Frequency Response Analysis of Fluid
Control Systems for Parabolic Trough Solar Collectors

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ABSTRACT

Previous studies of solar collector fluid control systems have utilized computer simulations of collector and piping dynamics to evaluate stability and response characteristics. To obtain reasonable simulation accuracy requires substantial computer memory and time, and is well beyond the capability of small desk-top computers. This paper derives a linearized steady state frequency response for parabolic trough collectors and for connecting piping, which can be used in standard gain-phase analyses to evaluate system stability and closed loop frequency response. The frequency response characteristics of a typical collector string and piping are used in a gain-phase analysis to get some insight into the effect on system stability of various system parameters such as controller gain, sensor and controller time constants, and sensor location.

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I. Introduction

Control of the heat transfer fluid in a parabolic trough solar collector field is necessary to prevent overheating and to maintain relatively constant output temperatures for varying solar input conditions. Tight fluid temperature control is particularly important in applications where electric power is being generated, since it is desirable to operate at as high a temperature as possible and with small variability in order to maximize heat engine performance. The control approach generally consists of measuring collector output temperature and then varying the fluid flow rate through the collectors in accordance with some control algorithm.

This fluid control problem has been addressed previously (Refs. 1, 2) by computer simulation studies which required substantial computer memory and time capability to simulate the temperature dynamics of the collectors and interconnecting piping. This rather cumbersome approach was necessary due to the lack of a more compact representation of the collector and piping dynamics. Conventional control analysis techniques typically use transfer function, frequency response, or state equation representations of the devices being controlled. The objective of this study was to derive frequency response characteristics of the collectors and piping which could be used in standard gain-phase analyses to evaluate system stability and closed loop frequency response.

The general approach was to linearize the system by considering only small variations from some operating point. General equations are derived for collector and piping frequency responses as a function of the operating point and the physical characteristics of the collectors and piping. Finally, frequency response characteristics of a typical collector system are derived and utilized to analyze fluid control system performance.

II. System Configuration

A typical collector field and the associated fluid control system is shown in Figure 1.

Fig. 1 System Configuration

The collector field has collectors arranged in parallel rows which are connected to a thermal storage tank through main supply and return manifolds. The centrifugal pump provides a relatively constant head across the collector field, and the flow through each row is modulated by a control valve operating off the output temperature of the row. Individual row controls are used rather than a single flow control on the entire field to accommodate differences in collector performance and partial shading of the field. The use of individual row controls also results in a faster responding and more accurate system since the temperature sensors can be placed right at the output of the collectors rather than at the end of the manifold.

The three-way valve at the output of the field is necessary to prevent pumping cold fluid into the storage tank during startup or during cloud-cover transients. This valve causes the field output to be recirculated to the field input for temperatures below some set point, which is typically about 50° below the desired operating temperature. The blending tank prevents sending hot fluid directly to the collector input. When the system output temperature approaches the normal operating temperature, the three-way valve connects the field output to the storage tank, and temperature control from this point on is determined completely by the individual row controls.

The length of the collector rows is determined by a design tradeoff between pumping power required and thermal efficiency.

(Ref. 3), and is a function of the temperature rise desired. A temperature rise of 160°F was considered for this study which resulted in an optimum row length of about 320 ft.

When the system is at operating temperature, the fluid returned to the collector field from the bottom of the thermal storage tank is essentially at a constant temperature. Under these conditions, the fluid temperatures in each row are independent of each other, and the analysis of one row will be representative of all the other rows. Figure 2 is a flow diagram of the temperature control loop to be studied in the remainder of this paper.

Fig. 2 Temperature Control Loop

In Figure 2, T_D is the desired fluid temperature, T_C is the temperature out of the collector, T_S is the fluid

temperature at the sensor location, and T_M is the temperature as measured by the sensor. The sensor is normally located at some distance downstream from the collector output, and therefore, the response characteristics of the pipe between the collector and sensor must be considered. A major portion of this paper involves the derivation of frequency response characteristics of the collector and pipe blocks in Figure 2 which can then be used to analyze the control loop.

