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**SPXCPL: Two-Dimensional Modeling Program of Self-Potential
Effects from Cross-Coupled Fluid and Heat Flow
(User's Guide and Documentation for Version 1.0)**

by

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PROGRAM DESCRIPTION

Statement of Problem

The self potential (SP) method is based on the measurement of naturally occurring potential differences generated mainly by electrochemical, electrokinetic and thermoelectric sources. The multiplicity of sources can be either an advantage or a disadvantage. On the one hand, a number of phenomena can be studied with the technique and, on the other hand, the possibility of a number of different sources can sometimes be confusing.

There has been a mild resurgence in the use of the SP method in geothermal exploration (Corwin and Hoover, 1979), in the study of earthquake related phenomena (Fitterman, 1978; Corwin and Morrison, 1977), and in engineering applications (Ogilvy et al., 1969; Bogoslovsky and Ogilvy, 1973).

Older methods of interpretation were mostly based on polarized spheres (de Witte, 1948; Yungul, 1950) or line dipole current sources (Meiser, 1962; Paul, 1965). Although these techniques are useful, they provide little information about the nature of the primary sources. Nourbehecht (1963), drawing on the earlier work of Marshall and Madden (1959), discussed the source mechanisms in detail and provided a technique for the solution of coupled flows which incorporated the primary driving potential. His solution is formulated in terms of a total (pseudo) potential composed of the electric potential and the weighted primary source potential (pressure, temperature, concentration). Sill (1981 and 1982) presented an alternative method for the solution of coupled flow problems that explicitly models both the primary flow and the induced secondary electric potentials (joint modeling). The use of this technique with a two-dimensional (2-D) algorithm for potential problems provides a new flexibility in the modeling of SP data.

Method of Solution

The general equation for coupled flows can be written (Onsager, 1931; Marshall and Madden, 1959; Nourbehecht, 1963)

$$\Gamma_i = \sum_j L_{ij} X_j, \quad (1)$$

where the fluxes Γ_i (charge, matter, heat, etc.) are related to the various forces X_j (gradients of electric potential, pressure, temperature, etc.) through the coupling coefficients ("conductivities") L_{ij} . For many practical applications of coupled flows, we are concerned with secondary electric current flows and potentials which are driven by some other primary flow. When the effects of the secondary electric potentials on the primary flow are small, the primary flow equation is decoupled and the resulting equations are

$$\Gamma_1 = -L_{11} \nabla \xi, \quad (2)$$

and
$$J_{\text{Total}} = \Gamma_2 = -L_{21} \nabla \xi - \sigma \nabla \phi, \quad (3)$$

where Γ_1 is the primary flow (solution flux, heat flux, etc.), L_{11} is the primary conductivity (permeability, thermal conductivity, etc.) and ξ is the primary potential (pressure, temperature, etc.), J_{Total} is the total electric current, L_{21} is the cross coupling "conductivity", σ is the ordinary electrical conductivity and ϕ is the electric potential. The decoupled primary flow problem (equation 2) can be solved separately and used in the solution of the electrical flow problem (equation 3). Starting with equation 3, we note that the first term is a "convection" current driven by the primary flow and the second term is the usual conduction current driven by the gradient of the electric potential. Using this approach, we can write equation 3 as

$$J_{\text{Total}} = J_{\text{conv}} + J_{\text{cond}} , \quad (4)$$

$$\text{where} \quad J_{\text{conv}} = -L_{21} \nabla \xi , \quad (5)$$

$$\text{and} \quad J_{\text{cond}} = -\sigma \nabla \phi . \quad (6)$$

If no external current sources are imposed and we have DC conditions (i.e., $\partial \rho / \partial t = 0$), then the total current is divergenceless ($\nabla \cdot J_{\text{Total}} = 0$) and

$$\nabla \cdot J_{\text{cond}} = - \nabla \cdot J_{\text{conv}} = \nabla \cdot (L_{21} \nabla \xi) = \nabla L_{21} \cdot \nabla \xi + L_{21} \nabla^2 \xi . \quad (7)$$

Thus there are sources (nonzero divergence) of conduction current wherever there are gradients of the cross-coupling coefficient parallel to the primary flow (flow perpendicular to boundaries) or wherever there are external or induced sources of the primary flow. The sources of the conduction current given by the right-hand side of equation 7 can then be used to determine the resultant electric potential, ϕ . In equation 7, we see from the $\nabla L_{21} \cdot \nabla \xi$ term that gradients in the same direction produce positive sources for the conduction current and gradients in opposite directions produce negative sources. Substitution of equation 2 into the $L_{21} \nabla^2 \xi$ term results in the two additional source terms $-L_{21} \nabla \cdot r_1 / L_{11}$ and $L_{21} \nabla L_{11} \cdot r_1 / L_{11}^2$. The first of these is nonzero at externally impressed sources of the primary flow. If $\nabla \cdot r_1 > 0$, a positive source for the primary flow, then the electrical source is negative for the usual case where $L_{21} > 0$. The second term is nonzero at boundaries where the primary flow conductivity changes. These boundaries are the locations of the secondary or induced sources for the primary flow.

