda_i^2} \right) \quad (9)$$

where ϵ is the dielectric function for the drift waves given by

$$\epsilon = 1 + \frac{1}{k^2 \lambda_e^2} + \frac{1-\Gamma}{k^2 \lambda_i^2} - \frac{\Gamma}{k^2 \lambda_e^2} \frac{\omega^*}{\omega}$$

In Eq. (9), the second and the third terms of the right hand side is the electron shielding energy associated with the parallel motion and the ion $c\tilde{E} \times \tilde{B}/B^2$ perpendicular kinetic energy respectively, both of them larger than the electric field energy. As the electric field energy grows due to drift instabilities, kinetic energy of ions also grows in agreement with

Fig. 10. The increase of ion kinetic energy is found to be $\sum_k (|E_k|^2/8\pi) (1 - \Gamma)/k^2 \lambda_i^2 \approx \omega_{pi}^2/\Omega_i^2 \sum_k |E_k|^2/8\pi$ for $b < 1$. Using the values of $|E_k|^2$ found in the simulations including the drift waves, convective cells and ambipolar electric field ($m = n = 0$), the ion kinetic energy is found to increase by a factor of two in good agreement with the simulation result.

C. Particle Diffusion and Generation of Convective Cells

As we have seen previously, particle diffusion is due to drift instabilities at early times and at later stages, it is caused by nonlinearly excited convective cells except for the last case (Case 4) where $L_s/L_n = 10$. Let us now estimate the diffusion rate analytically and compare with the simulation results. Let us assume a quasilinear theory may be applied for the diffusion due to drift waves at the early phase of the instabilities. This assumption is reasonable since there are many unstable modes present and the mode-coupling process is not important yet because the amplitude of the growing fluctuations is still small. Later the mode coupling process becomes of vital importance, and large amplitude convective cells are excited.

Writing the electron density

$$n(\underline{r}, t) = n_{0e}(\underline{r}, t) + n_1(\underline{r}, t)$$

where $n_{0e}(\underline{r}, t)$ is the particle density varying slowly compared with the drift wave time scale and $n_1(\underline{r}, t)$ is the density fluctuations associated with the drift waves. The continuity

equation for the average electron density may be given by

$$\frac{\partial n_{0e}}{\partial t} + \frac{\partial}{\partial x} (n_1 v_{1x}) = 0$$

v_{1x} is the electron x component velocity associated with the drift fluctuations and is given by $v_{1x} = cE_y/B$. Writing

$$n_1(\tilde{r}, t) = \sum_{\mathbf{k}} \left[n_{1\mathbf{k}}(\mathbf{x}) \exp[i(\omega t - k_y y - k_z z)] + \text{c.c.} \right]$$

and

$$\frac{n_{1\mathbf{k}}(\mathbf{x})}{n_{0e}} = \frac{e\phi}{T_e} \left[1 - \frac{\omega - \omega^*}{k_{\parallel} v_e} Z \left(\frac{\omega}{k_{\parallel} v_e} \right) \right],$$

we find, near marginal stability $\gamma_k/\omega_k \ll 1$,

$$\frac{\partial n_{0e}}{\partial t} = \left(\frac{e}{m_e} \right)^2 \sum_{\mathbf{k}} \frac{k_y}{\Omega_e} \frac{\partial}{\partial x} |\phi_{\mathbf{k}}|^2 \frac{\gamma_k (2 - \Gamma)}{v_e^2 \omega_k} n_{0e} \quad (10)$$

where Eq. (3) is used for γ_k and ω_k . Similarly, the equation for the ions will be

$$\frac{\partial n_{0i}}{\partial t} = \left(\frac{e}{m_i} \right)^2 \sum_{\mathbf{k}} \left(\frac{k_y}{\Omega_i} \right)^2 \frac{\partial}{\partial x} |\phi_{\mathbf{k}}|^2 \frac{\gamma_k \Gamma}{\omega_k^2} \frac{\partial n_{0i}}{\partial x}. \quad (11)$$

