**MATHEMATICAL MODELS
FOR UNDERGROUND INJECTION
OF GASEOUS WASTES
INTO THE VADOSE ZONE**

DECEMBER 1968

**AEC RESEARCH &
DEVELOPMENT REPORT**

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PACIFIC NORTHWEST LABORATORY

RICHLAND, WASHINGTON

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MATHEMATICAL MODELS FOR UNDERGROUND INJECTION
OF GASEOUS WASTES INTO THE VADOSE ZONE

by

L. G. King

Water and Land Resources Department
Environmental and Life Sciences Division

December, 1968

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ABSTRACT

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MATHEMATICAL MODELS FOR UNDERGROUND INJECTION
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INTRODUCTION

The mathematical models described in this report were developed as part of a research program to investigate the feasibility of underground disposal of gaseous wastes in the event of a nuclear reactor incident. The benefits from underground disposal of radioactive gases are due primarily to the spreading effect of the flow-system and the delay in reaching ground surface which allows for radioactive decay to significantly reduce activity reaching the atmosphere. The models as developed can be applied to two quite different philosophies of disposal: intermittent pulsed disposal or a single disposal. The latter type lends itself most readily to the case of a reactor incident. The intermittent pulsed disposal could prove satisfactory for more routine disposal of small volumes of gaseous wastes.

The physical system under consideration consists of a single well injecting gas into the partially saturated materials above the regional water table. The models apply only during the injection period when the flow is caused by the pressure driving force arising from the pumping of gas down the well. At the cessation of pumping, the results should give the distribution of gas in the ground. In the present state of model development, further movement of gas must be estimated by other means.

The report shows the development of the basic partial differential equations governing the flow of gas through the ground during the injection period. The different models resulting from solution of the boundary value problems are described and the underlying assumptions are given. Results are given in the appendices in graphical form for a rather wide range of conditions. An example of how these graphical results might be applied to a hypothetical disposal situation is given. Then the models are applied to two different field tracer tests conducted at the National Reactor Testing Station (NRTS) in Idaho. Suggestions for further research work needed for more complete model development and verification are given.

SUMMARY AND CONCLUSIONS

As part of a research program to investigate the feasibility of underground disposal of gaseous wastes in the event of a nuclear reactor incident, development of mathematical models for describing movement of gas through the ground was undertaken. The gas is assumed compressible and the flow is treated as isothermal and steady. Initial transient movement at the very beginning of injection is neglected. The injection is assumed to take place above the regional water table into partially saturated materials through a single well. Movement of soil moisture is considered as being insignificant in its effect upon the gas flow.

Three essentially different models were developed depending upon the type of media into which the gas is to be injected. The three models involve injection into media which, with respect to permeability, can be assumed to be homogeneous and isotropic, layered but homogeneous and isotropic within each layer, and homogeneous and anisotropic, respectively. Results for the homogeneous, isotropic model are presented graphically for a rather wide range of conditions. An example is presented showing the use of the graphical results in a hypothetical disposal situation.

The models were used to analyze results of two types of field tracer tests conducted at the National Reactor Testing Station (NRTS) in Idaho. The first test, using helium as a tracer, was conducted in shallow alluvium deposits of the Big Lost River. In the second test, Xe-133 was used as a tracer and the injection was into fractured basalt. The traced gas entered the basalt between 110 and 122.5 feet below ground surface.

For the helium test, the homogeneous, isotropic model gave results which compared satisfactorily with field measurements of flow rate and travel times. Problems with inexact matching of model and field geometry at the injection point raised doubt as to complete model verification and did not warrant attempts to model the helium test with the other two models.

For the xenon test, the homogeneous, isotropic model was not successful. It estimated gas movement much too rapidly in the vertical and too slowly in the horizontal direction. The layered model was also unsuccessful in that it gave too rapid movement in the vertical direction relative to the horizontal direction. The homogeneous, anisotropic model generally described the measured gas movement, although uncertainties of parameters precluded complete model verification.

It was concluded that further research should be undertaken to verify the models and to establish the importance of dispersion in flow of gas in the field. Although reasonable descriptions of the gas flow in the tracer tests were obtained, the models and methods of determining the needed parameters from the field require further development before they can be used for prediction with a high degree of confidence. However, the results should prove an aid to understanding the gas movement and should be of value for engineering estimates in cases where complete studies have not been undertaken.

DISCUSSION

GENERAL DESCRIPTION OF MATHEMATICAL MODELS

The basic approach to development of models for describing the movement of gas following its injection into the ground has been one of initially using simple models and progressing to more complicated ones as necessary. The physical system consists of a single well structure discharging gas into the vadose zone above the water table. (See Figure B-1 in Appendix B.) Any moisture existing in this zone is assumed immobile for the duration of the injection period. All models have been developed under the assumption that the flow of gas is both isothermal and steady; i.e., any initial transient motion of gas caused by startup of injection or variations of injection rate is neglected. The models are valid only during the injection period when the injection pressure creates the driving force. After injection ceases, the gas is moved by other phenomena such as diffusion, barometric pressure fluctuations, thermal forces, etc. This report does not consider such post-injection movement.

The models and their development are briefly described in this section. Details of development and graphical results for a rather wide range of variables are given in Appendices A through G.

The basis for the models is a differential equation obtained by combining Darcy's equation with the equation describing the conservation of mass for the steady flow of a compressible fluid. For the case when the flow is assumed in response to a gradient of potential energy with effects of gravity included, the resulting partial differential equation is

$$\text{div grad } \phi + \frac{2\alpha}{\gamma} [\text{grad } \phi \cdot \text{grad } \phi - g \frac{\partial \phi}{\partial z}] + \text{grad } \ln k \cdot \text{grad } \phi = 0 \quad (\text{A-26})$$

where the symbols are defined in Appendix A. Equation A-26 is a nonlinear equation which, in general, must be solved by a numerical method such as finite differences. Boundary value problems using Equation A-26 were solved with the aid of a computer program called STEADY⁽⁵⁾ after slight modification to accommodate the above type of nonlinearity. Application of this model is discussed later in this report (page 41). Note that for this model the porous medium is assumed to be isotropic with respect to permeability but the permeability may vary throughout the space (i.e., heterogeneous, isotropic media).

A second model is developed by neglecting the effects of gravity and assuming that the flow is in response to the gradient of pressure. An additional assumption that the porous medium is homogeneous results in Laplace's equation in the square of the absolute pressure

$$\text{div grad } P^2 = 0 \quad (\text{A-32})$$

Results from solution of boundary value problems involving Equation A-32 apply to homogeneous, isotropic media.

A third type of model is one in which the medium is assumed homogeneous but anisotropic with respect to permeability. In Appendix D it is shown that, for the coordinate system considered in this report, flow problems in anisotropic media can be solved by transforming the system into an equivalent isotropic system and completing the solution in the homogeneous, isotropic system. Hence, the models derived from Equation A-32 are treated in considerable detail in this report.

The actual mathematical model based on Equation A-32 involves further assumptions which allow the solution of the boundary value problem to be obtained as rapidly converging infinite products. The additional assumptions are discussed in Appendices B and C and are briefly listed here. Both the ground surface and the water table are assumed to be horizontal. The water table is a boundary to gas flow. The system is symmetric about the axis of the well. The gas is injected at constant pressure and, at ground surface, the pressure remains constant at atmospheric pressure. The well casing above the injection length is ignored. The shape of the injection length of the well is assumed as conforming to the shape of an equipressure line from a line source. This equipressure line passes through the outer radius of the injection length at the mid-depth of the injection length. An equipressure surface formed by revolving such an equipressure line about the well axis is nearly cylindrical in shape for any significant length-to-diameter ratio of the injection structure.

The flow rate, Q_s , in SCFM for the homogeneous, isotropic model is given by

$$Q_s = [1.17 \times 10^5] [A(T)] [B(p_o')] (kD) \quad (C-39)$$

where $A(T)$ and $B(p_o')$ are given on pages E-2 and E-3, respectively, of Appendix E. The dimensionless flow rate, $\dot{Q}$, is shown graphically on pages E-4 through E-15. k is the permeability in square feet (1 darcy = 1.09×10^{-11} square feet) and D is the depth in feet from ground surface to the water table. Thus, to determine total flow one must know L/D , H/D , and R_o/D (See Figure B-1), k , D , T (the temperature), and p_o' (the

injection pressure in psig).

If, during the injection period, the gas begins to enter the atmosphere at the ground surface, the graphs of Appendix F will be of value for determining the flow variation. Pages F-2 through F-9 give the fraction of the total flow that enters the atmosphere within a circle of radius R from the injection well. Pages F-10 through F-17 give the dimensionless flow rate per unit area (flux) entering the atmosphere as a function of the radius R from the injection well. Note that the values of dimensionless flux must be multiplied by the total flow rate in SCFM and divided by D^2 in order to obtain the actual flux in SCFM/ft².

It is expensive to determine the general pattern of travel time throughout the entire system for any range of variables. The dimensionless travel time depends not only upon R_0/D , L/D , and H/D , but also upon the injection pressure and the location (r/D , z/D) to which the time refers. Travel times must be calculated along flow paths, and although the path and the time of travel may be calculated simultaneously with a computer program called STREAM⁽⁴⁾, many paths are required to determine the entire pattern. For travel along the path $r = 0$ from the source back to ground surface, the path is known and calculations are simplified. This path gives the minimum time for gas travel back to ground surface for a given system. As discussed in detail in Appendix C, the minimum travel time can be determined from

$$t = \left(\frac{\mu f D^2 F(p_0')}{(1.555 \times 10^5) k} \right) b \quad (C-57)$$

where t is the minimum travel time in hours, μ is the viscosity shown graphically on page G-3 of Appendix G, $F(p_0')$ is given on page G-2, f is

effective porosity (expressed as a fraction), k is permeability in square feet, D is in feet, and b is given to within $\pm 5\%$ over the range of p_o' from 1 to 20 psig in graphical form on pages G-4 through G-15.

As an example of the application of this model, suppose it is desired to discharge 1,500,000 SCF of gas over a 24-hour period into a formation with $D = 200$ feet through a 12-inch diameter well at an injection pressure of 5 psig. Thus, $Q_s = 1050$ SCFM and $R_o/D = 2.5 \times 10^{-3}$. Assume that the following are also known: $k = 1.09 \times 10^{-10}$ ft² (10 darcys), $T = 20^\circ\text{C}$. Thus, from page E-2, $A(T) = 6,180$; from page E-3, $B(p_o') = 172$. Using the above values in Equation C-39 shows that for the desired conditions, $\dot{Q} = 0.387$. From page E-8, it is seen that this condition is met by a rather narrow range of H/D values and quite a wide range of L/D values. If it is desired to maximize the time of travel of gas back to ground surface, L/D should be as large as possible; i.e., $L/D = 0.68$ and $H/D = 0.32$. Values of $L/D = 0.6$ and $H/D = 0.3$ are also acceptable for obtaining the desired flow rate. Appendix G can be used with Equation C-57 to estimate the minimum time for gas to reach ground surface. Suppose $f = 0.2$. From page G-2, $F(p_o') = 0.0128$; from page G-3, $\mu = 3.784 \times 10^{-7}$; and from page G-8, $b = 12.9$ for $L/D = 0.68$ and $H/D = 0.32$. Hence, from Equation C-57, the minimum travel time under these conditions is about 29 hours. If $L/D = 0.6$ and $H/D = 0.3$, $b = 10.2$ and the minimum travel time is about 23 hours. For the remaining calculations of this example, values of $L/D = 0.6$ and $H/D = 0.3$ will be used.

In this example we know that $Q_s = 1050$ SCFM and $D = 200$ feet. Thus, $Q_s/D^2 = 2.6 \times 10^{-2}$ and the dimensionless flux values on page F-12 must be

multiplied by 2.6×10^{-2} in order to obtain the flux entering the atmosphere at ground surface in SCFM/ft². The flux has a maximum value at the well of about 9.6×10^{-3} SCFM/ft² and decreases to about 2.1×10^{-3} SCFM/ft² at $R = 200$ ft. If arrival time at ground surface were known as a function of the radius from the well, such information could be combined with the above to provide input to various mixing models with the atmosphere. The arrival times are obtained by calculating the general pattern of travel times in the system.

In calculation of the general pattern of flow, the graphs of Appendix F, such as page F-6 for this example, are helpful in starting numerical calculation of streamlines so as to obtain any desired flow between streamlines. Figure 1 shows the pattern for this example. Equation C-54 must be used to obtain the factor to convert $\dot{t}$ into travel time in hours. Equation C-54 becomes $t = 9.07 \dot{t}$ for this example.

Figure 2 gives the dimensionless travel time $\dot{t}$ at ground surface as a function of R/D (labeled curve 1). This curve and the graph on page F-12 provide the basis for a simple atmospheric mixing model. For the one-day injection of this example, the corresponding dimensionless travel time is $\dot{t} = 2.65$. Assume that, after 24 hours, clean air is pumped at the same rate. Curve 2 of Figure 2 gives the arrival of clean air at ground surface. For any given time, curves 1 and 2 bound the contaminated air emerging at ground surface. The amount of contaminated air entering the atmosphere per unit time is given by the average flux between R_1 and R_2 times the area bounded by R_1 and R_2 . Assume that this air is completely mixed with h feet of atmosphere above ground surface and transported away at a net wind speed v , and that the mixing occurs only with the air directly above the emerging contaminated

FIGURE 1. Flow Pattern with Lines of Equal Time of Travel, $\dot{t}$, Superimposed

FIGURE 2. *Dimensionless Travel Time to Ground Surface as a Function of R/D*

air. The mixing can be estimated to take place in a volume equal to h times the area bounded by R_1 and R_2 for an average time of $2 R_1/v$.