The collector and piping responses were derived in general terms, but a specific collector and pipe configuration was used for examples. The collector considered was an advanced design trough presently being developed by Sandia National Laboratories in conjunction with industry (Ref. 4). These collectors have a two-meter aperture and a peak efficiency of about 60%. The expected steady state performance of these collectors is shown in Figure 3 and the receiver tube configuration is shown in Figure 4.

Fig.3 Steady State Collector Performance

Fig. 4 Receiver Cross-Section

One inch schedule 40 pipe was chosen for the section of pipe between the collector and sensor, and the cross-section of the pipe and insulation is shown in Figure 5.

Fig. 5 Pipe Cross-Section

An insulation thickness approximately equal to the pipe diameter was chosen which results in a heat loss coefficient of about .2 BTU/hr-ft-°F. The heat transfer fluid considered was Therminol 66 (Ref. 5) which has a maximum bulk temperature of 650°, and, therefore, an operating temperature of 600° was chosen.

III. Collector Response

An incremental length of collector receiver tube with the pertinent heat energy inputs and outputs is shown in Figure 6.

Fig. 6 Incremental Length of Receiver Tube

By applying heat balance relations to the receiver tube and fluid, equations (1) and (2) are obtained.

$$\frac{\partial T_F}{\partial t} \rho_F C_F A_F \Delta X = \pi D h_T (T_R - T_F) \Delta X - (T_{F_{X+\Delta X}} - T_{F_X}) Q(t) \rho_F C_F \quad (1)$$

$$\frac{\partial T_R}{\partial t} \rho_R C_R A_R \Delta X = I E_O W \Delta X - h_L \Delta X (T_R - T_A) - \pi D h_T (T_R - T_F) \quad (2)$$

where ρ_R , C_R , A_R , T_R = receiver tube density, specific heat, cross-sectional area, and temperature.

ρ_F , C_F , A_F , T_F = fluid density, specific heat, cross-sectional area and temperature.

T_A = ambient temperature.

I = solar insolation (component normal to aperture plane).

E_O = optical efficiency.

W = aperture width

h_L = heat loss coefficient = $W \times$ slope of efficiency curve (Fig. 3).

D = receiver tube diameter.

h_T = heat transfer coefficient.

$Q(t)$ = volume flow rate (time variable).

X = distance along the collector

t = time

In these equations, the functional dependence of T_R and T_F on the distance along the collector, X , and time, t , is implied. The dynamic effect of the thermal mass of the glass envelope has

been neglected in these equations since the temperature variation of the glass is quite small. The steady state effect of the glass is included in the thermal loss coefficient, h_L . It has been shown in Ref. 1 that this approximation has negligible effect on the dynamic response obtained.

Allowing the incremental length to approach zero,

$$\frac{T_{F, X + \Delta X} - T_{F, X}}{\Delta X} \longrightarrow \frac{\partial T_F}{\partial X},$$

and the partial differential equations (3) and (4), which define the receiver tube and fluid temperature dynamics are obtained.

$$\frac{Q(t)}{A_F} \frac{\partial T_F}{\partial X} + \frac{\partial T_F}{\partial t} = \frac{\pi D h_T}{\rho_F C_F A_F} (T_R - T_F) \quad (3)$$

$$\frac{\partial T_R}{\partial t} = \frac{I E_O W}{\rho_R C_R A_R} - \frac{h_L}{\rho_R C_R A_R} (T_R - T_A) - \frac{\pi D h_T}{\rho_R C_R A_R} (T_R - T_F) \quad (4)$$

To obtain the collector response required, it is necessary to solve these equations for the fluid temperature at $x = L$ (L = collector length) with a variable flow rate, $Q(t)$.

In solving these equations, it will be assumed that the fluid physical characteristics are constant over the range of temperatures considered. If values for the fluid parameters are chosen at the middle of the temperature operating range, the variations will generally be quite small. Since

turbulent flow in the receiver tube is normally a design requirement (Ref. 3), the heat transfer coefficient will be given by (5).

$$h_T = K Q^{.8} \quad (\text{Ref. 6}) \quad (5)$$

Since the flow rate is a variable, several of the terms in equations (3) and (4) have variable coefficients, and as a result these equations are not amenable to general solution.

