BNL-28198

CONF-800763--1

MASTER

Contribution to the "Baryon 80" Conference

Toronto, July 14-16, 1980

The bag model, the hyperspherical formalism and the heavy baryons

J. M. Richard

Theory Division, CERN, CH 1211 Geneve 23

and Physics Department, Brookhaven National Laboratory, Upton, N.Y. 11973

The submitted manuscript has been authored under contract DE-AC02-76CH00016 with the U.S. Department of Energy. Accordingly, the U.S. Government retains a nonexclusive, royalty-free license to publish or reproduce the published form of this contribution, or allow others to do so, for U.S. Government purposes.

DISCLAIMER

DISCLAIMER

Contribution to the "Baryon 80" Conference

Toronto, July 14-16, 1980

The bag model, the hyperspherical formalism and the heavy baryons

J. M. Richard

Theory Division, CERN, CH 1211 Geneve 23

and Physics Department, Brookhaven National Laboratory, Upton, N.Y. 11973

The bag model, the hyperspherical formalism and the heavy baryons

J. M. Richard
Theory Division, CERN, Geneva 23
and Physics Department, B.N.L., Upton, N.Y. 11973

The spectroscopy of heavy baryons has been studied recently at CERN by P. Hasenfratz et al.[1]. Referring to their paper for more details, we restrict our attention here to discuss a few points concerning some questions raised during this conference.

By "heavy baryons", we mean here triple flavored objects like ccc, cbb, etc... i.e. states very difficult to detect experimentally. Theoretically, however, they are of great interest. A non-relativistic picture seems justified and, as in quarkonium, the spectrum hopefully reflects directly the dynamics. For light quarks on the other hand, there are additional uncertainties due to relativistic effects.

We use the framework of the bag model, basically the same as the M.I.T. version[2] apart from surface tension effects[3]. For light hadrons, the M.I.T. group has used a "cavity" approximation, i.e. they considered fast quarks oscillating inside a fixed bag; they obtained a good overall fit[4] with a bag constant $B^{1/4}$=145 Mev and a QCD coupling constant α_s=2.2. The cavity approximation is questionable for light quarks (e.g. the problem of center of mass motion) and is definitely inadequate for very heavy quarks. The latter have low average velocity and during their motion, the bag readjusts itself almost instantaneously to an optimal shape. The relevant calculational scheme has two steps and is similar to the Born-Oppenheimer method used in molecular physics, with the following correspondence: quark=nuclei, electron cloud=gluonic field.

i) First, the positions of the quarks are frozen. The bag energy is computed (numerically) for arbitrary shapes and the minimum of this energy with respect to the bag shape defines the potential of the quarks.

ii) The second step consists in solving the Schrodinger equation with the above potential.

This program has been first carried out in the case of quarkonium[5]. Some important points are:

a) The potential in Coulombic at short distances, linear for large separations and rather well interpolated by the simple formula

$$V(r) \sim -\frac{4}{3}\frac{\alpha}{4} + \lambda r \qquad (1)$$

b) The linear term corresponds, asymptotically, to an elongated cylindrical shape for the bag. However, in the range of distances relevant for J/ψ or Υ wave-functions, the potential is already almost linear whereas the bag is still quite spherical. This phenomenon is called "precocious" linearity.

c) Apart from quark masses, the potential has two parameters, α_s and the tension λ which is related to the bag constant B. A fit of quarkonium data gives m_c=1.35 GeV, m_b=4.75 GeV, α_s=0.385 and $B^{1/4}$=235 MeV, the two latters in disagreement with the M.I.T. results.

d) Beyond the scope of this note are some problems directly related to QCD, such as the spin forces in $Q\bar{Q}$ or the gluonic excitations $Q\bar{Q}g$, both discussed in ref. 5). Another problem is how to properly incorporate asymptotic freedom in this scheme, since present calculations has been carried out only to lowest order in α_s. It may not be sufficient to replace everywhere the constant α_s in the lowest order calculations by a running $\alpha_s(\frac{Q^2}{\Lambda^2})$, as done in ref. 6).

