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**REACTOR PRIMARY COOLANT SYSTEM
RUPTURE STUDY
QUARTERLY PROGRESS REPORT NO. 14
JULY-SEPTEMBER 1968**

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ATOMIC POWER EQUIPMENT DEPARTMENT
GENERAL ELECTRIC
SAN JOSE, CALIFORNIA

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ATOMIC POWER EQUIPMENT DEPARTMENT

SAN JOSE, CALIFORNIA

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NOTE *Correct all copies of the applicable publication as specified below.*

ITEM	REFERENCES (SECTION, PAGE, PARAGRAPH, LINE)	INSTRUCTIONS (CORRECTIONS AND ADDITIONS)
1	Section VII, pg. 7-5, Table 7-1, column 6	Column heading reads "Welded High Heat." Should read "Welded Low Heat (Normal)."
2.	Section VII, pg. 7-5, Table 7-1, column 7	Column heading reads "Welded Low Heat." Should read "Welded High Heat (Block)."
3.	Section VII, pg. 7-6, Table 7-2, column 6	Column heading reads "Welded High Heat." Should read "Welded Low Heat (Normal)."
4.	Section VII, pg. 7-6, Table 7-2, column 7	Column heading reads "Welded Low Heat." Should read "Welded High Heat (Block)."

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GEAP-5716
AEC Research and
Development Report
December 1968

REACTOR PRIMARY COOLANT SYSTEM RUPTURE STUDY

QUARTERLY PROGRESS REPORT NO. 14

JULY-SEPTEMBER 1968

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SECTION I

INTRODUCTION

The Reactor Primary Coolant System Rupture Study is being conducted for the Safety Branch, Division of Reactor Development and Technology, USAEC. This fourteenth quarterly report documents progress for the period of July 1 to September 30, 1968. The overall objective of the program is to improve the reliability of reactor system piping by increasing current knowledge of failure-causing mechanisms and by enhancing capability for design evaluation and analysis.

Toward the attainment of this objective, work is proceeding on the following tasks:

Task A—Reliability Engineering,
Task B—Stress Analyses,
Task C—Fracture Mechanics,
Task D—Fatigue, and
Task K—Effect of BWR Environment

The specific objectives of the tasks and technical progress are covered in the following sections. Previous reports issued as a part of this program are listed below.

GEAP-4445—A Survey of Water Reactor Primary System Conditions Pertinent to the Study of Pipe Rupture.

GEAP-4446—A Review of Fracture Modes as Related to Reactor Primary Coolant Pipe Rupture.

GEAP-4474—Experimental and Analytical Program Recommendations - Reactor Pipe Rupture Study.

GEAP-4574—Survey of Piping Failures for the Reactor Primary Coolant Pipe Rupture Study.

GEAP-4578—Mechanical Design Considerations in Primary Nuclear Piping.

GEAP-4678—Fracture Mechanics and the Stability of Engineering Structures.

GEAP-4911—Reactor Primary Coolant System Rupture Study Quarterly Progress Report Number 1, April—June 1965.

GEAP-4964—Reactor Primary Coolant System Rupture Study Quarterly Progress Report Number 2, July—August 1965.

GEAP-5014—Fracture Toughness of Some Engineering Alloys.

GEAP-5716

GEAP-5082—Reactor Primary Coolant System Rupture Study Quarterly Progress Report
Number 3, October—December 1965.

GEAP-5147—Reactor Primary Coolant System Rupture Study Quarterly Progress Report
Number 4, January—March 1966.

GEAP-4192—Reactor Primary Coolant System Rupture Study Quarterly Progress Report
Number 5, April—June 1966.

GEAP-5279—Reactor Primary Coolant System Rupture Study Quarterly Progress Report
Number 6, July—September 1966.

GEAP-5410—Low-Cycle Fatigue Strength Reduction in Notched Flat Plates.

GEAP-5427—Reactor Primary Coolant System Rupture Study Quarterly Progress Report
Number 7, October—December 1966.

GEAP-5471—PAPA—Structural Analysis of Plates and Shells Using Trapezoidal and
Triangular Plate Elements.

GEAP-5474—Reactor Primary Coolant System Rupture Study Quarterly Progress Report
Number 8, January—March 1967.

GEAP-5512—Reactor Primary Coolant System Rupture Study Quarterly Progress Report
Number 9, April—June 1967.

GEAP-5553—PAPA—The Digital Computer Program for the Static Analysis of Structures
Made Up of Plate and Panel Elements.

GEAP-5554—Reactor Primary Coolant System Rupture Study Quarterly Progress Report
Number 10, July—September 1967.

GEAP-5557—Stress Intensity Factor for a Circumferential Through-Wall Crack in a
Straight Pipe, January 1968.

GEAP-5620—Failure Behavior in ASTM A-106B Pipes Containing Axial Through-Wall
Flaws, April 1968.

GEAP-5607—Fatigue Crack Growth in Nuclear Reactor Piping Steels—March 1968.

GEAP-5587—Reactor Primary Coolant System Rupture Study Quarterly Progress Report
Number 11, October—December 1967.

GEAP-5637—Reactor Primary Coolant System Rupture Study Quarterly Progress Report
Number 12, January—March 1968.

GEAP-5622—A Failure Diagram for Axially Flawed Pipes, April 1968.

GEAP-5680—Reactor Primary Coolant System Rupture Study Quarterly Progress Report
Number 13, April—June 1968.

GEAP-5653—Status of Pipe Rupture Study at General Electric—II, July 1968.

SECTION II

SUMMARY

2.1 TASK A—RELIABILITY ENGINEERING

Analyses of the low-cycle fatigue data generated in Tasks D-1 and D-2 at Schenectady were completed. These experimental results, crack growth rate da/dn versus range of stress intensity factor, ΔK , were statistically treated to determine the variability for each of the three piping grades.

For estimating the probability of leakage at a pipe cross section due to low-cycle fatigue, a specific cross section is chosen, and design values and uncertainty distributions assembled. Two solutions are called for: the variance of cycles using the Taylor Series method, to show the relative contribution of each variable to the variance of cycles; and a Monte Carlo method to generate the distribution of the lower tail. The variance solution is reported herein.

2.2 TASK B—STRESS ANALYSIS

A topical report, "BWR Primary Piping Stresses for System Reliability Analyses," was submitted for publication, thus completing the major portions of stress analysis work for the Reliability Engineering task. Progress has been slow in completing problem solutions of piping components as a result of difficulties with the PAPA code.

2.3 TASK C—FRACTURE MECHANICS

Eleven flawed pipe specimens of A-106B were destructively tested, including nine by hydrostatic pressure loading and two by pressure plus bending loads. One of the latter, a 6-inch, Schedule-80 pipe with a through-wall circumferential flaw 2-1/2 inches long on the tension side, failed at 6000-psi pressure and a bending load sufficient to cause yielding at the flaw tips. A similar pipe with an axial flaw of the same size failed at less than 4000 psi without bending loading.

2.4 TASK D—FATIGUE

2.4.1 TASK D-1—Crack Growth Studies

Surface crack growth rates of part-through surface cracks tested under fully-reversed, strain-controlled loading have been determined for the bending mode. The specimen used was a cantilever beam identical to that used previously by others. These results are compared with those for A-212B and 2-1/4 Cr-1 Mo steel tested under fully reversed tension-compression loading. Crack growth rates in the bend specimen are substantially lower than those measured in tension-compression; prior results suggest that this is due, in part, to an effect of crack geometry.

2.4.2 TASK D-2—Crack Initiation in the Presence of Stress Concentration

Two topical reports, entitled "Notched High-Strain Fatigue Behavior of Three Low-Strength Structural Steels" and "The Influence of Stress/Strain Concentration and Mean Stress on the Low-Cycle Fatigue Behavior of Three Structural Steels at Room Temperature," were drafted and submitted for publication.

The cyclic deformation behavior of carbon, 2-1/4 Cr-1 Mo, and Type-304 stainless steel under completely reversed, strain-controlled conditions was investigated at 550°F.

2.4.3 TASK D-3—Prototype Pipe Fatigue Tests

Fatigue testing at room temperature of an elbow specimen and three cantilever notched piping straights was completed. In addition, two tests were completed at 550°F: a notched cantilever pipe straight and a "good" butt welded specimen. Setup of a hot (550°F), pressurized (1000 psi), notched stainless steel piping specimen was begun. This specimen with a growing crack will demonstrate "leak or break" under simulated BWR operating conditions.

2.4.4 TASK D-4—Cyclic Strain Aging

The results of transmission electron microscopy of specimens deformed in tension at room temperature, 300°F, and also in fatigue at 350°F, are described. Increasing dislocation tangling with increasing temperature at least up to 300°F was observed for the uniaxial tests. Cell formation occurred more readily during cyclic deformation and a well defined substructure was obtained in the ferritic areas of all the specimens examined.

The role of interstitial elements in contributing to the observed dislocation configurations is uncertain. However, the degree of strain aging will undoubtedly be influenced by the dislocation arrangement. It is speculated that a correlation may exist between the extent of dislocation tangling, as reflected in cell size, and the increase in transition temperature.

2.5 TASK K—EFFECT OF BWR ENVIRONMENT

With regard to equipment to be installed in the experimental loop at Dresden I: The dynamic test vessel was ordered for delivery in March 1969, the static test vessel was completed and the P&ID (piping and instrumentation drawing) was approved by Commonwealth Edison Company. Augmenting stress-bends for application as static or sustained loading specimens, modified WOL specimens will be included in the static test vessel. Four fatigue specimen designs are being evaluated in the laboratory to ensure uniformity of axial strain (by static loading) and consistent failure behavior (by cyclic loading).

SECTION III

TASK A—RELIABILITY ENGINEERING

(S. A. Wilson)

3.1 OBJECTIVES

The objectives of the reliability engineering task are to develop methods for:

- a. Estimation of the reliability of a reactor primary piping system in terms of leak rate and time of reactor service. In particular, to find 1 minus the probability that such a piping system will develop one or more

through-wall leaks or complete severance leaks	in	10 20 30 40	years of service.
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- b. Identification of the principal sources of real variability or uncertainty in service life.

3.2 SUMMARY AND CONCLUSIONS

Subsection 3.3 contains the algebraic modification to the low-cycle fatigue crack growth rate equation which arises from the analysis in subsection 3.5, where a mean-adjusted linear model is adopted for estimating intercept and slope parameter values from laboratory crack growth rate testing. Intercept and slope are now designated C and B rather than A and B.

Subsection 3.4 presents an approximate solution, by propagation of errors, to the variance of natural log of low-cycle fatigue cycles to grow a circumferential crack from depth a_0 through to leakage at a selected pipe cross section. This variance is a measure of dispersion of n_t . The contribution to this variance—due to uncertainty or variability—of each participating variable is found. This contribution depends on both the variance of the variable and on the way the variable appears in the n_t equation. Thus, the principal contributors to the dispersion of n_t are selected for Monte Carlo estimation of the probability that n_t is less than n_c , operating cycles in 40 years. These principal contributors are as follows:

<u>Variable</u>	<u>Source of Uncertainty or Variability</u>	<u>Percentage Contribution of Variance of this Variable to the Variance of $\ln$ Cycles to Leak</u>
Initial crack depth	Judgmental uncertainty distribution	71%
Slope parameter	Laboratory testing	17%
Stress	Judgmental uncertainty distributions	7%
Intercept parameter	Laboratory testing	4%
		99%

The remaining 1% was contributed by uncertainty in wall thickness, diameter, and pressure. While these four variables were thought, from previous sensitivity testing, to be the major contributors, the dominance of the initial crack depth distribution, even allowing for the fairly severe distribution assumed, was surprising, as were the relatively small contributions of the stress and intercept distributions. The relatively large contribution of slope variance assumes that the Tasks D-1, D-2 crack growth rate lines may be extended to lower ΔK values; i.e., assumes the same variability among slopes would have been obtained if the laboratory testing had started with smaller crack depths.

Subsection 3.5 outlines and extends the statistical analysis in Appendix A, which estimates mean values and variances of intercepts and slopes in the low-cycle fatigue crack growth rate equation, from laboratory tests under Tasks D-1 and D-2. The principal conclusions are as follows:

- a. There was no evidence that the tangent formula for stress intensity factor range did not give a $\log da/dn$ versus $\log \Delta K$ relationship adequately modeled by a straight line.
- b. Slopes obtained by using a nonsymmetrical crack growth ΔK equation, and lengths of the more rapidly growing crack, for cracks growing from one or both sides of an edge-notched plate were consistent with those obtained by using the tangent formula on the center-slotted plates; the slopes using a symmetrical crack growth ΔK equation and the same lengths were considerably higher.
- c. Single mean and variance values of slope are appropriate to all materials, temperatures, and minor test variables.
- d. Occasionally anomalous—low or even negative—slope runs occur.
- e. There was a decrease in slope due to cycling at 0.6 cpm rather than 6, 10, or 12 cpm; such testing was done only on carbon steel plates at 600 and 650° F.
- f. The variance in estimating slope for each run, due to variability among da/dn points and to number of points taken, accounts for approximately 40% of the variance among experimental lines, and so was subtracted from that variance to obtain the variance in slope without any experimental contribution, for use in the variance solution in subsection 3.4 described above.
- g. Using the same slope for all runs, different intercept mean and variance values were appropriate to each of the six material-temperature combinations. Lower values of mean and variance occurred for 2-1/4 Cr-1 Mo than for Type-304 stainless steel and carbon steel at both room and elevated temperatures. Tentatively, higher mean values occurred for elevated temperatures for all three materials, although this conclusion is possibly due to the effect of a specimen type change rather than temperature change. A between-heat component of variance of intercept appeared for carbon steel, but not for Type-304 stainless steel or 2-1/4 Cr-1 Mo.
- h. Higher mean intercept values occurred for elevated mean stress runs for all three materials.

- i. Tentatively, higher mean intercept values occurred for actual pipe specimens than for flat plate specimens of the same nominal material, although this conclusion is possibly due to the effect of a specimen type change rather than material configuration change.
- j. There was no effect on slope or intercept of the 10-minute hold time run on carbon steel at 600° F.

3.3 THE THROUGH-WALL CRACK GROWTH CYCLES EQUATION (G. T. Vernon)

The low-cycle fatigue crack growth rate equation used in these quarterlies has been:

$$\frac{da}{dn} = A \cdot (\Delta K)^B \quad (1)$$

(for notation, see Table 3-1).

After some modifications and changes of notation, we have settled on the following form for the cyclic variation of stress intensity, ΔK , [Equation (1), Thirteenth Quarterly, GEAP-5680]:

$$\begin{aligned} \Delta K = \sqrt{t} \cdot \left(\frac{1}{1 - a/t} \right)^{3/2} \sqrt{\frac{a}{t}} \cdot \left\{ \left[S_B \left(\frac{a}{t} \right) + 3 \left(\frac{a}{t} \right)^{3/2} \cdot S_M \left(\frac{a}{t} \right) \right] \cdot \sqrt{\frac{f(a/t)}{a/t}} \right. \\ \left. + \left(1 - \frac{a}{t} \right) \cdot S_M \left(\frac{a}{t} \right) \cdot \sqrt{\frac{1 - a/t}{a/t}} \cdot g \left(\frac{a}{t} \right) \right\} \cdot h \left(\frac{a}{t} \right) \end{aligned} \quad (2)$$

A case was made in subsections 4.3.4 and 4.3.5 [Equations (18) and (19) of the Twelfth Quarterly, GEAP-5637] for taking at least initially the following three combinations only of $S_B \left(\frac{a}{t} \right)$ and $S_M \left(\frac{a}{t} \right)$:

$$S_B \left(\frac{a}{t} \right) = 0, \quad S_M \left(\frac{a}{t} \right) = S_E + \frac{1}{2} S_P \quad (3)$$

$$S_B \left(\frac{a}{t} \right) = 0, \quad S_M \left(\frac{a}{t} \right) = S_P \quad (4)$$

$$S_B \left(\frac{a}{t} \right) = S_E, \quad S_M \left(\frac{a}{t} \right) = S_P \quad (5)$$

each of which is independent of crack length, a , (obtained from the assumption of negligible secondary stresses).

In the above Equation (1), the symbol A has been used both as a scaling constant and as a random parameter. Subsequently, it has been found (see subsection 3.5 and Appendix A) that it is convenient to use separate symbols and write (1) as

$$\frac{da}{dn} = C \cdot \left(\frac{\Delta K}{\sqrt{\Delta K^*}} \right)^B \quad (6)$$

* $\sqrt{\Delta K}$ is the geometric mean value of ΔK for a group of experimental runs at Schenectady under a given test condition.

TABLE 3-1
NOTATION FOR CRACK GROWTH CYCLES EQUATION

a	- Crack depth
n	- Number of cycles
n_t	- Number of cycles to grow through the wall
A	- Scaling constant and random parameter
B	- Equation parameter
ΔK	- Cyclic variation of stress intensity
t	- Pipe wall thickness
$S_B(x) = \sigma_{PB} + (1 - x) \cdot \sigma_{SB}$	- Bending stress
$S_M(x) = \sigma_{PM} + (1 - x) \cdot \sigma_{SM}$	- Membrane stress
σ_{PB}	- Primary bending stress
σ_{PM}	- Primary membrane stress
σ_{SB}	- Secondary bending stress
σ_{SM}	- Secondary membrane stress
$f(x) = \frac{\pi}{6} \cdot (1 - e^{-6x})$	- Bending coefficient (Bueckner)*
$g(x) = 2 \tan\left(\frac{\pi x}{2}\right)$	- Membrane coefficient
$h(x) = 1 + 0.0325 [1 + \cos(\pi x)]^2$	- Free surface correction factor**
C	- Random parameter
$\bar{\Delta K}$	- Geometric mean value of ΔK in experimental runs
$S_p = \frac{1}{2} \cdot \frac{pd}{t}$	- Hoop pressure stress
S_E	- Calculated expansion stress
p	- Pressure
d	- Pipe diameter
a_0	- Initial crack depth

*See subsection 3.3.2 of the Thirteenth Quarterly, GEAP-5680.

**See Equation (4), P. C. Paris and G. C. Sih, American Society for Testing and Materials, S. T. P. No. 381.

This should have the effect of minimizing the correlation between B and C so that the experimental determination of B and C can best be used for prediction. For details, see subsection 3.5. Equation (6) may be integrated over $\frac{a}{t} = x$ for the number of cycles to grow through:

$$n_t = \frac{t}{C} \cdot \int_{\frac{a_0}{t}}^1 \left(\frac{\tilde{\Delta K}}{\Delta K} \right)^B dx, \quad (7)$$

which may be expressed:

$$n_t = \frac{(\tilde{\Delta K})^B}{C \cdot t^{(B/2) - 1}} \cdot \int_{\frac{a_0}{t}}^1 \left[x^{-1/2} F(S_B, S_M; x) \right]^B dx, \quad (8)$$

with F defined as in Equation (10), subsection 3.3 of the last Quarterly, GEAP-5680:

$$F(S_B, S_M; x) = (1 - x)^{3/2} \left(\left\{ [S_B(x) + 3x^{3/2} S_M(x)] \cdot \sqrt{\frac{f(x)}{x}} + (1 - x) \cdot S_M(x) \cdot \sqrt{\frac{1 - x}{x}} \cdot g(x) \right\} \cdot h(x) \right)^{-1}, \quad (9)$$

where it was found to be well behaved for any combinations of stresses, σ_{PB} and σ_{SB} , and σ_{PM} and σ_{SM} . These forms are to be used in subsection 3.4, and in the future for Monte Carlo simulation. On top of these simple forms for the n_t equation with physical variable expressed as letters, we will have to superimpose uncertainty coefficients previously designated by starred terms; but this notation is avoidable with the concepts of statistics developed in subsection 3.4.

3.4 DISPERSION OF NUMBER OF CYCLES, n_t (G. T. Vernon)

3.4.1 Introduction

In the Thirteenth Quarterly, GEAP-5680, subsection 3.3, Equations (9), (10), and (11), we transformed the equation for the number of cycles, n_t , required for a crack to grow through a pipe wall to a form suitable for rapid numerical evaluation. This was preparatory to conducting a Monte Carlo simulation of the distribution of n_t under the hypothesized distributions of the physical and equation parameters.

In this Quarterly, the input distributions come under scrutiny. It is in fact possible—with the aid of the same transformation given in the Thirteenth Quarterly, subsection 3.3, Equation (7)—to estimate what spread of values of n_t to expect under given mean reactor conditions. Not only will a measure of dispersion, $\text{var } \ln n_t$, be given to good approximation, but the contribution of each of the causes of uncertainty to the total dispersion will be estimated. Thus, by eliminating the negligible contributors, it should be possible to characterize the lower tail of the n_t distribution with greater accuracy for a given outlay of resources in the Monte Carlo simulation.

But since many of our readers may have been exposed more to the general methods of applied mathematics, such as Taylor Series expansions and integration techniques (which form the backbone of this subsection 3.4), than to those of statistics, we will first briefly review the necessary concepts.*

3.4.2 Statistical Concepts

All continuous valued variables in all real situations are "random variables". In fact, when we say in an experiment that a variable, X , takes on a value, x , say 2×10^{-3} or whatever, we cannot know this value to better than, say, 10^{-8} . A statistical approach is to attempt to characterize our lack of perfect knowledge. The key to this is the "distribution function", written F with subscript X , which is defined to be the probability (relative frequency) of the random variable, X , taking on a value less than or equal to x :

$$F_X(x) = \Pr \{X \leq x\} = \int_{-\infty}^x dF_X(\xi)**;$$

note:

$$\Pr \{X < \infty\} = \int_{-\infty}^{\infty} dF_X(\xi)** = 1.$$

When several random variables, $X_1, X_2, \dots, X_I$, are involved the probability of the respective variables taking values less than or equal to $x_1, x_2, \dots, x_I$ may be denoted:

$F_{X_1, X_2, \dots, X_I}(x_1, x_2, \dots, x_I)$ or just: $F(x_1, x_2, \dots, x_I)$ with

$$\int dF(x_1, x_2, \dots, x_I)** = 1,$$

(the case of one random variable being included here with $I = 1$). Now let $\phi(X_1, \dots, X_I)$ and $\psi(X_1, \dots, X_I)$ be any two functions of the random variables, so that ϕ and ψ will distribute over a continuum of values or as many discrete values as is possible from the ranges of the variables, X_i . Then the "expected (or average) values" of $\phi(X_1, \dots, X_I)$ and $\psi(X_1, \dots, X_I)$ are defined:

$$E \phi(X_1, \dots, X_I) = \int \phi(x_1, \dots, x_I) dF(x_1, \dots, x_I),$$

$$E \psi(X_1, \dots, X_I) = \int \psi(x_1, \dots, x_I) dF(x_1, \dots, x_I),$$

respectively, while the "covariance" between ϕ and ψ is defined:

$$\begin{aligned} \text{cov}[\phi(X_1, \dots, X_I), \psi(X_1, \dots, X_I)] &= \int \{[\phi(x_1, \dots, x_I) - E\phi(X_1, \dots, X_I)] \\ &\quad \cdot [\psi(x_1, \dots, x_I) - E\psi(X_1, \dots, X_I)]\} dF(x_1, \dots, x_I). \end{aligned}$$

* See also subsection 3.3.3 of the Second Quarterly, GEAP-4964.

**See further for comment on the Stieltjes Integral.

With $\phi(X_1, X_2) = X_1$, $\psi(X_1, X_2) = X_2$ and distribution function $F(x_1, x_2)$, this definition specializes to:

$$\begin{aligned}\text{cov}(X_1, X_2) &= \text{cov}[\phi(X_1, X_2), \psi(X_1, X_2)] \\ &= \int [\phi(x_1, x_2) - E\phi(X_1, X_2)] \cdot [\psi(x_1, x_2) - E\psi(X_1, X_2)] dF(x_1, x_2) \\ &= \int [(x_1 - EX_1) \cdot (x_2 - EX_2)] dF(x_1, x_2) \\ &= E[(X_1 - EX_1) \cdot (X_2 - EX_2)];\end{aligned}$$

furthermore, with $\phi(X_1) = X_1 = \psi(X_1)$ and distribution function $F_{X_1}(x_1)$, the covariance, $\text{cov}(X_1, X_1)$, is known as the "variance":

$$\text{var } X_1 = \int (x_1 - EX_1)^2 dF_{X_1}(x_1) = E[(X_1 - EX_1)^2].$$

Those readers who are not familiar with the symbolism of the "Stieltjes Integral" are invited to restrict their attention to two special cases of the integral:

$$\int [\dots] dF(x_1, x_2, \dots, x_I) \equiv \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} [\dots] \cdot f(x_1, x_2, \dots, x_I) \cdot dx_1 dx_2 \dots dx_I$$

$$\int [\dots] dF(x_1, x_2, \dots, x_I) \equiv \sum_{j_1=1}^{J_1} \sum_{j_2=1}^{J_2} \dots \sum_{j_I=1}^{J_I} [\dots] \cdot dF(x_{1,j_1}, x_{2,j_2}, \dots, x_{I,j_I}),$$

where $f(x_1, x_2, \dots, x_I) = \frac{\partial^I F(x_1, x_2, \dots, x_I)}{\partial x_1 \cdot \partial x_2 \cdot \dots \cdot \partial x_I}$ denotes the "probability density function",

p.d.f. (analogous to mass density), corresponding to $F(x_1, x_2, \dots, x_I)$; the former expression holds when the derivative, f , exists for all arguments, $x_1, x_2, \dots, x_I$, while the latter holds in the case of a finite collection of probability mass points, which sum up to 1. Thus, the Stieltjes Integral notation may be thought of as a convenient mathematical shorthand for either or both of the above, in addition to being suitable for mixtures of the above two cases, as to be seen in subsection 3.4.5.

The Stieltjes Integral has properties similar to the ordinary integral. For example, with functions ϕ and ψ of random variables as above, "linearity" of the integral with respect to addition and constants a and b gives:

$$\begin{aligned}
E[a\phi(X_1, \dots, X_I) + b\psi(X_1, \dots, X_I)] &= \int [a\phi(x_1, \dots, x_I) + b\psi(x_1, \dots, x_I)] dF(x_1, \dots, x_I) \\
&= a \int \phi(x_1, \dots, x_I) dF(x_1, \dots, x_I) + b \int \psi(x_1, \dots, x_I) dF(x_1, \dots, x_I) \\
&= aE\phi(X_1, \dots, X_I) + bE\psi(X_1, \dots, X_I),
\end{aligned}$$

linearity of the expectation operator, E . For a more special property of distribution functions, F , we note:

$$\begin{aligned}
\int x_1 dF(x_1, \dots, x_I) &= \int x_1 dF_{X_1}(x_1) = EX_1 \\
\int (x_1 - EX_1)^2 dF(x_1, \dots, x_I) &= \int (x_1 - EX_1)^2 dF_{X_1}(x_1) = \text{var } X_1,
\end{aligned}$$

and, in general,

$$\int \phi(x_1) dF(x_1, \dots, x_I) = \int \phi(x_1) dF_{X_1}(x_1) = E\phi(X_1)$$

(which in fact justifies the use of the operator symbol " E ").

To carry the mass density—probability density analogy further, we would point out that the expected value of X , EX , is analogous to the center of mass and that the variance, $\text{var } X$, is analogous to the second moment about the center of mass.

When knowledge of the value taken on by one of the X_i 's, say X_j , would influence the distribution of values of others of the X_i , $i \neq j$, then we say that there is a "statistical dependency" between the X_i 's. For example, suppose we read just one of several meters in an experiment, and are able to make a better guess as to the reading of some of the other meters than we could have done before this first reading; then the meter readings are statistically interdependent. Otherwise we say that the X_i 's are "independent".

