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THERMONUCLEAR INVERSE MAGNETIC PUMPING POWER
CYCLE FOR STELLARATOR REACTORS

By

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SEPTEMBER 1985

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PHYSICS
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PRINCETON UNIVERSITY
PRINCETON, NEW JERSEY

PREPARED FOR THE U.S. DEPARTMENT OF ENERGY,
UNDER CONTRACT DE-AC02-76-CO-3073.

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THERMONUCLEAR INVERSE MAGNETIC PUMPING POWER
CYCLE FOR STELLARATOR REACTORS

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ABSTRACT. A novel power cycle for direct conversion of alpha-particle energy into electricity is proposed for an ignited plasma in a stellarator reactor. The plasma column is alternately compressed and expanded in minor radius by periodic variation of the toroidal magnetic field strength. As a result of the way a stellarator is expected to work, the plasma pressure during expansion is greater than the corresponding pressure during compression. Therefore, negative work is done on the plasma during a complete cycle. This work manifests itself as a back-voltage in the toroidal field coils, and direct electrical energy is obtained from this voltage. For a typical reactor, the average power obtained from this cycle (with a minor radius compression factor on the order of 50%) can be as much as 50% of the electrical power obtained from the thermonuclear neutrons without compressing the plasma. Thus, if it is feasible to vary the toroidal field strength, the power cycle provides an alternative scheme of energy conversion for a deuterium-tritium fueled reactor. The cycle may become an important method of energy conversion for advanced neutron-lean fueled reactors. By operating two or more reactors in tandem, the cycle can be made self-sustaining.

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1. INTRODUCTION

A novel power cycle for direct conversion of α -particle energy to electricity is proposed [1] for stellarator reactors. This power cycle provides an alternative scheme of energy conversion for deuterium-tritium (D-T) fueled reactors, and could become an important method of energy conversion for advanced neutron-lean fueled (e.g., D-He³) reactors. The direct energy conversion is achieved by alternating compression and expansion of the plasma minor radius. The cycle is composed of three stages as shown in Fig. 1.

At stage 0, ignited plasma is thermally stable and in thermal balance. The plasma β is slightly above β_c . [$\beta \equiv P/(B_z^2/8\pi)$, i.e., the ratio of plasma kinetic to the internal magnetic field pressure, and β_c , the maximum attainable β , is set by the onset condition for MHD ballooning instabilities.] During the compression phase (from stage 0 to stage 1), plasma is compressed adiabatically by increasing the external toroidal magnetic field strength, and the plasma β decreases. At stage 1, the end of the compression phase, β is less than β_c and the plasma is no longer in thermal balance since the thermonuclear α -particle heating power exceeds the rate of energy loss (due to neoclassical transport, turbulent convection, and bremsstrahlung radiation). During the heating phase (from stage 1 to stage 2), as the plasma temperature and β are driven up by the excess heating power, the plasma volume is kept constant by further increasing the external field strength. As β approaches β_c , the rate of energy loss increases until it balances the α -particle heating power when β reaches β_c . The plasma is again in thermal balance and is thermally stable. This is stage 2 of the cycle.

To complete the cycle, the external field is then reduced so that the plasma expands back to its original radius during the expansion phase (from

stage 2 to stage 0). When the plasma expands, the plasma β tends to increase. However, β is already at β_0 at stage 2, hence ballooning instabilities force β to stay at β_0 through turbulent convection during the entire expansion phase. As a result, the plasma pressure during the expansion is higher than the corresponding pressure during the compression. Therefore, negative work is done on the plasma during a complete cycle. Note that if there is no β limit, then more work can be obtained during a complete cycle. This work manifests itself as a back-voltage in the toroidal field coils, and direct electrical energy is obtained from this voltage. By operating two or more reactors in tandem, the cycle can be made self-sustaining.

As an alternative cycle, net work can also be done on the external system by letting the plasma expand at constant pressure between stages 1 and 2, rather than keeping the plasma at constant volume. The reason that these two methods (heating phase at constant plasma volume and at constant plasma pressure) are chosen for discussion in this paper is their simplicity for analytical study. There are many other possible methods that can also result in net work done, e.g., the plasma minor radius varies sinusoidally during the cycle.

If the external magnetic field is considered to be a "piston," then the external system absorbs work done by the pumping action of the plasma on the piston. This action is referred to as "magnetic pumping" in this paper. It is "inverse magnetic pumping" over a full cycle because work is done on the external system by the plasma. Note that this concept of magnetic pumping differs from the traditional definition which refers to "gyro-relaxation," as in Refs. [2]-[6].

This paper is organized as follows: In Sec. 2, the physics of the cycle is analyzed. In Sec. 3, the work done and performance of the cycle are

calculated, and an analogy to a conventional internal combustion engine is made. In Sec. 4, the mode of operation and a sample calculation are presented. The conclusion is given in Sec. 5. Appendix A gives an alternative method (via the voltage-current relation) for calculating the electrical work extracted. In Appendix B, the thermonuclear thermal stability is discussed. Finally, a detailed analysis of the temporal evolution of the plasma temperature after compression is presented in Appendix C.

2. PHYSICS OF THE CYCLE

The detailed physical processes at, and between, various stages of the cycle are analyzed in this section. To simplify the analysis, the plasma column is modeled by a straight cylinder. The plasma density, temperature, and the magnetic field are assumed to be uniform across the minor radius except for a sharp discontinuity across the boundary between the plasma and the confining vacuum magnetic field. Throughout this analysis, the plasma is assumed to behave like an ideal MHD fluid.

Before the compression, the ignited plasma is in thermal balance, at a stable thermal equilibrium (see Appendix B). The plasma β is slightly above β_c . Thus, the plasma pressure gradient exceeds the critical pressure gradient by a small amount. Turbulent convective cells are driven by this excess pressure gradient and carry part of the outward particle and energy fluxes [7]; the rest of the fluxes are carried by neoclassical transport [8]. This is stage 3 in the power cycle. The plasma and magnetic field parameters are:

Plasma β :	$\beta_0 \equiv \beta_c$,
Plasma pressure:	P_0 ,
Plasma minor radius:	a_0 ,
Internal magnetic field:	B_{i0} ,
External magnetic field:	B_{e0} .

The subscript 0 refers to the physical quantities at stage 0.

During the compression phase from stage 0 to stage 1, the plasma is compressed adiabatically as the external toroidal magnetic field strength is increased. Adiabatic compression means that the compression time is shorter than the burn time (the time for the thermonuclear α -particles to increase the plasma temperature by an amount comparable to itself). It is important for the compression to be adiabatic because otherwise, the α -particles could heat up the plasma and more work would be required during the compression phase. On the other hand, as explained later in this section, the compression phase must be carried out slowly compared with τ_i (the 90° "deflection time" for ions) so that the compression is three-dimensional.