Let us now return to baryons. The potential energy between three heavy quarks in a color singlet has the following properties [1,7]).

i) It has a Coulombic piece $V_c = -\frac{2}{3}\alpha_s \sum_{i<j} \frac{1}{r_{ij}}$, and, for large separation, it becomes a generalized linear potential[8]) $V_e = \lambda \mathrm{Min}(d_1 + d_2 + d_3)$, where d_i is the distance from an arbitrary point to the location of the i^{th} quark. The tension λ is the same as for $Q\bar{Q}$ in eq. (1).

ii) Correspondingly, the asymptotic shape of the bag is a γ-configuration, each arm having the same cross-section as the $Q\bar{Q}$ tube. With pure volume energy for the bag, a triangular Δ-shape would be degenerate with the γ. If a surface tension is also incorporated, the γ shape is clearly favored.

iii) We have again precocious linearity. The linear regime of the potential starts already when the bag is still almost spherical.

We want to insist on the fact that the long range interaction consists of a genuine 3-body force. This is obvious if one has in mind the γ-shape configuration. For the equivalent Δ shape, the reason is that the tension along each leg depends upon the location of the opposite quark. However, the generalized linear potential is accidentally - I would say "unfortunately" - rather well approximated numerically by a sum of 2-body terms[8]) $V(r) \sim 0.54\lambda\,(r_{12} + r_{23} + r_{32})$. So the relation $V_{QQ} = V_{Q\bar{Q}}\,(\tilde{\lambda}_1\tilde{\lambda}_2)_{QQ}/(\tilde{\lambda}_1\tilde{\lambda}_2)_{Q\bar{Q}}$, which is too often advocated and is valid only for one-gluon-exchange, is fortuitously satisfied in an approximate way for the entire effective potential. It's validity in other cases is not at all guaranteed. For instance, in the 6-quark problem of interest for the short range nuclear interaction, one may expect 6-body forces to be non-negligible.

The potential energy of the three quarks being determined, one has to solve the 3-body Schrodinger equation. The method of the hyperspherical expansion turns out to be very well suited for this problem[1]). This formalism leads in general to an infinite set of coupled equations. Our QQQ potential has the nice property to be almost reduced to its hypercentral component V_o. Each quantum state is dominated by a single hyperspherical harmonic and is governed by the simple radial equation

$$u''(\xi) - \frac{(L+3/2)\,(L+5/2)}{\xi^2}\,u(\xi) + \mu(E-V_o)\mu(\xi) = 0 \qquad (2)$$

with L=0,1,2..being the "grand" orbital quantum number. The ground state of ccc, for instance, has mainly L=0. The first correction corresponds to a coupling to L=4. It changes the results by less than 1%. Remember for comparison than in atomic helium, the L=4 admixture takes around 6% of the norm and increases the binding by almost 10%[9]). The difference is that the 3Q problem is very symmetric, whereas the $d^{++}e^-e^-$ case involves very different masses and mixes repulsion and attraction. Note that the relevance of the hyperspherical formalism for heavy baryons has also been pointed out by Chanda et al[10], who use however empirical pairwise potentials.

For illustration, we reproduce here the spectra displayed in Ref. 1).

Table 1

Heavy baryon mass spectrum

Spin	Total ℓ parity	Grand orbital L	Number of nodes n	Masses of ccc	bbb	ccb	cbb
3/2	0^+	0	0	4.79	14.30	8.03	11.20
3/2	0^+	0	1	5.30	14.75	8.52	11.66
1/2	1^-	1	0	5.14	14.66	8.37	11.54
1/2	0^+, 1^+, 2^+	2	0	5.42	14.91	8.64	11.80

Note that our hypercentral potential has almost all the degeneracies of the harmonic oscillator except the equality of the (L,n) and (L+2, n-1) energies. Remember that with pure 2-body forces treated as perturbation around the harmonic oscillator solution, one cannot shift downward the 0^+ with (L,n)=(0,1) without splitting also the (2,0) states according to the rule[11]

$$M[L=2,\ \ell^{\pi}=2^+]-M[L=2,\ell^{\pi}=0^+] = M[L=2,\ell^{\pi}=0^+]-M[L=0,n=1,\ell^{\pi}=0^+]$$

So, measuring all those splittings could in principle provide a test for the existence of 3-body forces.