When $X_1, X_2, \dots, X_I$ are statistically independent, the distribution function satisfies

$$F(x_1, x_2, \dots, x_I) = F_{X_1}(x_1) \cdot F_{X_2}(x_2) \cdot \dots \cdot F_{X_I}(x_I),$$

where the factoring is guaranteed by the lack of relation between the values that $X_1, X_2, \dots, X_I$ take on. When, in addition, the X_i 's are identically distributed, we may write simply:

$$F(x_1, x_2, \dots, x_I) = F_X(x_1) \cdot F_X(x_2) \cdot \dots \cdot F_X(x_I). \quad (10)$$

Independence of $X_1, X_2, \dots, X_I$ implies, for $i \neq j$:

$$\begin{aligned}
\text{cov}(X_i, X_j) &= \int [x_i - EX_i] \cdot [x_j - EX_j] dF(x_1, x_2, \dots, x_I) \\
&= \int \int \dots \int [(x_i - EX_i) \cdot (x_j - EX_j)] dF_{X_1}(x_1) dF_{X_2}(x_2) \dots dF_{X_I}(x_I)
\end{aligned}$$

$$\begin{aligned}
&= \int (x_1 - EX_1) dF_{X_1}(x_1) \cdot \int (x_j - EX_j) dF_{X_j}(x_j) \cdot \prod_{\substack{k \neq j \\ k \neq 1}} \int dF_{X_k}(x_k) \\
&= \left[\int x_1 dF_{X_1}(x_1) - EX_1 \cdot \int dF_{X_1}(x_1) \right] \cdot \left[\int x_j dF_{X_j}(x_j) - EX_j \cdot \int dF_{X_j}(x_j) \right] = 0 \quad (11)
\end{aligned}$$

from the definitions: $\int x_i dF_{X_i}(x_i) = EX_i$, $\int dF_{X_k}(x_k) \equiv 1$ for $1 \leq i, k \leq I$.

The later steps, above, are mainly a consequence of the linearity of the Stieltjes Integral. Similar reasoning yields:

$$\begin{aligned}
\text{var } \phi(X_1, \dots, X_I) &= \int [\phi(x_1, \dots, x_I) - E\phi(X_1, \dots, X_I)]^2 dF(x_1, \dots, x_I) \\
&= \int [\phi(x_1, \dots, x_I)]^2 dF(x_1, \dots, x_I) - [\int \phi(x_1, \dots, x_I) dF(x_1, \dots, x_I)]^2 \\
&= E[\phi(X_1, \dots, X_I)]^2 - [E\phi(X_1, \dots, X_I)]^2. \quad (12)
\end{aligned}$$

3.4.3 Variance of a Function of Random Variables Approximated

In many physical situations, the function $\phi(X_1, X_2, \dots, X_I)$ has as many derivatives as one would care to evaluate. For instance, with the function, ϕ , of two variables, (X_1, X_2) , defined:

$$\phi(X_1, X_2) = X_1 + X_1 \cdot X_2,$$

the leading partials are found to be:

$$\frac{\partial \phi}{\partial x_1}(x_1, x_2) = 1 + x_2$$

$$\frac{\partial \phi}{\partial x_2}(x_1, x_2) = x_1$$

$$\frac{\partial^2 \phi}{\partial x_1^2} = \frac{\partial^2 \phi}{\partial x_2^2} = 0$$

$$\frac{\partial^2 \phi}{\partial x_1 \partial x_2} = \frac{\partial^2 \phi}{\partial x_2 \partial x_1} = 1,$$

with all other partials zero, identically over every pair (x_1, x_2) .

Under such circumstances, the Taylor Series identity is valid:

$$\begin{aligned}
\phi(x_1, x_2, \dots, x_I) &= \phi(\mu_1, \mu_2, \dots, \mu_I) + \sum_{i=1}^I \left[\frac{\partial \phi}{\partial x_i}(\mu_1, \mu_2, \dots, \mu_I) (x_i - \mu_i) \right] \\
&\quad + \frac{1}{2} \sum_{i=1}^I \sum_{j=1}^I \left[\frac{\partial^2 \phi}{\partial x_i \partial x_j}(\mu_1, \mu_2, \dots, \mu_I) (x_i - \mu_i) \cdot (x_j - \mu_j) \right] + \dots
\end{aligned}$$

for any set of points $(\mu_1, \mu_2, \dots, \mu_I)$, where the right-hand side is convergent. We put:

$\mu_i = \int x_i dF(x_i) = E X_i$ for $1 \leq i \leq I$ to obtain:

$\phi(x_1, \dots, x_I)$

$$\begin{aligned} -E\phi(X_1, \dots, X_I) &= \sum_{i=1}^I \left[\frac{\partial \phi}{\partial x_i}(EX_1, \dots, EX_I) \left((x_i - EX_i) - 0 \right) \right] \\ &+ \frac{1}{2} \sum_{i=1}^I \sum_{j=1}^I \left[\frac{\partial^2 \phi}{\partial x_i \partial x_j}(EX_1, \dots, EX_I) \left[(x_i - EX_i)(x_j - EX_j) \right. \right. \\ &\quad \left. \left. - \text{cov}(X_i, X_j) \right] \right] + \dots, \end{aligned}$$

so that the variance of $\phi(X_1, \dots, X_I)$ is found:

$E[\phi(X_1, \dots, X_I)]$

$$\begin{aligned} -E\phi(X_1, \dots, X_I)]^2 &= \sum_{i=1}^I \sum_{j=1}^I \left[\frac{\partial \phi}{\partial x_i}(EX_1, \dots, EX_I) \frac{\partial \phi}{\partial x_j}(EX_1, \dots, EX_I) \text{cov}(X_i, X_j) \right] \\ &+ \sum_{i=1}^I \sum_{j=1}^I \sum_{k=1}^I \left[\frac{\partial \phi}{\partial x_i}(EX_1, \dots, EX_I) \frac{\partial^2 \phi}{\partial x_j \partial x_k}(EX_1, \dots, EX_I) \right. \\ &\quad \left. \cdot \int (x_i - EX_i) \cdot (x_j - EX_j) \cdot (x_k - EX_k) dF(x_1, \dots, x_I) \right] \\ &+ \frac{1}{4} \sum_{i=1}^I \sum_{j=1}^I \sum_{k=1}^I \sum_{\ell=1}^I \left[\frac{\partial^2 \phi}{\partial x_i \partial x_j}(EX_1, \dots, EX_I) \frac{\partial^2 \phi}{\partial x_k \partial x_\ell}(EX_1, \dots, EX_I) \right. \\ &\quad \left. \cdot \int (x_i - EX_i) \cdot (x_j - EX_j) \cdot (x_k - EX_k) \cdot (x_\ell - EX_\ell) dF(x_1, \dots, x_I) \right. \\ &\quad \left. - \text{cov}(X_i, X_j) \cdot \text{cov}(X_k, X_\ell) \right] \quad (13) \end{aligned}$$

When the random variables, $X_1, \dots, X_I$, are independent, we may put [cf. Equation (10)]:

$$dF(x_1, \dots, x_I) = dF_{X_1}(x_1) \cdot \dots \cdot dF_{X_I}(x_I)$$

which, with the aid of Equation (11), leads to considerable simplification of Equation (13):

$$\begin{aligned}
\text{var } \phi(X_1, \dots, X_I) &= \sum_{i=1}^I \left[\frac{\partial \phi}{\partial x_i}(EX_1, \dots, EX_I) \right]^2 \cdot \text{var } X_i \\
&+ \sum_{i=1}^I \left[\frac{\partial \phi}{\partial x_i}(EX_1, \dots, EX_I) \cdot \frac{\partial^2 \phi}{\partial x_i^2}(EX_1, \dots, EX_I) \right] \cdot \int (x_i - EX_i)^3 dF_{X_i}(x_i) \\
&+ \frac{1}{4} \sum_{i=1}^I \left[\frac{\partial^2 \phi}{\partial x_i^2}(EX_1, \dots, EX_I) \right]^2 \left[\int (x_i - EX_i)^4 dF_{X_i}(x_i) - (\text{var } X_i)^2 \right] \\
&+ \frac{1}{2} \sum_{i=1}^I \sum_{j \neq i} \left[\frac{\partial^2 \phi}{\partial x_i \partial x_j}(EX_1, \dots, EX_I) \right]^2 \text{var } X_i \cdot \text{var } X_j + \dots, \quad (14)
\end{aligned}$$

since all terms containing factors of the form: $\int (x_i - EX_i) dF_{X_i}(x_i)$ vanish.

Two special cases of Equation (14) deserve mention:

1. $X_1, \dots, X_I$ are independent and $\phi(X_1, \dots, X_I) = X_1 + \dots + X_I$,

2. $X_1, \dots, X_I$ are independent and $\phi(X_1, \dots, X_I) = X_1 \cdot \dots \cdot X_I$.

1. $\text{var}(X_1 + \dots + X_I) = \text{var } X_1 + \dots + \text{var } X_I$ (15)

2. $\text{var}(X_1 \cdot \dots \cdot X_I) = \sum_{i=1}^I \left(\frac{EX_1 \cdot \dots \cdot EX_I}{EX_i} \right)^2 \cdot \text{var } X_i$
 $+ \frac{1}{2} \sum_{i=1}^I \sum_{j \neq i} \left(\frac{EX_1 \cdot \dots \cdot EX_I}{EX_i \cdot EX_j} \right)^2 \cdot \text{var } X_i \cdot \text{var } X_j + \dots$ (16)

In particular, it will be seen below that we sometimes have a choice between 1. and 2.; because 1. is exact, we naturally would prefer the use of it where possible.

3.4.4 Application of Variance Approximation to Measuring the Dispersion of n_t

If the crack growth cycles, n_t , equation were of an independent multiplicative form, then Equation (16) consisting of many terms would be appropriate. If, further, all of the random variables making up the n_t equation were positive, then we could use Equation (16), or alternatively, take the $\ln$ of the n_t equation and find the variance of $\ln n_t$ by using Equation (15), which consists of first terms only. An advantage of the latter is that the variance of $\ln n_t$ would be merely the sum of the variances of the $\ln$'s of the random variables in the n_t equation.

Let us examine the $\ln$ of the crack growth cycles, n_t , Equation (8); for illustration, we define the bending and membrane stresses, $S_B(x)$ and $S_M(x)$, respectively, as in Equation (3):

$$\ln n_t = B \ln \tilde{\Delta K} - \ln C - \left(\frac{B}{2} - 1\right) \ln t + \ln \int_{a_0}^1 \left[\frac{x^{1/2} F(0, 1; x)}{S_E + \frac{1}{2} S_p} \right]^B dx. \quad (17)$$

It will be seen in subsection 3.5 that B and $\ln C$ are almost independent random variables. Further, S_E and S_p (see subsection 3.4.5.6) have some multiplicative uncertainty coefficients in common. Though this is not of the independent additive form exactly, so that Equation (15) would be appropriate, there is reason to believe that the variance approximation (14) will be more accurate for $\ln n_t$ than for n_t itself.

Some values of n_t have been plotted on a log scale in Figure 3-6 of the Eleventh Quarterly, GEAP-5587, using typical outlying values of the equation random variables, one variable at a time. These are, by and large, evenly scattered about the reference value on a log scale, but highly skewed to the lower side when plotted on a natural scale. This may indicate that the distribution of $\ln n_t$ is closer to normal than that of n_t . Normality of the distribution of $\ln n_t$ was, in fact, a hypothesis of J. L. Jaech in Appendix A to the Sixth Quarterly, GEAP-5279.

For these reasons, we surmise that $\text{var } \ln n_t$ should be our main measure of dispersion of n_t . The first expression alone on the right-hand side of Equation (14) for $\text{var } \ln n_t$ is retained here in a first approximation to $\text{var } \ln n_t$ (a common practice in analysis of the propagation of error). For the major contributors to $\text{var } \ln n_t$, we may have to investigate the effect of ignoring subsequent terms. Only in a Monte Carlo simulation of n_t will we discover how close to normality $\ln n_t$ may be distributed; that is, how useful $\text{var } \ln n_t$ is for estimating crudely the tail of the n_t distribution.

In the next two subsections 3.4.5 and 3.4.6, we estimate the variance of each of the physical random variables and the partial derivative coefficients, respectively, to include in the approximation from Equation (14) for the variance of $\ln n_t$. In subsection 3.4.7, we apply the results to the pipe cross section selected in subsection 3.4 of the Thirteenth Quarterly, GEAP-5680.

3.4.5 Variance of the Random Variables in the n_t Equation

While the uncertainty distributions introduced for a number of the pipe design specifications, mainly in the Sixth and Eighth Quarterlies, GEAP-5279, and GEAP-5474, were subjective (judgmental) and must remain tentative and subject to revision in the light of any reader's superior judgment, nonetheless these will be adopted by and large in the present quarterly for analyzing the major sources of variation in n_t .

Many of the random physical and parameter variables in the n_t equation have been denoted in subsection 3.3 of the Sixth Quarterly, GEAP-5279, and subsequently, as: $X = U \cdot v$, where v is the plan value of X and U , represented previously by v^* (the uncertainty coefficient on v), is the ratio of actual value to planned value. From Equation (12) and the linearity of E , the variance of y is given simply by

$$\text{var } X = EX^2 - (EX)^2 = v^2 \cdot EU^2 - (v \cdot EU)^2 = v^2 \cdot \text{var } U. \quad (18)$$

Now, if the uncertainty coefficient, U , derives multiplicatively from I independent influences, $U_1, U_2, \dots, U_I$:

$$U = U_1 \cdot U_2 \cdot \dots \cdot U_I,$$

then the variance of U is given by

$$\begin{aligned} \text{var } U = & \sum_{i=1}^I \left[\frac{EU_1 \cdot EU_2 \cdot \dots \cdot EU_I}{EU_i} \right]^2 \cdot \text{var } U_i \\ & + \frac{1}{2} \sum_{i=1}^I \sum_{j \neq i}^I \left[\frac{EU_1 \cdot EU_2 \cdot \dots \cdot EU_I}{EU_i \cdot EU_j} \right]^2 \cdot \text{var } U_i \cdot \text{var } U_j + \dots \end{aligned}$$

from Equation (16). This equation is somewhat awkward to use in practice, and there is no easy way to be assured that errors in taking the first terms only are small enough.

An alternative approach is to find the variance of $\ln U$:

$$\text{var } \ln U = \text{var } \ln U_1 + \text{var } \ln U_2 + \dots + \text{var } \ln U_I, \text{ from Equation (15),}$$

where we are still making the assumption of independence of the U_i . With the aid of Equation (14), with $I = 1$, we find, putting $\phi(X) = e^X$:

$$\begin{aligned} \text{var } U &= e^{2E \ln U} \text{var } \ln U + \dots \\ &= e^{2E \ln U} (\text{var } \ln U_1 + \text{var } \ln U_2 + \dots + \text{var } \ln U_I) + \dots \end{aligned} \quad (19)$$

In case a reader is skeptical about the rapidity of convergence, for the case of $I=1$, we shall derive more terms in the Taylor Series expansion for $\text{var } \phi(X)$. In fact, the Taylor Series identity is:

$$\phi(x) = \phi(\mu) + \phi'(\mu)(x - \mu) + \dots = \sum_{i=0}^{\infty} \phi^{(i)}(\mu) \frac{(x - \mu)^i}{i!},$$

where $\phi^{(0)}$ represents ϕ and $0!$ is taken to be equal to 1.

Then, putting $\mu = EX$, we obtain successively:

$$\phi(x) = \phi(EX) + [\phi'(EX)] \cdot (x - EX) + \dots = \sum_{i=0}^{\infty} \phi^{(i)}(EX) \frac{(x - EX)^i}{i!}$$

$$E\phi(X) = \phi(EX) + \dots = \sum_{i=0}^{\infty} \phi^{(i)}(EX) \frac{E(X - EX)^i}{i!} \quad (20)$$

$$\phi(x) - E\phi(X) = [\phi'(EX)] \cdot (x - EX) + \dots = \sum_{i=0}^{\infty} \phi^{(i)}(EX) \left[\frac{(x - EX)^i}{i!} - \frac{E(X - EX)^i}{i!} \right]$$

$$[\phi(x) - E\phi(X)]^2 = [\phi'(EX)]^2 (x - EX)^2 + \dots$$

$$\begin{aligned} &= \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \phi^{(i)}(EX) \phi^{(j)}(EX) \left\{ \frac{[(x - EX)^i - E(X - EX)^i][x - EX)^j - E(X - EX)^j]}{i! j!} \right\} \\ &= \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \phi^{(i)}(EX) \phi^{(j)}(EX) \left[\frac{(x - EX)^{i+j} - (x - EX)^i E(X - EX)^j - E(X - EX)^i (x - EX)^j + E(X - EX)^i E(X - EX)^j}{i! j!} \right] \end{aligned}$$

and hence:

$$\begin{aligned} \text{var } \phi(X) &= E[\phi(X) - E\phi(X)]^2 = [\phi'(EX)]^2 E(X - EX)^2 + \dots \\ &= \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \phi^{(i)}(EX) \phi^{(j)}(EX) \left[\frac{E(X - EX)^{i+j} - E(X - EX)^i E(X - EX)^j}{i! j!} \right] \\ &= \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \phi^{(i)}(EX) \phi^{(j)}(EX) \left[\frac{E(X - EX)^{i+j} - E(X - EX)^i E(X - EX)^j}{i! j!} \right], \end{aligned} \quad (21)$$

since the bracketed term cancels out in the numerator at $k = i = 0$ or $k = j = 0$ from

$E(X - EX)^{n+k} = E(X - EX)^n$ and $E(X - EX)^k = 1$. With $\phi(x) = e^x$ we have from Equations (20) and (21):

$$E e^X = e^{EX} \sum_{i=0}^{\infty} \frac{E(X - EX)^i}{i!} \quad (22)$$

$$\text{var } e^X = e^{2EX} \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \frac{E(X - EX)^{i+j} - E(X - EX)^i E(X - EX)^j}{i! j!} \quad (23)$$

Rapid convergence is ensured in Equations (22) and (23) when $-1 < X - EX < 1$, since E funcn. $(X) = \int$ funcn. $(x) dF(x)$ is a weighted average. In particular, for the uncertainty distributions considered below, $X = \ln U$ is heavily concentrated both at $x = 0$ and in the range: $-1 + EX < x < 1 + EX$.

As a matter of fact it would do little harm, when U is clustered about 1, to assume: $E \ln U \approx 0$ and from (19) put:

$$\text{var } U \approx \text{var } \ln U_1 + \text{var } \ln U_2 \quad (24)$$

although we have not done this below.

We may now take the random variables in the n_t equation, (8), and evaluate the variances by the foregoing procedure (or by more direct methods), one by one.

3.4.5.1 Thickness, t

The variability of thickness, t , was ascribed in the Sixth Quarterly, GEAP-5279, as arising from two causes of uncertainty. First, the average to design ratio on thickness varies

about 1 for an in situ piping component, with the variability dependent on the degree of site inspection; we denote this ratio U_1 . Second, the local thickness varies about this average by a ratio which we denote U_2 . Hence,

$$t = U_1 U_2 v,$$

where v represents the design thickness in inches. From Equations (18) and (19) we find:

$$\text{var } t = v^2 \text{ var } U = v^2 e^{2E \ln U} (\text{var } \ln U_1 + \text{var } \ln U_2) + \dots \quad (25)$$

where $U = U_1 U_2$, under the assumption that U_1 and U_2 are not dependent. This assumption of independence is equivalent to saying that local thick and thin places are described by a ratio whose distribution is not dependent on the overall thickness of the pipe.

For the distribution of $\ln U_1$ we must take into consideration the rejection of some piping components at the welding site due to mismatch of thicknesses and welders' inspection. We denote by S the average to design thickness ratio in the supplied components, by r_i and r_{i+1} two endpoints of the i th interval into which $\ln S$ may fall, and by $\underline{W}$ the event of a pipe being welded into place. Then,

$$\Pr \{ \underline{W}, r_i < \ln S \leq r_{i+1} \} = \Pr \{ r_i < \ln S \leq r_{i+1} \} \Pr \{ \underline{W} \mid r_i < \ln S \leq r_{i+1} \}, \quad (26)$$

where $\Pr \{ \underline{W} \mid r_i < \ln S \leq r_{i+1} \}$ is the conditional probability of a component being welded into place given that $\ln S$ falls between r_i and r_{i+1} . The last probability is closely approximated by $\Pr \{ \underline{W} \mid \ln S = r_i \}$, when r_{i+1} is sufficiently close to r_i , since we shall see that the approximation is assumed continuous in r_i . Further, a welded component must have been supplied with $\ln S$ falling in one of the nonoverlapping ranges $r_i < \cdot \leq r_{i+1}$, when the intervals span all possible values of $\ln S$. Hence, the summation of Equation (26) yields

$$\Pr \underline{W} = \int_{-\infty}^{\infty} \Pr \{ \underline{W} \mid \ln S = r' \} dF_{\ln S}(r'), \quad (27)$$

in the limit of small intervals, when as usual we define:

$$F_{\ln S}(r) = \Pr \{ \ln S \leq r \}.$$

We now use Equation (27) to find the distribution of $\ln U_1$ (by the Bayes Formula technique):

$$\begin{aligned} \Pr \{ \ln U_1 \leq r \} &= \Pr \{ \ln S \leq r \mid \underline{W} \} \\ &= \frac{\Pr \{ \ln S \leq r, \underline{W} \}}{\Pr \underline{W}} \\ &= \frac{\int_{-\infty}^r \Pr \{ \underline{W} \mid \ln S = r' \} dF_{\ln S}(r')}{\int_{-\infty}^{\infty} \Pr \{ \underline{W} \mid \ln S = r' \} dF_{\ln S}(r')} \end{aligned} \quad (28)$$

Figures 3-1 and 3-2 illustrate the two contributing probabilities, and have been compounded from information in the Sixth Quarterly, GEAP-5279. A reader of that Quarterly will observe that Figure 3-1 has been modified here to include both the information on pipe supplied after hydrostatic testing, and that on pipe fortuitously bypassing the test. It would seem that the welder is not to know the difference.

With the following inputs (read from Figures 3-1 and 3-2, respectively, as functions):

$$\Pr(\underline{W} | S = \rho) = \begin{cases} (\rho - 2)^2 \cdot \rho^{2-3 \ln \rho} & 0 \leq \rho \leq 2 \\ 0 & \text{otherwise} \end{cases} \quad (29)$$

$$\left\{ \begin{aligned} dF_{\ln S}(r) &= \begin{cases} 0 & r \leq -2 \\ 0.0005 (r + 2) dr & -2 \leq r \leq -1 \\ [0.0005 + 0.0105 (r + 1)] dr & -1 \leq r < 0 \\ 0.012 (1 - r) dr & 0 < r \leq 1 \\ 0 & r \geq 1 \end{cases} \\ dF_{\ln S}(0) &= 0.988 \end{aligned} \right\}, \quad (30)$$

when $dF_{\ln S}$ is defined for the case of no site inspection, numerical integration has been used to compute Equation (27), $\int_{-\infty}^{\infty} r' \cdot \Pr\{\underline{W} | \ln S = r'\} dF_{\ln S}(r')$ and $\int_{-\infty}^{\infty} (r')^2 \cdot \Pr\{\underline{W} | \ln S = r'\} dF_{\ln S}(r')$ to yield, by Equations (28) and (12),

$$E \ln U_1 = -1.58 \times 10^{-4},$$

$$\text{var } \ln U_1 = 4.43 \times 10^{-4}.$$

Diagram (1), page 3-15 of the Sixth Quarterly, GEAP-5279, has been retained for computing the distribution and hence the variance of $\ln U_2$:

$$dF_{\ln U_2}(r) = \begin{cases} 0 & r \leq -\ln 3 \\ \frac{0.01}{(\ln 3)^2} \cdot (r + \ln 3) dr & -\ln 3 \leq r < 0 \\ 0.99 & 0 \\ \frac{0.01}{(\ln 3)^2} (\ln 3 - r) dr & 0 < r \leq \ln 3 \\ 0 & r \geq \ln 3 \end{cases}$$

In the defining equation,

$$\begin{aligned} E \ln U_2 &= \int_{-\infty}^{\infty} r dF_{\ln U_2}(r) \\ &= \int_{-\ln 3}^0 r \cdot \frac{0.01}{(\ln 3)^2} \cdot (r + \ln 3) dr + 0 \times 0.99 + \int_0^{\ln 3} r \cdot \frac{0.01}{(\ln 3)^2} (\ln 3 - r) dr, \end{aligned}$$

FIGURE 3-1. PROBABILITY OF PIPE OF VARIOUS THICKNESSES BEING FIELD WELDED

FIGURE 3-2. PIPE THICKNESSES AS SUPPLIED BY THE MANUFACTURER

we substitute in the first term on the right-hand side (for $-\ln 3 \leq r < 0$)

$s = -r$, $ds = -dr$, to obtain first:

$$\int_{-\ln 3}^0 r \cdot \frac{0.01}{(\ln 3)^2} (r + \ln 3) dr = - \int_0^{\ln 3} s \cdot \frac{0.01}{(\ln 3)^2} (\ln 3 - s) ds, \text{ or } E \ln U_2 = 0,$$

second: $E(\ln U_2)^2 = 2 \left(\frac{0.01}{\ln 3} \cdot \frac{r^3}{3} - \frac{0.01}{(\ln 3)^2} \cdot \frac{r^4}{4} \right) \Big|_0^{\ln 3}$; so that by Equation (12) we obtain:

$$\text{var } \ln U_2 = E[(\ln U_2)^2] - (E \ln U_2)^2 = 0.02 \left(\frac{1}{3} - \frac{1}{4} \right) \cdot (\ln 3)^2 = 0.002012 ;$$

and by Equation (25): $\text{var } t \approx v^2 \cdot e^{-2 \times 0.000158} (4.43 + 20.12) \times 10^{-4} = 24.54 \times 10^{-4} \cdot v^2$.

3.4.5.2 Initial Crack Depth as Fraction of Wall Thickness, $\frac{a_0}{t}$ *

Since we are dealing with as small as 1-inch pipe cross sections, it has been thought appropriate to characterize the cross-section defect depth as the depth of the largest crack in an area, α , of maximum stress of the cross section; that is, cracks will only propagate at the deepest defect present.

Suppose the crack frequency is random and averages λ defects per square inch over any area of piping of a given material and manufacture. Then the assumption of independence of adjoining areas' defect frequency is mathematically equivalent to the Poisson Distribution Law:

$$\Pr \left\{ m \text{ cracks occur in an area of } \alpha \text{ in.}^2 \right\} = e^{-\alpha \lambda} \cdot \frac{(\alpha \lambda)^m}{m!} \quad (31)$$

(which happens to be a term in the exponential Taylor expansion, normalized to 1).

Further, we suppose that no crack is to be differentiated from or related to any other crack, so that any crack defect depth, X , is distributed according to $F_X(x)$ independent of any other crack depth. Given that exactly m cracks happen to occur in an area, then

$$\begin{aligned} F_{\max X_i}(x) &= \Pr \{X_1 \leq x, X_2 \leq x, \dots, X_m \leq x\} \\ &= \Pr \{X_1 \leq x\} \cdot \Pr \{X_2 \leq x\} \cdot \dots \cdot \Pr \{X_m \leq x\} = [F_X(x)]^m \end{aligned} \quad (32)$$

is the distribution of the maximum, $\max X_i$, of the defect depths, X_i , of the m cracks.

*Closely following suggestions contained in a note (August 1966, unpublished) by J. L. Jaech.

