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Effects of Scattered Radiation on Scintillation Detector Response*

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ABSTRACT

A complete analysis of the response of a collimated scintillation detector to an in vivo distribution of radioactivity must include the responses to both unscattered γ -rays and the continuous spectrum of photons with reduced energy, produced by Compton interactions in the tissue. The amount of scattered radiation entering the detector is a function of the γ -ray energy, the distribution of radioactivity in the tissue, and the tissue attenuation coefficient. The ratio of responses to scattered and unscattered photons, called the scatter fraction, S , is also a function of the energy resolution of the scintillation detector, and the base line setting, E_b , of the pulse height analyzer. To evaluate S as a function of E_b , it is necessary to know the shape of the scatter spectrum within the unscattered photopeak region. Smoothed scatter spectra were computed for four γ -energies in the range of 140 to 511 keV by smoothing the Klein-Nishina equation for the energy spectrum of scattered photons. In each case the amplitude of the smoothed scatter spectrum is normalized to, and subtracted from, the pulse amplitude spectrum observed for a brain phantom to obtain the photopeak due to unscattered γ -rays, and a measure of S vs. E_b .

The effects of scattered radiation on the detector sensitivity, line spread function, and modulation transfer function (MTF) are evaluated. In general, sensitivity is decreased and spatial resolution is improved as E_b is increased. When the statistical figure of merit (which is proportional to the product of the sensitivity and the MTF^2) is used as a criterion of system quality, an optimum base line setting can be found for specific application. For brain scanning with γ -ray emitters in the energy range of 140 to 511 keV, the optimum window setting almost precisely brackets the unscattered photopeak.

It is shown that the response to complex objects in a scattering medium can be computed from the line spread function or the modulation transfer function.

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1. INTRODUCTION

In every medical radioisotope imaging procedure the object to be imaged is a three-dimensional distribution of radioactive material, which can be thought of as a low-intensity, self-luminous object embedded in a turbid medium which scatters and absorbs the primary radiation emitted by the object. An adequate theory of radioisotope imaging systems must enable us to predict the images of such objects, including the effects of scattered radiation. The purpose of this paper is to describe concepts and procedures for quantifying the effects of scatter on the sensitivity and spatial resolution, and a criterion for obtaining an optimum compromise between these effects.

In this analysis, everything between the object and image that affects the image in any way is regarded as part of the generalized imaging system; this includes the intervening tissue in which the object is embedded. [The term "source" and phrases such as "scattering in the source" are nevertheless used in the usual sense to refer to the distribution of radioactivity and the medium containing it.] Although this notion may appear bizarre, it greatly simplifies the treatment of scattered radiation, the existence of which is attributed to the scattering medium, and the necessary response to which is attributed to the imperfect energy resolution of the detector. This point of view is developed in more detail elsewhere [1].

The example of brain scanning with systems employing collimated scintillation detectors is used to illustrate the theory, which can be applied to other radioisotope imaging procedures.

¹Computation was accomplished with the assistance of the Biological Sciences Computation Center, University of Chicago, under USPHS Grant FR00013 from the Division of Research Facilities and Resources of the National Institutes of Health.

2. SENSITIVITY

It has been shown elsewhere [2] that a large volume distribution of radioactivity can be reduced to an equivalent sheet distribution, σ [γ -rays emitted/sec-cm²], by taking attenuation into account. The concept of sensitivity, S [cps/ γ -rays emitted per sec-cm²], to be discussed here can be defined by the relation

$$C[\text{counts/sec}] = S\sigma \quad (1)$$

which describes the response to the equivalent sheet σ , including the magnitude of the response to scattered radiation.

2.1. Analysis of the Input Radiation and the Detector Response

The total response of a collimated scintillation detector viewing a large distributed source of radioactive material consists of several components. The situation is depicted in Fig. 1 for brain scanning. The most important component is that due to γ -rays (designated a) which originate within the collimator field of view, penetrate the overlying tissues without interaction, and enter the collimated detector properly. In addition to these, some γ -rays (designated b) interact with the brain tissue, are scattered through small angles, and enter the detector with somewhat less energy (designated b'), while others (designated c and d) are scattered through larger angles, and enter the detector as relatively low energy photons (designated c' and d'). The energy-angle relationship is given [3] by Compton's equation

$$E_s = \frac{E_o}{1 + \frac{E_o}{mc^2} (1 - \cos \theta)} \quad (2)$$

where E_o is the energy of the original γ -ray, E_s is the energy of the scattered photon, θ is the angle of the scattered photon with respect to the direction of the original γ -ray, and mc^2 is the rest mass-energy of an electron, .511 MeV. Thus the scattered photon has minimum energy when it is backscattered through $\theta = 180^\circ$ and maximum energy in the limit when $\theta = 0^\circ$; i.e., when it is unscattered.

