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DISPERSION CORRECTION THROUGH MOVEMENT OF THE CLOSED ORBIT*

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ABSTRACT

The closed orbit correction system can be used to correct the vertical dispersion by displacing the orbit at the quadrupoles and sextupoles. The accuracy of the results have been verified by comparison with exact calculations. Results for correcting the horizontal dispersion are also given.

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1. INTRODUCTION

The presence of magnetic field errors can introduce vertical dispersion¹ of the closed orbit. In accelerators with superconducting magnets, the exceptionally large skew quadrupole error field in the dipole is a strong source of vertical dispersion. In the ISABELLE accelerator, a peak increase in the beam size at the intersection points, due to vertical dispersion, of 0.7 mm is expected at 30 GeV where the beam size is 3 mm. This would cause a loss in luminosity of about 25%, and, perhaps more important, would cause the beam-beam interaction to vary about 25% at the different crossing points.

The vertical dispersion can be corrected by vertically displacing the closed orbit^{2,3}, rather than using the more direct method of exciting skew quadrupoles⁴ that have to be especially provided for this purpose. The orbit is displaced vertically at the sextupoles, which generates skew quadrupole fields, and at the quadrupoles which generates horizontal dipole fields; these skew quadrupole fields and horizontal dipole fields both contribute to the dispersion. This method of correcting the vertical dispersion is feasible if the vertical displacement of the closed orbit required is not excessive. This is difficult to determine by analytical means and requires a computer simulation study. It was found for ISABELLE, that the smallest vertical displacement, 5 mm, that would correct the vertical dispersion required more dipole field correction than is available in the ISABELLE closed orbit correction system. The dipole field correction required could be reduced by allowing a larger vertical displacement of the orbit, which led to a required displacement of 8 mm.

2. THE DISPERSION MATRIX AND THE CORRECTION PROCEDURE

Correction of the vertical closed orbit may be done by writing the vertical orbit displacement at the beam monitors labeled by the index K as

$$Y(K) = Y_A(K) + \Delta Y(K), \quad (2.1)$$

where $Y_A(K)$ is the initial or measured displacement and $\Delta Y(K)$ is the correction produced by the dipole correction coils. The dipole corrections may be computed by using the orbit displacement matrix $A(K,J)$ which gives the orbit displacement at point K due to a corrector at point J. Thus,

$$\Delta Y(K) = \sum_J A(K,J) \cdot D(J), \quad (2.2)$$

$$D(J) = L(J) \Delta B(J) / B_p.$$

$L(J)$ and $\Delta B(J)$ are the length and field of the corrector at point J. The orbit correction matrix $A(K,J)$ is given by

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$$A(K,J) = \frac{[\beta_y(K) \beta_y(J)]^{1/2}}{2 \sin \pi \nu_y} \cos(\pi \nu_y - |\psi(K) - \psi(J)|), \quad (2.3)$$

where $\psi(K)$ is the vertical betatron phase at point K.

The strength of the required correctors, $D(J)$, can be determined by the minimization equation,

$$\delta \sum_K Y^2(K) = \delta \sum_K [Y_A(K) + \sum_J A(K,J) D(J)]^2 = 0 \quad (2.4)$$

The vertical dispersion can be corrected in a similar manner by introducing the dispersion matrix $C(L,J)$ which gives the dispersion at point L, $\Delta Y_p(L)$ due to the corrector at point J. Thus,

$$\Delta Y_p(L) = \sum_J C(L,J) D(J). \quad (2.5)$$

The dispersion matrix $C(L,J)$ is generated by moving the orbit vertically in magnets which may have either a sextupole or quadrupole field. $C(L,J)$ is given by

$$C(L,J) = \sum_M A(L,M) (L(M)/B\rho) (2B_2 X_p - G) A(M,J) - A(L,J). \quad (2.6)$$

$L(M)$, $B_2 X_p^2$, and G are the length, the sextupole field and the gradient of the magnet at point M. The points labeled by L are the points where the dispersion is to be corrected. For ISABELLE these points are the 6 crossing points where the beams cross.