Now, the event that the largest crack depth, $X_{\max}$, is less than or equal to x arises from the mutually exclusive events $\{\text{No. cracks} = m, \max X_i \leq x\}$ with respective probabilities,

$$\Pr \{m \text{ cracks occur}\} \cdot \Pr \{\max X_i \leq x \mid m \text{ cracks occur}\},$$

so that the probability of this event is given from Equations (31) and (32) as

$$F_{\max X}(x) = \sum_{m=0}^{\infty} e^{-\alpha\lambda} \frac{(\alpha\lambda)^m}{m!} [F_X(x)]^m = e^{-\alpha\lambda} \sum_{m=0}^{\infty} \frac{[\alpha\lambda F_X(x)]^m}{m!} = e^{\alpha\lambda [F_X(x) - 1]}$$

by expansion of the last. It may be noted that $F_{\max X}(x)$ takes on a value of $e^{-\alpha\lambda}$ at $x=0$ and we may define $F_{\max X}(x)$ as 0 for $x < 0$, so that $dF_{\max X}(0) = e^{-\alpha\lambda}$ is a jump at $x=0$, and the Stieltjes Integral now appears to be the most natural form for this case. In fact, $e^{-\alpha\lambda}$ is the probability of a defect-free surface area of α square inches.

The expected value of $X_{\max}$ and its variance are derived as follows:

$$\begin{aligned} EX_{\max} &= \int_0^1 x dF_{\max X}(x) = 0 \cdot e^{-\alpha\lambda} + \int_0^1 x d \left\{ e^{\alpha\lambda [F_X(x) - 1]} \right\} \\ &= \left\{ x e^{\alpha\lambda [F_X(x) - 1]} \right\} \Big|_0^1 - \int_0^1 e^{\alpha\lambda [F_X(x) - 1]} dx \\ &= 1 - \int_0^1 e^{\alpha\lambda [F_X(x) - 1]} dx, \end{aligned} \quad (33)$$

(where the integral limits signify that we are measuring crack depth as a fraction of wall thickness),

$$\begin{aligned} E(X_{\max})^2 &= \int_0^1 x^2 dF_{\max X}(x) \\ &= 0 \cdot e^{-\alpha\lambda} + \left(x^2 e^{\alpha\lambda [F_X(x) - 1]} \right) \Big|_0^1 - \int_0^1 2xe^{\alpha\lambda [F_X(x) - 1]} dx \\ &= 1 - 2 \int_0^1 x e^{\alpha\lambda [F_X(x) - 1]} dx; \end{aligned}$$

and from Equations (12) and (33):

$$\begin{aligned} \text{var } X_{\max} &= E(X_{\max})^2 - (EX_{\max})^2 \\ &= 2 \int_0^1 (1-x) e^{\alpha\lambda [F_X(x) - 1]} dx - \left\{ \int_0^1 e^{\alpha\lambda [F_X(x) - 1]} dx \right\}^2 \end{aligned} \quad (34)$$

A p.d.f. of $\log \frac{a_0}{t}$ has been proposed on page 3-10 of the Sixth Quarterly, GEAP-5279:

$$\frac{dF_{\log X}(z)}{dz} = \begin{cases} 0 & z < -\log 100 \\ \frac{2}{\log 0.8 + \log 100} \left(1 - \frac{z + \log 100}{\log 0.8 + \log 100} \right) & -\log 100 \leq z \leq \log 0.8 \\ 0 & z \geq \log 0.8 \end{cases}$$

which would imply:

$$1 - F_{\log X}(y) = \begin{cases} 0 & y \geq \log 0.8 \\ \int_y^{\log 0.8} \frac{2}{(\log 0.8 + \log 100)^2} (\log 0.8 - z) dz & \log 0.8 \geq y \geq -\log 100 \\ 1 & y \leq -\log 100 \end{cases}$$

By use of the transformation $w = \log 0.8 - z$, $dw = -dz$ for $\log 0.8 \geq y \geq -\log 100$:

$$1 - F_{\log X}(y) = \int_0^{\log 0.8 - y} \frac{2w}{(\log 0.8 + \log 100)^2} dw$$

integrates to: $\frac{(\log 0.8 - y)^2}{(\log 0.8 + \log 100)^2}$

But the equality: $1 - F_{\log X}(y) = \Pr \{ \log X > y \}$ follows from the definition. Hence, $1 - F_X(x) = \Pr \{ X > x \}$ is equal to:

$$\Pr \{ \log X > \log x \} = 1 - F_{\log X}(\log x) = \frac{(\log 0.8 - \log x)^2}{(\log 0.8 + \log 100)^2} \quad \text{for } 0.8 \geq x \geq \frac{1}{100}$$

∴ from the above in Equations (33) and (34), we obtain:

$$EX_{\max} = 1 - \left[\int_0^{0.01} e^{-\alpha x} dx + \int_{0.01}^{0.8} e^{-\alpha x} \left(\frac{\log 0.8 - \log x}{\log 0.8 + \log 100} \right)^2 dx + \int_{0.8}^{1.0} dx \right] = 0.0364^*$$

$$\begin{aligned} \text{var } X_{\max} &= 2 \left[\int_0^{0.01} (1-x) e^{-\alpha x} dx + \int_{0.01}^{0.8} (1-x) e^{-\alpha x} \left(\frac{\log 0.8 - \log x}{\log 0.8 + \log 100} \right)^2 dx \right. \\ &\quad \left. + \int_{0.8}^{1.0} (1-x) dx \right] - \left[\int_0^{0.01} e^{-\alpha x} dx + \int_{0.01}^{0.8} e^{-\alpha x} \left(\frac{\log 0.8 - \log x}{\log 0.8 + \log 100} \right)^2 dx \right. \\ &\quad \left. + \int_{0.8}^{1.0} dx \right]^2 = 7.0 \times 10^{-3}^* \end{aligned}$$

* Evaluated with $\alpha = \pi d/4 = 1.656\pi$, $\lambda = 0.1$ (Thirteenth Quarterly, GEAP-5680, Table 3-1).

3.4.5.3 Diameter, d

The diameter of the cross section of interest of a pipe is thought to have three major causes of uncertainty. First, the whole line of pipe may average a ratio, U_1 , to the design value of diameter. Second, a pipe component diameter may average a ratio, U_2 , to the line average. Third, the diameter of the pipe cross section of interest may average a ratio, U_3 , to the component average. Hence, we have

$$d = U_1 U_2 U_3 v,$$

where v is the design diameter and we assume that each cause of uncertainty is independent of the other.

The distributions of U_1 and U_2 are given in the Sixth Quarterly, GEAP-5279, (sub-section 3.3, Cause 10.1), while U_3 is assumed (here for the first time) to have a standard deviation of $\frac{1}{2}\%$ and $\ln U_3$ to be normally distributed.

From Equations (18) and (19), we have

$$\text{var } d = v^2 \text{ var } U \approx v^2 e^{2E \ln U} (\text{var } \ln U_1 + \text{var } \ln U_2 + \text{var } \ln U_3) \quad (35)$$

where $U = U_1 U_2 U_3$. $F_{\ln U_1}(w)$ is given by (no site inspection):

$$\Pr\{\ln U_1 \leq w\} = \Pr\{U_1 \leq e^w\} = \begin{cases} 0 & w \leq \ln 0.9 \\ 5 \times 10^{-4} (e^w - 0.9) & \ln 0.9 \leq w \leq 0 \\ 0.99945 + 5 \times 10^{-4} e^w & 0 \leq w \leq \ln 1.1 \\ 1 & w \geq \ln 1.1 \end{cases}$$

yielding first:

$$\begin{aligned} E \ln U_1 &= \int_{-\infty}^{\infty} w dF_{\ln U_1}(w) = \int_{\ln 0.9}^{\ln 1.1} w dF_{\ln U_1}(w) \\ &= \left[w F_{\ln U_1}(w) \right]_{\ln 0.9}^{\ln 1.1} - \int_{\ln 0.9}^{\ln 1.1} F_{\ln U_1}(w) dw \\ &= \ln 1.1 - \left[5 \times 10^{-4} (e^w - 0.9) \right]_{\ln 0.9}^0 - \left(0.99945 w + 5 \times 10^{-4} e^w \right) \Big|_0^{\ln 1.1} \\ &= \ln 1.1 - 5 \times 10^{-4} [1 + 0.9 (\ln 0.9 - 1)] - [0.99945 \ln 1.1 + 5 \times 10^{-4} (1.1 - 1)] \\ &= 5.5 \times 10^{-4} \ln 1.1 - 10^{-4} - 4.5 \times 10^{-4} \ln 0.9 = -0.17 \times 10^{-6} \end{aligned}$$

second:

$$E(\ln U_1)^2 = \int_{-\infty}^{\infty} w^2 dF_{\ln U_1}(w) = 0.335 \times 10^{-6}$$

and hence from (12) third:

$$\text{var } \ln U_1 = E(\ln U_1)^2 - (E \ln U_1)^2 = 0.335 \times 10^{-6}.$$

$$\text{Similarly, } F_{\ln U_2}(w) = \begin{cases} 0 & w \leq \ln 0.9 \\ 5 \times 10^{-3} (e^w - 0.9) & \ln 0.9 \leq w \leq 0 \\ 0.9945 + 5 \times 10^{-3} e^w & 0 \leq w \leq \ln 1.1 \\ 0 & w \geq \ln 1.1 \end{cases}$$

gives

$$E \ln U_2 = -1.7 \times 10^{-6}, \text{ and from Equation (12):}$$

$$\text{var } \ln U_2 = E(\ln U_2)^2 - (E \ln U_2)^2 = 3.35 \times 10^{-6}.$$

$$\text{But we assumed: } \text{var } \ln U_3 = (\ln 1.005)^2 = 0.2475 \times 10^{-4}.$$

Hence, from Equation (35) we have finally derived:

$$\text{var } d \approx v^2 e^{-2} \times 1.8 \times 10^{-6} (0.34 + 3.35 + 24.88) \times 10^{-6} = 0.29 \times 10^{-4} v^2.$$

3.4.5.4 Pressure, p

Two major causes of uncertainty are attributed to pressure. First, there is uncertainty in the cycle operating pressure, U_1 ; and second, with low probability, overpressures may occur due to water hammer. Noting that the design value, v , is bigger by Δv than the expected pressure value, we may put

$$p = (v - \Delta v) U_1 U_2. \quad (36)$$

The distributions of U_1 and U_2 are taken from the Sixth Quarterly, GEAP-5279 (sub-section 3.3, Causes 5.1 and 5.2):

$$\begin{aligned} F_{\ln U_1}(w) &= \Pr\{\ln U_1 \leq w\} = \Pr\{U_1 \leq e^w\} \\ &= F_{U_1}(e^w) = \begin{cases} 0 & \text{for } w \leq \ln 0.96 \\ \int_{0.96}^{e^w} (25)^2 (x - 0.96) dx & \text{for } \ln 0.96 \leq w \leq 0 \\ 1 - \int_{e^w}^{1.04} (25)^2 (1.04 - x) dx & \text{for } 0 \leq w \leq \ln 1.04 \\ 1 & \text{for } w \geq \ln 1.04 \end{cases} \end{aligned} \quad (37)$$

3.4.5.3 Diameter, d

The diameter of the cross section of interest of a pipe is thought to have three major causes of uncertainty. First, the whole line of pipe may average a ratio, U_1 , to the design value of diameter. Second, a pipe component diameter may average a ratio, U_2 , to the line average. Third, the diameter of the pipe cross section of interest may average a ratio, U_3 , to the component average. Hence, we have

$$d = U_1 U_2 U_3 v$$

where v is the design diameter and we assume that each cause of uncertainty is independent of the other.

The distributions of U_1 and U_2 are given in the Sixth Quarterly, GEAP-5279, (subsection 3.3, Cause 10.1), while U_3 is assumed (here for the first time) to have a standard deviation of $\frac{1}{2}\%$ and $\ln U_3$ to be normally distributed.

From Equations (18) and (19), we have

$$\text{var } d = v^2 \text{ var } U \approx v^2 e^{2E \ln U} (\text{var } \ln U_1 + \text{var } \ln U_2 + \text{var } \ln U_3) \quad (35)$$

where $U = U_1 U_2 U_3$. $F_{\ln U_1}(w)$ is given by (no site inspection):

$$\Pr \{ \ln U_1 \leq w \} = \Pr \{ U_1 \leq e^w \} = \begin{cases} 0 & w \leq \ln 0.9 \\ 5 \times 10^{-4} (e^w - 0.9) & \ln 0.9 \leq w \leq 0 \\ 0.99945 + 5 \times 10^{-4} e^w & 0 \leq w \leq \ln 1.1 \\ 1 & w \geq \ln 1.1 \end{cases}$$

yielding first:

$$\begin{aligned} E \ln U_1 &= \int_{-\infty}^{\infty} w dF_{\ln U_1}(w) = \int_{\ln 0.9}^{\ln 1.1} w dF_{\ln U_1}(w) \\ &= \left[w F_{\ln U_1}(w) \right]_{\ln 0.9}^{\ln 1.1} - \int_{\ln 0.9}^{\ln 1.1} F_{\ln U_1}(w) dw \\ &= \ln 1.1 - \left[5 \times 10^{-4} (e^w - 0.9w) \right]_{\ln 0.9}^0 - \left(0.99945 w + 5 \times 10^{-4} e^w \right) \Big|_0^{\ln 1.1} \\ &= \ln 1.1 - 5 \times 10^{-4} [1 + 0.9 (\ln 0.9 - 1)] - [0.99945 \ln 1.1 + 5 \times 10^{-4} (1.1 - 1)] \\ &= 5.5 \times 10^{-4} \ln 1.1 - 10^{-4} - 4.5 \times 10^{-4} \ln 0.9 = -0.17 \times 10^{-6} \end{aligned}$$

second:

$$E(\ln U_1)^2 = \int_{-\infty}^{\infty} w^2 dF_{\ln U_1}(w) = 0.335 \times 10^{-6}$$

and hence from (12) third:

$$\text{var } \ln U_1 = E(\ln U_1)^2 - (E \ln U_1)^2 = 0.335 \times 10^{-6}.$$

$$\text{Similarly, } F_{\ln U_2}(w) = \begin{cases} 0 & w \leq \ln 0.9 \\ 5 \times 10^{-3} (e^w - 0.9) & \ln 0.9 \leq w \leq 0 \\ 0.9945 + 5 \times 10^{-3} e^w & 0 \leq w \leq \ln 1.1 \\ 0 & w \geq \ln 1.1 \end{cases}$$

gives

$$E \ln U_2 = -1.7 \times 10^{-6}, \text{ and from Equation (12):}$$

$$\text{var } \ln U_2 = E(\ln U_2)^2 - (E \ln U_2)^2 = 3.35 \times 10^{-6}.$$

$$\text{But we assumed: } \text{var } \ln U_3 = (\ln 1.005)^2 = 0.2475 \times 10^{-4}.$$

Hence, from Equation (35) we have finally derived:

$$\text{var } d \approx v^2 e^{-2 \times 1.8 \times 10^{-6}} (0.34 + 3.35 + 24.88) \times 10^{-6} = 0.29 \times 10^{-4} v^2.$$

3.4.5.4 Pressure, p

Two major causes of uncertainty are attributed to pressure. First, there is uncertainty in the cycle operating pressure, U_1 ; and second, with low probability, overpressures may occur due to water hammer. Noting that the design value, v , is bigger by Δv than the expected pressure value, we may put

$$p = (v - \Delta v) U_1 U_2. \quad (36)$$

The distributions of U_1 and U_2 are taken from the Sixth Quarterly, GEAP-5279 (sub-section 3.3, Causes 5.1 and 5.2):

$$\begin{aligned} F_{\ln U_1}(w) &= \Pr\{\ln U_1 \leq w\} = \Pr\{U_1 \leq e^w\} \\ &= F_{U_1}(e^w) = \begin{cases} 0 & \text{for } w \leq \ln 0.96 \\ \int_{0.96}^{e^w} (25)^2 (x - 0.96) dx & \text{for } \ln 0.96 \leq w \leq 0 \\ 1 - \int_{e^w}^{1.04} (25)^2 (1.04 - x) dx & \text{for } 0 \leq w \leq \ln 1.04 \\ 1 & \text{for } w \geq \ln 1.04 \end{cases} \end{aligned} \quad (37)$$

and $F_{\ln U_2}(y) = \Pr\{\ln U_2 \leq y\} = \Pr\{U_2 \leq e^y\}$

$$= F_{U_2}(e^y) = \begin{cases} 0 & \text{for } y \leq 0 \\ 1 - 10^{-3} \cdot \int_{e^y}^{1.1} 2(10)^2 (1.1 - z) dz & \text{for } 0 \leq y \leq \ln 1.1 \\ 1 & \text{for } y \geq \ln 1.1 \end{cases} \quad (38)$$

respectively, where the scale for U_1 in the Quarterly has been normalized to 1000 psi = 1.0 so that U_1 may be treated as any other uncertainty coefficient.

From Equations (37) and (38) we obtain, respectively,

$$F_{\ln U_1}(w) = \begin{cases} 0 & \text{for } w \leq \ln 0.96 \\ \frac{(25)^2}{2} (e^w - 0.96)^2 & \text{for } \ln 0.96 \leq w \leq 0 \\ 1 - \frac{(25)^2}{2} (1.04 - e^w)^2 & \text{for } 0 \leq w \leq \ln 1.04 \\ 1 & \text{for } w \geq \ln 1.04 \end{cases}$$

and

$$F_{\ln U_2}(y) = \begin{cases} 0 & \text{for } y \leq 0 \\ 1 - 10^{-1} \cdot (1.1 - e^y)^2 & \text{for } 0 \leq y \leq \ln 1.1 \\ 1 & \text{for } y \geq \ln 1.1 \end{cases}$$

By identical methods to those used for the same calculations on diameter uncertainty, the variance of $U = U_1 U_2$ is obtained:

$$E \ln U_1 = -0.13 \times 10^{-3}$$

$$\text{var } \ln U_1 = 0.267 \times 10^{-3}$$

$$E \ln U_2 = 32.5 \times 10^{-6}$$

$$\text{var } \ln U_2 = 1.57 \times 10^{-6}$$

By Equations (18), (19), and (36):

$$\text{var } p \approx (v - \Delta v)^2 e^{2E \ln U} \text{var } U = 0.268 \times 10^{-3} (v - \Delta v)^2.$$

3.4.5.5 Expansion Stress, S_E , Other Than from p , d , and t

The uncertainties in the calculated expansion stress, S_E , other than those caused directly by the other physical variables, are listed:

U_1 - due to arrangement error

U_2 - due to calculation method error

U_3 - due to calculation error

U_4 - due to loss of flexibility

U_5 - due to loop redesign error

and have had probability densities proposed in the Sixth and Tenth Quarterlies, GEAP-5279 and GEAP-5554, respectively, both in Section 3.3 (the latter have been adopted here preferentially). With these we calculate:

Arrangement error [directly from the graph (GEAP-5279)] :

$$\begin{aligned} E \ln U_1 &= 10^{-2} \cdot \left[\int_{-\ln 2}^0 \frac{2(\ln 2 + u)u}{(\ln 2 + \ln 10) \ln 2} du + \int_0^{\ln 10} \frac{2(\ln 10 - u)u}{(\ln 2 + \ln 10) \ln 10} du \right] \\ &= 10^{-2} \cdot \left[\frac{-(\ln 2)^2 + \frac{2}{3}(\ln 2)^2 + (\ln 10)^2 - \frac{2}{3}(\ln 10)^2}{\ln 2 + \ln 10} \right] \\ &= \frac{1}{3} \cdot 10^{-2} (\ln 10 - \ln 2) = 5.3648 \times 10^{-3} \end{aligned}$$

$$E(\ln U_1)^2 = \frac{1}{6} \cdot 10^{-2} \left[\frac{(\ln 2)^3 + (\ln 10)^3}{\ln 2 + \ln 10} \right] = 6.9772 \times 10^{-3}$$

$$\text{var } \ln U_1 = E(\ln U_1)^2 - (E \ln U_1)^2 = 6.948 \times 10^{-3}$$

Calculation method error [directly from the graph (GEAP-5279)]

similarly:

$$E \ln U_2 = \frac{1}{3} \cdot (\ln 1.1 - \ln 2) = -199.279 \times 10^{-3}$$

$$E(\ln U_2)^2 = \frac{1}{6} \cdot \left[\frac{(\ln 2)^3 + (\ln 1.1)^3}{\ln 2 + \ln 1.1} \right] = 70.58 \times 10^{-3}$$

$$\text{var } \ln U_2 = 30.867 \times 10^{-3}$$

Calculation error (subsection 3.3.2, GEAP-5554):

$$E \ln U_3 = \frac{1}{3} \cdot 10^{-1} \cdot (\ln 7.5 - \ln 2) = 44.058 \times 10^{-3}$$

$$E(\ln U_3)^2 = \frac{1}{6} \cdot 10^{-1} \cdot \left[\frac{(\ln 2)^3 + (\ln 7.5)^3}{\ln 2 + \ln 7.5} \right] = 52.394 \times 10^{-3}$$

$$\text{var } \ln U_3 = 50.453 \times 10^{-3}$$

Loss of flexibility error [directly from the graph (GEAP-5279)]:

$$\begin{aligned}
 E \ln U_4 &= 10^{-2} \cdot \left[\int_0^{\ln 3} \frac{2u}{\ln 3 + \ln 10} \cdot du + \int_{\ln 3}^{\ln 10} \frac{2(\ln 10 - u)u}{(\ln 10)^2 - (\ln 3)^2} \cdot du \right] \\
 &= 10^{-2} \cdot \left[\frac{(\ln 3)^2 (\ln 10 - \ln 3) + (\ln 10)^3 - \frac{2}{3} (\ln 10)^3 - \ln 10 \cdot (\ln 3)^2 + \frac{2}{3} (\ln 3)^3}{(\ln 10)^2 - (\ln 3)^2} \right] \\
 &= \frac{1}{3} \cdot 10^{-2} \cdot \frac{[(\ln 10)^3 - (\ln 3)^3]}{[(\ln 10)^2 - (\ln 3)^2]} = 8.858 \times 10^{-3} \\
 E(\ln U_4)^2 &= 10^{-2} \cdot \left[\frac{\frac{2}{3} (\ln 3)^3 (\ln 10 - \ln 3) + \frac{2}{3} (\ln 10)^4 - \frac{1}{2} (\ln 10)^4 - \frac{2}{3} \ln 10 \cdot (\ln 3)^3 + \frac{1}{2} (\ln 3)^4}{(\ln 10)^2 - (\ln 3)^2} \right] \\
 &= \frac{1}{6} \cdot 10^{-2} \cdot \frac{[(\ln 10)^4 - (\ln 3)^4]}{[(\ln 10)^2 - (\ln 3)^2]} = 10.848 \times 10^{-3}
 \end{aligned}$$

$$\text{var } \ln U_4 = 10.770 \times 10^{-3}$$

Loop redesign error [subsection 3.3, Cause 1.5 (GEAP-5279)]:

$$E \ln U_5 = 0.5365 \times 10^{-3}$$

$$E(\ln U_5)^2 = 0.6977 \times 10^{-3}$$

$$\text{var } \ln U_5 = 0.695 \times 10^{-3}.$$

We obtain the expectation and variance of $\ln U$ by summing:

$$E \ln U = 10^{-3} (5.365 - 199.279 + 44.058 + 8.858 + 0.536) = -140.5 \times 10^{-3}$$

$$\text{var } \ln U = 10^{-3} (6.948 + 30.867 + 50.453 + 10.770 + 0.695) = 99.7 \times 10^{-3},$$

which yield by Equations (22), (18), and (19):

$$ES \approx v \cdot e^{E \ln U} \left(1 + \frac{1}{2} \text{var } \ln U \right) = 0.9123 v$$

$$\text{var } S \approx v^2 \cdot e^{2E \ln U} \cdot \text{var } \ln U = 75.3 \times 10^{-3} v^2,$$

where S is used for the random variable S_E , excluding the effect of the physical variables, p , d , and t .

3.4.5.6 Other Sources of Uncertainty in the n_t Equation

The variances of B and $\ln C$, the equation parameters, are estimated from the Schenectady experimental runs. A full discussion of the methods of estimation is contained in subsection 3.5 and Appendix A.

Pressure stress, S_p , should have exactly the uncertainty due to its components, p, d, and t. By expressing S_p as $\frac{1}{2} pd/t$ wherever it occurs in the n_t equation, we pick up all the uncertainty through the uncertainties in p, d, and t.

Expansion stress, S_E , likewise has additional uncertainties due to p, d, and t.

p:

Since the thermal expansion in a pipe dominates the pressure expansion, and the pressure uncertainties are relatively small, we ignore here the cause of uncertainty in S_E due to p.

d:

The uncertainty in the diameter of an entire line of pipe is thought to cause a proportionate change in expansion stresses, the uncertainty in that of a piping component to cause an inversely proportionate change in expansion stresses (with other components experiencing a less than proportionate change in expansion stress, here ignored), and the uncertainty in that of a cross section within a component to cause a change in expansion stresses inversely proportional to its square.

t:

The expansion stress, S_E , has uncertainty thought to be inversely proportional to that of t.

3.4.5.7 Comment on Independence of Uncertainties

A comment here may be appropriate. We have broken down the causes of uncertainty in the n_t equation so as to identify what independent influences are at work. If, subsequently, it appears that any of these uncertainties do not arise independently, then we will attempt to evaluate the covariance between these for use in Equation (13) or rearrange the n_t equation to obviate the difficulty. However, we feel that there are intuitive grounds for believing the uncertainties detailed in this subsection (3.4.5) to have minimal interdependence.

3.4.6 Coefficients on Variances for Approximation of var $\ln n_t$

In subsection 3.4.4, we gave some justification for using—with the first expression alone on the right-hand side retained—Equation (14):

$$\text{var } \phi(X_1, \dots, X_I) \approx \sum_{i=1}^I \left[\frac{\partial \phi}{\partial x_i}(EX_1, \dots, EX_I) \right]^2 \cdot \text{var } X_i \quad (39)$$

for $\phi = \ln n_t$ with the X_i identified as the variables, B , $\ln C$, t , etc. This approximate equation, (39), is known as the propagation of errors approximation since uncertainties in the X_i reveal themselves as contributors to the uncertainty in $\phi(X_1, \dots, X_I)$. It is widely used to compare the relative importance of the uncertainties in the X_i so as, for example, to reduce the uncertainty in $\phi(X_1, \dots, X_I)$ most efficiently through possible reductions in the uncertainties of the X_i .

From Equation (8), we obtain

$$\ln n_t = (\ln \tilde{\Delta K}) \cdot B - \ln C - \left(\frac{B}{2} - 1\right) \cdot \ln t + \ln \int_{\frac{a_0}{t}}^1 \left[x^{-1/2} F(S_B, S_M; x) \right]^B dx,$$

from which we now derive the coefficients for insertion into the approximate equation, (39), with S_B and S_M defined for each of the three combinations of stresses picked out in subsection 3.3, Equations (3), (4), and (5).