Numerical Modeling

The transmission surface algorithm for the DC potential problem is given in Madden (1971). A brief review is given here starting with the general potential flow equations,

$$\Gamma = -L \nabla \xi, \quad (8)$$

and
$$\nabla \cdot \Gamma = S \quad (9)$$

where Γ is the flux, ξ is the potential, S is the source and L is the "conductivity". When L is independent of y (strike direction), equations 8 and 9 can be Fourier (cosine) transformed in the y -direction giving

$$-L(x,z) \frac{\partial \xi(x,\lambda,z)}{\partial x} = \Gamma_x(x,\lambda,z) \quad (10)$$

$$-L(x,z) \frac{\partial \xi(x,\lambda,z)}{\partial z} = \Gamma_z(x,\lambda,z) \quad (11)$$

$$\frac{\partial \Gamma_x}{\partial x} + \frac{\partial \Gamma_z}{\partial z} + \lambda^2 L \xi = S(x,\lambda,z). \quad (12)$$

The flux and the potential are, in general, 3 dimensional, depending on the nature of the source. Equations 10, 11, and 12 can be approximated by lumped-element rectangular network (Figure 2) and the difference equation at a node is

$$\begin{aligned} & Y_x(i,j-1)[\xi(i,j-1) - \xi(i,j)] + Y_x(i,j)[\xi(i,j+1) - \xi(i,j)] \\ & + Y_z(i-1,j)[\xi(i-1,j) - \xi(i,j)] + Y_z(i,j)[\xi(i+1,j) - \xi(i,j)] \quad (13) \\ & + Y(i,j)\xi(i,j) = S(i,j) \Delta x \Delta z \end{aligned}$$

where

$$Y_x = L \Delta z / \Delta x \quad (14)$$

$$Y_z = L \Delta x / \Delta z \quad (15)$$

$$Y = \lambda^2 \Delta x \Delta z, \quad (16)$$

$$i = 1 \dots n,$$

and

$$j = 1 \dots m.$$

The set of $n \cdot m$ node equations can be reorganized into the matrix equation

$$C \xi = S \quad (17)$$

where

C = coefficient matrix $[n \cdot m \times n \cdot m]$,

ξ = node potential vector $(n \cdot m)$,

and

S = node source vector $(n \cdot m)$.

One method of solution of equation 17 is outlined by Swift (1967), who also provided the code (Swift, personal comm.). The final solution in x, y, z space is obtained by an inverse Fourier transform of a suite of $\xi(\lambda)$ solutions.

The solution of the coupled problem requires three models for the physical properties, the primary flow resistivity (L_{11}^{-1}), the voltage cross-coupling coefficients ($C_{21} = \rho L_{21}$) and the electrical resistivity (ρ). For each λ value, the primary flow potentials (ξ) are computed for the primary model given a source distribution. From the divergence equation 7, the electrical source terms are calculated at each node. In the lumped circuit, the divergence equation 13 is used with L replaced by the cross-coupling coefficients L_{21} . In the last step, the electrical source terms calculated in the previous step are used with the resistivity model to determine the electrical potentials (ϕ).

The solution of these problems is formulated in equation 17 in terms of a

flow source which is commonly taken as a point source. Finite-length line source distributions can be easily computed by simple convolution of the point source solution along the source line. In other applications, a specified potential distribution (Dirichlet boundary condition) rather than flow source distributions may be desirable. A simple procedure for accomplishing this, without disturbing the formal solution of equation 17, is outlined in Killpack and Hohmann (1979).

Application

The program is applicable to the calculation of self-potential effects due to fluid flow (electrokinetic effects) and heat flow (thermoelectric effects). The geological structure is two dimensional but the sources can be either finite line sources or point sources.

The accuracy of the calculated potentials depends on the model discretization and the distance from the source(s). For the default mesh, the accuracy is usually a few percent at a distance of about one unit from the source.

Surface boundary conditions for the primary problem require careful consideration as the form of the flow near the air-earth interface can have a profound effect on the resultant electric potentials. For temperature problems the appropriate boundary condition is a constant temperature, which is taken as zero. With this boundary condition there is a normal flux of heat at the surface and there will be induced electrical sources here, if the surface medium has a nonzero coupling coefficient. In the models, zero temperature at the surface is produced by giving the air a very large thermal conductivity.

While it is correct that the excess pressure is zero from the water table up to the surface, uncritical use of the zero surface pressure boundary

condition in potential-flow problems often results in a nonzero normal gradient and therefore a fluid flow at the air-earth interface. As was noted in Sill and Johng (1979), it is much more important to model the flow geometry which is predominantly horizontal near the surface or the water table. Horizontal fluid flow at the surface in a potential-flow problem requires a zero vertical gradient of the pressure. In the models, zero vertical gradients are produced by giving the air a vanishingly small hydraulic permeability. Zero vertical gradients at a water table below the surface can be modeled with a thin, very low permeability layer overlying the saturated material at the position of the water table. In effect the modeled flow is confined by impermeable layers rather than having the flow deviated by variations in the height of the water table.