The plasma β decreases during the compression because the internal magnetic field pressure increases faster than the plasma kinetic pressure. This can be shown explicitly if the plasma β is expressed in terms of p_0 and B_{i0} . Using the adiabatic compression law ($pV^\gamma = \text{const}$ with $\gamma = 5/3$), the definition of β , and the frozen flux condition, the plasma β can be expressed as

$$\begin{aligned}\beta(t) &= \frac{p_0}{(B_{i0}^2/8\pi) r_v} \frac{1}{r_v^{2/3}} \\ &= \frac{\beta_c}{r_v^{2/3}}.\end{aligned}\quad (1)$$

Here, $r_v(t) \equiv a_0/a(t)$ is the compression ratio. During the compression, r_v increases from unity to a_0/a , and thus β decreases. If β_c is roughly constant, then the plasma β is below β_c after compression. This is stage 1 in the power cycle. The plasma and magnetic field parameters are:

Plasma β :	$\beta_1 < \beta_0 \equiv \beta_c$,
Plasma pressure:	$P_1 > P_0$,
Plasma minor radius:	$a_1 < a_0$,
Internal magnetic field:	$B_{i1} > B_{i0}$,
External magnetic field:	$B_{e1} > B_{e0}$.

The subscript 1 refers to the physical quantities at stage 1.

After the adiabatic compression, the increase in α -particle heating power per unit volume is larger than the increase in neoclassical energy diffusion and bremsstrahlung radiation losses per unit volume. Also, turbulent convective transport can be assumed to be absent after compression since β is less than β_c . This imbalance between the heating power and the power loss causes the plasma temperature to increase while the plasma volume is kept constant by further increasing the external magnetic field strength. The increase in temperature can be understood in more detail from the volume-averaged energy balance equation

$$3n \frac{dT}{dt} = P_\alpha(t) - [P_{\text{neo}}(t) + P_{\text{conv}}(t) + P_X(t)] . \quad (2)$$

The first term on the right-hand side of this equation is the heating power from the thermonuclear α -particles. The first term inside the bracket is the energy loss rate due to neoclassical diffusion, i.e.,

$$P_{\text{neo}} = \frac{3}{2} \frac{nT_e}{\tau_e} + \frac{3}{2} \frac{nT_i}{\tau_e} . \quad (3)$$

From Ref. [8] the electron energy confinement time is

$$\tau_e^e = 10^{-5} \frac{n_{14} B_4^2 R_m^2 a_m^2}{(5.4 + C)(2\epsilon_h r)^{3/2} T_4^{7/2}} \text{ sec} \quad (4)$$

and the ion energy confinement time is

$$\tau_e^i \approx \tau_e^e.$$

Here, the minor radius a_m and the major radius R_m are in meters, B_4 is in 10 kG, electron or ion density n_{14} is in 10^{14} cm^{-3} , plasma temperature T_4 is in 10 keV, ϵ_h is the depth of the helical magnetic well caused by external helical windings, and the normalized ambipolar electric field strength $C \equiv (e/T)(\partial\phi/\partial r)n(\partial n/\partial r)^{-1}$ can be approximated by unity. The second term inside the bracket in Eq. (2) is the convective energy loss rate. The last term is the energy loss rate due to bremsstrahlung radiation and is equal to $cn^2\sqrt{T}$, with c a constant. Note that density is not a function of time during the heating phase since the plasma volume is held fixed. Also note that before the compression, thermal balance means that $dT/dt = 0$. The ratio of the value of each term on the right-hand side of Eq. (2) after the adiabatic compression to the value of the corresponding term before the compression will now be studied. Because of the increase in plasma density and temperature after the adiabatic compression, the rate of thermonuclear α -particle energy production is larger than that before the compression by the ratio

$$\begin{aligned} \frac{P_\alpha(t_1)}{P_\alpha(t_0)} &= \frac{n_1^2 \langle \sigma v \rangle_1 3.5 \text{ MeV}}{n_0^2 \langle \sigma v \rangle_0 3.5 \text{ MeV}} \\ &= r^4 \frac{\langle \sigma v \rangle_1}{\langle \sigma v \rangle_0}. \end{aligned} \quad (5)$$

Here, 3.5 MeV is the energy of an α -particle generated from D-T reactions, and

$\langle \sigma v \rangle_0$ and $\langle \sigma v \rangle_1$ are the Maxwellian reactivities at stage 0 and stage 1, respectively. If $r_v = 1/0.6$ and $T_0 = 10$ keV, then $T_1 = 19.8$ keV. Using the formula for $\langle \sigma v \rangle_{D-T}$ given in Ref. 9, Eq. (5) has a value of 32.1. To obtain the ratio of the neoclassical energy loss rate per unit volume after the compression to that before the compression, we let $P_{neo} \equiv 3nT/r_c^e$. Then, it can be shown that

$$\frac{P_{neo}(t_1)}{P_{neo}(t_0)} = r_v^4, \quad (6)$$

which has a value of 7.72 for $r_v = 1/0.6$. The turbulent convective transport is assumed to vanish at the end of the compression phase since β drops below β_c . Finally, the ratio of the bremsstrahlung radiation energy loss rate after compression to that before the compression is

$$\frac{P_x(t_1)}{P_x(t_0)} = r_v^{14/3}, \quad (7)$$

which has a value of 10.85 for $r_v = 1/0.6$. From Eqs. (5)-(7), we can conclude that α -particle heating will further increase the plasma temperature after the adiabatic compression since the relative increase in the α -particle heating power is larger than the relative increase in the energy loss rates. This phenomenon of plasma temperature rise after compression is of fundamental importance to the success of the power cycle, since the plasma must have a higher pressure during the expansion phase than during the compression. As the temperature rises, the plasma β increases since the plasma volume is held fixed. As β approaches β_c , the convective energy loss re-emerges and the convective energy loss rate gradually catches up with the increase in the α -particle heating power. After the compression, the plasma temperature

asymptotically approaches the limit at which $\beta = \beta_c$ in a characteristic time defined as the "thermal relaxation time" (see Fig. 8). (In Appendix C, the temporal evolution of the plasma temperature is treated in detail and the thermal relaxation time is estimated.) At stage 2 in the power cycle, the plasma is again thermally stable and in thermal balance at a stable equilibrium. The plasma and magnetic field parameters are:

Plasma β :	$\beta_2 \cong \beta_c$,
Plasma pressure:	$P_2 > P_1$,
Plasma minor radius:	$a_2 = a_1$,
Internal magnetic field:	$B_{i2} = B_{i1}$,
External magnetic field:	$B_{e2} > B_{e1}$.