Equations (3) and (4) can be converted to constant coefficient equations by considering only small variations from some nominal operating point. Substituting $T_R = T_{R_0} + \Delta T_R$, $T_F = T_{F_0} + \Delta T_F$, $Q = Q_0 + \Delta Q$ and $h_T = h_{T_0} + \Delta h_T$ into equations (3) and (4), and then subtracting out the steady state solution and ignoring second order terms, equations (6) and (7) are obtained. These partial differential equations define the temperature variations from nominal and have constant coefficients.

$$\begin{aligned} \frac{Q_0}{A_F} \frac{\partial \Delta T_F}{\partial X} + \frac{\partial \Delta T_F}{\partial t} - K_F (\Delta T_R - \Delta T_F) = \frac{K_F}{h_{T_0}} (T_{R_0} - T_{F_0}) \Delta h_T \\ - \frac{\partial T_{F_0}}{\partial X} \frac{\Delta Q}{A_R} \end{aligned} \quad (6)$$

$$\frac{\partial \Delta T_R}{\partial t} + K_L \Delta T_R + K_R (\Delta T_R - \Delta T_F) = - \frac{K_R}{h_{T_0}} (T_{R_0} - T_{F_0}) \Delta h \quad (7)$$

$$\text{where } K_F = \frac{\pi D h_{T_0}}{\rho_F C_F A_F}$$

$$K_R = \frac{\pi D h_{T_O}}{\rho_R C_R A_R}$$

$$K_L = \frac{h_L}{\rho_R C_R A_R}$$

Equalities (8), (9) and (10) can be used to simplify the temperature dynamic equations.

$$\frac{\partial T_{F_O}}{\partial X} = \frac{T_2 - T_1}{L} \quad (8)$$

where $T_2 - T_1$ = nominal collector temperature rise

L = collector length

$$\frac{\Delta h_T}{h_{T_O}} = \frac{.8 K Q_O^{-.2} \Delta Q}{K Q_O^{.8}} = \frac{.8 \Delta Q}{Q_O} \quad (9)$$

$$T_{R_O} - T_{F_O} = \frac{Q_O \rho_F C_F}{\pi D h_{T_O} L} (T_2 - T_1) \quad (10)$$

In equations (8) and (10), it was assumed that $\frac{\partial T_{F_O}}{\partial X}$ and $T_{R_O} - T_{F_O}$ are constant throughout the length of the receiver tube and are equal to their average values. Substituting (8), (9) and (10) into (6) and (7), the final form of the partial differential equations defining temperature dynamics are obtained.

$$\frac{Q_O}{A_F} \frac{\partial \Delta T_F}{\partial X} + \frac{\partial \Delta T_F}{\partial t} - K_F (\Delta T_R - \Delta T_F) = K_1 \frac{\Delta Q}{A_F} \quad (11)$$

$$\frac{\partial \Delta T_R}{\partial t} + K_L \Delta T_R + K_R (\Delta T_R - \Delta T_F) = 4 K_1 \frac{K_R}{K_F} \frac{\Delta Q}{A_F} \quad (12)$$

$$\text{where } K_1 = -.2 \frac{T_2 - T_1}{L}$$

The steady state temperature response to a unit sinusoidal variation in ΔQ was obtained by assuming the general sinusoidal solution given in equations (13) and (14).

$$\Delta T_F(t, x) = F_1(x) \sin \omega t + F_2(x) \cos \omega t \quad (13)$$

$$\Delta T_R(t, x) = F_3(x) \sin \omega t + F_4(x) \cos \omega t \quad (14)$$

These assumed solutions were substituted into equations (11) and (12) and then solved for $F_1(x)$ and $F_2(x)$. The details of the solution are presented in the Appendix and the resulting collector frequency response is given by equations (15) and (16).