The results of a particular model calculation can be scaled up or down by noting that

$$\xi \propto \rho_{\xi} I_{\xi} / a,$$

and
$$\phi \propto C_{21} \xi,$$

or
$$\phi \propto C_{21} \rho_{\xi} I_{\xi} / a,$$

where ξ = primary potential (pressure or temperature)

ρ_{ξ} = primary resistivity (hydraulic resistivity or thermal resistivity)

I_{ξ} = primary source strength (in units of $\rho_{\xi} \nabla \xi \times \text{Area}$))

a = model length unit

and C_{21} = voltage cross-coupling coefficient (in volts per unit of primary potential).

Probably the safest procedure is to stick to MKS units for all parameters.

Miscellaneous

Author - W. R. Sill, 1980

The code was adapted from a transmission surface algorithm code provided by C. M. Swift, Jr. Many of the subroutines are from the same source.

Language - FORTRAN IV

Execution time - on the Prime system the execution time (CPU) is about
4.5 minutes

PROGRAM OPERATION

The main program SPXCPL requires the following subroutines:

RMDL - reads coded models from input file
NETFOR - calculates the lumped circuit elements
AFORM - forms the coefficient matrix A as in $A \cdot V = S$
ROY6 - solves $A \cdot V = S$ for V given A and S
SOURCE - calculates electrical sources
TRANS - transforms voltage from Lamda to space domain
INTEG3 - transforms voltage from Lamda domain to space domain, off axis
CONV - calculates potentials on the surface for a finite line source
MINV - matrix inverse
KMM - matrix multiply

Input

Input for the program consists of scale and option parameters, the locations of the primary flow sources and three physical properties models. The input data is in a file and an example of an input file is shown in Figure 1 and the explanation is given below.

line 1 Program execution command for PRIME 400 computer system
line 2 Contains the parameters
 , LU, IYOF, ICON, LYS FORMAT (4I2)
LU = Logical unit number of printer
IYOF = 0, on axis voltage only, vertical section
 = 1, vertical section plus planview of surface voltage
ICON = 0, point source only
 = 1, finite line source, surface voltage

INPUT MODEL

52 columns by 11 rows

HORIZONTAL DISTANCE MODEL UNITS

* - Source Location

Horizontal Node # = NTX = 26

Verticle Node # = NDX = 3

FIGURE 2 INPUT MODEL MESH

LYS = half length of line source in model units
 line 3 Ascale = length of one model unit FORMAT (1E9.3)
 line 4 to 7 Primary source parameters
 line 4 NT = # of sources FORMAT (I2)
 line 5 NTX = horizontal node location FORMAT (17I3)
 line 6 NDX = vertical node location FORMAT (17I3)
 (for NTX and NDX see Figure 2)
 line 7 TS = source strength FORMAT (17F3.0)
 line 8 to 66 contains the input for the three physical
 parameter models
 line 8 header for primary resistivity model (32 characters)
 line 9 NRES, IDELH, IDELV, RESAIR FORMAT (3I2, 1E9.2)
 NRES = # of media, up to 9
 IDELH - control to change horizontal mesh (=0, no; = 1, yes)
 IDELV - control to change vertical mesh (=0, no; = 1, yes)
 line 10 new values for DELV(2) through DELV(12) FORMAT (16F4.2)
 line 11 to 17 code #, value of primary resistivity FORMAT (I2,E9.3)
 line 18 to 28 model in code form FORMAT (2X,52I1)
 see also Figure 2
 line 29 to 66 continues with the cross-coupling model and electrical
 resistivity model in the same form as lines 8 to 28.

The input parameter for the primary model is the primary resistivity.
 For a pressure problem this is the hydraulic resistivity or the
 "impermeability". Also for a pressure problem the hydraulic resistivity for
 the air should be large. For a thermal problem the primary resistivity is the
 thermal resistivity and the air should be given a small thermal resistivity.

For the cross-coupling parameter the input should be the voltage cross-coupling coefficient in volts per unit of primary potential. The main program will convert this to a current cross-coupling parameter by using the resistivity model.

The electrical model parameter is the electrical resistivity. As is usual, the air should be given a large value.

The safest way to keep out of trouble with all these units is to stick to the MKS system.

Output

The output, Figure 3, consists of an image of the input file showing the three physical properties models. This is followed by four vertical cross sections ($x, y=0, z$) of the primary potential in the plane of the point sources. These four tables contain the values of the potential as a function of depth and horizontal distance in model units. The first two tables show the potential at unit intervals from -7 units to +7 units. The second two tables show the potentials at .25 unit intervals from -1.75 units to +1.75 units.

The next four tables are similar vertical cross sections for the electric potential when the primary sources are points.

This is followed by two tables which display the values of the surface potential ($x, y, z=0$) for point sources of the primary field. This potential distribution is symmetric about the line $y=0$.

The last two tables contain a similar display for the surface potential ($x, y, z=0$) for a primary line source of the stated length. The line source is symmetric about the line $y=0$ as are the potentials.