Much work remains to be done on the production and decay properties of those baryons, in view of future experimental studies. We strongly emphasize here that most of them should be rather narrow. For instance, the threshold for the Zweig allowed decay of the upsilon is experimentally

$$\Delta M_1=M(b\bar{q}) + M(bq) - M(b\bar{b}) \sim 1.1 \text{ GeV}$$

The corresponding quantity for triple beauty is

$$\Delta M_2=M(b\bar{q}) + M(bbq) - M(bbb)$$

The light quark q should have roughly the same binding energy in $\bar{b}q$ than in bbq, since in both cases it feels a small source of color $\bar{3}$. This means $\Delta M_2 \sim \Delta M_1$ can reasonably be expected, i.e. all bbb states in Table 1 and even some further excitations are very narrow. The heavy baryon spectrum is even more rich than the $Q\bar{Q}$ sector and its study in our potential model is as reliable as quarkonium physics.

I would like to thank N. Isgur and G. Karl for the organization of "Baryon 80", the Nuclear Theory group of B.N.L. for its hospitality after

the Conference and C. B. Dover for his useful comments.

References

1. P. Hasenfratz, R. Horgan, J. Kuti, J. M. Richard Cern-Th 2838 (1980) to appear in Physics Letters B
2. A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, V. F. Weisskopf Phys. Rev. D9 (1974) 3471
3. P. Hasenfratz, J. Kuti, A. S. Szalay, in Proc. 10th Rencontres de Moriond, (Meribel, France, 1975), ed. by J. Tran Thanh Van
   P. Hasenfratz, J. Kuti Phys. Rep. 40C (1978) 75
4. T. DeGrand, R. L. Jaffe, K. Johnson, J. Kiskis Phys. Rev. D12 (1975) 2060
5. P. Hasenfratz, R. Horgan, J. Kuti, J. M. Richard Cern-Th 2837 (1980) submitted to Physics Letters B
6. W.C. Haxton, L. Heller Los Alamos preprint (7 March 80). to appear in Phys. Rev. D.
7. A. T. Aerts, L. Heller Los Alamos preprint July 1980.
8. H. G. Dosch, V. F. Muller Nucl. Phys. B116 (1976) 470
9. J. L. Ballot, J. Navarro J. Phys. B 8 (1975) 172
10. F. S. Moraes, M. T. Cuelho, R. Chanda Lett. Nuovo Cim. 26 (1979) 466
    A. B. Guimares, M. T. Cuelho, R. Chanda Rio de Janeiro preprint 1980
11. N. Isgur, G. Karl Phys. Rev. D19 (1979) 2653

BNL-28198

Contribution to the "Baryon 80" Conference

Toronto, July 14-16, 1980

The bag model, the hyperspherical formalism and the heavy baryons

J. M. Richard

Theory Division, CERN, CH 1211 Geneve 23

and Physics Department, Brookhaven National Laboratory, Upton, N.Y. 11973

The submitted manuscript has been authored under contract DE-AC02-76CH00016 with the U.S. Department of Energy. Accordingly, the U.S. Government retains a nonexclusive, royalty-free license to publish or reproduce the published form of this contribution, or allow others to do so, for U.S. Government purposes.

Contribution to the "Baryon 80" Conference

Toronto, July 14-16, 1980

The bag model, the hyperspherical formalism and the heavy baryons

J. M. Richard

Theory Division, CERN, CH 1211 Geneve 23

and Physics Department, Brookhaven National Laboratory, Upton, N.Y. 11973

The bag model, the hyperspherical formalism and the heavy baryons

J. M. Richard
Theory Division, CERN, Geneva 23
and Physics Department, B.N.L., Upton, N.Y. 11973

The spectroscopy of heavy baryons has been studied recently at CERN by P. Hasenfratz et al.[1]. Referring to their paper for more details, we restrict our attention here to discuss a few points concerning some questions raised during this conference.

By "heavy baryons", we mean here triple flavored objects like ccc, cbb, etc... i.e. states very difficult to detect experimentally. Theoretically, however, they are of great interest. A non-relativistic picture seems justified and, as in quarkonium, the spectrum hopefully reflects directly the dynamics. For light quarks on the other hand, there are additional uncertainties due to relativistic effects.