To complete the cycle, the external magnetic field is decreased so that the plasma expands back to its original minor radius. This is the expansion phase. When the plasma expands, the plasma β tends to increase. [According to Eq. (1), β increases as r_v decreases.] However, β is already at β_c at stage 2, hence ballooning instabilities force β to stay at β_c (recall that β_c is assumed to be a constant) through turbulent convection during the entire expansion phase. Consequently, the plasma pressure during the expansion phase can be expressed as

$$P(t) = \beta_c \frac{B_i^2(t)}{8\pi} .$$

Upon applying the frozen flux condition, this expression can be written as

$$P(t) = P_0 \frac{v_0^2}{v^2(t)} , \quad (8)$$

which varies exactly as though it had an adiabatic index $\gamma = 2$. As a result, the plasma pressure during the expansion is higher than the corresponding pressure during the compression. Therefore, negative work is done on the plasma during a complete cycle. This work manifests itself as a mean back-voltage in the toroidal field coils, and direct electrical energy is obtained from this voltage. Using the pressure-volume (P-V) relation, the amount of work done on the coils and the thermal efficiency of the cycle are calculated in Sec. 3. This work is also calculated by using the voltage-current (V-I) relation in Appendix A, and the result agrees with that obtained from using the P-V relation. It is now clear that if the compression phase had been carried out faster than τ_i , then the compression would be two-dimensional ($\gamma = 2$) and the pressure variation during the compression would follow Eq. (8) instead of $PV^{5/3} = \text{const.}$ Consequently, no net work would be done on the external system since the plasma pressure during the compression phase would equal the corresponding pressure during expansion.

Finally, the cycle described here satisfies the Kelvin-Planck statement of the second law of thermodynamics [10] by losing heat (obtained from thermonuclear α -particles) to the outside through turbulent convection.

3. CYCLE ENERGETICS AND PERFORMANCE

During the heating phase, between stages 1 and 2, the plasma can either be held at constant volume or be allowed to expand at constant pressure while receiving energy from the thermonuclear α -particles. In a complete cycle, both methods result in net work done on the external system. The work done and thermal efficiency for each of these methods are calculated in this section.

3.1. The thermonuclear inverse magnetic pumping power cycle with heating phase at constant plasma volume

To calculate the amount of work delivered during this cycle, it is only necessary to consider the work performed by the plasma during the compression and expansion phases. No work is performed during the heating phase because the plasma volume is constant.

The work done on the plasma during the adiabatic compression is

$$\begin{aligned} {}_0W_1 &= \int_{v_0}^{v_1} P \, dv \\ &= -\frac{3}{2} P_0 v_0^{5/3} \left(\frac{1}{v_1^{2/3}} - \frac{1}{v_0^{2/3}} \right). \end{aligned}$$

Invoking $P_0 = \beta_c B_{i0}^2 / 8\pi$ and the pressure balance equation $P_0 + B_{i0}^2 / 8\pi = B_{e0}^2 / 8\pi$, P_0 can be expressed as

$$P_0 = \frac{\beta_c}{1+\beta_c} \frac{B_{e0}^2}{8\pi}. \quad (9)$$

Using this expression, ${}_0W_1$ can be written as

$${}_0W_1 = -\frac{3}{16} a_0^2 L \left(r_v^{4/3} - 1 \right) \frac{\beta_c}{1+\beta_c} B_{e0}^2, \quad (10)$$

where $L = 2\pi R$ is the length of the system. The work done by the plasma during the expansion phase is calculated by using Eq. (9). The result is

$$\begin{aligned} {}_2W_0 &= \int_{v_2}^{v_0} P \, dv \\ &= \frac{1}{8} a_0^2 L \left(r_v^2 - 1 \right) \frac{\beta_c}{1+\beta_c} B_{e0}^2. \end{aligned} \quad (11)$$

Therefore, the amount of net work done on the coils during a complete cycle is

$$W_{\text{net}} = {}_0W_1 + {}_2W_0$$

$$= \frac{1}{8} a_0^2 L \left(r_v^2 - \frac{3}{2} r_v^{4/3} + \frac{1}{2} \right) \frac{\beta_c}{1+\beta_c} B_{e0}^2. \quad (12)$$

This expression shows that either a higher compression ratio or a larger β_c will give a larger amount of delivered work. The net work done normalized by the machine dependent parameters $(1/8)a_0^2 L \beta_c [1/(1+\beta_c)] B_{e0}^2$ is

$$W_{\text{norm}} = r_v^2 - \frac{3}{2} r_v^{4/3} + \frac{1}{2},$$

which is plotted versus the compression ratio in Fig. 2. Although it is desirable to operate the cycle at a high compression ratio, the attainable compression ratio may depend on the highest achievable toroidal magnetic field. This field is limited by the allowable static and dynamic loading on the machine structure. The amount of work delivered by the cycle can be represented by the area of the triangle 0-1-2 in the pressure-volume diagram shown in Fig. 3.

The performance of this cycle is characterized by the thermal efficiency η_{th} , which is defined as

$$\eta_{\text{th}} = \frac{W_{\text{net}}}{Q_H}, \quad (13)$$

where Q_H represents the heat obtained by the cycle (engine) from a heat source (thermonuclear α -particles). To calculate the heat received by the plasma during the heating phase, use the first law of thermodynamics,

$$Q_H = {}_1U_2 + {}_1W_2 . \quad (14)$$

Here, ${}_1U_2$ is the change in plasma internal energy between stages 1 and 2 and can be expressed as

$${}_1U_2 = 3\pi a_0^2 \ln_0 (T_2 - T_1) .$$

Since the plasma volume is held fixed during the heating phase, $Q_H = {}_1U_2$. From $P_2 = \beta_c B_{12}^2 / 8\pi$, the frozen flux condition, and the adiabatic compression law, it can be shown that

$$Q_H = \frac{3}{16} a_0^2 L (r_v^2 - r_v^{4/3}) \frac{\beta_c}{1+\beta_c} B_{e0}^2 . \quad (15)$$

Using Eqs. (12) and (15), the thermal efficiency can be expressed as

$$\eta_{th} = 1 - \frac{1}{3} \frac{r_v^2 - 1}{r_v^2 - r_v^{4/3}} . \quad (16)$$

The important thing to note is that the efficiency of this cycle is a function only of the compression ratio and increases with it. The thermal efficiency is plotted versus the compression ratio in Fig. 4. As the compression ratio approaches infinity, the thermal efficiency approaches a value of 2/3.

The thermal efficiency can be visualized by looking at the temperature-entropy (T-S) diagram for the cycle, Fig. 5. The thermal efficiency is the ratio of the area of the triangle 0-1-2 to the area beneath the path 1-2.