FIGURE 3 (Continued)

PRIMARY POTENTIAL

	-7	-6	-5	-4	-3	-2	-1	0
0.00	0.19E 10	0.23E 10	0.28E 10	0.35E 10	0.47E 10	0.68E 10	0.11E 11	0.14E 11
0.25	0.19E 10	0.23E 10	0.28E 10	0.35E 10	0.47E 10	0.67E 10	0.11E 11	0.15E 11
0.50	0.19E 10	0.23E 10	0.28E 10	0.35E 10	0.47E 10	0.67E 10	0.11E 11	0.17E 11
0.75	0.19E 10	0.23E 10	0.28E 10	0.35E 10	0.46E 10	0.66E 10	0.11E 11	0.22E 11
1.00	0.19E 10	0.23E 10	0.27E 10	0.34E 10	0.45E 10	0.65E 10	0.11E 11	0.35E 11
1.50	0.19E 10	0.22E 10	0.27E 10	0.33E 10	0.43E 10	0.60E 10	0.94E 10	0.15E 11
2.50	0.18E 10	0.21E 10	0.25E 10	0.30E 10	0.37E 10	0.48E 10	0.62E 10	0.72E 10
3.50	0.17E 10	0.19E 10	0.23E 10	0.26E 10	0.31E 10	0.37E 10	0.42E 10	0.45E 10
5.50	0.14E 10	0.16E 10	0.18E 10	0.20E 10	0.22E 10	0.23E 10	0.25E 10	0.25E 10
10.50	0.92E 09	0.97E 09	0.10E 10	0.11E 10	0.11E 10	0.11E 10	0.12E 10	0.12E 10
20.50	0.44E 09	0.45E 09	0.46E 09	0.47E 09	0.48E 09	0.48E 09	0.49E 09	0.49E 09
40.50	0.21E 09	0.22E 09	0.22E 09	0.22E 09	0.22E 09	0.22E 09	0.22E 09	0.23E 09

PRIMARY POTENTIAL

	0	+1	+2	+3	+4	+5	+6	+7
0.00	0.14E 11	0.11E 11	0.68E 10	0.47E 10	0.35E 10	0.28E 10	0.23E 10	0.19E 10
0.25	0.15E 11	0.11E 11	0.67E 10	0.47E 10	0.35E 10	0.28E 10	0.23E 10	0.19E 10
0.50	0.17E 11	0.11E 11	0.67E 10	0.47E 10	0.35E 10	0.28E 10	0.23E 10	0.19E 10
0.75	0.22E 11	0.11E 11	0.66E 10	0.46E 10	0.35E 10	0.28E 10	0.23E 10	0.19E 10
1.00	0.35E 11	0.11E 11	0.65E 10	0.45E 10	0.34E 10	0.27E 10	0.23E 10	0.19E 10
1.50	0.15E 11	0.94E 10	0.60E 10	0.43E 10	0.33E 10	0.27E 10	0.22E 10	0.19E 10
2.50	0.72E 10	0.62E 10	0.48E 10	0.37E 10	0.30E 10	0.25E 10	0.21E 10	0.18E 10
3.50	0.45E 10	0.42E 10	0.37E 10	0.31E 10	0.26E 10	0.23E 10	0.19E 10	0.17E 10
5.50	0.25E 10	0.25E 10	0.23E 10	0.22E 10	0.20E 10	0.18E 10	0.16E 10	0.14E 10
10.50	0.12E 10	0.12E 10	0.11E 10	0.11E 10	0.11E 10	0.10E 10	0.97E 09	0.92E 09
20.50	0.49E 09	0.49E 09	0.48E 09	0.48E 09	0.47E 09	0.46E 09	0.45E 09	0.44E 09
40.50	0.23E 09	0.22E 09	0.22E 09	0.22E 09	0.22E 09	0.22E 09	0.22E 09	0.21E 09

PRIMARY POTENTIAL

	-1.75	-1.50	-1.25	-1.00	-0.75	-0.50	-0.25	0.00
0.00	0.75E 10	0.84E 10	0.94E 10	0.11E 11	0.12E 11	0.13E 11	0.14E 11	0.14E 11
0.25	0.75E 10	0.84E 10	0.94E 10	0.11E 11	0.12E 11	0.13E 11	0.15E 11	0.15E 11
0.50	0.75E 10	0.84E 10	0.95E 10	0.11E 11	0.12E 11	0.14E 11	0.16E 11	0.17E 11
0.75	0.74E 10	0.83E 10	0.94E 10	0.11E 11	0.13E 11	0.16E 11	0.19E 11	0.22E 11
1.00	0.72E 10	0.81E 10	0.93E 10	0.11E 11	0.13E 11	0.17E 11	0.23E 11	0.35E 11
1.50	0.66E 10	0.74E 10	0.83E 10	0.94E 10	0.11E 11	0.12E 11	0.14E 11	0.15E 11
2.50	0.51E 10	0.54E 10	0.58E 10	0.62E 10	0.65E 10	0.69E 10	0.71E 10	0.72E 10
3.50	0.38E 10	0.40E 10	0.41E 10	0.42E 10	0.43E 10	0.44E 10	0.45E 10	0.45E 10
5.50	0.24E 10	0.24E 10	0.24E 10	0.25E 10	0.25E 10	0.25E 10	0.25E 10	0.25E 10
10.50	0.11E 10	0.11E 10	0.12E 10	0.12E 10	0.12E 10	0.12E 10	0.12E 10	0.12E 10