If the source is small compared to the mean free path length of the scattered photons, then little multiple scattering (designated e, e', e'') occurs and the source appears, approximately, as an isotropic source of (single) scattered radiation. In that case, the energy spectrum of photons emanating from the patient can be described by an unscattered component of γ -rays and a continuous distribution given by the Klein-Nishina equation [3] which describes the relative abundance of (single) scattered photons. The shape (but not the absolute magnitude) of this distribution is given by

$$f(E_s) = \frac{1}{E_o^2} \left\{ \frac{E_s}{E_o} + \frac{E_o}{E_s} - 2 \left(\frac{mc^2}{E_s} - \frac{mc^2}{E_o} \right) + \left(\frac{mc^2}{E_s} - \frac{mc^2}{E_o} \right)^2 \right\} \quad (3)$$

On the assumption that equation (3) is applicable, the spectrum of photons emanating from a 16 cm diameter cylindrical source of ^{203}Hg (uniformly distributed) is shown just below the schematic of the patient's head in Fig. 1D.

A photon from the source may enter the detector properly, or by penetrating the collimator material (designated p) or by scattering on the collimator walls (designated f). For γ energies below 1 MeV, it is usually possible to limit the penetration component to a negligible level by proper choice of collimator material and septum thickness [4]. In addition, Mather [5] has shown that the collimator scatter fraction is less than .07 for this energy range.

On entering the NaI(Tl) crystal scintillator, the photon energy may (designated g) or may not (designated h) be totally absorbed. The Compton electron component [6,7] (the spectrum of pulses due to Compton interactions in the crystal in which the energy of the scattered electron is absorbed, while the scattered photon (h') escapes from the crystal) can be reduced by using only the central portion of a large diameter, thick crystal.

Under these conditions, the observed spectrum of output pulses consists of three principle components, which can be seen in the pulse amplitude spectrum of ^{203}Hg in Fig. 1C.

1. The geometrical component due to unscattered, properly collimated, and totally absorbed γ -rays which appear in the smoothed photopeak.
2. The scatter component, which is approximated by the smoothed Klein-Nishina spectrum of scattered photons emanating from the source.
3. An X-ray component, characteristic of the collimator material, due to photoelectric interactions of the incoming photons with the collimator walls. The detector used to obtain the ^{203}Hg spectrum in Fig. 1C had a Pb collimator, so that the peak at approximately 74 keV (also in Fig. 2) is due to fluorescent Pb K X-rays (as well as some Tl X-rays from internal conversion).

2.2 Effect of Finite Energy Resolution on the Output Pulse Amplitude Spectrum

If the scintillation detector had uniform sensitivity and perfect energy resolution, the spectrum of output pulse amplitudes would correspond precisely to the spectrum of input photon energies. In reality, the resolution is imperfect and the spectra due to scattered and

unscattered radiation overlap. To determine the fraction of pulses within the photopeak due to scattered radiation, it is necessary to know the shape of the scatter spectrum in the region of overlap.

Assuming that equation (3) is an adequate approximation in this region, the pulse amplitude spectrum due to scattered photons is given by

$$F(E) = \int_{E_b}^{E_0} f(E_s) p(E_s, E) dE_s \quad (4)$$

where $p(E_s, E)$ is the energy spread function of the scintillation detector, which is approximately gaussian in form, with a standard deviation σ_s that is a function of energy [2,8].

We can make use of $F(E)$ to separate the components due to scattered and unscattered radiation in an observed spectrum by normalizing its amplitude to fit the observed spectrum at a point, E_n , just below the unscattered photopeak ($E_n = E_0 - 3.5 \sigma_s$) where it is assumed that the only contribution to the observed spectrum is due to scattered radiation. By subtracting this normalized curve from the observed spectrum only at points $E > E_n$, we obtain the theoretical photopeak of unscattered γ -rays. This is shown in Fig. 1C for ^{203}Hg and Fig. 2 for $^{99\text{m}}\text{Tc}$, ^{113}Sn and ^{85}Sr .

Indirect evidence supporting the assumptions used to obtain $F(E)$ is provided by the fact that these theoretical photopeaks are almost identical to photopeaks obtained with point sources of radioactivity in air. The largest discrepancy occurs for $^{99\text{m}}\text{Tc}$ where the point source spectrum shows the energy resolution to be $R_s = .161$ while the theoretical photopeak shows $R_s = .156$. For the other isotopes the discrepancy in R_s is less than .002.

2.3. The Tissue Scatter Fraction, S

Ordinarily, a pulse height analyzer (PHA) is used in scanning systems to select from the observed spectrum the photopeak due to unscattered γ 's for recording. The upper level discriminator is set just above the photopeak to reject pulses due to cosmic rays or other high energy, uncollimated, background sources. The base line discriminator is usually set below the photopeak to reject some of the scattered radiation.

