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OF PERMANENT MAGNET SODIUM FLOWMETERS

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Abstract

Permanent magnet flowmeters are used to measure flow in sodium cooled power plants both in the core and in plant piping. In this paper the cross-correlation technique and in particular the transfer function method is used to process the flow turbulence signals from electrodes spaced along the pipe in the vicinity of the magnetic field. Results from a loop flowmeter the EBR-II secondary flowmeter and a 400 mm diameter flowmeter will be presented.

Introduction

The permanent magnet sodium flowmeter is a rugged, reliable instrument for use in sodium cooled nuclear power plants, both in the core as well as in plant piping. This paper covers a method of in-place calibration that may be applied while the flowmeter is performing its normal function. The calibration of a permanent magnet flowmeter depends directly on its magnetic field strength which can drift with time and temperature.^{1,2} The cross-correlation technique involves determining the cross-correlation function of signal variations caused by flow turbulence from two sets of electrodes spaced along the flow channel of permanent magnet flowmeters. These electrode sets may include the ones normally used to obtain the flow signal. The cross-correlation technique should be especially valuable for in-core flowmeters that are relatively inaccessible or for large pipe flowmeters that are too large to be conveniently calibrated in flow loops.

Flow turbulence signals from a small 12 mm loop flowmeter through the 300 mm EBR-II secondary sodium flowmeter to the 400 mm flowmeter of the Sodium Pump Test Facility at LMEC will be analyzed.^{3,4,5} An impulse response method will be used to develop the cross-correlation functions. The peak of the cross-correlation function yields the transit time between electrode sets and consequently the flow since the spacing between electrodes is known. Therefore:

$$\text{Sodium Vel.} = \frac{\text{Electrode Separation (d)}}{\text{Transit time over distance (d)}} = \frac{d}{\tau} \quad (1)$$

and

$$\text{Vol. Flow} = \frac{\text{Vol. of pipe between electrodes}}{\text{Transit time over distance (d)}} = \frac{V}{\tau} \quad (2)$$

Theory

Consider a generalized measurement situation shown in Fig. 1, where, in general, there are two accessible observation points X and Y where a measurement can be made. The observation points X and Y are related through some linear transfer quantity, H, and Y is perturbed by some uncorrelated noise source. Usually the main interest lies in the nature of the transmission path and in particular what this path does to the signal. In some applications an excitation is applied and the interest is in the response and its strength. These two approaches are measurements of

Fig. 1 Generalized System Representation.

relationship and causality respectively and both are implied in the word correlation.

The initial approach to the problem was to find the cross-correlation of the two signals obtained. Mathematically, the cross-correlation of two signals $y(t)$ and $x(t)$ over a period of time, T , is given by:

$$R_{yx}(\tau) = \frac{1}{T} \int_0^T y(t)x(t+\tau)dt, \quad (3)$$

where $R_{yx}(\tau)$ is simply the averaged product of $y(t)$ delayed with respect to $x(t)$. When the value of $R_{yx}(\tau)$ is high for some delay τ , it can be said that x and y are similar, in some sense, at this delay value. The problem with this type of analysis lies in the interpretation of the given value of the cross-correlation function $R_{yx}(\tau)$. The use of the normalized cross-correlation function, for some particular cases, makes the interpretation easier. The normalized cross-correlation function is given by:

$$\rho_{yx}(\tau) = \frac{R_{yx}(\tau)}{\sqrt{R_{xx}(0) R_{yy}(0)}}; \quad -1 \leq \rho_{yx}(\tau) \leq 1, \quad (4)$$

where $R_{xx}(0)$ and $R_{yy}(0)$ are the auto-correlation functions for $x(t)$ and $y(t)$ for zero delay respectively. In the special case where the value of $\rho_{yx}(\tau)$ is equal to ± 1 , it can be said that the correlation is clear and we can interpret the delay as the true one, which indicates that the transmission path, as shown in Fig. 1 is lossless and there is no noise contamination. However, physical situations like the one under consideration do not exhibit the above ideal characteristics. Usually transmission path losses exist and noise contamination is present. These considerations will yield a normalized cross-correlation somewhere between zero and one. The measuring of such a result is difficult to interpret, because two situations can occur: either the transmission path has a gain less than unity or the contaminating noise has reduced the degree of causality between signals $x(t)$ and $y(t)$. This distinction between the two cases is not clear and therefore some other method of obtaining the cross-correlation function must be implemented.

