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The Mechanism of Current Modulation
by Optic Phonon Emission
in Heterojunction Tunneling Experiments

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Abstract

We explain recent observations by Hickmott *et al.* of sequential longitudinal optic phonon emission in tunneling currents of GaAs-Al_xGa_{1-x}As heterojunctions in terms of inhomogeneous tunneling and a magnetopolaronic mass correction.

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I. Introduction

Hickmott *et al.*¹ recently reported periodic structure in the tunneling current of GaAs-Al_xGa_{1-x}As heterojunctions which they identified with sequential longitudinal-optic phonon emission. Sequential phonon emission was previously observed in point-junctions by Cavenett *et al.*² and in recent experiments on In-InGaAs contacts by Lu *et al.*³ The heterojunction experiment differs from these in that it happens only when a strong magnetic field (>4T) is applied and occurs in the controlled environment of a sample grown by molecular beam epitaxy⁴. This experiment has attracted a great deal of theoretical interest^{5,6} because it is difficult to understand the mechanism by which sequential phonon emission affects the tunneling current. The tunnel barrier is only about 200Å thick. Electrons are believed to tunnel through this barrier and accelerate ballistically through a one micron lightly doped region. Each time an accelerated electron's kinetic energy reaches the longitudinal optic phonon energy of 0.036eV, the electron is thought to emit a phonon and stop. This happens repeatedly until the electron traverses the region through which the voltage is dropped. The difficulty with this picture is that the phonon

emissions occur after the electrons have tunneled. Since the measured current is equal to the number of electrons per unit time that traverse the tunnel barrier, inelastic events which occur after tunneling should not affect the current. To understand this experiment, we must explain how phonon emission communicates with the tunnel barrier. Two possibilities immediately present themselves:

- 1) The emission of phonons quantum-mechanically reflects the electrons back through the barrier.
- 2) Electrons stopped by phonon emission produce an electric field at the tunnel barrier through space-charging.

The first possibility may be ruled out in this experiment because the barrier transmittance is too low. Using a tunnel barrier height of approximately $\Delta = 0.4 \text{ eV}$ and defining $\hbar^2 Q^2 / 2m = \Delta$, where $m \approx 0.07 m_0$ is the band effective mass for GaAs, gives a barrier transmittance of $T \sim \exp(-2Qd) \sim 3 \times 10^{-16}$, where $d \sim 200 \text{ \AA}$ is the barrier thickness.¹ Hence electrons reflected by phonon emission have an exceedingly low

probability of tunneling back through the barrier. This leaves the space charge effect as the prime candidate. Paradoxically, an elementary estimate of the size of the space charge effect shows that it would be 10^6 times too small. The purpose of this paper is to point out that there is strong evidence that the space charge mechanism is in quantitative agreement with the existing experimental data. We shall argue that the current is not tunneling uniformly, but rather through a small "window" approximately one micron in size. This window might be an extrinsic defect such as a thin region of the barrier, or an intrinsic feature like a corner. The concentration of the current to a small area enhances the size of the space charge effect by a factor of approximately 10^4 . We attribute the remaining factor of 100 to a magnetopolaronic mass correction effect suggested by one of us in a previous paper.⁷

The plan of this paper is as follows. In Part II, we review the experimental geometry and the depletion physics of the heterojunction device in the presence of magnetic fields. In Part III, we discuss the physics of tunneling and present evidence that the real tunnel barrier conducts in a highly ideal manner. This will imply that the "magnetic freezeout" reported by Hickmott *et al.*¹ is most likely a rate effect

irrelevant to the tunneling experiment. In Part IV, we introduce a model to explain the tunneling curves, and infer from this model that the current is being channelled through a small window. In Part V, we discuss the polaronic correction to the electron mass when it approaches its phonon emission threshold in the presence of a magnetic field. In Part VI, we combine our understanding of the depletion physics with the polaronic mass correction to make a quantitative theory of the periodic oscillations in the current. We summarize our results in Part VII.

II. Magnetic Freezeout

Hickmott *et al.*¹ reported the observation of magnetic freezeout of the lightly doped region of the device and invoked it to help explain the role of the magnetic field in producing phonon oscillations of the current. We believe that freezeout does not occur on the time scale of the experiment, and thus that it does not facilitate the observation of sequential phonon emission. The evidence for this is as follows.

Figure 1 shows the geometry of the experiment.¹ Electrons go from

right to left, tunneling from a metallic n^+ region of Si-doped GaAs through a 200Å barrier of $Al_xGa_{1-x}As$ into a one micron lightly doped region of GaAs bounded by another heavily doped n^+ region. The doping density p in the lightly doped region is approximately 10^{15} cm^{-3} .

