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MASTER

ELECTRON-BEAM ENVELOPES AND MATCHING FOR A COMBINED WIGGLER AND ALTERNATING-GRADIENT QUADRUPOLE CHANNEL*

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ABSTRACT

This work studies the electron-beam envelopes and matching for a combined wiggler and alternating-gradient quadrupole field for a free-electron laser (FEL) that will be operated in the VUV or XUV wavelength region. The quadrupole field is assumed to vary continuously along the symmetry axis. The linearized equations of electron motion are solved analytically by using the two-scale perturbation method for a plane polarized wiggler. The electron-beam envelopes and the envelope equations, as well as the matching conditions in phase space, are obtained from the electron trajectories. A comparison with the numerical solution is presented.

1. Introduction

Recently, there has been a growing interest in the study of using FELs to generate coherent and tunable XUV radiation at high intensity for various basic physics and material studies [1]. It has been pointed out that in order to achieve an equal gain in the VUV or XUV regime, an FEL needs much higher density in the electron beam and a longer wiggler length than an FEL operated in the infrared or visible light region [2]. One is led then to consider using an external focusing channel along with the wiggler in the VUV or XUV FEL design.

In this paper, we will study the electron-beam envelopes and matching for combined wiggler and quadrupole fields. Our study will be limited to

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plane-polarized wigglers for FELs to be operated in the VUV or XUV ranges only. The quadrupole channel considered here is one in which the quadrupole field gradient alternates sign and is continuous along the symmetry axis. The wiggler's field is assumed to be coaxial with that of the quadrupole channel To make the analysis tractable, the self-field of the electron beam is ignored, the nonlinear components of the wiggler field are neglected, and the axial speed of electrons is assumed to be constant. The tedious details of solving the trajectory equations, omitted here, have been documented in internal reports by the authors [3,4,5].

2. Electron Trajectories

We choose a cartesian coordinate system so that the electrons are traveling in the z-direction and the wiggler field is in the y-direction. The z-axis is assumed to coincide with the symmetry axis of the coaxial wiggler and quadrupole fields. The origin of the coordinate system is chosen at the entrance of the wiggler/quadrupole channel. The variation of the y-component of the wiggler field along the z-direction can be written as $B_w \cos(2\pi z/\lambda_w)$, where B_w is the amplitude of the y-component of the magnetic field of the wiggler, λ_w is the wavelength of the wiggler, and we have neglected the fringe field at the entrance. The quadrupole field considered here can be represented as $B'y \sin(2\pi z/\lambda_Q)$ and $-B'x \sin(2\pi z/\lambda_Q)$, in the x- and y-directions, respectively, where B' is the field gradient and λ_Q is the quadrupole wavelength.

Neglecting the self-field of the beam and assuming the laser's field is absent, the linearized equations of motion for an electron are

$$\frac{d^2x}{dz^2} = k_w[a_w \cos(k_w z) - k_w x b \sin(K k_w z)] \quad , \text{ and} \tag{1}$$

$$\frac{d^2y}{dz^2} = k_w^2[-a_w \sin(k_w z)(\frac{dx}{dz}) + b \sin(K k_w z)]y \quad , \tag{2}$$

where $K = k_Q/k_w$, $a_w = qB_w/(m\gamma ck_w)$, $b = qB'/(m\gamma ck_w^2)$, m is the rest mass of an electron, γ is the ratio of an electron's total energy to the quantity mc^2 (c is the speed of light), q is the unit charge, $k_w = 2\pi/\lambda_w$ and $k_Q = 2\pi/\lambda_Q$. In obtaining Eqs. (1) and (2), we have neglected the dy/dt term in the x-motion and the approximations $\sinh(k_w y) \approx k_w y$, $\cosh(k_w y) \approx 1$, $v_z \approx c$, and $ct \approx v_z t = z$ have been used, where t is the time and v_z is the axial speed of the electron.