Consider the two time varying signals $x(t)$ and $y(t)$ received at two points along the flow channel of a permanent magnet flowmeter Fig. 2. The portion of the channel between these two signals can be characterized as a system with input $x(t)$ and output $y(t)$.

Fig. 2 Experimental Setup.

Fig. 1. Knowing the input and output of this system we can completely characterize it, that is, we can find the relationship of the output to the input by obtaining the transfer function of the system. The transfer function is obtained by considering frequency domain techniques.^{6,7}

The Fourier Transforms of the input and output signals are given by:

$$F_x(\omega) = F\{x(t)\}, \quad (5)$$

$$F_y(\omega) = F\{y(t)\}.$$

Using Eqs. (5), the following power spectra can be obtained:

$$G_{xx}(\omega) = F_x(\omega) F_x^*(\omega), \quad (6)$$

$$G_{yy}(\omega) = F_y(\omega) F_y^*(\omega), \quad (7)$$

$$G_{yx}(\omega) = F_y(\omega) F_x^*(\omega), \quad (8)$$

where $G_{xx}(\omega)$, $G_{yy}(\omega)$, $G_{yx}(\omega)$ are the power spectra of the input, output and the cross-power spectra of input-output respectively, and where * indicates the conjugate of the quantity. The output is related to the input through the following relationship

$$Y(t) = h(t) \Delta x(t) + n(t) \quad (9)$$

where Δ indicates convolution, $h(t)$ is the impulse response of the system and $n(t)$ is the additive uncorrelated noise. In the frequency domain Eq. (9) is written as

$$F_y(\omega) = H(\omega)F_x(\omega) + F_n(\omega) \quad (10)$$

$H(\omega)$ is the Fourier Transform of $h(t)$ called the transfer function of the system. $F_n(\omega)$ is the Fourier Transform of the additive uncorrelated noise $n(t)$. Under the assumption that $F_n(\omega)$ is small or $G_{yx}(\omega)$ is smoothed enough to make the cross-spectrum of the input $x(t)$ and the noise $n(t)$ negligible, then we can write

$$G_{yx}(\omega) = H(\omega)G_{xx}(\omega), \quad (11)$$

or

$$H(\omega) = \frac{G_{yx}(\omega)}{G_{xx}(\omega)}. \quad (12)$$

Since $G_{yx}(\omega)$ can be obtained from the input-output signals $x(t)$ and $y(t)$, Eq. (12) yields the transfer function of the system. The inverse transform of Eq. (11) yields:

$$R_{yx}(\tau) = h(\tau) \Delta R_{xx}(\tau) \quad (13)$$

where $R_{xx}(\tau)$ is the auto-correlation function of $x(t)$ and $R_{yx}(\tau)$ is the cross-correlation function of $x(t)$ and $y(t)$. The above equation was obtained by making use of the fact that correlation and power spectra form a transform pair and that multiplication in the frequency domain implies convolution in the time domain. Equation (13) will yield the desired cross-correlation function.

Experiment

The experimental set-up at the sodium flowmeter is shown in Fig. 2. It consists of a permanent magnet flowmeter, four electrodes, differential amplifier for noise cancellation and signal amplitude enhancement, filters, a magnetic tape recorder and an oscilloscope to monitor the input and recorded signals. The operating temperatures for the various tests were in the range between 400°F and 800°F. The differential amplifier was used to amplify the time varying component of the flow signal. This signal is proportional to the flow, i.e., the signal level is higher when the flow is high and lower when the flow is low. In the output signal, a 60 Hz component, due mainly to peripheral instrumentation, is present. This component is eliminated by using a low pass filter with a cutoff set at 20 Hz. Both the input and output of the filter is monitored on the scope for examination purposes. The output of the filters is recorded on the tape recorder.

