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CBEAM.2-D,
A TWO-DIMENSIONAL BEAM FIELD CODE

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May 1985

Lawrence
Livermore
National
Laboratory

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Work performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore Laboratory under Contract W-7405-Eng-48.

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MASTER

CBEAM.2-D, A TWO-DIMENSIONAL BEAM FIELD CODE

ABSTRACT

CBEAM.2-D is a two-dimensional solution of Maxwell's equations for the case of an electron beam propagating through an air medium. Solutions are performed in the beam-retarded time frame. Conductivity is calculated self-consistently with field equations, allowing sophisticated dependence of plasma parameters to be handled. A unique feature of the code is that it is implemented on an IBM PC microcomputer in the BASIC language. Consequently, it should be available to a wide audience.

I. INTRODUCTION

In recent years, several problems in applied physics have been identified that require a detailed knowledge of electromagnetic fields associated with a cylindrical energy pulse. Many of the problems examined include the formation of relativistic electrons which then pass through some medium. Characteristics of the electromagnetic field associated with the electron pulse are usually of interest for several reasons--such as diagnostic content and feedback into transport of the main energy pulse. Thus, calculation of the electromagnetic fields becomes a critical component of the overall problem examination.

In this report I present a numerical program called CBEAM.2-D, which calculates the electromagnetic fields associated with a cylindrically symmetric relativistic pulse of electrons passing through air. Similar programs have been developed and

reported on in the past,¹⁻³ but these have all been produced with particular applications in mind and with correspondingly developed mathematical and numerical modeling. Usually these programs are available only on the mainframe computer system upon which they were developed. This program, however, was written to operate on an IBM PC and with a more general audience in mind.

The electrodynamic model equations are derived in Section II. Fundamental features of the equations are:

1. Radial and axial components of the electric field and the azimuthal magnetic field are calculated.
2. Beam-retarded time coordinates and the "rigid beam" approximation are used, i.e., derivatives with respect to z in the retarded time coordinate system are set to zero.
3. A high-gamma approximation is employed.

Model equations describing the state of the plasma electrons and their formation by relativistic electrons as they pass through air are given in Section III. The plasma electron model is similar to that originally developed by Wittwer⁴ and modified by Dreyer.¹ Features of the plasma model equations are:

1. The scalar form of Ohm's law is applied,

$$\vec{J}_p = \sigma \vec{E} \quad (1)$$

where $\vec{J}_p$ is the plasma current density, $\vec{E}$ is the electric field strength, and σ is the scalar conductivity.

2. Plasma electron production modes are by relativistic electron impact ionization and avalanche ionization.
3. Plasma subtraction modes are two- and three-body attachment processes and by three-body recombination.
4. Plasma electron momentum loss rate and energy loss rate are characterized by a momentum transfer collision frequency and an energy transfer collision frequency, respectively.
5. All reaction and collision frequencies are assumed to be functions of plasma electron energy and background neutral gas energy.
6. The plasma is only weakly ionized.

Finite difference equations used in the program are derived in Section IV and sample results are provided in Section V. A program listing is given in the Appendix.

II. ELECTROMAGNETIC MODEL EQUATIONS

Electron beams moving at relativistic speeds through the air have three characteristics that allow important approximations to the Maxwell field equations, greatly simplifying numerical calculation. These are

1. Terms involving $1/\gamma^2$ in the model equations can be ignored.
2. Because field disturbances propagate from the rear to the front of the pulse extremely slowly and propagate from the front of the beam to the rear of the beam at nearly the speed

of light, the electric field quickly achieves an equilibrium and the field is essentially "frozen" with the beam as it moves through the air. Thus, one can safely ignore z variations in the beam-retarded time coordinate system (to be defined later).