In the absence of a magnetic field, the lightly doped region is expected to deplete classically. This is illustrated in figure 2. In classical depletion, donors are totally ionized whenever an electric field is present. This divides the lightly doped region into two parts: the voltage drop occurs across a totally ionized layer of width w next to the barrier, while the remaining volume is free of electric fields and remains un-ionized. The depletion width w adjusts itself to make the total potential drop across the sample equal to the applied voltage V :

$$\frac{2\pi p w^2}{\epsilon} = V \quad (1)$$

We take the static dielectric constant of GaAs to be $\epsilon \approx 13$. The electric field E at the tunnel barrier is then

$$E = \frac{4\pi\rho w}{\epsilon} = (8\pi\rho V/\epsilon)^{1/2} \quad (2)$$

This leads to a differential capacitance C of the depletion layer:

$$C = \frac{d}{dV} \frac{\epsilon EA}{4\pi} = A \left(\frac{\epsilon\rho}{8\pi(V+V_{SH})} \right)^{1/2} \quad (3)$$

where $A = 4 \times 10^{-4} \text{ cm}^2$ is the sample area and we have added a flat-band shift voltage V_{SH} . Depletion causes the the charge accumulated on the capacitor (and thus electric field at the tunnel barrier) to be larger than it would be in the absence of depletion.

The solid lines of figure 3 show the capacitance as a function of voltage as measured by Hickmott *et al.*¹ in the presence of magnetic fields from 0 to 14T oriented perpendicular to the sample faces. These measurements were made using a derivative technique in which a DC bias V is applied to the sample together with a small oscillatory component $\delta V \cos(\omega t)$. The current response was analyzed with a circuit model to determine the capacitance. The capacitance was reported to be independent of frequency between 1kHz and 1MHz.⁴ The curve at 0T exhibits classical depletion. The X's are a fit to the data using equation 3

with $p = 1.1 \times 10^{15} \text{ cm}^{-3}$ and $V_{\text{SK}} = 0.03 \text{ V}$, so that the depletion width will have grown to consume the entire one micron lightly doped region when the applied voltage V reaches roughly one volt. One concludes from the quality of this fit that there is nothing unusual about the depletion physics at OT and that indeed, one knows the size of the depletion width w with some confidence.

As the magnetic field increases, the correction to the experimental capacitance due to depletion diminishes until, in the high-field limit, it becomes constant at the value

$$C = \frac{\epsilon A}{4\pi L} \approx 4.7 \text{ pF} \quad (4)$$

here $L = 1 \mu\text{m}$ is the length of the lightly doped region. This is exactly the behavior one would expect if donors were not ionizing because of magnetic freezeout.

A magnetic field produces freezeout by increasing the binding energy of the donors, thereby lowering the rates of thermal and field ionization. The binding energy increase becomes significant when the cyclotron

energy $\hbar\omega_c$ becomes comparable to the impurity ionization energy E_B :

$$\hbar\omega_c \sim E_B \quad (5)$$

This occurs when the magnetic field is approximately 4T. If the donor ionization rate is sufficiently small that the donors do not ionize over the time scale of the experiment measurements, then they are said to be frozen out.

Hickmott *et al.*¹ inferred from the data in figure 3 that the lightly doped region was frozen out. This is conceivable, since the temperature of the sample was $\sim 1.6^\circ\text{K}$. However, it is significant that at intermediate field strengths, where the depletion behavior is less clear-cut, the capacitance data is fit well by an expression of the form

$$C = \frac{\epsilon A}{4\pi L} \frac{1}{[(V+V_{SK})/V_I + D^2/L^2]^{1/2}} \quad (6)$$

where D is a length that grows continuously from zero to one micron as the

magnetic field increases. $V_1 = 2\pi\rho L^2/\epsilon = 0.75V$ is the voltage at which the $1\mu m$ lightly doped region becomes completely ionized. The X's in figure 3 are plots of equation 6 with the values for D given in Table 1.

Table 1

<u>H (Tesla)</u>	<u>D(microns)</u>
0	0.00
2	0.04
4	0.25
6	0.56
8	0.72
10	0.80
12	0.88
14	0.98

Equation 6 is the result of a model calculation in which a region of size D adjacent to the tunnel barrier is prevented from depleting. That is, one has

$$V = \frac{2\pi\rho w^2}{\epsilon} + \frac{4\pi\rho wD}{\epsilon} - V_{SH} \quad (7)$$

While it would be inappropriate to interpret the capacitance model literally, it is significant that the purported freezeout effect is much larger at low voltages than at large ones. This is peculiar, since there is no reason for the donors near the tunnel barrier to be more difficult to ionize than

those far from the barrier. One would actually expect the opposite to be true, since the electric fields near the tunnel barrier are larger. The easiest way to understand this behavior is to assume that the system is not frozen out even when the magnetic field is large, but that instead there is a series resistance between the ohmic contact and the depletion front which increases with magnetic field. This slows down the flow of positive charge to and from the depletion front. For example, positive charge could be diffusing into the depletion front as holes hopping from one donor to the next. If this were the case, then donors far from the ohmic contact would take longer to ionize than those close to the contact, and the disparity would increase with the strength of the magnetic field.

