DOE/CS/30254-1

THE CONTINUING DEVELOPMENT
OF THE DEROB SYSTEM

Quarterly Report
for Period July 1, 1980-September 30, 1980

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October 1980

Prepared for

THE U. S. DEPARTMENT OF ENERGY
AGREEMENT NO. DE-AC02-80CS30254

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ABSTRACT

The last module of the DEROB System has been re-programmed and re-coded in an effort to reduce the computational time and cost associated with using DEROB. As of the writing of this report some preliminary tests have been carried out on the new program, and the tentative results show that the time of execution can be reduced anywhere from 7% to 40% of the time required by the previous version of DEROB. The variability in the improvement arises from the options specified by the user.

We are now in the process of carrying out additional tests to debug the program. When these are completed a copy of the new code will be sent to the technical monitors at SERI. This report: outlines the structure of the new program; derives the general form of the heat transfer equations used in this new version; and describes the properties of the convergence accelerator derived for this new version of DEROB.

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INTRODUCTION

The current cycle of funding started on July 1st, 1980 retroactive to June 1st, 1980.

The Principal Investigator and the Technical Monitor for the project, Mr. Ronald Judkoff of the Solar Energy Research Institute, agreed that the first task to be undertaken and completed was to find a way to reduce the execution time of the DEROB System.

The time allocated to this project by the principal investigator has been exclusively dedicated to this task.

This effort has resulted in a complete re-programming and re-coding of the DBROLEN module of DEROB. The resultant module which has the working name of HUEVADP is undergoing tests at the moment. Preliminary results show that the time of execution can be reduced anywhere from 50% to 10% of the DBROLEN execution time.

The variability of the reduction in the time of execution is due to the different levels of accuracy the user may select. As soon as the present set of tests are completed, a copy of the program will be forwarded to the technical monitor at SERI so that they may run their tests on it.

THE PROGRAMMING STRATEGY TO SOLVE THE HEAT BALANCE EQUATION IN DEROB

The HUEVADP module is the last module of the 4th version of DEROB. Its main task is to solve the algebraic equations that represent the numerical integration of the heat balance equations of each node in the simulation.

These equations can be cast in the matrix form

$$\underline{T} = \underline{Z} + \underline{A} \cdot \underline{T} \quad (1)$$

where the i th element of the vectors $\underline{T}$ and $\underline{Z}$ represent the resultant and driving temperatures of the i th node. The component A_{ij} of the matrix $\underline{A}$ represent the dimensionless factor coupling node j to i th node i . In general these coupling factors are functions of the temperatures of the nodes being coupled; thus the equations are in general non-linear.

Not all of the non-linear effects have the same relative importance. Their importance is determined by the mode of heat transfer they represent. Conduction coefficients are purely linear in the DEROB treatment; convection coefficients between fluid and surfaces exhibit a temperature dependence but its importance is tempered by the fluid velocity contribution to the coefficient; the Stephen-Boltzman radiation exchange coefficient is purely non-linear, but since they are functions of the temperature in the absolute scale they are linearizable; the advection

exchange due to buoyancy of fluids is non-linear and in many cases non-linearizable. Coefficients arising from step functions, i.e. on-off conditions for fans and auxiliary equipment, present a different type of problem and they will be addressed separately in a later report.

The most straightforward approach to solve equation (1) is by iteration where the elements of $\underline{A}$ are recalculated at each iteration step. Since the elements of $\underline{A}$ are always positive numbers less than 1 the iteration sequence converges to the solution.

In general the calculation simulates the energy behavior of a structure over a period of N days. For each day of the simulation the calculation is repeated once for each hour of the day, although different step sizes for the time integration are permitted by the programs. For each hour solution there is an iteration loop carried out to obtain the solution. For the purpose of this report we can define three major loops: the daily advance; the hourly advance; and the iteration sequence. The hourly loop is contained within the daily loop, and the iteration loop is contained within the hourly loop.