Up to this point, the discussions have been restricted to the case in which there is a limiting β . However, the cycle will become more efficient and more work can be obtained if β_c is very high or if there is actually no β

limit (this situation may occur in Helic, Ref. [11]). For this case the work required to compress the plasma is given by Eq. (10). The plasma pressure at stage 2 may now reach a higher value than in the case where there is a β limit. [The temperature at stage 2 can be obtained by the graphical method discussed in Appendix B. Note that turbulent convection is absent here since there is no β limit.] During the expansion, the plasma pressure variation follows $pV^{5/3} = \text{const}$ since β will no longer be prevented from rising. Hence,

$$2W_0 = \frac{3}{2} \pi a_0^2 L (r_v^{4/3} - 1) P_{0'} ,$$

where the subscript 0' denotes conditions at stage 0' - the moment when the expansion phase is completed and the plasma volume returns to its original value at state 0. At stage 0', $P_{0'} > P_0$ and the power losses are greater than the α -particle heating power. Thus, the plasma temperature will decrease. While the temperature is decreasing, the external magnetic field is reduced in order to keep the plasma at constant volume, and heat diffuses to the outside by neoclassical transport. The cycle is completed when the plasma temperature and external magnetic field return to their original values at stage 0 (point C in Fig. 7).

The net work is

$$W_{\text{net}} = \frac{3}{2} \pi a_0^2 L (r_v^{4/3} - 1) n_0 (T_{0'} - T_0) , \quad (17)$$

where $T_{0'}$ can be related to T_2 using the adiabatic law. The thermal efficiency is

$$\eta_{\text{th}} = 1 - \frac{1}{r_v^{4/3}} , \quad (18)$$

which is the Otto cycle efficiency. Note that this cycle is identical to the Otto cycle.

3.2. The thermonuclear inverse magnetic pumping power cycle with heating phase at constant plasma pressure

In this power cycle, the compressed plasma is allowed to expand at constant pressure until the plasma β reaches β_c . At this point (stage 2), the plasma radius is between the minor radius before the compression and that after the compression. Next, the plasma column is expanded further with β staying at β_c until the minor radius reaches the precompression value. The cycle is now complete.

To calculate the amount of work delivered during this cycle, note that the work required to compress the plasma is the same as that of the previous cycle [see Eq. (10)]. To calculate the work done by the plasma on the confining field (${}_1W_2 + {}_2W_0$), it is necessary to know the plasma minor radius at the end of the constant pressure heating phase. Starting with $P_2 = \beta_c B_{12}^2 / 8\pi$, using the frozen flux condition, and noting that $P_2 = P_1$, it can be shown that

$$a_2 = a_0 r_v^{-5/6} . \quad (19)$$

The work done by the plasma on the coils during the constant pressure heating phase is

$$\begin{aligned} {}_1W_2 &= \int_{v_1}^{v_2} P \, dv \\ &= \pi L r_v^{10/3} (a_2^2 - a_1^2) P_0 \end{aligned}$$

$$= \frac{1}{8} a_0^2 L r_v^{4/3} (r_v^{1/3} - 1) \frac{\beta_c}{1+\beta_c} B_{e0}^2 .$$

Using Eq. (9), the work done by the plasma during the final constant β_c expansion phase is

$$\begin{aligned} 2W_0 &= \int_{v_2}^{v_0} p \, dv \\ &= \frac{1}{8} a_0^2 L (r_v^{5/3} - 1) \frac{\beta_c}{1+\beta_c} B_{e0}^2 . \end{aligned}$$

Therefore, the amount of net work done on the coils during a complete cycle is

$$\begin{aligned} W_{\text{net}} &= 0W_1 + 1W_2 + 2W_0 \\ &= \frac{1}{8} a_0^2 L \frac{\beta_c}{1+\beta_c} B_{e0}^2 W_{\text{norm}} , \end{aligned} \quad (20)$$

where $W_{\text{norm}} = 2 r_v^{5/3} - (5/2)r_v^{4/3} + 1/2$ is plotted versus the compression ratio in Fig. 2. Again, the amount of work delivered by this cycle is represented by the area of the triangle 0-1-3 in the pressure-volume diagram, Fig. 3.

The thermal efficiency, which again depends only on the compression ratio, is

$$\eta_{\text{th}} = 1 - \frac{1}{5} \frac{r_v^{5/3} - 1}{r_v^{5/3} - r_v^{4/3}} . \quad (21)$$

The thermal efficiency is plotted versus the compression ratio in Fig. 4. As the compression ratio approaches infinity, the thermal efficiency approaches a value of 4/5. Now, the thermal efficiency is the ratio of the area of triangle 0-1-3 to the area beneath the path 1-3 in Fig. 5. Thus, for the same

compression ratio, this cycle delivers less work but at a higher thermal efficiency than the previous cycle. From the standpoint of reactor economics, it is probably the net work done, rather than the thermal efficiency, that is important. Hence, the cycle described in Sec. 3.1 appears to be an appropriate choice.

4. MODE OF OPERATION AND SAMPLE CALCULATION OF OUTPUT POWER

4.1. Mode of operation

To make the power cycle self-sustaining, part of the work done by the plasma on the coils during the expansion phase must be stored in order to supply power for the next compression phase. However, the amount of energy to be stored is large compared with the net work done by the plasma during the cycle. Thus, even if the ratio of the energy lost (during the transfer to compensate for the loss in the storage system) to the total energy transferred from the reactor to the storage system (this ratio is defined as the recirculation inefficiency) is only a few percent, the total energy lost in recirculation may still exceed the work done during the cycle. Consequently, the power cycle may require a lower recirculation inefficiency than can be provided by conventional energy storage systems, e.g., capacitive, inductive, and inertial (motor-generator-flywheel). To obviate this, we propose to operate two (or more) stellarator reactors in tandem. The major electrical energy loss in this system is the resistive dissipation in the coils, but this loss is small compared with the net work performed by the cycle as is shown in the second part of this section.

This proposed reactor system consists of two identical stellarator reactors A and B. The cycle with heating phase at constant volume will be used for discussions here. The operation scenario is as follows (see

Fig. 6). When the plasma in reactor A is expanding, some of the resulting work is used to compress the plasma in reactor B. When the expansion phase in reactor A is completed, the plasma in that reactor returns to its pre-compression state (stage 0) while the plasma in reactor B completes the compression phase and is at stage 1. The plasma in reactor A is now maintained at stage 0 temporarily while the external magnetic field strength in reactor B is increasing so that the plasma volume in reactor B can be kept constant as the plasma temperature is increasing. During this period, the power used to increase the magnetic field strength in reactor B comes either from the electrical power generated by the thermonuclear neutrons in reactor A or from that in reactor B itself. When the plasma in reactor B reaches thermal balance (stage 2), it is allowed to expand. Some of the resulting work obtained from reactor B during its expansion phase is used to compress the plasma in reactor A. The cycle continues in this manner. The current in the coils of reactor A and B as a function of time is illustrated in Fig. 6. Note that this proposed coupled reactor system is analogous to a two-cylinder, internal combustion engine.