The strongest evidence against freezeout is the incompatibility of the capacitance and tunneling measurements. The tunnel current is depressed by less than 20% at $V=0.75V$ in the presence of a 14T magnetic field normal to the surface.^{1,9} This is consistent with one's expectation that the tunnel barrier conductance should be fairly insensitive to the magnetic field. It is not consistent with the occurrence of freezeout, however, because the electric field at the tunnel barrier of a frozen sample is vastly smaller than that of an unfrozen one. This may be quantified in the

following way. Let us assume that the sample at 14T is frozen out and thus that the electric field at the tunnel barrier is proportional to the applied voltage V:

$$E_{14T} = V_{14T}/L \quad (8)$$

The measured tunnel current $I_{14T}^{\text{experiment}}(V)$ is then a readout of the barrier conductance which one may use with the known depletion physics to predict the tunnel current at 0T:

$$I_{0T}^{\text{frozen}} = I_{14T}^{\text{experiment}} [2(V_1 V)^{1/2}] \quad (9)$$

This theoretical current is compared with the measured one^{1,9} in figure 4. The poor agreement leads one to conclude that either the sample is not frozen out or the tunnel conductance is strongly magnetic field dependent. The latter possibility is unlikely on theoretical grounds.

III. Tunnel Barrier

In this section we discuss the conductance of an ideal tunnel barrier and compare it with the tunneling current measured in the experiment. We shall argue that the tunneling current is proportional to a simple power of the electric field E in the barrier: $I \sim E^\alpha$ where $\alpha \sim 2$, and that the effect of the magnetic field is simply to subtract a constant from E .

One expects the tunnel current to be a power law rather than an exponential because the voltage V_B dropped across the barrier is so much smaller than its height Δ . The change to the WKB tunneling amplitude T of a barrier of height $V(x)$ due to a perturbation $\delta V = V_B x/d$, where d is the barrier thickness, is

$$\begin{aligned} \delta T/T &= (2m/\hbar^2)^{1/2} \left[\delta \int V^{1/2} dx \right] \\ &= \frac{1}{4} \frac{V_B}{\Delta} \ln(T) \end{aligned} \quad (10)$$

with $\Delta=0.4\text{V}$, $d=200\text{\AA}$, and $V_B < 0.02\text{V}$, one obtains $\delta T/T < 0.2$.

Given that changes to the tunnel conductance are small, one can model the tunneling problem with a flat barrier as shown in figure 5. The density of donors in the n^+ region is $\sim 10^{18}\text{cm}^{-3}$, so that $E_F \approx 0.07\text{eV} \ll \Delta$. The flat-barrier idealization has a simple exact solution. Within an accuracy of $E_B/\Delta \sim 0.15$, the current density is

$$J = e^{-2Qd} \frac{(1+x)}{16} \left[x^{1/2} + x^{3/2} + \frac{(1-x)^2}{2} \ln \frac{1+\sqrt{x}}{1-\sqrt{x}} \right] - \frac{x^{3/2}}{6} \quad (11)$$

where $x=V_B/E_F$, $\hbar^2 Q^2/2m=\Delta \sim 0.4\text{eV}$, $d \sim 200\text{\AA}$, and

$$J_0 = \frac{16}{\pi^2} \frac{emE_F^2}{\hbar^3 \Delta} E_F \quad (12)$$

The low-voltage limit of this expression is the power law $I \sim V_B^{2.5}$. In summary, the current is expected to behave as a power law rather than an exponential, because the barrier transmittance is nearly independent of voltage, for the voltages of interest.

Equation 11 was derived by assuming that the barrier height shown in figure 5 drops abruptly from $\Delta=0.4\text{V}$ to zero at the barrier edge. In reality the barrier edge is probably sloped, due to structural irregularities or charged impurities. If the slope is sufficiently gradual so that the fractional change in the tunnelling electron's momentum over one de Broglie wavelength is small, a WKB treatment of the problem becomes appropriate.¹⁰ This condition will be satisfied in this experiment providing that the length scale over which the barrier height drops from Δ to zero is larger than about $10\text{\AA}$, or approximately two atomic layers. This is plausible since the tunnel barrier is a random alloy of $\text{Al}_x\text{Ga}_{1-x}\text{As}$, and it is difficult to engineer the barrier edge to within $10\text{\AA}$. The WKB treatment yields

$$J = \frac{2\pi em}{(2\pi\hbar)^3} e^{-2Qd} V_B^2 \quad (13)$$

Since the data are more consistent with the V_B^2 law, we conclude that the WKB model is more applicable to the experiment. The reason that the current goes as V_B^2 instead of V_B , as is the case in metal-insulator-metal

junctions, is that at small V_B electrons leave the barrier at speeds $(2V_B/m)^{1/2}$ rather than at the Fermi velocity.

