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Numerical Treatment of Interior Boundary Conditions
on the Onsager Equation

J. Viecelli

September 2, 1982

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on the Onsager Equation

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Introduction

It has been shown that the extra condition needed to fix the stream function on an interior boundary in an incompressible liquid flow is that the integral of the pressure gradient along any path enclosing the boundary should vanish [1,2]. More recently this constraint has been used to solve for a few special cases of compressible centrifuge flows [3]. Here one way in which this constraint can be easily incorporated in a numerical scheme for solving the Onsager equation for the gas flow in a centrifuge is described.

Equation for the Interior Boundary Stream Function

The geometry and notation are the same as used in the derivation of the Onsager equation given in [4]. Assuming a baffle similar to that in [3] but having some axial thickness, I use the paths enclosing the baffle shown in Fig. 1. Along the sections $x = \text{constant}$ the axial pressure gradient can be gotten from the axial momentum equation yielding;

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$$(e^x p)_y = \frac{8A^6}{Re} e^x w_{xx} = - \frac{16A^8}{Re} e^x (e^x \psi_x)_{xx} \quad , \quad (1)$$

where ψ is the stream function. Along the sections $y = \text{constant}$ the radial pressure gradient is obtained from the radial momentum equation,

$$(e^x p)_x = \phi \quad . \quad (2)$$

Substituting for ϕ from the Ekman boundary conditions along the boundaries $y=y_1$, and $y=y_2$ gives, along the upper surface,

$$(e^x p)_x = \bar{\phi}(x, y_2) + 4S^{3/4} Re^{1/2} e^{x/2} (\psi(x, y_2) - k) \quad (3)$$

and along the lower surface

$$(e^x p)_x = \bar{\phi}(x, y_1) - 4S^{3/4} Re^{1/2} e^{x/2} (\psi(x, y_1) - k) \quad (4)$$

where k is the unknown stream function value on the baffle, equal to the net mass flow circulating around the baffle, and $\psi(x, y_2)$ and $\psi(x, y_1)$ are the interior stream function values above and below the baffle.

Integrating the pressure gradient along the two paths enclosing the baffle yields, for path 1 along the upper side,

$$\begin{aligned} e^{x_2} p(x_2, y_2) &= e^{x_1} p(x_1, y_1) + \int_{y_1}^{y_2} (e^x p)_y \big|_{x=x_1} dy \\ &+ \int_{x_1}^{x_2} \bar{\phi}(x, y_2) dx + 4S^{3/4} Re^{1/2} \int_{x_1}^{x_2} e^{x/2} (\psi(x, y_2) - k) dx \end{aligned} \quad (5)$$

and for path 2 along the lower side,

$$e^{x_2} p(x_2, y_2) = e^{x_1} p(x_1, y_1) + \int_{y_1}^{y_2} (e^x p)_y \Big|_{x=x_2} dy \quad (6)$$

$$+ \int_{x_1}^{x_2} \bar{\phi}(x, y_1) dx - 4S^{3/4} Re^{1/2} \int_{x_1}^{x_2} e^{x/2} (\psi(x, y_1) - k) dx$$

Subtracting Eq. (6) from Eq. (5) and noting that continuity of pressure around the baffle requires the same result independent of path one gets,

$$0 = \int_{y_1}^{y_2} [e^x p_y \Big|_{x=x_1} - e^x p_y \Big|_{x=x_2}] dy + 4S^{3/4} Re^{1/2} \int_{x_1}^{x_2} e^{x/2} [\psi(x, y_2) + \psi(x, y_1) - 2k] dx$$

$$+ \int_{x_1}^{x_2} [\bar{\phi}(x, y_2) - \bar{\phi}(x, y_1)] dx \quad (7)$$

Rearranging terms, substituting Eq. (1), and solving for k then yields,

$$k = \left\{ 4S^{3/4} Re^{1/2} \int_{x_1}^{x_2} [\psi(x, y_2) + \psi(x, y_1)] dx \right.$$