For a VUV or an XUV FEL to be focused by relatively long quadrupoles ($50\ \text{cm} < \lambda_Q < 100\ \text{cm}$), we need only to consider the region where $K << 1$, $a_w << K^{3/2}$ and $b < K^2$. In this range, λ_w and λ_Q are two rather distinct scales; therefore, good approximations to the solutions of Eqs. (1) and (2) can be found by using the two-scale perturbation method [5]. Retaining the first few lower order terms in the perturbation series, we have

$$x(z) = Ce^{U_x(z)} \sin[\phi + V_x(z)] + \left(\frac{a_w}{k_w \sin\psi}\right) e^{U_x(z)} \sin[\psi - V_x(z)] + W(z) \ , \quad (3)$$

$$\text{and } y(z) = De^{U_y(z)} \sin[\theta + V_y(z)] \ , \quad (4)$$

where C, ϕ, D and θ are constants dependent on the initial conditions,

$$U_x(z) = \left(\frac{b}{K^2}\right)\left[\sin(k_Q z) + \left(\frac{b}{8K^2}\right)\cos(2k_Q z)\right] + \dots \ , \quad (5)$$

$$V_x(z) = \frac{1}{\sqrt{2}}\left(\frac{b}{K^2}\right) k_Q z + \sqrt{2}\left(\frac{b}{K^2}\right)^2 \cos(k_Q z) + \dots \ , \quad (6)$$

$$U_y(z) = -\frac{a_w^2}{8}\cos(2k_w z) - \frac{b}{K^2}\left(1 + \frac{2a_w^2}{K^2} + \frac{7b^2}{4K^4}\right)\sin(k_Q z) + \frac{b^2}{8K^4}\cos(2k_Q z) + \dots \ , \quad (7)$$

$$V_y(z) = \sqrt{\frac{1}{2}(a_w^2 + \frac{b^2}{K^2})} \left\{ \left[1 + \left(-\frac{b^2}{a_w^2 K^2 + b^2} \right) \left(\frac{a_w^2}{K^2} + \frac{25b^2}{32K^4} \right) \right] k_w z - \frac{5b^2}{8K^5} \sin (2k_Q z) - \frac{2b}{K^3} \cos (k_Q z) \right\} + \ldots \quad , \tag{8}$$

$$W(z) = -(\frac{a_w}{k_w}) \cos (k_w z) + \frac{a_w b}{2k_w} \left[\frac{\sin (k_w - k_Q)z}{(1 - K)^2} - \frac{\sin (k_w + k_Q)z}{(1 + K)^2} \right] + \ldots \quad , \tag{9}$$

$$\text{and } \psi = \tan^{-1} \left(\frac{1 - K^2}{\sqrt{2}} \right) + \ldots \quad . \tag{10}$$

In Eq. (3), the general solution for the x-motion is contained in the first term and the particular solution is written as the second and third terms. The first and second terms describe the betatron oscillations induced by the quadrupole and the wiggler, respectively. The third term describes the fast oscillation excited by the wiggler. Note that the particular solution is independent of C and ϕ; therefore, the oscillations induced by the wiggler should be coherent motions for all electrons in the beam as we expected. Also, because the second term in Eq. (3) has the same form as the general solution, the quantity W(z) must satisfy Eq. (1).

The quantities $U_x(z)$ and $V_x(z)$ in the x-solution are related by

$$\frac{dV_x}{dz} = \frac{bk_w}{K\sqrt{2}} e^{-2U_x(z)} [1 - \frac{7}{32} (\frac{b}{K^2})^2 + \ldots] \quad , \tag{11}$$

$$\text{and } \frac{d^2U_x}{dz^2} + \left(\frac{dU_x}{dz}\right)^2 = \left(\frac{dV_x}{dz}\right)^2 - bk_w^2 \sin (k_Q z) \quad . \tag{12}$$

For the y-solution, the corresponding relations between $U_y(z)$ and $V_y(z)$ are

$$\frac{dV_y}{dz} = \sqrt{\frac{k_w^2}{2} (a_w^2 + \frac{b^2}{K^2})} e^{-2U_y(z)} \left[1 - \frac{7b^4}{32 K^4(a_w^2 K^2 + b^2)} \right] + \ldots \quad , \tag{13}$$

and

$$\frac{d^2U_y}{dz^2} + \left(\frac{dU_y}{dz}\right)^2 = \left(\frac{dV_y}{dz}\right)^2 - (a_w k_w)^2 \sin^2 (k_w z) + bk_w^2 \sin (k_Q z) \quad . \tag{14}$$