Thus, the main thrust of the revision of DEROB was to pull out of the iteration loop as many of the calculations that did not need to be repeated. The first calculation to be pulled out was the determination of the elements of the driving temperature $\underline{Z}$. The next step was to pull out of the iteration sequence the calculation of the correlation coefficients (matrix $\underline{A}$). The contributions to these coefficients due to conduction are calculated prior to entering the daily loop.

For the first level of accuracy (LEVEL=1) all the other contributions to the correlation coefficients are calculated during the first two hourly cycles and they are never repeated again. This level results in the greatest economy. The results, however, are likely to be valid for calculations where the non-linear terms are not important such as in the standard calculations of conventional structures.

For the second level of accuracy (LEVEL=2) the correlation coefficients are recalculated on an hourly basis, but they are not updated within the iteration loop. This level can be described as the linear approximation and yields excellent results whenever the system under simulation is not dominated by non-linear terms, especially those due to advection.

For the third level of accuracy (LEVEL=3) the correlation coefficients are again recalculated on an hourly basis, and those due to advection are updated within the iteration loop. This level will under most conditions yield good results.

The highest level of accuracy (LEVEL=4) goes beyond what level 3 does in that all the non-linear contributions to the correlation coefficients are updated within the iteration loop.

A logical sequence has been written into the program to select the LEVEL of execution. The user will be provided with the option to override the program in the selection of the accuracy level.

In addition to the pre-programming of the solution, a convergence accelerator has been programmed. A more detailed description of the accelerator is given in the last section of this report.

Preliminary tests of the program show that substantial savings can be derived from this new approach. By way of an example consider the case on which many of the tests have been carried out: a two-volume structure, one of them a rock-bin advectively connected to a living area. The simulation is carried out with 48 nodes. A LEVEL=3 calculation requires 39% of the time required by DEROB3 while a LEVEL=1 reduces the calculation time to 7% of DEROB3 time.

THE GENERAL FORM OF THE HEAT TRANSFER EQUATION

All the heat transfer equations used in DEROB are special cases of the general equation

$$C_i \frac{dT_i}{dt} = Q_i + \sum_{j=1}^N G(T_i, T_j) (T_j - T_i) \quad (1 \leq i \leq V) \quad (1)$$

where

i is the node index

Q_i is the external heat source applied to node i

T_i is the temperature of node i

$G(T_i, T_j) \equiv G_{ij}$ is the coupling coefficient between nodes i and j . In general, G_{ij} is a function of the temperatures of the nodes i and j . Consequently, equation 1 is non-linear in the temperatures.

C_i is the heat capacitance of node i

$\frac{d}{dt}$ is the time derivative

V is the number of unknown nodes

N is the total number of nodes, both known and unknown

Equation 1 can be recast into the form

$$\frac{dT_i}{dt} + v_i T_i = v_i q_i + v_i \sum_{j=1}^N g_{ij} T_j \quad (1 \leq i \leq V) \quad (2)$$

where $v_i = \frac{\phi_i}{C_i}$ relaxation rate

$g_{ij} = G_{ij}/\phi_i$ dimensionless coupling coefficient

$q_i = Q_i/\phi_i$ temperature representation of the heat source

$$\phi_i = \sum_{j=1}^N G_{ij}$$

If we number the nodes sequentially so that the unknown nodes come first in the sequence, and the known (air temp., sky temp., ground temp., etc.) nodes are the last in the sequence, then equation (2) can be cast in the form

$$\frac{dT_i}{dt} + v_i T_i = v_i H_i S_i + v_i \sum_{j=1}^V g_{ij} T_j \quad (3)$$