4.2. Sample calculation of output power

The averaged total electrical output power obtained from direct conversion is defined as

$$\overline{P}_{\text{direct}} = \frac{W_{\text{net}} - \oint P_L dt}{\tau_P} \quad (22)$$

Here, P_L is the resistive power dissipation in the coils, τ_P is the period of one complete cycle, and $\oint dt$ is the cyclic time integral, i.e., time integration over the period τ_P . In this section, only the power cycle with

heating phase at constant plasma volume is considered. Also note that the neoclassical energy loss P_{neo} , which could in principle be converted into electricity, is not included in Eq. (22).

As an example, for a stellarator reactor with reactor parameters given in Table I, the thermal relaxation time required by the plasma to reach stage 2 after the adiabatic compression with $r_v = 1/0.6$ is approximately 0.16 sec as indicated in Appendix C. If the compression and expansion phases are each carried out in 0.01 sec [note that $\tau_i = 0(10^{-3})$ sec for the plasma with parameters given in Table I], then $\tau_p \approx 0.18$ sec. With this information and using Eq. (12), it can be shown that $W_{net}/\tau_p = 2.2$ GW.

If the plasma radius remains constant, then at $r = a$, the thermonuclear neutron power $P_n \approx 12.2$ GW. The electrical power obtained from the neutrons at a conversion efficiency $\eta_c = 1/3$ is $\eta_c P_n \approx 4.1$ GW. Thus, W_{net}/τ_p is about 50% of $\eta_c P_n$. Note that W_{net}/τ_p for each reactor in the coupled stellarator system may be lower than the 2.2 GW since τ_p for the coupled system is longer (see Fig. 6). Also note that the actual values of W_{net}/τ_p and P_n should be lower than the results obtained here since the radial profiles of density and temperature must be taken into account properly.

To calculate the resistive power dissipation in the coils, note that

$$P_i = \frac{T_{room}}{T_{coil}} I^2 R. \quad (23)$$

The factor T_{room}/T_{coil} is an approximation for the thermodynamic efficiency of the refrigeration cycle, i.e.,

$$\eta_{th} = \frac{T_{room} - T_{coil}}{T_{coil}}$$

where T_{room} and T_{coil} are the room temperature and the cryogenic temperature of the cooled coils, respectively. The current in the coils is given by

$$I = \frac{c}{4\pi} \frac{B_e}{N},$$

where c is the speed of light and N is the number of turns of the coil per meter. The coil resistance is expressed as

$$R = \frac{\ell}{A} \eta,$$

where $\ell = 2\pi a_c L N$ is the total length of the coil (a_c is the distance between the coil and the center of the plasma), A is the coil cross-sectional area, and η is the resistivity. If b is the width of the coil (measured along the direction of the minor radius), then $A = b/N$. Thus, Eq. (23) can be written as

$$P_L = \frac{c^2}{4} \frac{a_c L}{b} \frac{T_{\text{room}}}{T_{\text{coil}}} \eta B_e^2. \quad (24)$$

Using the reactor parameters in Table I and setting $a_c = 3$ m, $b = 0.5$ m, $T_{\text{room}} = 293^\circ\text{K}$, and $T_{\text{coil}} = 77^\circ\text{K}$ (the boiling point for liquid nitrogen), it can be shown that the total resistive power dissipation in a copper coil during a complete cycle is on the order of a few percent of W_{net} . Hence, $\delta P_L dt$ can be neglected and $P_{\text{direct}} = 2.2$ GW.

It may be thought that more neutron power could be obtained by increasing the confining magnetic field strength and operating the plasma at a higher pressure without pulsing the minor radius. However, this mode of operation is undesirable since the neutron wall load would exceed the present day engineering limit which is below $10 \text{ MW}\cdot\text{m}^{-2}$.

5. CONCLUSIONS

We have proposed a power cycle for direct conversion of α -particle energy into electricity for stellarator reactors. The direct energy conversion is achieved by alternating compression and expansion of the plasma minor radius.

In analyzing each phase of the power cycle, there appear to be no major physical or engineering obstacles that would make the cycle impractical. For the stellarator reactor with parameters given in Table I, the averaged power obtained from the cycle with compression ratio 1/0.6 is about 50% of the electrical power obtained from the thermonuclear neutrons from the same reactor without compressing the plasma. Thus, the cycle may provide an alternative scheme for extracting energy from D-T fueled reactors. This scheme can be used either along or as a supplement to the electrical power generated from thermonuclear neutrons. For advanced neutron-lean fueled reactors, this power cycle may become an important scheme for energy conversion. The possibility of using this cycle for tokamak reactors, however, has not been pursued in this paper because of the uncertainty in the behavior of anomalous transport in tokamaks.

ACKNOWLEDGMENTS

It is a great pleasure to thank D. L. Jassby, J. L. Johnson, D. W. Kerst, R. G. Mills, S. Yoshikawa, D. A. Buchenauer, R. G. Hay, B. J. Mcklich, and E. R. Salberta for illuminating discussions. This work is supported by United States DoE Contract No. DE-AC02-76-CHO-3073.

APPENDIX A

CALCULATION OF THE POWER CYCLE WORK
DONE FROM THE VOLTAGE-CURRENT RELATION

The net work delivered by the power cycle with heating phase at constant plasma volume is calculated in Sec. 3 using the pressure-volume relation. In this appendix, it is shown that the same result can be obtained by using the voltage-current relation, i.e., $w_{\text{net}} = \oint v i dt$. Work done during each of the three phases in the power cycle will now be calculated.

Using the pressure balance equation, the external magnetic field at time t during the compression phase can be expressed in terms of the internal magnetic field at stage 0 and the compression ratio at time t . The resulting expression is

$$B_e(t) = B_{i0} r_v^2(t) \left[1 + \frac{\beta_c}{r_v^{2/3}(t)} \right]^{1/2} \quad (A1)$$

The current in the coil is

$$I(t) = \frac{c}{4\pi} \frac{B_e(t)}{N} \quad (A2)$$

The total magnetic flux inside the original plasma radius a_0 is

$$\Phi(t) = \Phi_i + \pi a_0^2 B_e(t) \left(1 - \frac{1}{r_v^2(t)} \right), \quad (A3)$$

where Φ_i is the flux inside the plasma. Note that Φ_i is time independent because of the frozen flux condition. The total induced voltage is

$$V(t) = \frac{1}{c} L N \frac{d\phi}{dt} \quad . \quad (A4)$$

Using Eqs. (A1)-(A4), the work required from the external electrical system to compress the plasma can now be calculated. Let the cycle start with stage 0 at time 0 and reach stage 1 at time t_1 . Then the work done during the compression phase is

$$\begin{aligned} {}_0W_1 &= \int_0^{t_1} VI \, dt \\ &= \frac{L}{4\pi} \int_0^{t_1} B_e \frac{d\phi}{dt} \, dt \quad . \end{aligned}$$