FIGURE 3 (Continued)

20.50	0.49E	09	0.49E	09	0.49E	09	0.49E	09	0.49E	09	0.49E	09	0.49E	09	0.49E	09
40.50	0.22E	09	0.22E	09	0.22E	09	0.22E	09	0.22E	09	0.23E	09	0.23E	09	0.23E	09

PRIMARY POTENTIAL

	0.00	+0.25	+0.50	+0.75	+1.00	+1.25	+1.50	+1.75
0.00	0.14E 11	0.14E 11	0.13E 11	0.12E 11	0.11E 11	0.94E 10	0.84E 10	0.75E 10
0.25	0.15E 11	0.15E 11	0.13E 11	0.12E 11	0.11E 11	0.94E 10	0.84E 10	0.75E 10
0.50	0.17E 11	0.16E 11	0.14E 11	0.12E 11	0.11E 11	0.95E 10	0.84E 10	0.75E 10
0.75	0.22E 11	0.19E 11	0.16E 11	0.13E 11	0.11E 11	0.94E 10	0.83E 10	0.74E 10
1.00	0.35E 11	0.23E 11	0.17E 11	0.13E 11	0.11E 11	0.93E 10	0.81E 10	0.72E 10
1.50	0.15E 11	0.14E 11	0.12E 11	0.11E 11	0.94E 10	0.83E 10	0.74E 10	0.66E 10
2.50	0.72E 10	0.71E 10	0.69E 10	0.65E 10	0.62E 10	0.58E 10	0.54E 10	0.51E 10
3.50	0.45E 10	0.45E 10	0.44E 10	0.43E 10	0.42E 10	0.41E 10	0.40E 10	0.38E 10
5.50	0.25E 10	0.25E 10	0.25E 10	0.25E 10	0.25E 10	0.24E 10	0.24E 10	0.24E 10
10.50	0.12E 10	0.12E 10	0.12E 10	0.12E 10	0.12E 10	0.12E 10	0.11E 10	0.11E 10
20.50	0.49E 09	0.49E 09	0.49E 09	0.49E 09	0.49E 09	0.49E 09	0.49E 09	0.49E 09
40.50	0.23E 09	0.23E 09	0.23E 09	0.22E 09	0.22E 09	0.22E 09	0.22E 09	0.22E 09

VOLTAGE

	-7	-6	-5	-4	-3	-2	-1	0
0.00	-.96E 01	-.11E 02	-.14E 02	-.18E 02	-.24E 02	-.34E 02	-.53E 02	-.72E 02
0.25	-.96E 01	-.11E 02	-.14E 02	-.18E 02	-.24E 02	-.34E 02	-.53E 02	-.76E 02
0.50	-.96E 01	-.11E 02	-.14E 02	-.18E 02	-.23E 02	-.33E 02	-.54E 02	-.87E 02
0.75	-.95E 01	-.11E 02	-.14E 02	-.17E 02	-.23E 02	-.33E 02	-.55E 02	-.11E 03
1.00	-.95E 01	-.11E 02	-.14E 02	-.17E 02	-.23E 02	-.32E 02	-.54E 02	-.18E 03
1.50	-.93E 01	-.11E 02	-.13E 02	-.17E 02	-.22E 02	-.30E 02	-.47E 02	-.75E 02
2.50	-.90E 01	-.10E 02	-.12E 02	-.15E 02	-.19E 02	-.24E 02	-.31E 02	-.36E 02
3.50	-.84E 01	-.97E 01	-.11E 02	-.13E 02	-.16E 02	-.18E 02	-.21E 02	-.23E 02
5.50	-.72E 01	-.80E 01	-.89E 01	-.99E 01	-.11E 02	-.12E 02	-.12E 02	-.13E 02
10.50	-.46E 01	-.48E 01	-.51E 01	-.53E 01	-.55E 01	-.57E 01	-.58E 01	-.58E 01
20.50	-.22E 01	-.23E 01	-.23E 01	-.24E 01	-.24E 01	-.24E 01	-.24E 01	-.24E 01
40.50	-.11E 01	-.11E 01	-.11E 01	-.11E 01	-.11E 01	-.11E 01	-.11E 01	-.11E 01

VOLTAGE

	0	+1	+2	+3	+4	+5	+6	+7
0.00	-.72E 02	-.53E 02	-.34E 02	-.24E 02	-.18E 02	-.14E 02	-.11E 02	-.96E 01
0.25	-.76E 02	-.53E 02	-.34E 02	-.24E 02	-.18E 02	-.14E 02	-.11E 02	-.96E 01
0.50	-.87E 02	-.54E 02	-.33E 02	-.23E 02	-.18E 02	-.14E 02	-.11E 02	-.96E 01
0.75	-.11E 03	-.55E 02	-.33E 02	-.23E 02	-.17E 02	-.14E 02	-.11E 02	-.95E 01
1.00	-.18E 03	-.54E 02	-.32E 02	-.23E 02	-.17E 02	-.14E 02	-.11E 02	-.95E 01
1.50	-.75E 02	-.47E 02	-.30E 02	-.22E 02	-.17E 02	-.13E 02	-.11E 02	-.94E 01
2.50	-.36E 02	-.31E 02	-.24E 02	-.19E 02	-.15E 02	-.12E 02	-.10E 02	-.90E 01
3.50	-.23E 02	-.21E 02	-.18E 02	-.16E 02	-.13E 02	-.11E 02	-.97E 01	-.84E 01
5.50	-.13E 02	-.12E 02	-.12E 02	-.11E 02	-.99E 01	-.89E 01	-.80E 01	-.72E 01
10.50	-.58E 01	-.57E 01	-.57E 01	-.55E 01	-.53E 01	-.51E 01	-.48E 01	-.46E 01