3.4.6.1 Evaluation of Partial Derivatives for var $\ln n_t$ Approximation

The partial derivatives, $\frac{\partial \phi}{\partial x_i} (EX_1, \dots, EX_I)$ for the var $\ln n_t$ approximate equation, (39), are as follows:

Equation parameter, B :

$$\ln \tilde{\Delta K} - \frac{1}{2} \ln Et + \frac{\int_{\frac{a_0}{t}}^1 \ln(x^{-1/2} F) \cdot \left[(x^{-1/2} F)^{EB} \right] \cdot dx}{\int_{\frac{a_0}{t}}^1 x^{-\frac{EB}{2}} [F]^{EB} dx}$$

Equation parameter, $\log C$:

$$-2.3026$$

Thickness, t :

$$-\frac{\frac{EB}{2} - 1}{Et} + \frac{\int_{\frac{a_0}{t}}^1 x^{-\frac{EB}{2}} \left(\frac{\partial S_B}{\partial t} \cdot \frac{\partial F}{\partial S_B} + \frac{\partial S_M}{\partial t} \cdot \frac{\partial F}{\partial S_M} \right) \cdot [EB \cdot [F]^{EB-1}] dx}{\int_{\frac{a_0}{t}}^1 x^{-\frac{EB}{2}} [F]^{EB} dx}$$

Initial crack depth as fraction of wall thickness, $\frac{a_o}{t}$:

$$\frac{\left(E \frac{a_o}{t}\right)^{-EB/2} \cdot \left[F \left(\begin{array}{l} \left(0, ES_E + (1/4) \cdot Ep \cdot Ed/Et \text{ in (3)} \right) \\ \left(0, (1/2) Ep \cdot Ed/Et \text{ in (4)} \right) \\ \left(ES_E, (1/2) Ep \cdot Ed/Et \text{ in (5)} \right) \end{array} \right); E \frac{a_o}{t} \right]^{EB}}{\int_{E \frac{a_o}{t}}^1 x^{-EB/2} [F]^{EB} dx}$$

Pressure, p:

$$\frac{\frac{\partial S_M}{\partial p} \cdot \int_{E \frac{a_o}{t}}^1 x^{-EB/2} \frac{\partial F}{\partial S_M} \cdot [EB [F]^{EB-1}] dx}{\int_{E \frac{a_o}{t}}^1 x^{-EB/2} [F]^{EB} dx}$$

Diameter, d:

$$\frac{\int_{E \frac{a_o}{t}}^1 x^{-EB/2} \left(\frac{\partial S_B}{\partial d} \cdot \frac{\partial F}{\partial S_B} + \frac{\partial S_M}{\partial d} \cdot \frac{\partial F}{\partial S_M} \right) [EB \cdot [F]^{EB-1}] \cdot dx}{\int_{E \frac{a_o}{t}}^1 x^{-EB/2} [F]^{EB} dx}$$

Expansion stresses, S_E , other than from p, d, and t:

$$\frac{\int_{E \frac{a_o}{t}}^1 x^{-EB/2} \left(\frac{\partial S_B}{\partial S_E} \cdot \frac{\partial F}{\partial S_B} + \frac{\partial S_M}{\partial S_E} \cdot \frac{\partial F}{\partial S_M} \right) [EB \cdot [F]^{EB-1}] \cdot dx}{\int_{E \frac{a_o}{t}}^1 x^{-EB/2} [F]^{EB} dx}$$

With:

$$F = F \left(\begin{array}{l} \left(0, ES_E + (1/4) \cdot Ep \cdot Ed/Et \text{ in (3)} \right) \\ \left(0, (1/2) \cdot Ep \cdot Ed/Et \text{ in (4)} \right) \\ \left(ES_E, (1/2) \cdot Ep \cdot Ed/Et \text{ in (5)} \right) \end{array} \right); x$$

$$\frac{\partial F}{\partial S_B} = - (1-x)^{-3/2} [F]^2 \sqrt{f(x)/x} \cdot h(x)$$

$$\frac{\partial F}{\partial S_M} = - (1-x)^{-3/2} [F]^2 \left[3x^{3/2} \sqrt{f(x)/x} + (1-x) \sqrt{\frac{1-x}{x}} \cdot g(x) \right] \cdot h(x)$$

$$\frac{\partial S_B}{\partial t} = \begin{Bmatrix} 0 & \text{in (3)} \\ 0 & \text{in (4)} \\ -\frac{ES_E}{Et} & \text{in (5)} \end{Bmatrix}$$

$$\frac{\partial S_M}{\partial t} = \begin{Bmatrix} \frac{-ES_E}{Et} + \frac{-Ep \cdot Ed}{4 \cdot (Et)^2} & \text{in (3)} \\ -(1/2) \cdot Ep \cdot Ed / (Et)^2 & \text{in (4)} \\ -(1/2) \cdot Ep \cdot Ed / (Et)^2 & \text{in (5)} \end{Bmatrix}$$

$$\frac{\partial S_M}{\partial p} = \begin{Bmatrix} (1/4) \cdot Ed / Et & \text{in (3)} \\ (1/2) \cdot Ed / Et & \text{in (4)} \\ (1/2) \cdot Ed / Et & \text{in (5)} \end{Bmatrix}$$

Diameter, whole pipe line, d_1 :

$$\frac{\partial S_B}{\partial d_1} = \begin{Bmatrix} 0 & \text{in (3)} \\ 0 & \text{in (4)} \\ \frac{ES_E}{Ed_1} & \text{in (5)} \end{Bmatrix}$$

$$\frac{\partial S_M}{\partial d_1} = \begin{Bmatrix} \frac{ES_E}{Ed_1} + \frac{1}{4} \cdot \frac{Ep \cdot Ed}{Ed_1 \cdot Et} & \text{in (3)} \\ \frac{1}{2} \cdot \frac{Ep \cdot Ed}{Ed_1 \cdot Et} & \text{in (4)} \\ \frac{1}{2} \cdot \frac{Ep \cdot Ed}{Ed_1 \cdot Et} & \text{in (5)} \end{Bmatrix}$$

Diameter, component, d_2 :

$$\frac{\partial S_B}{\partial d_2} = \begin{Bmatrix} 0 & \text{in (3)} \\ 0 & \text{in (4)} \\ -\frac{ES_E}{Ed_2} & \text{in (5)} \end{Bmatrix}$$

$$\frac{\partial S_M}{\partial d_2} = \begin{Bmatrix} -\frac{ES_E}{Ed_2} + \frac{1}{4} \cdot \frac{Ep \cdot Ed}{Ed_2 \cdot Et} & \text{in (3)} \\ \frac{1}{2} \cdot \frac{Ep \cdot Ed}{Ed_2 \cdot Et} & \text{in (4)} \\ \frac{1}{2} \cdot \frac{Ep \cdot Ed}{Ed_2 \cdot Et} & \text{in (5)} \end{Bmatrix}$$

Diameter, cross-section, d_3 :

$$\frac{\partial S_B}{\partial d_3} = \begin{Bmatrix} 0 & \text{in (3)} \\ 0 & \text{in (4)} \\ -2 \frac{ES_E}{Ed_3} & \text{in (5)} \end{Bmatrix}$$

$$\frac{\partial S_M}{\partial d_3} = \begin{Bmatrix} -2 \frac{ES_B}{Ed_3} + \frac{1}{4} \cdot \frac{Ep \cdot Ed}{Ed_3 \cdot Et} & \text{in (3)} \\ \frac{1}{2} \cdot \frac{Ep \cdot Ed}{Ed_3 \cdot Et} & \text{in (4)} \\ \frac{1}{2} \cdot \frac{Ep \cdot Ed}{Ed_3 \cdot Et} & \text{in (5)} \end{Bmatrix}$$

$$\frac{\partial S_B}{\partial S_E} = \begin{Bmatrix} 0 \text{ in (3)} \\ 0 \text{ in (4)} \\ 1 \text{ in (5)} \end{Bmatrix} \quad \frac{\partial S_M}{\partial S_E} = \begin{Bmatrix} 1 \text{ in (3)} \\ 0 \text{ in (4)} \\ 0 \text{ in (5)} \end{Bmatrix}$$

These may be evaluated numerically with the aid of the transformation, subsection 3.3, Equation (7), in the Thirteenth Quarterly, GEAP-5680. Each partial derivative of diameter is evaluated separately.

3.4.7 Application of Results to a Particular Cross Section

In subsection 3.4.6, Equation (39), we found:

$$\text{var } \ln n_t \approx \sum_{i=1}^I \left[\frac{\partial \ln n_t}{\partial x_i} \bigg|_{(EX_1, EX_2, \dots, EX_I)} \right]^2 \cdot \text{var } X_i. \quad (40)$$

The coefficients, $\frac{\partial \ln n_t}{\partial x_i}$, are evaluated at $(EX_1, \dots, EX_I)$. To compute the expected values,

EX_i , we need to know the plan values, since EX_i is the product of one of these with EU, the respective bias in uncertainty. Similarly, in subsection 3.4.5, we evaluated $\text{var } X_i$ for each i as a multiple of the square of the respective plan value.

The coefficients in the approximate equation, (40), have been evaluated by the formulae of subsection 3.4.6.1 with S_B and S_M defined as in Equation (3) for the cross section detailed in subsection 3.4 of the Thirteenth Quarterly, GEAP-5680. These coefficients—together with distribution parameters of the uncertainty coefficients under their cause code numbers (from subsection 3.3, GEAP-5279 and subsection 3.2, GEAP-5474, the Sixth and Eighth Quarterlies), design values, plan values, expected values, variances and contributions to the linear part of $\text{var } \ln n_t$ [given in the approximate equation, (40)]—are listed in Table 3-2.

It should be noted that each term in the approximate equation, (40), may be found either as the product of $\left(\frac{\partial \ln n_t}{\partial x_i} \right)^2$ with $\text{var } X_i$ or as the product of $\left[(\text{plan})^2 \cdot \left(\frac{\partial \ln n_t}{\partial x_i} \right)^2 \right]$ with $\text{var } U$,

the variance of the uncertainty coefficient. But $\text{var } U$ measures the spread of X_i in ratio form, irrespective of the original units of X_i or the value of the expected value of X_i in these units, and this allows us to compare $\text{var } U$ between the random variables, X_i . Then the size of

$\left[(\text{plan})^2 \cdot \left(\frac{\partial \ln n_t}{\partial x_i} \right)^2 \right]$ determines how strongly the functional form of the n_t equation will

transmit uncertainty in a particular X_i into uncertainty in the number of cycles to grow through, n_t . For instance, in computing the contribution of uncertainty in expansion stress (other than from p , d , and t) to the uncertainty in n_t , from (40), we may either multiply $57 \text{ E } \ell b^2 \text{ in.}^{-4}$ by $2.4 \text{ E-8 } c^2 \ell b^{-2} \text{ in.}^4$ or multiply 0.0753 by 17.9 c^2 , reading from Table 3-2; the latter, however, tells us, in scanning the respective columns in the table, that stress contributes to the uncertainty in n_t rather because what stress is realized is highly uncertain than because the n_t equation is sensitive to changes in stress.

TABLE 3-2
CONTRIBUTIONS TO DISPERSION OF n_t AT CHOSEN CROSS SECTIONS

	Symbol	Cause Code No.	Design	Δ Design	Plan	$E \ln U_i \times 10^{-3}$	Expected Value	$\text{var } \ln U_i \times 10^{-3}$	$\text{var } U \times 10^{-3}$	Variance	$\frac{\partial \ln n_t}{\partial x_i}$	$\left(\frac{\partial \ln n_t}{\partial x_i}\right)^2$	$(\text{plan})^2 \cdot \left(\frac{\partial \ln n_t}{\partial x_i}\right)^2$	Contribution to var $\ln n_t$	Contribution to var (n_t) as percentage
Equation parameter	B	--	--	--	--	--	4.45	--	--	1.016	1.808	3.27		3.3231	17.49
Equation parameter	log C	--	--	--	--	--	-3.96	--	--	0.138	-2.303	5.30		0.7318	5.05
Thickness, in.	$\left. \begin{matrix} U_1 \\ U_2 \\ U \end{matrix} \right\}$	$\left\{ \begin{matrix} 1.9 \\ 1.11 \\ 9.1 \end{matrix} \right\}$	$\left\{ \begin{matrix} \\ \\ \end{matrix} \right\}$			$\begin{matrix} -0.158 \\ 0 \\ -0.158 \end{matrix}$		$\begin{matrix} 0.443 \\ 2.012 \\ 2.455 \end{matrix}$							
	t	--	0.432	--	0.432	--	0.432	--	--	0.46E-3	8.324	69.29	12.9	0.0317	0.17
Initial crack depth	$\frac{a_o}{t}$	2.2	--	--	--	--	0.0364	--	--	0.007	-43.992	1935.30		13.5595	71.39
Diameter, in.	$\left. \begin{matrix} U_1 \\ U_2 \\ U_3 \\ U \end{matrix} \right\}$	$\left\{ \begin{matrix} 1.12 \\ \\ 10.1 \end{matrix} \right\}$	$\left\{ \begin{matrix} -- \\ -- \\ -- \end{matrix} \right\}$	--		$\begin{matrix} -0.0003 \\ -0.0033 \\ 0 \\ -0.004 \end{matrix}$		$\begin{matrix} 0.000 \\ 0.003 \\ 0.025 \\ 0.029 \end{matrix}$	$\begin{matrix} 0.000 \\ 0.003 \\ 0.025 \\ 0.029 \end{matrix}$		$\begin{matrix} -4.821 \\ 3.640 \\ 7.870 \end{matrix}$	$\begin{matrix} 23.24 \\ 13.25 \\ 61.93 \end{matrix}$	$\begin{matrix} 23.2 \\ 13.2 \\ 61.9 \end{matrix}$	$\begin{matrix} 0.0000 \\ 0.0000 \\ 0.0015 \end{matrix}$	$\begin{matrix} 0.00 \\ 0.00 \\ 0.01 \end{matrix}$
	d		6.624		6.624		6.624			1.5E-3				0.0016	0.01
Pressure, lb/in. ⁻²	$\left. \begin{matrix} U_1 \\ U_2 \\ U \end{matrix} \right\}$	$\begin{matrix} 5.1 \\ 5.2 \end{matrix}$				$\begin{matrix} -0.134 \\ 0.033 \\ -0.101 \end{matrix}$		$\begin{matrix} 0.267 \\ 0.002 \\ 0.269 \end{matrix}$	$\begin{matrix} \\ 0.268 \end{matrix}$						
	p	--	1200	-200	1000	--	1000	--	--	268	-0.0006	35E-8	0.35	0.0001	0.00
Expansion stress (other than from p, d, and t), lb/in. ⁻²	$\left. \begin{matrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U \end{matrix} \right\}$	$\begin{matrix} 1.1 \\ 1.2 \\ 1.3 \\ 1.4 \\ 1.5 \end{matrix}$				$\begin{matrix} 5.4 \\ -199.3 \\ 44.1 \\ 8.9 \\ 0.5 \\ -140.5 \end{matrix}$		$\begin{matrix} 6.95 \\ 30.85 \\ 50.45 \\ 10.77 \\ 0.70 \\ 99.7 \end{matrix}$	$\begin{matrix} \\ 75.3 \end{matrix}$						
	S	--	27450	--	27450	--	25042	--	--	57E 6	1.5E-4	2.4E-8	17.9	1.3476	7.09
Total Variance	$\ln n_t$													18.9944	100.00

The expected value of n_t for this cross section is computed to be 1.08 Mc, with spread characterized from Table 3-2, $(\sqrt{\text{total variance}})$, as a factor of $\exp(\sqrt{18.9944}) = 78$ —an indication that 65 to 70% of tests under similar physical conditions would have fatigue lives lying between 14 kc and 84 Mc. Table 3-2 also shows that the contribution to the total $\text{var } \ln n_t$ of initial crack depth, $\frac{a_0}{t}$, is predominant. Thus, we must expect the skewness of the distribution of $\frac{a_0}{t}$ to skew the distribution of $\ln n_t$. This must be taken to indicate, at least for this welded joint, that finding a method for generating the tail distribution is mandatory for any approximation of the probability of leakage within the number of operating cycles expected in 40 years.

Further possibilities are to assume ultrasonic testing of welds and to look at the variance approximation for a cross section of a piping component.

3.5 OUTLINE OF THE ANALYSIS OF CRACK GROWTH DATA FROM TASKS D-1 AND D-2

3.5.1 Introduction

- 1.* This analysis was introduced in subsection 3.4 of the Twelfth Quarterly, GEAP-5673, to which the reader should refer for important introductory details. The main objective of the analysis is to estimate mean values and variances of parameters A and B appropriate to the materials and temperatures found in a reactor piping system, for use in the low-cycle fatigue crack growth rate model,

$$da/dn = A(\Delta K)^B. \quad (41)$$

This model is modified in step 3, given in subsection 3.5.2.

A secondary objective is to determine whether differences in the major variables of material, temperature, and specimen type, and the minor variables of heat of material, stress (always gross-section cyclic stress range), cyclic frequency, hold time at stress, and stress pattern (zero to tension or tension-tension) account for any significant differences in A or B. The analysis was performed by John L. Jaech of Battelle-Northwest, and is given in Appendix A. Mr. Jaech was associated with Task A of the Pipe Rupture Study from April 1965 to October 1966. The following subsections are an outline of the analysis, and give some extensions. Designations in brackets (e.g., [A.1]) refer to subsections in Appendix A.

The characteristics of the 44 specimens analyzed are summarized in Table 3-3, and are given in detail in Table A-1.

3.5.2 Statement of Model; Data Fitting and Refitting

2. [A.1, A.2] The analysis first justifies the use of the (ordinary) linear unweighted least-squares method of fitting the experimental $\log \Delta a/\Delta n$ versus $\log \Delta K$ data to obtain slope and intercept estimates. The equation that is fitted for each specimen is

$$\log \Delta a/\Delta n = \log A + B \log \Delta K, \text{ or } y = A' + Bx \quad (42)$$

*These step numbers run sequentially through all subsections in subsection 3.5.

TABLE 3-3

SUMMARY: SPECIMEN COUNT AND RUN NUMBERS

Specimen Type	A			B			D	F		
Configuration	14-in. wide center-slotted plate			5-in. wide center-slotted plate			6 in. diameter Pipe Halves	2-1/2-in. wide edge-notched plate		
Frequency, OT, except as shown	10 cpm			12 cpm			10 cpm	6 cpm		
Material	212	304	2-1/4	212	304	2-1/4	106	212	304	2-1/4
Temperature										
R. T.	3 5-7	3 1-3	2 11,12				3 30-32	5 36-40	3 33-35	4 41-44
TT	1 8	2 4,29	1 13							
485°F	2 9-10									
540°F			1 14							
550°F					3 15-17					
600°F				4 20,22-24						
0.6 cpm				2 21,26						
10 min. hold				1 25						
650°F				2 27-28	1 18	1 19				

Thus, slope is B and intercept is $A' = \log A$. It can be seen that this equation is obtained by taking common logs in the crack growth rate model. The techniques of the least-squares method are justified if the data have the following properties: (1) errors in y values are independent among each other, (2) x values are known with error small (or zero) compared to the error in y values, and (3) errors in x are not correlated with errors in y. The (ordinary) unweighted procedure is justified if the variance of y given x is constant for all x; that is, the scatter of points vertically about the line at low x is the same as at high x, or errors in y values are independent of x values. Since y is a log scale and each Δn is known without error, it can be shown that this is the same as requiring that the standard deviation of Δa be proportional to Δa . A linear model is justified by absence of a pattern of a long series of residuals of the same sign (data points on the same side of line) in the runs. Each of these properties is either demonstrated or made plausible in the analysis.

3. The linear model as written has very high correlation between slope and intercept. This correlation is undesirable because it requires introducing a large term for covariance between A and B, along with the variances of A and B, in the variance solution for n_t , thus distorting the picture of the real contributions of variabilities of A and B to the variability of n_t . This problem can be avoided by writing the linear model in the mean-adjusted form,

$$y = C' + B (x - \bar{x}), \quad (43)$$

where $\bar{x}$ is the grand mean of x for all runs. Now C' and B are uncorrelated because of model construction, and analysis shows that they are physically uncorrelated; i.e., high slopes do not occur principally with high mean-adjusted intercepts, for example. The mean-adjusted intercept C' is simply the intercept at $x = \bar{x}$, whereas A' is the intercept at $x = 0$. To see the crack growth rate model for the mean-adjusted linear model, write

$$\log \Delta a / \Delta n = \log C + B (\log \Delta K - \overline{\log \Delta K}), \quad (44)$$

$$\Delta a / \Delta n = C \left(\frac{\Delta K}{\overline{\Delta K}} \right)^B, \quad (45)$$

where $\overline{\Delta K}$ is the antilog of $\overline{\log \Delta K}$, the grand mean $\log \Delta K$. This is Equation (6) of subsection 3.3.1. Thus, we shall actually estimate mean values and variances of C and B for use in the variance of n_t solution.

4. Analysis showed that while the $\sigma_{\Delta a} / \Delta a = \text{constant}$ assumption was not severely violated, the greatest deviations from this assumption were caused by the successive crack length measurements being made to the nearest 0.01 inch, to the difficulty in seeing the exact crack tip, and to uneven crack growth. The deviations appeared (a) in the low- ΔK region characterized by small Δa values for ordinary Δn 's, and (b) for some data points taken after a relatively small number of additional cycles, Δn . No compensation was made in the analysis for the relative portion of crack-growth life which each $\Delta a / \Delta n$ point represented.

5. [A. 2] da/dn points were obtained by successive differences of the experimental a and n values, giving $\Delta a/\Delta n$ points. This method properly carries the real experimental variability—such as from uneven crack growth rate and fairly coarse crack-length measurements—ahead into uncertainties on the parameter estimates. For discussion of effects of some data smoothing methods, see subsection 3.6 of the Eleventh Quarterly, GEAP-5587.
6. All data points for each of the 44 specimens were fitted. Because of their strong influence on slope and intercept in a least-squares analysis, a few (between 0 and 3) severely out-lying points in each run were deleted and the lines refitted. In nearly all cases, large numbers of points remained for refitting, as given in the ninth column of Table A-1. The slope values now obtained were used for slope estimating and significance testing in subsection 3.5.3. Estimating and significance testing of intercepts must wait until it is determined whether slope differences found are not significant, and common slopes—parallel lines—for at least certain categories of runs can be assumed. Then intercept estimates at $x = 0$ for each line are obtained and intercept testing can proceed on a common basis for all lines. The mean-adjusted intercepts at $x = \bar{x}$ can be obtained subsequently.
7. [A. 3] It was necessary to choose the more appropriate of two methods of calculating ΔK for the edge-notched specimen type F runs. The non-symmetric definition of ΔK [Equation (9) Twelfth Quarterly, GEAP-5637] was adopted because of better agreement of the slope values with those from the center-slotted plate runs, specimen types A and B. Agreement was good enough to warrant using the specimen type F results in the parameter estimates. The symmetric definition of ΔK [Equation (8), Twelfth Quarterly] gave slopes that were considerably higher than for the center slotted plates; this definition assumed that the crack growth rate actually seen from only one edge of the specimen was growing from both edges.

3.5.3 Slope Estimating and Significance Testing

8. A slope estimate is now on hand for each run. The values are given in the tenth column of Table A-1, are plotted versus temperature in Figure 3-3, and are used throughout this subsection 3.5.3. Significance testing began with the minor variables and concluded with the major variables. Mean slope values for the different levels of the variables on which testing was performed in the following steps are displayed for comparison in Figure 3-4. A discussion of the variance used for significance testing is called for. Significance testing will recognize that there are four possible successive stages of combining results from the major test variables. The four stages can be called individual lines, test conditions, groups, and all runs. An "individual line" represents a run on a material at a temperature for one specimen type at one stress range. A "test condition" is a material at a temperature for one specimen type and all stress ranges. A "group" is a material at a temperature for all specimen types and stress ranges. "All runs" is obvious.

It was found that the slope of each individual line was well enough estimated to reveal that there were significant differences among slopes of individual lines within test conditions for physical, but unknown, reasons. On the other hand, it was found that there was no consistent effect due to changes in stress. Therefore the runs at different stresses in

FIGURE 3-3. ESTIMATES OF SLOPE B VERSUS TEMPERATURE

ZERO-TENSION VERSUS TENSION-TENSION:
 2 RUNS WERE ANOMALOUS
 2 RUNS WERE INCLUDED AS NOT SIGNIFICANTLY DIFFERENT
 B = 4.45 MEAN SLOPE, ALL RT RUNS AT 10-12 CPM ON SPECIMEN TYPES A AND F

FIGURE 3-4. SUMMARY OF MEAN SLOPES, B

each test condition could be considered replicates, and the variances among slopes within test conditions were estimated for each test condition. These variances were found to be just significantly different, but to give stability to this estimate in the face of small sample sizes, a pooled estimate of this variance was used for all test conditions and is the key measure for significance testing of slopes. This pooled variance of slope among individual lines within test conditions is 1.463. The 5% level was used for significance testing throughout the analysis.

9. [A.4] For the tests on B, a few runs were discarded outright because of having very large uncertainties on B. Some of these runs also had anomalous values of B, and covered very short distances on the log ΔK scale. These runs are identified in Figure 3-3.
10. [A.5] The 0.6-cpm specimens were shown to have a slope just significantly lower than the 6-10-12 cpm specimens, and were not used further in the slope analysis.
11. [A.6] There were four TT (tension-tension, elevated mean stress) runs. Two were discarded in step 9; the other two were retained as consistent with the OT (zero-tension) runs.
12. [A.7] Slope of the 10-minute hold time run was not anomalous; the run was included. Slopes for the runs remaining were used for all further slope testing, steps 13-17, and are identified in Figure 3-3. Runs discarded were used again for estimating and tests on intercept C'.
13. [A.7] The sequence of major significance tests on slope was materials, specimen types, and temperatures. This sequence was chosen based on the following discussion.

From an analysis viewpoint, the laboratory testing was performed in an unbalanced experiment design (that is, unequal numbers of runs were made per test condition). Consequently, all combinations of levels of the variables did not get tested (full factorial design); moreover, each level of the variables did not appear an equal number of times throughout all test conditions (which would be true of a fractional factorial design). In some cases, a level of one variable occurred only with one level of another variable, making the effects of going to each of the two levels indistinguishable; e.g., specimen type B was tested at only elevated temperatures. It was not a "planned experiment design" from a statistical viewpoint. A sequence of one-way analyses of variance appeared to be the most straightforward way to proceed, to make significance tests on slope for the major variables.

In general, in significance testing it is desirable to include as many results as possible so that estimates are made with greatest numbers of degrees of freedom and thus have smallest uncertainty (variance). To achieve this for unbalanced designs, one chooses, by inspection, a sequence of the test variables such that for the first one there appears to be the greatest chance that no significant differences will be shown among the estimates (of parameter B) from the different levels of the variable; and also, if possible, so that conclusions from the experiment are least compromised if no significant differences are mistakenly shown. The latter criterion should not be overlooked, because the first tests made

will ordinarily use the fewest degrees of freedom and thus be least powerful, leading to the conclusion that no significant differences exist for estimates that are relatively widely scattered. Because of the pattern of the experimental runs, the most appropriate sequence of variables was only moderately successful in meeting the objective of successively increasing the number of runs with each test.