It is well known that the diffusion coefficients of ions and electrons are the same for this case [4]. The diffusion coefficient may be written as, from Eq. (11),

$$D_{\perp} = \left(\frac{e}{m_i} \right)^2 \sum_{\vec{k}} \left(\frac{k_y}{\Omega_i} \right)^2 |\phi_{\vec{k}}|^2 \frac{\gamma_{\vec{k}} \Gamma}{\omega_{\vec{k}}^2} = \sum_{\vec{k}} \frac{c^2 |E_{ky}|^2}{B^2} \frac{\gamma_{\vec{k}} \Gamma}{\omega_{\vec{k}}^2} .$$

One can estimate the diffusion coefficient using $|E_{ky}|^2$ from Fig. 3 to find $D_{\perp} \approx 10^{-2} \rho_i^2 \omega_{pe}$ at $\omega_{pe} t \approx 800$ which is close to but smaller than the observed value by a factor of 2 or 3. We believe the reason for this is the convective cells are already excited and able to cause a comparable diffusion even though their amplitudes are relatively small at this stage.

The nonlinear excitation of convective cells due to drift instabilities has been considered earlier [7]. Since the dispersion relation of drift instabilities is highly dispersive at $k_{\perp} \rho_s \approx 1$ as seen in Fig. 2, a resonant coupling takes place between two drift modes with the same frequency but different $k_{\perp}$ such as mode 1 and 2 in Fig. 2 so that $\omega_1 - \omega_2 \approx 0$. The importance of this process is that the drift instabilities have the maximum growth rate at $k_{\perp} \rho_s \approx 1$ and the mode coupling among drift waves is rather weak due to the presence of strong dispersion.

Writing the density perturbations and potentials associated with the drift waves as

$$n_i(\vec{x}, t) = \sum_{\vec{k}} (n_{\vec{k}}^d \exp[i\omega t + i\vec{k} \cdot \vec{x}] + c.c.)$$

and

$$\phi(\vec{x}, t) = \sum_{\vec{k}} [\phi_{\vec{k}}^d \exp(i\omega t + i\vec{k} \cdot \vec{x}) + c.c.]$$

and similar expressions for the convective cells, we find from the ion continuity equation [31],

$$\begin{aligned} \frac{\partial n_i^c}{\partial t} + i\omega^* \frac{en_0}{T_e} \phi^c + k_{1s}^2 \frac{en_0}{T_e} \frac{\partial \phi^c}{\partial t} + \frac{n_0 c^2}{2\Omega_i B^2} \sum_{\tilde{k}_1 = \tilde{k}_1 + \tilde{k}_2} (k_{x_2} k_{y_1} - k_{x_1} k_{y_2}) \\ (k_1^2 - k_2^2) \phi_{k_1}^d \phi_{k_2}^{d*} + \frac{c}{2B} \sum_{\tilde{k}_1 = \tilde{k}_1 + \tilde{k}_2} (k_{x_1} k_{y_2} - k_{x_2} k_{y_1}) \phi_{k_1}^d \eta_{ik_2}^{d*} = 0 \end{aligned} \quad (12)$$

where the second and third terms represent nonlinear polarization drift $\nabla \cdot (n_0 \tilde{v}_2)$ and $c \tilde{E} \times \tilde{B} / B^2$ drift $\nabla \cdot (n_1 \tilde{v}_1)$. Here,

$$\tilde{v}_1 = \tilde{v}_E + \tilde{v}_p,$$

$$\tilde{v}_2 = \frac{c (\tilde{v}_E \cdot \nabla) \tilde{E}_\perp}{\Omega_i B},$$

$$\tilde{v}_E = \frac{c \tilde{E} \times \tilde{B}}{B^2},$$

and

$$\tilde{v}_p = \frac{c}{B\Omega_i} \frac{\partial \tilde{E}_\perp}{\partial t}.$$

Here ions are assumed cold for simplicity and hence $\omega_i^* = 0$.