FIGURE 3 (Continued)

20.50	-.24E	01	-.24E	01	-.24E	01	-.24E	01	-.24E	01	-.23E	01	-.23E	01	-.22E	01
40.50	-.11E	01	-.11E	01	-.11E	01	-.11E	01	-.11E	01	-.11E	01	-.11E	01	-.11E	01

VOLTAGE

	-1.75	-1.50	-1.25	-1.00	-0.75	-0.50	-0.25	0.00
0.00	-.38E 02	-.42E 02	-.47E 02	-.53E 02	-.59E 02	-.65E 02	-.70E 02	-.72E 02
0.25	-.37E 02	-.42E 02	-.47E 02	-.53E 02	-.60E 02	-.67E 02	-.73E 02	-.76E 02
0.50	-.37E 02	-.42E 02	-.47E 02	-.54E 02	-.62E 02	-.71E 02	-.81E 02	-.87E 02
0.75	-.37E 02	-.41E 02	-.47E 02	-.55E 02	-.64E 02	-.78E 02	-.95E 02	-.11E 03
1.00	-.36E 02	-.41E 02	-.46E 02	-.54E 02	-.65E 02	-.83E 02	-.11E 03	-.18E 03
1.50	-.33E 02	-.37E 02	-.41E 02	-.47E 02	-.53E 02	-.61E 02	-.69E 02	-.75E 02
2.50	-.25E 02	-.27E 02	-.29E 02	-.31E 02	-.33E 02	-.34E 02	-.36E 02	-.36E 02
3.50	-.19E 02	-.20E 02	-.21E 02	-.21E 02	-.22E 02	-.22E 02	-.22E 02	-.23E 02
5.50	-.12E 02	-.12E 02	-.12E 02	-.12E 02	-.12E 02	-.13E 02	-.13E 02	-.13E 02
10.50	-.57E 01	-.57E 01	-.58E 01	-.58E 01	-.58E 01	-.58E 01	-.58E 01	-.58E 01
20.50	-.24E 01	-.24E 01	-.24E 01	-.24E 01	-.24E 01	-.24E 01	-.24E 01	-.24E 01
40.50	-.11E 01	-.11E 01	-.11E 01	-.11E 01	-.11E 01	-.11E 01	-.11E 01	-.11E 01

VOLTAGE

	0.00	+0.25	+0.50	+0.75	+1.00	+1.25	+1.50	+1.75
0.00	-.72E 02	-.70E 02	-.65E 02	-.59E 02	-.53E 02	-.47E 02	-.42E 02	-.38E 02
0.25	-.76E 02	-.73E 02	-.67E 02	-.60E 02	-.53E 02	-.47E 02	-.42E 02	-.37E 02
0.50	-.87E 02	-.81E 02	-.71E 02	-.62E 02	-.54E 02	-.47E 02	-.42E 02	-.37E 02
0.75	-.11E 03	-.95E 02	-.78E 02	-.64E 02	-.55E 02	-.47E 02	-.41E 02	-.37E 02
1.00	-.18E 03	-.11E 03	-.83E 02	-.65E 02	-.54E 02	-.46E 02	-.41E 02	-.36E 02
1.50	-.75E 02	-.69E 02	-.61E 02	-.53E 02	-.47E 02	-.41E 02	-.37E 02	-.33E 02
2.50	-.36E 02	-.36E 02	-.34E 02	-.33E 02	-.31E 02	-.29E 02	-.27E 02	-.25E 02
3.50	-.23E 02	-.22E 02	-.22E 02	-.22E 02	-.21E 02	-.21E 02	-.20E 02	-.19E 02
5.50	-.13E 02	-.13E 02	-.13E 02	-.12E 02	-.12E 02	-.12E 02	-.12E 02	-.12E 02
10.50	-.58E 01	-.58E 01	-.58E 01	-.58E 01	-.58E 01	-.58E 01	-.57E 01	-.57E 01
20.50	-.24E 01	-.24E 01	-.24E 01	-.24E 01	-.24E 01	-.24E 01	-.24E 01	-.24E 01
40.50	-.11E 01	-.11E 01	-.11E 01	-.11E 01	-.11E 01	-.11E 01	-.11E 01	-.11E 01