3. Electrons, even those moving at relativistic speeds, are highly ionizing and rapidly convert the air from a dielectric to a highly conductive medium. In that region of high conductivity, conduction currents overwhelm displacement currents. The axial component of the displacement current is set to zero for the purpose of this study and is a highly accurate approximation within the high conductivity regions. This approximation is also extended throughout the calculational region, but the approximation does not appear to substantially affect the accuracy of the calculations since the axial component of the field also scales as $1/\gamma^2$ in the absence of conductivity.¹

We begin with Maxwell's curl equation for the fields,

$$\vec{\nabla} \times \vec{E} = - \frac{1}{c} \frac{\partial \vec{B}}{\partial t} \quad (2-1)$$

$$\vec{\nabla} \times \vec{B} = \frac{4\pi}{c} \vec{J} + \frac{1}{c} \frac{\partial \vec{E}}{\partial t} \quad (2-2)$$

where $\vec{E}$ is the electric field strength, $\vec{B}$ is the magnetic field intensity, $\vec{J}$ is the conduction current density, and c is the speed of light. Employing a cylindrical coordinate system and assuming symmetry about the cylinder axis, the equations become:

$$-\frac{\partial E_z}{\partial r} - \frac{\partial E_r}{\partial z} = \frac{1}{c} \frac{\partial B_\theta}{\partial t}, \quad (2-3)$$

$$-\frac{\partial E_r}{\partial t} + 4\pi\sigma E_r = -c \frac{\partial B_\theta}{\partial z}, \quad (2-4)$$

$$-\frac{\partial E_z}{\partial t} + 4\pi (J_{bz} + \sigma E_z) = \frac{c}{r} \frac{\partial}{\partial r} (r B_\theta). \quad (2-5)$$

Ohm's law is used to relate plasma current density and field strength,

$$\vec{J}_p = \sigma \vec{E}_z \quad (2-6)$$

and the longitudinal component of the beam current density is J_{bz} . The subscripts r , z , and θ indicate radial, axial, and azimuthal components, respectively. Independent variables r , z , and t are radial distance, longitudinal distance, and time, respectively.

Now, transform the independent variables (t, z) to (τ, λ) according to

$$\tau = t - z/\beta c \quad (2-7)$$

$$\lambda = z/\beta c. \quad (2-8)$$

Employing an arbitrary function, f , derivatives transform according to:

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial \tau} \frac{\partial \tau}{\partial t} + \frac{\partial f}{\partial \lambda} \frac{\partial \lambda}{\partial t} = \frac{\partial f}{\partial \tau}, \quad (2-9)$$

$$\frac{\partial f}{\partial z} = \frac{\partial f}{\partial \tau} \frac{\partial \tau}{\partial z} + \frac{\partial f}{\partial \lambda} \frac{\partial \lambda}{\partial z} = \frac{1}{\beta c} \left(\frac{\partial f}{\partial \lambda} - \frac{\partial f}{\partial \tau} \right). \quad (2-10)$$

Setting the λ derivative equal to zero (rigid beam approximation) in equation (2-10) and applying the derivative transformations to equations (2-3), (2-4), and (2-5):

$$-\frac{\partial E_z}{\partial r} + \frac{1}{\beta c} \frac{\partial E_r}{\partial \tau} = \frac{1}{c} \frac{\partial B_\theta}{\partial \tau}, \quad (2-11)$$

$$-\frac{\partial E_r}{\partial t} + 4\pi\sigma E_r = \frac{1}{\beta} \frac{\partial B_\theta}{\partial \tau}, \quad (2-12)$$

$$-\frac{\partial E_z}{\partial \tau} + 4\pi (J_{bz} + \sigma E_z) = \frac{c}{r} \frac{\partial}{\partial r} (r B_\theta). \quad (2-13)$$

These are the fundamental field equations in the retarded time coordinate system.

Our goal is to decouple the fields between equations. This can be done by deriving a second-order differential equation (DE) in E_z (with respect to r and τ) then requiring that the axial displacement current is zero, and replacing E_r by a function of E_z . The resulting equation is essentially a diffusion equation for E_z .