Thus, the dilution may be expressed as

$$\frac{(\text{flux})(\text{Area}) 2 R_1}{(\text{Area}) h v} = \text{Dilution}$$

If we consider radioactive decay, then the average activity $\bar{A}$ as seen by an observer sufficiently far downstream may be expressed as a ratio of the injected activity A_0 as

$$\frac{\bar{A}}{A_0} = (e^{-\lambda t})(\text{Dilution})$$

or since we use dimensionless terms

$$\frac{\bar{A}}{A_0} = e^{-\lambda t} \left[\frac{(\text{Ave flux}) 2 R_1/D}{h v} \right] \frac{Q_s}{D^2} D$$

or

$$\frac{\bar{A}}{A_0} = \frac{2 Q_s e^{-\lambda t} (\text{Ave flux})(R_1/D)}{D h v}$$

where the Ave flux is given by

$$\text{Ave flux} = \frac{\int_{R_2/D}^{R_1/D} \text{flux } d(R/D)}{R_1/D - R_2/D}$$

and flux denotes the dimensionless flux given on page F-12 for this example. Assume that the injected gas were Xe-133 with a half-life of 5.27 days, $\lambda = 0.1315$ for t in days. Further assume that $h = 3$ feet and $v = 0.1$ MPH or 8.8 ft/min. The procedure for using Figure 2 and

the graph on page F-12 is as follows: Choose a value of t in days and compute the corresponding $\bar{t}$. Enter Figure 2 to determine R_1/D and R_2/D . Using these values in the graph on page F-12, determine the average flux. Compute $\bar{A}/A_0$ for the chosen value of t and repeat. Results of this mixing model are shown in Figure 3. It should be pointed out that dispersion has been neglected in all the calculations. Also, the assumed mixing model is simple and approximate. The curves resulting from application of the homogeneous, isotropic flow model provide a basis for other atmospheric mixing models. The results of Appendices E, F, and G and Figure 1 and Curve 1 of Figure 2 are independent of the assumed atmospheric mixing model.

GENERAL DESCRIPTION OF NRTS FIELD TESTS

The models described in this report have been applied in connection with field tracer tests conducted at the National Reactor Testing Station (NRTS) in Idaho. Two types of tests will be considered here: helium tests and xenon tests. The helium tests were conducted in shallow alluvium above the basalt while the xenon tests involved injection of gas into the fractured basalt. This section gives a general description of the field tests and summarizes the measured data. Application of the models is discussed in the next section of this report.

Helium Tests

The helium tests were designed to investigate the feasibility of small-scale disposals in the alluvium. Battelle-Northwest was not involved in the design or conduction of the tests, but was asked to help evaluate the tests after all data were collected.

FIGURE 3. Results by Assuming Mixing Only in Lower 3 Feet of the Atmosphere with No Lateral Mixing and an Average Wind Speed of 0.1 MPH for One-Day Pulse Injection of Xe-133 Followed with Continuous Pumping of Clean Air

The test site was located in the central part of NRTS in soils deposited by the Big Lost River⁽³⁾. A hole, 8 inches in diameter and 23 feet deep, was drilled with a power auger through the alluvium to the surface of the basalt. It was impossible to maintain an open hole below the 14-foot depth because of caving of dry unconsolidated sands and gravels. A 4-inch steel casing was installed to this depth and packed to 12.7-foot depth with 2-inch gravel. Cement was poured into the annular space between the casing and the hole wall, forming a grout throughout the entire cased depth. Sixteen hand-augered observation holes were drilled symmetrically about the injection hole as shown in Figure 4. Also shown in Figure 4 are the locations of holes from which soil samples were taken after completion of the tracer tests. From the soil sample holes, a total of 21 disturbed samples were taken for laboratory measurements. In the laboratory, values of air permeability were measured at the field moisture content. The field moisture content ranged from 4.1 to 14.1, with an average of 8.6 percent by volume. The total porosity of the samples ranged from 0.409 to 0.592 with an average of 0.504. On 2 of the 21 samples the laboratory reported that the permeability was "too high to measure on our apparatus." On the remaining 19 samples, the measured values of air permeability ranged from 1.34 to 4.44 with an average of 2.50 darcys.

Two separate injection tests were performed in the alluvium using air traced with helium. In each test, the injection of gas was intermittent.⁽³⁾ Since the models apply only to continuous injection of gas, only the first period of each test can be modeled. The two tests referred to here are called Part I and Part II of Test No. 3 by Mudra and Schmalz⁽³⁾.

FIGURE 4. Layout of Holes for Helium Tests at NRTS

For the first period (285 min) of Part I, the average measured flow rate was 149.0 SCFM, the well head pressure varied from 11.4 to 12.2 psig with an average of about 12.0 psig, the temperature of injected air was measured only over the last two-thirds of the first period and varied between 66 and 75°F. For the first period (218 min) of Part II, the average measured flow rate was 141.8 SCFM, the well head pressure varied from 12.0 to 12.4 psig with an average of about 12.2 psig, the temperature of the injected air varied from about 108 to 112°F.

Samples of gas were withdrawn periodically at each of the observation holes shown on Figure 4. For Part I, the average first-arrival time measured at the 10-ft radius was about 17 minutes and at the 25-ft radius it was about 76 minutes. For Part II, the corresponding arrival times were about 28 and 89 minutes, respectively. During the continuous injection period, no helium was detected at the 50- and 100-ft radii in either test.

Xenon Tests

It was anticipated that a large quantity of gas could be injected into the fractured basalt above the water table before any significant amount began to emerge at ground surface. The xenon tests were designed with this thought in mind. After the desired quantity of gas was injected, the pumping was stopped and the injection well was sealed off. Thus, the artificial pressure driving force was removed such that further movement of gas would take place under natural convective and diffusive phenomena.

The site was chosen in the Birch Creek Playa near Test Area North in the northern part of NRTS. In this area, the water table exists at about 205 feet below ground surface. The surface of the playa deposits is extremely flat and the basalt-playa interface is on the order of 40 to 60 feet deep.

Since the original concept for this test was that the playa deposits would act as a retarding layer, the first well was drilled where the playa deposits were thickest. Preliminary tests on the first well, No. 6 (see Figure 5), indicated that the formation was tight and that the well would not be satisfactory for injection of the desired quantity of gas. Well No. 7 was drilled and rejected as the injection well for similar reasons. Well No. 8 was then drilled and on the basis of well logs and preliminary tests, it was selected for further development as the injection well. The remaining wells shown on Figure 5 were drilled as observation wells. In each of the observation wells, sampling ports were established as indicated in Table I. Details of construction of the sampling ports are described by Schmalz⁽⁷⁾. In addition to the sampling ports, geiger tubes were installed in Wells 7, 18, 19, and 20.

The injection well, No. 8, was tested by pumping air into various zones before the well was completed for the tracer injection tests. The total depth of the well was 203 feet. Packers were used to test the following zones: 170'-203', 150'-170', 130'-150', 110'-130', and 110'-203'. Results of the air injection tests are given in Table II. Since the results indicated that the 110'-130' zone was very permeable compared to the rest of the hole, the injection of tracer was limited to this zone by filling the hole with sediments to 131 feet and with a plug of expanding cement between the depths of 131 and 122.5 feet. The hole was cased to the

FIGURE 5. Layout of Holes for Xenon Tests at NRTS

TABLE I

Location of Sampling Port for Subsurface Samples
During Xenon Tracer Test

Well No.	Radial Distance from Injection Well (feet)	Depth Basalt- Playa Interface (feet)	Depth of Sampling Port Below Ground Surface (feet)			
6	1,372	55	10	57		
7	798	38	10	35	63	103
9	1,651	53	10	54	80	
10	787	46.5	10	44	80	
11	782	48	10	46	120	
12	1,200	47	10	49	80	
13	1,006	8	7	60	120	
14	394	51.5	5	10	57	
15	200	47	5	10	53	
16	101	40	5	10	26	63
17	50	45	5	10	41	65
18	415	50	10	118	164	190
19	408	48	10	104	139	190
20	400	8	7	69	111	190

Note: Depth to Basalt-Playa interface at the injection well (Well No. 8)
is 50 feet.

TABLE II

Air Injection Tests on Well No. 8 Prior to Final
Construction for Xenon Tracer Test

<u>Test No.</u>	<u>Zone Tested</u>	<u>Ave. Hole Dia. from Caliper Log (inches)</u>	<u>Approximate Formation Pressure (psig)</u>	<u>Flow Rate (SCFM)</u>
1	170'-203'	8.25	86	360
2	150'-170'	8.5	37 54 76	345 450 575
3	130'-150'	8.5	40 51 60	415 520 830
4	110'-130'	8.8	<1 2 4-6	340 650 1,070
5	110'-203'	8.5	0.75 0.9 1.8 2.5	415 515 860 1,030

110-foot depth so that the final injection length was 12.5 feet. Full details of the well construction are given by Schmalz⁽⁷⁾. It should be noted that air injection tests were not conducted in the basalt between the basalt-playa interface (50 feet deep) and the bottom of the casing (110 feet deep), nor were such tests conducted in the holes drilled for location of the sampling ports.

In order to obtain permeability values for the playa deposits, air injection tests were conducted in an area about 170 feet east of Well No. 8. Three holes about 15 feet apart were drilled with an auger. The first five feet of each hole were drilled with a 5-inch diameter auger. Below this depth, a smaller auger making a 3.5-inch diameter hole was used. In each hole, a packer was placed at a depth of 5 feet and the hole above was filled with expanding cement. A tube, 1.25 inches in diameter, penetrated the cement and the packer to the open hole below. Hole number one was 15 feet deep with 10 feet of open hole below the packer. Hole number two was 10.3 feet deep leaving 5.3 feet of open hole, while hole number three had a total depth of 12.2 feet and an open hole length of 7.2 feet. Table III summarizes the data collected from these tests in the playa deposits.

In addition to the injection tests in the playa deposits, four different core samples were taken from these deposits. Results from air permeability measurements in the laboratory on the four cores are summarized in Table IV. Note that the measurements on the cores essentially give the vertical permeability without the contribution of vertical fractures or fissures.

TABLE III

Air Injection Tests in Playa Deposits at NRTS

<u>Replicate No.</u>	<u>Pressure (psi)</u>	<u>Flow (SCFM)</u>	<u>Temp (°F)</u>
<u>Hole Number 1</u>			
1	3.17	69	52
	5.4	140	52
	7.0	223	52
2	3.7	125	54
	5.3	183	54
	7.2	254	54
3	3.0	116	54
	4.6	172	54
	5.3	201	54
	7.1	253	54
	7.2	287	54
4	3.2	147	52
	5.0	215	52
	6.4	264	52
	7.1	288	52
5	3.4	159	50
	5.3	169	50
	6.2	252	50
<u>Hole Number 2</u>			
1	4.1	101	50
	7.0	191	50
	10.0	306	49
	9.0	287	47
	9.2	299	47
2	4.5	148	51
	7.0	229	51
	8.5	295	51
3	4.2	182	51
	7.3	256	51
4	3.9	187	52
	7.3	256	51
	7.9	290	50

TABLE III (Continued)

<u>Replicate No.</u>	<u>Pressure (psi)</u>	<u>Flow (SCFM)</u>	<u>Temp (°F)</u>
	<u>Hole Number 3</u>		
1	2.6	57	58
	3.2	59	58
	3.7	95	58
	5.2	129	58
2	2.8	79	57
	3.4	86	57
	4.3	108	57
	5.2	141	57
3	2.6	74	56
	3.4	94	56
	4.0	120	55
	5.3	151	55
	6.2	187	55
	7.3	225	55

TABLE IV

Air Permeability Measurements on Core Samples from the
Playa Deposits at NRTS

<u>Depth of Core</u>	<u>Permeability in Darcys</u>
10'6" - 11'0"	1.665
12'2" - 12'8"	1.776
13'8" - 14'2"	0.781
15'11" - 16'3"	0.097

The actual tracer test using Xe-133 began at 0924 on September 30, 1967, with injection continuing until 0205 on October 1, 1967. During this period of 16 hours and 41 minutes, a total of 1,000 curies of Xe-133 was injected in a total volume of 1,004,500 SCF of air. Thus, the concentration at injection, C_0 , was about 3.5×10^{-2} $\mu\text{Ci/cc}$. Details of flow rate and pressure during injection are summarized in Table V. The flow rate averaged about 1,000 SCFM with an average pressure and temperature of 1.59 psig and 65.2°F, respectively. At 0205 on October 1, 1967, the well was capped and no further injection took place.

The observation program began with the beginning of injection and continued through most of the month of October. Details of the sampling programs are summarized by Schmalz⁽⁷⁾, Bowman and Bennett⁽¹⁾, and Hardaway, et al.⁽²⁾. Hardaway, et al.⁽²⁾ discuss measurements of the flux across ground surface and dispersion in the atmosphere. Bowman and Bennett⁽¹⁾ discuss the subsurface sampling at locations given in Table I and measurements of flux across ground surface. Schmalz⁽⁷⁾ summarizes the results of measurements using geiger tubes installed at some of the sampling ports. Analysis of the sampling results and geiger tube data gave estimates of earliest arrival time at the sampling ports as summarized in Table VI. These values of arrival time are of primary interest in modeling the gas movement during the injection period. Applications of the models are discussed in the next section of this report.

APPLICATION OF MODELS TO NRTS FIELD TESTS

In describing the application of the mathematical models to the field tests, the two types of tests, helium tests and xenon tests, are considered separately. As noted earlier, the helium tests were conducted in shallow sediments overlying basalt while the xenon tests involved

TABLE V

Test Data at Injection Well During Injection of Xe-133
Traced Air Into Basalt at NRTS

<u>Date</u>	<u>Time</u>	<u>Flow Rate (SCFM)</u>	<u>Well Head Pressure (psig)</u>	<u>Temperature (°F)</u>
9-30-67	0924	Injection began		
"	0930	1042	1.51	66
"	1100	1030	1.64	70
"	1300	995	1.65	75
"	1430	965	1.65	78
"	1630	973	1.60	69
"	1830	985	1.61	68
"	2030	997	1.57	60
"	2200	989	1.56	56
"	2400	980	1.55	55
10-1-67	0159	980	1.52	55
"	0205	Injection ended		

TABLE VI

Estimates of Earliest Arrival of Gas at Subsurface
Sampling Ports

<u>Well No.</u>	<u>Radial Distance from Injection Well (feet)</u>	<u>Depth of Sampling Port (feet)</u>	<u>Measured Earliest Arrival in Hours After Injection Began</u>
6	1,372	10	N.D.
6	1,372	57	N.D.
7	798	10	N.D.
7	798	35	290-360 or later
7	798	63	N.D.
7	798	103	72
9	1,651	10	170
9	1,651	54	N.D.
9	1,651	80	340
10	787	10	N.D.
10	787	44	27
10	787	80	< 27
11	782	10	145
11	782	46	74
11	782	120	< 24
12	1,200	10	N.D.
12	1,200	49	76
12	1,200	80	76
13	1,006	7	72
13	1,006	60	6.8
13	1,006	120	6.8
14	394	5	N.D.
14	394	10	N.D.
14	394	57	76
15	200	5	N.D.
15	200	10	N.D.
15	200	53	4.8

TABLE VI (Continued)

Well No.	Radial Distance from Injection Well (feet)	Depth of Sampling Port (feet)	Measured Earliest Arrival in Hours After Injection Began
16	101	5	315
16	101	10	120
16	101	26	N.D.
16	101	63	< 3.3
17	50	5	N.D.
17	50	10	N.D.
17	50	41	< 26
17	50	65	< 5.2
18	415	10	*
18	415	118	1.9-2.5
18	415	164	72
18	415	190	76
19	408	10	300
19	408	104	1.8-2.0
19	408	139	1.9-4.0
19	408	190	72
20	400	7	6.7
20	400	69	1.75-10.3
20	400	111	1.8-2.3
20	400	190	74

N.D. - Xe-133 was not detected above background during 24 days of observation

* - No report for this location

injection of traced air into the fractured basalt. Both injections were made above the regional water table at NRTS.