The tissue scatter fraction, S , is defined by the ratio of scattered to unscattered radiation within the window of the PHA. Thus, if N_s and N_u are the numbers of pulses within the window due to scattered and unscattered photons, respectively, then

$$S \equiv \frac{N_s}{N_u} \quad (5)$$

This corresponds to the ratio of areas under the two spectra within the window.

2.4. The counting Efficiency, Ψ , for Unscattered γ -Rays

As the base line discriminator is raised above E_n , pulses due to unscattered radiation are rejected. If N_p is the number of pulses in the entire photopeak due to unscattered radiation, and N_u is the number of these within the window of the PHA, then we can define the counting efficiency for unscattered γ 's by the ratio

$$\Psi \equiv \frac{N_u}{N_p} = \frac{C_u}{C_p} \quad (6)$$

where C_p and C_u are the corresponding count rates. Both S and Ψ are plotted as functions of base line setting (BLS) in Fig. 3 for ^{203}Hg . Thus, for a BLS = 225 keV, we find $S = .45$ and $\Psi = 1$; for BLS = 254 keV, $S = .24$ and $\Psi = .97$. This shift in BLS reduces the response to scattered radiation by a factor of almost 2, while reducing the response to unscattered γ -rays only 3%.

2.5. The Net Effect of Scattered Radiation on Sensitivity

From equations (5) and (6) it follows that the number of pulses accumulated within the PHA window during a counting time of τ seconds is given by

$$N = N_u + N_s = N_u(1 + S) = C_p \tau \Psi(1 + S) \quad (7a)$$

which can also be written

$$N = C\tau = G(1 + S)\eta\Psi\sigma\tau \equiv \$\sigma\tau \quad (7b)$$

where C is the count rate, G is the geometrical efficiency of the collimator, η is the crystal efficiency, σ is the effective object intensity when attenuation in the tissue is taken into account [2], and $\$$ is the sensitivity of the imaging system.

If no scattered radiation were present, or if the energy resolution were perfect so that all scatter could be rejected, then $S = 0$ and the base line would be set for maximum efficiency, $\Psi = 1$. Equation (7b) indicates that the response to scattered radiation alters the sensitivity of the imaging system by the factor $\Psi(1 + S)$.

3. SPATIAL RESOLUTION

In section 2, attention was focused on the magnitude of the response to scattered photons to determine the effect on sensitivity. The effect of scatter on spatial resolution is determined by the shape of the response function, defined for a point or line element of the object.

3.1. FWHM as a Measure of Resolution

To simplify the specification of the spatial response, various measures of the width of the response function have been used to define resolution. Perhaps the most familiar and widely used measures of resolution are the full width at the half-maximum height (FWHM) of the response to a point source or to a line source. Fig. 4 shows two such response curves, or line spread functions, $L(x)$, obtained from a line source² of ^{99m}Tc in air and in water, at the focal distance (7.5 cm) of a collimator designed for brain scanning with this isotope. The base line was set to 126 keV for these measurements.

The line spread function for the source in air consists primarily of a "geometrical" component, $L_g(x)$, due to γ -rays emitted within the collimator field of view, which has a diameter of $2R = 1.72$ cm, and a small component $L_s(x)$, due to septal penetration. The magnitude of this line spread function has been normalized in Fig. 4 to the magnitude with which $L_g(x) + L_s(x)$ occurs when the line source is in water.

The effect of radiation scattered in the water is to add a component $L_s(x)$ to the (suitably attenuated) spread function measured in air. This increases FWHM by $.85 - .82 = .03$ cm. Thus, if FWHM is used as a measure of spatial resolution one might with some justification conclude that the effect of scattered radiation on resolution is negligible and can be ignored for all practical purposes. However, the question immediately arises as to whether or not we can account for the response to extended sources of radioactivity on the basis of this measure of resolution. The answer appears to be that we cannot, even for quite simple distributions.

One of the simplest extended sources we might use as a test object is the plane step-source³ or edge, analogous to the knife-edge

²The line source consists of ^{99m}Tc in polyethylene tubing, 30 cm in length [(PE-20), I.D. = 0.015", O.D. = 0.043"].

³The plane of ^{99m}Tc was .8 mm thick and 30 cm X 15 cm, in a lucite container.

used in optics. The measured response to the step-source in water in the focal plane is indicated by dots in Fig. 5. The solid curve through these dots indicates the theoretical response to the step-source in water, and the fit is reasonably good. This supports the validity of the theoretical procedure, which is just a convolution of the line spread function with a step function. The same procedure was used to compute the theoretical response to the step-source in air, indicated by the dashed curve in Fig. 5. The two theoretical curves have quite different shapes, which we believe cannot be accounted for by the small difference in resolution, as measured by FWHM. Clearly, the reason is that the principal effect of the response to scattered photons is to add the long tails of $L_s(x)$ to the line spread function measured in air. Although these tails have little effect on FWHM, they have a pronounced effect on the shape of the response to this extended source. From this, we conclude that FWHM is an insensitive measure of the effect of scattered radiation on spatial response or resolution.