First, eliminate B_θ from equations (2-11) and (2-13):

$$\frac{c}{r} \frac{\partial}{\partial r} \left(r \frac{\partial B_\theta}{\partial \tau} \right) = \frac{\partial}{\partial \tau} \left[-\frac{\partial E_z}{\partial r} + 4\pi J_z \right] = \frac{c^2}{r} \frac{\partial}{\partial r} \left[r \left(-\frac{\partial E_z}{\partial r} + \frac{1}{\beta c} \frac{\partial E_r}{\partial \tau} \right) \right]. \quad (2-14)$$

Next, eliminate B_θ from equations (2-11) and (2-12):

$$-\frac{\partial B_\theta}{\partial \tau} \quad c \frac{\partial E_z}{\partial r} + \frac{1}{\beta} \frac{\partial E_r}{\partial \tau} = \beta \frac{\partial E_r}{\partial \tau} + 4\pi\sigma\beta E_r,$$

$$\text{or} \quad -\frac{1}{\beta^2 \gamma^2} \frac{\partial E_r}{\partial \tau} + 4\pi \sigma E_r = c \frac{\partial E_z}{\partial \tau} . \quad (2-15)$$

Make the high-gamma approximation,

$$|4\pi \sigma E_r| \gg \left| -\frac{1}{\beta^2 \gamma^2} \frac{\partial E_r}{\partial \tau} \right|$$

and solve for E_r ,

$$E_r = \frac{c}{4\pi \beta \sigma} \frac{\partial E_z}{\partial \tau} . \quad (2-16)$$

Replace E_r and set the axial displacement current density equal to zero in equation (2-14):

$$\frac{1}{r} \frac{\partial}{\partial r} \left\{ r \left[-\frac{\partial E_z}{\partial r} + \frac{\partial}{\partial \tau} \left(-\frac{1}{4\pi \beta^2 \sigma} \frac{\partial E_z}{\partial \tau} \right) \right] \right\} = \frac{4\pi}{c^2} \frac{\partial}{\partial \tau} (J_{bz} + \sigma E_z) . \quad (2-17)$$

Remaining equations are

$$B_\theta = \frac{4\pi}{cr} \int_0^r r' dr' (J_{bz} + \sigma E_z) , \quad (2-18)$$

$$-\frac{\partial E_r}{\partial \tau} + 4\pi \sigma E_r = \frac{1}{\beta} \frac{\partial B_\theta}{\partial \tau} . \quad (2-19)$$

III. SWARM EQUATIONS

A pulse of relativistic electrons passing through the air interacts strongly with air molecules to produce many low-energy electrons. Because of the many different chemical and electrical species comprising air, it is difficult if not impossible to completely describe the state of the air molecules, ions, and electrons altered by the pulse of high-energy electrons.

Nonetheless, simple treatments do exist that provide a substantial degree of accuracy for short pulses in clear air.

In this section, I present a simple treatment of air chemistry and dynamics called the swarm equations. Several different versions have been developed, but the one presented here is a simplified version of the model developed by Wittwer⁴ and later modified by Dreyer.¹ It is a single-species model because it tracks only the state of low-energy plasma electrons; however, effects of many different species are hidden in the reaction rates. The model is valid only for weakly ionized plasmas. For a derivation of the swarm equations and a discussion of the dominant chemical reactions, see Ref. 1.

Fundamental equations are:

$$\frac{dn_e}{dt} = R_I + (\alpha_I(W) - \alpha_A(W)) n_e - \beta_R(W) n_e^2, \quad (3-1)$$

$$\frac{dW}{dt} = \frac{e^2 E^2}{m v_e} - \nu_m(W - \epsilon), \quad (3-2)$$

$$\frac{d\epsilon}{dt} = f \left(\frac{dW}{dt} \right) + \nu_e(W - \epsilon) (n_e/N), \quad (3-3)$$