Given the similar shapes of the current-voltage curves, one might suspect that these curves are simply shifted from one another in an elementary fashion. If the barrier conductance is not magnetic field dependent, we would expect the shifts to enter through the electric field E at the barrier, since $I \sim E^2$. This is in fact case. In figure 6 we have mapped the various I - V curves^{1,9} in figure 6a onto the single curve of figure 6b via the transformation

$$I \rightarrow (I^{1/2} - \beta)^2 \quad (14)$$

where β is a constant depending only on magnetic field. Because of the nonlinearity of the curves, the quality of this fit is highly significant. We interpret this result in two ways:

1. It corroborates the $I \sim V_B^2$ law.

2. It identifies the cause of the current suppression to be a static shift in V_B proportional to β .

The magnetic field dependence of β is shown in figure 7. It is a parabola. Converting β to a potential shift δV_B and comparing it with the diamagnetic energy $E_{dis} = m\omega_c^2 a^2/2$ where $a \approx 100\text{\AA}$ is the Bohr radius of the donor electron in GaAs, we obtain $\delta V_B/E_{dis} \approx 2.6 \times 10^{-3}$. Without additional experimental information it is difficult to identify the mechanism which produces such a quadratic shift in the electric field, although there are two obvious possibilities. It might be due to the effect of the magnetic field on the correlation energy of the degenerate regions from which the electrons are injected. It might also be an incipient localization effect.¹¹ In the case of an isolated atom in a magnetic field H , the ionization energy is directly proportional to H , to lowest order. This is because the continuum energy is shifted by $\hbar\omega_c/2 \sim H$, while the shift in the ground state is of order H^2 . On the other hand, the magnetic field produces no change in the ionization energy of jellium, since the electrons are not localized. The presence of impurities would presumably produce a situation intermediate between a

localized electron in an atom and an extended one in jellium and perhaps produce a shift proportional to H^2 .

IV. Inhomogeneous tunneling

In this section, we examine the experimental current-voltage curves and argue that the electric field at the point of tunneling is smaller than the average value, as measured by the capacitance. We explain the disparity with a model in which the electrons tunnel only through a window approximately $1\mu\text{m}$ in diameter and neutralize the impurities in their path.

The combined current and capacitance data are inconsistent with uniform tunneling. This is shown in figure 8 for the current^{1,9} at OT. The dashed line results from combining the uniform depletion behavior of the OT capacitance data, which predicts (see equation 2):

$$E \sim V^{1/2} \quad (15)$$

with the current law $I \sim E^2$, to give $I \sim V$. The actual current, drawn as a solid line, is quadratic in the voltage for $V < 0.4V$, implying that $E \sim V$. Hence the electric fields measured by the current and capacitance data behave very differently:

$$E_{I-V} \sim V \neq E_{C-V} \sim V^{1/2} \quad (16)$$

Equation 16 implies that at low voltages the local electric field producing the current (proportional to V) is smaller than the average electric field measured by the capacitance (proportional to $V^{1/2}$). This has two important implications:

1. Electrons are tunneling through a small fraction of the barrier surface. Otherwise the tunneling electric field would equal the average field.
2. The barrier at the point of tunneling is thinner than average. There is no other reason for electrons to tunnel at a point with smaller than

average electric field.

One can account for both behaviors with a simple model. Let us assume that electrons tunnel only through a single small circular region of radius R where the tunnel barrier is thinner than average, as illustrated in figure 9. Let us also assume that impurity atoms in the cylindrical path of the tunneling electrons are neutralized because the electron current density is so high in that region. The electric field in the center of the cylinder at the tunnel barrier will be

$$E = \frac{V_R}{R} (KV/V_R + 1)^{1/2} - 1 \quad (17)$$

where $V_R = 2\pi\rho R^2/\epsilon$. Since $I \sim E^2$, equation 17 implies that

$$I = I_0 [(1 + V/V_R)^{1/2} - 1]^2 \quad (18)$$

Using $I_0 = 1.63 \times 10^{-7} \text{ A}$ and a cylinder radius of $0.75 \mu\text{m}$, we obtain the close fit to the OT current data^{1,9} shown in figure 10. The parameter V_R describes the voltage at which the current-voltage curve changes from

quadratic to linear in V , and is a direct measure of the cylinder radius R . Although this model does not necessarily correspond in detail to the experimental geometry, it does demonstrate how inhomogeneous tunneling helps us to make sense of the experimental data. It describes the shape of the I - V curves quite well, explains why the electrons tunnel through a region of smaller electric field, and provides us with an estimate of the area of the tunneling region.

Note that a cross-sectional area of $1\mu\text{m}^2$ increases the charge density of the tunneling electrons by a factor of more than 10^4 , compared to the case of uniform tunneling. This dramatically increases the space charge effects of tunneling electrons stopped by phonon emission.

