

MASTER

RECENT EXPERIMENTAL RESULTS ON THE BEAM-BEAM EFFECTS  
IN STORAGE RINGS AND AN ATTEMPT OF THEIR INTERPRETATION\*

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SUMMARY

The latest available experimental results on the luminosity, the space charge parameters, and the beam blowup as functions of particle energy and beam current are reviewed. The comparison with the phenomenological diffusion theory are done and useful scaling laws are derived. Some implications for  $\bar{p}p$  storage rings are discussed.

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## Introduction

Although there are a number of excellent papers<sup>1-3,14</sup> on the beam-beam phenomena, the importance of the problem which implies the most severe limitation on the beam currents of the storage ring as well as recent availability of new experimental results<sup>4-6</sup> and theoretical approach<sup>7</sup> make it quite feasible to add to the list.

The problem has also an important practical impact on many storage rings of the immediate future. For an electron-positron storage ring it can give, by applying the appropriate scaling laws, some insight on the acceptable magnitude of the space charge parameter. The same is also true for  $\bar{p}p$  machine which can be considered, with respect to the beam-beam effect, as  $e^-e^+$  ring with extremely small particle energy.

Although the beam-beam effect itself is rather crude and well pronounced, a theoretical description of it is very difficult to give both analytically and numerically. The main difficulty lies in the nonlinear character of the forces involved and to some extent in the complicated dependence on many beam and machine parameters interlacingly influencing each other.

In this situation a phenomenological approach seems to be adequate. A proper parameterization of the problem and description of many functional dependencies by a few fitting parameters can supply us with needed scaling laws. The behavior of such a fitting parameter with energy for example cannot be explained by a theory. This dependence will be found from an experiment. But after it is established it



can have certain predictive power and will give some insight for the future accelerators.

There is also some hope to find suitable theoretical ground for the accepted dependencies in the numerical analysis of the problem. Much work is needed in this respect.

In this work I suggest some scaling laws for the luminosity, space charge parameters, and beam size as functions of particle energy, maximum beam current, and the number of bunches. These scaling laws are derived from the latest experimental data available now.

The biggest drawback of the description suggested here, as I see it, lies, contrary to the observations, in the complete absence of the fitting parameter dependence on the machine tune. This drawback can be attributed to an averaging procedure needed for a diffusion-like description of the process. By this averaging all resonance structure of the particle motion is completely lost. It is probable that the resonance and diffusion approaches could be complementary to each other. Again much work is needed here.

Section 1 of this work is devoted to the recent experimental results from SPEAR,<sup>4,8</sup> ADONE,<sup>5</sup> and PETRA.<sup>6</sup> In Section 2 the diffusion theory is used to derive main relationships and, together with the experimental results, to get main scaling laws. In Section 3 we summarize these scaling laws, and in Section 4 some predictions for future storage rings are done based upon the scaling laws.

## 1. Experiment

Before discussing recent experimental results observed on different electron storage rings it is useful to look first at the conditions in which they are obtained and the assumptions under which they are interpreted.

### 1.1 Main relationships and assumptions

First of all let us discuss relevant storage ring parameters as well as experimental conditions under which they are usually measured. I will list the main parameters and relationships between them although the latter are all well known.

1.1.1 Luminosity of the storage ring for the head-on collision of two identical beams is usually assumed to be

$$\mathcal{L} = \frac{i^2}{4\pi e^2 f B \sigma_x \sigma_y} \quad (1)$$

where  $i$  is the current in either of two beams,  $B$  is the number of bunches in each of the beams,  $f$  is the revolution frequency of the particle with the charge  $e$ ,  $\sigma_x$  and  $\sigma_y$  are horizontal and vertical dimensions of the bunch (rms widths if the distribution is Gaussian) at the interaction point.