3.2. MTF(v) as a Measure of Resolution

A measure of spatial resolution that can be defined for linear systems, and which takes into account the shape (but not the magnitude) of the entire line spread function $L(x)$, is the Modulation Transfer Function, $MTF(v)$, or spatial frequency response [1,2,9]. Briefly, the rationale for this description of spatial response arises in a natural manner from the fact that any object and its image can be described in terms of the amplitudes and phases of their respective spatial frequency components; the $MTF(v)$ is simply a measure of the efficiency with which modulation or contrast at each frequency v [cycles/cm] is transferred by the imaging system from the object to the image. This is analogous to the temporal frequency response of an electronic amplifier or filter.

It is shown elsewhere [1] that $MTF(v)$ can be computed from the line spread function.

$$MTF(v) \equiv \frac{\int_{-\infty}^{\infty} L(x) e^{-2\pi i v x} dx}{\int_{-\infty}^{\infty} L(x) dx} = \frac{MTF_e(v) + P \cdot MTF_p(v) + S \cdot MTF_s(v)}{(1 + P + S)} \quad (8a)$$

(Here, P is the penetration fraction, analogous to the scatter fraction S .)

This function is shown in Fig. 6 (UPPER) for the two line spread functions in Fig. 4. Comparing MTF(ν) curves marked (In Air) and (In Water) it is clear that the principal effect⁴ of radiation scattered in the water is to reduce the efficiency of transfer of modulation or contrast.

Further insight into the MTF(ν) concept can be gained from a brief examination of its components. For example, for the collimator used in this study, $P \ll 1$ for the 140 keV γ -rays of ^{99m}Tc , and can be ignored. In addition, the magnitude of $\text{MTF}_o(\nu)$ and $\text{MTF}_s(\nu)$ depends on the shape, and especially the width, of $L_o(x)$ and $L_s(x)$ respectively, relative to the wavelength $\lambda = 1/\nu$. For an infinitely narrow line spread function (a delta function), $\text{MTF}(\nu) = 1$ for all ν . For a broad flat function such as $L_o(x)$, $\text{MTF}_o(\nu)$ decreases rapidly from a value of 1 at $\nu = 0$ to a value of 0 at $\nu \approx 0.2$ cycles/cm. For $\nu > 0.2$ cycles/cm, $\text{MTF}_s(\nu) \approx 0$ under the conditions for measuring $L(x)$; that is, sinusoidal structures with wave lengths less than $\lambda = 1/.2 = 5$ cm are not resolved by the response to scattered photons. For this frequency range, equation (8a) can be written

$$\text{MTF}(\nu) = \frac{\text{MTF}_o(\nu)}{1 + S} \quad (8b)$$

which indicates that the efficiency with which contrast is transferred from the object to the image is reduced by the factor $1/(1 + S)$ when the object is in water.

3.2.1. The Image of a Square-Wave Distribution, Computed from MTF(ν)

To determine whether or not the MTF(ν) enables us to account for the response to extended sources of radioactivity in a scattering medium, we have computed and measured the response to a plane square-wave distribution of ^{99m}Tc in water at the focal plane of the collimator. These curves are shown in Fig. 7 where the shape of the theoretical curve (solid line) normalized "by eye" fits the observations (dots) reasonably well. We shall outline the procedure by which the theoretical response curve was computed to illustrate the general principles involved in all such calculations.

⁴Note that scatter appears to have only a small effect on the high frequency cutoff point (which is sometimes associated with resolving power), which may be thought of as that frequency, ν_c , at which the MTF(ν) has decreased to some arbitrary level, such as $\text{MTF}(\nu_c) = .04$. In that case, Fig. 6 (UPPER) indicates that $\nu_c \approx 1.12$ cycles/cm (in air) and $\nu_c \approx 1.09$ cycles/cm (in water) for this study. From this small difference we conclude that the high frequency cutoff point is also an insensitive measure of the effect of scattered radiation on spatial response or resolution.

It is well known [10] that a very broad class of mathematical functions can be analyzed in terms of a spectrum of sinusoidal components. The distribution of ^{99m}Tc used in this experiment was in a thin lucite container consisting of strips, 2 cm in width and 30 cm in length, in which there was, alternately, a uniform concentration of radioactivity, and no radioactivity. Such a distribution can be described as a one-dimensional plane square-wave, with a period of 4 cm corresponding to the fundamental wavelength $\lambda_1 = 1/\nu_1$. A periodic function of this sort can be described by a discrete spectrum consisting of the fundamental frequency ν_1 and harmonics of this frequency.