V. Magnetopolarons

In this section, we explain how an electron moving through a polar semiconductor in the presence of a magnetic field forms a polaron which

may travel a thousand times slower than a free electron. We argue that a build-up of these charged polarons in the undepleted region of the device is responsible for the periodic modulation of the current, and that the role of the magnetic field in the experiment of Hickmott *et al.*¹ is to make the electron transport one-dimensional.

The origin of the magnetopolaron can be understood qualitatively as follows. Consider the motion of an electron through a lattice, in the absence of electron-phonon coupling. For kinetic energies below the phonon energy, the state of the system is that of a free electron with no phonons. When the kinetic energy of the electron reaches the phonon energy, the zero-phonon/one-electron state is degenerate with states consisting of one phonon and a zero energy electron. The electron-phonon interaction in a polar semiconductor such as GaAs mixes these two types of states to produce a new state of lower energy, pinned below the LO phonon energy of 0.036eV. We shall refer to this new state as a magnetopolaron. It should not be confused the usual meaning of the term associated with the cyclotron resonance condition $\hbar\omega_c = 0.036\text{eV}$.¹² For GaAs, the energy splitting induced by the electron-phonon interaction in the absence of a

magnetic field is negligibly small. However, when the free electrons are confined to one dimension by a magnetic field, the energy lowering is strongly enhanced because of the divergent density of states at zero energy.

The Hamiltonian for this system is most conveniently expressed in the Landau gauge, where the vector potential $\mathbf{A}$ is chosen to be

$$\mathbf{A} = -Hy \mathbf{x} \quad (19)$$

The magnetic field $\nabla \times \mathbf{A} = H \mathbf{z}$ then enters the single-electron Hamiltonian in the manner

$$\begin{aligned} H &= (\mathbf{p} - e\mathbf{A}/c)^2/2m \\ &= [p_y^2/2m + m\omega_c^2(y - y_0)^2/2] + p_x^2/2m \end{aligned} \quad (20)$$

where m is the band effective mass and the term in brackets describes a harmonic oscillator of frequency $\omega_c = eH/mc$, centered at $y_0 = -cp_x/eH$. The normalized eigenfunctions for this Hamiltonian are specified by three

quantum numbers:

$$|k_z, k_x, n\rangle = \frac{\exp(ik_z z)}{L_z^{1/2}} \frac{\exp(ik_x x)}{L_x^{1/2}} \psi_n(y-y_0) \quad (21)$$

where $\hbar k_x = p$, and ψ_n is a normalized harmonic oscillator wavefunction:

$$\psi_n(y) = \frac{e^{-t^2/2}}{(\pi 2^{n+1} n!)^{1/2}} \left(\frac{\partial}{\partial t}\right)^n e^{-t^2} \quad (22)$$

Here $t = y/a_H$, where a_H is the magnetic length $(\hbar/m\omega_c)^{1/2}$. The energy eigenvalues are degenerate in k_x and depend on the Landau level index n and parallel momentum k_z through

$$E_{k_z, k_x, n} = \hbar^2 k_z^2 / 2m + (n + 1/2) \hbar \omega_c \quad (23)$$

Hence the unperturbed many-body Hamiltonian for phonons and free electrons may be written as

$$H_0 = \sum_{k_z, k_x, n} E_{k_z, k_x, n} c_{k_z, k_x, n}^\dagger c_{k_z, k_x, n} + \sum_{\mathbf{q}} \hbar \omega_{\mathbf{q}} a_{\mathbf{q}}^\dagger a_{\mathbf{q}} \quad (24)$$

where $c^\dagger$ (c) creates (destroys) an electron described by the single-body wavefunction $|k_z, k_x, n\rangle$, and $a^\dagger$ (a) creates (destroys) a phonon of momentum $\mathbf{q}$ and frequency $\omega_{\mathbf{q}}$. The electron-phonon coupling is described by the Frohlich interaction¹³:

$$H_1 = \sum_{\mathbf{q}, n, n'} \sum_{k_z, k_x} V_{\mathbf{q}, nn'} c_{k_z - \mathbf{q}_z, k_x - \mathbf{q}_x, n}^\dagger c_{k_z, k_x, n'} (a_{\mathbf{q}}^\dagger + a_{-\mathbf{q}}) \quad (25)$$

where the matrix element $V_{\mathbf{q}, nn'}$ is given by

$$V_{\mathbf{q}, nn'} = U_{\mathbf{q}} \sum_{k_x, k_x'} \langle k_z, k_x, n | e^{-i\mathbf{q} \cdot \mathbf{r}} | k_z', k_x', n' \rangle \quad (26)$$

with

$$U_{\mathbf{q}} = \hbar \omega_{\mathbf{q}} \left(\frac{4\pi\alpha}{\Omega k_{LO}^3} \right)^{1/2} \frac{k_{LO}}{q} \quad (27)$$