1.1.2 Space charge parameters under the same conditions are given by the following formulae

a) for the vertical motion

$$\xi_y = \frac{e i \beta_y}{2\pi f B E \sigma_y (\sigma_x + \sigma_y)} \quad (2)$$

b) for the horizontal motion

$$\xi_x = \frac{e\beta_x}{2\pi f BE\sigma_x(\sigma_x + \sigma_y)} \quad (3)$$

In these formulae  $\beta_x$  and  $\beta_y$  are values of horizontal and vertical  $\beta$ -functions at the interaction point,  $E$  is particle energy. Both the luminosity  $\mathcal{L}$  and the space charge parameters  $\xi_y$  and  $\xi_x$  depend on the bunch size which is very difficult to measure directly. But it is clear that both values are sensitive to the charge distribution in the core of the beam rather than to the tails of it. At the same time it is known<sup>2</sup> that tails are affected by the beam-beam interaction much more strongly than the core.

1.1.3 The beam lifetime  $T$  for a single Gaussian bunch is given by<sup>9</sup>

$$T = \tau e^{c/2/c} \quad (4)$$

where  $\tau$  is the vertical damping time

$$\frac{1}{\tau} = C_Y f E^3 / 2\rho \quad (5)$$

$C_Y = 8.85 \times 10^{-5} \text{ m/GeV}^3$ ,  $\rho$  = bending radius in m,  $E$  the energy in GeV.

$$c = \Gamma^2 / \sigma^2 \quad (6)$$

$\Gamma$  is an effective aperture of the machine. The beam lifetime is sensitive to the distribution of the particles in the tails where the beam-beam interaction changes distributions significantly. That makes

the maximum luminosity strongly dependent upon the value of the maximum beam current which in turn happens to be a fast function of the particle energy.

1.1.4 Parameters of interest. Among the machine parameters entering into expressions (1-6), the energy  $E$ , the number of bunches  $B$ , and the revolution frequency  $f$  are known with great accuracy. The luminosity  $\mathcal{L}$  and the beam current  $i$  can be measured directly.

On the other hand, several other parameters such as  $\beta_x$ ,  $\beta_y$  are very difficult to measure. Although one can expect that  $\beta_x$ ,  $\beta_y$  should be modified by the beam-beam force, these functions are changed only in the second order of the perturbation theory and therefore usually are assumed to be equal to their theoretical value at the zero current. The same holds for the horizontal beam emittance  $\epsilon_x$  and consequently for the horizontal beam size  $\sigma_x = \sqrt{\epsilon_x \beta_x}$ .

1.1.5 Experimental conditions and assumptions. Experimental data on the beam-beam effect are obtained on different machines virtually in quite different conditions.

- a) The investigation of the beam-beam limitations. Measurements of this kind are done during special machine physics runs. The main goal of these measurements is to achieve the maximum possible luminosity for given parameters by increasing the currents to the point where the lifetime of the beam starts to decrease sharply. To maximize the luminosity of the ring both currents are usually maintained pretty much the same. For the SPEAR measurements<sup>4</sup>

$$2(i_+ - i_-)/(i_+ + i_-) \leq (2-3)\%$$

One tries to do the same with the vertical size of the beam. At least at SPEAR this condition was met by means of adjustment of the phase between the rf cavities positioned symmetrically around the interaction point.<sup>10</sup>

Experimental data obtained in this situation should be more sensitive to the particle distribution at large amplitudes (to the tails of distribution) rather than to the distribution in the core of the beam.

- b) The investigation of the storage ring performance. Measurements of this kind are usually done during high energy physics runs in a parasitic mode. Maximum luminosity is achieved in this case under a restrained condition of the beam lifetime being unaffected or almost unaffected by beam-beam phenomena. These measurements should be more sensitive to the distribution in the core of the beam.

In all of the storage rings the longitudinal size of the bunch  $\sigma_z$  is much less than  $\beta_y$ . If this condition were not fulfilled, different particles along the bunch would experience different focusing and the results could be distorted by this effect. As we shall see later, it is assumed usually that the distribution of the particles is Gaussian, at least in the core. This assumption one needs to be able to calculate the space charge parameters from the measured luminosity and current.