$$- \frac{16A^8}{Re} \int_{y_1}^{y_2} [e^x (e^x \psi_x)_{xx} \Big|_{x=x_1} - e^x (e^x \psi_x)_{xx} \Big|_{x=x_2}] dy$$

$$\left. + \int_{x_1}^{x_2} [\bar{\phi}(x, y_2) - \bar{\phi}(x, y_1)] dx \right\} \quad (8)$$

$$\div 16S^{3/4} Re^{1/2} (e^{x_2/2} - e^{x_1/2})$$

Equation (8) gives the net mass circulation around the baffle in terms of the interior stream function and its derivatives along the boundary and in terms of the temperature boundary conditions along the surface of the baffle specified by

$$\bar{\phi}(x, y_2) = T(x, y_2)/T_w \quad (9)$$

$$\bar{\phi}(x, y_2) = T(x, y_2)/T_w \quad (10)$$

Solution method

Discretization of the Onsager equation by the method described in [5] leads to a set of simultaneous linear equations for the stream function on a set of grid points which is easily solved by standard methods. Boundary condition (8) can be similarly discretized and included explicitly in the matrix of coefficients but this is algebraically very tedious and so susceptible to error.

An alternative method requires twice the computer time but is much easier to implement. Since k in Eq. (8) is linear in ψ one can solve for k in two steps. First, solve the Onsager equation with homogeneous boundary conditions except for a fixed input non zero stream function value k on the baffle, call it k_0 . After obtaining the solution then calculate a value of k from Eq. (8) using this solution, call it k_1 . Next compute the solution to the Onsager equation for the desired inhomogeneous boundary conditions, but with the stream function k on the baffle identically zero. After obtaining this solution use it to calculate a k from Eq. (8) call it k_2 . The solution for k consistent with the desired inhomogeneous boundary conditions is then given by

$$k = \frac{k_0 k_2}{k_0 + k_1} \quad (11)$$

Equation (11) results because k computed from Eq. (8) is linear in the input value k_0 and so can be solved for k [by Eq. (8)] = k [fixed input] in two steps. The penalty of doubled computational time is negligible as a single solution of the Onsager equation requires very little computer time.

Radial Flows into and out of the Ekman Layers

As shown in [3] the pressure continuity condition implies some radial flows into and out of the Ekman layers around the edges at x_1 and x_2 . In the numerical solution these flows are included automatically if normal boundary conditions along the edges between y_1 and y_2 are included. These boundary conditions, along the edges $x = x_1$ and $x = x_2$, are

$$\psi = k \quad (12)$$

$$\psi_x = 0 \quad (13)$$

$$(e^x(e^x\psi_x)_{xx})_x = -\frac{Re}{16A^8}\bar{\phi}_y \quad (14)$$

Equation (12) is necessary because the stream function at the baffle edge must equal the net mass circulation around the baffle. Equation (13) is the no slip boundary condition on the axial velocity component, and Eq. (14) gives the temperature boundary condition identical to that at the rotor wall where $\psi=0$. Figure 2 shows these boundary conditions as they apply along the baffle surfaces, along with a rough mesh to indicate the location of nodes in the difference scheme described in [5].

Conclusion

The pressure continuity constraint for flow around a baffle discussed in [3] can be quite easily applied to the Onsager equation. It can be used to derive a formula for the circulation around a baffle with some axial thickness, which can then be used in a two step solution method. The formula could also be incorporated explicitly into the matrix of coefficients resulting from discretization of the Onsager equation but this approach would require considerably more algebra. The first method is recommended. The radial flows into and out of the Ekman layers are included naturally when provision for axial thickness of the baffle is included. Boundary conditions along surfaces parallel to the rotor wall are similar to those at the rotor wall and can be treated by the same numerical methods.

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Fig. 1

Fig. 2

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