VOLTAGE PLAN VIEW X,Y,Z=0

	-7	-6	-5	-4	-3	-2	-1	0
0.00	-.89E 01	-.11E 02	-.13E 02	-.17E 02	-.22E 02	-.32E 02	-.51E 02	-.70E 02
0.50	-.90E 01	-.11E 02	-.13E 02	-.17E 02	-.22E 02	-.32E 02	-.49E 02	-.66E 02
1.00	-.90E 01	-.11E 02	-.13E 02	-.16E 02	-.21E 02	-.30E 02	-.43E 02	-.56E 02
2.00	-.86E 01	-.10E 02	-.12E 02	-.15E 02	-.19E 02	-.24E 02	-.29E 02	-.32E 02
3.00	-.80E 01	-.93E 01	-.11E 02	-.13E 02	-.16E 02	-.18E 02	-.21E 02	-.21E 02
4.00	-.73E 01	-.84E 01	-.97E 01	-.11E 02	-.13E 02	-.15E 02	-.16E 02	-.18E 02
5.00	-.68E 01	-.76E 01	-.86E 01	-.97E 01	-.11E 02	-.12E 02	-.12E 02	-.12E 02
6.00	-.63E 01	-.70E 01	-.77E 01	-.84E 01	-.91E 01	-.97E 01	-.98E 01	-.95E 01
7.00	-.58E 01	-.63E 01	-.69E 01	-.74E 01	-.79E 01	-.83E 01	-.87E 01	-.92E 01
8.00	-.54E 01	-.58E 01	-.62E 01	-.66E 01	-.69E 01	-.71E 01	-.72E 01	-.71E 01

FIGURE 3 (Continued)

9.00	-.49E 01	-.52E 01	-.56E 01	-.58E 01	-.60E 01	-.62E 01	-.60E 01	-.57E 01
10.00	-.45E 01	-.47E 01	-.50E 01	-.52E 01	-.53E 01	-.54E 01	-.55E 01	-.58E 01

VOLTAGE PLAN VIEW X,Y,Z=0

	0	+1	+2	+3	+4	+5	+6	+7
0.00	-.70E 02	-.51E 02	-.32E 02	-.22E 02	-.17E 02	-.13E 02	-.11E 02	-.90E 01
0.50	-.66E 02	-.49E 02	-.32E 02	-.22E 02	-.17E 02	-.13E 02	-.11E 02	-.90E 01
1.00	-.56E 02	-.43E 02	-.30E 02	-.21E 02	-.16E 02	-.13E 02	-.11E 02	-.90E 01
2.00	-.32E 02	-.29E 02	-.24E 02	-.19E 02	-.15E 02	-.12E 02	-.10E 02	-.86E 01
3.00	-.21E 02	-.21E 02	-.18E 02	-.16E 02	-.13E 02	-.11E 02	-.93E 01	-.80E 01
4.00	-.18E 02	-.16E 02	-.15E 02	-.13E 02	-.11E 02	-.97E 01	-.84E 01	-.73E 01
5.00	-.12E 02	-.12E 02	-.12E 02	-.11E 02	-.97E 01	-.86E 01	-.76E 01	-.68E 01
6.00	-.95E 01	-.96E 01	-.97E 01	-.91E 01	-.84E 01	-.77E 01	-.70E 01	-.63E 01
7.00	-.92E 01	-.87E 01	-.83E 01	-.79E 01	-.74E 01	-.69E 01	-.63E 01	-.58E 01
8.00	-.71E 01	-.72E 01	-.71E 01	-.69E 01	-.66E 01	-.62E 01	-.58E 01	-.54E 01
9.00	-.57E 01	-.60E 01	-.62E 01	-.61E 01	-.58E 01	-.56E 01	-.52E 01	-.49E 01
10.00	-.58E 01	-.55E 01	-.54E 01	-.53E 01	-.52E 01	-.50E 01	-.47E 01	-.45E 01

2D-VOLTAGE FOR SOURCE LENGTH 4.

SOURCE=S/TOTAL LENGTH

	-7	-6	-5	-4	-3	-2	-1	0
0.00	-.89E 01	-.11E 02	-.13E 02	-.16E 02	-.21E 02	-.28E 02	-.41E 02	-.52E 02
1.00	-.87E 01	-.10E 02	-.13E 02	-.16E 02	-.20E 02	-.27E 02	-.39E 02	-.48E 02
2.00	-.84E 01	-.99E 01	-.12E 02	-.14E 02	-.18E 02	-.24E 02	-.32E 02	-.39E 02
3.00	-.79E 01	-.92E 01	-.11E 02	-.13E 02	-.16E 02	-.19E 02	-.24E 02	-.27E 02
4.00	-.74E 01	-.85E 01	-.98E 01	-.11E 02	-.13E 02	-.15E 02	-.17E 02	-.18E 02
5.00	-.68E 01	-.77E 01	-.88E 01	-.99E 01	-.11E 02	-.12E 02	-.13E 02	-.14E 02
6.00	-.63E 01	-.70E 01	-.78E 01	-.86E 01	-.95E 01	-.10E 02	-.11E 02	-.11E 02
7.00	-.58E 01	-.64E 01	-.70E 01	-.76E 01	-.81E 01	-.85E 01	-.87E 01	-.87E 01
8.00	-.54E 01	-.58E 01	-.62E 01	-.67E 01	-.70E 01	-.73E 01	-.74E 01	-.74E 01
9.00	-.40E 01	-.43E 01	-.46E 01	-.49E 01	-.51E 01	-.52E 01	-.53E 01	-.53E 01
10.00	-.27E 01	-.29E 01	-.31E 01	-.33E 01	-.34E 01	-.35E 01	-.35E 01	-.34E 01

2D-VOLTAGE FOR SOURCE LENGTH 4.