As for the electrons, both the linear and nonlinear polarization drifts are very small in comparison with $\tilde{v}_E$. Then, using Poisson's equation for the convective cells, we find, for

$$\phi_{k_1}^c$$

$$\begin{aligned} \frac{\partial \phi_{k_1}^c}{\partial t} + \frac{c}{B} \frac{1}{k_1^2} \sum_{\substack{k_1 = k_1 + k_2 \\ \sim 1, \sim 2}} (k_{x_2} k_{y_1} - k_{x_1} k_{y_2}) (k_1^2 - k_2^2) \phi_{k_1}^d \phi_{k_2}^{d*} \\ + \frac{\Omega_i^2}{\omega_{pi}^2} \frac{c}{B k_1^2} \sum_{\substack{k_1 = k_1 + k_2 \\ \sim 1, \sim 2}} k_2^2 (k_{x_1} k_{y_2} - k_{x_2} k_{y_1}) \phi_{k_1}^d \phi_{k_2}^{d*} = -k_1^2 D_1 \phi_{k_1}^c \quad (13) \end{aligned}$$

where $\omega_{pi}^2 / \Omega_i^2 \gg 1$ is assumed. We note the second term due to nonlinear polarization drift, $\nabla \cdot (n_0 \mathbf{v}_2)$, is much greater than the third term $\nabla \cdot (n_1 \mathbf{v}_1)$. The right hand side is the viscous damping of the convective cells [13] and is added here. The coupling coefficient in the second term is large and scales as k^2 so that the excitation of convective cells becomes important even for small amplitude drift waves.

When the drift waves are still in the linearly unstable stage, one can solve Eq. (13) for the amplitude of the convective cells to find, for $\omega_{pi}^2 / \Omega_i^2 \gg 1$,

$$\begin{aligned} \phi_{k_1}^c(t) = \left[\phi_{k_1}^c(0) + \sum_{\substack{k_1, k_2 \\ \sim 1, \sim 2}} \frac{M_{12} |\phi_{k_1}^d(0) \phi_{k_2}^{d*}(0)|}{\gamma_{k_1} + \gamma_{k_2} + k_1^2 D_1} \right] \exp(-k_1^2 D_1 t) \\ - \sum_{\substack{k_1, k_2 \\ \sim 1, \sim 2}} \frac{M_{12} |\phi_{k_1}^d(0) \phi_{k_2}^{d*}(0)|}{\gamma_{k_1} + \gamma_{k_2} + k_1^2 D_1} \exp[(\gamma_1 + \gamma_2)t] \quad (14) \end{aligned}$$

where M_{12} is the coupling coefficient defined by

$$M_{12} = \frac{c}{B} \frac{(k_{x_2} k_{y_1} - k_{x_1} k_{y_2}) (k_1^2 - k_2^2)}{k_1^2}.$$

Equation (4) has two important characteristics. First of all, it predicts that the convective cells always grow regardless of linear damping mechanism as long as the drift instabilities are present. There is no threshold for the growth. The convective cells grow as $\exp(\gamma_1 + \gamma_2)t$ as a result of the beating of two unstable drift waves. The growth rate predicted here is close to the observations given in Sec. III. Secondly, since the coupling coefficient M_{12} is large, the nonlinear growth is important even for small amplitude drift waves. While the convective cells may be weakly unstable linearly [26,32], nonlinear excitation due to drift instabilities appears more important because of its large growth rate and coupling coefficient. Also the drift waves are more easily subject to instabilities and their localization is much wider suggesting the large amount of free energy available for the growth of the instabilities.

Let us emphasize here, however, our derivation of the mode-coupling processes is based on the fluid equations which do not include the finite gyroradius correction. This correction reduces the mode-coupling coefficient M_{12} to some extent for $k_\perp \rho_s \geq 1$ [33]. The reduction is not serious for $k_\perp \rho_s < 1$.

Let us now estimate the saturation amplitudes of the drift waves and convective cells when the shear is weak or absent. It is reasonable to assume that nonlinear saturation will take place when the amplitude of convective cells becomes comparable to that of drift waves. At that stage, the $c\tilde{E} \times \tilde{B}/B^2$ motion of

particles due to convective cells will destroy the coherent response associated with drift waves and the instability will saturate. This is certainly what has been observed in the numerical simulations shown in Figs. 3,7 and 14. Equating Eq. (14) to the drift waves, $|\phi_k^d(t)| = |\phi_k^d(0)|e^{\gamma t}$, we find the saturation amplitude to be

$$|\phi_k^d(t)| \approx |\phi_k^c(t)| \approx \frac{\gamma_k}{M_{12}} \approx \frac{\gamma_k}{\omega_k} \frac{\rho_i}{L_n} \frac{T_e}{e}$$

or

$$|e\phi_k^d(t)/T_e| \approx |e\phi_k^c(t)/T_e| \approx (\gamma_k/\omega_k) (\rho_i/L_n) \quad (15)$$

which gives $|e\phi_k^d(t)/T_e| \approx 12\%$ at saturation for the Case 1 and 2 and is close to the value of the simulations, $|e\phi_k^d(t)/T_e| \approx 10\%$. Equation (15) gives good agreement with the measurements for Q-devices [19] as well as large toroidal devices [15,16,17].