Helium Tests

As a first approximation to modeling the injection of helium traced air in the Big Lost River sediments, it was assumed that the sediments were homogeneous and isotropic. Because of the way in which the injection well was constructed, air entered the soil through the end of a 4-inch casing. This geometry could not be met exactly by the model. However, since the original augered hole had caved somewhat, it was decided that a nearly spherically shaped cavity would best model the field geometry.

Values of the parameters were chosen such that $L = 14.25$ ft, $H = 0.5$ ft, $R_0 = 0.5$ ft (See Figure B-1). Since $D = 23$ ft, the dimensionless parameters become $L/D = 0.62$, $H/D = R_0/D = 2.175 \times 10^{-2}$. The dimensionless flow rate, $\dot{Q}$, could be estimated by extrapolation from the graphs of Appendix E or calculated directly from Equation C-38. The directly calculated value for the chosen geometry was $\dot{Q} = 0.285$. In order to calculate the flow rate from Equation C-39, the temperature, permeability, and injection pressure must also be known. For Part I, the average injection pressure was 12.0 psig. The temperature varied from 66 to 75°F, and since it was assumed that some cooling would occur as the air flowed through the soil, a temperature of 68°F (or 20°C) was used for calculations. From page E-2, $A(T) = 6.18 \times 10^3$ and from page E-3, $B(p_0') = 497$. Thus, if the permeability is chosen* as 6 darcys (or

* This choice will be discussed later.

$6.54 \times 10^{-11} \text{ ft}^2$), the flow rate calculated from Equation C-39 is 154 SCFM. The calculated value compares favorably with the measured value of 149 SCFM for Part I.

The dimensionless travel time, $\dot{t}$, was calculated numerically to four locations in the system. The locations were 10 ft radially out from the injection well and 2 ft down from ground surface, 25 ft out and 5 ft down, 50 ft out and 10 ft down, and 100 ft out and 10 ft down (See Figure 4). The corresponding values of $\dot{t}$ were 1.91, 10.84, 99.1, and 4,960, respectively. The travel time can be obtained by using these values in Equation C-54. From page G-3, $\mu = 3.784 \times 10^{-7}$ and with the effective porosity chosen as $f = 0.2$, the values of travel time are 16.9 min, 96 min, 14.6 hr, and 30.4 days. The corresponding measured arrival times for Part I were 17 min for the 10-ft radius and 76 min for the 25-ft radius. During the continuous injection period (285 min or 4.75 hr) no helium was detected at the 50-ft and 100-ft radii. Thus agreement of calculated and measured values is quite good.

For Part II, the average injection pressure was 12.2 psig, and if it is assumed that cooling of injected air takes place, the temperature could be chosen as 32°C (or 89.6°F). The calculated flow rate is 147 SCFM, which compares favorably with the measured value of 141.8 SCFM. The dimensionless travel times are 1.89 and 10.69 for the 10-ft and 25-ft radii, respectively. These values give the calculated travel times as 17.1 min and 97 min which compare to the measured travel times of 28 min and 89 min.

Although agreement is fairly satisfactory, the above calculations cannot be taken as complete verification of the model. The calculations

have been performed by completely neglecting dispersion. It could be said that a better agreement would be expected by comparing computed travel times with 50% arrival times; i.e., with the time at which the concentration at the sampling location reaches 50% of the injection concentration. Such measured times are averaged to give 38 min and 161 min at the 10-ft and 25-ft radii, respectively, for Part I, and 51 min and 156 min for Part II. These values are roughly twice the measured first arrival times. Thus, if the effective porosity were used as 0.4 the corresponding calculated arrival times would be about 34 min and 192 to 194 min. Since the average field moisture content was 8.6% and the average total porosity was 50.4%, the total non-water-filled porosity would be $50.4 (1 - 8.6/50.4) = 41.8\%$. Thus a value of 0.4 for the effective porosity may not be unreasonable.

It should be noted that the value of permeability of 6 darcys is higher than the average value measured on the samples taken into the laboratory of 2.5 darcys. Such variation is entirely within reason, however, for comparison of sample measurements with in-situ measurements of permeability. The use of smaller values of permeability in the calculations would require use of a larger effective cavity discharging the gas. Since strict geometric similarity between injection well and model do not exist, further attempts to compare calculated and measured values for the helium tests are probably not warranted. Results of calculations are summarized in Table VII. It was concluded that the model was sufficiently good to be of benefit in the design, conduction, and analysis of the xenon tracer tests described below.

TABLE VII

Summary of Results of Application of Homogeneous,
Isotropic Model to Helium Tests at NRTS

<u>Test</u>	<u>Sampling Radius (ft)</u>	<u>First Arrival</u>		<u>50% Arrival</u>	
		<u>Measured</u>	<u>Calculated</u>	<u>Measured</u>	<u>Calculated</u>
Part I	10	17 min	16.9 min	38 min	34 min
"	25	76 min	96 min	161 min	192 min
"	50	*	14.6 hr	*	**
"	100	*	30.4 days	*	**
Part II	10	28 min	17.1 min	51 min	34 min
"	25	89 min	97 min	156 min	194 min

* Gas did not arrive during continuous injection period.

** No calculations made.

Xenon Tests

Prior to description of the application of models to actual injection of air traced with Xe-133 into the basalt, use of the models for obtaining estimates of permeability will be discussed.

Estimating Permeability with Homogeneous, Isotropic Model

If the medium into which gas was injected were truly homogeneous, the model should give the right value for permeability from a match of measured and calculated flow rates. If the medium is layered, the model might be used to estimate permeability of various zones. Such use of the model is expected to give fairly good results, since the major pressure loss in the system is near the injection zone (See Figure 1). For the case where a very permeable zone is bounded above and below by much less permeable material, application of the model is expected to under-estimate the true permeability of the very permeable zone.

The homogeneous, isotropic model was used to estimate permeabilities for the various basalt zones from results of air injection tests summarized in Table II. For these calculations, the total depth of the system, D , was taken equal to 205 ft and the temperature was assumed to be 20°C. The results of these calculations are summarized in Table VIII. $\dot{Q}$ was calculated directly from Equation C-38 and k was calculated from Equation C-39.

The model was also used to estimate permeability of the playa deposits from results of air injection tests summarized in Table III. For the calculations, it was assumed that the basalt would be less permeable

TABLE VIII

Estimates of Permeability of Basalt Zones by Application of
Homogeneous, Isotropic Model to Air Injection Results*

Test No.	Zone Tested	L/D	H/D	R_o/D	$\dot{Q}$	Permeability	
						ft ²	Darcys
1	170'-203'	.829	.161	1.68×10^{-3}	.205	1.20×10^{-12}	.110
2	150'-170'	.732	.097	1.73×10^{-3}	.150	6.45×10^{-12} 4.58×10^{-12} 3.29×10^{-12}	.560 .421 .303
3	130'-150'	.634	.097	1.73×10^{-3}	.152	6.69×10^{-12} 5.68×10^{-12} 6.96×10^{-12}	.617 .520 .637
4	110'-130'	.536	.097	1.79×10^{-3}	.154	5.54×10^{-10} 4.59×10^{-10} 2.77×10^{-10}	50.8 42.2 25.4
5	110'-203'	.536	.454	1.73×10^{-3}	.495	2.53×10^{-10} 2.70×10^{-10} 2.17×10^{-10} 1.84×10^{-10}	23.2 24.7 19.9 16.8

* See Table II.

than the playa deposits. Hence, D was taken as the depth of the basalt-playa interface below ground surface. A contour map of the basalt surface showed that D could range between 35 and 45 feet in the area of the tests (170 feet east of Well No. 8). Dimensionless flow rates were calculated for D equal to 35, 40, and 45 feet and are summarized in Table IX. In the estimation of k from Equation C-39, the effects of different values of D enter through the product, $\dot{Q}D$, which is tabulated as the last column of Table IX. Permeability values calculated for these tests are very insensitive to changes in value of D between 35 and 45 feet. A value of D equal to 40 feet was used for calculations summarized in Table X. The insensitivity of permeability estimates to different values of D lessens the importance of any assumption of relative permeabilities of basalt to playa deposits. The estimates based upon the above tests are probably fairly good, especially for the top 15 or 20 feet of playa deposits. Note that Table X shows a general increase of estimated permeability with each test on a given hole; i.e., there appears to be a "development" of the deposits by each succeeding test. Hence, it is believed that the first test on each hole represents the most reliable values.

Application of Homogeneous, Isotropic Model to Xenon Injection

As a first approximation, the injection of air traced with Xe-133 was modeled using the homogeneous, isotropic model. Although good agreement was not expected, the results provide interesting comparisons and aid in understanding the flow of gas through the fractured NRTS basalts.

From the results given in Table VIII, it might be expected that a permeability value of about 50 darcys should be used. In the actual tracer

TABLE IX

Effect of D on Estimates of Permeability of Playa Deposits

<u>Hole No.</u>	<u>D</u>	<u>L/D</u>	<u>H/D</u>	<u>R_o/D</u>	<u>$\dot{Q}$</u>	<u>$\dot{QD}$</u>
1	35	.143	.286	4.17×10^{-3}	.4510	15.78
	40	.125	.250	3.64×10^{-3}	.3945	15.78
	45	.111	.222	3.24×10^{-3}	.3507	15.78
2	35	.143	.151	4.17×10^{-3}	.2775	9.71
	40	.125	.132	3.64×10^{-3}	.2425	9.70
	45	.111	.118	3.24×10^{-3}	.2166	9.75
3	35	.143	.206	4.17×10^{-3}	.3504	12.26
	40	.125	.180	3.64×10^{-3}	.3062	12.25
	45	.111	.160	3.24×10^{-3}	.2724	12.26

TABLE X

Estimates of Permeability of Playa Deposits by Application of
Homogeneous, Isotropic Model to Air Injection Results*

Hole No.	Zone Tested	Replicate No.	Q	Permeability	
				ft ²	Darcys
1	5' - 15'	1	.394	5.43 x 10 ⁻¹¹	4.98
				6.18 x 10 ⁻¹¹	5.67
				7.26 x 10 ⁻¹¹	6.66
		2		8.51 x 10 ⁻¹¹	7.81
				8.31 x 10 ⁻¹¹	7.62
				8.06 x 10 ⁻¹¹	7.39
		3		9.96 x 10 ⁻¹¹	9.14
				9.18 x 10 ⁻¹¹	8.43
				9.12 x 10 ⁻¹¹	8.37
				8.15 x 10 ⁻¹¹	7.48
				9.05 x 10 ⁻¹¹	8.30
		4		1.17 x 10 ⁻¹⁰	10.7
				1.04 x 10 ⁻¹⁰	9.51
				9.56 x 10 ⁻¹¹	8.77
				9.22 x 10 ⁻¹¹	8.46
		5		1.17 x 10 ⁻¹⁰	10.7
				7.57 x 10 ⁻¹¹	6.94
				9.40 x 10 ⁻¹¹	8.64

* See Table III.

TABLE X (Continued)

Hole No.	Zone Tested	Replicate No.	Q	Permeability	
				ft ²	Darcys
2	5' - 10.3'	1	.242	9.85 x 10 ⁻¹¹	9.04
				1.00 x 10 ⁻¹⁰	9.21
				1.04 x 10 ⁻¹⁰	9.54
				1.10 x 10 ⁻¹⁰	10.1
				1.11 x 10 ⁻¹⁰	10.2
		2		1.31 x 10 ⁻¹⁰	12.0
				1.21 x 10 ⁻¹⁰	11.1
				1.23 x 10 ⁻¹⁰	11.3
		3		1.74 x 10 ⁻¹⁰	15.9
				1.28 x 10 ⁻¹⁰	11.8
		4		1.94 x 10 ⁻¹⁰	17.8
				1.28 x 10 ⁻¹⁰	11.8
				1.32 x 10 ⁻¹⁰	12.1
3	5' - 12.2'	1	.306	7.48 x 10 ⁻¹¹	6.86
				6.17 x 10 ⁻¹¹	5.66
				8.46 x 10 ⁻¹¹	7.83
				7.83 x 10 ⁻¹¹	7.19
		2		9.54 x 10 ⁻¹¹	8.75
				8.39 x 10 ⁻¹¹	7.70
				8.11 x 10 ⁻¹¹	7.44
				8.53 x 10 ⁻¹¹	7.82
		3		9.62 x 10 ⁻¹¹	8.83
				9.12 x 10 ⁻¹¹	8.37
				9.71 x 10 ⁻¹¹	8.91
				8.88 x 10 ⁻¹¹	8.14
				9.17 x 10 ⁻¹¹	8.41
	9.09 x 10 ⁻¹¹	8.34			

injection, the average injection pressure was 1.59 psig and the average temperature was 65.2°F (18.4°C). The gas was injected between the depths of 110 ft and 122.5 ft ($H = 12.5$ ft, $L = 110$ ft). With $R_0/D = 1.79 \times 10^{-3}$ and $D = 205$ ft, $\dot{Q} = 0.109$ and the expected flow rate from Equation C-39 is about 440 SCFM. The average measured flow rate during injection was about 1000 SCFM. This result is evidence that during the air tests with packers (Table II), the air entered the basalt nonuniformly along the hole between depths of 110 ft and 130 ft. In order to calculate a flow rate near 1000 SCFM, a permeability of about 115 darcys (1.25×10^{-9} ft²) must be used. Using this value, travel times to some of the sampling ports were calculated. Results of these calculations are shown in Table XI. The travel time, t , is obtained from the dimensionless travel time, $\dot{t}$, through Equation C-54 using $k = 1.25 \times 10^{-9}$ ft², $p_0' = 1.59$ psig, $D = 205$ ft, $T = 18.4^\circ\text{C}$, and $f = 0.005$. This choice of f will be discussed later. Since the travel time is directly proportional to f , results for other porosity values are easily obtained from Table XI.