3. Beam Envelopes and Matching

We consider the x-envelope first. For convenience, we rewrite the x-solution as $x(z) = h \exp[U_x(z)] \sin [\phi' + V_x(z)] + W(z)$, where h and ϕ' are constants depending on the initial conditions. Eliminating the trigonometric functions in x(z) and dx(z)/dz, one obtains an invariant quantity similar to the one previously obtained by Courant and Snyder [6] as

$$\left\{(x - W)^2 + \frac{2K^2 e^{4U_x}}{(\Gamma_x bk_w)^2}\left[\frac{d}{dz}(x - W) - (x - W)\frac{dU_x}{dz}\right]^2\right\} \Big/ e^{2U_x} = h^2 \quad , \tag{15}$$

where $\Gamma_x = 1 - 7b^2/(32K^4) + \ldots \approx 1$. From this, we can infer that the beam envelope is

$$X(z) = \pm h_m e^{U_x(z)} + W(z) \quad , \tag{16}$$

where h_m is the maximal h among all the electrons. We note that because of the coherent wiggling of electrons, the x-envelope will not be symmetric with respect to the z-axis.

In the phase plane of x-dx/dz, Eq. (15) represents an ellipse with area $\pi\Gamma_x bk_w h^2/(\sqrt{2}K)$ and the center at [W(z), dW(z)/dz]. Assuming that the x-dx/dz phase plane occupied by the beam particles is uniformly populated, then the quantity h_m is related to the beam emittance ϵ_x by $h_m = [\sqrt{2}\epsilon_x K/(\Gamma_x bk_w)]^{1/2}$.

By the same process, the Courant-Snyder invariant in the y-motion is

$$\left\{y^2 + \frac{2K^2 e^{4U_y}}{k_w^2(a_w^2K^2 + b^2)\Gamma_y^2}\left[\frac{dy}{dz} - (\frac{dU_y}{dz})y\right]^2\right\} \Big/ e^{2U_y} = D^2 \quad , \tag{17}$$

where $\Gamma_y = 1 - 7b^2/[32K^4(a_w^2K^2 + b^2)] + \cdots \approx 1$, and the y-envelope Y(z) is

$$Y(z) = \pm D_m e^{U_y(z)} \quad . \tag{18}$$

In Eq. (18), the quantity D_m is related to the y-emittance ϵ_y by $D_m = [\sqrt{2}K\epsilon_y/(\Gamma_y k_w)]^{1/2}(a_w^2K^2 + b^2)^{-1/4}$.

In principle, the conditions to match an electron beam into this combined wiggler and quadrupole channel are given by the phase-plane ellipses [Eqs. (15) and (17)] evaluated at the entrance (z = 0) with h and D replaced by h_m and D_m, respectively. For practical reasons, one may also use the conditions of matching beam envelopes and slopes at z = 0, which can be obtained easily from Eqs. (16) and (18).

Using Eqs. (11)-(14), and the fact that W(z) satisfies Eq. (1), we can derive the following envelope equations:

$$\frac{d^2X}{dz^2} + bk_w^2 X \sin(k_Q z) - \frac{\epsilon_x^2}{(X - W)^3} = a_w k_w \cos(k_w z) \quad , \tag{19}$$

$$\text{and } \frac{d^2Y}{dz^2} + k_w^2[a_w^2 \sin^2(k_w z) - b \sin(k_Q z)]Y - \frac{\epsilon_y^2}{Y^3} = 0 \quad , \tag{20}$$

where X and Y stand for the beam envelopes in the x- and y-direction respectively.

4. Numerical Examples and Comparison with Numerical Solutions

The parameter values used in the numerical calculation are λ_w = 1.6 cm, λ_Q = 80 cm, B_w = 0.75 T, B' = 5 T/m, and γ = 400. From these values, we have $a_w \approx 2.8 \times 10^{-3}$, $b \approx 4.75 \times 10^{-5}$, and K = 0.02.

An example of a trajectory calculation is shown in fig. 1 for x-motion only, where the initial values of $x(0) = 2 \times 10^{-3}/k_w$ and $dx/dz|_0 = 0$ were used. For the relatively short axial distance shown in the figure, the betatron oscillation cannot be fully displayed, but the fast oscillation induced by the wiggler can be seen clearly. The result shown in fig. 1 was actually calculated from Eq. (3). However, when compared with the numerical solution of Eq. (1) by the Runge-Kutta method, we found no discernible difference in the plots. Therefore, fig. 1 also represents the numerical solution of Eq. (1). The y-motion, not shown here, also has excellent agreement between the numerical and analytical solutions.