We use the framework of the bag model, basically the same as the M.I.T. version[2] apart from surface tension effects[3]. For light hadrons, the M.I.T. group has used a "cavity" approximation, i.e. they considered fast quarks oscillating inside a fixed bag; they obtained a good overall fit[4] with a bag constant $B^{1/4}$=145 Mev and a QCD coupling constant α_s=2.2. The cavity approximation is questionable for light quarks (e.g. the problem of center of mass motion) and is definitely inadequate for very heavy quarks. The latter have low average velocity and during their motion, the bag readjusts itself almost instantaneously to an optimal shape. The relevant calculational scheme has two steps and is similar to the Born-Oppenheimer method used in molecular physics, with the following correspondence: quark=nuclei, electron cloud=gluonic field.

i) First, the positions of the quarks are frozen. The bag energy is computed (numerically) for arbitrary shapes and the minimum of this energy with respect to the bag shape defines the potential of the quarks.

ii) The second step consists in solving the Schrodinger equation with the above potential.

This program has been first carried out in the case of quarkonium[5]. Some important points are:

a) The potential in Coulombic at short distances, linear for large separations and rather well interpolated by the simple formula

$$V(r) \sim -\frac{4}{3}\frac{\alpha}{4} + \lambda r \qquad (1)$$

b) The linear term corresponds, asymptotically, to an elongated cylindrical shape for the bag. However, in the range of distances relevant for J/ψ or Υ wave-functions, the potential is already almost linear whereas the bag is still quite spherical. This phenomenon is called "precocious" linearity.

c) Apart from quark masses, the potential has two parameters, α_s and the tension λ which is related to the bag constant B. A fit of quarkonium data gives m_c=1.35 GeV, m_b=4.75 GeV, α_s=0.385 and $B^{1/4}$=235 MeV, the two latters in disagreement with the M.I.T. results.

d) Beyond the scope of this note are some problems directly related to QCD, such as the spin forces in $Q\bar{Q}$ or the gluonic excitations $Q\bar{Q}g$, both discussed in ref. 5). Another problem is how to properly incorporate asymptotic freedom in this scheme, since present calculations has been carried out only to lowest order in α_s. It may not be sufficient to replace everywhere the constant α_s in the lowest order calculations by a running $\alpha_s(\frac{Q^2}{\Lambda^2})$, as done in ref. 6).

Let us now return to baryons. The potential energy between three heavy quarks in a color singlet has the following properties [1,7].

i) It has a Coulombic piece $V_c = -\frac{2}{3}\alpha_s \sum_{i<j} \frac{1}{r_{ij}}$, and, for large separation, it becomes a generalized linear potential[8] $V_e = \lambda \mathrm{Min}(d_1 + d_2 + d_3)$, where d_i is the distance from an arbitrary point to the location of the i^{th} quark. The tension λ is the same as for $Q\bar{Q}$ in eq. (1).

ii) Correspondingly, the asymptotic shape of the bag is a γ-configuration, each arm having the same cross-section as the $Q\bar{Q}$ tube. With pure volume energy for the bag, a triangular Δ-shape would be degenerate with the γ. If a surface tension is also incorporated, the γ shape is clearly favored.

iii) We have again precocious linearity. The linear regime of the potential starts already when the bag is still almost spherical.

We want to insist on the fact that the long range interaction consists of a genuine 3-body force. This is obvious if one has in mind the γ-shape configuration. For the equivalent Δ shape, the reason is that the tension along each leg depends upon the location of the opposite quark. However, the generalized linear potential is accidentally - I would say "unfortunately" - rather well approximated numerically by a sum of 2-body terms[8] $V(r) \sim 0.54\lambda\,(r_{12} + r_{23} + r_{32})$. So the relation $V_{QQ} = V_{Q\bar{Q}}\,(\tilde{\lambda}_1\tilde{\lambda}_2)_{QQ}/(\tilde{\lambda}_1\tilde{\lambda}_2)_{Q\bar{Q}}$, which is too often advocated and is valid only for one-gluon-exchange, is fortuitously satisfied in an approximate way for the entire effective potential. It's validity in other cases is not at all guaranteed. For instance, in the 6-quark problem of interest for the short range nuclear interaction, one may expect 6-body forces to be non-negligible.

The potential energy of the three quarks being determined, one has to solve the 3-body Schrodinger equation. The method of the hyperspherical expansion turns out to be very well suited for this problem[1]. This formalism leads in general to an infinite set of coupled equations. Our QQQ potential has the nice property to be almost reduced to its hypercentral component V_o. Each quantum state is dominated by a single hyperspherical harmonic and is governed by the simple radial equation

$$u''(\xi) - \frac{(L+3/2)\,(L+5/2)}{\xi^2}\,u(\xi) + \mu(E-V_o)\mu(\xi) = 0 \qquad (2)$$

with L=0,1,2..being the "grand" orbital quantum number. The ground state of ccc, for instance, has mainly L=0. The first correction corresponds to a coupling to L=4. It changes the results by less than 1%. Remember for comparison than in atomic helium, the L=4 admixture takes around 6% of the norm and increases the binding by almost 10%[9]. The difference is that the 3Q problem is very symmetric, whereas the $d^{++}e^-e^-$ case involves very different masses and mixes repulsion and attraction. Note that the relevance of the hyperspherical formalism for heavy baryons has also been pointed out by Chanda et al[10], who use however empirical pairwise potentials.