14. [A. 7] Materials seemed to meet the above criteria best as the variable to be tested first, because there was clearly not a significant difference among slopes for the three materials. To perform this test, sets of results were selected where the only differences were among materials; i. e., specimen type and temperature were the same. In particular, specimen types A and F results at room temperature and specimen type B results at 650°F were used. Results of 8, 9, and 7 runs were available for Type-304 stainless steel, carbon steel, and 2-1/4 Cr-1 Mo, respectively. No significant differences among materials were found.
15. [A. 8] The second variable chosen for slope significance testing was specimen type. Now, results from different materials could be combined, but test conditions had to be selected where specimen type was the only other variable; i. e., temperature was constant. The only temperature at which more than one specimen type was tested was room temperature, using specimen types A, D, and F (omitting B, used only at elevated temperatures). Results of 9, 3, and 12 runs were available for specimen types A, D, and F, respectively. No significant differences were found in slope among these three specimen types at room temperature, although it is noted that slopes increased as specimen thickness decreased (A, 0.4 in.; B, 0.25 in; and F, 0.10 in.), raising the question of the possible effect of the time in crack life at which plane strain conditions gave way to plane stress conditions. Since specimen types A and B were taken from different heats than specimen type F for each material, this was also a test for an effect due to heat; and since specimen type F results were calculated by a different ΔK equation, this was also a test for an effect due to calculation method.
16. [A. 8] Now, all results at each temperature could be combined for testing for significant differences in slope among temperatures. Here, significant differences were found, but the pattern of which differences were significant seemed to be technically anomalous since a high slope was found at 600°F (as determined by carbon steel, specimen B), and a low slope was found at 650°F (as determined by all three materials, specimen B). No such large difference in slope between these two nearby temperatures was expected, and we hesitated to be immediately responsive to it.
17. [A. 9] It was decided to use slope $B = 4.45$, the weighted mean slope of the room temperature runs only of all three specimen types, as characterizing all three materials at both room and elevated temperatures. This value was found by weighting the mean slope for each specimen type by the inverse of the variance of estimate of its mean slope; thus, well-defined mean slope estimates contributed more to determining the weighted mean. The variance of estimate of this weighted mean slope was 0.0610. The weighted mean of all slopes, including elevated temperature runs, was nearly the same at 4.39. Thus, the erratic behavior of the elevated temperature runs did not greatly affect the average. Such a weighted mean for room temperature specimen type A runs only was 4.09, possibly

implying $B = 4.0$ when ΔK is well known and plane strain conditions prevail during much of crack life; but 4.45 was chosen as a common slope for study of intercepts and use in the n_t equation.

The variance of individual specimen slopes, about the grand mean of 4.45, was chosen as 1.016 and was arrived at as follows. The pooled variance of individual slopes within test conditions was given in step 8 as 1.463. This variance is about the mean of each test condition. Adding the variance among test conditions raises this to 1.563, the variance of individual slopes about the grand mean of 4.45. The grand mean is not exactly known; the variance of estimate of 4.45 is 0.061, and when added to 1.563 is 1.624. But the mean of variances of estimate of each individual slope for the 29 runs in Table A-2 is 0.608; this is the mean of the squares of σ_b in the tenth column of Table A-1, for the 29 runs. This fairly large contribution to 1.463—and thus to 1.624—is associated strictly with the measurement process as described in step 4., and can be removed from 1.624 to give 1.016, the variance of a true individual slope value omitting measurement process variation. It will be seen in step 24 that for illustrating variances of B and C' by confidence intervals, $\text{var } B = 1.624$ will be used to bound the region within which measured lines fall, whereas $\text{var } B = 1.016$ will be used for the variance of B in the solution for variance of n_t , since measurement process variation need not be considered in the variation in how cracks actually grow in piping.

3.5.4 Intercept Estimating and Significance Testing

18. [A.9] Using a common slope of 4.45, data of all specimens were refit, obtaining a new estimate of intercept for each of the resulting parallel lines. The refit lines pass through the point $(\bar{x}, \bar{y})$ of the previous lines. The choice of slope will affect the mean C values estimated, but will have very little effect on significance testing of intercepts. Steps 18 through 21 describe significance testing of intercepts. Intercept values are plotted in Figure 3-5; mean intercept values for the different levels of the variables on which testing was performed are displayed for comparison in Figure 3-6.
19. As for slopes, significance testing among intercepts of individual lines within test conditions showed that intercepts of individual lines were well enough estimated to reveal that there were significant differences among intercepts of individual lines within test conditions for physical, but unknown, reasons. Also, again there was no consistent effect on intercept due to stress. Therefore, it was concluded that a test condition has a single expected value of C' , about which there is variation among individual specimens.
20. [A.9] The first intercept tests were on possible effects of the minor variables. No significant effects were found due to cyclic frequency. There was a significant effect due to TT loading compared to OT, however; the TT intercepts were higher, and were omitted from further analysis. Thus, elevated mean stress runs are subject to higher crack growth rates. The relationship of intercept, obtained by the usual method of using cyclic stress range in ΔK , to two ways of accounting for mean stress is shown in Figure 3-8. Figure 3-8B shows, in particular, a nearly linear relationship between intercept and mean stress/stress range. There was a predictable mean stress effect on intercept for all three materials. Intercept of the 10-minute hold time run was not anomalous and was included.

FIGURE 3-5. ESTIMATES OF INTERCEPT—LOG A VERSUS TEMPERATURE USING B = 4.45

FIGURE 3-6. SUMMARY OF MEAN INTERCEPTS, LOG A, USING SLOPE B = 4.45

21. [A.9] On the major variables, it was shown that there were significant differences in intercept among test conditions. Examination of the data indicated that it would be appropriate to treat separately the specimen type D pipe halves since the mean intercept was high for this test condition, implying high crack growth rate. Physically, Task D-1 indicates this may be due to the application of the center-slotted flat-plate ΔK formula in this unusual configuration of two half cylinders tested face-to-face, each having a center slot, to measurements of crack growth for only the half in which the crack grew most rapidly and in which failure ultimately occurred. While only carbon steel pipe (A-106B) results were included in the analysis, subsequent testing of SS 304 and 2-1/4 Cr -1 Mo pipe also shows the high intercepts; see PP1 and PP2 lines on Figures 3-4 and 3-5 of the Twelfth Quarterly, GEAP-5637.
22. Having segregated the specimen type D results, it was appropriate to estimate a mean intercept for each material-temperature group for specimen types A, B, and F only. For each group, two components of variance were estimated: within-heat and between-heat. The within-heat component was taken among the individual lines, and pooled for specimen types A and F at room temperature, and B at elevated temperature. The between-heat component was taken between specimens types A and F at room temperature since different heats of plate were used for these two specimen types. There was a substantial between-heat component of variance for the carbon steel and no such component for the SS 304 and 2-1/4 Cr-1 Mo. Task D-1 has tentatively ascribed this pattern to the greatly different yield strengths of the two carbon steel heats, while the difference in yield strengths between heats of the other two materials was small. Thus, the difference between heats for carbon steel is apparently a genuine heat difference, not due just to specimen type difference. See Figure 3-7 which illustrates these relationships. The 0.2% yield strength data and sources are given in Table 3-4.

FIGURE 3-7. MEAN INTERCEPTS AT ROOM TEMPERATURE VERSUS 0.2% YIELD STRENGTH FOR SPECIMEN TYPES A, B AND F

FIGURE 3-8. EFFECT ON INTERCEPT OF ELEVATED MEAN STRESS

TABLE 3-4

VALUES AND SOURCES OF 0.2% YIELD STRENGTHS, ksi, FOR
TASKS D-1 AND D-2 TEST MATERIALS

Material	Specimen Type	
	A and B	F
Carbon Steel; A-212-B	45.0, M&P Lab Subsection 6.2.2 Second Quarterly, GEAP-4964	29.4*, M&P Lab Subsection 6.3.1 Second Quarterly, GEAP-4964
SS 304; A-240, TP-304	36.4, Mill 34.5, M&P Lab Subsection 6.2.5 First Quarterly, GEAP-4911	38.8, M&P Lab Subsection 6.3.1 Second Quarterly, GEAP-4964
2-1/4 Cr -1 Mo; A-387-D	35.6, M&P Lab Subsection 6.2.1 Third Quarterly, GEAP-5082	36.7, M&P Lab Subsection 6.3.1 Second Quarterly, GEAP-4964

*In direction of rolling.

The exact role played in accounting for the differences in intercepts due to specimen type or to temperature is clouded because the apparent increase in intercept with temperature could possibly also be accounted for by the change to specimen type B, which was tested only at elevated temperatures. Specimen type B was not judged beforehand to offer any reasons for results not being strictly comparable with those of the larger specimen type A. Both specimen types A and B are center-slotter plates: A is 14 in. wide $\times$ 0.4 in. thick, and B is 5 in. wide $\times$ 0.25 in. thick; B was better adapted to cyclic testing in a furnace. The tangent ΔK formula was used for both specimen types. But there were also three type A specimens tested at elevated temperatures (485 and 540°F): It can be seen on Figures 3-5 and 3-6 that the mean of these three tests is significantly below that of the 14 type B specimen tests. However, the tentative conclusion is to take the type B specimen results as truly indicative of the temperature effect, and to estimate separate mean intercept values for the six groups: three different materials at room and elevated temperatures. These group estimates are also significantly different.

The variances among intercepts of individual lines within test conditions were found to be significantly different; they have been retained for application to piping. As was done for variance of slope in step 17, the variance of intercept for an individual pipe location is taken as the sum of the variance among intercepts within test conditions, and the variance of estimate of the intercept for each test condition (taken same as for group). Both of these components are separated as to the between- and within-heat contributions. (For slope, these contributions were not separated, per step 17 which showed no significant difference in slope due to specimen type; i. e., heat.)

3.5.5 Final Results and Confidence Intervals

23. All estimates of mean value and variance of slope and intercept are tabulated in Table 3-5. The values which will be used for computing a mean and variance of $\ln n_t$ per subsection 3.4 of this Quarterly are headed with (*). It can be seen that a single value of slope is given for all groups, and a single value of variance of slope. The variance value of 1.016 is found in step 19, subsection 3.5.4. Different values of intercept are given for each group, along with different values of variance of intercept found from Equation (A-33). For comparison of the "best lines" so estimated for the six groups, see Figure 3-9.
24. For illustration of the relationship of means and variances of slope and intercept to the experimental lines fitted, 2σ confidence intervals for the case of carbon steel at room temperature are shown in Figure 3-10. Two kinds of confidence intervals are given: the inner limits show the uncertainty on the location of the mean line found using means of slope and intercept; the outer limits show the uncertainty on the location of any single future line. It would be expected that the outer limits would contain nearly all of the experimental lines, and they appear to do so. The values of variance of slope and intercept used to find the outer limits are $\text{var } B = 1.624$ and $\text{var } C' = 0.13425$. The limits are referred to as " 2σ ", and are only approximately 95% limits, because of the few data available; in particular, the t value much larger than 2 associated with the one degree of freedom available for estimating between-heat variance was not imposed.
25. The question arose during the course of the analysis as to whether there was a possible effect on either slope or intercept of the extent of crack life over which the crack growth rate data were taken. Figure 3-11 shows the relative portions of crack life over which data were reported for each specimen. No consistent effect on slope or intercept was found. It is noted that most cracks were allowed to grow to between 25% and 50% of total length available for crack length a in the ΔK equation used. Thus, regions of questionable validity of the ΔK equations used were avoided. It is also noted that four of the runs were continuations of previous runs at other conditions on the same specimens. There is no consistent effect on C or B due to this practice; in particular, there is no increase in intercept for the second test attributable to pre-fatiguing. Second test conditions were always at higher stress range, so second test lines are always further right on the ΔK scale.
26. An arbitrary, but apparently unavoidable, feature is introduced into the analysis when variances among intercepts of the parallel lines are assigned to individual lines, or to groups of lines, at the grand mean of x for all runs, $\bar{\bar{x}}$, rather than at the mean x for the individual line, or for groups of lines, which are actually slightly displaced from each other. Another effect of this fixing of intercept variance at $\bar{\bar{x}}$ is that, when applied to real piping situations in which cracks may start at lower ΔK values than in the laboratory testing, the possible crack growth rate lines given by mean C and B values, surrounded by variances of C and B , "pivot"—due to variance of B —at $\bar{\bar{x}}$. Thus, steeper slopes will certainly have smaller crack growth rates as ΔK moves up through the low- ΔK values; and lower slopes will have higher crack growth rates in the low- ΔK region. The effect of this pivoting at $\bar{\bar{x}}$, which is relatively high along the ΔK scale for cracks growing from small initial sizes, is to increase n_t for higher slopes, and decrease n_t for lower slopes. No more rational choice of pivoting position seems to be available, however. This $\bar{\bar{x}}$ value is introduced into the crack growth model by $\tilde{\Delta K} = \text{antilog } \bar{\bar{x}} = \text{antilog log } \Delta K$.

FIGURE 3-9. LOW-CYCLE FATIGUE CRACK GROWTH RATE TESTS, TASKS D-1 AND D-2

FIGURE 3-10. LOW-CYCLE FATIGUE CRACK GROWTH RATE TESTS, TASKS D-1 AND D-2

FIGURE 3-11. EXTENT OF CRACKS

TABLE 3-5
ESTIMATES OF MEAN VALUE AND VARIANCE OF SLOPE AND INTERCEPT
FOR A PIPE LOCATION OF KNOWN MATERIAL AND TEMPERATURE

	**		**	*	*	$\log \Delta K$	*	**		*	Number of			
	Log A	A	Log C	C	B	$\bar{x}$	$\frac{\Delta K}{\bar{\Delta K}}$	Var log C	Var C	Var B	Specimens of Type			
								V_2 at $x = \bar{x}$			A	B	F	Total
Room Temperature														
Carbon Steel	-25.378	4.188×10^{-26}	-4.080	0.8318×10^{-4}	4.45	4.786	6.109×10^4	0.13425	0.493×10^{-8}	1.016	3		5	8
SS 304	-25.260	5.495×10^{-26}	-3.962	1.091×10^{-4}	4.45	4.786	6.109×10^4	0.13802	0.871×10^{-8}	1.016	3		3	6
2-1/4 Cr-1 Mo	-25.653	2.223×10^{-26}	-4.355	0.4416×10^{-4}	4.45	4.786	6.109×10^4	0.00753	0.00779×10^{-8}	1.016	2		4	6
Elevated Temperature														
Carbon Steel	-24.888	12.94×10^{-26}	-3.590	2.570×10^{-4}	4.45	4.786	6.109×10^4	0.14695	5.146×10^{-8}	1.016	2	9		11
SS 304	-24.953	11.10×10^{-26}	-3.655	2.213×10^{-4}	4.45	4.786	6.109×10^4	0.07038	1.828×10^{-8}	1.016		4		4
2-1/4 Cr-1 Mo	-25.410	3.891×10^{-26}	-4.112	0.7727×10^{-4}	4.45	4.786	6.109×10^4	0.20280	0.642×10^{-8}	1.016	1	1		2

Unadjusted Linear Model: $\log da/dn = \log A + B \log \Delta K$
Unadjusted Crack Growth Rate Model: $da/dn = A(\Delta K)^B$
Mean-Adjusted Linear Model: $\log da/dn = \log C + B (\log \Delta K - \log \bar{\Delta K})$
Mean-Adjusted Crack Growth Rate Model: $da/dn = C \left(\frac{\Delta K}{\bar{\Delta K}} \right)^B$

*Values that are used directly in finding the variance of $\ln n_f$ per Equation (40).
Log A = Intercept in unadjusted linear model, at $\log \Delta K = 0$
A = Antilog intercept in unadjusted model, at $\log \Delta K = 0$
Log C = Intercept in mean-adjusted linear model, at $\log \Delta K = \log \bar{\Delta K}$
C = Antilog intercept in mean-adjusted linear model, at $\log \Delta K = \log \bar{\Delta K}$
Var B = Variance of slope.
B = Slope
Log $\bar{\Delta K}$ = Grand mean $\log \Delta K$ for the experimental runs.
 $\bar{\Delta K}$ = Antilog $\log \bar{\Delta K}$
Var log C = Variance of intercept in mean-adjusted linear model, at $\log \Delta K = \log \bar{\Delta K}$.
Var C = $\left[\frac{E(C)}{0.4343} \right]^2$ var log C, by propagation of error; for E(C) use C.

** Summary of run numbers from Table A-10 used in finding mean values of log A and log C per Equation (A-16b), and var log C per Equation (A-33):

Room Temperature: Carbon steel: 5-7, 36-40
SS 304: 1-3, 33-35
2-1/4 Cr-1 Mo: 11-12, 41-44

Elevated Temperature: Carbon steel: 9-10, 22-28
SS 304: 15-18
2-1/4 Cr-1 Mo: 14, 19

SECTION IV

TASK B—STRESS ANALYSIS

4.1 OBJECTIVE

The purpose of the stress analysis task is to develop a method for the stress and strain analyses of those piping components which have been intractable problems. The designer must be able to determine the detailed stress and strain distributions in complex piping geometry under complex loadings for avoidance of leaks and severance in piping and to take into account low-cycle fatigue, corrosion, and critically sized flaws.

4.2 TASK B-1—COMPUTER PROGRAM DEVELOPMENT (E. North)

The primary objective of the computer code development, Task B-1, is to develop the analytical tools required to define the state of stress and strain that exists in complex piping when in service. At present, through use of the PAPA computer code a designer can analyze complex piping geometries; i.e., tees, wyes, and laterals for stresses in the elastic range of material behavior.

4.2.1 Summary

The RESTART program for inclusion into the PAPA computer code has been coded and checkout of this capability is in progress. It is estimated that this portion of the program is 80% complete. Work on the input-output portions of the EPS program is in progress.

4.2.2 Computer Code Development—PAPA (RESTART)

Coding of the RESTART portion of the PAPA program has been completed. Checkout consists of purposely aborting the solution of a problem, writing necessary information on tape, and then initiating a restart of the problem to completion. At present, this work is in checkout.

The RESTART procedure as presently defined has the capability of handling aborts at five points in the program. The five restart checkpoints are as follows:

- a. After each substructure stiffness matrix has been decomposed, the appropriate information is stored on tape. This will permit the program to have a restart initiated during the matrix decomposition phase of the PAPA program.
- b. After the decomposition of the substructure stiffness matrices has been successfully completed, sets of external joint unit deflections appropriate to each substructure are back-substituted. For each pass of the external joint deflection vector, the appropriate external joint stiffness matrix is written on tape. For each substructure, sets of external joint deflections are back-substituted until the external joint stiffness matrix for each substructure is complete.

- c. After back-substitutions of all the external joint deflections for each substructure have been completed, this information is the external joint matrix for each substructure.
- d. After all substructures have been processed by applying unit deflections at the external joints, all the data needed to construct the external joint stiffness matrix for the total structure have been successfully saved.

(The above four checkpoints are based on ease of initiating a convenient point of saving data, and ability to permit the highest degree of reliability in saving meaningful data for the RESTART program.)

- e. The last checkpoint is during the calculation of the forces, moments, and stress for each plate element.

All data generated during the solution portion of the PAPA program have been saved for a restart of the program.

Using the five points, as specified, restart during use of the PAPA program will have precise meaning for analysis having a large number of substructures. If an abort takes place during the running of an analysis program, the data needed for the restart initiation is saved on tape, with messages printed out specifying: (a) the portion of the PAPA program during which the malfunction took place, (b) what is needed to initiate a restart procedure on the data generated and saved on tape, and (c) the number and name of the restart tape.

To reiterate the philosophy of the restart procedure in the PAPA program: This restart procedure will be a permanent part of the PAPA program, and is of primary concern for structures having more than one substructure. It has been set up to handle only those aborts that are associated with machine malfunctions (not aborts associated with input errors of the user).

4.2.3 Problem Solution—PAPA

Errors found in the curved, saddle-shaped trapezoidal element have all been corrected. Test analyses of portions of cones and torus have been formulated and analyzed with good results. The 180 degree piping elbow will be reanalyzed.

4.2.4 Computer Code Development—EPS

Coding of the input-output portions of the elastic-plastic portions of the EPS program are in progress.

4.3 TASK B-2—PROBLEM SOLUTIONS (J. D. Gilman)

4.3.1 Summary

A topical report, entitled "BWR Primary Piping Stresses for System Reliability Analysis," was submitted for publication, thus concluding the major portions of stress analysis work for the

reliability task. Continued development of the model piping system will be necessary as the information is utilized by the Reliability task. With regard to problem solutions for piping components, progress has been delayed by problems with the PAPA code, which are discussed in this report.

4.3.2 Problem Solutions

The major objective of Task B-2 is to verify the capability of the PAPA code to calculate stresses in complex piping components. Progress during the past year and a half has been hampered by (1) inaccuracies which have required modifications of the program, and (2) difficulties in running large test cases at San Jose, which has a computer system differing in some respects from that at Evendale.

As a result of Item (2), above, it was decided about a year ago to formulate test cases at San Jose and run them on the Evendale computer. The first test case (documented in the 11th Quarterly Report) produced results for a piping tee which were interpreted as satisfactory, in view of large deviations of the reference photoelastic model from an ideal thin shell. The second test case (a piping elbow in bending) revealed fundamental problems in the curved trapezoidal element, which have been corrected. Meanwhile, a small test case run at San Jose has shown an apparently incorrect behavior of the triangular element under in-plane loading. This test case has been forwarded to Evendale for further investigation.

4.4 SUBTASK B-3—EXPERIMENTAL STRESS ANALYSIS THREE-DIMENSIONAL PHOTOELASTIC ANALYSIS (E. Kiss)

General Electric comments and suggestions forwarded to the University of Illinois will be incorporated into the final report of the photoelastic investigation of five "cracked" piping components. In addition, the following has occurred:

- a. Analysis of an additional stress distribution regarded as significant for the pressurized tee has been completed.
- b. The original data sheets and stress distributions have been sent to GE for information.

The final report is being duplicated by the University and will be sent to GE for distribution.

Crack tip stress intensity factors have been evaluated for each of the photoelastic models, using the experimentally determined stress distributions. Two methods have been used: evaluation on a point-by-point basis, and evaluation on an average basis.

$$K/S = (2\pi x)^{1/2} \left(\frac{\sigma_y}{S} \right)_x \quad \dots \text{point by point.}$$

$$K/S = \sqrt{\frac{\pi}{2x}} \int_0^x \left(\frac{\sigma_y}{S} \right) dx \quad \dots \text{average.}$$

Typical results will be shown for each of the models tested in the following Quarterly Report.

SECTION V

TASK C—FRACTURE MECHANICS

(M. B. Reynolds)

5.1 OBJECTIVE

The objective of the task is the investigation of the relationship between flaw configuration and pressure load limit in pipe for nuclear reactor coolant circuits. Task efforts to date have been almost entirely limited to ASTM Type A-106B steel pipes in sizes up to 12 inches.

5.2 EXPERIMENTAL APPROACH

In the experimental program, capped segments of pipe containing flaws of various configurations were hydrostatically pressurized to failure with an air-operated piston pump capable of generating pressures up to 300 times the pressure of the driving air. Applied pressure, as determined by an electrical strain gage type pressure transducer, was recorded as a function of the number of pump strokes in each test. The volume of liquid (water) delivered per pump stroke was found to be reasonably independent of pressure; hence, the total number of pump strokes was taken to be indicative of deformation of the specimen.

Flaw configurations investigated included longitudinal and circumferential flaws, both through-wall and part-through, in varying lengths and depths. Most flaws were simply machined; in a few cases the machined flaw was extended by fatigue prior to testing. The recorder and the technique for patching the through-wall flawed pipes has been described previously.*

5.3 CIRCUMFERENTIALLY FLAWED PIPES UNDER PRESSURE AND BENDING STRESSES

Because the axial stress in a pressurized, thin-walled, cylindrical vessel is but one-half the circumferential stress, it may be expected that the reduction in pressure load limit due to a circumferential flaw will be less than that due to an axial flaw of comparable size. This is indeed the case, as seen in Table 5-1 which lists the pressure load limits for a number of specimens of ASTM A106B pipes containing circumferential flaws of various configurations. In each case, the specimen was tested using water, or a mixture of water and ethylene glycol, as the pressurizing medium. With the exception of the tests involving bending moments, all tests were conducted at approximately 60° F. In all specimens, except those containing the most severe flaws, the hoop stress reached levels sufficient to cause general yielding before the stress intensity in the direction normal to the flaw reached the instability level.

The effect of simultaneously applying a bending stress to the specimen was to lower the pressure load limit somewhat. The experimental configuration used for testing specimens under combined bending and pressure loading is shown schematically in Figure 5-1. In each case, the

* GEAP-5620, "Failure Behavior in ASTM A-106B Pipe Containing Axial Through-Wall Flaws", Reynolds, M. B., April 1968

TABLE 5-1
PRESSURE LOAD LIMITS FOR CIRCUMFERENTIALLY FLAWED PIPES

Specimen Number	Size and Schedule	Outside Diameter	Wall Thickness, T (in.)	Burst Pressure (ksi)	Pressure Load Limit**	Flaw Configuration	Flaw Length (degrees)	x/T *	Remarks
6-20-8	6 in. - 80	6.66	0.434	9.79	1.00	0.020-in. Milled slot, outside	360	0.72	General yielding prior to failure
6-21-8	6 in. - 80	6.66	0.429	8.60	0.98	0.020-in. Milled slot, outside	360	0.55	General yielding prior to failure
6-22-8	6 in. - 80	6.67	0.429	4.81	0.55	0.020-in. Milled slot, outside	360	0.18	
6-23-8	6 in. - 80	6.66	0.435	9.20	1.00	Through-wall, 0.050 in. wide	34	0.00	Split axially after general yielding
6-24-8	6 in. - 80	6.66	0.432	7.74	0.88	0.020-in. Milled slot, outside	360	0.38	General yielding prior to failure
6-25-8	6 in. - 80	6.66	0.436	8.20	0.93	Through-wall, 0.050 in. wide	68	0.00	Split axially after general yielding
5-2 6-26-8	6 in. - 80	6.66	0.428	9.42	--	0.020-in. Milled slot, outside	360	0.52	Type-304 stainless steel, 22% circumferential strain at flaw
8-20-4	8 in. - 40	8.66	0.321	4.275	0.76	0.020-in. Milled slot, outside	360	0.39	General yielding
6-29-I-8	6 in. - 80	6.68	0.449	8.58		45° Vee, inside	180	0.33	Split axially and circumferentially
<u>PRESSURE PLUS BENDING</u>									
6-27-8-B	6 in. - 80	6.68	0.424	6.00		Through-wall, 0.045 in. wide	42.7	0.00	Superposed bending moment of 5.49×10^5 lb-in. to produce tension normal to flaw. Visible yielding at flaw ends prior to failure.
6-28-8-B	6 in. - 80	6.68	0.425	6.82		0.020-in. Milled slot, outside	360	0.70	Bending moment 6.60×10^5 lb-in. at flaw. Considerable yielding in ligament on tensile side prior to failure.

*T = Wall thickness
x = Ligament thickness

**Expressed as fraction of value for an unflawed pipe.

FIGURE 5-1. TEST ASSEMBLY, SCHEMATIC

bending load was applied at a point approximately 33 inches from the flaw location. The hydraulic cylinder, of a 3.25-inch piston diameter, was actuated by an electrical pump in conjunction with a spring-loaded pressure relief valve to maintain the bending load at any desired constant value.

In the first specimen tested under a bending load (6-28-8-B in Table 5-1), the flaw took the form of a 0.020-inch-wide slot milled to a constant depth completely around the specimen. Due to breakage of the yoke assembly between the hydraulic cylinder and the specimen, it was necessary to load the specimen several times before a combined load level sufficient to break the specimen was achieved. Because of the unusually high ambient temperature prevailing this test was carried out at 89° F. Using an expression derived by Harris,* it was possible to estimate the maximum applied elastic stress intensity factor which would have existed in the region of the flaw, subject to the maximum tensile stress had not local yielding occurred. This value was found to be approximately 58 ksi $\sqrt{\text{in}}$. The expression is as follows:

* Harris, D. O., "Stress Intensity Factors for Hollow Circumferentially Notched Round Bars", UCRL-14859, Lawrence Radiation Laboratory, University of California, Livermore, California, April 1966

$$K_I = \frac{F}{\pi \left[(c+a)^2 - c^2 \right]} \sqrt{\frac{\pi t}{0.80 + \frac{t}{a+c} \left(4 + 1.08 \frac{c}{a} \right)}} + \frac{4 M (c+a) \cos \phi}{\pi \left[(c+a)^4 - c^4 \right]} \sqrt{\frac{\pi t}{0.80 + \frac{t}{a+c} \left(7.12 + 1.08 \frac{c}{a} \right)}}$$

in which

F = applied tensile load,
M = applied bending moment,
a = ligament depth,
t = flaw depth,
c = pipe inside radius,
 ϕ = angular coordinate,
a+t = pipe wall thickness, and
 K_I = opening-mode stress intensity factor.