Examples of matching an electron beam into the quadrupole/wiggler channel are shown in figs. 2-4 for $\epsilon_x = \epsilon_y = 1.3 \times 10^{-7}$ m•rad. Fig. 2 shows the matched phase-plane ellipses at z = 0 for x- and y-motions, where the results are computed from Eqs. (15) and (17) with $h = h_m$ and $D = D_m$. The matched x-envelope for a matched beam is shown in fig. 3, where the result is obtained from the numerical solution of Eq. (19) for the initial conditions [computed from Eq. (16) and the derivative of that at z = 0] of $X(0) = 0.1713/k_w$, and $dX/dz|_0 = 4.2523 \times 10^{-4}$. Note that the coherent motion of wiggling appears in the envelope. An example of the unmatched beam is shown in fig. 4, where the initial conditions $X(0) = 0.16352/k_w$, $dX/dz|_0 = 4.2523 \times 10^{-4}$ were used. The mismatch was introduced by decreasing the envelope in the previous example of matched case by 5% at z = 0. Note that for the same emittance, the unmatched envelope is larger than the matched one. All the qualitative results of the x-envelope except for the coherent wiggling, also appear in the y-envelope which is omitted here for brevity.

5. Conclusion

We have studied the electron motion, the beam envelopes and matching for a combined wiggler and alternating-gradient quadrupole channel for a VUV or an XUV FEL. The excellent agreement between the analytical and numerical solutions to the equations of motion suggests that the results obtained here can be useful for further study of the physical properties of a VUV or an XUV FEL with external alternating-gradient quadrupole focusing.

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[6] E. D. Courant and H. S. Snyder, "Theory of the Alternating Gradient Synchrotron," Ann. of Phys., 3, 1 (1958).

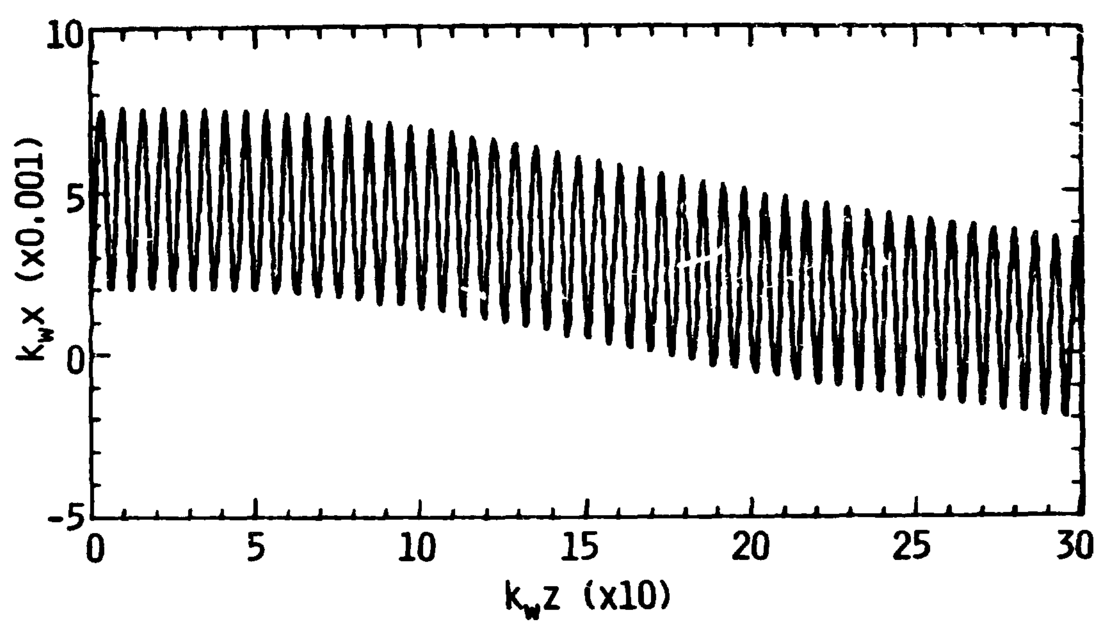
Figure Captions

Fig. 1. Analytical and numerical results of x-trajectory for $0 \leq z \leq 300/k_w$, $a_w = 2.8 \times 10^{-3}$, $b = 4.75 \times 10^{-5}$, $K = 0.02$, $x(0) = 2 \times 10^{-3}/k_w$, and $dx/dz|_0 = 0$.