For illustration, we reproduce here the spectra displayed in Ref. 1).

Table 1

Heavy baryon mass spectrum

Spin	Total ℓ parity	Grand orbital L	Number of nodes n	Masses of ccc	bbb	ccb	cbb
3/2	0^+	0	0	4.79	14.30	8.03	11.20
3/2	0^+	0	1	5.30	14.75	8.52	11.66
1/2	1^-	1	0	5.14	14.66	8.37	11.54
1/2	0^+, 1^+, 2^+	2	0	5.42	14.91	8.64	11.80

Note that our hypercentral potential has almost all the degeneracies of the harmonic oscillator except the equality of the (L,n) and (L+2, n-1) energies. Remember that with pure 2-body forces treated as perturbation around the harmonic oscillator solution, one cannot shift downward the 0^+ with (L,n)=(0,1) without splitting also the (2,0) states according to the rule[11])

$$M[L=2,\ \ell^{\pi}=2^+]-M[L=2,\ell^{\pi}=0^+] = M[L=2,\ell^{\pi}=0^+]-M[L=0,n=1,\ell^{\pi}=0^+]$$

So, measuring all those splittings could in principle provide a test for the existence of 3-body forces.

Much work remains to be done on the production and decay properties of those baryons, in view of future experimental studies. We strongly emphasize here that most of them should be rather narrow. For instance, the threshold for the Zweig allowed decay of the upsilon is experimentally

$$\Delta M_1=M(b\bar{q}) + M(bq) - M(b\bar{b}) \sim 1.1\ \text{GeV}$$

The corresponding quantity for triple beauty is

$$\Delta M_2=M(b\bar{q}) + M(bbq) - M(bbb)$$

The light quark q should have roughly the same binding energy in $\bar{b}q$ than in bbq, since in both cases it feels a small source of color $\bar{3}$. This means $\Delta M_2 \sim \Delta M_1$ can reasonably be expected, i.e. all bbb states in Table 1 and even some further excitations are very narrow. The heavy baryon spectrum is even more rich than the $Q\bar{Q}$ sector and its study in our potential model is as reliable as quarkonium physics.

I would like to thank N. Isgur and G. Karl for the organization of "Baryon 80", the Nuclear Theory group of B.N.L. for its hospitality after the Conference and C. B. Dover for his useful comments.

References

1. P. Hasenfratz, R. Horgan, J. Kuti, J. M. Richard Cern-Th 2838 (1980) to appear in Physics Letters B
2. A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, V. F. Weisskopf Phys. Rev. D9 (1974) 3471
3. P. Hasenfratz, J. Kuti, A. S. Szalay, in Proc. 10th Rencontres de Moriond, (Meribel, France, 1975), ed. by J. Tran Thanh Van
   P. Hasenfratz, J. Kuti Phys. Rep. 40C (1978) 75
4. T. DeGrand, R. L. Jaffe, K. Johnson, J. Kiskis Phys. Rev. D12 (1975) 2060
5. P. Hasenfratz, R. Horgan, J. Kuti, J. M. Richard Cern-Th 2837 (1980) submitted to Physics Letters B
6. W.C. Haxton, L. Heller Los Alamos preprint (7 March 80). to appear in Phys. Rev. D.
7. A. T. Aerts, L. Heller Los Alamos preprint July 1980.
8. H. G. Dosch, V. F. Muller Nucl. Phys. B116 (1976) 470
9. J. L. Ballot, J. Navarro J. Phys. B 8 (1975) 172
10. F. S. Moraes, M. T. Cuelho, R. Chanda Lett. Nuovo Cim. 26 (1979) 466
    A. B. Guimares, M. T. Cuelho, R. Chanda Rio de Janeiro preprint 1980
11. N. Isgur, G. Karl Phys. Rev. D19 (1979) 2653