Comparison of results of Table XI with measured arrival times of Table VI shows that the homogeneous, isotropic model is not satisfactory for modeling the injection into the basalt. In general, the calculated movement is too rapid in the vertical and too slow in the radial direction.

Application of Layered Model to Xenon Injection

The results of Tables VIII and X indicate that a layered model with different permeability values for each layer, but homogeneous within

TABLE XI

Results of Travel Time Calculations for Homogeneous,
Isotropic Model Applied to Xenon Test

<u>Well No.</u>	<u>Radial Distance from Injection Well (ft)</u>	<u>Sampling Depth (ft)</u>	<u>Dimensionless Time, $\bar{t}$</u>	<u>Travel* Time, t (hrs)</u>	<u>Measured Time (hr)</u>
14	394	5	1,511	37.8	N.D.
		10	1,465	36.6	N.D.
		57	1,162	29.0	76
15	200	10	165.1	4.13	N.D.
		53	123.8	3.10	4.8
16	101	5	37.01	0.925	315
		10	34.61	0.865	120
		26	28.13	0.703	N.D.
		63	18.02	0.450	<3.3
17	50	5	17.14	0.429	N.D.
		10	15.67	0.392	N.D.
		41	8.17	0.204	< 26
		65	4.38	0.110	<5.2
20	400	111	1,038	25.9	1.8-2.3
		190	942	23.6	74

* f assumed equal to 0.005 (This choice will be discussed later).

each layer, might model the measured flow satisfactorily. Such a model was set up using the permeability values from the air injection results. This model predicted a flow-rate which was much too small. The permeability values were adjusted until the predicted flow rate was about 1000 SCFM and hence agreed with measured results of the xenon injection. Travel times for this model had to be calculated by hand from graphical data resulting from finite difference solution of Equation A-26. The results were similar to the homogeneous, isotropic model in that vertical movement was in general too rapid compared to radial movement. Since calculations using the layered model were expensive and chances for good agreement with measured results appeared rather poor, attention was focused on modeling with the use of a homogeneous, anisotropic model.

Application of Homogeneous, Anisotropic Model to Xenon Injection

In Appendix D, it is shown that problems of flow in homogeneous, anisotropic media can be handled by transforming the problem into an equivalent flow in homogeneous, isotropic media. The problems of input data are compounded by the need to know both the horizontal and vertical components of the permeability. Of course, such measurements were not taken in the field. The approach for this report was to investigate several ratios of horizontal permeability to vertical permeability, k_H/k_V , and determine if calculated results were reasonable. As a guide to possible ratios, consider that the values of permeability from Table IV represent k_V in the playa deposits and that those from Table X represent k_H . The average of the first test in each hole of Table X gives $k_H = 7$ darcys. With these approximations, k_H/k_V might be expected to vary from about 4 to about 70 in the playa deposits.

For conditions to be modeled, Equation C-39 reduces to

$$Q_S = (7.38 \times 10^{12}) k_H \dot{Q} \quad (1)$$

where $T = 18.4^\circ\text{C}$, $p_o' = 1.59$ psig, $D = 205$ ft. Or since $Q_S = 1000$ SCFM,

$$k_H = \frac{1.355 \times 10^{-10}}{\dot{Q}} \quad (2)$$

Equation C-54 becomes

$$t = (6.25 \times 10^{-9}) \frac{ft}{k_V} \quad (3)$$

where t is the travel time in hours.

The procedure for arriving at reasonably good values of k_H and k_V was as follows. For a range of k_H/k_V , the curve of $\dot{Q}$ as a function of k_H/k_V shown on Figure 6 was calculated from Equation C-38. The curve of Figure 7 was calculated from Equation C-53 using the computer program STREAM⁽⁴⁾. From Figure 6 for a given value of k_H/k_V , $\dot{Q}$ was found. Equation 2 then gave k_H such that the proper flow rate was obtained. Then since the value of k_H/k_V was known, k_V was determined and the travel time to any location could be determined if f and t were known. Namely, to the 104-ft depth of Well No. 19, the travel time was determined from Figure 7 and an assumed value of porosity, f . If this value of t did not agree with the measured value, a new value of k_H/k_V was chosen and the process was repeated.

In following the above procedure, it was noted from Equation 3 that the travel time is directly proportional to t/k_V . For the sampling port at the 104-ft depth of Well No. 19, Figure 8 gives the curve for t/k_V as

FIGURE 6. Dimensionless Flow Rate as a Function of Permeability Ratio

FIGURE 7. Dimensionless Travel Time to 104-ft Depth of Well No. 19 as a Function of Permeability Ratio

FIGURE 8. Ratio of Dimensionless Travel Time to Vertical Permeability for 104-ft Depth of Well No. 19 as a Function of Permeability Ratio

a function of k_H/k_V . For this travel path, it is noted that the curve has a minimum value of 5.5×10^{10} at a permeability ratio of about 700 to 800. Thus, continual increase of permeability ratio does not result in continual decrease in travel time along this path. The measured travel time along this path was about 2 hours (See Table VI). A value of $f = 0.0058$ would be needed to calculate a travel time of 2 hours with $\dot{t}/k_V = 5.5 \times 10^{10}$. At any other value of k_H/k_V , a smaller effective porosity would be needed to give a calculated travel time of 2 hours. It is further noted, from Figure 8, that in the range of k_H/k_V from 100 to 1000, the calculated travel time for this path varies only by a factor of about 1.5. Preliminary evaluations of travel times to other subsurface sampling ports indicated that values of $k_H/k_V = 75$ and $f = 0.005$ might give a reasonable description of gas movement during injection.

The results of Table XII were computed for the following conditions: $k_H/k_V = 75$, $k_H = 2 \times 10^{-9} \text{ ft}^2$ (183 darcys), $k_V = 2.67 \times 10^{-11} \text{ ft}^2$ (2.45 darcys), $f = 0.005$, $T = 18.4^\circ\text{C}$, $D = 205 \text{ ft}$. For these conditions, $\dot{Q} = 0.0676$ and from Equation 1, $Q_s = 998 \text{ SCFM}$; also, $t = (1.17) \dot{t}$. Included in Table XII for comparison purposes are the measured first arrival times (See Table VI). It should be noted that the injection time was 16.7 hr and that the model calculations apply only to this time period. Hence any calculated time greater than 16.7 hr indicates the time at which gas would be expected to arrive under continuous injection conditions. It is expected that actual measured arrivals for such locations would be considerably greater than the calculated time shown in Table XII.

By way of comparison, travel times were calculated for k_H/k_V equal to 200 and 800. For $k_H/k_V = 200$, with $k_H = 2.2 \times 10^{-9} \text{ ft}^2$, $k_V = 1.1 \times 10^{-11}$

TABLE XII

Results of Travel Time Calculations for Homogeneous,
Anisotropic Model with $k_H/k_V = 75$ Applied to Xenon Test

Well No.	Radial Distance from Injection Well (ft)	Sampling Depth (ft)	Dimensionless Time $\bar{t}$	Travel Time, t (hr)	Measured Time (hr)
7	798	63	23.4	27.4	N.D.
7	798	103	16.7	19.5	72
10	787	80	18.6	21.8	<27
11	782	120	15.6	18.2	<24
13	1,006	120	32.7	38.2	6.8
14	394	57	7.83	9.15	76
15	200	53	5.64	6.60	4.8
16	101	63	3.05	3.57	<3.3
17	50	41	7.54	8.82	<26
17	50	65	2.57	3.01	<5.2
18	415	118	2.38	2.79	1.9-2.5
18	415	164	7.13	8.34	72
18	415	190	19.2	22.5	76
19	408	104	2.44	2.86	1.8-2.0
19	408	139	3.13	3.66	1.9-4.0
19	408	190	18.9	22.1	72
20	400	69	5.61	6.57	1.75-10.3
20	400	111	2.14	2.50	1.8-2.3
20	400	190	18.6	21.8	74

N.D. - Xe-133 was not detected above background during 24 days of observation

ft^2 , $Q_s = 1010$ SCFM. For $k_H/k_V = 800$, with $k_H = 2.4 \times 10^{-9} \text{ ft}^2$, $k_V = 3 \times 10^{-12} \text{ ft}^2$, $Q_s = 990$ SCFM. Results of these comparisons calculated with $f = 0.005$ are summarized in Table XIII. The measured first arrival times from Table VI are also included for comparison. It is believed that the results for $k_H/k_V = 75$ give the best overall agreement with field measured results.

All calculations for application of the models to the xenon test have neglected dispersion since calculated arrival times were compared with earliest measured arrival times. Data were not available for comparing calculated travel times with 50% arrival times. The 50% concentration corrected back to time of injection would have been about $1.75 \times 10^{-2} \mu\text{Ci/cc}$. Only one sampling port (111-ft depth, Well No. 20) indicated a concentration of this magnitude during the test. The indication was from the instrument system only--not from the samples⁽⁷⁾. This 50% arrival time was between 7 and 13 hours after injection began. All other conditions the same, such a travel time (10 hours) would result from the calculations for $k_H/k_V = 75$ if the porosity were chosen as $f = 0.02$.

The unexpected small values of effective porosity, f , required in the calculations could arise because of two independent factors. First, the models have been used as an attempt to calculate first arrival times neglecting dispersion. Data from both the helium test and the xenon test indicated that if calculated times were compared to 50% arrival times, larger values of f would give fairly good agreement. For the flow in the basalt, the small values of f could be indicative of primarily a "fracture-type" movement; i.e., the gas moves through fractures which occupy only a very small part of the total cross-section.

TABLE XIII

Comparison of Calculated Travel Times for Various Permeability Ratios with Measured Arrival Times for Xenon Test

Well No.	Radial Distance from Injection Well (ft)	Sampling Depth (ft)	Measured Time (hr)	Calculated Travel Times (hr)		
				$\frac{k_H}{k_V} = 75$	$\frac{k_H}{k_V} = 200$	$\frac{k_H}{k_V} = 800$
7	798	63	N.D.	27.4	27.6	52.4
7	798	103	72	19.5	12.6	8.21
10	787	80	<27	21.8	18.1	23.8
11	782	120	<24	18.2	11.3	5.12
13	1,006	120	6.8	38.2	23.7	12.5
14	394	57	76	9.15	16.2	52.6
15	200	53	4.8	6.60	15.5	59.8
16	101	63	<3.3	3.57	9.05	35.8
17	50	41	< 26	8.82	23.7	94.2
17	50	65	<5.2	3.01	8.01	31.8
18	415	118	1.9-2.5	2.79	1.75	1.04
18	415	164	72	8.34	12.3	35.1
18	415	190	76	22.5	44.6	156
19	408	104	1.8-2.0	2.86	2.05	1.74
19	408	139	1.9-4.0	3.66	3.32	4.92
19	408	190	72	22.1	44.3	155
20	400	69	1.75-10.3	6.57	9.97	28.8
20	400	111	1.8-2.3	2.50	1.63	1.04
20	400	190	74	21.8	44.0	155

N.D. - Xe-133 was not detected above background during 24 days of observation

SUGGESTIONS FOR FURTHER MODEL DEVELOPMENT AND VERIFICATION

As explained earlier, the models described in this report were developed for application only during the period of continuous injection of gas into the ground above the regional water table. Once the injection ceases, the models can no longer be expected to describe further movement of gas under natural convective or diffusive forces. Comparison of calculated and measured results for the field tests described herein seems to justify neglecting the initial transient motion of gas as pumping begins.

Since the field tests were not designed primarily for model verification, several questions as to the adequacy of the models remain unanswered. The importance of dispersion in the field for gas flow needs further study and clarification. Indications are that the models presented herein would best be used to predict 50% arrival times. The Darcian velocity field obtained from the models could be used in transport equations to incorporate the effects of dispersion. Once such models were fully developed, concentration distributions could be obtained. These distributions would serve as initial conditions for prediction of further gas movement after cessation of injection.

It is suggested that further research work include field and laboratory tests specifically designed and conducted to allow determination of the importance of dispersion and to develop models capable of describing both Darcian flow and first arrival of gas. It is important that thought be given to measurement of needed parameters in the field if use of the models for prediction instead of description is to be fully exploited.

The graphs presented in Appendices E, F, and G appear to cover a fairly wide range of geometric variables for homogeneous, isotropic systems. However, in applying the results to anisotropic systems, it was found that the range needed extension to smaller values of R_0/D . It is suggested that the range be extended to allow the graphs to be used for first engineering calculations for a wider variety of problems.

Although this report and the research leading to it were concerned with disposal of radioactive gaseous wastes, the results can be used to investigate the benefits to be derived from underground discharge of other types of gaseous wastes. This method of disposing of more conventional noxious waste gases shows considerable promise, particularly due to the relatively long soil-gas contact time which can exist throughout much of the system. In many instances, final gas clean-up kinetics would be favored by long periods of contact between the gas and the soil moisture. Also, it is well known that soil bacteria (and possibly other mechanisms) can play a major role in reducing specific gaseous pollutants to their elemental constituents.