$S_i = [Q_i + \sum_{j=V+1}^N g_{ij} T_j] / H_i$ is the effective driving temperature, (e.g. sol-air temperature for external surfaces)

$$H_i = \sum_{j=V+1}^N g_{ij}$$

In the case where the i th node is massless, equation 3 becomes

$$T_i = H_i S_i + \sum_{j=1}^V g_{ij} T_j \quad (4)$$

Note that the coefficients H_i and g_{ij} are positive, less than unity, and dimensionless and they satisfy the relation

$$1 = H_i + \sum_{j=1}^V g_{ij} \quad (5)$$

Temporal Integration

For economy in writing, let the right-hand side of equation (3) be represented by a single symbol

$$F_i = H_i S_i + \sum_{j=1}^V g_{ij} T_j \quad (6)$$

Therefore, equation 3 becomes

$$\frac{dT_i}{dt} + v_i T_i = v_i F_i \quad (7)$$

Integrate equation 7 from time $t=t$ to time $t \rightarrow t+\Delta t$ assuming that v_i stays constant over this time interval to obtain

$$T_i^1 = T_i x_i + (F_i^1 - F_i x_i) - x_i \int_t^{(t+\Delta t)} \left(\frac{dF_i}{dt}\right) e^{+v_i t} dt \quad (8)$$

$x_i \equiv \exp(-v_i \Delta t)$

Prime quantities are evaluated at time $t+\Delta t$, and the unprimed at $t=t$.

Assuming that the time derivative of the driving function F can be treated as a constant, the result of the time integration can be cast in the form

$$T_i^1 = T_i x_i + h_i F_i + f_i F_i^1 \quad (9)$$

where

$$f_i = 1 - (1 - x_i) / (v_i \Delta t)$$

$$h_i = (1 - x_i) / (v_i \Delta t) - x_i$$

note:

$$h_i + f_i + x_i = 1 \quad (10)$$

in the massless limit $v_i \Delta t \rightarrow \infty$ $x_i \rightarrow 0$

$$h_i \rightarrow 0$$

$$f_i \rightarrow 1$$

and

$$T_i^1 \rightarrow F_i^1$$

in the infinite mass limit, $v_i \Delta t \rightarrow 0$ $x_i \rightarrow 1$

$$h_i \rightarrow 0$$

$$f_i \rightarrow 0$$

$$\text{and } T_i^{-1} \rightarrow F_i$$

Use equation 5 to write out explicitly the value of F_i^{-1} in equation 9 yields

$$T_i^{-1} = A_i Z_i^{-1} + \sum_{j=1}^V b_{ij} T_j \quad (11)$$

where

$$b_{ij} = f_i g_{ij}$$

$$A_i = x_i + h_i + f_i H_i^{-1}$$

$$Z_i = [T_i x_i + h_i F_i + f_i H_i^{-1} S_i^{-1}] / A_i$$

effective

source temperature.

Note, of course, $A_i + \sum_{j=1}^V b_{ij} = 1$

Equations 11 and 4 are formally the same. In the massless limit

$$A_i \rightarrow H_i' ; \quad Z_i \rightarrow S_i'$$

$$b_{ij} \rightarrow g_{ij}'$$

THE LINEAR APPROXIMATION TO THE SOLUTION

The matrix form of the equations is

$$\underline{T} = \underline{a} \cdot \underline{z} + \underline{b} \cdot \underline{T} \quad (12)$$

where $\underline{a}$ is diagonal in the representation of equation 11.

Assuming that $\underline{a}$ and $\underline{b}$ are independent of $\underline{T}$, the solution to equation 12 is

$$\underline{T} = (\underline{I} - \underline{b})^{-1} \cdot \underline{a} \cdot \underline{z} \quad (13)$$

Iteratively, the solution to equation 12 is obtained by making an initial guess, $\underline{T}^{(0)}$, and applying it to equation 12 to obtain the result of the 1st iteration

$$\underline{T}^{(1)} = \underline{a} \cdot \underline{z} + \underline{b} \cdot \underline{T}^{(0)} \quad (14)$$