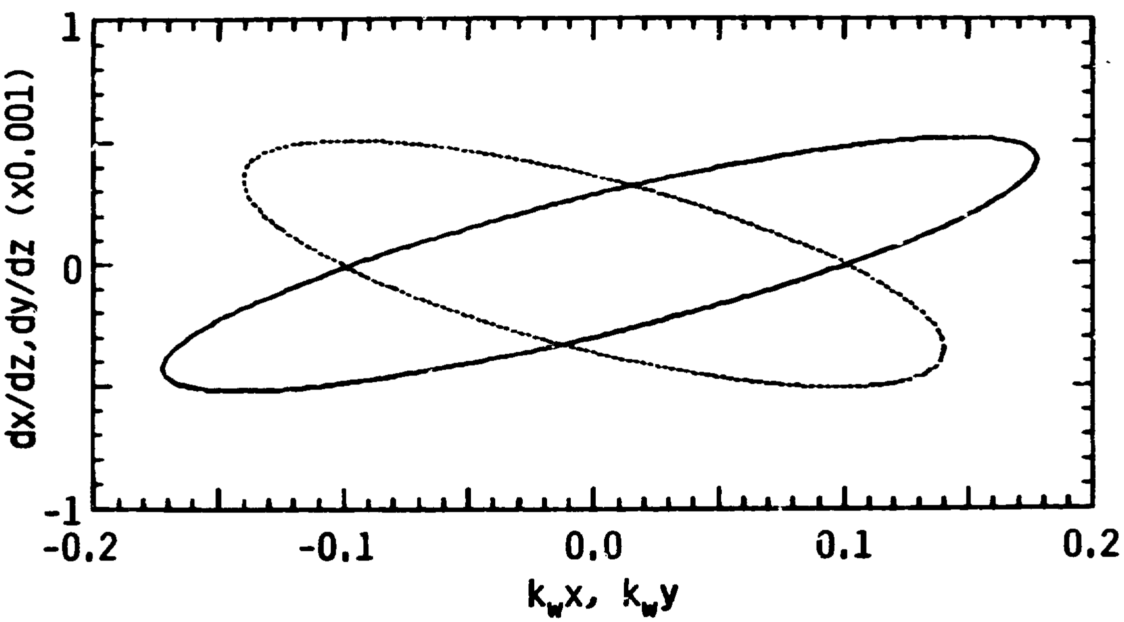
Fig. 2. Phase-space ellipses for a matched beam at z = 0 for the same parameter values of a_w, b, and K used in fig. 1 and $\epsilon_x = \epsilon_y = 1.3 \times 10^{-7}$ m•rad, where the solid line represents the x-ellipse and the dotted line represents the y-ellipse.

Fig. 3. The x-envelope for a matched beam for the same parameter values used in fig. 2. Note the coherent motion of wiggling is shown in the envelope.