In these expressions, $\hbar^2 k_{LO}^2/2m = \hbar\omega_q \approx 0.036\text{eV}$, Ω is the volume of sample, and α is the coupling constant for GaAs:

$$\alpha = \frac{1}{2} \frac{e^2 k_{LO}}{\hbar \omega_q} (1/\epsilon_\infty - 1/\epsilon_0) \approx 0.07 \quad (28)$$

In GaAs, the optical and static dielectric constants have the values $\epsilon_\infty \approx 11$ and $\epsilon_0 \approx 13$, respectively. In general, the interaction Hamiltonian H_I couples states of a given Landau level to all the others. However, in strong magnetic fields such that $\hbar\omega_c \gg \alpha \hbar\omega_q$, the contribution of the lowest Landau level will predominate, and we choose a variational wavefunction composed only of states in the lowest ($n=0$) Landau level. This makes the problem one-dimensional and greatly simplifies the calculation.

The simplest possible trial wavefunction consists of a superposition of zero- and one-phonon states:

$$|\Psi_{\mathbf{k}}\rangle = (\alpha_{\mathbf{k}} c'_{\mathbf{k}} + \sum_{\mathbf{q}} \beta_{\mathbf{k}\mathbf{q}} c'_{\mathbf{k}-\mathbf{q}} a'_{\mathbf{q}}) |0\rangle \quad (29)$$

In the presence of a magnetic field, the quantum index $\mathbf{k}$ is shorthand for the quantum numbers (k_x, k_y, n) . We wish to choose the coefficients $\alpha_{\mathbf{k}}$ and $\beta_{\mathbf{k}\mathbf{q}}$ so that the variational energy estimate

$$E(\mathbf{k}_z) = \langle \Psi_{\mathbf{k}} | H_0 + H_1 | \Psi_{\mathbf{k}} \rangle \quad (30)$$

is minimized, subject to the normalization condition

$$1 = \langle \Psi_{\mathbf{k}} | \Psi_{\mathbf{k}} \rangle = |\alpha_{\mathbf{k}}|^2 + \sum_{\mathbf{q}} |\beta_{\mathbf{k}\mathbf{q}}|^2 \quad (31)$$

One obtains

$$E(\mathbf{k}_z) = E_0(\mathbf{k}_z) + \sum_{\mathbf{q}} \frac{|\mathbf{V}_{\mathbf{q}}^{00}|^2}{E(\mathbf{k}_z) - [E_0(\mathbf{k}_z - \mathbf{q}_z) + \hbar\omega_{\mathbf{q}}]} \quad (32)$$

where $E_0(\mathbf{k}_z) = \hbar^2 \mathbf{k}_z^2 / 2m$ and

$$|\mathbf{V}_{\mathbf{q}}^{00}|^2 = U_{\mathbf{q}}^2 \exp\left\{-\frac{\hbar}{2m\omega_c} (\mathbf{q}_x^2 + \mathbf{q}_y^2)\right\} \quad (33)$$

The lowest-energy solution of equation 32 is a polaron with a "binding energy" $\Delta E = \hbar\omega_{\mathbf{q}} - E_0(k_z) < 0$ below 0.036eV. Because the energy $E(k_z=0)$ of the polaron at rest is lowered below zero, the emission threshold is also lowered. A more careful treatment of the problem using a variational wavefunction which includes states with more than one phonon¹⁴ alleviates this problem and gives a value of ΔE which is about 40% lower than that of equation 32.⁷ The results of doing this for various magnetic field strengths are the dashed dispersion curves shown in Figure 11. We emphasize that because the energy calculation is variational, it underestimates the strength of the magnetopolaron binding energy. The sloped line in figure 11 shows the energy-momentum relation of a free electron state, while the one-phonon state is represented by a flat line, corresponding to the dispersion curve of an LO phonon in GaAs. An important consequence of the polaron dispersion relation is that the region just below the phonon energy has a negative effective mass. This means that a small electric field parallel to the magnetic field can have the paradoxical effect of slowing the polaron under conditions that would accelerate a free electron. This is because the electric field changes the wave vector k of the particle, while the velocity of the polaron is given by

the slope of the dispersion curve:

$$v = \frac{1}{\hbar} \frac{\partial E}{\partial k} \quad (34)$$

Figure 12 shows a plot of the calculated group velocities versus wave-vector at 14T, in units of $v_{LO} = (2\hbar\omega_{LO}/m)^{1/2}$. The velocity peaks near $k = k_{LO} = mv_{LO}$, and then quickly falls asymptotically to zero.