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APPENDIX A

DEVELOPMENT OF EQUATIONS DESCRIBING
STEADY, ISOTHERMAL FLOW OF GAS THROUGH POROUS MEDIA

EQUATION CONTAINING A POTENTIAL WITH GRAVITY INCLUDED

The model considers the flow of a compressible gas through a porous medium under isothermal conditions. The basic dynamic equation is Darcy's equation which can be written in the form

$$\underline{q} = - \frac{k\rho}{\mu} \left(\frac{1}{\rho} \text{grad } p + g \underline{k} \right) \quad (\text{A-1})$$

where $\underline{q}$ is the volume flux of gas (Darcian velocity), k is the permeability of the medium to gas flow, ρ is the density of the gas, μ is the viscosity of the gas, p is the gas pressure, g is the acceleration of gravity, and $\underline{k}$ is a unit vector in the vertical direction. The density is a function of pressure only. The pressure and permeability are functions of space.

Equation A-1 can be written

$$\underline{q} = - \frac{k\rho}{\mu} \text{grad } \phi \quad (\text{A-2})$$

provided the potential ϕ exists and is such that

$$\text{grad } \phi = \frac{1}{\rho} \text{grad } p + g \underline{k} \quad (\text{A-3})$$

From vector analysis, a necessary and sufficient condition for the existence of ϕ is that

$$\text{curl} \left(\frac{1}{\rho} \text{grad } p + g \underline{k} \right) = \underline{0} \quad (\text{A-4})$$

since the curl of every gradient is identically zero. Expanding Equation A-4 gives

$$\text{curl} \left(\frac{1}{\rho} \text{grad } p + g \underline{k} \right) = \nabla \frac{1}{\rho} \times \nabla p + \frac{1}{\rho} \nabla \times \nabla p + \nabla \times g \underline{k} \quad (\text{A-5})$$

The curl of any gradient is zero, hence $\nabla \times \nabla p = \underline{0}$. The curl of the constant vector $\underline{g}$ is also equal to zero. Thus, only the first term on the right hand side of Equation A-5 remains. In general curvilinear coordinates ⁽⁸⁾ with arc length given by

$$d\underline{s} = (h_1 dx_1, h_2 dx_2, h_3 dx_3) \quad (A-6)$$

the gradient of a scalar u is

$$\text{grad } u = \left(\frac{1}{h_1} \frac{\partial u}{\partial x_1}, \frac{1}{h_2} \frac{\partial u}{\partial x_2}, \frac{1}{h_3} \frac{\partial u}{\partial x_3} \right) \quad (A-7)$$

Thus

$$\begin{aligned} \nabla \left(\frac{1}{\rho} \right) &= \left(\frac{1}{h_1} \frac{\partial \left(\frac{1}{\rho} \right)}{\partial x_1}, \frac{1}{h_2} \frac{\partial \left(\frac{1}{\rho} \right)}{\partial x_2}, \frac{1}{h_3} \frac{\partial \left(\frac{1}{\rho} \right)}{\partial x_3} \right) \\ &= \left(-\frac{1}{h_1 \rho^2} \frac{\partial \rho}{\partial x_1}, -\frac{1}{h_2 \rho^2} \frac{\partial \rho}{\partial x_2}, -\frac{1}{h_3 \rho^2} \frac{\partial \rho}{\partial x_3} \right) \\ &= \left(-\frac{1}{h_1 \rho^2} \frac{d\rho}{dp} \frac{\partial p}{\partial x_1}, -\frac{1}{h_2 \rho^2} \frac{d\rho}{dp} \frac{\partial p}{\partial x_2}, -\frac{1}{h_3 \rho^2} \frac{d\rho}{dp} \frac{\partial p}{\partial x_3} \right) \\ &= -\frac{1}{\rho^2} \frac{d\rho}{dp} \nabla p \end{aligned}$$

Substituting the last result into the first term on the right hand side of Equation A-5 gives

$$\nabla \frac{1}{\rho} \times \nabla p = -\frac{1}{\rho^2} \frac{d\rho}{dp} \nabla p \times \nabla p = \underline{0}$$

since the cross product of any vector with itself is identically zero. Hence, the entire right hand side of Equation A-5 is zero.

The potential ϕ exists and is given by Equation A-3. To find the expression for ϕ explicitly, dot both sides of Equation A-3 with the arc length $d\underline{s}$

$$\text{grad } \phi \cdot d\underline{s} = \left(\frac{1}{\rho} \text{grad } p + g\underline{k} \right) \cdot d\underline{s} \quad (\text{A-8})$$

From equation A-6 and A-7,

$$\text{grad } \phi \cdot d\underline{s} = \frac{\partial \phi}{\partial x_1} dx_1 + \frac{\partial \phi}{\partial x_2} dx_2 + \frac{\partial \phi}{\partial x_3} dx_3 = d\phi \quad (\text{A-9})$$

Also,

$$\text{grad } p \cdot d\underline{s} = dp \quad (\text{A-10})$$

If consideration is limited to curvilinear coordinate systems in which $x_3 = z$ (i.e., the vertical axis), then

$$g\underline{k} \cdot d\underline{s} = g dz \quad (\text{A-11})$$

Combining Equation A-8, A-9, A-10, and A-11 gives

$$d\phi = \frac{1}{\rho} dp + g dz \quad (\text{A-12})$$

Conditions are satisfied to allow integration of Equation A-12 giving

$$\phi = \int \frac{dp}{\rho} + gz + C_1 \quad (\text{A-13})$$

To complete the integration, the function $\rho = \rho(p)$ must be specified. Since for many waste gases, the physical properties influencing flow do not significantly differ from the properties of air, the equation of state for dry air is used throughout the development and results of this report. The equation is given as

$$\text{density (in gm/ml)} = \left(\frac{0.0012929}{1 + \frac{T}{273.13}} \right) \left(\frac{H}{76} \right) \quad (\text{A-14})$$

where T is the temperature in degrees Centigrade and H is the absolute pressure in centimeters of mercury. Using handbook conversion factors gives $1 \text{ gm/ml} = 1.94 \text{ slugs/ft}^3$ and

$$\rho = \frac{\alpha}{\gamma} (p + \gamma) \quad (\text{A-15})$$

where

$$\alpha = \frac{0.002509}{1 + 0.00366 T} \quad (A-16)$$

and $\gamma = (14.696)(144)$ psf, p is the gage pressure in psf, ρ is the density in slugs/ft³. Using Equation A-15 to perform the integration indicated in Equation A-13 gives

$$\phi = \frac{\gamma}{\alpha} \ln (p + \gamma) + gz + C \quad (A-17)$$

where C is a constant of integration which must be determined from the boundary conditions imposed by the particular flow problem to be solved.

If the model is limited to steady flow of gas, the equation of continuity is

$$\text{div } (\rho \underline{q}) = 0 \quad (A-18)$$

Assuming that μ is constant and substituting Equation A-2 into Equation A-18 gives

$$\text{div } (k\rho^2 \text{ grad } \phi) = 0 \quad (A-19)$$

Expansion of Equation A-19 results in

$$k\rho^2 \text{ div grad } \phi + k \text{ grad } \rho^2 \cdot \text{ grad } \phi + \rho^2 \text{ grad } k \cdot \text{ grad } \phi = 0 \quad (A-20)$$

From Equation A-7,

$$\begin{aligned} \text{grad } \rho^2 &= \left(\frac{1}{h_1} \frac{\partial \rho^2}{\partial x_1}, \frac{1}{h_2} \frac{\partial \rho^2}{\partial x_2}, \frac{1}{h_3} \frac{\partial \rho^2}{\partial x_3} \right) \\ &= \left(\frac{1}{h_1} 2\rho \frac{\partial \rho}{\partial x_1}, \frac{1}{h_2} 2\rho \frac{\partial \rho}{\partial x_2}, \frac{1}{h_3} 2\rho \frac{\partial \rho}{\partial x_3} \right) \\ &= 2\rho \left(\frac{1}{h_1} \frac{\partial \rho}{\partial x_1}, \frac{1}{h_2} \frac{\partial \rho}{\partial x_2}, \frac{1}{h_3} \frac{\partial \rho}{\partial x_3} \right) = 2\rho \text{ grad } \rho \end{aligned}$$

Using this result, Equation A-20 becomes

$$\text{div grad } \phi + \frac{2}{\rho} \text{grad } \rho \cdot \text{grad } \phi + \frac{1}{k} \text{grad } k \cdot \text{grad } \phi = 0 \quad (\text{A-21})$$

or

$$\text{div grad } \phi + 2 \text{grad } \ln \rho \cdot \text{grad } \phi + \text{grad } \ln k \cdot \text{grad } \phi = 0 \quad (\text{A-22})$$

From Equation A-15

$$\ln \rho = \ln \frac{\alpha}{\gamma} + \ln (p + \gamma) \quad (\text{A-23})$$

Solving Equation A-17 for $\ln (p + \gamma)$ and substituting the result into Equation A-23 gives

$$\ln \rho = \ln \frac{\alpha}{\gamma} + \frac{\alpha}{\gamma} (\phi - gz - C) \quad (\text{A-24})$$

Hence,

$$\text{grad } \ln \rho = \frac{\alpha}{\gamma} (\text{grad } \phi - g \underline{k}) \quad (\text{A-25})$$

Equation A-22 becomes

$$\text{div grad } \phi + \frac{2\alpha}{\gamma} [\text{grad } \phi \cdot \text{grad } \phi - g \frac{\partial \phi}{\partial z}] + \text{grad } \ln k \cdot \text{grad } \phi = 0 \quad (\text{A-26})$$

Equation A-26 is a nonlinear partial differential equation which, in general, must be solved by a numerical method such as finite differences. This equation will not be discussed further in this appendix. Instead attention will be focused on a linearized form of the equation which can be solved in closed form.

EQUATION DERIVED BY NEGLECTING GRAVITY EFFECTS

The model developed in this study applies to the injection of gases into the vadose zone; i.e., into the ground above the regional water table. Rarely does the depth from ground surface to the water

table exceed a few hundred feet. Since the density of the gas is small, the contribution of gravity to the potential causing flow is likewise small and can be neglected for the elevation differences encountered in the disposal problems to be analyzed with the model.

Neglecting the gravity component of Equation A-1 gives

$$\underline{q} = - \frac{k}{\mu} \text{grad } p \quad (\text{A-27})$$

i.e., the flow is in response to the gradient of pressure. Since the absolute pressure P is given by $P = p + \gamma$,

$$\text{grad } P = \text{grad } p$$

and Equation A-27 becomes

$$\underline{q} = - \frac{k}{\mu} \text{grad } P \quad (\text{A-28})$$

Substituting Equation A-28 into Equation A-18 gives

$$\text{div} \left(\frac{k\rho}{\mu} \text{grad } P \right) = 0 \quad (\text{A-29})$$

Since the flow is isothermal, μ is constant. If the porous medium is assumed to be homogeneous, k is constant. Equation A-15 can be written

$$\rho = \frac{\alpha P}{\gamma} \quad (\text{A-30})$$

where, for isothermal flow, both α and γ are constants. Hence, Equation A-29 becomes

$$\text{div} (P \text{ grad } P) = 0 \quad (\text{A-31})$$

As shown following Equation A-20, $\text{grad } P^2 = 2P \text{ grad } P$.

Equation A-31 can be written as

$$\text{div grad } P^2 = 0 \quad (\text{A-32})$$

That is, the problem reduces to solving Laplace's equation in p^2 for the particular boundary conditions imposed by the injection system.

APPENDIX B

THE PHYSICAL SYSTEM AND SIMPLIFYING ASSUMPTIONS

The physical structure considered for the injection of waste gases into the ground consists of a single well as shown schematically in Figure B-1. The injection length, H , could, in reality, be either a perforated casing or an open hole depending upon local conditions. The injection length may be located at any desired distance, L , below ground surface. D is the total depth of the water table below ground surface, and R_0 is the radius of the injection structure.

In order to obtain mathematical solutions in closed form, the following simplifying assumptions were made. Both the ground surface and the water table were assumed to be level. Any moisture existing in the ground above the water table was assumed immobile. The permeability of the ground to air flow was assumed uniform. The assumption of uniform permeability is equivalent to assuming either that the soil was homogeneous with uniform moisture content or that any nonuniformity of moisture content was just balanced by a change of saturated water permeability.

It was further assumed that the gas does not flow across the boundary formed by the water table. The injection rate was assumed to remain constant and initial transient flow was neglected. That is, the gas was injected at a constant pressure, P_0 , and at ground surface, the pressure remained constant at atmospheric pressure, γ .

FIGURE B-1. Definition Sketch for Underground Injection of Gas into Homogeneous Media

APPENDIX C

MATHEMATICAL MODEL OF THE PHYSICAL SYSTEM

STATEMENT OF BOUNDARY VALUE PROBLEM

A possible approach to obtaining a mathematical solution of the injection problem would be to determine the proper distribution of point sources to give an equipressure line which would approximate the cylindrical shape of the injection length. Such an approach would involve lengthy integrations to determine the distribution of point sources. It was assumed that the shape of an equipressure line resulting from the solution of a line source potential problem would sufficiently approximate the shape of the injection length. The well casing above the injection length was neglected. Thus, the partial differential equation to be solved in cylindrical coordinates was Equation A-32.

It is convenient to make the problem dimensionless by scaling all lengths to D (See Figure B-1) and the pressures to the injection pressure, P_0 . The scaled variables will be denoted by a "dot" above the variable name. Thus, for $\dot{P} = \dot{P}(\dot{r}, \dot{z})$, the boundary value problem consists of the equation

$$\text{div grad } \dot{P}^2 = 0 \quad (\text{C-1})$$

and the boundary conditions

$$\dot{P}(\dot{r}, 0) = \gamma/P_0 \quad (\text{C-2})$$

$$\dot{P}(\infty, \dot{z}) = \gamma/P_0 \quad (\text{C-3})$$

$$\frac{\partial \dot{P}(\dot{r}, 1)}{\partial \dot{z}} = 0 \quad (\text{C-4})$$

$$\dot{P}(\dot{r}_0, \dot{z}_0) = 1 \quad (\text{C-5})$$

where $\dot{z}_0 = L/D + H/2D$ and $\dot{r}_0 = R_0/D$. The boundary condition given as Equation C-5 is used to specify which equipressure line of the line source solution is to be used as an approximation of the injection length of the well. The above boundary value problem is represented in Figure C-1.