In some aspects there is also a difference between the strong beam-strong beam and the strong beam-weak beam interactions.

## 1.2 Recent experimental results

An experimental fact observed on all the machines is that the horizontal size of the bunch is not influenced by the beam-beam interaction<sup>2,6</sup> with the accuracy  $\leq 10\%$ .

### 1.2.1 Procedure of calculating values of interest

It is instructive first to see how one can derive the relevant parameters from the measured ones.

a) First of all assuming  $\sigma_x$  to be equal to  $\sqrt{c_x \beta_x}$ , one can find beam aspect ratio  $\sigma_y/\sigma_x$  from the measured luminosity (1):

$$\sigma_y/\sigma_x = 1^2/4\pi e^2 f B \sigma_x \mathcal{L} \quad (7)$$

b) Formula (3) then allows us to find the horizontal space charge parameter

$$\xi_x = e i \beta_x / 2\pi f B E \sigma_x^2 (1 + \sigma_y/\sigma_x) \quad (8)$$

c) After eliminating  $\sigma_y$  from (1) and (2) one gets:

$$\xi_y = 2e^3 \mathcal{L} \beta_y / E i (1 + \sigma_y/\sigma_x) \quad (9)$$

Let us review the recent experimental results obtained on different storage rings.

### 1.2.2 SPEAR. Dependence on energy (H. Wiedemann<sup>4</sup>)

Recently a set of new measurements of the maximum luminosity and the beam current versus machine energy was undertaken by H. Wiedemann. The range of energy variation was from 0.6 to 3.7 GeV and is much wider than in all previous experiments. The data were taken during the special runs of the SPEAR dedicated to machine physics. Much work was done to adjust all the machine parameters to achieve maximum luminosity. Special attention was paid to balance the vertical sizes of electron and positron bunches to avoid the loss of the luminosity due to the flip-flop effect.

The fit by a power law to recent data seems to give quite different slopes, especially for the vertical space charge parameter, than ones in the previous measurements.<sup>2</sup> The difference may be attributed to the fact that the energy range in the work<sup>2</sup> was much narrower (from approximately 1.2 to 2.5 GeV). Although the measurements are still in progress, the data are quite reliable in the opinion of the experimenter.<sup>4</sup> Table 1 summarizes the results of fitting to these measured and calculated data.

### 1.2.3 SPEAR. Dependence on the beam current

Table 2 summarizes the data picked up from SPEAR logbooks by M. Cornacchia.<sup>8</sup> The data were mostly taken during regular physics runs of the machine. The fits to the data taken at high energy physics run are recalculated. Instead of fitting data by the least square method the maximum luminosity was fitted.

### 1.2.4 ADONE (S. Tazzari<sup>5</sup>)

Table 3 summarizes the dependencies of the maximum luminosity and the beam current versus energy which were taken from the report by S. Tazzari.<sup>5</sup> The space charge parameters of this machine were kept approximately equal to each other. The fit for the space charge parameters is derived from the calculated values plotted in the work.<sup>5</sup> The number of bunches in ADONE can be and was changed. The data taken with 1 and 3 bunches do not contradict the assumption

$$\xi_y \sim 1/\sqrt{B}$$

### 1.2.5 PETRA (G. Voss<sup>6</sup>)

The data from the measured specific luminosity  $\mathcal{L}/i^2$  during high energy physics experiments were fitted with the help of the blowup function  $\sigma_y$  assumed to behave according to the following:

$$\sigma_y^2 = \sigma_0^2 + \left(\frac{ai}{\sigma_y}\right)^2 \quad (10)$$

Here  $\sigma_0$  is the value of  $\sigma_y$  at zero current  $i$  and  $a$  is a parameter. From the data taken at different energies,  $a$  is found to be:

$$a = \text{const}/E^4 \quad (11)$$

The values of aspect ratio of the beam emittances are estimated to be of the order of several percent at all energies.