In the presence of strong electric fields, impurity scattering, or phonon decay mechanisms, the magnetopolaron eventually decays into a free electron plus a real phonon. We expect that the polaron will field-ionize when the electric field E is large enough so that

$$eEr > \frac{\hbar^2}{2mr^2} \equiv \Delta E \quad (35)$$

where r is a measure of the size of the polaron, as estimated from its binding energy ΔE . Impurity scattering at low magnetic fields relaxes momentum conservation, allowing a polaron of momentum k_z to decay into a phonon of momentum q plus a free electron. Since the phonon energy

$\hbar\omega_q \approx 0.036\text{eV}$ actually decreases with $|q|$, it can become lower than the polaron energy $E(k_z)$. When the magnetic field is sufficiently strong to suppress scattering, then in the absence of electric fields we expect that the emission of a real phonon occurs when the energy uncertainty for phonon decay equals the energy gap between the polaron and free electron states. Since the rate for an LO phonon in GaAs to decay into acoustic phonons is about 7ps^{15} , the energy uncertainty is roughly half of $0.1\text{meV} = \hbar/7\text{ps}$. This energy uncertainty is small enough so that at the magnetic field strengths used in this experiment, the velocity of the polarons can be decreased by a factor of several hundreds compared to the peak velocity. Since the tunneling current is continuous, this implies that a large charge density can accumulate in the field-free region. The resultant space charge then feeds back to the tunnel barrier and modulates the current.

VI. Phonon Oscillations

In this section we explain the mechanism for the periodic modulation of the tunneling current in terms of the inhomogeneous tunneling of magnetopolarons. To do this, we add space charging to the simple "circular

"window" model of nonuniform tunneling used in Part IV. As before, we assume that electrons tunnel only through a small circular window of radius R , neutralizing all impurity atoms in their path. When polarons are injected into the undepleted region with an energy just below the phonon energy, they drift with very small group velocities v_D , producing a polaron charge density $\delta\rho$. The electric field δE due to $\delta\rho$ is responsible for the periodic dips δI in the current. The equations needed to calculate δI are given as follows:

$$\delta I/I = 2\delta E/E \quad (36)$$

$$\delta E = (V_I/L)(1-w/L)^2 \delta\rho/\rho \quad (37)$$

$$\delta\rho = \frac{I/\pi R^2}{v_D} \quad (38)$$

$$I = I_0(EL/V_I)^2 \quad (39)$$

$$E = V_R/R [(V/V_R+1)^{1/2} - 1] \quad (40)$$

$$v_D/L = (V/V_I)^{1/2} \quad (41)$$

The relevant constants are $\epsilon=13$, $\rho=1.1 \times 10^{15} \text{cm}^{-3}$, $L=1\mu\text{m}$, $R=0.75\mu\text{m}$, and

$$V_I = 2\pi\rho L^2/\epsilon = 0.75\text{V} \quad (42)$$

$$V_R = (R/L)^2 V_I = 0.42\text{V} \quad (43)$$

The origins of these equations are as follows. Equation 36 is a consequence of the current law $I \sim E^2$. To obtain equation 37, we estimate the strength of the space charge electric field at the tunnel barrier by calculating the electric field due to an infinite plane of charge of thickness $L-w$ at a distance w from the barrier, remembering that the voltage drop across the device is fixed at a given value V . Equation 38 follows from the definition of the current density. Equations 39, 40, and 41 were obtained from the current law $I \sim E^2$ and from equations 17 and 1, respectively.

The size of the current dip depends on the magnetic field through the drift velocity v_D . This velocity is difficult to estimate at low magnetic fields when impurity scattering is important, without a reliable value for the

collision time. However, at high magnetic fields, we can estimate the lifetime of the polaron by using the LO phonon decay time of 7ps. This gives a value of δI at 14T which is in good agreement with the experiment. If ΔV is the total voltage drop across the device modulo the phonon energy $\hbar\omega_{LO}=0.036\text{eV}$, then polarons emerge from the depletion region with an energy E such that

$$\hbar\omega_{LO} - E = \Delta V \quad (44)$$

We assume that such that the smallest value ΔV can have is equal to the energy uncertainty ΔE_{gap} , corresponding to the decay time due to collisions or phonon decay:

$$\Delta E_{\text{gap}} \sim 0.5(\hbar/7\text{ps}) = .05 \text{ meV} \quad (45)$$

In the absence of faster decay mechanisms, the polaron decay mechanism provides a rough cut-off which gives the minimum drift velocity and therefore the maximum current dip.

Since the values of v_{LO}/v_D at low magnetic fields cannot be estimated using equation 45, we have approximated them by linearly interpolating from zero at 4T, where the current oscillations are first visible, to the value of 1000 at 14T predicted by using equation 45. In figure 13, we have plotted the differences in the current as a function of voltage, by subtracting the current at 4T, in order to make the phonon bumps clearly visible. Figure 13a shows the experimental data^{1,9}, while figure 13b is obtained by using the interpolated values, divided by two. We have modeled the bumps as gaussians with a maximum amplitude δI calculated from equation 36, with an artificial width of 7meV to match the experiment. This is *ad hoc* but not unreasonable in our opinion. We think it likely that the width is due to coulombic repulsions in the polaron gas or to the diffusion of polarons out of the cylindrical "window" region, both of which are difficult to estimate reliably. Figures 13a and 13b are closely matched, up to a phase which we believe is not reliable. Experiments done by Guimares *et al.*¹⁶ have observed minima near multiples of the phonon energy, as predicted by our theory. It is especially noteworthy that the bumps die off near 0.6V, with a maximum around 0.3V. This is a

consequence of equations 37 and 38, which say that the current dip δI decreases with the depletion width and increases with the current. The theoretical values for the size of the current dips are in good agreement with the experiment. That one must divide the values of Table 2 by two to fit the data is not unreasonable, given the uncertainty of the polaron cut-off mechanism.