Briefly, the effective intensity of an object distribution that is an even periodic function can be described by an equation of the form

$$\sigma(x) \left[\frac{\text{y-rays emitted}}{\text{sec-cm}^2} \right] = \bar{\sigma} + \sum_{n=1}^{\infty} \tilde{\sigma}(n\nu_1) \cos 2\pi n\nu_1 x \quad (9)$$

where $\bar{\sigma}$ is the average or DC component and the quantities $\tilde{\sigma}(n\nu_1)$ are the amplitudes of the sinusoidal or AC components.

In general, the object modulation or contrast $m_o(\nu)$ is defined by the ratio of AC to DC components.

$$m_o(\nu) = \frac{\tilde{\sigma}(\nu)}{\bar{\sigma}} \quad (10)$$

so that the object can also be described by

$$\sigma(x) = \bar{\sigma} \left[1 + \sum_{n=1}^{\infty} m_o(n\nu_1) \cos 2\pi n\nu_1 x \right] \quad (11)$$

If the imaging system is linear, the image can be described by an equation of the same form as equation (9);

$$N(x) \left[\frac{\text{counts}}{\text{cm}^2} \right] = \bar{N} + \sum_{n=1}^{\infty} \tilde{N}(n\nu_1) \cos 2\pi n\nu_1 x \quad (12)$$

Similarly, the image modulation or contrast $m_i(\nu)$ is defined by the ratio of AC to DC components,

$$m_i(\nu) = \frac{\tilde{N}(\nu)}{\bar{N}} = \frac{\tilde{C}(\nu)}{\bar{C}} \quad (13)$$

so that the image can also be described by

$$N(x) = \bar{N} [1 + \sum_{n=1}^{\infty} m_I(nv_1) \cos 2\pi n v_1 x] \quad (14)$$

The bracketed expressions in equations (11) and (14) describe the shape of the object and its image, respectively; thus, an undistorted image of the object can be formed only if $m_I(nv_1) = m_O(nv_1)$ for all components in the object. Image modulation can also be defined by

$$m_I(v) = \text{MTF}(v) m_O(v) \quad (15)$$

so that the image can also be described by

$$N(x) = \bar{N} [1 + \sum_{n=1}^{\infty} \text{MTF}(nv_1) m_O(nv_1) \cos 2\pi n v_1 x] \quad (16)$$

Comparing equations (11) and (16), it is clear that an undistorted image can be produced only if $\text{MTF}(v) = 1$ for all v in the object.

For the square-wave object distribution,

$$m_O(nv_1) = \frac{4}{n\pi} \sin \frac{n\pi}{2}, \text{ for } n = 1, 3, 5, \dots \quad (17)$$

The first four components in this object modulation spectrum are indicated in the lower portion of Fig. 6, along with the image modulations for the object in air and in water, reduced by the appropriate MTF factors from the curves above. When the corresponding sinusoids are summed, we obtain the theoretical response curves in Fig. 7, where the amplitude of the theoretical curve (in water) has been normalized to fit the points indicating the observed response to the square-wave (in water). The amplitude of the theoretical curve (in air) was then normalized by the same factor. Thus, the lower contrast in the image of the object in water is not the result of reduced average intensity due to attenuation of γ -rays; rather, it is due to reduced $\text{MTF}(v)$ because of scattered radiation.

From this study, it appears that the image of a plane one-dimensional object in a scattering medium can be computed from the line spread function for that plane, or from the associated MTF; it is not evident how, in general, the image can be computed from a measure of the width of the line spread function, such as FWHM.

4. COMPROMISE BETWEEN SENSITIVITY AND RESOLUTION

The effects of scattered radiation on sensitivity and spatial resolution are described quantitatively by equations (7b) and (8) respectively. Since both properties are affected by the base line discriminator setting, the question arises as to where the base line should be set to obtain the best compromise between sensitivity and resolution. The optimum setting might be defined as that which maximizes the likelihood of a correct diagnosis; however, it is not evident how the criterion implied by this statement should be formulated.

Many less ambitious alternative criteria exist, though each is more limited and somewhat arbitrary. To illustrate one approach, we shall use the statistical criterion called the Figure of Merit, Q , which has been discussed in detail elsewhere [2,11]. An outline of the derivation of $Q(v)$ for the detection of sinusoidal plane waves appears in Appendix A. There it is shown that when $MTF_s(v) \approx 0$ (this is true, approximately, for $v > .2$ cycles/cm), the Figure of Merit is maximum when the quantity $(1 + S)/\psi$ is minimum. The latter quantity is the factor by which the amount of radioactivity or the scanning time must be increased to obtain the Figure of Merit of an ideal system, for which $S=0$ and $\psi=1$.