Substituting Eqs. (A1) and (A3) into this expression, it can be shown that

$$\begin{aligned} {}_0W_1 &= \frac{1}{8} a_0^2 L B_{i0}^2 \int_0^{t_1} \frac{d}{dt} \left[r_v^2(t) \left(1 + \frac{\beta_c}{r_v(t)} \right)^{1/2} \right] dt \\ &\quad + \frac{1}{8} a_0^2 L B_{i0}^2 \beta_c \int_0^{t_1} \frac{d}{dt} r_v^{4/3}(t) dt \\ &= \frac{3}{16} a_0^2 L \left(r_v^{4/3} - 1 \right) \frac{\beta_c}{1+\beta_c} B_{e0}^2 + \frac{1}{8} a_0^2 L \left[\left(1 - \frac{1}{r_v^2} \right) B_{e1}^2 + r_v^2 B_{i1}^2 \right] \\ &\quad - \frac{1}{8} a_0^2 L B_{i0}^2 \quad . \quad (A5) \end{aligned}$$

The first term on the right-hand side of this equation is identical to the $p dV$ work given by Eq. (11). The difference between the second and the third term represents the change in magnetic energy of the system.

Since the plasma volume is constant during the heating phase, the work done on the plasma by the external system during this phase is just the

difference between the energy of the external magnetic field at stage 1 to that at stage 0. Hence,

$${}_1W_2 = \frac{1}{8} a_0^2 L \left(1 - \frac{1}{2} \right) \left(B_{e2}^2 - B_{e1}^2 \right) \quad (A6)$$

During the expansion phase, β stays at β_c . Thus, using the pressure balance equation, the external magnetic field can be expressed as

$$B_e(t) = B_{i0} r_v^2(t) (1 + \beta_c)^{1/2} \quad (A7)$$

Then, in exactly the same manner as in obtaining Eq. (A5), the negative work exerted by the external system on the plasma during the expansion phase is

$$\begin{aligned} {}_0W_2 = & -\frac{1}{8} a_0^2 L (r_v^2 - 1) \frac{\beta_c}{1+\beta_c} B_{e0}^2 - \frac{1}{8} a_0^2 L \left[\left(1 - \frac{1}{2} \right) B_{e1}^2 + r_v^2 B_{i1}^2 \right] \\ & + \frac{1}{8} a_0^2 L B_{i0}^2 \quad (A8) \end{aligned}$$

The physical interpretation of each term on the right-hand side of Eq. (A8) is exactly the same as the corresponding term of Eq. (A5). Note that the first term on the right-hand side of Eq. (A8) is identical to the PdV work given by Eq. (12).

Adding Eqs. (A5), (A6), and (A8) gives the net work done during a complete cycle,

$$\begin{aligned} W_{\text{net}} &= {}_0W_1 + {}_1W_2 + {}_2W_0 \\ &= -\frac{1}{8} a_0^2 L \left(r_v^2 - \frac{3}{2} r_v^{4/3} + \frac{1}{16} \right) \frac{\beta_c}{1+\beta_c} B_{e0}^2 \quad (A9) \end{aligned}$$

which is identical to Eq. (12).

APPENDIX B

THERMONUCLEAR THERMAL STABILITY

Due to the nonlinear temperature dependence of the α -particle heating power term and the power loss terms on the right-hand side of Eq. (2), the thermonuclear plasma in a stellarator can have either more than one thermal equilibrium, i.e., the thermonuclear power can balance the power losses at different temperatures, or have no equilibrium at all. This appendix gives a qualitative discussion about the possible thermal equilibria and their stability. The reactor parameters listed in Table I are used in the discussion here.

For a fixed plasma density ($n = 6 \times 10^{14} \text{ cm}^{-3}$), the thermonuclear α -particle heating power per unit volume is plotted versus plasma temperature as the solid curve in Fig. 7. In the same figure, the power loss per unit volume [due to neoclassical diffusion [given by Eq. (3)], and bremsstrahlung radiation] as a function of temperature appears as the dashed curve. (The power loss due to turbulent convection is not included in this dashed curve, but its effect on thermal equilibrium is discussed later in this section.) Figure 7 indicates that there are two equilibrium solutions. The first (point A) is thermally unstable. If the plasma temperature decreases slightly, the neoclassical diffusion and Bremsstrahlung radiation losses will begin to dominate and hence the plasma temperature will decrease to zero. On the other hand, if there is a slight increase in plasma temperature, the α -particle

heating power will dominate and thus the plasma temperature will proceed on a thermal excursion until the loss rate matches the heating power at a stable thermal equilibrium, which is the second equilibrium solution (point C). (Similar discussions for tokamak reactors have been given by Conn [12], and by Kolesnichenko and Reznik [13].) Thus, to start a stellarator reactor, it is only necessary to raise the plasma temperature to the unstable equilibrium at point A, even if the desired operating condition at the stable equilibrium is at a higher temperature. It is interesting to note that if a_0 is reduced to 1.5 m, but all the other reactor parameters remain unchanged, then the stable thermal equilibrium occurs at some temperature between 7-8 keV. The stable thermal equilibrium (point C) in Fig. 7 requires a plasma temperature of approximately 11 keV. If $\beta_C = 19.3$, then β at point C exceeds β_C . As a result, pressure driven turbulent convection will increase the power loss, raise the loss curve, and force the stable equilibrium to occur at a lower plasma temperature (point B) such that $\beta \approx \beta_C$. (As β approaches β_C , the plasma will follow the broken curve in Fig. 7.)

After adiabatic compression, due to the increase in both the α -particle heating power and the power losses, the curves in Fig. 7 shift toward higher plasma temperature and power (the upper-right hand corner). As was mentioned in Sec. 2, the increase in the α -particle heating power is greater than the increase in the power losses. This implies that, immediately after compression, the plasma temperature is located somewhere between the corresponding thermal equilibria (points A and B) of the shifted curves.

Consequently, the plasma temperature continues to rise until it reaches a new stable thermal equilibrium. The temporal evolution of the plasma temperature after the compression is studied in Appendix C.

APPENDIX C

TEMPORAL EVOLUTION OF THE PLASMA TEMPERATURE AFTER ADIABATIC COMPRESSION

The power delivered by the power cycle via direct energy conversion depends on the period of the cycle. One of the major factors that governs the period is the thermal relaxation time, i.e., time for the plasma β to reach β_0 after adiabatic compression. In this appendix, the rate of energy transfer of an individual α -particle to the background plasma is studied. Then, the time evolution of the plasma temperature after the compression is studied using the volume-averaged energy equation for electrons. From the numerical solution of this equation, the thermal relaxation time is estimated.