The second iteration is obtained by applying equation 12 on $\underline{T}^{(1)}$

$$\begin{aligned} \underline{T}^{(2)} &= \underline{a} \cdot \underline{z} + \underline{b} \cdot \underline{T}^{(1)} \\ &= (\underline{I} + \underline{b}) \cdot \underline{a} \cdot \underline{z} + \underline{b}^2 \cdot \underline{T}^{(0)} \end{aligned} \quad (15)$$

$$(\underline{b}^2 = \underline{b} \cdot \underline{b})$$

Repeating the process m times we obtain

$$\underline{T}^{(m)} = \left(\sum_{i=1}^{m-1} \underline{b}^i \right) \cdot \underline{a} \cdot \underline{z} + \underline{b}^m \cdot \underline{T}^{(0)} \quad (16)$$

$$\underline{b}^i = \underbrace{\underline{b} \cdot \underline{b} \cdot \dots \cdot \underline{b}}_{i \text{ times}}$$

The series $\sum_{i=1}^{m-1} \underline{b}^i = (\underline{I} - \underline{b}^m) \cdot (\underline{I} - \underline{b})^{-1}$

Therefore, after m iterations (equation 16)

$$\underline{T}^{(m)} = \underline{T} + \underline{b}^m \cdot (\underline{T}^{(0)} - \underline{T}) \quad (17)$$

where we used equation 13

Using equation 17 to express the results of two consecutive iterations we obtain

$$\begin{aligned}\underline{T}^{(m)} &= \underline{T} + \underline{b}^m \cdot (\underline{T}^{(0)} - \underline{T}) \\ \underline{T}^{(m-1)} &= \underline{T} + \underline{b}^{m-1} \cdot (\underline{T}^{(0)} - \underline{T})\end{aligned}\tag{18}$$

thus, they are related

$$\underline{T}^{(m)} - \underline{T} = \underline{b} \cdot (\underline{T}^{(m-1)} - \underline{T})\tag{19}$$

The mth and (m+1)th iteration results are similarly related

$$\underline{T}^{(m+1)} - \underline{T} = \underline{b} \cdot (\underline{T}^{(m)} - \underline{T})\tag{20}$$

Thus the results of successive iterations are simply related

$$\underline{T}^{(m+1)} - \underline{T}^{(m)} = \underline{b} \cdot (\underline{T}^{(m)} - \underline{T}^{(m-1)})\tag{21}$$

Defining the vectors

$$\begin{aligned}\underline{\Delta}^{(m)} &= \underline{T}^{(m)} - \underline{T}^{(m-1)} \\ \underline{\Delta}^{(m+1)} &= \underline{T}^{(m+1)} - \underline{T}^{(m)}\end{aligned}\tag{22}$$

equation 21 becomes

$$\underline{\Delta}^{(m+1)} = \underline{b} \cdot \underline{\Delta}^{(m)}\tag{23}$$

Now convert the vector $\underline{\Delta}^{(m+1)}$ into a diagonal matrix

$$\begin{aligned}\underline{0} \cdot \underline{\Delta}^{(m+1)} &\equiv \underline{\Delta}^{(m+1)} \\ &= \begin{pmatrix} \Delta_1 & 0 & \\ 0 & \Delta_2 & \\ & & \Delta_3 \end{pmatrix}\end{aligned}\tag{24}$$

Thus the ratio of two successive iteration differences is given by

$$\underline{\Delta}^{(m+1)} \cdot \underline{\Delta}^{(m)^{-1}} = [(\underline{\theta} \cdot \underline{b} \cdot \underline{\Delta}^{(m)}) \cdot \underline{\Delta}^{(m)^{-1}}] = \underline{R}^{(m+1)} \quad (25)$$

where the ratio matrix $\underline{R}^{(m+1)}$ is diagonal.

The properties of the elements of this matrix will be described later. For the moment we will make use of the fact that the value of the elements converge rapidly toward a limiting value within the first few iterations

$$\underline{R}^{(m)} \xrightarrow{m \rightarrow \infty} \underline{R} \quad (26)$$

Usually by the time $m=2$, or 3 the value of $\underline{R}^{(m)}$ has reached the value of $\underline{R}$ within operational tolerances.