## 2. Theory

The word "theory" is probably an exaggeration in application to the beam-beam phenomena, at least in its present state. What I really mean is a kind of phenomenological theory which helps to make parametrization of the experimental data in a suitable way and to derive some scaling laws by means of a few fitting parameters. The behavior of these fitting parameters is not described by a theory and should be taken from the comparison with an experiment.

It is useful first to go through main assumptions under which the theory is developed as well as those which will be used in the following considerations.

### 2.1 Assumptions

2.1.1 First of all we shall consider one dimensional model of the beam-beam interaction. Although the phenomenon is essentially multidimensional, the justification of this model at least in the first approximation comes from the experimental observations that the vertical size of the bunch is most strongly affected by the interaction while the horizontal size of the bunch seems to be affected very little if any.

One may argue about the loss of some particular multidimensional features like the Arnold diffusion, sideband resonances, and the like. All of these effects seem to be small compared to the main rough effect.

2.1.2 Secondly, we assume that at least some number of particles behave stochastically. The reason for such a behavior can be nonlinearities in the machine lattice, nonlinearity of the electromagnetic

beam-beam force, combined action of many close-lying resonances, presence of a stochastic layer in the phase space of particle motion, etc. Note that I do not include in this list the change of particle amplitude due to radiation quantum fluctuations making thus the consideration equally applicable to proton storage rings.

2.1.3 We shall use in forthcoming considerations an assumption that both beams are identical. This assumption is not mandatory for the derivations but is justified by experimental conditions and makes all formulae more straightforward.

2.1.4 Also everywhere where it is appropriate I will simplify the calculations using Gaussian distribution, linear force, etc. Although more exact calculations can be fulfilled sometimes they do not seem to be necessary due to oversimplifying assumptions made above already.

## 2.2 Beam blowup according to diffusion theory

At each interaction the vertical coordinate  $y$  and the angle in vertical plane  $y'$  are changed as follows:

$$\Delta y = 0 \quad (12)$$

$$\Delta y' = 2\pi r_y \frac{\sigma_0}{\beta_y} K_b \phi_b(u) \quad (13)$$

where  $b = (\sigma_y/\sigma_x) / \sqrt{1 - (\sigma_y/\sigma_x)^2}$ ,  $u = y/\sigma_0$

and  $K_b \phi_b$  is a function describing the electromagnetic force of the opposite bunch. For Gaussian distribution<sup>7</sup>

$$K_b = \sqrt{\frac{\sqrt{1+b^2}+b}{\sqrt{1+b^2}-b}} \quad (14)$$

$$\phi_b(u) = u \int_0^1 \frac{dv}{\sqrt{u+b^2}} e^{-vu^2} \quad (15)$$

According to the main assumption a certain part of the motion due to the interaction (13) can be described as stochastic and hence can be considered as an additional source of diffusion (in addition to all other sources which do not depend on the beam-beam force).

We know that at least the linear part of the force cannot cause the stochasticity. It can be considered as an additional focusing force and hence should be included in the regular part of particle motion. Probably the same is true also for some nonlinear parts of the force.

That is why for the purpose of calculating beam blowup as a consequence of a diffusion-like process we should consider not all the force  $\phi_b(u)$ , but only some nonlinear part of it  $\tilde{\phi}_b(u)$ . The way to get  $\tilde{\phi}_b$  out of  $\phi_b$  is not clear and should be considered here only as a way to introduce in the theory a phenomenological fitting parameter. It can be done in different manners:

$$\tilde{\phi}_b(u) = \begin{cases} \phi_b(u) - (1-h)\phi_b\left(\frac{u}{1-h}\right) & , \quad (\text{S. Kheifets}^7) \\ h\phi_b'(u) & , \quad (\text{A. Ruggiero}^{11}) \end{cases} \quad (16)$$

One can find still other possibilities. For a small value of  $h$  both procedures give essentially the same result.