The rate of thermonuclear α -particle energy transferred to the background plasma is given by Trubnikov [14] in the form

$$\frac{d\epsilon_\alpha}{dt} = -\frac{\epsilon_\alpha}{\tau^{\alpha/D}} - \frac{\epsilon_\alpha}{\tau^{\alpha/e}}, \quad (C1)$$

where ϵ_α is the α -particle energy, and $\tau^{\alpha/D}$ and $\tau^{\alpha/e}$ are, respectively, the characteristic energy transfer time between the α -particle and electrons, and

the characteristic energy transfer time between the α -particle and deuterons. In this analysis, we assume $T = T_e = T_i$. To obtain the characteristic energy transfer times, note that after the compression (at stage 1), the plasma with stage 0 parameters given in Table I has $n_1 = 16.7 \times 10^{14} \text{ cm}^{-3}$ and $T_i = 19.8 \text{ keV}$ if $r_v = 1/0.6$. Thus,

$$\frac{\epsilon_\alpha(0)}{T_1} = 176.8 \gg 1$$

and

$$\frac{m_e \epsilon_\alpha(0)}{m_\alpha T_1} = 0.02 \ll 1,$$

where $\epsilon_\alpha(0) = 3.5 \text{ MeV}$ is the energy of an α -particle generated from D-T reactions, and m_e and m_α are the electron and α -particle mass, respectively. Using these limits, the expressions for $\tau^{\alpha/e}$ can be simplified accordingly. Trubnikov [14] showed that

$$\tau^{\alpha/D} = \frac{m_\alpha^{1/2} \epsilon_\alpha^{3/2}}{4\sqrt{2} \pi e^4 n \ln \Lambda} \quad (C2)$$

and

$$\tau^{\alpha/e} = \left[\frac{\sqrt{\pi}/4}{2\epsilon_\alpha/3T} \left(\frac{m_\alpha T}{m_e \epsilon_\alpha} \right)^{1/2} \right] \tau^{\alpha/D} \quad (C3)$$

For $\epsilon_\alpha(0) = 3.5$ MeV, it can be shown that $\tau^{\alpha/e} \approx \tau^{\alpha/D}/57$. Thus, at the time of birth of an α -particle, the rate of energy transfer to the electrons is about sixty times faster from an α -particle than to the deuterons. At the critical energy given by

$$\epsilon_{\text{acrit}} = \left(\frac{3\sqrt{\pi}}{8}\right)^{2/3} \left(\frac{m_\alpha}{m_e}\right)^{1/3} T, \quad ,$$

the rate of energy transfer from an α -particle to the electrons is equal to that to the ions. At $T_i = 19.8$ keV, $\epsilon_{\text{acrit}} \approx 0.29$ MeV. Thus, the thermonuclear α -particles lose most of their energy to electrons and for practical purposes, the α -particle energy transfer rate can be approximated by

$$\frac{d\epsilon_\alpha}{dt} = - \frac{\epsilon_\alpha}{\tau^{\alpha/e}}. \quad (C4)$$

Using Eqs. (C2) and (C3), the characteristic energy transfer time can be expressed as

$$\tau^{\alpha/e} = 1.7 \times 10^{-1} \frac{T_4^{3/2}}{n_{14}} \text{ sec}. \quad (C5)$$

Substituting the values of density and temperature of the plasma immediately after the compression into Eq. (C5) gives $\tau^{\alpha/e} \approx 0.05$ sec. Note that using Eq. (4), it can be shown that after the compression, the plasma energy confinement time $\tau_e \approx 0.32$ sec. Hence $\tau^{\alpha/e} < \tau_e$.

To obtain the temporal evolution of the plasma temperature after the compression, assume $T_e = T_i$ and use the volume-averaged energy balance equation for the plasma:

$$3n_1 \frac{dT}{dt} = P_\alpha(t) - [P_{neo}(t) + P_X(t)] \quad (C6)$$

In this equation, the turbulent convective energy loss is not included since we have assumed that there is no convective loss for $\beta < \beta_c$ (in reality, however, convective loss emerges gradually as β approaches β_c , as was mentioned in Appendix B). Let the time at the end of the compression phase be zero. Then, the power deposited by the α -particles to the background plasma per unit volume, $P_\alpha(t)$, at any time t after the compression can be expressed as

$$P_\alpha(t) = - \int_0^t dt' r_v^4 \frac{n_0^2}{4} \langle \sigma v \rangle_{t'} \frac{d\epsilon_\alpha}{dt}(t, t') \\ - \int_{-\infty}^0 dt' \left(\frac{1+2r_v^2}{3} \right) r_v^4 \frac{n_0^2}{4} \langle \sigma v \rangle_{t'} \frac{d\epsilon_\alpha}{dt}(t, t') \quad (C7)$$

Here, the first (second) term on the right-hand side of this equation represents the α -particle power deposition at any time $t > 0$ by all the α -particles generated in some intermediate time between $0 \leq t' \leq t$ ($0 \geq t' \geq -\infty$); n_0 is the density before the compression, and $d\epsilon_\alpha(t, t')/dt$ [given by Eq. (C4)] represents the rate of energy transfer at time t , of an individual α -

particle generated at some previous time t' . The factor $(1+2r_v^2)/3$ in the second term is the increase in α -particle energy due to two-dimensional adiabatic compression (compression is two-dimensional if the compression phase is carried out faster than $\tau_{\alpha 1}$ - the 90° deflection time between an α -particle and background ions).

Equation (C6) is an integrodifferential equation, but it can be converted into a second order differential equation by taking a time derivative. Using Eqs. (C4) and (C5), it can be shown that

$$\frac{d^2 \epsilon_\alpha}{dt^2} = - \frac{1}{\tau^{\alpha/e}} \frac{d\epsilon_\alpha}{dt} - \frac{3}{2} \left(\frac{1}{T} \frac{dT}{dt} \right) \frac{d\epsilon_\alpha}{dt} .$$

Using this expression, we find that the time derivative of Eq. (C7) can be written as,

$$\frac{dp_\alpha}{dt} = - \frac{P_\alpha}{\tau^{\alpha/e}} - \frac{3}{2} \left(\frac{1}{T} \frac{dT}{dt} \right) P_\alpha + r_v^4 \frac{n_0^2}{4} \langle \sigma v \rangle_t \frac{3.5 \text{ MeV}}{\tau^{\alpha/e}} ,$$

where $\langle \sigma v \rangle_t$ is the Maxwellian reactivity at time t . Then, after taking the time derivative of Eq. (C6), using the above expression for dp_α/dt , and performing some straightforward algebra, it can be shown that

$$\begin{aligned} & \tau^{\alpha/e} \frac{d^2 T}{dt^2} + \frac{3}{2} \frac{\tau^{\alpha/e}}{T} \left(\frac{dT}{dt} \right)^2 + \left[1 + \frac{\tau^{\alpha/e}}{n_1 T} (P_{\text{neo}} + P_x) \right] \\ & + \frac{1}{3} \frac{\tau^{\alpha/e}}{n_1} \frac{\partial}{\partial T} (P_{\text{neo}} + P_x) \left] \frac{dT}{dt} \right. \\ & = \frac{1}{3n_1} \left[(3.5 \text{ MeV}) \frac{n_1^2}{4} \langle \sigma v \rangle_t - (P_{\text{neo}} + P_x) \right] . \end{aligned} \quad (\text{C8})$$

This equation can be solved numerically. Two initial conditions are needed.