The second specimen tested contained a circumferential through-wall flaw subtending an angle of 43 degrees on the tension side of the specimen. This specimen (6-28-8-B in Table 5-1) was tested at a temperature of 74° F. The appearance of the specimen after testing is shown in Figure 5-2. The applied elastic stress intensity factor for this flaw at failure was estimated, using the relation developed by Gilman.* The value obtained was 141 ksi $\sqrt{\text{in.}}$. This is consistent with the value of approximately 96 ksi $\sqrt{\text{in.}}$ reported in the Twelfth Quarterly Report for axial through-wall flaws in this material, considering that Charpy impact tests indicated a substantially greater toughness for crack propagation in the circumferential direction than in the axial direction.**

In the pipe containing the 360-degree part-through flaw, considerable plastic thinning of the ligament at the base of the flaw occurred prior to failure. In this case, the applicability of an equation based on linear elasticity is very questionable. Thinning of the ligament prior to failure would lead to a low value for the stress intensity factor, in any case. It should be re-emphasized that estimated stress intensity factors for thin-section materials such as ASTM A-106B pipe at room temperature, while useful for comparison of relative toughness, do not represent the plane strain toughness of the material in thick section.

* Gilman, J. D., "Stress Intensity Factor for a Circumferential Through-Wall Crack in a Straight Pipe", GEAP-5557, General Electric Company, APED, San Jose, California January 1968.

** Quarterly Report No. 6, GEAP-5279.

**FIGURE 5-2. CIRCUMFERENTIALLY-FLAWED SPECIMEN TESTED UNDER COM-
BINED BENDING AND PRESSURE (SPECIMEN 6-28-8-B IN TABLE 5-1)**

5.4 FAILURE DIAGRAM FOR AXIALLY FLAWED PIPES

A failure diagram for predicting the pressure limit loads of axially-flawed pipes was initially presented in the Twelfth Quarterly Report. This was drawn by plotting limit load, expressed as fraction of the burst pressure for a similar unflawed pipe, against the reciprocal of flaw length. On such a plot, all data points from pipes containing part-through flaws should lie between the curve representing pipes with axial through-wall flaws and a straight line at load equal to unity for unflawed pipes. During the present reporting period, additional burst pressure data on ASTM A-106B pipe at 60°F has been obtained—chiefly for pipes containing internal 45-degree vee flaws. The new data have been added to that previously reported, and are shown in Figure 5-3 in which relative burst pressure is plotted against reciprocal flaw length.

In the work on pipes containing axial through-wall flaws, it was found that the dependence of load limit on pipe size could be accommodated to some extent by plotting burst pressure against the ratio of flaw length to pipe diameter. The same burst pressure data noted above from pipes with part-through flaws was therefore plotted against the reciprocal of this ratio in Figure 5-4. For any given fixed flaw length, the limit load should vary smoothly and monotonically with the thickness x of the ligament at the base of the flaw. Also, it should be possible to draw a smooth curve through any set of points, all of which represent behavior of flaws having the same value of x/T (T =pipe wall thickness). As the value of x/T increases, these curves should shift in position smoothly from the curve for through-wall flaws ($x/T=0$) to the upper line representing no flaw. Examination of the values for x/T associated with each plotted point in Figures 5-3 and 5-4 confirms this trend and illustrates the scatter in experimental data resulting from variation in properties of the ASTM A-106B material. Based on the experimental data presently available, there appears to be no reason to base the failure diagram on the variation of limit load with D/L rather than simply $1/L$. Resolution of this question would require a larger quantity of material of very uniform properties than is presently available.

FIGURE 5-3. PROPOSED FAILURE DIAGRAM—PLOT OF LOAD LIMIT VERSUS $1/L$

FIGURE 5-4. ALTERNATE FAILURE DIAGRAM—PLOT OF LOAD LIMIT VERSUS D/L

SECTION VI

TASK D—FATIGUE

6.1 OBJECTIVE

The objectives of the fatigue task are (a) to extend the understanding of low-cycle fatigue cracking in high-pressure piping components which have mechanical or defect flaws, and (b) to apply this knowledge in developing rational design bases for prevention or control of fatigue crack initiation and growth in reactor piping systems.

6.2 TASK D-1—FATIGUE CRACK GROWTH STUDIES (A. J. BROTHERS)

6.2.1 Summary

Surface crack growth rates of part-through surface cracks tested under fully-reversed strain-controlled loading have been determined for the bending mode. The specimen used was a cantilever beam identical to that used previously by others. These results are compared with those for A-212B and 2-1/4 Cr - 1 Mo steel tested under fully-reversed tension-compression loading. Crack growth rates in the bend specimen are substantially lower than those measured in tension-compression; prior results suggest that this is due, in part, to an effect of crack geometry.

6.2.2 Crack Growth of Part-Through Surface Cracks in Bending Under Fully-Reversed Strain-Controlled Loading

To determine fatigue crack growth rates under conditions of completely-reversed strain-controlled loading, tests are being conducted in both tension-compression and reversed bend loading. The former results presented in Quarterly Reports Nos. 12 and 13 indicate that fatigue crack growth rates for through-crack, centrally-slotted plates are exponentially dependent on total strain range with no significant effect of crack length. However, crack growth rates for part-through surface cracks prior to penetration of the thickness were dependent on the initial defect geometry, and were somewhat less than rates for through-plate cracks. To determine whether the crack rates in tensile loading correspond to those measured in bending, for which considerable published information is currently available, * tests have been conducted on A-212B carbon steel and the 2-1/4 Cr - 1 Mo alloy using a specimen and test procedure identical to those of Crooker and Lange. * The specimen configuration and experimental setup are shown in Figure 6-1. The 2.5-inch-wide by 0.5-inch-thick reduced test section contains a 0.25-inch-long by 0.060-inch-deep, electric-discharged-machined, part-through surface slot. The specimen is cycled in reversed tension-bending over deflection-controlled limits to obtain constant nominal total strain values. The strain range is measured by two adhesively-bonded strain gages at locations of about 0.25 inch from the edge of the specimen. The testing speed is 2 to 3 cpm.

* Crooker, T. W., and Lange, E. A., "Low Cycle Fatigue Crack Propagation Resistance of Materials for Large Welded Structures," Fatigue Crack Propagation ASTM STP 415.

FIGURE 6-1. EXPERIMENTAL SETUP FOR REVERSE-LOADED CANTILEVER BEND SPECIMEN

The crack growth curves from these tests for the carbon steel and the 2-1/4 Cr - 1 Mo alloy steel are shown in Figures 6-2 and 6-3, respectively. The crack growth curves for carbon steel (Figure 6-2), though nominally linear for most strain levels, do exhibit some effect of increasing crack growth rates with increasing crack length. However, the crack growth curves for the 2-1/4 Cr - 1 Mo alloy clearly show an increasing growth rate as a function of increasing crack length.

The fracture surfaces of the specimens tested to date are shown in Figure 6-4. These features indicate that the surface crack length-to-depth ratios vary for the two alloys. The primary difference is that associated with lower total growth through the thickness in the carbon steel. This appears to result from a crack-stopping influence of transverse fissuring associated with inclusion stringers in the carbon steel plate. The specific values of crack aspect ratio for these tests are listed in Table 6-1; these values for carbon steel are greater than those for the alloy steel. However, in both materials the final aspect ratio appears to be independent of strain level. The crack length-to-depth ratio for a single test varies, but over a fairly narrow range.

Comparison of the dependence of crack growth rate on total strain range for these tests for the carbon steel with those obtained previously in tension-compression are shown in Figure 6-5. Although the dependence of crack growth rate on strain range as reflected in the slopes of these curves is generally similar, the level of crack growth rates for the bending mode is considerably less than those measured in tension-compression tests. Since the rates for the 2-1/4 Cr - 1 Mo alloy are not constant for a given strain range, these results should not be compared directly with those measured previously on the basis of strain range; however, their general magnitude is also much less than those measured in tension (Figure 6-6).

The apparent dependence of crack growth rate on crack length for the 2-1/4 Cr - 1 Mo alloy in these bend tests, in addition to effects of crack geometry measured previously in tension tests of part-through cracks, indicates conclusively that total strain range is an inadequate parameter with which to characterize growth rates for various geometries.

In these current tests, the differences reflected in crack growth rate characteristics of the two alloys can be rationalized in part by consideration of the influence of crack shape on the crack stress field as represented by the stress intensity factor, K ; i.e.,

$$K = M_B \sigma \sqrt{\pi} \sqrt{a}/Q \quad (\text{see Figure 6-7}).^*$$

Quantitative examination of the relative influence of these variables indicate that the primary factor is the crack length term. For example, for $a/2c$ values in the range examined, all terms with the exception of $\sqrt{a}$ remain constant with $\pm 6.5\%$. However, for a given surface crack length, the crack depth term differs for the two alloys as a result of the more shallow

* Smith, F. W., "Stress Intensity Factors for a Semi-Elliptical Surface Flaw," Structural Development Res. Memo No. 17, Boeing Airplane Corp., August 1966.

FIGURE 6-2. CRACK GROWTH CURVES FOR A-212B CARBON STEEL AT ROOM TEMPERATURE

FIGURE 6-3. CRACK GROWTH CURVES FOR 2- $\frac{1}{4}$ Cr-1 Mo STEEL AT ROOM TEMPERATURE

GEAP-5716

$\pm 1.0\%$

$\pm 0.5\%$

$\pm 0.3\%$

A-212B CARBON STEEL

$\pm 1.0\%$

$\pm 0.5\%$

$\pm 0.3\%$

2-1/4 Cr-1 Mo ALLOY

FIGURE 6-4. FRACTURE SURFACES OF BEND SPECIMENS

FIGURE 6-5. COMPARISON OF TENSION-COMPRESSION VERSUS BEND LOADING ON CORRELATION OF CRACK GROWTH RATE AND TOTAL STRAIN RANGE FOR A-212B STEEL AT ROOM TEMPERATURE

FIGURE 6-6. COMPARISON OF TENSION-COMPRESSION VERSUS BEND LOADING ON CORRELATION OF CRACK GROWTH RATE AND TOTAL STRAIN RANGE FOR 2-1/4 Cr-1 Mo ALLOY AT ROOM TEMPERATURE

FIGURE 6-7. APPROXIMATE STRESS INTENSITY FACTORS AT $\alpha = 0^\circ$ AND $\alpha = 90^\circ$ FOR SURFACE FLAWS IN BENDING

TABLE 6-1

CRACK-ASPECT RATIO IN BEND SPECIMENSA-212B

<u>Strain Level (%)</u>	<u>Final Crack Depth (in.)</u>	<u>Final Surface Crack Length (in.)</u>	<u>Final Crack Length-To-Depth Ratio, 2c/a</u>
±0.3	0.17	1.99	11.1
±0.5	0.19	1.90	10.0
±1.0	0.21	2.18	10.9

2-1/4 Cr - 1 Mo

±0.3	0.27	2.11	7.5
±0.5	0.28	2.03	7.6
±1.0	0.33	2.00	6.45

Crack Length-To-Depth Ratio for 2-1/4 Cr - 1 MoAlloy at ± 0.3% Strain Level

<u>Surface Crack Length 2c (in.)</u>	<u>Crack Depth, a (in.)</u>	<u>Crack Length-To-Depth Ratio, 2c/a</u>	<u>a/2c</u>
0.25	0.06	4.1	0.244
0.80	0.14	5.65	0.177
1.63	0.24	6.8	0.147
2.11	0.28	7.5	0.133

crack depth in the carbon steel specimens. This effectively decreases the stress intensity factor for a given surface length and hence results in a lower increase in stress intensity factor throughout the test life. This presumably accounts for the lower dependence of crack growth rate on crack length for these specimens.

6.3 TASK D-2—CRACK INITIATION (E. Krempl)

6.3.1 Summary

Two topical reports, entitled "Notched High-Strain Fatigue Behavior of Three Low-Strength Structural Steels" and "The Influence of Stress/Strain Concentration and Mean Stress on the Low-Cycle Fatigue Behavior of Three Structural Steels at Room Temperature," were drafted and submitted for publication.

The cyclic deformation behavior of carbon 2-1/4 Cr - 1 Mo, and Type-304 stainless steel under completely reversed, strain-controlled conditions was investigated at 550° F.

6.3.2 Deformation Behavior of Smooth-Bar Specimens Under Completely Reversed, Strain-Controlled Conditions at 550° F

6.3.2.1 Test Method

To accomplish true strain control at elevated temperature, a special specimen design had to be adopted. Ridges were placed on the gage length of the specimen so that the strain could be accurately measured. The specimen configuration is shown in Figure 6-8. It was recognized that the ridges enhance fatigue crack initiation due to the stress concentration. Since the accurate strain control was deemed important, this design was chosen. In a previous study,* cracking had been observed in the gage length while the main crack was forming at the ridges. This occurrence of cracking was considered to be adequate.

The ridges serve as an anchor for screwed-on cross bars which, in turn, are connected to extensometer rods to carry the relative displacement out of the furnace. The core and body of a Hewlett Packard 7DCDT-050 displacement transducer are attached to the upper and lower extensometer rod, respectively. An electrical signal proportional to the displacement could thus be obtained. A dial gage next to the transducer measures the same displacement as the transducer, and is used for calibration purposes. A photo of the setup is given in Figure 6-9.

With this arrangement, elevated temperature, strain-controlled cyclic tests could be easily performed. The output of the transducer is directly connected to the servo control of the 200,000-lb Tinius Olsen servo-controlled Universal testing machine.

* Krempl, E., and Walker, C. D., "The Effect of Creep-Rupture Ductility and Hold-Time on the 1000F Strain Fatigue Behavior of a 1 Cr - 1 Mo - .25 V Steel." Paper presented at ASTM Annual Meeting, June 1968, ASTM STP 459, 1969.

The test procedure was such that the specimen was kept at 550° F for about an hour before the test was started. The strain limits were preset so that the desired strain range was maintained from the very beginning to the end of the test. Figures 6-10 and 6-11 show hysteresis loops at the beginning of a test with a stainless steel and a carbon specimen. The desired strain range is achieved and maintained from the first quarter-cycle to the end of the test.

The load range is free to vary in the strain-controlled test. To obtain information on the variation of the load range with cycles, hysteresis loops were recorded at suitable cycle intervals. In addition, the load was monitored continuously against time on a separate recorder.

6.3.2.2 Test Results

The mechanical properties of the three materials (carbon, steel, Type-304 stainless steel, and 2-1/4 Cr - 1 Mo alloy steel) at 550° F are listed in Table 6-2. The room temperature properties are listed for comparison purposes. Table 6-3 lists the constants of the true stress-strain equation which was discussed in Appendix B of Quarterly Progress Report No. 8, GEAP-5474.

Examples of the variation of the stress amplitudes with cycles are given in Figures 6-12a, b, c. The cyclic strain-hardening characteristics of the three materials can be readily distinguished. In general, carbon steel showed considerably more cyclic strain hardening than the other two materials. This can be readily seen if the cyclic and the monotonic stress-strain curves are compared, as shown in Figures 6-13a, b, and c. Stable stress range values are used to construct the cyclic stress-strain curves.

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FIGURE 6-8. TEST SPECIMEN

FIGURE 6-9. TEST SETUP SHOWING EXTENSOMETRY AND HALF OF FURNACE

FIGURE 6-10. HYSTERESIS LOOPS AT START OF TEST WITH TYPE 304 STAINLESS STEEL SPECIMEN

FIGURE 6-11. HYSTERESIS LOOPS AT START OF TEST WITH CARBON STEEL SPECIMEN

FIGURE 6-12a. VARIATION OF STRESS AMPLITUDES AND STRESS RANGE FOR CARBON STEEL AT 550°F. COMPLETELY REVERSED, STRAIN-CONTROLLED TESTING.

FIGURE 6-12b. VARIATION OF STRESS AMPLITUDES AND STRESS RANGE FOR 2 1/4 Cr-1 Mo AT 500°F. COMPLETELY REVERSED, STRAIN-CONTROLLED TESTING.

FIGURE 6-12c. VARIATION OF STRESS AMPLITUDES AND STRESS RANGE FOR TYPE-304 STAINLESS STEEL AT 550°F. COMPLETELY REVERSED, STRAIN-CONTROLLED TESTING.

FIGURE 6-13a. CYCLIC AND MONOTONIC STRESS-STRAIN DIAGRAMS AT 550°F FOR CARBON STEEL

FIGURE 6-13b. CYCLIC AND MONOTONIC STRESS-STRAIN DIAGRAM AT 550°F FOR 2½ Cr-1 Mo ALLOY STEEL

FIGURE 6-13c. CYCLIC AND MONOTONIC STRESS-STRAIN DIAGRAMS AT 550°F FOR TYPE-304 STAINLESS STEEL

TABLE 6-2

ROOM TEMPERATURE AND 550° F
TENSILE PROPERTIES OF THREE PIPING MATERIALS

<u>Material</u>	<u>Temperature (° F)</u>	<u>Prop. Limit (10³ psi)</u>	<u>0.2% Y. S. (10³ psi)</u>	<u>U. T. S. (10³ psi)</u>	<u>R. A. (%)</u>	<u>Elongation (2 inches Gage Length) (%)</u>
Carbon Steel	75	21.0	29.4	62.4	55.8	35.0
	550	-	-	72.0	47.0	30.0
	550	15.5	25.9	72.1	49.0	31.0
	550	11.5	25.5	72.5	45.0	29.0
2-1/4 Cr - 1 Mo Alloy Steel	75	23.6	36.7	74.6	67.5	31.5
	550	18.0	32.0	69.5	59.0	23.0
	550	24.0	32.0	69.5	58.0	21.0
	550	-	-	70.1	58.0	20.0
Type-304 Stainless Steel	75	26.9	38.8	85.0	76.1	66.8
	550	-	-	68.5	70.0	45.0
	550	15.0	24.0	67.0	70.0	50.0
	550	14.0	25.9	68.0	68.0	46.0

TABLE 6-3

CONSTANTS OF TRUE STRESS-STRAIN EQUATION
AT ROOM TEMPERATURE AND 550° F

<u>Material</u>	<u>Temperature (° F)</u>	<u>Strain Hardening Exponent</u>	<u>Const. K (10³ psi)</u>	<u>Uniform Strain (%)</u>	<u>Plastic Work (10³ lb-in./in. ³)</u>
Carbon Steel	75	0.22	112	42	77
	550	0.19	114	16	35
2-1/4 Cr - 1 Mo Alloy Steel	75	0.15	117	16	105
	550	0.14	103	14	62
Type-304 Stainless Steel	75	0.43	193	51	219
	550	0.45	163	-	17

The great degree of cyclic strain hardening of carbon steel (Figure 6-13a) is very evident. The stress amplitude approaches and actually reaches the ultimate tensile strength in the monotonic tests; however, no instabilities were observed in the specimen. Strain aging is very likely responsible for the considerable increase in stress amplitude (Figures 6-11 and 6-12a) with cyclic loading of carbon steel.

In contrast to the other two materials, Type-304 stainless steel exhibits strongly amplitude dependent strain hardening, Figure 6-13c. The difference between the monotonic and cyclic stress-strain diagram increases considerably with increasing strain. Neither carbon nor 2-1/4 Cr - 1 Mo steel show this strong amplitude dependence of the cyclic strain hardening.

If the monotonic and the cyclic stress-strain curves at 550° F are compared with those at room temperature, which were reported in Appendix B of GEAP-5474, the following observations can be made.

- a. Neither the monotonic nor the cyclic stress-strain curves of 2-1/4 Cr - 1 Mo alloy steel are appreciably affected by the increase in temperature.
- b. The monotonic stress-strain diagram of Type-304 stainless steel is lower at 550° F than at room temperature. The cyclic stress-strain curves, however, are very similar at both temperatures.

By far, the greatest influence of temperature is on the cyclic stress-strain diagram of carbon steel. The 550° F diagram is considerably higher than the one at room temperature. The monotonic curves show much less difference than the cyclic curves.

6.4 TASK D-3—PROTOTYPE PIPE FATIGUE TESTING (E. Kiss, J. D. Heald)

6.4.1 Summary

Fatigue testing, at room temperature, of an elbow and three notched pipe straights was completed. In addition, two tests were completed at 550° F: a notched pipe straight and "good" butt-welded specimen. Table 6-4 summarizes the pertinent test parameters.

Setup of a hot (550° F), pressurized (1000 psi), notched stainless steel piping specimen was begun. This specimen (HSNP-80-I) will demonstrate "leak or break" under simulated BWR operating conditions.

6.4.2 Data Obtained

6.4.2.1 Crack Initiation

Figures 6-14, 6-15, 6-16, and 6-17 show the crack initiation data from prototype pipe specimens compared to the data of cylinders tested at the M&P Laboratory at Schenectady. These figures, presented in previous progress reports, have had the most recent data points added.

TABLE 6-4
DATA SUMMARY

Sample Description	Test Temperature (°F)	Internal Pressure (hydraulic) (psig)	Nominal Stress Range (psi)	Cantilever End Deflection (in.)	Cycles to Crack Initiation (N_I)	Cycles to Fully Penetrate Wall (N_w) (2)	Total Cycles (N_ϕ) (3)
CSLS-1, ASTM A182F304 Schedule 40, Short Radius S. S. Elbow	57-75	1000	44,200	± 10.78	Not obtained	907 (907) See Figure 6-29	4202 (4202)
HW-1, ASTM A-106 Gr. B, Schedule 40, Carbon to Carbon Circumferential Butt Weld Sample	550	None	58,200	± 5.45	25000 ⁽¹⁾	36000-36240 (10500-10740)	38500 (38000)
CSS-109, ASTM A-106 Gr. B, Schedule 80, Type I (External) Defect	50-70	1000	72,300	± 7.69	600	6000 (3400)	16850 (14850)
HSN-40-1, ASTM A-312, Type 304 S.S. Schedule 40, Type I (External) Notch	550	None	58,400	± 6.34	800	5250 (3550)	20500 (14250)
CSS-119, ASTM A-312, Type 304 S.S. Schedule 80, Type I (External) Notch	66-78	1000	74,600	± 8.125	425	5156 (4231)	20603 (15047)
CSS-107, ASTM A-106, Gr B, Schedule 80, Type I (External) Notch	59-80	1000	74,100	± 7.859	200	4575 (3575)	14205 (12705)

(1) Crack was found by ultrasonics in the top of the pipe.

(2) N_w = Total cycles when crack penetrated through wall (includes N_I).
Number in parentheses is full amplitude cycles included in $N_w - N_I$.

(3) N_ϕ = Total cycles after crack has grown to length where test terminated (includes N_w).
Number in parentheses is full amplitude cycles included in $N_\phi - N_w$.

FIGURE 6-14. NOMINAL STRESS RANGE VERSUS CYCLES FOR CRACK INITIATION, TYPE 304 STAINLESS STEEL AT 550°F

FIGURE 6-15. NOMINAL STRESS RANGE (psi) VERSUS CYCLES-TO-CRACK INITIATION, ASTM A-106B CARBON STEEL, 550°F

FIGURE 6-16. TYPE 304 STAINLESS STEEL, ROOM TEMPERATURE, STRESS RANGE VERSUS INITIATION CYCLES

FIGURE 6-17. CARBON STEEL, ROOM TEMPERATURE, STRESS RANGE VERSUS INITIATION CYCLES

6.4.2.2 Crack Growth

a. Through-Wall Crack Growth on Pipe Straight

Following crack initiation, fully reversed loading is continued until the crack propagates through the wall. Stress reduction techniques are used to mark the fracture surface during the growth of the crack. Figure 6-18 illustrates, for specimen HSN 40-1, stainless steel at 550° F, an excellent example of a marked fracture surface. Figure 6-19 shows, for through-wall crack growth, the stress intensity factor range versus crack growth rate for stainless steel piping specimens at 550° F. The solid line represents the average of data obtained from laboratory specimens in Task D-1.

b. Circumferential Crack Growth on Pipe Straight

After a crack has proceeded through the wall, cycling is continued as zero to tension loading, causing the crack to grow circumferentially around the pipe.

Figures 6-20 through 6-23 show typical data and curve fits for both crack length and crack growth rate versus cycles of loading. The range of stress intensity factor is calculated at various crack lengths from the measured cyclic loading, and is shown tabulated with the corresponding crack growth rate in Table 6-5.

These values are shown plotted in Figures 6-24 through 6-27, along with previous data obtained for prototype piping.

c. Axial Crack Growth on Piping Elbow

Surface crack length measurements were made on an elbow (CSLS-1) under cyclic in-plane bending. On both sides of the elbow, cracks propagated axially and symmetrically after through-wall penetration. Figures 6-28 through 6-33 show the crack length history for this specimen.

FIGURE 6-18. STAINLESS STEEL SPECIMEN HSN-40-1, TESTED AT 550°F, SHOWING A FATIGUE FRACTURE WITH GROWTH MARKINGS PRODUCED BY PERIODIC REDUCTIONS OF LOADING

FIGURE 6-19. THROUGH-WALL CRACK GROWTH RATE, da/dN , VERSUS RANGE OF STRESS INTENSITY FACTOR, ΔK , FOR TYPE-304 STAINLESS STEEL AT 550°F

FIGURE 6-20. CIRCUMFERENTIAL CRACK GROWTH VERSUS CYCLES, A-106 CARBON STEEL, ROOM TEMPERATURE SPECIMEN No. CSS-109

FIGURE 6-21. CIRCUMFERENTIAL CRACK GROWTH RATE VERSUS CYCLES, ASTM A-106 CARBON STEEL, SPECIMEN No. CSS-109

FIGURE 6-22. CIRCUMFERENTIAL CRACK GROWTH VERSUS CYCLES, ASTM A-312 TYPE 304 STAINLESS STEEL, SPECIMEN No. HSN-40-1

FIGURE 6-23. CIRCUMFERENTIAL CRACK GROWTH RATE VERSUS CYCLES ASTM A-312 TYPE 304 STAINLESS STEEL, SPECIMEN No. HSN-40-1

TABLE 6-5

CALCULATED STRESS INTENSITY FACTOR RANGE,
 ΔK VERSUS MEASURED CRACK GROWTH RATE, da/dn

<u>Cycles</u>	<u>1/2 Crack Length (in.)</u>	<u>β°</u>	<u>ΔK (ksi $\sqrt{\text{in.}}$)</u>	<u>Crack Growth Rate (μ in./cycle)</u>
<u>Specimen HW-1</u>				
36,430	2.828	48.9	109.7	1,671
36,510	3.037	52.5	126.9	3,408
36,549	3.172	54.9	138.8	4,306
36,610	3.500	60.6	156.7	5,930
<u>Specimen CSS-109</u>				
8,000	1.200	20.8	62.7	293
9,300	1.705	29.5	78.4	571
9,800	2.067	35.8	90.0	976
10,100	2.410	41.7	104.9	1,446
10,400	3.212	55.6	174.0	3,799
<u>Specimen HSN-40-1</u>				
7,100	0.667	11.5	40.3	92
10,300	1.230	21.3	58.4	274
12,100	2.055	35.6	81.8	825
12,600	2.558	44.2	103.0	1,305
12,925	3.100	53.6	139.3	1,858
<u>Specimen CSS-119</u>				
8,100	1.000	17.3	59.2	90
10,350	1.355	23.4	72.6	229
12,500	2.058	35.6	94.6	575
13,000	2.430	42.0	111.2	1,110
<u>Specimen CSS-107</u>				
8,250	1.350	23.4	68.2	251
9,000	1.580	27.3	73.8	346
10,000	2.125	36.8	93.2	940
10,300	2.490	43.1	109.4	1,523
10,700	3.380	58.5	183.2	3,262

FIGURE 6-24. CIRCUMFERENTIAL CRACK GROWTH RATE VERSUS STRESS INTENSITY RANGE FOR A-106B CARBON STEEL AT ROOM TEMPERATURE

FIGURE 6-25. CIRCUMFERENTIAL CRACK GROWTH RATE VERSUS STRESS INTENSITY RANGE A-106-B CARBON STEEL MATERIAL AT 550°F

FIGURE 6-26. CRACK GROWTH RATE VERSUS STRESS INTENSITY RANGE FOR TYPE 304 STAINLESS STEEL AT ROOM TEMPERATURE

FIGURE 6-27. CIRCUMFERENTIAL CRACK GROWTH RATE VERSUS STRESS INTENSITY RANGE FOR TYPE 304 STAINLESS STEEL AT 550°F

FIGURE 6-28. CARBON STEEL ELBOW, CSLS-I, N = 0 CYCLE

FIGURE 6-29. CARBON STEEL ELBOW, CSLS-1, N = 907 CYCLES, SHOWING FIRST WALL PENETRATION

FIGURE 6-30. CARBON STEEL ELBOW, CSLS-1, N = 1005 CYCLES

FIGURE 6-31. CARBON STEEL ELBOW, CSLS-1, N = 1054 CYCLES

FIGURE 6-32. CARBON STEEL ELBOW, CSLS-1, $N = 1253$ CYCLES

FIGURE 6-33. CARBON STEEL ELBOW, CSLS-1, N = 4202 CYCLES

6.5 TASK D-4—CYCLIC STRAIN AGING OF CARBON STEEL (A. J. Brothers)

6.5.1 Summary

The results of transmission electron microscopy of specimens deformed in tension at room temperature, 300° F, and 490° F, and also in fatigue at 350° F, are described. Increasing dislocation tangling with increasing temperature up to at least 300° F was observed for the uniaxial tests. Cell formation occurred more readily during cyclic deformation, and a well defined substructure was obtained in the ferritic areas of all the specimens examined.