$$\left| \frac{\Delta T_F(\omega)}{\Delta Q(\omega)} \right| = \sqrt{F_1(L)^2 + F_2(L)^2} \quad (15)$$

$$\text{Phase of } \frac{\Delta T_F(\omega)}{\Delta Q(\omega)} = \tan^{-1} \frac{F_2(L)}{F_1(L)} \quad (16)$$

$$\text{where } F_1(L) = \frac{ac + bd}{a^2 + b^2} \left(1 - e^{-aL} \cos bL \right) - \frac{ad - cb}{a^2 + b^2} e^{-aL} \sin bL$$

$$F_2(L) = \frac{ad - cb}{a^2 + b^2} \left(1 - e^{-aL} \cos bL \right) + \frac{ac + bd}{a^2 + b^2} e^{-aL} \sin bL$$

$$a = \frac{A_F}{Q_O} \left[K_F - \frac{K_F (K_L + K_R) K_R}{\omega^2 + (K_L + K_R)^2} \right]$$

$$b = \frac{A_F}{Q_O} \left[\omega + \frac{\omega K_F K_R}{\omega^2 + (K_L + K_R)^2} \right]$$

$$c = \frac{K_1}{Q_0} \left[1 + \frac{4 K_F (K_L + K_R)}{\omega^2 + (K_L + K_R)^2} \right]$$

$$d = \frac{K_1}{Q_0} \left[- \frac{4 K_F \omega}{\omega^2 + (K_L + K_R)^2} \right]$$

The gain and phase response of a 320 ft. collector with the characteristics described in Section II is shown in Figure 7.

Fig. 7 Collector Frequency Response

The phase angle plotted is the phase lag above the nominal 180° static phase reversal (flow increase produces temperature decrease). It can be seen that both the amplitude and phase responses have pronounced resonances, and are quite different from more conventional process frequency responses. Frequency responses are given for nominal flow rates of 2 and 10 gpm which correspond to solar inputs of 100 and 300 BTU/hr-ft² normal to the aperture plane. These are approximately the minimum and maximum expected insolation values, and a fluid control loop that operates satisfactorily under these two conditions will probably be satisfactory for all solar inputs.

IV. Pipe Response

The partial differential equations describing fluid temperature dynamics in a pipe are of the same form as for the collector (equations (3) and (4)) except that the solar input term is missing.

$$\frac{Q(t)}{A_F} \frac{\partial T_F}{\partial X} + \frac{\partial T_F}{\partial t} = \frac{\pi D h_T}{\rho_F C_F A_F} (T_P - T_F) \quad (17)$$

$$\frac{\partial T_P}{\partial t} = - \frac{h_L}{\rho_P C_P A_P} (T_P - T_A) - \frac{\pi D h_T}{\rho_P C_P A_P} (T_P - T_F) \quad (18)$$

Using the same approximations as for the collector derivation, the small variation equations (equivalent to equations (11) and (12)) for the pipe dynamics are obtained.

$$\frac{Q_o}{A_F} \frac{\partial \Delta T_F}{\partial X} + \frac{\partial \Delta T_F}{\partial t} - K_F (\Delta T_P - \Delta T_F) = 0 \quad (19)$$

$$\frac{\partial \Delta T_P}{\partial t} + K_L \Delta T_P + K_P (\Delta T_P - \Delta T_F) = 0 \quad (20)$$

In this case, the solution desired is the response of pipe output temperature to a unit sinusoidal change in input temperature. Details of the solution are presented in the Appendix and the resulting pipe frequency response is given by equations (21) and (22).

$$\left| \frac{\Delta T_{out}(\omega)}{\Delta T_{in}(\omega)} \right| = e^{-aL} \quad (21)$$

$$\text{Phase of } \frac{\Delta T_{out}(\omega)}{\Delta T_{in}(\omega)} = -bL \quad (22)$$

The parameters a and b in equations (21) and (22) are as defined for the collector response except that the physical characteristics of the pipe are used in place of those for the receiver tube.

The amplitude and phase responses of a one inch insulated pipe as described in Section II are plotted in Figure 8.

Fig. 8 One Inch Pipe Frequency Response

The amplitudes (in db) and phases shown are for a one foot length of pipe and must be multiplied by the length of pipe to obtain the complete response. Responses are shown for nominal flow rates of 2 and 10 gpm which are the expected extremes of flow rate.

As expected, the pipe phase response is similar to that of a pure transport delay, exhibiting a continuously increasing phase lag with increasing frequency. Unlike a

pure transport delay, the pipe response exhibits some attenuation which approaches an asymptotic limit as frequency increases. The rather rapid phase lag increase with frequency in conjunction with a relatively small attenuation tends to make the pipe a destabilizing influence on the fluid control system.