Particle diffusion due to convective cells may be estimated using Eq. (15).

$$D_\perp = \sum_k \frac{c^2 E_{k\perp}^2}{B^2} \frac{1}{k_\perp^2 D_\perp} \approx \left(\frac{cT_e}{eB} \right)^2 \frac{\rho_i^2}{L_n^2} \sum_k \left(\frac{\gamma_k}{\omega_k} \right)^2 \frac{1}{D_\perp}$$

or

$$D_\perp \approx \frac{cT_e}{eB} \frac{\rho_i}{L_n} \left[\sum_k \left(\frac{\gamma_k}{\omega_k} \right)^2 \right]^{1/2} \approx \frac{cT_e}{eB} \frac{\rho_i}{L_n} \quad (16)$$

Equation (16) gives the well-known Bohm diffusion scaling

$D_\perp \sim cT_e/eB$ with the numerical coefficient $\sim \rho_i/L_n$ instead of

1/16 [34]. Note the diffusion coefficient, Eq. (16), is estimated to be $D_{\perp} \sim 2 \times 10^{-2} \rho_i^2 \omega_{pe}$ for the shearless case (Case 1) in good agreement with the simulation result. For modern toroidal confinement devices such as tokamaks $\rho_i/L_n = 10^{-2} \sim 10^{-3}$ so that the particle diffusion will be estimated $10^{-1} \sim 10^{-2}$ of the Bohm diffusion and is not too far from the experimental observations. Since the diffusion given by Eq. (16) scales as $B^{-2} L_n^{-1}$, the confinement should improve as the device becomes larger with stronger field. This is encouraging for construction of larger devices for fusion since the particle confinement time is estimated from Eq. (16) to be a few second for $T_e = 10$ keV, $B = 40$ kG and the plasma radius ~ 2 m.

When the shear is very strong (Case 4), it was found that the drift instabilities are stabilized by the quasilinear diffusion in real space and therefore the convective cells saturate at very small amplitude. Note the damping rate due to shear, $\langle k_{\parallel} v_e \rangle$, must replace the viscosity damping $k^2 D_{\perp}$ in Eq. (14). Shear damping does not inhibit nonlinear growth of the convective cells, however, so long as drift waves are unstable. In this case the flattening of the density gradient due to the quasilinear diffusion lead the system to stability and it appears one can apply the standard quasilinear theory for estimating particle diffusion in this case as shown in Eqs. (10) and (11).

V. CONCLUDING REMARKS

We have seen the nonlinear processes and the resultant anomalous diffusion of particles and energy associated with the collisionless drift instabilities are quite versatile and complex in that many different physical processes take place simultaneously. It is of great help therefore to have simulation results available for searching the dominant processes and constructing nonlinear theories. We believe the simulation results presented here are rather close to small scale real laboratory experiments and the physical processes observed in the numerical simulations must be going on in large confinement experiments as well.

As far as the physical processes are concerned, quasilinear diffusion in both real and velocity space, nonlinear mode-coupling of drift waves to excite convective cells and the particle diffusion due to convective cells are the prominent nonlinear processes. While we found the particle diffusion is large, electron heat diffusion is smaller than the particle diffusion since the drift waves carry low-energy electrons preferentially and high-energy electrons are confined. This process tends to generate electron temperature gradient in the direction of density gradient.

When the simulation results are compared with laboratory experiments, both show the presence of strong plasma turbulence and hence broad spectrum in the drift wave frequency range. This broadening takes place due to the presence of strong mode-coupling

of drift waves which generate convective cells. On the other hand, laboratory experiments show that the electron heat diffusion is always larger than the particle diffusion considerably while the simulations show the opposite. Since the simulations can be explained reasonably well by the analytic theory, we believe the anomaly of electron heat diffusion may be caused by some processes not involved in the present simulations. Such possibilities are the braiding of the magnetic surface [35] and the trapped particle turbulence [2]. These possibilities are now under investigations using three dimensional toroidal code [13] and the magnetostatic simulation code [20].