The role of interstitial elements in contributing to the observed dislocation configurations is uncertain. However, the degree of strain aging will undoubtedly be influenced by the dislocation arrangement. It is speculated that a correlation may exist between the extent of dislocation tangling, as reflected in cell size, and the increase in transition temperature. Further work is planned to confirm this.

6.5.2 Electron Microscopy

In a previous quarterly report (No. 12), the results of replica and thin film studies of tensile specimens deformed at room temperature, 300° F, and 490° F, were described. No discrete carbide precipitation was detected within the ferrite grains. Examination of the foils taken from close to the fracture of these specimens, where the local strains were between 10 and 20%, revealed some dislocation tangling and the beginning of cell formation. Figure 6-34 illustrates the type of dislocation structure obtained at the three test temperatures. In general, after deformation at room temperature, the dislocations tended to be relatively straight. A number of fairly well-defined sub-boundaries consisting of dislocation nets were observed [Figure 6-34(a)]. At the higher test temperatures [Figure 6-34(b) and (c)], far more dislocation tangling was obtained, and small areas of fairly dense dislocation structures indicated an early stage of cell formation. No significant difference was observed in the specimens deformed at 300 and 490° F. These observations are rather similar to previous reports of the effect of deformation temperature on the dislocation structure of α - iron^{(1, 2, 3)*} where the tendency to form cells increased with increasing strain and increasing temperature up to about 500° F. An examination of the decarburized foils⁽²⁾ indicated similar structures, suggesting that the type of dislocation configuration was dependent on the deformation temperature and not a result of dislocation-interstitial interactions. However, it seems probable that the strong interactions which are known to occur in this temperature range are at least partly responsible.⁽⁴⁾ This is a difficult point to resolve, but undoubtedly the segregation of interstitials to the dislocations in the tangles contributes to the observed increase in work-hardening rate.

Since, in these studies, maximum Δ FATT was observed after fatigue at 350° F, thin foils were prepared from parallel to the initial rolling plane for specimens tested under the following conditions:

* See subsection 6.5.3 for references.

(a)

(b)

(c)

FIGURE 6-34. EFFECT OF TEMPERATURE ON DISLOCATION ARRANGEMENT AFTER TENSILE DEFORMATION — (a) R.T. ~ 20%, (b) 300°F ~ 10%, (c) 490°F ~ 15% 75,000X

<u>Specimen No.</u>	<u>Strain Range (%)</u>	<u>No. of Cycles</u>	<u>Δ FATT($^{\circ}$ F)</u>
5	1.5	37	110
6	0.5	200	100
7	0.38	200	85

Figure 6-35 shows the dislocation structure and illustrates very well-defined cells approximately $0.5 \mu\text{m}$ diameter in all cases. No significant structural differences were observed for the different cyclic exposures. The similar FATT is perhaps significant in this respect, suggesting that the cell size and degree of strain aging following 37 cycles at 1.5% strain is equivalent to 200 cycles at the lower strain range and, in turn, results in a similar FATT. To examine further the possible relation between FATT and cell formation (involving both type of cell and cell size), additional specimens cycled at room temperature and at 550°F will be examined and the results presented in the next report.

The general conclusions are that dislocation tangling increases in tensile tests at 300 and 550°F compared with room temperature. This, together with associated strain aging, seems likely to be responsible for the increased work hardening rate and reduced ductility. After cyclic deformation at 350°F , a well defined cellular dislocation structure is observed. It is uncertain whether interstitial elements affect the cell formation per se, although it seems probable that they do play a part. However, segregation of interstitials to dislocations leading to strain age hardening is not in doubt and it seems reasonable that the type of dislocation arrangement (e.g., cell size) will influence the degree of strain aging and hence the increase in transition temperature.

6.5.3 References for Subsection 6.5

1. Carrington, W., Hale, K. F., and McLean, D., Proc. Roy. Soc., 259, 1960, 203.
2. Brandon, D. G., and Nutting, J., J. I. S. I., 190, 1960, 160.
3. Keh, A. S., Weissman, S., "Electron Microscopy and Strength of Crystals", Interscience, New York, 1963, 231.
4. McLean, D., "Mechanical Properties of Metals", John Wiley and Sons, New York, 1962, 128.

FIGURE 6-35. CELL FORMATION FOLLOWING FATIGUE AT 350°F — (a) 37 CYCLES AT 1.5%, (b) 200 CYCLES AT 0.5%, (c) 200 CYCLES AT 0.38% 75,000X

SECTION VII

TASK K—EFFECT OF BWR ENVIRONMENT

7.1 OBJECTIVE

The objective of Task K is to determine quantitatively the effect of BWR primary coolant on the low-cycle fatigue life of piping steels. Measurements of cycles-to-failure will be obtained employing a specially-designed fatigue testing unit in an experimental piping loop at a boiling water reactor. The planned test duration is 2-1/2 years, allowing time for corrosion, if any, to affect crack initiation and growth.

7.2 SUMMARY

The status of key items was as follows:

- a. The DNPS-1 (Dresden Nuclear Power Station No. 1) dynamic test vessel has been ordered from National Forge Company.
- b. The DNPS-1 Static Test Vessel has been completed.
- c. Static testing of three types of fatigue specimens has been completed.
- d. Design drawings for both the DNPS loop piping and instrumentation were close to completion.
- e. Startup of the test loop containing specimens is scheduled for July 1969.

7.3 DRESDEN SITE EQUIPMENT

7.3.1 Dynamic Fatigue Test Vessel (DNPS-1)

The dynamic fatigue test vessel has been ordered from National Forge Company with delivery scheduled for March 1969. The modified vessel features include inlet and outlet flow distribution manifolds, support legs, and increased head thickness. A drawing is shown in GEAP-5680, Quarterly No. 13. The manifolds have been sized to provide uniform flow distribution, with a maximum total vessel pressure drop of 5 psi full flow (10 gpm).

7.3.2 Static Test Vessel (DNPS-1)

The static test vessel (Figure 7-1) fabricated from 6-inch pipe, was hydrotested and delivered to the Vallecitos site for temporary storage. It will be shipped to the Dresden site with the dynamic test vessel. A review of experimental programs in stress corrosion conducted by others has indicated the general successful application of precracked specimens for measurement of susceptibility to stress corrosion cracking of materials in aqueous environments.

FIGURE 7-1. STATIC TEST VESSEL (DNPS-1)

Consequently, it has been decided to expose WOL* specimens in addition to stress bend specimens. For illustration, a WOL specimen of the 1-T type is shown in Figure 7-2 (Figure 14 in Reference 2). The test plan is defined in Tables 7-1 and 7-2; the layout is patterned after the base program as defined in Table 7-1, GEAP-5637.

7.3.3 Fatigue Machine

The existing fatigue machine hold-down flange assembly has been modified to be compatible with the modified dynamic test vessel. The fatigue jig is now interchangeable with either a teflon chevron seal or a metallic bellows seal. This was done as a hedge against the possibility of unsatisfactory operational performance of the teflon seals. A photo is shown in Figure 7-3.

The hydraulic pump needed to drive the fatigue machine has been ordered.

7.3.4 Site Activities

7.3.4.1 Loop Piping

The system P&ID has been approved by Commonwealth Edison Company, and the test cubicle piping design is nearly completed.

7.3.4.2 Loop Instrumentation

Instrumentation layout drawings have been completed for both the loop operating panel and fatigue test operating panel. Fabrication work has begun on the former at the Vallecitos Lab. It is anticipated that the loop operating panel will be installed at Dresden during the early part of 1969 and be used for loop pre-startup testing.

7.4 PREOPERATIONAL ACTIVITIES

7.4.1 Shakedown Test Vessel

To proceed prior to delivery of the DNPS-1 dynamic fatigue vessel with shakedown testing of all mechanical equipment and instrumentation to be used for the fatigue testing, an AEC surplus stainless steel pressure vessel has been obtained, and has been modified to provide

- a. Inlet and outlet flow nozzles,
- b. Top head which will be compatible with the fatigue test fixture and instrumentation, and
- c. Support legs.

This vessel will be used to check assembly, instrumentation, and operation of the fatigue machine at the operating environmental conditions of 550° F and 1000 psi.

* Wedge Opening Loading

FIGURE 7-2. MODIFIED WOL SPECIMEN AFTER ARREST

TABLE 7-1

TASK K STRESS-BEND SPECIMENS

Material				Material Condition								No. of Specimens
304 Stainless Steel	304-L Stainless Steel	Inconel- 600	Carbon Steel	Material Condition Code	Welded, High Heat	Welded, Low Heat	Machined (Ground)	Solution Annealed	Pickled	Sensitized	Stress Relieved	
X				1			X	X	X			2
X				2			X					2
X				3		X	X					2
X				4	X		X					2
X				5			X	X	X	X		2
	X			1			X	X	X			2
	X			2			X					2
	X			4	X		X					2
	X			5			X	X	X	X		2
		X		1			X	X	X			2
		X		4	X		X					2
		X		5			X	X	X	X		2
			X	2			X					2
												26

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TABLE 7-2

TASK K MODIFIED WOL SPECIMENS

Material				Material Condition								No. of Specimens
304 Stainless Steel	304-L Stainless Steel	Inconel- 600	Carbon Steel	Material Condition Code	Welded, High Heat	Welded, Low Heat	Machined (Ground)	Solution Annealed	Pickled	Sensitized	Stress Relieved	
X				2			X					2
X				3		X	X					2
X				4	X		X					2
X				5			X	X		X		2
	X			2			X					2
	X			4	X		X					2
	X			5			X	X		X		2
		X		1			X					2
		X		4	X		X					2
		X		5			X	X		X		2
			X	2			X					2
			X	6			X				X	2
			X	4	X		X					2
			X	7	X		X				X	2
												28

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FIGURE 7-3a. FATIGUE MACHINE BOLTED TO DYNAMIC VESSEL HEAD RESTING ON INSPECTION STAND

FIGURE 7-3b. A CLOSE-UP OF THE FATIGUE MACHINE IN FIGURE 7-3a, TO SHOW THE SPECIMEN ARRANGEMENT

7.4.2 Test Head Support Assembly Stand

A test head support assembly stand (Figure 7-4) has been designed and fabricated. This stand will be used to support the shakedown pressure vessel head and DNPS-1 pressure vessel head during fatigue unit assembly, specimen installation, strain range adjustment, and checkout. In addition, it will be sent to the Dresden site where it will be used for interim operating inspections of the fatigue specimens.

7.5 FATIGUE SPECIMEN EVALUATION

Four different fatigue specimens have been tested statically to determine the surface strain—length distribution and strain—end deflection relation. A photograph of the specimens tested is shown in Figure 7-5. Specifically, the specimens shown are:

- i. Modified taper - constant thickness, triangular-shaped sides.
- ii. Radiused sides - constant thickness, radius-shaped sides.
- iii. Modified NRL-0.5 - Reduced thickness by 50%, triangular-shaped sides
- iv. Modified NRL-0.75 - Reduced thickness by 25%, triangular-shaped sides.

The surface strain distribution was determined by strain gages affixed along the specimen length. Figure 7-6 shows typical strain distributions for the Types i. and ii. specimens, while Figure 7-7 shows the maximum surface strain versus end deflection for each specimen tested as well as that theoretically predicted for the triangular specimen with constant thickness (see Appendix A, GEAP-5680).

Preliminary fatigue testing of several types of specimens, using a General Electric Coffin fatigue machine specifically modified for this purpose, has begun. These tests will demonstrate the dynamic behavior of the fatigue specimens during the crack initiation and crack propagation stages. In addition, this will provide a preliminary test of the DNPS-1 loading screw and electrical cycle counting system. A photo of the Coffin fatigue machine is shown in Figure 7-8.

It is planned to generate, for the same specimens used for BWR environmental testing in DNPS-1, cycles-to-failure data in air at room temperature, using this fatigue machine. Present plans are to determine for each basic Task K material (i.e., 304, 304-L, Inconel-600, and carbon steel) a reference air cyclic life versus strain range history. These data are necessary in order to be able to provide meaningful "BWR environmental fatigue life reduction factors."

FIGURE 7-4. TEST HEAD AND INSPECTION SUPPORT STAND

MODIFIED TAPER

RADIUSUSED SIDES

MODIFIED NRL - 0.5

MODIFIED NRL - 0.75

FIGURE 7-5. ALTERNATE FATIGUE SPECIMENS

FIGURE 7-6. MEASURED STRAIN-LENGTH DISTRIBUTION

FIGURE 7-7. MAXIMUM MEASURED STRAIN VERSUS SPECIMEN END DEFLECTION

FIGURE 7-8. MODIFIED COFFIN-TYPE FATIGUE TESTING MACHINE

7.6 REFERENCES FOR SECTION VII

1. Piper, D. E., Smith, S. H., and Carter, R. V., "Corrosion Fatigue and Stress Corrosion Cracking in Aqueous Environments," Metals Engineering Quarterly, Vol. 8, No. 3, p. 50-63, August 1968.
2. Novak, S. R., and Rolfe, S. T., "Modified WOL Specimen for K_{ISCC} Environmental Testing," U. S. Steel Corp., submitted to ASTM for publication.
3. Dahlberg, E. P., "Evaluation of Stress-Corrosion Cracking Characteristics of Aluminum Alloys Using Precracked Specimens," Naval Research Laboratory, ASTM Symposium on Stress Corrosion and Corrosion Principles, Atlanta, Ga., October 3, 1968.

APPENDIX A

ANALYSIS OF CRACK GROWTH RATE DATA

(John L. Jaech, Battelle-Northwest)

This appendix contains an analysis of the crack growth rate data from Tasks D-1 and D-2, conducted by A. J. Brothers and E. Krempl, respectively, at the Materials and Processes Laboratory, Schenectady.

A.1 THE MODEL

The mathematical model assumed to hold for these data is as follows.

$$da/dn = A (\Delta K)^B, \quad (A-1)$$

where

da/dn = crack growth rate,

ΔK = stress intensity factor, and

A, B = parameters.

Four specimen shapes were used in the study, defined as follows:

<u>Specimen Type</u>	<u>Description</u>
A	Wide plate
B	Narrow plate
D	Prototype pipe
F	Flat plate

Four materials were included in the study:

Carbon steel plate, A-212B
 Stainless steel plate, SS-304
 Low alloy steel plate, 2-1/4 Cr - 1 Mo
 Carbon steel pipe, A-106B

The stress intensity factor, ΔK , is defined differently for the different specimen types (geometries). For types A, B, and D, this is defined:

$$\Delta K = \Delta \sigma_g \left(W \tan \frac{\pi a_1}{W} \right)^{1/2}, \quad (A-2)$$

where

$\Delta \sigma_g$ = gross stress range on undefected section (psi),

W = undefected plate, width (inches), and
 a_i = crack half-length of i^{th} observation for a particular specimen, including initial notch (inches).

For specimen F, two definitions of ΔK are proposed. For the symmetrical case,

$$\Delta K = \Delta \sigma_g \left(2b \tan \frac{\pi a_i}{2b} \right)^{1/2} h_i, \quad (\text{A-3})$$

where

$2b$ = undefected plate width (inches), and
 $h_i = f_1(a_i)$, f_1 a known function.

For the nonsymmetrical case,

$$\Delta K = \Delta \sigma_g (\pi a_i)^{1/2} k_i, \quad (\text{A-4})$$

where

$k_i = f_2(a_i)$, f_2 a known function.

The problem is to use the data from each run to estimate the A and B parameters in Equation (A-1). First, da/dn is replaced by $\Delta a/\Delta n$, and Equation (A-1) is rewritten:

$$\ln (\Delta a/\Delta n) = \ln A + B \ln (\Delta K). \quad (\text{A-5})$$

A.2 UNWEIGHTED LEAST-SQUARES METHOD

Note from Equation (A-5) that the intercept and slope of the straight line represented by this equation are functions of the parameters of interest. In estimating these parameters from the data, the unweighted least-squares estimating procedure appears to be the correct procedure. This will be true if the assumptions underlying this technique are valid (or nearly so). Specifically, it requires that

- The variance be homogeneous.
- The variance of $\ln (\Delta a/\Delta n)$ dominates.
- The errors in the left-hand side and the right-hand side have correlation near zero.

These assumptions are examined for validity. To examine condition a., write

$$\text{variance } \ln (\Delta a/\Delta n) = \text{variance } \ln (\Delta a) \quad (\text{A-6})$$

$$= \frac{\text{variance } (\Delta a)}{*(E \Delta a)^2} \quad (\text{A-7})$$

*E - expected (or true mean) value of (see subsection 3.4).

It is not unreasonable to assume that $\text{var}(\Delta a) = C(E \Delta a)^2$; i.e., the standard deviation associated with an increase in crack length of (Δa) units is directly proportional to this increase in length. Then, Equation (A-7) becomes

$$\frac{\text{var}(\Delta a)}{(E \Delta a)^2} = \frac{C(E \Delta a)^2}{(E \Delta a)^2} = C, \text{ a constant,} \quad (\text{A-8})$$

satisfying condition a.

Examine condition b. for specimen types A, B, and D. From an error analysis viewpoint, in the region corresponding to the bulk of the data, ΔK may be written:

$$\Delta K \approx \Delta \sigma_g (\pi a_i)^{1/2}, \quad (\text{A-9})$$

$$\ln(\Delta K) \approx \ln(\Delta \sigma_g) + 1/2 \ln \pi + 1/2 \ln a_i, \quad (\text{A-10})$$

$$\text{var} \ln(\Delta K) \approx \frac{\text{var} \ln a_i}{4} \approx \frac{\text{var} a_i}{4(E a_i)^2}. \quad (\text{A-11})$$

Now,

$$a_i = a_0 + \Delta a_1 + \dots + \Delta a_i, \text{ where } a_0 = \text{initial crack length.}$$

Under the same assumption on the variance of Δa_i ,

$$\begin{aligned} \text{var} a_i &= C \Sigma (E \Delta a_i)^2 \\ \frac{\text{var} a_i}{4(E a_i)^2} &= C \left(\frac{\Sigma (E \Delta a_i)^2}{4 [a_0 + \Sigma (E \Delta a_i)]^2} \right) \end{aligned} \quad (\text{A-12})$$

Now, the coefficient of C is much less than 1 because the denominator is the square of not only a sum, but a sum plus a value a_0 , with a_0 and all $E \Delta a_i$ being positive, while the numerator is the sum of squares. The variance of $\ln(\Delta K)$ is therefore negligible compared with the variance of $\ln(\Delta a/\Delta n) = C$, and condition b. is met. A similar argument holds for specimen type F.

With respect to condition c., it is clear that with the exception of the first few points, errors in $\ln(\Delta a/\Delta n)$ and $\ln(\Delta K)$ are essentially uncorrelated; $\ln(\Delta a/\Delta n)$ is affected only by the single term (Δa_i) , whereas the effect of this single term on $\ln(\Delta K)$ is very small.

Thus, the conditions necessary to apply an unweighted least-squares analysis are not grossly violated, and the analysis proceeds on that basis.

A.3 ESTIMATES OF THE B PARAMETER

The B parameter in Equation (A-1) was estimated for each of the 44 runs. The estimation was accomplished in two stages. First, all the data were included for each run. Second, "outlying" points were deleted, and the remaining data were used to obtain a new estimate of B. The process

of detecting these outlying points was somewhat subjective, but in general, points lying more than two standard deviations from the fitted line were deleted. With so many points available for each line in general, the deletion process was not considered crucial, and it was not deemed necessary to be completely objective in this matter.

The results for the 44 runs are summarized in Table A-1. In addition to the estimate of B , its one standard deviation uncertainty is also given. Other columns in the table give the final numbers of data points used, data characterizing each run, and the residual standard deviation for each run. For the F specimens, results are shown separately for (ΔK) defined in the symmetric and nonsymmetric cases.

From this point on in the analysis, it was decided to use the nonsymmetrical definition of (ΔK) for the F specimens. The B values for this set are in better agreement with the corresponding values for the other sets. Judging from the sizes of the residual standard deviations, it makes little difference which definition of (ΔK) is used; one provides just as good a fit as the other.

A.4 COMPARISON OF THE SLOPES

The intercept estimates ($\ln A$) are not shown in Table A-1. This is because they are meaningful only when paired with given B values. The first aim of the analysis was to determine to what extent the B parameters differed among themselves. Common estimates of B would then be used for the appropriate cases, and the intercepts recomputed for these common B values.

To determine the extent to which these B values differ, it is first necessary to estimate the variance between specimens having the same characteristics (i.e., replicate specimens). It is this "between-specimen" variance which is of importance, and not the σ_B 's listed in Table A-1. The between-specimen variance will, of course, reflect the contribution from σ_B of Table A-1.

There are no true replicate specimens in the sense that all associated characteristics are the same. However, there are many sets of replicates in which they are the same except for the gross stress range $\Delta\sigma_g$. An analysis demonstrated that B was independent of $\Delta\sigma_g$. Therefore, the between-specimen variance can be estimated from these sets of replicates. The data are given in Table A-2.

Runs 5, 15, and 20 were deleted because of the very large uncertainties associated with the slope estimates. The estimate of the between-specimen variance, based on the remaining 19 degrees of freedom, was then 1.463. As a matter of interest, the average σ_B^2 listed in Table A-1, and computed from the same 29 runs given in Table A-2 was 0.608.

With the estimate of 1.463 now found, it is possible to determine the effects on B of the various factors identified in Table A-1.

TABLE A-1
LOW-CYCLE FATIGUE LABORATORY TEST CONDITIONS,
TASKS D-1 (SPECIMEN TYPES A, B, D) AND D-2 (SPECIMEN TYPE F)

Run (4)	Specimen Shape	Specimen Width, in.	Material	Temperature, °F	$\Delta\sigma_g$, Gross Stress Range, 1000s psi	OT or TT (1)	Frequency, cpm	No. of Data Points Final	Slope $b \pm 1\sigma_b$	$\sigma_{y/x}$ (3)
1	A	13.7	Type 304 stainless steel plate	RT	23.2	OT	10	15	6.70±1.70	0.168
2	A	13.6	Type 304 stainless steel plate	RT	33.9	OT	10	45	2.37±0.44	0.161
3	A	13.7	Type 304 stainless steel plate	RT	45.0	OT	10	46	3.14±0.31	0.163
4	A	14.0	Type 304 stainless steel plate	RT	33.9-23.2=10.7	TT	10	54	0.11±0.61	0.198
5	A	14.0	CS A-212B plate	RT	27.4	OT	10	14	-1.14±4.50	0.216
6	A	14.0	CS A-212B plate	RT	31.9	OT	10	26	3.86±0.49	0.142
7	A	13.7	CS A-212B plate	RT	38.1	OT	10	40	4.77±0.45	0.208
8	A	14.0	CS A-212B plate	RT	38.2-25.7=12.5	TT	10	45	4.74±0.36	0.211
9	A	14.0	CS A-212B plate	485	38.1	OT	10	14	4.48±0.69	0.171
10	A	13.7	CS A-212B plate	485	26.5	OT	10	43	2.63±0.64	0.209
11	A	14.0	2½ Cr-1 Mo plate	RT	30.4	OT	10	46	3.63±0.16	0.122
12	A	14.0	2½ Cr-1 Mo plate	RT	35.7	OT	10	28	3.56±0.23	0.142
13	A	14.0	2½ Cr-1 Mo plate	RT	35.7-26.8=8.9	TT	10	56	0.25±0.60	0.169
14	A	14.0	2½ Cr-1 Mo plate	540	30.4	OT	10	38	3.39±0.29	0.183
15	B	5.0	Type 304 stainless steel plate	550	25.1	OT	12	5	2.47±5.60	0.379
16	B	5.0	Type 304 stainless steel plate	550	33.9	OT	12	8	5.18±1.03	0.171
17	B	5.0	Type 304 stainless steel plate	550	36.0	OT	12	12	5.92±0.81	0.224
18	B	5.0	Type 304 stainless steel plate	650	35.7	OT	12	20	2.36±0.50	0.218
19	B	5.0	2½ Cr-1 Mo plate	650	30.4	OT	12	31	1.72±0.96	0.315
20	B	5.0	CS A-212B plate	600	23.8	OT	12	5	1.08±8.28	0.335
21	B	5.0	CS A-212B plate	600	28.8	OT	0.6	34	2.85±0.56	0.256
22	B	5.0	CS A-212B plate	600	28.8	OT	12	7	6.06±0.54	0.118
23	B	5.0	CS A-212B plate	600	31.6	OT	12	14	6.48±0.96	0.252
24	B	5.0	CS A-212B plate	600	34.4	OT	12	11	5.38±0.51	0.149
25	B	5.0	CS A-212B plate	600	34.4	OT	hold (2)	42	4.12±0.40	0.225
26	B	5.0	CS A-212B plate	600	34.4	OT	0.6	19	3.80±0.63	0.196
27	B	5.0	CS A-212B plate	650	30.0	OT	12	16	3.29±0.54	0.134
28	B	5.0	CS A-212B plate	650	30.0	OT	0.6	32	1.81±1.36	0.207
29	A	14.0	Type 304 stainless steel plate	RT	33.9-12.5=21.4	TT	10	20	4.06±0.43	0.088
30	D	10.2	CS A-106B pipe	RT	22.1	OT	10	23	3.45±0.26	0.118
31	D	10.2	CS A-106B pipe	RT	25.2	OT	10	6	4.67±0.71	0.100
32	D	10.2	CS A-106B pipe	RT	29.4	OT	10	5	4.84±0.91	0.233
33A SF3	F	2.5	Type 304 stainless steel plate	RT	32.0	OT	6	30	4.55±0.24	0.256
33B									4.16±0.15	0.178
34A SF2	F	2.5	Type 304 stainless steel plate	RT	39.9	OT	6	20	6.73±0.34	0.176
34B									4.79±0.25	0.182
35A SF5	F	2.5	Type 304 stainless steel plate	RT	48.7	OT	6	10	10.22±1.77	0.381
35B									7.25±1.31	0.394
36A CF11	F	2.5	CS A-212B plate	RT	44.0	OT	6	18	4.51±2.02	0.342
36B									3.72±1.65	0.342
37A CF7	F	2.5	CS A-212B plate	RT	34.9	OT	6	15	7.28±1.19	0.233
37B									5.49±0.86	0.225
38A CF8	F	2.5	CS A-212B plate	RT	48.0	OT	6	6	5.06±1.75	0.371
38B									3.68±1.23	0.362
39A CF6	F	2.5	CS A-212B plate	RT	34.0	OT	6	22	7.49±0.67	0.306
39B									5.33±0.46	0.294
40A CF5	F	2.5	CS A-212B plate	RT	39.9	OT	6	7	3.90±1.33	0.384
40B									2.89±0.99	0.385
41A AF4	F	2.5	2½ Cr-1 Mo plate	RT	42.0	OT	6	17	6.76±0.50	0.197
41B									4.80±0.37	0.202
42A AF3	F	2.5	2½ Cr-1 Mo plate	RT	50.4	OT	6	10	6.99±0.48	0.081
42B									5.45±0.45	0.096
43A AF6	F	2.5	2½ Cr-1 Mo plate	RT	46.0	OT	6	15	7.40±0.82	0.222
43B									5.78±0.62	0.216
44A AF7	F	2.5	2½ Cr-1 Mo plate	RT	32.3	OT	6	28	4.87±0.31	0.184
44B									3.72±0.28	0.214
PP1	D	10.2	Type 304 stainless steel pipe	RT	(5)	OT	(5)	(5)	(5)	(5)
PP2	D	10.2	2½ Cr-1 Mo pipe	RT	(5)	OT	(5)	(5)	(5)	(5)

(1) OT: Zero-to-tension; TT: Tension-tension loading, mean stress test.
(2) 10 minute holdtime at maximum load.
(3) Standard deviation of points about the line
(4) Runs designated A used Equation (8) for ΔK to arrive at b and $\sigma_{y/x}$ (symmetrical case). Runs designated B used Equation (9) (nonsymmetrical case).
(5) Runs designated PP1 and PP2 are lines taken directly from the Tenth Quarterly, GEAP-5554, Figure 6-1, and Eleventh Quarterly, GEAP-5587, Figure 6-2, respectively. The lines were based on data from three and four tests, respectively.