$$\sigma = -\frac{e^2 n_e}{m v_e}. \quad (3-4)$$

where

n_e = plasma electron number density (no./cm³),

R_I = impact ionization rate (no./cm³-sec),

α_I = avalanche coefficient (sec⁻¹),

α_a = attachment coefficient (sec^{-1}),
 β_r = recombination coefficient ($\text{cm}^3/\text{no.}-\text{sec}$),
 W = characteristic plasma electron energy (eV),
 N_0 = air molecular density at STP ($= 2.69 \times 10^{19}/\text{cm}^3$),
 N = air molecular density at pressure ($\text{no.}/\text{cm}^3$),
 E = electric field strength (sV/cm),
 ϵ = characteristic energy of background neutral gas (eV),
 f = fraction of beam energy loss that goes directly to neutral particles ($= 0.3$),
 $\frac{dT}{dt}$ = energy loss rate of beam particles (eV/sec),
 ν_m = energy transfer collision frequency (sec^{-1}),
 ν_e = momentum transfer collision frequency (sec^{-1}), and
 e = electronic charge (4.8×10^{-10} statcoulombs).

Collision frequencies and reaction rates are all functions of the electron swarm energy, W , and are:

electron-neutral momentum transfer collision frequency, ν_{mn} ,

$$\frac{\nu_{mn}}{N} = 1.25 \times 10^{-7} W^{0.0935} (1 + 0.93 W^{1.5})^{0.405} / (1 + 1.26 W^{1.67})^{0.373}; \quad (3-5)$$

electron-ion momentum transfer collision frequency, ν_{mi} ,

$$\nu_{mi} = 1.46 \times 10^{-6} (n_e/W^{3/2}) \text{Ln} \left(1.55 \times 10^{10} (W^3/n_e)^{1/2} \right); \quad (3-6)$$

electron-neutral energy transfer collision frequency,

$$\frac{v_c}{N} = 1.3 \times 10^{-11} (1 + 4.4 \times 10^4 W^5)^{0.32} / \\ (1 + 1.93 \times 10^3 W^{10})^{0.138} + 4.13 \times 10^{-10} W^{5.22} / \\ (1 + 0.06 W^7)^{0.456} (1 + 1.68 \times 10^{-3} W^4)^{0.097} ; \quad (3-7)$$

avalanche coefficient,

$$\frac{\alpha_I}{N} = 3.76 \times 10^{-15} W^{8.7} / (1 + 2.92 \times 10^{-4} W^5)^{1.412} ; \quad (3-8)$$

three-body dissociative attachment, α_{3a} ,

$$\frac{\alpha_{3a}}{N^2} = 2.78 \times 10^{-31} / (1 + 12.5 W) ; \quad (3-9)$$

two-body dissociative (electron-ion) recombination,

$$\beta_r = 2.1 \times 10^{-7} (0.026/W)^{0.63} ; \quad (3-10)$$

two-body dissociative attachment, α_{2a} ,

$$\frac{\alpha_{2a}}{N} = 4.95 \times 10^{-12} / (1 + 3.45 W^{-4.8})^{1.29} ; \quad (3-11)$$

$$v_m = v_{mn} + v_{mi} ; \quad (3-12)$$

$$\alpha_a = \alpha_{3a} + \alpha_{2a} . \quad (3-13)$$

The impact ionization rate is not a function of the electron swarm energy, but is a function of the beam electron energy and air molecular density. It is

$$R_I = \frac{4\pi e^4 Z N}{(1.6 \times 10^{-12}) \phi_e m_e \beta^2 c^2} \left[\ln \left(\frac{m_e c^2 \gamma (\gamma^2 - 1)}{\phi_e} \right) - \beta^2 \right] \quad (3-14)$$

electrons/cm³/sec, where

T_e = beam electron energy,
 Z = 7.2, effective atomic number for air,
 m_e = electron rest mass (9.11×10^{-28} g),
 c = speed of light (3×10^{10} cm/sec),
 v = electron speed
 β = v/c ,
 γ = $(1 - \beta^2)^{-1/2}$, and
 ϕ_e = 86 eV, energy loss per ionization.