SOLUTION OF BOUNDARY VALUE PROBLEM

To obtain the solution of the problem, it is convenient to consider first the solution of a point source-sink potential problem and then integrate to obtain a solution for a line source-sink problem. From the latter solution, the desired solution of Equations C-1 through C-5 can be obtained.

Consider the following point source-sink boundary value problem (The "dots" will be dropped from the notation for simplicity) in axisymmetric cylindrical coordinates for the region $(-1 \leq z \leq 1, r \geq 0)$

$$\nabla^2 u = \frac{S}{2\pi} \frac{\delta(r)}{r} [\delta(z - \lambda) - \delta(z + \lambda)] \quad (C-6)$$

$$\frac{\partial u(r, \pm 1)}{\partial z} = 0 \quad (C-7)$$

$$u(\infty, z) = 0 \quad (C-8)$$

where δ is the Dirac delta function, S is the strength of the point source, and the problem is sketched in Figure C-2.

The solution of the point source boundary value problem results in three separate expressions, each valid for the region indicated below. The solution is

FIGURE C-1. Graphical Representation
 of Boundary Value Problem in P

FIGURE C-2. Graphical Representation of Point Source Boundary Value Problem

$$u = u_1(r, z) \quad \text{for} \quad (\lambda \leq z \leq 1, r > 0) \quad (C-9)$$

$$\text{and} \quad (\lambda < z \leq 1, r = 0)$$

$$u = u_2(r, z) \quad \text{for} \quad (-\lambda \leq z \leq \lambda, r > 0) \quad (C-10)$$

$$\text{and} \quad (-\lambda < z < \lambda, r = 0)$$

$$u = u_3(r, z) \quad \text{for} \quad (-1 \leq z \leq -\lambda, r > 0) \quad (C-11)$$

$$\text{and} \quad (-1 \leq z < -\lambda, r = 0)$$

To obtain the line source solution, $U(r, z)$, perform the integration indicated below with $\sigma = L/D$ and $\tau = L/D + H/D$.

$$U_1(r, z) = \int_{\sigma}^{\tau} u_1(r, z) dh \quad (C-12)$$

$$U_2(r, z) = \int_{\sigma}^z u_1(r, z) dh + \int_z^{\tau} u_2(r, z) dh \quad (C-13)$$

$$U_3(r, z) = \int_{\sigma}^{\tau} u_2(r, z) dh \quad (C-14)$$

The separate parts of the line source solution are valid for the regions:

$$U_1(r, z) \quad \text{for} \quad (\tau \leq z \leq 1, r > 0) \quad (C-15)$$

$$\text{and} \quad (\tau < z \leq 1, r = 0);$$

$$U_2(r, z) \quad \text{for} \quad (\sigma \leq z \leq \tau, r > 0); \quad (C-16)$$

$$U_3(r, z) \quad \text{for} \quad (0 \leq z \leq \sigma, r > 0) \quad (C-17)$$

$$\text{and} \quad (0 \leq z < \sigma, r = 0)$$

By inspection, the potential is symmetric about the r axis ($z = 0$), hence $U(r, -z) = -U(r, z)$ and the r axis is an equipotential ($U(r, 0) = 0$). For the gas flow problem, the solution is needed only for the region $0 \leq z \leq 1$.

The solution $U(r,z)$ satisfies the following conditions

$$U(r,0) = 0 \quad (C-18)$$

$$U(\infty, z) = 0 \quad (C-19)$$

$$\frac{\partial U(r,1)}{\partial z} = 0 \quad (C-20)$$

The equation

$$\nabla^2 U = 0 \quad (C-21)$$

is satisfied everywhere within the region ($0 \leq z \leq 1, r > 0$).

Equation C-21 is also satisfied along the lines ($r = 0,$

$0 \leq z < \sigma$) and ($r = 0, \tau < z \leq 1$). Returning to the "dot"

notation to denote scaled variables, it can be shown that

the solution to the boundary value problem given by Equations

C-1 through C-5 is of the form

$$\dot{P}(\dot{r}, \dot{z}) = \left(\dot{U}(\dot{r}, \dot{z}) + (\gamma/P_0)^2 \right)^{1/2} \quad (C-22)$$

Substituting Equation C-18 into Equation C-22 gives

$$\dot{P}(\dot{r}, 0) = \gamma/P_0$$

Thus, Equation C-2 is satisfied. Using Equation C-19 and C-22,

shows that Equation C-3 is satisfied. Squaring both sides of

Equation C-22 and differentiating the result with respect to

$\dot{z}$ gives

$$2\dot{P} \frac{\partial \dot{P}}{\partial \dot{z}} = \frac{\partial \dot{U}}{\partial \dot{z}} \quad (C-23)$$

Since $\dot{P}(\dot{r}, 1) \neq 0$, Equation C-20 and C-23 give

$$\frac{\partial \dot{P}(\dot{r}, 1)}{\partial \dot{z}} = 0$$

i.e., Equation C-4 is satisfied.

From Equation C-22,

$$\dot{P}^2(\dot{r}, \dot{z}) = \dot{U}(\dot{r}, \dot{z}) + (\gamma/P_0)^2 \quad (C-24)$$

and

$$\frac{\partial \dot{P}^2}{\partial \dot{r}} = \frac{\partial \dot{U}}{\partial \dot{r}}$$

$$\frac{\partial \dot{P}^2}{\partial \dot{z}} = \frac{\partial \dot{U}}{\partial \dot{z}}$$

Thus, $\nabla^2(\dot{P}^2) = \nabla^2 \dot{U} = 0$; i.e., Equation C-1 is satisfied. Hence, the problem is solved except for specification of the source strength by using Equation C-5.

The solution $\dot{P}(\dot{r}, \dot{z})$ consists of three separate parts, each valid over different regions. The complete solution is:

$$\dot{P}(\dot{r}, \dot{z}) = \dot{P}_1(\dot{r}, \dot{z}) = \left(\frac{\dot{S}}{4\pi} \ln \Pi_1(\dot{r}, \dot{z}) + (\gamma/P_0)^2 \right)^{1/2} \quad (C-25)$$

valid for $(\dot{r} > 0, \dot{\tau} \leq \dot{z} \leq 1)$ and $(\dot{r} = 0, \dot{\tau} < \dot{z} \leq 1)$;

$$\dot{P}(\dot{r}, \dot{z}) = \dot{P}_2(\dot{r}, \dot{z}) = \left(\frac{\dot{S}}{4\pi} \ln \frac{1}{\dot{r}^2} \Pi_2(\dot{r}, \dot{z}) + (\gamma/P_0)^2 \right)^{1/2} \quad (C-26)$$

valid for $(\dot{r} > 0, \dot{\sigma} \leq \dot{z} \leq \dot{\tau})$;

$$\dot{P}(\dot{r}, \dot{z}) = \dot{P}_3(\dot{r}, \dot{z}) = \left(\frac{\dot{S}}{4\pi} \ln \Pi_3(\dot{r}, \dot{z}) + (\gamma/P_0)^2 \right)^{1/2} \quad (C-27)$$

valid for $(\dot{r} > 0, 0 \leq \dot{z} \leq \dot{\sigma})$ and $(\dot{r} = 0, 0 \leq \dot{z} < \dot{\sigma})$; where

$\dot{\sigma} = L/D$, $\dot{\tau} = L/D + H/D$, and for $n = 1, 3, 5, \dots$

$$\begin{aligned}
\Pi_1(\dot{r}, \dot{z}) = & \prod_{n=1}^{\infty} \left\{ \left(\frac{2(n-1)+\dot{z}-\dot{\sigma}+\sqrt{[2(n-1)+\dot{z}-\dot{\sigma}]^2+\dot{r}^2}}{2(n-1)+\dot{z}-\dot{\tau}+\sqrt{[2(n-1)+\dot{z}-\dot{\tau}]^2+\dot{r}^2}} \right) \left(\frac{2(n+1)-\dot{z}-\dot{\tau}+\sqrt{[2(n+1)-\dot{z}-\dot{\tau}]^2+\dot{r}^2}}{2(n+1)-\dot{z}-\dot{\sigma}+\sqrt{[2(n+1)-\dot{z}-\dot{\sigma}]^2+\dot{r}^2}} \right) \right. \\
& \left(\frac{2(n-1)+\dot{z}+\dot{\sigma}+\sqrt{[2(n-1)+\dot{z}+\dot{\sigma}]^2+\dot{r}^2}}{2(n-1)+\dot{z}+\dot{\tau}+\sqrt{[2(n-1)+\dot{z}+\dot{\tau}]^2+\dot{r}^2}} \right) \left(\frac{2(n+1)-\dot{z}+\dot{\tau}+\sqrt{[2(n+1)-\dot{z}+\dot{\tau}]^2+\dot{r}^2}}{2(n+1)-\dot{z}+\dot{\sigma}+\sqrt{[2(n+1)-\dot{z}+\dot{\sigma}]^2+\dot{r}^2}} \right) \\
& \left(\frac{2n-\dot{z}-\dot{\sigma}+\sqrt{(2n-\dot{z}-\dot{\sigma})^2+\dot{r}^2}}{2n-\dot{z}-\dot{\tau}+\sqrt{(2n-\dot{z}-\dot{\tau})^2+\dot{r}^2}} \right) \left(\frac{2n+\dot{z}-\dot{\tau}+\sqrt{(2n+\dot{z}-\dot{\tau})^2+\dot{r}^2}}{2n+\dot{z}-\dot{\sigma}+\sqrt{(2n+\dot{z}-\dot{\sigma})^2+\dot{r}^2}} \right) \\
& \left. \left(\frac{2n-\dot{z}+\dot{\sigma}+\sqrt{(2n-\dot{z}+\dot{\sigma})^2+\dot{r}^2}}{2n-\dot{z}+\dot{\tau}+\sqrt{(2n-\dot{z}+\dot{\tau})^2+\dot{r}^2}} \right) \left(\frac{2n+\dot{z}+\dot{\tau}+\sqrt{(2n+\dot{z}+\dot{\tau})^2+\dot{r}^2}}{2n+\dot{z}+\dot{\sigma}+\sqrt{(2n+\dot{z}+\dot{\sigma})^2+\dot{r}^2}} \right) \right\} \\
\Pi_2(\dot{r}, \dot{z}) = & \left(\dot{z}-\dot{\sigma}+\sqrt{(\dot{z}-\dot{\sigma})^2+\dot{r}^2} \right) \left(\dot{\tau}-\dot{z}+\sqrt{(\dot{\tau}-\dot{z})^2+\dot{r}^2} \right) \prod_{n=1}^{\infty} \left\{ \left(\frac{2(n+1)+\dot{z}-\dot{\sigma}+\sqrt{[2(n+1)+\dot{z}-\dot{\sigma}]^2+\dot{r}^2}}{2(n+1)+\dot{z}-\dot{\tau}+\sqrt{[2(n+1)+\dot{z}-\dot{\tau}]^2+\dot{r}^2}} \right) \right. \\
& \left(\frac{2(n+1)-\dot{z}-\dot{\tau}+\sqrt{[2(n+1)-\dot{z}-\dot{\tau}]^2+\dot{r}^2}}{2(n+1)-\dot{z}-\dot{\sigma}+\sqrt{[2(n+1)-\dot{z}-\dot{\sigma}]^2+\dot{r}^2}} \right) \left(\frac{2(n-1)+\dot{z}+\dot{\sigma}+\sqrt{[2(n-1)+\dot{z}+\dot{\sigma}]^2+\dot{r}^2}}{2(n-1)+\dot{z}+\dot{\tau}+\sqrt{[2(n-1)+\dot{z}+\dot{\tau}]^2+\dot{r}^2}} \right) \\
& \left(\frac{2(n+1)-\dot{z}+\dot{\tau}+\sqrt{[2(n+1)-\dot{z}+\dot{\tau}]^2+\dot{r}^2}}{2(n+1)-\dot{z}+\dot{\sigma}+\sqrt{[2(n+1)-\dot{z}+\dot{\sigma}]^2+\dot{r}^2}} \right) \left(\frac{2n-\dot{z}-\dot{\sigma}+\sqrt{(2n-\dot{z}-\dot{\sigma})^2+\dot{r}^2}}{2n-\dot{z}-\dot{\tau}+\sqrt{(2n-\dot{z}-\dot{\tau})^2+\dot{r}^2}} \right) \\
& \left(\frac{2n+\dot{z}-\dot{\tau}+\sqrt{(2n+\dot{z}-\dot{\tau})^2+\dot{r}^2}}{2n+\dot{z}-\dot{\sigma}+\sqrt{(2n+\dot{z}-\dot{\sigma})^2+\dot{r}^2}} \right) \left(\frac{2n-\dot{z}+\dot{\sigma}+\sqrt{(2n-\dot{z}+\dot{\sigma})^2+\dot{r}^2}}{2n-\dot{z}+\dot{\tau}+\sqrt{(2n-\dot{z}+\dot{\tau})^2+\dot{r}^2}} \right) \\
& \left. \left(\frac{2n+\dot{z}+\dot{\tau}+\sqrt{(2n+\dot{z}+\dot{\tau})^2+\dot{r}^2}}{2n+\dot{z}+\dot{\sigma}+\sqrt{(2n+\dot{z}+\dot{\sigma})^2+\dot{r}^2}} \right) \right\}
\end{aligned}$$

$$\pi_3(\dot{r}, \dot{z}) = \prod_{n=1}^{\infty} \left\{ \frac{2(n+1) + \dot{z} - \dot{\sigma} + \sqrt{[2(n+1) + \dot{z} - \dot{\sigma}]^2 + \dot{r}^2}}{2(n+1) + \dot{z} - \dot{\tau} + \sqrt{[2(n+1) + \dot{z} - \dot{\tau}]^2 + \dot{r}^2}} \right\} \left(\frac{2(n+1) - \dot{z} - \dot{\tau} + \sqrt{[2(n+1) - \dot{z} - \dot{\tau}]^2 + \dot{r}^2}}{2(n+1) - \dot{z} - \dot{\sigma} + \sqrt{[2(n+1) - \dot{z} - \dot{\sigma}]^2 + \dot{r}^2}} \right)$$

$$\left(\frac{2(n-1) + \dot{z} + \dot{\sigma} + \sqrt{[2(n-1) + \dot{z} + \dot{\sigma}]^2 + \dot{r}^2}}{2(n-1) + \dot{z} + \dot{\tau} + \sqrt{[2(n-1) + \dot{z} + \dot{\tau}]^2 + \dot{r}^2}} \right) \left(\frac{2(n-1) - \dot{z} + \dot{\tau} + \sqrt{[2(n-1) - \dot{z} + \dot{\tau}]^2 + \dot{r}^2}}{2(n-1) - \dot{z} + \dot{\sigma} + \sqrt{[2(n-1) - \dot{z} + \dot{\sigma}]^2 + \dot{r}^2}} \right)$$

$$\left(\frac{2n - \dot{z} - \dot{\sigma} + \sqrt{(2n - \dot{z} - \dot{\sigma})^2 + \dot{r}^2}}{2n - \dot{z} - \dot{\tau} + \sqrt{(2n - \dot{z} - \dot{\tau})^2 + \dot{r}^2}} \right) \left(\frac{2n + \dot{z} - \dot{\tau} + \sqrt{(2n + \dot{z} - \dot{\tau})^2 + \dot{r}^2}}{2n + \dot{z} - \dot{\sigma} + \sqrt{(2n + \dot{z} - \dot{\sigma})^2 + \dot{r}^2}} \right)$$

$$\left(\frac{2n - \dot{z} + \dot{\sigma} + \sqrt{(2n - \dot{z} + \dot{\sigma})^2 + \dot{r}^2}}{2n - \dot{z} + \dot{\tau} + \sqrt{(2n - \dot{z} + \dot{\tau})^2 + \dot{r}^2}} \right) \left(\frac{2n + \dot{z} + \dot{\tau} + \sqrt{(2n + \dot{z} + \dot{\tau})^2 + \dot{r}^2}}{2n + \dot{z} + \dot{\sigma} + \sqrt{(2n + \dot{z} + \dot{\sigma})^2 + \dot{r}^2}} \right) \Bigg\}$$