A plot of $(1 + S)/\psi$ vs. base line setting is shown in Fig. 8 for the ^{203}Hg brain phantom. In this case the optimum base line setting is at 254 keV, where $\psi = .97$, and $S = .24$ [see Fig. 3]. For this setting the factor $(1 + S)/\psi = 1.28$, which indicates that the amount of radioactivity or the counting time must be increased by 28% to obtain the Figure of Merit of an ideal system. On the other hand, if the base line were set at 225 keV, one of these factors must be increased by 45%. The optimum setting is shown in Fig. 9 in relation to the spectrum of ^{203}Hg . In Fig. 2, the vertical dashed lines indicate the optimum settings for $^{99\text{m}}\text{Tc}$, ^{113}Sn , and ^{85}Sr .

5. RESULTS

Table I contains the results of the study of scattering in the brain phantom for the four isotopes used. In each case the optimum base line setting results in an asymmetrical window, in qualitative agreement with the findings of Ross et al. [12]. Such settings increase the need for stable electronic components.

In general, the scatter fraction increases with decreasing γ -energy, the range being $S = .31$ to $.68$ for 510 and 140 keV γ -rays, respectively, when symmetrical window settings are used; for optimum, asymmetrical settings, the range is $S = .18$ to $.38$ for 510 and 140 keV respectively.

A plot of the optimum base line setting vs. γ -energy is close enough to a straight line that optimum settings for intermediate γ -ray energies might be obtained by interpolation.

In a previous study [13,14] of the Figure of Merit vs. γ -energy for detecting midline brain lesions, the effect of scattered radiation was not considered; thus the results applied to ideal systems, with $S = 0$ and $\Psi = 1$. When scatter is considered, Q is reduced by the factor $\Psi/(1 + S)$. In this case, it appears in Fig. 10 that the γ -energy for which the Figure of Merit is maximum is shifted from ~ 100 keV (when $S = 0$) to ~ 150 keV (when $S \neq 0$).

It should be noted, however, that this conclusion is reached on the assumption of equal numbers of γ -rays emitted per second per cm^3 ; i.e., equal values of ρ . A more significant comparison might be based on the assumption of equal deleterious effects; that is, values of ρ in the organ to be imaged which result from quantities of isotope that produce what are regarded as equally deleterious radiation dosages to the whole body or to a critical organ, whichever is limiting. Studies of this type have been carried out by Kuhl [15] and Matthews [16].

In any attempt to generalize these results to include other detectors and source distributions, we must keep in mind that the magnitude of S is related to γ -energy in a complex, twofold way. We can think of S as determined by two factors: (1) the degree of overlap of the spectra of scattered and unscattered radiation, and (2) the relative amounts of scattered and unscattered radiation leaving the source. These factors are affected by: the γ -energy, the energy resolution, and source size, but are otherwise unaffected by the crystal or collimator. For example, in a more extensive study using a 3-5/8" diameter X 1-3/4" thick NaI(Tl) crystal, four collimators having quite different characteristics resulted in almost identical scatter fractions for the ^{203}Hg brain phantom, all falling within the range $S = .23 \pm .01$. ($S = .24$ for ^{203}Hg with the 5" diameter X 5" thick crystal used in the study summarized in Table I.)

In general, more accurate results could be obtained by replacing equation (3) with an equation which takes into account multiple scattering [17].

6. SUMMARY

A radioisotope imaging system can be described in terms of its sensitivity and resolution, both of which are affected by septal penetration, and scattering within the patient. The effect of scattered radiation on these properties depends on the magnitude and shape, respectively, of this component of response. The compromise between sensitivity and resolution is regarded as optimum with respect to base line setting when the quantity $(1 + S)/\Psi$ is minimum.

The sensitivity of the imaging system is given by equation (7b) and the spatial resolution is described by the modulation transfer function in equation (8).

These properties enable us to account for the images of certain simple objects embedded in a scattering medium. For example, a periodic one-dimensional plane wave distribution of radioactivity can be described by its Fourier series, which can be written in terms of the effective average intensity and the modulation of its frequency components, as in equation (11). Similarly, the image of this object can be described by its Fourier series, which can be written in terms of the mean count density and the modulation of its frequency components, as in equation (14).

The relations between these quantities can be summarized in two equations which relate to magnitude and shape: If $t[\text{sec}/\text{cm}^2]$ is the observation time per unit area,

Object Intensity (Mean) X Sensitivity X t = Image Density (Mean)

$$\bar{\sigma} \times \bar{\phi} \times t = \bar{N} \quad (18)$$

and

Object Modulation X Modulation Transfer Function = Image Modulation

$$m_o(v) \times \text{MTF}(v) = m_I(v) \quad (19)$$

These equations can be combined to form

Object Spectrum X Spectral Sensitivity X t = Image Spectrum

$$\bar{\sigma} \cdot m_o(v) \times \bar{\phi} \cdot \text{MTF}(v) \times t = \bar{N} m_I(v) \quad (20)$$

or

$$M_o(v) \times \bar{\phi}(v) \times t = M_I(v)$$

This formulation is intended only for objects that are periodic one-dimensional plane wave distributions which have Fourier series representations and has been illustrated by the square wave distribution. A program for generalizing the formulation might include objects that are periodic two-dimensional distributions in a plane, nonperiodic one- and two-dimensional distributions in a plane, and volume distributions.