Assume that by the m th iteration the value of the $\underline{R}^{(m)}$ has achieved the desired convergence requirement

$$\underline{\Delta}^{(m+1)} \cdot \underline{\Delta}^{(m)^{-1}} = \underline{R} \quad (27)$$

$$\text{or} \quad \underline{\Delta}^{(m+1)} = \underline{R} \cdot \underline{\Delta}^{(m)}$$

Since all these matrices are diagonal, the elements obey the scalar relation

$$\Delta_i^{(m+1)} = R_i \Delta_i^{(m)} \quad (28)$$

Thus, at this point the equations have become effectively de-coupled, and each variable can be treated as a single equation in one variable.

Equation 28 is the approximation equivalent to equation 23. By similarity, the approximation equivalent to equation 20 (or 19) is

$$T_i^{(m)} - T_i = R_i (T_i^{(m-1)} - T_i) \quad (29)$$

From this result we obtain the convergence acceleration formula

$$T_i = \frac{T_i^{(m)} - R_i T_i^{(m-1)}}{1 - R_i} \quad (30)$$

The resultant value of T_i is in general quite close to the true solution.

The Interpretation of the Accelerator

The meaning of the ratio R_i , and of the accelerator can be illustrated using one equation in one variable.

$$T = S + RT \quad (31)$$

The solution is given by

$$T = S/(1-R)$$

The iterative solution follows

$$T^{(m)} = T + R^m (T_0 - T) \quad (32)$$

Applying the accelerator formula (equation 30) to equation 32 we obtain the exact result.

Graphically, the accelerator can be described identifying the left hand side of equation 31 as the identity function. The right hand side as the equation of a line

$$F = S + RT \quad (33)$$

The arrows in the figure show the results of two consecutive iterations, T_1 and T_2 after the initial guess T_0 . Clearly, the slope of the function F is given by

$$R = \frac{T_2 - T_1}{T_1 - T_0} \quad (34)$$

Properties of the Ratio R_i

The limiting value of $R_i^{(m)}$ as $m \rightarrow \infty$ can be interpreted as the slope of the function F_i after the coupled equation had been decoupled through iterations.

By way of illustration consider the 3 equations

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} .1 & 0 & 0 \\ 0 & .167 & 0 \\ 0 & 0 & .214 \end{pmatrix} \begin{pmatrix} 10 \\ 0 \\ 3 \end{pmatrix} + \begin{pmatrix} 0 & .4 & .50 \\ .333 & 0 & .50 \\ .357 & .429 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \quad (35)$$

Their solution is

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3.734 \\ 2.842 \\ 3.194 \end{pmatrix}$$

Their equivalent set of decoupled equations is

$$\begin{aligned} x_1 &= 1.120 + .700x_1 \\ x_2 &= .853 + .700x_2 \\ x_3 &= .958 + .700x_3 \end{aligned} \quad (36)$$

The slopes (R_1, R_2, R_3) of the three functions is identically the same.

This result is consistent with the requirement that the solutions to a set of linear equations be unique. If the slopes were different the straight line function would intersect with one another in addition to intersecting the identity line.

Figure 2 illustrates the iterative solution for x_2 . The iteration results for x_1 and x_3 have to be carried out at the same time, but their results are not shown in order to keep the figure simple.

<u>Iteration</u>	<u>x</u>	<u>Ratio</u>	<u>Accel. value</u>
0	0	—	
1	.333	—	
2	1.140	2.432	
3	1.645	.626	2.49
4	2.005	.713	2.89
5	2.256	.697	2.83
6	2.432	.701	2.84
7	2.555	.699	2.84

Thus, depending on the desired accuracy one can activate the convergence accelerator after the iteration where the slope (Ratio) has reached the desired tolerance.