It is reasonable to assume that for particles which behave erratically there is a complete mixing of phases within the bunch and in the long run each particle can be expected to acquire any value of coordinate  $y$ . In this case the beam blowup can be found by averaging the value  $(\Delta y')^2$  over the distribution function

$$\sigma_y^2 = \sigma_0^2 \left( 1 + \eta \langle K_b^2 \phi_b^2 \rangle \right) \quad (17)$$

where the brackets  $\langle \rangle$  mean averaging over the distribution function.

In expression (17)

$$\eta = 2B f \tau (2\pi \xi_y)^2, \quad (18)$$

where  $\tau$  is the vertical damping time (5).

For Gaussian distribution

$$\langle K_b^2 \phi_b^2 \rangle = \frac{K_b^2}{\sqrt{\pi} \sigma_y / \sigma_0} \int_{-\infty}^{\infty} \phi_b^2(u) e^{-\frac{\sigma_0^2 u^2}{\sigma_y^2}} du \quad (19)$$

Instead of doing actual calculations we substitute in the following

$$\phi_b(u) = h \phi'(0) = 2h(\sqrt{1+b^2} - b) \quad (20)$$

Then we get:

$$\sigma_y^2 = \sigma_0^2 + \frac{2\pi^2 e^2 \tau^2 B^2 h^2 \sigma_0^2 i^2}{f B E^2 \sigma_y^2 \sigma_x^2 (1 + \sigma_y/\sigma_x)^2} \quad (21)$$

First of all we see here exactly the same formula (10) that was postulated in the work.<sup>6</sup> Comparing (21) with (10), we find

$$h = \frac{\pi e \beta_y h \sigma_0}{E \sigma_x (1 + \sigma_y/\sigma_x)} \sqrt{\frac{2\tau}{f B}} \quad (22)$$

An expression similar to (21) can also be found in the paper<sup>11</sup> (see Eq. (39) of this work) which gives to parameter h the physical meaning of the probability of finding the particle in a stochastic layer.

Expression (21) was also derived by J. Recc<sup>12</sup> from the assumption

$$\sigma_y^2 = \sigma_0^2 + f \text{Br} \beta_y^2 \theta_{\text{rms}}^2$$

where  $\theta_{\text{rms}}$  is the effective r.m.s. scattering angle of a particle in the vertical plane.

### 2.3 Scaling laws

Expressions (21,22) contain only one unknown parameter h. Let us consider it as a phenomenological parameter which should be determined from experimental data. One way to do this is to use PETRA results<sup>6</sup> (11). It is easy to see that to satisfy  $E^{-4}$  decrease for the value a we need the following dependence of h on energy:

$$h \sim E^{-3/2} \quad (23)$$



Since we are interested now in maximum values of the luminosity and the current, we derive from (10) that asymptotically at large current  $i$  (for the case  $\sigma_y \ll \sigma_x$ , one can get results for the opposite limit in a similar way)  $\sigma_y^4 \approx a^2 i^2$  or

$$\sigma_y \sim \sqrt{i/E^2} \quad (24)$$

The maximum possible value of  $\sigma_y$  limited by particle losses and beam lifetime should be some constant which can be written as  $\sqrt{A_y \beta_y}$  where  $A_y$  is an effective vertical acceptance of the storage ring. From formula (4) for Gaussian distribution we would find that  $c_v$  is constant with the logarithmic accuracy. Let us see now what consequences follow from these assumptions.

### 2.3.1 Dependence on energy

Consider first the situation where the limitation arises from the beam lifetime. Assuming  $\sigma_y = \text{const}$  in expression (24) we immediately get

$$i_{\text{max}} \sim E^4 \quad (25)$$

With the help of this expression we also get the following scaling laws (note that for the electron storage ring  $\sigma_x \sim E$ ):

$$\mathcal{L}_{\text{max}} \sim E^7 \quad (26)$$

$$f_{y\text{max}} \sim E^2 \quad (27)$$

$$E_{\text{xmax}} \sim E \quad (28)$$

$$\sigma_y/\sigma_x \sim 1/E \quad (29)$$

### 2.3.2 Dependence on current

Let us now turn to experiments in which beam lifetime limit has not been reached yet. At a given energy one gets from the same expressions

$$\sigma_{x,y} \sim i^{1/2} \quad (30)$$

$$\epsilon_{y\max} \sim i^{1/2} \quad (31)$$

$$\mathcal{L}_{\max} \sim i^{3/2} \quad (32)$$

### 2.3.3 Dependence on the number of bunches B

We should distinguish between the strong beam-weak beam and the strong beam-strong beam cases.