APPENDIX A

The Figure of Merit Criterion

Let C_t and C_o be the true mean count rates over two regions which we may wish to distinguish. The signal to be detected is the true mean difference $(C_t - C_o)$. The noise accompanying this signal is simply the random fluctuations in the signal itself, which can be measured by the

standard deviation of the difference, $S.D.(C_t - C_o) = \sqrt{\frac{C_t + C_o}{\tau}}$, where τ is the observation time for C_t and for C_o . The criterion for detection is that the signal to noise ratio be equal to or greater than some assigned value;

$$SNR = \frac{SIGNAL}{NOISE} = \frac{(C_t - C_o)}{\sqrt{\frac{C_t + C_o}{\tau}}} \geq 2z \quad (21)$$

The observation time required to accumulate enough counts to resolve C_t and C_o with a statistical reliability associated with z standard deviations (treating false positives and false negatives as equally serious) is

$$\tau \geq \frac{4z^2(C_t + C_o)}{(C_t - C_o)^2} = \frac{4z^2}{Q} \quad (22)$$

This equation defines Q , which can be written in several useful forms:

$$Q = \frac{(C_t - C_o)^2}{(C_t + C_o)} = \frac{C_o \left(\frac{C_t}{C_o} - 1 \right)^2}{\left(\frac{C_t}{C_o} + 1 \right)} = \frac{(SNR)^2}{\tau} = \frac{4z^2}{\tau} \quad (23)$$

This Figure of Merit is a measure of the rate at which (Signal to Noise Ratio)² is built up by the imaging system. Alternatively, Q can be described as the statistical reliability that can be achieved per unit of observation time for distinguishing between two count rates, C_t and C_o . In particular, if C_t and C_o are the true mean maximum and minimum count rates in response to a sinusoidal plane wave of radioactivity with frequency ν , then $Q(\nu)$ can be interpreted as a measure of the detectability of object sinusoids. In that case

$$Q(\nu) = (C_t + C_o) \left[\frac{C_t - C_o}{C_t + C_o} \right]^2 = 2\bar{C}[m_I(\nu)]^2$$

$$= 2\bar{\sigma} \cdot m_o^2(\nu) MTF^2(\nu) = 2\bar{\sigma} \cdot m_o^2(\nu) \times \bar{\$} \cdot MTF^2(\nu) \quad (24)$$

In the last form, the factors relating to the object and to the imaging system have been separated, and can be used independently.

Making use of equation (7b) for sensitivity, $\bar{\$}$, (generalized to include the penetration fraction) and equation (8a) for $MTF(\nu)$, we can write

$$Q(\nu) = \frac{2G\eta\bar{\sigma}m_o^2(\nu)[MTF_o(\nu) + P \cdot MTF_p(\nu) + S \cdot MTF_s(\nu)]^2}{1 + P + S} \quad (25)$$

For an ideal imaging system (with respect to penetration and scatter) $P = 0$, $S = 0$, $\Psi = 1$ and $Q(\nu) \equiv Q_I(\nu)$.

$$Q_I(\nu) = 2G\eta\bar{\sigma}m_o^2(\nu)MTF_o^2(\nu) \quad (26)$$

Such a system is superior to a real system by the factor

$$\frac{Q_I(\nu)}{Q(\nu)} = \frac{(1 + P + S)}{\Psi \left[1 + P \frac{MTF_p(\nu)}{MTF_o(\nu)} + S \frac{MTF_s(\nu)}{MTF_o(\nu)} \right]^2} \quad (27)$$

Near $\nu \approx 0$, the wave length $\lambda = \frac{1}{\nu}$ is very long compared to the width of all components of $L(x)$. As a consequence, $MTF_o(\nu) = MTF_p(\nu) = MTF_s(\nu) \approx 1$, and

$$\frac{Q_I(\nu)}{Q(\nu)} = \frac{1}{\Psi[1 + P + S]} = \frac{\bar{\$}_I}{\bar{\$}} < 1; \quad (28)$$

that is, the system we have called "ideal" is actually inferior to a real system by a factor equal to the ratio of their sensitivities. This simply means that, so long as penetration and scatter do not degrade

MTF(ν) very much, the contribution of these responses to sensitivity increases $Q(\nu)$. However, this is not the range of ν that is of primary interest to nuclear medicine, where structures having dimensions of a few centimeters must be detected.