V. Fluid Control Loop Analysis

Using the collector and piping responses derived in Sections III and IV, the fluid control loop shown in Figure 2 can be analyzed using standard open loop gain-phase analysis techniques. In the following example it will be assumed that the temperature sensor response characteristic is a simple first order lag with a time constant of three seconds, and that the sensor is located 5 feet from the end of the collector (at the end of the flex hose connecting the receiver tube to the fixed piping). A simple proportional controller will be considered initially, and it will be assumed that a fast-acting valve positioner is used which has a negligible response time compared to the other system elements.

The combined frequency response defining the temperature sensor output for a change in flow rate is shown in Figure 9 for nominal flow rates of 2 and 10 gpm.

Fig. 9 System Open Loop Response - 5 Ft. Sensor Location

The objective is to choose a value for the controller proportional band (gain) which will result in a stable system for the two extremes of flow rates, and which also results in acceptable accuracy and response characteristics. From Figure 9 it can be seen that a controller gain of -20 db will result in a phase margin of about 45° at gain crossover for a flow rate of 10 gpm, and a phase margin of about 40° at a flow rate of 2 gpm. A controller gain of -20 db (.1 cu ft/min/ $^\circ\text{F}$) requires a temperature change of about 10° to change the

flow rate from 2 gpm to 10 gpm (range of about 1 cu ft/min) resulting in a steady state accuracy of about $\pm 5^\circ$. Gain crossover occurs in the vicinity of 3-4 radians per minute for both flow rates indicating a closed loop frequency response on the order of .5 cycles per minute. This accuracy and frequency response is adequate for most applications, and it appears that a simple proportional controller is adequate for this case.

In some cases it might be desirable to put the temperature sensor and controller at some central location for ease of monitoring and maintenance purposes. Under these conditions, the pipe length between the collector and temperature sensor will be relatively long and the frequency response of the flow control system will be quite different. Figure 10 shows the open loop frequency response for a sensor location that is 50 feet downstream from the collector.

Fig. 10 System Open Loop Response - 50 Ft. Sensor Location

It can be seen that the phase lag in this case is considerably larger due to the additional phase lag in the pipe, and the use of the same proportional gain would result in an unstable system. In order to achieve a phase margin of about 45° , the proportional gain must be reduced to -36 db (.016 cu ft/min/ $^\circ$ F). This gain will result in a static accuracy of about $\pm 30^\circ$ F and a closed loop frequency response in the order of .2 cycles per minute. This accuracy and response is inadequate for many applications, and in particular is inadequate for electric power generation. The use of integral or lag-lead compensation in the controller can reduce the static error, but the dynamic response will not be improved significantly. For these reasons, it is desirable that the temperature sensor be located as close as possible to the collector output if tight temperature control is desired.

It should be noted that a 1 inch pipe was used between the sensor and collector in the previous example. If a larger pipe had been used, the situation would have been worse since the phase lag in the pipe is inversely proportional to the fluid flow velocity. For example, if a two-inch pipe had been used, a sensor location of only 12.5 feet from the collector would have resulted in the same response as for the 50 ft and one-inch pipe case.

VI. Conclusions

General frequency response characteristics for parabolic trough solar collectors and associated interconnecting piping have been derived. The use of these frequency responses in analyzing a fluid control system for a typical string of collectors has been demonstrated. The analysis indicated that a simple proportional controller will result in a stable system with good accuracy provided that the temperature sensor is located near the collector output. These are essentially the same conclusions reached in previous studies (Ref. 1) using complex computer simulations of the collectors and piping.

The frequency response technique is particularly valuable in obtaining a rapid insight into how controller gains and compensations should be chosen for a specified collector configuration. It should be noted, however, that this approach is only valid for small changes in the system variables, and obtaining the response to large changes or to obtain detailed transient responses still requires a computer simulation of the collectors and piping.

The configuration studied in this paper was a relatively simple one, but the technique can be extended to more complex collector and piping configurations by combining the frequency responses of the individual elements to obtain an overall response. More complex control system responses such as those due to sampling in a digital collector, various types of compensation, and valve actuator response can also be included.