TABLE A-2

INDIVIDUAL AND MEAN SLOPE ESTIMATES BY GROUP
FOR GROUPS HAVING MORE THAN ONE RUN

<u>Group</u>	<u>Runs</u>	<u>Slopes (B)</u>	<u>Arithmetical Average</u>
A-304-RT	1, 2, 3	6.70, 2.37, 3.14	4.07
A-212-RT	(5), 6, 7	(-1.14), 3.86, 4.77	4.31
A-212-485	9, 10	4.48, 2.63	3.56
A-2-1/4-RT	11, 12	3.63, 3.56	3.60
B-304-550	(15)16, 17	(2.47), 5.18, 5.92	5.55
B-212-600	(20), 22, 23, 24	(1.08), 6.06, 6.48, 5.38	5.97
D-106-RT	30, 31, 32	3.45, 4.67, 4.84	4.32
F-304-RT	33, 34, 35	4.16, 4.79, 7.25	5.40
F-212-RT	36, 37, 38, 39, 40	3.72, 5.49, 3.68, 5.33, 2.89	4.22
F-2-1/4-RT	41, 42, 43, 44	4.80, 5.45, 5.78, 3.72	4.94

A.5 EFFECT OF FREQUENCY

The direct comparisons that can be made come from runs 21-28, and involve comparing frequencies of 0.6 cpm with frequencies of 10 or 12 cpm. Two temperatures are involved, and the data are kept separate for these two temperatures (see Table A-3).

TABLE A-3

EFFECT OF CYCLIC FREQUENCY ON SLOPE

<u>Runs 21-26</u>				<u>Runs 27-28</u>			
<u>212 at 600° F</u>				<u>212 at 650° F</u>			
<u>Runs</u>	<u>0.6 cpm</u>	<u>Runs</u>	<u>10-12 cpm</u>	<u>Runs</u>	<u>0.6 cpm</u>	<u>Runs</u>	<u>12 cpm</u>
21	2.85	22	6.06	28	1.18	27	3.29
26	3.80	23	6.48				
		24	5.38				
		25	4.12 (10 min. hold)				
Average =	3.32		5.51				

When it is recalled that the residual variance between slopes is 1.463, the weighted average difference between the 0.6 cpm frequency specimens and the 10-12 cpm frequency specimens is 2.00, with a standard deviation of 0.89. This results in a "t" value of 2.25, and is evidence that the 0.6-cpm specimens have a smaller slope than the 10-12 specimens. From this point on, the 0.6-cpm specimens will be removed from the analysis.

In this connection, it should be noted that the A and D type specimens are all at a frequency of 10 cpm, the B type are mostly at 12 cpm, and the F type are all at 6 cpm. If the frequency effect is real, as just indicated, then differences between these specimen types will be confounded with differences in frequencies. This fact should be kept in mind, but it should also be pointed out that even though the 0.6-cpm specimens appear to differ from the 10-12 cpm ones, it may well be that this frequency effect does not obtain in the range from 6-12 cpm.

A.6 OT VERSUS TT

Most of the runs were cycled zero-to-tension (OT); only four were cycled tension to tension (TT). A glance at the Table A-1 data shows no obvious differences between OT and TT. It is noted that two of the "peculiar" runs (4 and 13) were both TT runs, but the significance of this is questionable. Henceforth, these two runs are omitted, while the other two TT runs are retained.

At this point in the analysis, the runs are excluded from the remainder of the analysis of the slopes are listed with comments (Table A-4).

TABLE A-4
RUNS NOT USED IN SLOPE ESTIMATE

<u>Discard Run</u>	<u>Comment</u>
4	TT run; very small slope
5	Negative slope with very high uncertainty
13	TT run; very small slope
15	Very high uncertainty in slope
20	Very high uncertainty in slope
21	0.6 cpm frequency
26	0.6 cpm frequency
28	0.6 cpm frequency

The remaining data are grouped by specimen type, material, and temperature as an aid to following the remainder of the analysis (see Table A-5).

A.7 DIFFERENCES BETWEEN MATERIALS

The materials can be compared with respect to slopes (B) by the following two-way table (Table A-6). Entries in the table are the average B values. The parentheses enclose the numbers of runs composing each average.

The unweighted averages $\pm 2\sigma$ are shown in Table A-6. These uncertainties are based on the between-specimen variance of 1.463, and do not include possible differences between the three rows of Table A-6. " σ " is σ of estimate of the average.

It is clear that no evidence exists to suggest that the materials may differ with respect to the slope (B) values. Therefore, materials may be combined at this stage.

TABLE A-5
RUNS USED IN SLOPE ESTIMATE

<u>Type</u>	<u>Material</u>	<u>Temperature</u>	<u>Run No.</u>	<u>B Value</u>
A	304	RT	1	6.70
			2	2.37
			3	3.14
			29	4.06
A	212	RT	6	3.86
			7	4.77
			8	4.74
A	212	485° F	9	4.48
			10	2.63
A	2-1/4	RT	11	3.63
			12	3.56
A	2-1/4	540° F	14	3.39
B	304	550° F	16	5.18
			17	5.92
B	304	650° F	18	2.36
B	2-1/4	650° F	19	1.72
B	212	600° F	22	6.06
			23	6.48
			24	5.38
			25	4.12
			27	3.29
D	106	RT	30	3.45
			31	4.67
			32	4.84
			33	4.16
F	304	RT	34	4.79
			35	7.25
			36	3.72
F	212	RT	37	5.49
			38	3.68
			39	5.33
			40	2.89
			41	4.80
F	2-1/4	RT	42	5.45
			43	5.78
			44	3.72

TABLE A-6
EFFECT OF MATERIAL ON SLOPE

Material Type:	304	212	2-1/4
RT - "A"	4.07(4)	4.46(3)	3.60(2)
RT - "F"	5.40(3)	4.22(5)	4.94(4)
650 - "B"	2.36(1)	1.72(1)	3.29(1)
Average	3.94	3.47	3.94
$\pm 2\sigma$	± 1.01	± 1.00	± 1.08

A.8 DIFFERENCES BETWEEN SPECIMEN TYPES, AND BETWEEN TEMPERATURES

Table A-7 gives the average slopes for the specimen type—temperature combinations after combining the different materials.

TABLE A-7
MEAN SLOPES BY TEMPERATURE AND SPECIMEN TYPE

Type	Temperature	No. of Obs.	Average B $\pm 2\sigma$
A	RT	9	4.09 $\pm$ 0.81
A	485	2	3.56 $\pm$ 1.71
A	540	1	3.39 $\pm$ 2.42
B	550	2	5.55 $\pm$ 1.71
B	600	4	5.51 $\pm$ 1.21
B	650	3	2.46 $\pm$ 1.40
D	RT	3	4.32 $\pm$ 1.40
F	RT	12	4.76 $\pm$ 0.69

In comparing the specimen types at room temperature, no significant differences are found.

The possible temperature effects are puzzling. No explanation is offered for the differences between the various temperature groups. The erratic behavior suggests some experimental difficulties existed between the various groups as opposed to any real temperature effect influencing the results. This is further strengthened by noting that when a weighted average of the room temperature data is taken, this comes out to $B = 4.45$, with a variance of 0.0610 ($\sigma B = 0.247$), while if all the data are included, the estimate remains virtually unchanged at $B = 4.39$.

In summarizing the slope analyses, no differences in average slopes due to the identified independent variables were detected with the possible exception of the low frequency (0.6 cpm) cycling. Each group of specimens may be regarded as having the same average slope. However, individual specimens within a group will have slopes that differ significantly. This is evident

from the paragraph before Table A-2, where it was pointed out that whereas the between-specimen variance of the slope estimate is 1.463, the average variance of an estimated specimen slope is only 0.608.

A primary aim of this entire analysis is the development of a prediction equation relating crack growth rate to stress intensity factor for each group of specimens, and the construction of confidence intervals around each relationship. The grouping is to be made partly on the basis of the data; i.e., groupings will be combined if the data indicate there are no significant differences between the groups, and if there is no reason why groupings should not be combined (one would want to keep the material types separate, for example).

It has been already concluded that one overall average slope may be used for all groupings. The next question to pursue is the extent to which the specimens differ in their intercepts. Intercepts cannot be compared directly since they are closely correlated with the slopes. We choose to make this evaluation by adjusting all intercepts to the common overall average slope. The other alternative would be to compare predicted values of $\log (\Delta a / \Delta n)$, henceforth called y , at given values of $\log (\Delta K)$, henceforth called x . With different slopes, this would have to be done at a number of different x values, which would be a confusing analysis. In view of the fact that the slopes do not differ drastically in spite of the statistical significance, it is considered appropriate to make the intercept analysis by adjusting to a common slope. In placing confidence intervals about an average line to embrace any future line for the material and temperature, the true variation between specimen slopes will be taken into account.

A.9 COMPARISON OF INTERCEPTS

Using the common slope of 4.45, each intercept $\log A = A'$ at $x = 0$ is found by subtracting 4.45 times the average $\log (\Delta K)$ value ($\bar{x}$) from the average $\log (\Delta a / \Delta n)$ value ($\bar{y}$). One could add $B \cdot \bar{x}$ to the resulting $(\bar{y} - B\bar{x})$ values to give the mean-adjusted intercept $\log C = C'$ at $x = \bar{x}$ and make the comparisons on this basis, but for parallel line segments, this is an equivalent exercise. The intercepts denoted by $\log A$ are given in Table A-8. All runs are included at this stage of the analysis.

TABLE A-8
INTERCEPTS LOG A FOR SLOPE = 4.45

Run	Log A	Run	Log A	Run	Log A	Run	Log A
1	-24.84	12	-25.66	23	-24.93	34	-25.17
2	-25.43	13	-23.88	24	-24.70	35	-25.00
3	-25.69	14	-25.67	25	-24.65	36	-25.21
4	-24.28	15	-24.93	26	-24.72	37	-25.24
5	-25.59	16	-24.96	27	-24.82	38	-25.00
6	-25.74	17	-24.67	28	-24.92	39	-25.23
7	-25.55	18	-25.25	29	-25.22	40	-25.46
8	-24.04	19	-25.15	30	-24.45	41	-25.57
9	-25.00	20	-24.79	31	-24.28	42	-25.64
10	-25.30	21	-25.00	32	-24.32	43	-25.65
11	-25.67	22	-24.94	33	-25.43	44	-25.73

Analyses of Table A-8 data show no differences due to $\Delta\sigma_g$ or to frequency. A difference between OT and TT does exist, however, with TT intercepts higher, as indicated by Table A-9.

TABLE A-9

EFFECT OF TT VERSUS OT TESTING ON INTERCEPT

304 at RT				212 at RT				2-1/4 at RT			
Runs	OT	Runs	TT	Runs	OT	Runs	TT	Runs	OT	Runs	TT
1	-24.84	4	-25.22	5	-25.59	8	-24.04	11	-25.67	13	-23.88
2	-25.43	29	-24.28	6	-25.74			12	-25.66		
3	-25.69			7	-25.55						

Because of this difference, all results from this point on relate to the zero-to-tension situation. The OT-TT effect is very large, compared to other differences in intercepts, as noted in Table A-9.

The intercept data are now grouped by specimen type, material, and temperature in Table A-10.

Average curves relating (da/dn) to (ΔK) will be found for each of the material-temperature combinations. Differences between the A and F specimens will be used to estimate the "between-heat" variance. For each material type, there is only one degree of freedom associated with this estimate, which means that the estimate has considerable uncertainty associated with it. Since "between-heat" variance is an important component of variation when trying to assess the variation between piping that may be experienced, the experimental program was weak in the estimation of this quantity. Clearly, it was not geared to accomplish this.

Estimates of the "within-heat" variance are also made separately for each material. Bartlett's test for homogeneity of variance indicated that the materials did indeed differ with respect to this quantity, and so the individual estimates were required. Estimates of the two components of variance are given in Table A-11.

A.10 CONFIDENCE LIMITS AND VARIANCE OF INTERCEPT

With these quantities estimated, it is now possible to establish confidence limits about the average line for each material-temperature group. Variances of intercepts among individual lines within groups will also be found. Two kinds of confidence limits are found:

- a. Limits indicating how well the average line is known. Values to plot such limits will be found from var y_a in an expression for y_a for the average line for the group. This var y_a is designated V_1 .

TABLE A-10
RUNS USED IN INTERCEPT ESTIMATES

<u>Specimen Type</u>	<u>Material</u>	<u>Temperature</u>	<u>Runs</u>	<u>Log A</u>
A	304	RT	1	-24.84
			2	-25.43
			3	-25.69
A	212	RT	5	-25.59
			6	-25.74
			7	-25.55
A	2-1/4	RT	11	-25.67
			12	-25.66
A	212	485° F	9	-25.00
			10	-25.30
A	2-1/4	540° F	14	-25.67
			15	-24.93
B	304	550-650° F	16	-24.96
			17	-24.67
			18	-25.25
B	212	600-650° F	20	-24.79
			21	-25.00
			22	-24.94
			23	-24.93
			24	-24.70
			25	-24.65
			26	-24.72
			27	-24.82
			28	-24.92
			19	-25.15
D	106	RT	30	-24.45
			31	-24.28
			32	-24.32
F	304	RT	33	-25.43
			34	-25.17
			35	-25.00
F	212	RT	36	-25.21
			37	-25.24
			38	-25.00
			39	-25.23
			40	-25.46
F	2-1/4	RT	41	-25.57
			42	-25.64
			43	-25.65
			44	-25.73

Note: TT runs 4, 8, 13 and 29 are omitted.

TABLE A-11
COMPONENTS OF VARIANCE OF INTERCEPTS

Material	RT Within-Heat Variance (σ_w^2)	RT Between-Heat Variance (σ_ξ^2)
304	0.0874	Negative*
212	0.0219	0.0736***
2-1/4	0.00323	Negative*
A-106	0.00790	Can't estimate**

*The estimate of σ_ξ^2 was negative; use zero.

**No estimate possible; use the same as for 212, arbitrarily.

***Assume same between-heat variance applies at elevated temperatures. Within-heat variances can be found for elevated temperatures.

- b. Limits indicating where a line for an individual specimen selected at random from a randomly chosen heat will fall. Values to plot such limits will be found from var y_1 in an expression for y_1 for individual lines. This var y_1 is designated V_2 . Since confidence limits diverge at x values both larger and smaller than the mean x , both V_1 and V_2 will be seen to be functions of x .

Notation may be summarized as follows.

For x and y :

Raw data points are designated (x, y) .

Points on the line for an individual run, fitted to these points, are designated (x, y_1) . The means of x and y , through which this line passes, are $(\bar{x}, \bar{y})$.

Points on the average line for a material-temperature group are designated (x, y_a) . This line passes through the grand means of x and y , designated $(\bar{\bar{x}}, \bar{\bar{y}})$ which are the means of $\bar{x}$ and $\bar{y}$ (strictly, the weighted means).

For intercepts, using the grand mean slope $\hat{B}$,

the model for an individual line is

$$y_i = A'_i + \hat{B}x, \quad (A-13)$$

so that A'_i for an individual line is estimated from

$$A'_i = \bar{y} - \hat{B}\bar{x} \quad (A-14)$$

once $\hat{B}$ is known, since the line passes through $(\bar{x}, \bar{y})$.

The model for the average line for a group is

$$y_a = A'_a + \hat{B}x, \quad (A-15)$$

so that A'_a for an average line is estimated from

$$A'_a = \bar{\bar{y}} - \hat{B}\bar{\bar{x}} \quad (A-16a)$$

once $\hat{B}$ is known, since the line passes through $(\bar{\bar{x}}, \bar{\bar{y}})$ for the group.

Equivalently, A'_a can be found as the mean of the A'_i of the k individual lines in the group, as

$$A'_a = \frac{A'_1 + A'_2 + \dots + A'_k}{k} \text{ (strictly, the weighted mean).} \quad (A-16b)$$

Confidence limits in a , on the average line, use Equation (A-15) for y_a . Term y_a is seen to be a linear function of two terms, but the terms are not independent (A'_a and $\hat{B}$ are not independent in this formulation of the linear model), so the covariance term is required. Thus,

$$V_1 = \text{var } y_a = \text{var } A'_a + 2 \text{cov } (A'_a, \hat{B}x) + \text{var } (\hat{B}x). \quad (A-17)$$

The variance as observed among the A'_i for a given group includes variability due only to differences in the $\bar{y}$ of the individual lines in the group, which is found from Table A-11. But we required $\text{var } A'_a$, so this part of the variance in estimate of the intercept of an average line can be written

$$\text{var } A'_a \text{ due to } \bar{y} \text{ only} = \frac{\sigma_w^2}{n} + \frac{\sigma_\xi^2}{m}, \quad (A-18)$$

where

m = number of heats represented in the average line, and

n = total number of specimens in the average line.

Terms m and n are tabulated in Table A-12. But variability due to uncertainty in $\hat{B}$ must also be included in $\text{var } A'_a$; this contribution can be found from Equation (A-16), treating $\bar{\bar{x}}$ and $\bar{\bar{y}}$ as constants, so

$$\text{var } A'_a \text{ due to uncertainty in } \hat{B} = \bar{\bar{x}}^2 \text{var } \hat{B}. \quad (A-19)$$

Thus,

$$\text{total var } A'_a = \frac{\sigma_w^2}{n} + \frac{\sigma_\xi^2}{m} + \bar{\bar{x}}^2 \text{var } \hat{B}. \quad (A-20)$$

TABLE A-12
NUMBER OF HEATS AND NUMBER OF SPECIMENS
IN VARIANCES OF INTERCEPT MEANS

<u>Material</u>	<u>Temperature</u>	<u>m</u>	<u>n</u>
2-1/4	RT	2	6
304	RT	2	6
212	RT	2	8
2-1/4	High	1	2
304	High	1	4
212	High	1	11
106	RT	1	3

The covariance between A'_a and $\hat{B}\bar{x}$ is found from $E(A'_a - EA'_a)(\hat{B}\bar{x} - E\hat{B}\bar{x})$ as

$$\text{cov}(A'_a, \hat{B}\bar{x}) = -2\bar{x} \text{ var } \hat{B}. \quad (\text{A-21})$$

Then, for the last term in Equation (A-17),

$$\text{var } \hat{B}x = x^2 \text{ var } \hat{B}. \quad (\text{A-22})$$

Thus, Equation (A-17) becomes

$$V_1 = \text{var } y_a = \underbrace{\frac{\sigma_w^2}{n} + \frac{\sigma_\xi^2}{m}}_{\text{var } y_a} + \underbrace{\bar{x}^2 \text{ var } \hat{B}}_{\text{var } \hat{B}\bar{x}} - \underbrace{2\bar{x}x \text{ var } \hat{B}}_{\text{cov}(A'_a, \hat{B}\bar{x})} + \underbrace{x^2 \text{ var } \hat{B}}_{\text{var } \hat{B}x} \quad (\text{A-23})$$

$$= \frac{\sigma_w^2}{n} + \frac{\sigma_\xi^2}{m} + (x - \bar{x})^2 \text{ var } \hat{B} \quad (\text{A-24})$$

$$= \frac{\sigma_w^2}{n} + \frac{\sigma_\xi^2}{m} + 0.0610 (x - \bar{x})^2. \quad (\text{A-25})$$

Perhaps the foregoing discussion is made more straightforward if the model is rewritten in its equivalent mean-adjusted form. Equation (A-15) for a point (x, y_a) on a group average line may be written:

$$y_a = C'_a + \hat{B}(x - \bar{x}). \quad (\text{A-26})$$

In Equation (A-26), C'_a and $\hat{B}$, the two x random variables, are uncorrelated, so there is no concern about the covariance between them. Equation (A-17) simplifies to

$$\begin{aligned} \text{var } y_a &= \text{var } C'_a + \text{var } [\hat{B}(x - \bar{x})] \\ &= \text{var } C'_a + (x - \bar{x})^2 \text{ var } \hat{B}. \end{aligned} \quad (\text{A-27})$$

Since the average line goes through $(\bar{x}, \bar{y})$, at $x = \bar{x}$ $y = C'_a$, then $C'_a = \bar{y}$.

Now $\text{var } \bar{y}$ is not found by observing the variance between the $\bar{y}$'s directly since a part of this variance is attributed to the fact that the specimens do not all have the same $\bar{x}$. This contribution must be removed from the total variance between the $\bar{y}$'s and this is accomplished by adjusting the $\bar{y}$ values to the same $\bar{x}$ value, $\bar{x}$. The resulting variance is simply $(\sigma_w^2/n + \sigma_\xi^2/m)$. Thus, Equation (A-27) becomes

$$V_1 = \text{var } y_a = \frac{\sigma_w^2}{n} + \frac{\sigma_\xi^2}{m} + (x - \bar{x})^2 \text{var } \hat{B} \quad (\text{A-28})$$

which is the same as Equation (A-24).

Confidence limits in b. are for an individual specimen selected at random from a random heat. The total variance, $V_2 = \text{var } y_i$, is made up of $\text{var } y_a$ —the above uncertainty in position of the average line—plus uncertainty associated only with an individual line. The latter can be found from the variance of a mean-adjusted expression for y_i , similar to that in Equation (A-26) for y_a but reflecting also the fact that individual lines have their own slopes, B_i , as follows:

$$y_i = C'_i + B_i (x - \bar{x}). \quad (\text{A-29})$$

The variance of this expression, due to independence of C'_i and B_i , is $\text{var } y_i$, without considering uncertainty on location of average line

$$\text{var } y_i = \text{var } C'_i + (x - \bar{x})^2 \text{var } B_i. \quad (\text{A-30})$$

Thus, the total variance of y_i is

$$V_2 = \text{var } y_i = \text{var } C'_a + \text{var } C'_i + (x - \bar{x})^2 (\text{var } \hat{B} + \text{var } B_i) \quad (\text{A-31})$$

$$= \sigma_w^2/n + \sigma_\xi^2/m + \sigma_w^2 + \sigma_\xi^2 + (x - \bar{x})^2 (\text{var } \hat{B} + \text{var } B_i)$$

$$= \sigma_w^2 (1 + 1/n) + \sigma_\xi^2 (1 + 1/m) + (x - \bar{x})^2 (0.061 + 1.463). \quad (\text{A-32})$$

Using Equations (A-25) and (A-32), plus the data in Tables A-11, A-12, and the balance of this subsection, confidence intervals can be constructed for the seven cases. The numbers to be used in constructing these confidence intervals are given in Table A-13; $\sqrt{V_1}$ and $\sqrt{V_2}$ refer to the two kinds of standard deviations computed: one for the average line by Equation (A-25) or (A-22), and the other by Equation (A-32) for a randomly selected specimen. The range of values for x in Table A-13 covers the experimental range of the data. Both confidence intervals are shown for illustration for the carbon steel room temperature case in Figure 3-10. These are 2σ limits; they cannot be interpreted strictly as 95% confidence limits because of the very large uncertainty associated with the estimate of the parameters, σ_w^2 in particular.

The variance of intercept C' for an individual line, including uncertainty in position of the group average line, is seen from Equation (A-32) to be

$$\text{var } C'_1 = \sigma_w^2 (1 + 1/n) + \sigma_\xi^2 (1 + 1/m) . \quad (\text{A-33})$$

This equation was used to calculate the "var log C " ($\equiv \text{var } C'$) column of Table 3-4.

TABLE A-13
STANDARD DEVIATIONS OF y FOR CONFIDENCE INTERVALS

Material	Temperature	x	y	$\sqrt{V_1}$	$\sqrt{V_2}$
2-1/4	RT	4.59	-5.224	0.060	0.283
		4.7		0.037	0.154
		4.8		0.023	0.063
		4.9	-3.845	0.031	0.123
		5.0		0.051	0.238
		5.1		0.074	0.358
304	RT	5.23	-2.376	0.105	0.517
		4.46	-5.413	0.145	0.514
		4.6		0.129	0.394
		4.7		0.123	0.336
		4.8		0.121	0.319
		4.9		0.124	0.349
		5.0	-3.010	0.132	0.415
		5.1		0.143	0.502
		5.2		0.158	0.602
		5.3		0.175	0.711
212	RT	5.42	-1.141	0.198	0.845
		4.50	-5.355	0.207	0.472
		4.6		0.202	0.406
		4.7		0.199	0.370
		4.8	-4.020	0.199	0.375
		4.9		0.203	0.417
		5.0		0.209	0.488
2-1/4	High	5.10	-2.685	0.218	0.517
		4.48	-5.474	0.069	0.286
		4.5		0.065	0.262
		4.6		0.048	0.147
		4.7	-4.495	0.040	0.070
		4.8		0.047	0.137
304	High	4.89	-3.650	0.061	0.239
		4.39	-5.414	0.157	0.426
		4.5		0.150	0.357
		4.6	-4.480	0.148	0.331
		4.7		0.150	0.349
		4.8		0.155	0.407
212	High	4.85	-3.368	0.159	0.446
		4.36	-5.488	0.283	0.537
		4.5		0.277	0.447
		4.6		0.275	0.415
		4.7	-3.975	0.275	0.421
		4.8		0.277	0.459
		4.9		0.282	0.526
106	RT	5.0	-2.640	0.289	0.610
		4.46	-4.503	0.281	0.476
		4.5		0.279	0.451
		4.6		0.277	0.407
		4.7	-3.435	0.276	0.398
		4.8		0.278	0.428
		4.9		0.282	0.487
		4.95	-2.322	0.284	0.525

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