Collimators for radioisotope imaging systems are typically designed to have a geometrical diameter of view in the range of 1 to 2 times the diameter of the object to be detected. In that case, $MTF_o(\nu)$ is typically < 1 over most of the object spectrum, but what is more important is the fact that $MTF_p(\nu)/MTF_o(\nu) \ll 1$ and $MTF_s(\nu)/MTF_o(\nu) \ll 1$ over most of the object spectrum. In this case equation (27) is reduced to

$$\frac{Q_I(\nu)}{Q(\nu)} = \frac{(1 + P + S)}{\Psi} \quad (29)$$

The conclusion from this is that, insofar as the penetration and scatter responses do not resolve the object structures, but contribute only to sensitivity, they are detrimental. Equation (29) is a generalization of the criterion for setting the base line discriminator. If the collimator septa are so thick that $P \ll 1$, these criteria are equivalent.

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Fig. 1.--From a collimated scintillation detector (A) viewing a distributed source of radioactivity (B) we obtain an output pulse amplitude spectrum (C) that is a smoothed version of the input spectrum (D), which consists of unscattered γ -rays and a continuous spectrum of scattered photons. The dashed line in C is an observed spectrum of ^{203}Hg . The solid lines are theoretical constructs.

Fig. 2.--Spectra observed from uniform volume distributions of radioactivity (16 cm in diam. X 16 cm high). A large detector [5" diam. X 5" thick NaI(Tl)] was used for A, B, and C. Spectrum D was obtained with a small detector [3-5/8" diam. X 1-3/4" thick NaI(Tl)] having better energy resolution. Note the prominent Compton electron spectrum in D compared to C, due to escape of photons scattered in the small crystal.

Fig. 3.--Both the scatter fraction, S, and the counting efficiency, Ψ , decrease with increasing Base Line Energy Setting. The dashed line indicates the optimum setting, based on Fig. 8.

Fig. 4.--The width of $L(x)$ for the line source in air is slightly greater than $2R$ (the diameter of the collimator field of view) due to the penetration component, $L_P(x)$. When the line source is in water, the width of $L(x)$ is greatly extended by the tails of $L_S(x)$, which describes the response to scattered photons. $L_S(x)$ has little effect on FWHM. The base line was set at 126 keV for both measurements.

Fig. 5.--The shape of the response to a step-source in water can be predicted from the shape of $L(x)$ measured in water, but not from $L(x)$ measured in air or from FWHM.

Fig. 6.--(UPPER) $MTF(\nu)$'s computed from $L(x)$'s in Fig. 4. Long wavelength sinusoids are well resolved by all components of $L(x)$; hence, $MTF(0) = 1$ for the line source in air and in water. For $\nu > 0$ the $MTF(\nu)$ for the line source in water is reduced by the response to scattered radiation.

(LOWER) The modulation for the first four sinusoidal components of a square-wave distribution of ^{99m}Tc at the focal distance is shown reduced in the image by the corresponding value of $MTF(\nu)$ above.

Fig. 7.--The shape of the image of a square-wave object is found by summing the responses to its frequency components, depicted in Fig. 6 (LOWER). The lower image contrast for the square-wave in water than in air is due entirely to the lower $\text{MTF}(\nu)$, as shown in Fig. 6 (UPPER). Normalization has eliminated the lower average response due to attenuation in the water.

Fig. 8.-- $\frac{1+S}{\Psi}$ is the factor by which the radiation dosage or the scanning time must be increased to obtain the Signal to Noise Ratio of an ideal system ($S = 0$, $\Psi = 1$). The optimum base line setting minimizes this factor.

Fig. 9.--Using the optimum base line setting, a portion of the unscattered photopeak is rejected (dark shading), and a portion of the scatter spectrum is counted (light shading) so that $\Psi = .97$, $S = .24$ for ^{203}Hg .

Fig. 10.--When the scattered radiation is not considered ($S = 0$) the theoretical Figure of Merit for brain scanning (2.5 cm diam. midline lesion, with an uptake ratio of 2 to 1, et cetera) is maximum at ~ 100 keV. When scatter is taken into account ($S \neq 0$) the Figure of merit is reduced by the factor $\frac{\Psi}{1+S}$ and is maximum at ~ 150 keV.

Isotope	γ Energy (keV)	Energy Resolution R_t	Base Line Setting, E_n , for counting Entire Unscattered Photopack (keV)	Scatter Fraction S for E_n Setting	"Optimum" Base Line Energy Setting, E_{opt} (keV)	Scatter Fraction S for E_{opt} Setting	Counting Efficiency for Unscattered γ 's, ψ , for E_{opt} Setting	$\frac{1+S}{\psi}$ for E_{opt} Setting
Tc^{99m}	140	.161	106	.66	126	.38	.95	1.45
Hg^{203}	279	.130	225	.45	254	.24	.97	1.28
Sn^{113}	393	.118	325	.38	357	.21	.97	1.24
Sr^{85}	510	.110	430	.31	467	.18	.97	1.21

Table I.--Summary of results from an analysis of the spectra in Figs. 2A, B, and C, and Fig. 9 using a 5" X 5" NaI(Tl) crystal.