SOURCE=S/TOTAL LENGTH

	0	+1	+2	+3	+4	+5	+6	+7
0.00	-.52E 02	-.41E 02	-.28E 02	-.21E 02	-.16E 02	-.13E 02	-.11E 02	-.89E 01
1.00	-.48E 02	-.39E 02	-.27E 02	-.20E 02	-.16E 02	-.13E 02	-.10E 02	-.87E 01
2.00	-.39E 02	-.32E 02	-.24E 02	-.18E 02	-.15E 02	-.12E 02	-.99E 01	-.84E 01
3.00	-.27E 02	-.24E 02	-.19E 02	-.16E 02	-.13E 02	-.11E 02	-.92E 01	-.79E 01
4.00	-.18E 02	-.17E 02	-.15E 02	-.13E 02	-.11E 02	-.98E 01	-.85E 01	-.74E 01
5.00	-.14E 02	-.13E 02	-.12E 02	-.11E 02	-.99E 01	-.88E 01	-.77E 01	-.68E 01
6.00	-.11E 02	-.11E 02	-.10E 02	-.95E 01	-.86E 01	-.78E 01	-.70E 01	-.63E 01
7.00	-.87E 01	-.88E 01	-.85E 01	-.81E 01	-.76E 01	-.70E 01	-.64E 01	-.58E 01
8.00	-.74E 01	-.74E 01	-.73E 01	-.70E 01	-.67E 01	-.63E 01	-.58E 01	-.54E 01
9.00	-.53E 01	-.53E 01	-.52E 01	-.51E 01	-.49E 01	-.46E 01	-.43E 01	-.40E 01
10.00	-.34E 01	-.35E 01	-.35E 01	-.34E 01	-.33E 01	-.31E 01	-.29E 01	-.27E 01

FIGURE 3 (Continued)

TESTING AND VALIDATION

Simple models with analytic solutions can be used for testing. The results of the sample job in Figure 3, a point source in a half-space, is one example. The accuracy of the calculation depends mostly on the model discretization, the suite of Lamda values and the integration over Lamda. Two routines, TRANS and INTEG3, are used for the Lamda integration. TRANS is used for the on-axis ($y=0$) integration and is usually the more accurate, while INTEG3 is used for the off-axis ($y \neq 0$) integration. INTEG3 is somewhat less accurate for $y=0$ with the usual suite of Lamda values but it is a more stable integrator for the off-axis positions. As is shown below, much of the error is a DC bias error which is relatively unimportant.

A comparison of the analytic and calculated pressures and electric potentials for the sample job of Figure 3 are given in Table 1. In this case the errors range from 4 percent to about 15 percent. At horizontal distances greater than one unit from the source, the error is largely a constant difference term. This constant difference term corresponds to a small DC bias which is relatively unimportant in self-potential measurements. If the bias term is corrected the errors reduce to a few percent. A finer discretization in the vertical direction near the surface can also be used to reduce the error.

Table 1. A comparison of the analytic (P_a, ϕ_a) and calculated (P_c, ϕ_c) results for the model of the sample job in Figure 3.

Surface location	Pressure/ 10^{11}		Potential/ 10^2		
	P_a	P_c	ϕ_a	ϕ_c	$\phi_a - \phi_c$
0	.159	.14	.795	.72	.075
1	.113	.11	.565	.53	.035
2	.071	.068	.356	.34	.016
3	.050	.047	.252	.24	.012
4	.039	.035	.193	.18	.013
5	.031	.028	.156	.14	.016
6	.026	.023	.131	.11	.021
7	.023	.019	.113	.096	.017

PROGRAMMER REFERENCE MANUAL

General Information

The program is set up to run on the ESL PRIME system and is written in FORTRAN IV. The required subroutines are listed in section 2. The general purpose library is needed for COS and SIN. The program and subroutines are loaded and executed with the use of CO C_LOAD and CO C_RUN commands as shown in the sample job of Figure 3. The program expects a hard copy terminal for the output. The output can also be saved in a file using the COMO command.

Program Organization

The program flow is documented with comments in the program. In addition, the subroutines and brief description of their function are given in section 2. The file structure and format are also described in section 2.

Maintenance, Modifications

The size of the mesh is fixed at 11 x 66 cells. Note that the input mesh (11x52) is automatically extended by 7 cells in the horizontal direction at either end. The discretization of the mesh can be changed by reading in the appropriate data (subroutine RMDL). All the vertical mesh increments can be changed through RMDL, however, in the horizontal direction only the central portion of the mesh (19 to 34 in Figure 2) can be changed. Further changes in the mesh increments must be made in SPXCPL. The spectrum of Lamda values can only be changed in the main program (SPXCPL).

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