To determine $\dot{S}$, substitute Equation C-5 into Equation C-26 to give

$$\dot{S} = \frac{4\pi[1 - (\gamma/P_0)^2]}{\ln\left(\frac{1}{r_0^2} \pi_2(\dot{r}_0, \dot{z}_0)\right)} \quad (C-28)$$

where $\dot{r}_0 = R_0/D$ and $\dot{z}_0 = L/D + H/2D$.

DETERMINATION OF FLOW RATES

The mass of gas flowing per unit time across a surface enclosing the line source is given by

$$\dot{Q}_m = \oint \rho \underline{q} \cdot d\underline{A} \quad (C-29)$$

Substituting Equations A-28 and A-30 into Equation C-29 gives

$$\dot{Q}_m = \frac{\alpha k}{\mu \gamma} \left(- \oint P \text{ grad } P \cdot d\underline{A} \right) \quad (C-30)$$

Scaling the length dimensions to D and the pressure to P_0 results in

$$\rho_m = \frac{\alpha k D P_0^2}{\mu \gamma} \left(- \oint \dot{P} \text{grad } \dot{P} \cdot d\dot{\underline{A}} \right) \quad (\text{C-31})$$

However, $\dot{P} \text{grad } \dot{P} = \frac{1}{2} \text{grad } \dot{P}^2 = \frac{1}{2} \text{grad } \dot{U}$ and Equation C-31 becomes

$$\rho_m = \frac{\alpha k D P_0^2}{\mu \gamma} \left(- \frac{1}{2} \oint \text{grad } \dot{U} \cdot d\dot{\underline{A}} \right) \quad (\text{C-32})$$

A line source can be considered as a continuous distribution of infinitesimal point sources. The point source solution has the form

$$\dot{U} = \dot{S} F(\dot{r}, \dot{z})$$

Where $\dot{S}$ is the strength of a single point source. The strength of an infinitesimal length of source is $\dot{S} d\dot{h}$. After integration of the solution, $\dot{U}$, to obtain the line source solution, $\dot{S}$ is the strength of the line source per unit length. Thus the total mass of fluid flowing per unit time from the line source of the potential problem given by Equation C-18 through C-21 is given by

$$- \oint \text{grad } \dot{U} \cdot d\dot{\underline{A}} = \dot{S}(\dot{\tau} - \dot{\sigma}) \quad (\text{C-33})$$

because this flow may be visualized as the flow of an incompressible fluid in space according to the equation

$$\underline{q} = - \text{grad } \dot{U}$$

Thus, from Equation C-32 and C-33

$$Q_m = \frac{\alpha k D P_o^2 (\dot{\tau} - \dot{\sigma}) \dot{S}}{2 \mu \gamma} \quad (C-34)$$

In equation C-34, Q_m would be given in slugs/sec for all variables in the ft-lb-sec dimension set. It is convenient to express flow rates in ft^3/min at standard conditions of temperature and pressure (SCFM). This is accomplished by multiplying Equation C-34 by 60 and dividing by ρ_s , where ρ_s is the density of air at standard conditions of temperature and pressure (0 degrees Centigrade and atmospheric pressure). Thus, with Q_s denoting the flow rate in SCFM, Equation C-34 becomes

$$Q_s = \frac{30 \alpha k D P_o^2 (\dot{\tau} - \dot{\sigma}) \dot{S}}{\mu \gamma \rho_s} \quad (C-35)$$

which after substituting from Equation C-28 becomes

$$Q_s = \frac{30 \alpha k D P_o^2 (\dot{\tau} - \dot{\sigma}) 4\pi [1 - (\gamma/P_o)^2]}{\mu \gamma \rho_s \ln \left(\frac{1}{r_o^2} {}_2F_2(\dot{r}_o, \dot{z}_o) \right)} \quad (C-36)$$

Equation C-36 can be grouped as follows

$$Q_s = \left(\frac{30}{\gamma \rho_s} \right) \left(\frac{\alpha}{\mu} \right) \left(P_o^2 - \gamma^2 \right) k D \dot{\hat{Q}} \quad (C-37)$$

where $\dot{\hat{Q}}$ may be regarded as a dimensionless source strength given by

$$\dot{\hat{Q}} = \frac{4\pi (\dot{\tau} - \dot{\sigma})}{\ln \left(\frac{1}{r_o^2} {}_2F_2(\dot{r}_o, \dot{z}_o) \right)} \quad (C-38)$$

If the injection pressure is given in psig instead of psfg ($P_0 = 144 p_0' + \gamma$), Equation C-37 becomes

$$Q_s = [1.17 \times 10^5] [A(T)] [B(p_0')] k D \dot{Q} \quad (C-39)$$

where

$$A(T) = \alpha/\mu \quad (\text{a function of } T) \quad (C-40)$$

$$B(p_0') = (p_0' + 14.7)^2 - 216.1 \quad (\text{a function of } p_0') \quad (C-41)$$

T is the temperature in degrees Centigrade

p_0' is the injection pressure in pounds per square inch gage

k is the permeability of the ground to air in square feet

D is the total depth of the system in feet

$\dot{Q}$ is the dimensionless source strength given by Equation C-38

Note that $\dot{Q}$ depends only upon the dimensionless geometry of the system and is independent of T , p_0' , k , and D .

DETERMINATION OF FLOW VARIATION AT GROUND SURFACE

It is quite easy to determine the fraction of the total flow which enters the atmosphere across the ground surface within a circle of radius R from the well axis. Curves of the fraction plotted as a function of R/D are useful for determining starting locations for the numerical generation of streamlines.

It is convenient to calculate the flow fraction in dimensionless terms which depend only upon the relative geometry of the flow system.

The integration is performed on the plane, $\dot{z} = 0$. From equation C-31, define $\dot{Q}_R$ as

$$\dot{Q}_R = - 2 \int_0^{\dot{R}} \dot{P} \frac{\partial \dot{P}}{\partial \dot{z}} (2\pi \dot{r} d\dot{r}) \quad (C-42)$$

or

$$\dot{Q}_R = - 2\pi \int_0^{\dot{R}} \frac{\partial \dot{P}^2}{\partial \dot{z}} \dot{r} d\dot{r} \quad (C-43)$$

From equation C-27,

$$\frac{\partial \dot{P}^2(\dot{r}, 0)}{\partial \dot{z}} = \frac{\dot{S}}{4\pi} \frac{\partial}{\partial \dot{z}} \left[\ln \pi_3(\dot{r}, 0) \right] \quad (C-44)$$

If the integration is carried to $\dot{r} \rightarrow \infty$, Equation C-33 gives the total flow as

$$\dot{Q}_T = \dot{S} (\tau - \sigma) \quad (C-45)$$

Combining Equations C-43, C-44, and C-45 gives the flow fraction as

$$\dot{Q}_R / \dot{Q}_T = - \frac{1}{2(\tau - \sigma)} \int_0^{\dot{R}} \frac{\partial}{\partial \dot{z}} (\ln \pi_3(\dot{r}, 0)) \dot{r} d\dot{r} \quad (C-46)$$

The integration indicated in Equation C-46 was performed numerically for the results of this report. Note that the flow fraction, $\dot{Q}_R / \dot{Q}_T$, is independent of the radius of the well, $\dot{r}_0$. Thus, for various values of $\dot{R}$, the flow fraction can be plotted as a function of $\dot{R}$ with L/D and H/D as parameters.

For some applications it may be of interest to know the flow per unit area (flux) at ground surface. The flux is easily obtained by dividing the incremental flow fraction arising in the numerical integration of Equation C-46 by the appropriate incremental area; i.e., the equation

$$\text{flux} = \frac{\left(\dot{Q}_R/\dot{Q}_T\right)_2 - \left(\dot{Q}_R/\dot{Q}_T\right)_1}{\pi(\dot{R}_2^2 - \dot{R}_1^2)} \quad (\text{C-47})$$

gives the flux at $\frac{1}{2}(\dot{R}_1 + \dot{R}_2)$. To obtain the actual flux in SCFM/ft² for any particular problem, Equation C-47 must be multiplied by the total flow rate in SCFM and divided by D^2 .

DETERMINATION OF TRAVEL TIMES

Darcy's equation, Equation A-28, gives the velocity of gas as if the entire space rather than only the pores were transmitting gas. That is, $\underline{q}$ is expressed as the flow rate per total cross-sectional area. In order to obtain an average velocity of flow through the pores, $\underline{q}$ must be divided by the porosity. Thus,

$$\frac{ds}{dt} = \frac{|\underline{q}|}{f} = \frac{k}{\mu f} |\underline{q} \text{ grad } P| \quad (\text{C-48})$$

where s is the distance down the flow path, t is the time of travel, f is the porosity, and $|\underline{q}|$ and $|\underline{q} \text{ grad } P|$ denote the magnitude of the two vectors $\underline{q}$ and $\text{grad } P$. Integrating Equation C-48 with $t = 0$ at $s = 0$ gives

$$t = \int_0^s \frac{\mu f \, ds}{k |\text{grad } P|} \quad (\text{C-49})$$

If length dimensions are scaled to D and pressure is scaled to P_o , Equation C-49 becomes

$$t = \frac{\mu D^2}{k P_o} \left(\int_0^s \frac{f \, ds}{|\text{grad } P|} \right) \quad (\text{C-50})$$

where t is the travel time in seconds for the ft-lb-sec dimension set.

If the path of flow is known, Equation C-50 may be used to obtain the time of gas travel along the flow path. The straight line $r = 0$ is such a path in the model presented in this report; and, in fact, is the shortest path with the shortest time for gas travel from the injection point to ground surface. Other paths were calculated numerically using the computer program STREAM⁽⁴⁾ which both determines the path of flow (streamline) and performs the integration indicated in Equation C-50. In the computer programs a porosity value of 0.3 was used. Hence, for any value of porosity, Equation C-50 becomes

$$t = \frac{\mu f D^2}{0.3 k P_o} \left(\int_0^s \frac{0.3 \, ds}{|\text{grad } P|} \right) \quad (\text{C-51})$$

or

$$t = \left(\frac{\mu f D^2}{0.3 k P_o} \right) \dot{t} \quad (\text{C-52})$$

where $\dot{t}$ is the dimensionless travel time given by

$$\dot{t} = \int_0^{\dot{s}} \frac{0.3 \, d\dot{s}}{|\text{grad } \dot{P}|} \quad (\text{C-53})$$

Equation C-52 gives the travel time in seconds. After dividing by 3600 and converting the pressure to psig, Equation C-52 becomes

$$t = \left(\frac{\mu f D^2}{(1.555 \times 10^5)(p_o' + 14.7)k} \right) \dot{t} \quad (\text{C-54})$$

where t is the travel time in hours, μ is the viscosity of air in lb-sec/ft², f is the porosity, $\dot{t}$ is the dimensionless travel time given by Equation C-53, and the other variables are described following Equation C-41.

The dimensionless travel time, $\dot{t}$, is a function of the dimensionless geometry of the system and the injection pressure. From Equation C-24,

$$\text{grad } \dot{P} = \frac{\text{grad } \dot{U}}{2\sqrt{\dot{U} + (\gamma/P_o)^2}} \quad (\text{C-55})$$

From Equations C-53 and C-55 it is evident that P_o cannot be removed from under the radical and hence must remain within the integral defining $\dot{t}$.

PRESENTATION OF MINIMUM TRAVEL TIME

The dimensionless travel time, $\dot{t}$, is a function of L/D , H/D , R_o/D , and P_o . Graphical presentation of minimum travel time

(along the path $r = 0$) for a very wide range of the variables results in a large number of graphs. For ease of use, it is desirable to reduce the number of graphs required to display the results. By plotting $\dot{t}$ as a function of injection pressure, p_o' (expressed in psia), it was found that all data could be closely approximated by an equation of the form

$$\dot{t} = b(p_o')^a \quad (C-56)$$

where a was nearly constant and b depended only upon L/D , H/D , and R_o/D . For constant a , combining Equations C-54 and C-56 results in

$$t = \left(\frac{\mu f D^2 F(p_o')}{(1.555 \times 10^5) k} \right) b \quad (C-57)$$

where

$$F(p_o') = \frac{(p_o')^a}{p_o' + 14.7} \quad (C-58)$$

APPENDIX D

TRANSFORMATION OF PROBLEMS INVOLVING
FLOW IN ANISOTROPIC MEDIA

Consider the single-phase, isothermal, steady flow of a compressible gas in a homogeneous porous medium which has horizontal permeability, k_H , different from the vertical permeability, k_V . In cylindrical coordinates, let the z -axis be vertical. The assumption of homogeneity implies symmetry about the axis, $r = 0$. Let $\underline{h}$ and $\underline{k}$ be the unit base vectors in the r and z directions, respectively. The volume flux is given by

$$\underline{q} = -\frac{1}{\mu} \underline{h} k_H \frac{\partial P}{\partial r} + \underline{k} k_V \frac{\partial P}{\partial z} \quad (D-1)$$

where $P = P(r, z)$ and the gravity component of potential is neglected (See Appendix A, Equation A-28).