Given the dispersion matrix $C(L,J)$ the dipole corrections necessary to correct the vertical dispersion can be computed in the following 2 ways. For ISABELLE, it is important to correct the vertical dispersion at the crossing point and the vertical dispersion around the rest of the ring is not as important. The strength of the correctors, $D(J)$, can be determined by the following minimization equation for the orbit displacement plus side conditions that correct the dispersion (for ISABELLE, there are 12 side conditions),

$$\begin{aligned} \delta \sum_K Y^2(K) &= \delta \sum_K [Y_A(K) + \sum_J A(K,J) D(J)]^2 = 0, \\ Y_p(L) &= Y_{p,A}(L) + \sum_J C(L,J) D(J) = 0, \\ Y(L) &= Y_A(L) + \sum_J A(L,J) D(J) = 0, \end{aligned} \quad (2.7)$$

$Y(L)$ is the vertical orbit displacement at the L labeled points where the dispersion is being corrected. We are requiring that the vertical orbit at these points not be changed when we correct the vertical dispersion.

Another possible formulation of the minimization equation to correct the vertical dispersion is

$$\begin{aligned} \delta \sum_K [Y^2(K) + \lambda \sum_L Y_p^2(L)] &= 0, \\ \text{or} \\ \delta \sum_K [Y_A(K) + \sum_J A(K,J) D(J)]^2 + \lambda \sum_L [Y_{p,A}(L) + \sum_J C(L,J) D(J)]^2 &= 0, \end{aligned} \quad (2.8)$$

to which we can add the constraints

$$Y(L) = Y_A(L) + \sum_J A(L,J) D(J) = 0.$$

The parameter λ has to be chosen empirically to get the correct balance between correcting

the orbit displacement and correcting the vertical dispersion.

The dispersion matrix $C(L,J)$ is given for the vertical dispersion by Eqs. (2.5), (2.6). The dispersion matrix for the horizontal dispersion is given by

$$\Delta X_p(L) = \sum_J C(L,J) D(J),$$

$$C(L,J) = - \sum_M A(L,M) (L(M)/B_0) (2B_2 X_p - G)_M A(M,J) - A(L,J),$$

where in this case $D(J)$ is given by

$$D(J) = - L(J) \Delta B(J) / B_0.$$

3. APPLICATION TO ISABELLE

The computer program CLOSORB was used to simulate ISABELLE with a set of random field errors that produce errors in the vertical orbit displacement and in the vertical dispersion. The orbit errors were corrected in 3 steps. In the first step, the vertical orbit displacement was corrected by moving the quadrupoles; in the second step, the vertical orbit displacement was corrected by exciting the 90 dipole correctors; and in the third step, the vertical dispersion at the 6 crossing points was corrected by exciting the 90 dipole correctors, while minimizing the resulting orbit displacement, and while not moving the orbit at the 6 crossing points.

Table I lists the results of computer simulation runs that simulate 10 ISABELLE accelerators having different initial field errors. Table I shows $Y_{\max}$, the maximum closed orbit displacement remaining when the vertical orbit is corrected by exciting the dipole correctors (the result of step 2), and $B_{\max}$ is the maximum dipole correction needed. Table I also shows $Y_{\max}$, and the corresponding $B_{\max}$, when the vertical dispersion is corrected by exciting the dipole correctors (the result of step 3).

Table 1

10 computer simulation runs giving results for correcting the orbit and then the vertical dispersion. $B_{\max}$ is the field correction required and $\Delta p/p = 0.01$.

Run	Y corrected (step 2)			Y _p corrected (step 3)		
	$Y_{\max}$ (mm)	$Y_{p,\max} \frac{\Delta p}{p}$ (mm)	$B_{\max}$ (gauss)	$Y_{\max}$ (mm)	$Y_{p,\max} \frac{\Delta p}{p}$ (mm)	$B_{\max}$ (gauss)
1	0.433	0.355	86	2.35	0	473
2	0.423	0.388	85	1.98	0	486
3	0.422	0.733	86	4.81	0	1058
4	0.491	0.329	86	4.28	0	946
5	0.411	0.785	86	5.52	0	1173
6	0.459	0.419	85	3.15	0	568
7	0.558	0.557	84	2.38	0	544
8	0.535	0.712	86	5.55	0	1069
9	0.473	0.565	85	3.28	0	745
10	0.380	0.595	85	4.13	0	785

One sees from Table I, that the orbit is corrected, as a result of step 2, with a peak error of 0.56 mm. When the orbit is displaced to correct the vertical dispersion, according to Eq. (2.7), then the maximum orbit displacement increases to 5.6 mm, and a field

in the field errors.