The permanent magnet establishes the magnetic field. The sodium flow acts as a conductor cutting the magnetic field lines creating an emf. This emf is the signal picked up by a set of electrodes which are attached to opposite sides of the flow pipe. The signal is composed of a dc component plus an ac component caused by flow turbulence. The output signal from the filter is random in nature as can be seen from typical output signals, Fig. 3, and has a peak to peak value (referred back to the electrodes) ranging from 1% of the dc component for small flowmeters to 10% for the largest.

Fig. 3 Actual Experimental Signal.

Data Processing

The data from the experiments were initially processed by using a hard-wired correlator. The operation of the hard-wired correlator is based on Eq. (3). The results gave all indications that a delay does exist, but the accuracy of the obtained delay was not good for the purposes of the investigation. The delay was within 10% of the actual for high flowrates, but for low flow rates was off by greater than 50% of the actual. The discrepancy can be attributed to the analog to digital (A/D) conversion of the data and the resolution of the displayed waveform.

Based on Eqs. (5,6,7,8,9,10,11, and 12) a program was written for the HP-5451A Fourier analyzer. A block diagram of this program is shown on Fig. 4. The first step of this process is the A/D conversion of the signals under consideration. Two of these signals are shown on Fig. 3. The similarities in the characteristics of the two signals are very obvious.

Fig. 4 Block Diagram Representation of the Transfer Function Method.

The second step calculates the Fourier Transforms of the given signals, while step three calculates the power spectrum of the given signals as well as their cross-power spectra. The results of step three are shown in Fig. 5. For better accuracy and stability, segments of the given signal are averaged. The purpose of this averaging is to extract the signal from the contaminating noise. If a great number of averages is taken, then the unwanted noise contribution will be virtually eliminated. Unfortunately, due to the memory limitations of the analyzer, the number of averages is limited and hence poor cancelation of noise occurs. At this point digital filtering may be applied if necessary to smooth the impulse response function.

Step four performs the evaluation of Eqs. (11 and 12) where the transfer function of the system is obtained, while step five takes the inverse Fourier Transform of the transfer function which yields the impulse response of the system. The last step convolves the auto-correlation function of the input signal with the impulse response of the system. The result is the cross-correlation function $R_{xy}(\tau)$. A typical auto-correlation function, impulse response and cross-correlation function are shown in Fig. 6. If the system represented a pure delay operation, the transfer function would have been a delta function

Fig. 5 Auto and Cross-Power Spectra of the Input Output Signals.

located at a time equal to the actual delay of the system. From Fig. 6 it is seen that the impulse response function is more complicated and this complication is the main reason that direct correlation fails and the transfer function technique is necessary.

Fig. 6 Representative Auto-correlation Impulse Response and Cross-correlation Functions.

Results and Conclusions

Using the transfer function method to process the data, transit delay times were obtained for the flow loop permanent magnet with 12 mm diameter pipe, the EBR-II secondary system flowmeter with 300 mm diameter pipe and the LMEC permanent magnet flowmeter with 400 mm diameter pipe. Data was processed for various flows and temperatures. The results are shown in Fig. 7. The solid lines in the Figure represent the expected time delay for the reference velocity, the dots are the time delays obtained by using the cross-correlation technique. It can be seen from the Figure that the technique is more accurate

for higher flow velocities. This observation is more evident with the 400 mm flowmeter. These factors must be investigated and the results must be related to the transit delay times.

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Fig. 7 Bulk Velocity vs Transit Delay Time.

In processing the data there is a trade-off between sample rate and record length for a 1024 point analyzer. To adequately account for the low frequencies in the power spectrum a long record length is needed. However, this reduces the time resolution. A higher capacity analyzer is needed so that resolution is not a limiting factor. Further study is needed on the relation between the flow profile in the pipe and the turbulent flow pattern. Also, the optimum electrode spacing has not been established.

Results to date have demonstrated that this technique yields transit delays that correlate with the actual flow within about 10 percent. A more complete study of the variables holds promise of developing a useful self-calibrating technique for permanent magnet sodium flowmeters.

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