Combining Equations D-1 and A-18, using Equation A-30 gives

$$k_H \frac{\partial^2 P^2}{\partial r^2} + \frac{k_H}{r} \frac{\partial P^2}{\partial r} + k_V \frac{\partial^2 P^2}{\partial z^2} = 0 \quad (D-2)$$

Let $(\bar{r}, \bar{z})$ denote a coordinate system related to the (r, z) system by

$$\begin{aligned} \bar{r} &= ar \\ \bar{z} &= z \end{aligned} \quad (D-3)$$

where a is constant. Equation D-2 written in the $(\bar{r}, \bar{z})$ system becomes

$$k_H a^2 \frac{\partial^2 \bar{P}^2}{\partial \bar{r}^2} + \frac{k_H a^2}{\bar{r}} \frac{\partial \bar{P}^2}{\partial \bar{r}} + k_V \frac{\partial^2 \bar{P}^2}{\partial \bar{z}^2} = 0 \quad (D-4)$$

where $\bar{P} = \bar{P}(\bar{r}, \bar{z}) = P\left(\frac{1}{a} \bar{r}, \bar{z}\right) = P(r, z)$. Thus, if $a^2 = k_V/k_H$, Equation D-4 reduces to

$$\frac{\partial^2 \bar{P}^2}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \frac{\partial \bar{P}^2}{\partial \bar{r}} + \frac{\partial^2 \bar{P}^2}{\partial \bar{z}^2} = 0 \quad (D-5)$$

which is the expansion of

$$\text{div grad } \bar{p}^2 = 0 \quad (\text{D-6})$$

in the $(\bar{r}, \bar{z})$ cylindrical coordinate system. Comparison of Equations A-32 and D-6 shows that Equation D-5 can be interpreted as describing flow through an equivalent isotropic media.

Flow rates are calculated as follows. Any curve in the (r, z) system can be described by the parametric equations

$$\begin{aligned} r &= F(\lambda) \\ z &= G(\lambda) \end{aligned} \quad (\text{D-7})$$

Suppose it is desired to determine the mass flow across the surface formed by rotating through 2π radians a segment of a curve between the points $(F(0), G(0))$ and $(F(1), G(1))$. In general the mass flow Q_m across a surface S is given by

$$Q_m = \int_S \rho \underline{q} \cdot d\underline{A} \quad (\text{D-8})$$

For the particular surface considered, $d\underline{A} = 2\pi r ds (\underline{n}_r \underline{h} + \underline{n}_z \underline{k})$ where ds is the arc length given by

$$ds = \sqrt{(F'(\lambda))^2 + (G'(\lambda))^2} d\lambda$$

and $\underline{n}_r$ and $\underline{n}_z$ are the radial and vertical components, respectively, of the unit vector normal to the curve. Thus the mass flow is

$$Q_m = - \frac{2\pi\alpha}{\mu\gamma} \int_0^1 \left(k_H P \frac{\partial P}{\partial r} \underline{n}_r + k_V P \frac{\partial P}{\partial z} \underline{n}_z \right) r \sqrt{(F'(\lambda))^2 + (G'(\lambda))^2} d\lambda \quad (\text{D-9})$$

The tangential unit vector is given by $[F'(\lambda) \underline{h} + G'(\lambda) \underline{k}] / \sqrt{(F'(\lambda))^2 + (G'(\lambda))^2}$, and the normal unit vector is $[-G'(\lambda) \underline{h} + F'(\lambda) \underline{k}] / \sqrt{(F'(\lambda))^2 + (G'(\lambda))^2}$

Substituting for n_r and n_z in Equation D-9 and simplifying gives

$$Q_m = - \frac{2\pi\alpha}{\mu\gamma} \int_0^1 \left(-k_H P \frac{\partial P}{\partial r} G'(\lambda) + k_V P \frac{\partial P}{\partial z} F'(\lambda) \right) F(\lambda) d\lambda \quad (D-10)$$

The equations

$$\begin{aligned} \bar{r} &= f(\lambda) \\ \bar{z} &= g(\lambda) \end{aligned} \quad (D-11)$$

specify the image of the curve in the $(\bar{r}, \bar{z})$ system. Hence, from Equations D-3, D-7, and D-11

$$\begin{aligned} f(\lambda) &= aF(\lambda) \\ g(\lambda) &= G(\lambda) \end{aligned} \quad (D-12)$$

Equation D-10 becomes

$$Q_m = - \frac{2\pi\alpha}{\mu\gamma} \int_0^1 \left(k_H \bar{P} a \frac{\partial \bar{P}}{\partial \bar{r}} g'(\lambda) + k_V \bar{P} \frac{\partial \bar{P}}{\partial \bar{z}} \frac{1}{a} f'(\lambda) \right) \frac{1}{a} f(\lambda) d\lambda$$

or since $a^2 = k_V/k_H$,

$$Q_m = - \frac{2\pi\alpha k_H}{\mu\gamma} \int_0^1 \left(\bar{P} \frac{\partial \bar{P}}{\partial \bar{r}} g'(\lambda) + \bar{P} \frac{\partial \bar{P}}{\partial \bar{z}} f'(\lambda) \right) f(\lambda) d\lambda \quad (D-13)$$

Equation D-13 is identical to the equation which would be obtained for the mass flow in the isotropic system if the permeability of the isotropic system were equal to k_H . Thus, the true flow rate can be calculated in the isotropic system $(\bar{r}, \bar{z})$ by using k_H as the permeability of the equivalent isotropic media.

In order to calculate time of travel along the path taken by the fluid particle, consider a streamline in the (r, z) system described by the parametric equations

$$\begin{aligned} r &= F(\lambda) \\ z &= G(\lambda) \end{aligned} \quad (D-14)$$

The equation for such a streamline is

$$v_z \frac{dr}{d\lambda} - v_r \frac{dz}{d\lambda} = 0$$

or

$$F'(\lambda) k_V \frac{\partial P(F(\lambda), G(\lambda))}{\partial z} - G'(\lambda) k_H \frac{\partial P(F(\lambda), G(\lambda))}{\partial r} = 0 \quad (D-15)$$

According to Equation D-3, let $f(\lambda) = aF(\lambda)$ and $g(\lambda) = G(\lambda)$.

Equation D-15 becomes

$$\frac{1}{a} f'(\lambda) k_V \frac{\partial \bar{P}(f(\lambda), g(\lambda))}{\partial \bar{z}} - g'(\lambda) k_H a \frac{\partial \bar{P}(f(\lambda), g(\lambda))}{\partial \bar{r}} = 0$$

or after dividing through by $\sqrt{k_V k_H}$,

$$f'(\lambda) \frac{\partial \bar{P}(f(\lambda), g(\lambda))}{\partial \bar{z}} - g'(\lambda) \frac{\partial \bar{P}(f(\lambda), g(\lambda))}{\partial \bar{r}} = 0 \quad (D-16)$$

which is the equation for a "streamline" in the equivalent isotropic system $(\bar{r}, \bar{z})$, where $\bar{r} = f(\lambda)$ and $\bar{z} = g(\lambda)$ are the parametric equations for the "streamline." Thus, streamlines transform according to Equation D-3 into streamlines in the equivalent isotropic system.

The travel time along a streamline ($r = F(\lambda)$, $z = G(\lambda)$) from the point $(F(0), G(0))$ to the point $(F(1), G(1))$ can be obtained by integrating the expression

$$\frac{ds}{dt} = \frac{1}{f} \sqrt{k_H^2 \left(\frac{\partial P(F(\lambda), G(\lambda))}{\partial r} \right)^2 + k_V^2 \left(\frac{\partial P(F(\lambda), G(\lambda))}{\partial z} \right)^2}$$

Since the arc length ds is given by

$$ds = \sqrt{(F'(\lambda))^2 + (G'(\lambda))^2} d\lambda$$

the time is

$$T = f \int_0^1 \frac{\sqrt{(F'(\lambda))^2 + (G'(\lambda))^2}}{\sqrt{k_H^2 P_r^2 + k_V P_z^2}} d\lambda \quad (D-17)$$

where f is the effective porosity and P_r and P_z denote partial differentiation with respect to r and z , respectively. Transforming Equation D-17 to the $(\bar{r}, \bar{z})$ system results in

$$T = \frac{f}{k_V} \int_0^1 \frac{\sqrt{\frac{k_H}{k_V} (f'(\lambda))^2 + (g'(\lambda))^2}}{\sqrt{\frac{k_H}{k_V} \bar{P}_r^2 + \bar{P}_z^2}} d\lambda \quad (D-18)$$

or

$$T = \frac{f}{k_V} \int_0^1 \frac{\sqrt{\frac{k_H}{k_V} + \left(\frac{g'(\lambda)}{f'(\lambda)}\right)^2}}{\sqrt{\frac{k_H}{k_V} + \left(\frac{\bar{P}_z}{\bar{P}_r}\right)^2}} \frac{f'(\lambda) d\lambda}{\bar{P}_r} \quad (D-19)$$

From Equation D-16

$$\frac{g'(\lambda)}{f'(\lambda)} = \frac{\bar{P}_z}{\bar{P}_r} \quad (D-20)$$

and Equation D-19 becomes

$$T = \frac{f}{k_V} \int_0^1 \frac{f'(\lambda) d\lambda}{\bar{P}_r} \quad (D-21)$$

In the $(\bar{r}, \bar{z})$ coordinates, let k_e be the "equivalent" permeability of the isotropic system. The travel time would be

$$T = \frac{f}{k_e} \int_0^1 \frac{\sqrt{(f'(\lambda))^2 + (g'(\lambda))^2}}{\sqrt{\bar{p}_r^2 + \bar{p}_z^2}} d\lambda \quad (D-22)$$

or

$$T = \frac{f}{k_e} \int_0^1 \frac{\sqrt{1 + \left(\frac{g'(\lambda)}{f'(\lambda)}\right)^2} f'(\lambda) d\lambda}{\sqrt{1 + \left(\frac{\bar{p}_z}{\bar{p}_r}\right)^2} \bar{p}_r} \quad (D-23)$$

Substituting Equation D-20 into D-23 gives

$$T = \frac{f}{k_e} \int_0^1 \frac{f'(\lambda) d\lambda}{\bar{p}_r} \quad (D-24)$$

Comparison of Equations D-21 and D-24 shows that the permeability of the equivalent isotropic system should be set equal to k_V for calculation of travel times.

The procedure is now formulated for solving problems of flow in homogeneous, anisotropic media in cylindrical coordinates. The procedure is outlined as follows:

1. Transform the anisotropic system (r, z) to an equivalent isotropic system $(\bar{r}, \bar{z})$ by multiplying the radial distances, r , by $\sqrt{k_V/k_H}$ obtaining $\bar{r}$.
2. Calculate the flow pattern (i.e., streamlines and equipotential lines) in the equivalent isotropic system.
3. Calculate flow rates using k_H as the permeability of the isotropic system.
4. Calculate travel times using k_V as the permeability of the isotropic system.

APPENDIX E

GRAPHS GIVING DIMENSIONLESS FLOW RATE $\dot{Q}$

FOR USE IN EQUATION C-39

In order to use Equation C-39 to calculate the flow rate in CFM at standard conditions of temperature and pressure, it is necessary to evaluate Equations C-40 and C-41. For convenience, the latter two Equations are presented in graphical form on the next two pages.

By inspection of Equation C-38, it is obvious that $\dot{Q}$, the dimensionless source strength or dimensionless flow rate, is a function only of R_o/D , L/D , and H/D . Twelve graphs giving $\dot{Q}$ are presented in this appendix (pages E-4 through E-15). A single value of R_o/D applies to each graph. $\dot{Q}$ is plotted as the ordinate with L/D as the abscissa, and each solid curve applies for the stated value of H/D . Note that the dashed curve connecting end points of the solid curves represents the limit of physically possible geometries since $L/D + H/D \leq 1$ (See Figure B-1). Linear interpolation between curves yields quite good results as does linear interpolation between different values of R_o/D .

E-2

E-4

E-5

E-9

E-10

APPENDIX F

GRAPHS GIVING FLOW VARIATION

AT GROUND SURFACE

The graphs presented in this appendix are valid only for the duration of the injection. The first eight graphs (pages F-2 through F-9) give the fraction of the total flow which enters the atmosphere across the ground surface within a circle of radius R from the axis of the injection well. A single value of L/D applies to each graph. The flow fraction, $\dot{Q}_R/\dot{Q}_T$, is plotted against R/D with each curve representing one value of H/D (See Equation C-46).

For some applications, it may be of interest to know the flow rate per unit area (flux) at ground surface. The dimensionless flux calculated from Equation C-47 is presented in graphical form on pages F-10 through F-17. Note that in order to obtain the actual flux in SCFM/ft², the values taken from the graphs must be multiplied by the total flow rate in SCFM and divided by D^2 .

F-5

F-7

F-8

F-9

F-10

F-15

APPENDIX G

GRAPHS GIVING DIMENSIONLESS MINIMUM TIME FOR
GAS TRAVEL FROM INJECTION WELL TO GROUND SURFACE

It was found that over the range of 1 to 20 psig injection pressure, minimum travel times could be represented to within about $\pm 5\%$ using Equation C-56 with $a = -0.8567$ and the values of b as shown on pages G-3 through G-14 of this appendix.

For convenience, $F(p_o')$ given by Equation C-58 is shown graphically on page G-2 and the viscosity⁽⁶⁾ of air is given on page G-3.

G-3

G-4

G-5

G-6

G-7

G-8

G-9

G-10

G-11

G-12

G-13

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