To establish the accuracy of the dispersion matrix, results found using the dispersion matrix and the CLOSORS program for the settings of the 90 vertical dipole correctors to correct the vertical dispersion were entered into the SYNCH program which computed the vertical dispersion at the crossing points due to these same settings of the 90 vertical dipole correctors. The computation done by the SYNCH program is relatively exact and there is no throwing away of higher order terms. One approximation that does remain is the treatment of the sextupole coils as point sextupoles.

Table III compares the results for vertical dispersion at the crossing points found in a computation using the SYNCH program and as found using Eq. (2.6) for the dispersion matrix for a particular setting of the 90 vertical dipole correctors in ISABELLE. The error in the vertical dispersion found using the dispersion matrix is about 3%. In correcting the vertical dispersion using the dispersion matrix this 3% error can be further reduced by iterating the correction process.

The result for the horizontal dispersion matrix, Eq. (2.9) was tested in the same way. The results are given in Table IV. A 7% error in the dispersion matrix results was found in this case.

Table III

Comparison of result for the vertical dispersion as computed using SYNCH, and using the dispersion matrix. $\Delta p/p = 0.01$.

L Crossing Point	$y \frac{\Delta p}{p p}$	(mm)
	Computed	Dispersion Matrix
1	0.290	0.294
2	- 0.115	- 0.119
3	0.061	0.066
4	- 0.732	- 0.753
5	- 0.089	- 0.091
6	0.761	0.785

$y_{p,max}$ in QD = 2.3 mm
Error = 3%
at crossing points.

Table IV

Comparison of results for the horizontal dispersion as computed using SYNCH, and using the dispersion matrix, $\Delta p/p = 0.01$.

L Crossing Point	$x \frac{\Delta p}{p p}$	(mm)
	Computed	Dispersion Matrix
1	0.520	0.443
2	0.139	0.093
3	0.785	0.727
4	1.389	1.325
5	- 0.684	- 0.652
6	1.427	1.324

$x_{p,max}$ in QF = 2 mm
Error = 7%
at crossing points.

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correction in the dipole correctors of about 1200 gauss is required. As the ISABELLE dipole correctors have a maximum correction of 400 G, the field correction found above to correct Y_p is too large to be acceptable.

The dipole field corrections required to correct Y_p is reduced by two procedures which are done in sequence. The first procedure, is to change the minimization equation in Eq. (2.7), so that instead of minimizing just the orbit displacement, one also minimizes the corrections required. The minimizing equation in Eq. (2.7) is replaced by

$$S \cdot \left[\sum_K Y(K)^2 + \lambda \sum_J DB(J)^2 \right] = 0, \quad (3.1)$$

where $DB(J)$ is the strength of the additional field correction provided by the dipole corrector at point J . The parameter λ has to be chosen to get the correct balance between correcting the orbit displacement and minimizing the size of the corrections involved. The result for the first procedure for reducing the size of the dipole correctors required is shown in the left side of Table II. The maximum dipole correction required is reduced from 1200 to 609 gauss. However, the maximum vertical orbit displacement is increased from 5.6 mm to 8.3 mm.

Table II

10 computer simulation runs using two procedures for reducing the field correction, B_{max} , required to correct the vertical dispersion.

Run	$S \cdot \left[\sum_K Y(K)^2 + \lambda \sum_J DB(J)^2 \right] = 0$				
	$\lambda = 2 \times 10^{-3}$			Fix $B(J) \approx 300$ at 300	
	Y_{max} (mm)	B_{max} (gauss)		Y_{max} (mm)	B_{max} (gauss)
1	3.99	249		4.0	249
2	3.18	248		3.2	248
3	5.04	474		5.5	300
4	4.88	454		5.1	300
5	7.44	609		8.1	300
6	4.81	285		4.8	285
7	3.37	277		3.4	277
8	8.29	448		8.3	300
9	4.31	416		5.4	300
10	6.63	449		6.5	300

The second procedure for reducing $DB(J)$ is then applied, and consists of taking those correctors which in the first procedure were found to require more than 300 gauss and fixing these correctors at exactly 300 gauss. The remaining unfixed correctors are then used to correct the vertical dispersion according to the first procedure and Eq. (3.1). The results of the second procedure are given on the right side of Table II. The net result is that one can correct the vertical dispersion with a maximum dipole correction of 300 G, and a maximum vertical displacement of 8.3 mm.

4. ACCURACY OF THE DISPERSION MATRIX

The derivation of the result for the dispersion matrix, Eq. (2.6) involves making various approximations. All magnets are treated as point magnets, and the result for the dispersion matrix is obtained by neglecting certain terms which are higher order in $\Delta p/p$ and