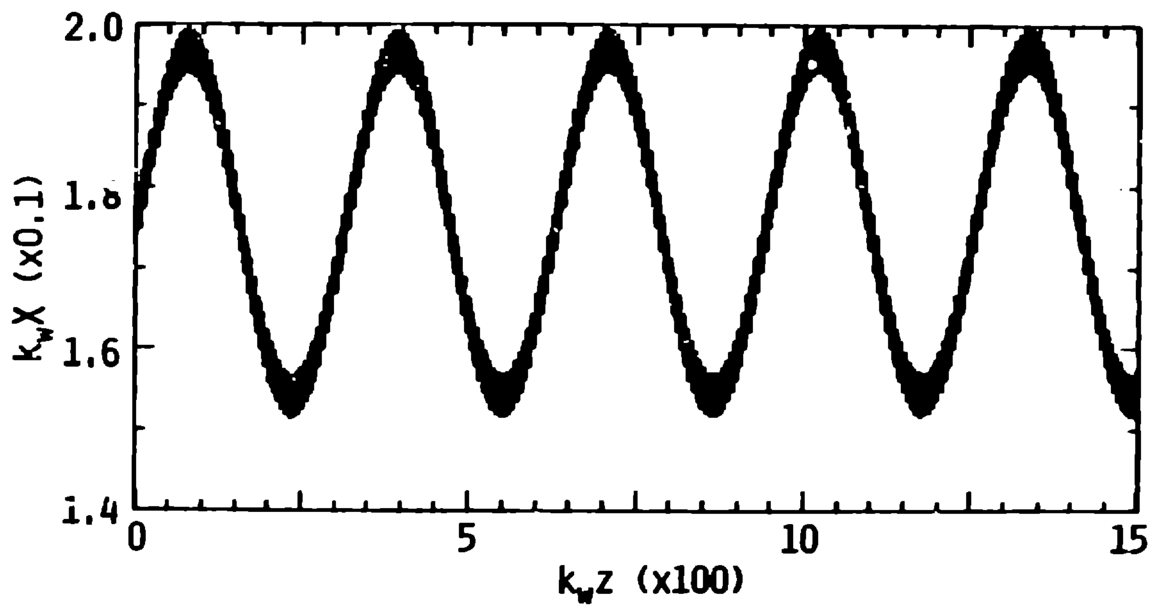
Fig. 4. The x-envelope for an unmatched beam for the same parameter values used in fig. 3. The mismatch is introduced by decreasing the beam envelope by 5% at z = 0.



10
5
0
-5
0
5
10
15
20
25
30
$k_w x$ (x0.001)
$k_w z$ (x10)



1
0
-1
dx/dz,dy/dz (x0.001)
-0.2
-0.1
0.0
0.1
0.2
k$_w$x, k$_w$y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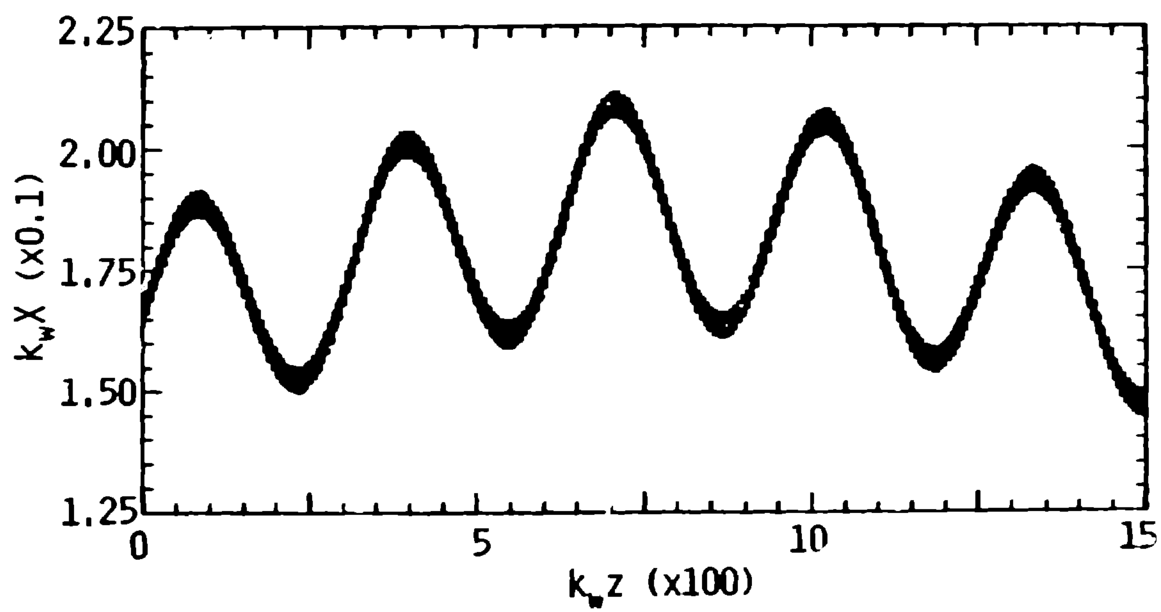
1.8
1.6
1.4
k_wX (x0.1)
0
5
10
15
k_wz (x100)



2.25
2.00
1.75
1.50
1.25
0
5
10
15
$k_w X$ (x0.1)
$k_w z$ (x100)