- a) For the strong beam-strong beam case an attempt to measure the dependence on B has been made on PETRA.<sup>13</sup> From expression (21) we have  $\sigma_y^4 = i^2/BE^8$  or

$$\sigma_y \sim i^{1/2}/E^2 B^{1/4} \quad (33)$$

Defining in accord with the work<sup>13</sup> the specific luminosity

$$\mathcal{L}_{sp} = \frac{\mathcal{L}}{R(1/B)^2} = \frac{\mathcal{L}B}{i^2} \quad (34)$$

we have

$$\mathcal{L}_{sp} \sim EB^{1/4}/\sqrt{i} \quad (35)$$

The dependence on B seems to be too weak to be in agreement with PETRA observations. The space charge parameters in this case should scale like:

$$\xi_y \sim \sqrt{1/B}^{3/4} \quad (36)$$

$$\xi_x \sim 1/E^3 B \quad (37)$$

Data on these dependencies are still not available.

- b) For the strong beam-weak beam case we have observations made on ADONE.<sup>5</sup> Expression (21) in this case should be rewritten for the blowup of the weak beam by an unperturbed strong beam:

$$\sigma_y^2 = \sigma_0^2 + \frac{2\pi^2 e^2 \tau B^2 h^2 I^2}{f BE^2 \sigma_x^2 (1 + \sigma_y/\sigma_x)^2} \quad (38)$$

Assuming the same dependence of h on E we have in this case

$$\sigma_y^2 \propto \frac{1^2}{BE^{10}} = \text{const} \quad (39)$$

The last equality corresponds to conditions of the ADONE experiment<sup>5</sup>. Hence

$$I_{\max} \sim E^5 \sqrt{B} \quad (40)$$

$$\mathcal{L}_{\max} \sim E^9 B \quad (41)$$

$$\xi_{y\max} \sim E^3 / \sqrt{B} \quad (42)$$

$$\xi_{x\max} \sim E^2 / \sqrt{B} \quad (43)$$

The scaling (40) seems to be in quite good agreement with the experimental data<sup>5</sup> on the strong beam-weak beam

interaction at ADONE both on E and on B. On the other hand,  $\xi_y$  and  $\xi_x$  were maintained equal. That makes the comparison of the energy dependence meaningless. The dependence on B is not contradictory to the experiment.

### 3. Summary of the experiment and theory comparison

Tables 4-6 present the summary of the theoretical and experimental values for different parameters relevant for the beam-beam interaction. Keeping in mind the number of assumptions and the approximations made the agreement seem to be astonishingly good.

### 4. Some speculations on a $p\bar{p}$ storage ring

There are two main dissimilarities between electron and proton storage rings relevant to our consideration. The first one is the absence of radiation damping of particle oscillations in the latter ring. Consequently the damping time constant  $\tau$  should be substituted by real time  $t$  in the expression for the beam blowup.

The second one is the energy dependence of the beam emittance. In a proton machine both  $\sigma_x$  and  $\sigma_y$  are proportional to  $1/\sqrt{E}$ .