The first one is

$$T(0) = r_v^{4/3} T_0 \quad (C9a)$$

from the adiabatic law. [Note that $T(0)$ denotes the plasma temperature at time 0 (stage 1) and T_0 denotes the plasma temperature at stage 0.] To determine the second initial condition $dT/dt|_{t=0}$, we need to know $P_\alpha(0)$. If the adiabatic compression time is faster than $\tau_{\alpha i}$, then the α -particle heating power per unit volume immediately after the compression is increased from the corresponding heating power before the compression by the ratio

$$\begin{aligned} \frac{P_\alpha(0)}{P_\alpha(0^-)} &= \left(\frac{1+2r_v^2}{3}\right) \frac{n_1^2 [\epsilon_\alpha / \tau^{\alpha/e}(0)]}{n_0^2 [\epsilon_\alpha / \tau^{\alpha/e}(0^-)]} \\ &= \left(\frac{1+2r_v^2}{3}\right) r_v^6 . \end{aligned}$$

Here, 0^- denotes the time at the beginning of the compression. The second initial condition for Eq. (C8) can now be written as

$$\left. \frac{dT}{dt} \right|_{t=0} = \frac{1}{3r_v^2 n_0} \left[\left(\frac{1+2r_v^2}{3}\right) r_v^6 P_\alpha(0^-) - (P_{neo} + P_K) \right]_{t=0} \quad (C9b)$$

Using the initial conditions given by Eqs. (C9a) and (C9b), the solution of Eq. (C8) for the reactor with parameters given in Table I with $r_v = 1/0.6$ is obtained and plotted versus time as the solid curve in Fig. 8. This figure shows that the plasma temperature asymptotically approaches a limit of 35.5 keV.

If we let the thermal relaxation time for the plasma β to reach β_c be the time that it takes the plasma temperature to reach 90% of the limiting temperature at 35.5 keV, then this time is approximately 0.16 (see Fig. 8). This condition would represent the end of the heating phase.

Note that the thermal relaxation time is somewhat larger than $\tau^{a/e}$. In the limit that the relaxation time is much greater than $\tau^{a/e}$, Eq. (C7) can be simplified by assuming $\langle \sigma v \rangle_t = \langle \sigma v \rangle_c$. Furthermore, the second term on the right-hand side of Eq. (C7) is generally small compared to the first term and thus can be neglected. The power delivered to the plasma now becomes

$$P_a(t) = r_v \frac{4}{4} \frac{n_0^2}{4} \langle \sigma v \rangle_c t_a^e, \quad (C10)$$

which is the instantaneous fusion power. This result simplifies Eq. (C6) to a first-order equation. The temporal evolution of temperature described by this simplified equation is plotted versus time as the dashed curve in Fig. 8.

Finally, note that if the turbulent convective loss were included in the calculation, then the resulting power loss would be higher due to the turbulent convection as β approaches β_c and the rate of temperature rise would become slower. However, the thermal relaxation time would probably be about the same, since turbulent convection also forces the asymptotic limit of the plasma temperature to drop to a lower value at which $\beta = \beta_c$.

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TABLE I. STELLARATOR REACTOR PARAMETERS

Plasma density (cm^{-3})	6×10^{14}
Plasma temperature (keV)	10
Toroidal magnetic field (kG)	50
Plasma β (%)	19.3
Major radius (m)	10
Minor radius (m)	2
ϵ_h	0.1

FIGURE CAPTIONS

FIG. 1. Minor cross section of a stellarator reactor at various stages during the thermonuclear inverse magnetic pumping power cycle.

FIG. 2. Net work done normalized to machine dependent parameters, $w_{\text{norm}} = w_{\text{net}} / [((1/8)a_0 L \beta_c \{1/(1+\beta_c)\} B_{e0}^2)]$, versus compression ratio r_v . (a) Power cycle with heating phase at constant plasma volume. (b) Power cycle with heating phase at constant plasma pressure.

FIG. 3. Pressure-volume (P-V) schematic. Path 0 + 1 is the compression phase. Paths 1 + 2 and 1 + 3 are, respectively, the heating phase at constant plasma volume and constant plasma pressure. Path 2 + 0 (3 + 0) is the expansion phase.

FIG. 4. Thermal efficiency η_{th} versus compression ratio r_v . (a) Power cycle with heating phase at constant plasma volume. (b) Power cycle with heating phase at constant plasma pressure.

FIG. 5. Temperature-entropy (T-S) schematic. Path 0 + 1 is the compression phase. Paths 1 + 2 and 1 + 3 are, respectively, the heating phase at constant plasma volume and constant plasma pressure. Path 2 + 0 (3 + 0) is the expansion phase.

FIG. 6. Schematic representation of currents in coils and electrical energy flow between reactors as a function of time for a coupled reactor system (for the power cycle with heating phase at constant plasma volume). I_0 is the current in the coils when the reactor is at stage 0 and τ_p is the period of one complete cycle.

FIG. 7. Comparison of heating power with power losses for the plasma in the stellarator with machine parameters given in Table I. The solid line (—) represents α -particle heating power per unit volume, the dashed line (---) represents power loss per unit volume due to neoclassical diffusion and bremsstrahlung radiation, and the broken line (- - -) represents the total power loss per unit volume (---- plus turbulent convection). Point A is an unstable thermal equilibrium, points B and C are stable thermal equilibria.

FIG. 8. Temporal evolution of the plasma temperature after adiabatic compression for the reactor with parameters given in Table I. The solid curve is obtained using Eq. (C8) and the dashed curved is obtained using the approximation for $P_\alpha(t)$ given by Eq. (C10).

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Fig. 1

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Fig. 2

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Fig. 3

Fig. 4

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Fig. 5

#85T0158

Fig. 6

Fig. 7

Fig. 8

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