Appendix

A. Collector Response

By substituting the assumed solutions (13) and (14) and $\Delta Q = \sin \omega t$ into equation (12) and equating the sine and cosine terms, a pair of algebraic equations relating F_1 , F_2 , F_3 , and F_4 are obtained.

$$\omega F_3 + (K_L + K_R) F_4 = K_C F_2 \quad (23)$$

$$(K_L + K_R) F_3 - \omega F_4 = K_C F_1 + 4 \frac{K_L K_R}{K_F A_F} \quad (24)$$

Solving (23) and (24) for F_3 and F_4

$$F_3 = \frac{\omega K_R F_2 + (K_L + K_R) K_R F_1 + 4(K_L + K_R) \frac{K_L K_R}{K_F A_F}}{\omega^2 + (K_L + K_R)^2} \quad (25)$$

$$F_4 = \frac{(K_L + K_R) K_R F_2 - \omega K_R - 4\omega \frac{K_L K_R}{K_F A_F}}{\omega^2 + (K_L + K_R)^2} \quad (26)$$

Substituting the assumed solutions (13) and (14), relationships (25) and (26), and $\Delta Q = \sin \omega t$ into equation (11) and equating the sine and cosine terms, a pair of ordinary linear differential equations defining F_1 and F_2 are obtained.

$$\underbrace{\frac{dF_1}{dx} + \frac{A_F}{Q_0} \left[K_F - K_F \frac{(K_L + K_R) K_R}{\omega^2 + (K_L + K_R)^2} \right]}_a F_1 - \underbrace{\frac{A_F}{Q_0} \left[\omega + \frac{\omega K_F K_R}{\omega^2 + (K_L + K_R)^2} \right]}_b F_2 = \underbrace{\frac{K_L}{Q_0} \left[1 + \frac{4 K_F (K_L + K_R)}{\omega^2 (K_L + K_R)^2} \right]}_c \quad (27)$$

$$\frac{dF_2}{dx} + \frac{A_F}{Q_0} \underbrace{\left[K_F - K_F \frac{(K_L + K_R) K_R}{\omega^2 + (K_L + K_R)^2} \right]}_a F_2 + \frac{A_F}{Q_0} \underbrace{\left[\omega + \frac{\omega K_F K_R}{\omega^2 + (K_L + K_R)^2} \right]}_b F_1 = \underbrace{\frac{K_1}{Q_0} \left[- \frac{4 K_F \omega}{\omega^2 + (K_L + K_R)^2} \right]}_d \quad (28)$$

These differential equations can be solved for F_1 and F_2 using standard Laplace Transform techniques.

$$F_1(x) = \left[\frac{ac + bd}{a^2 + b^2} (1 - e^{-ax} \cos bx) - \frac{ad - cb}{a^2 + b^2} e^{-ax} \sin bx \right] \quad (29)$$

$$F_2(x) = \left[\frac{ad - cb}{a^2 + b^2} (1 - e^{-ax} \cos bx) + \frac{ac + bd}{a^2 + b^2} e^{-ax} \sin bx \right] \quad (30)$$

Substituting (29) and (30) into the assumed solution (11) and setting $x = L$ the required frequency response is obtained.

$$\left| \Delta T_F(\omega) \right| = \sqrt{F_1(L)^2 + F_2(L)^2} \quad (31)$$

$$\text{Phase of } \Delta T_F = \tan^{-1} \frac{F_2(L)}{F_1(L)} \quad (32)$$

B. Pipe Response

The initial part of the development for the pipe response is identical to that of the collector response except that $K_1 = 0$. The differential equations to be solved for F_1 and F_2 in this case are as follows:

$$\frac{dF_1}{dx} + a F_1 - b F_2 = 0 \quad (33)$$

$$\frac{dF_2}{dx} + a F_2 + b F_1 = 0 \quad (34)$$

To obtain the output temperature response to a sinusoidal change in input temperature, these equations must be solved for $F_1 = 1$ and $F_2 = 0$ at $X = 0$. Using standard Laplace Transform techniques, the following solutions are obtained.

$$F_1(X) = e^{-ax} \cos bX \quad (35)$$

$$F_2(X) = e^{-ax} \sin bX \quad (36)$$

Substituting (35) and (36) into the assumed solution (11) and setting $X = L$, the required frequency response is obtained.

$$\left| \Delta T_F(\omega) \right| = e^{-aL}$$

$$\text{Phase of } \Delta T_F(\omega) = -bL$$

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