VII. Summary

In this paper, we presented a detailed analysis of the current and capacitance data of the experiment of Hickmott *et al.*⁹ From our careful examination of the data, we drew several important conclusions.

We argued that magnetic freezeout is not relevant to understanding the experiment. The strongest evidence for this is the incompatibility of the current and capacitance data when freezeout is assumed, as illustrated in figure 4. Furthermore, we were able to fit the capacitance curves quite closely with a simple model which behaves in a manner contrary to the behavior expected from freezeout. We concluded that either the sample

depletes classically or the tunnel conductance depends strongly on the magnetic field.

We found that the tunneling current is proportional to the square of the barrier electric field, and that the only effect of the magnetic field on the tunnel conductance is to depress the current by reducing the voltage drop across the tunnel barrier. We found that the tunneling curves at different magnetic fields were all related by a simple transformation, which was interpreted physically as indicating a shift in the barrier voltage. We also found the remarkable fact that the barrier voltage shift is quadratic in the magnetic field strength, as illustrated in figure 7.

We argued that electrons tunnel through a region of the sample that is different from the rest. Comparison of the current and capacitance data led us to conclude that the electric field producing the tunneling must be smaller than the average electric field. Hence we hypothesized that the electrons tunnel through a small "window" which is thinner than the rest of the tunnel barrier. Using a heuristic model, we obtained the excellent fit to the experimental current curve at 0T shown in figure 10, and deduced

that the area of the tunneling region should be $\sim 1\mu\text{m}^2$.

We explained the role of the magnetic field in producing phonon oscillations of the current by combining our analysis of the depletion physics and tunnel conductance with the magnetopolaronic effect suggested by Hellman and Harris⁷. We developed a simple model that accurately reproduces the current-voltage curves and explains the approximate size of their periodic modulation. In this model, the magnetic field enables the free electron and phonon states to mix with sufficient amplitude to produce a slow polaron. The slow polaron generates a large space charge in the undepleted region, and due to the small tunneling window, it has a strong effect on the tunneling current.

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Captions

1. Schematic representation of the experiment. Electrons tunnel from right to left, through a thin $\text{Al}_x\text{Ga}_{1-x}\text{As}$ tunnel barrier into a lightly doped region. The cross-sectional area of the device is much larger than shown.
2. Energy diagram of the heterojunction device at an applied voltage V . Most of the voltage drop is across the depletion layer w , with the remainder V_b falling across the barrier.
3. Capacitance versus voltage as a function of magnetic field. The experimental data is taken from reference 1. The points marked by an "X" are a plot of equation 6.
4. Current expected at 0T if freezeout occurs, compared with the experimental 0T and 14T curves.^{1,9} The poor agreement implies that freezeout does not occur.
5. Idealized flat barrier adjacent to an artificially flat substrate, used to

estimate the tunnel conductance.

- 6a. Experimental tunnel current measurements for magnetic fields increasing from 0T to 14T, taken from references 1 and 9. The current decreases with increasing field strength.
- b. Same data mapped onto a single curve via equation 12. This corresponds to shifting the barrier voltage V_B by an amount proportional to H^2 .
7. Shift in barrier voltage V_B , proportional to β , as a function of magnetic field. The data points shown with an "X" are fit well by a parabola, plotted as a solid line.
8. The dashed line shows the expected current at 0T, according to the depletion behavior reflected in the capacitance data. The solid curve is the measured 0T current.^{1,9} The discrepancy indicates that tunnelling occurs in a region of the sample which is different from the average.

9. Illustration of the "window" model of inhomogeneous tunnelling.
Electrons tunnel through a small thin region of the barrier and neutralize a cylindrical volume of radius R .
10. Current at 0T as calculated from the "window" model, shown by "X"s, with the measured 0T curve in solid.^{1,9} The window radius is 0.75μ .
11. Calculated energy-momentum dispersion relation taken from reference 7 for a polaron at various magnetic field strengths.
12. Velocity versus wave-vector for a magnetopolaron at 14T. The velocity is just the slope of the 14T polaron dispersion curve shown in figure 11. The dashed curve is the free-electron result.
- 13a. Current differences, $J(4T)-J(H)$, measured in the experiment.^{1,9}
 - b. Current differences, $J(4T)-J(H)$, as calculated from theory.