Several analytical estimates are given on the particle diffusion and heat transfer based on the mode-coupling and quasi-linear theories. A unified analytic treatment of the physical processes discussed here will be carried out to construct nonlinear theories and anomalous transport associated with the collisionless drift instabilities [33].

ACKNOWLEDGMENT

The authors would like to thank Dr. P. Guzdar for useful discussions.

This work is supported by the U. S. Department of Energy Contract No. EY-76-C-02-3073.

REFERENCES

- [1] KADOMTSEV, B. B., Plasma Turbulence, (Academic, New York, 1965), p. 79.
- [2] KADOMTSEV, B. B., POGUTSE, O. P., Nuclear Fusion, 11 (1971) 67.
- [3] TANG, W. M., Nucl. Fusion (to be published).
- [4] SAGDEEV, R. Z., GALEEV, A. A., Nonlinear Plasma Theory, (Benjamin, New York, 1969), pgs. 73 & 103.
- [5] HORTON, C. W., Phys. Rev. Lett. 37 (1976) 1269.
- [6] HASEGAWA, A., MIMA, K., Phys. Rev. Lett. 39 (1977) 205.
- [7] OKUDA, H., CHENG, C. Z., Phys. Rev. Lett., (submitted).
- [8] DUPREE, T. H., Phys. Fluids 11 (1968) 2680; 10 (1967) 1049; and 9 (1966) 1773.
- [9] LEE, W. W., OKUDA, H., Phys. Rev. Lett. 36 (1976) 870.
- [10] MATSUDA, Y., OKUDA, H., Phys. Rev. Lett. 36 (1976) 303.
- [11] CHENG, C. Z., OKUDA, H., Phys. Rev. Lett. 38 (1977) 708.
- [12] LEE, W. W., OKUDA, H., J. Compt. Phys. (to be published).
- [13] CHENG, C. Z., OKUDA, H., J. Compt. Phys. (to be published).
- [14] OKUDA, H., DAWSON, J. M., Phys. Fluids 16 (1973) 408.
- [15] MAZZUCATO, E., Phys. Rev. Lett. 36 (1976) 792.
- [16] SURKO, C. M., SLUSHER, R. E., Phys. Rev. Lett. 37 (1976) 1747.
- [17] OKABAYASH, M., ARUNASALAM, V., Nucl. Fusion 17 (1977) 497.
- [18] FURTH, H. P., RUTHERFORD, P. H., SELBERG, H., Phys. Fluids 16 (1973) 1054.
- [19] HENDEL, H. W., CHU, T. K., PALITZER, P. A., Phys. Fluids 11

- (1968) 2426; also PALITZER, P. A., Phys. Fluids 14 (1971) 2410.
- [20] LEE, W. W., OKUDA, H., J. Compt. Phys. (to be published).
- [21] FRIED, B. D., CONTE, S. D., The Plasma Dispersion Function, (Academic, New York, 1961).
- [22] PEARLSTEIN, D. L., BERK, H. L., Phys. Rev. Lett. 23 (1969) 220.
- [23] REWOLDT, G., TANG, W. M., FRIEMAN, E. A., Phys. Fluids 20 (1977) 402.
- [24] TAYLOR, J. B., in Plasma Physics and Controlled Nuclear Fusion Research (International Atomic Energy Agency, Vienna, 1977) Proc. 6th Int. Conf., Berchtesgaden, FRG, 1976 Paper CN-35/D10.
- [25] TRUE, M. A., LEE, W. W., OKUDA, H., Princeton Plasma Physics Laboratory Report, Matt-1270 (1977).
- [26] DAWSON, J. M., OKUDA, H., ROSEN, B., in Methods in Computational Physics, (Academic, New York, 1976), Vol. 16, 281.
- [27] TOMANO, T., PRATER, R., OHKAWA, T., Phys. Rev. Lett. 30 (1973) 431.
- [28] DRAKE, J. R., GREENWOOD, J. R., NAVRATIL, G. A., POST, R. S., Phys. Fluids 20 (1977) 148; NAVRATIL, G. A., POST, R. S., EHRHARDT, A. B., Phys. Fluids 20 (1977) 156.
- [29] OKUDA, H., CHU, C., DAWSON, J. M., Phys. Fluids 18 (1975) 243.
- [30] LEE, W. W., KUO, Y. Y., OKUDA, H., Phys. Fluids (to be published).