Hence for a  $p\bar{p}$  storage ring we should expect the following relations

$$\mathcal{Q} \sim \frac{1^2 \sqrt{E}}{\sigma_y} \quad (44)$$

$$\xi_y \sim 1/\sqrt{E} \sigma_y \quad (45)$$

$$\xi_x \sim 1 \quad (46)$$

$$a^2 \sim h^2 \tau / E^2 \quad (47)$$

For the case when the blowup is strong enough to influence the lifetime

$$\sigma_y^4 \sim \frac{h^2 i^2 t}{E^2 B} \sim \text{const} \quad (48)$$

If the dependence of  $h$  on  $E$  is the same as for an electron storage ring

$$i_{\text{max}} \sim E^{3/2} B^{1/2} / t^{1/2} \quad (49)$$

$$\varphi_{\text{max}} \sim E^{11/2} B / t \quad (50)$$

$$\xi_{y\text{max}} \sim E^2 \sqrt{B} / \sqrt{t} \quad (51)$$

$$\xi_{x\text{max}} \sim E^{5/2} \sqrt{B} / t \quad (52)$$

The quadratic dependence of  $\xi_y$  on energy differs from the 3/2 law which is obtained by L. Teng<sup>15</sup> from fitting the electron ring data.

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Table 1

Dependence of SPEAR parameters on the particle energy  $E$  (in GeV). The fit is done<sup>4</sup> by a function  $f = kE^q$ .

$f$	$k$	$q$	Comment
$\mathcal{L}_{\max}$	0.033	6.6	in $10^{30} \text{ cm}^{-2} \text{ sec}^{-1}$
$i_{\max}$	1.2	3.6	in ma
$\sigma_y/\sigma_x$	0.5	-1.0	--
$\epsilon_x$	0.022	0.87	--
$\epsilon_y$	0.011	2.3	--

Table 2

Dependence of SPEAR parameters on the beam current  $i$  (in ma). The fit is done by a function  $f = ki^q$ .

$f$	E GeV	k	q	Comment
$\mathcal{L}_{\max}$ ( $10^{30} \text{ cm}^{-2} \text{ sec}^{-1}$ )	1.5	0.030	1.95	} high } energy physics } runs
	2.5	0.046	1.55	
	3.7	0.054	1.45	
	1.95	0.052	1.41	} machine } physics runs
	1.95		1.45	
$a_y$			0.59	
$a_x$			0	
$\epsilon_y$	2.4		0.33	

Table 3

Dependence of ADONE parameters on the particle energy E (in GeV). The fit is done<sup>5</sup> by a function  $f = kE^q$ .

f	k	q	Comment:
$\varphi_{\max}$	0.64	7	$\text{in } 10^{30} \text{ cm}^{-2} \text{ sec}^{-1}$
$\xi_x = \xi_y$	0.068	1.57	--
$i_{\max}$ (in mA)	105	4.34	3 bunches } strong
	42.4	4.12	1 bunch } weak beam

Table 4

The power  $q$  in the power law

$$f(E) \sim E^q$$

Parameter $f$	Experiment			Theory	Comment
	SPEAR	ADONE	PETRA		
$h$				-3/2	(23)
$\mathcal{Q}_{\max}$	6.6	7		7	(26)
$i_{\max}$	3.6	4.5		4	strong - strong (25)
$i_{\max}$		4.12; 4.34		5	weak - strong (40)
$\xi_y$	2.3	1.5		2	(27)
$\xi_x$	0.9			1	(28)
$\sigma_y/\sigma_x$	-1			-1	(29)
$n$			-4	-4	(11)

Table 5

The power  $q$  in the power law  $\varepsilon(l) \sim l^q$ 

Parameter $f$	Experiment			Theory	Comment
	SP:AR	ADONE	PETR:		
$\mathcal{L}_{\max}$	1.4			1.5	(32)
$\mathcal{L}_{\text{spmax}}$			-0.5	-0.5	(35)
$\sigma_x \sigma_y$	0.6			0.5	(30)
$\xi_y$	0.4			0.5	(31)

Table 6

The power  $q$  in the power law  $f(B) \sim B^q$ .

Parameter $f$	Experiment			Theory	Comment
	SPEAR	ADONE	PETRA		
$\rho_{\max}$				-0.25	(35) { strong beam strong beam
$\rho_{\text{spmax}}$				0.25	
$i_{\max}$		0.8		0.5	(40) { strong beam weak beam
$\xi_{y\max}$		-0.8		-0.5	