- [31] LIU, C. S., ROSENBLUTH, M. N., TANG, W. M., Phys. Fluids 19 (1976) 1040.
- [32] MAHAJAN, S., GUZDAR, P., KAW, P., OBERMAN, C., Princeton Plasma Physics Laboratory Rept. PPPL-1351 (1977).
- [33] GUZDAR, P., CHENG, C. Z., KAW, P., OKUDA, H., Bull. Am. Phys. Soc. 32 (1977) 1127.
- [34] BOHM, D., in The Characteristics of Electrical Discharge in Magnetic Fields, (McGraw-Hill, New York 1949).
Chapt. 2, Sec. 5.
- [35] STIX, T. H., Phys. Rev. Lett. 36 (1976) 521.

772047
Fig. 1. A sketch of the simulation model. The plasma is imbedded in an external magnetic field. The model is periodic in Z direction while the conducting walls are assumed in x , y directions.

772300

Fig. 2. Numerical solution of the local dispersion for collisionless drift instability for a plasma of $m_i/m_e = 1800$. Similar curves are found for $m_i/m_e = 100$ and 400 used for simulations.

Fig. 3. Time development of the several Fourier modes associated with the drift instabilities. Case 1.

772434

772445
 Fig. 4. Time development of the radial mode structure of the most unstable mode (m,n) = (5,1). Case 1.

Fig. 5. Fluctuation spectrum with respect to m (a) and n (b) for different time steps. Case 1.

772439

772438
 Fig. 5. Frequency spectrum of the drift wave and convective cells measured at $r/a = 0.5$ for $m = 1$ (a) and $m = 3$ (b) modes. Case 1.

Fig. 7. Evolution of the convective cells ($n = 0$) for different m modes. Case 1.

772435

Fig. 8. Radial mode structure of the largest convective cells and its time evolution. Case 1.

772436

772447
 Fig. 9. Steepening of the electron parallel velocity distribution as a result of drift instabilities. Case 1.

Fig. 10. Electron parallel $T_{e||}$, ion perpendicular $T_{i\perp}$ and parallel $T_{i||}$ temperatures. Case 1. 772446

Fig. 11. Particle diffusion due to drift wave and convective cells. Case 1.

772433

762383
 Fig. 12. Diffusion of test ions sampled initially to form a ring. Note the development of cell structures. Case 1.

762384
 Fig. 13. Particle diffusion in the absence of convective cells.
 Fig. 11 is also plotted for comparison.

772123

Fig. 14. Evolution of the electric field fluctuations associated with the drift and convective modes in the presence of weak shear, $L_s/L_n = 50$. Case 2.

772441
 Fig. 15. Radial mode structure of drift $(m,n) = (4,1)$ (a) and convective $(1,0)$ (b) modes with time. Case 2.

772431
 Fig. 16. Diffusion of ions and electrons for $L_s/L_n = 25$.
 Case 3.

Fig. 17. Radial mode structure for (1,0) and (1,1) mode. Case 3. 772440

Fig. 18. Frequency spectrum for the (1,0) mode. Note two peaks near $\omega = 0$. Case 3.

772444

Fig. 19. Evolution of several m modes with $n = 0$. Case 3. .772432

772448
 Fig. 20. Orbits of test electrons at $\omega_{pe} t = 1500$ (a), 2500 (b), and 3500 (c) and orbits of test ions (d) $\omega_{pe} t = 1500$ (e), 2500 (f), and 3500 (g). Case 3.

772442
 Fig. 21. Evolution of density profile for the strongest shear.
 Case 4.

772443
 Fig. 22. Summary of the measured particle diffusion coefficient with respect to L_s/L_n . Note the diffusion for $L_s/L_n = 10$ drops essentially to zero after the instability saturates at $L_s/L_n = 8$.

772437
Fig. 23. Time evolution of the electron velocity distribution for $r/a \lesssim 0.5$ (a) and $r/a \gtrsim 0.5$ (b). Case 1.