IV. FINITE DIFFERENCE EQUATIONS

Considerable attention was paid to the form of the differential equation (DE) involving the axial field, E_z , equation (2-17) of Section II:

$$\frac{1}{r} \frac{\partial}{\partial r} \left\{ r \left[-\frac{\partial E_z}{\partial r} + \frac{\partial}{\partial t} \left(-\frac{1}{4\pi B^2 \sigma} \frac{\partial E_z}{\partial r} \right) \right] \right\} = \frac{4\pi}{c^2} \frac{\partial}{\partial t} (J_{bz} + \sigma E_z) . \quad (2-17)$$

The form of the DE is parabolic rather than hyperbolic, as it would have been if the axial component of the displacement were not set to zero. Also, an unstable tendency would have resulted if E_r were not replaced in the DE.

Explicit solution schemes of hyperbolic finite difference equations are usually only conditionally stable, and in some cases are absolutely unstable.⁵ Considerable effort must be spent devising stable schemes for hyperbolic finite difference

equations that are usually not universal and are hard to implement.

On the other hand, explicit solution schemes do exist for second-order parabolic finite difference equations that are nearly universal and absolutely stable.⁵ One such method, the Crank-Nicholson method, is used here. Time derivatives are taken as

$$\frac{\partial g}{\partial t} + \frac{g_{i+1}^j - g_i^j}{\Delta t} \quad (4-1)$$

where the subscript i is the i th discrete value of time and the superscript j is the j th discrete value in r . Time derivatives are thus centered at $t = t_{i+1/2}$. Spatial derivatives are centered on the j th radial point, but averaged over the i -1th and i th time points using a weighting factor f ,

$$\frac{\partial g}{\partial r} + f \left(\frac{g_{i+1}^{j+1} - g_{i+1}^{j-1}}{2\Delta r} \right) + (1 - f) \left(\frac{g_i^{j+1} - g_i^{j-1}}{2\Delta r} \right) \quad (4-2)$$

where

$$0 < f < 1. \quad (4-3)$$

Normally, f is given the value 0.5.

The complete set of finite difference equations is:

$$E_{zi+1}^{j+1} \left[f\Delta t + t_{ci+1/2}^{j+1/2} \right] / \Delta_+^2 + E_{zi+1}^j \left\{ \left[f\Delta t + t_{ci+1/2}^{j+1/2} \right] / \Delta_+^2 - \left[f\Delta t + t_{ci+1/2}^{j-1/2} \right] / \Delta_-^2 - \frac{4\pi\sigma_{i+1}^j}{c^2} \right\}$$

$$\begin{aligned}
& + E_{zi+1}^{j-1} \left[f \Delta t + t_{ci+1/2}^{j-1/2} \right] / \Delta_-^2 \\
& = \frac{4\pi}{c^2} \left(J_{bi+1}^j - J_{bi}^j \right) \\
& + E_{zi}^{j+1} \left[(1-f) \Delta t - t_{ci}^{j+1/2} \right] / \Delta_+^2 \\
& + E_{zi}^j \left\{ \left[(1-f) \Delta t - t_{ci}^{j+1/2} \right] / \Delta_+^2 - \left[(1-f) \Delta t - t_{ci}^{j-1/2} \right] / \Delta_-^2 - \frac{4\pi \sigma_i^j}{c^2} \right\} \\
& + E_{zi}^{j-1} \left[(1-f) \Delta t - t_{ci}^{j-1/2} \right] / \Delta_-^2 \quad (4-4)
\end{aligned}$$

where

$$\Delta_+ = \frac{r_j (\Delta r)^2}{r_j + \Delta r/2}$$

$$\Delta_- = \frac{r_j (\Delta r)^2}{r_j - \Delta r/2}$$

$$t_{ci}^j = \frac{1}{4\pi \beta^2 \sigma_i^j}.$$

$$B_{\theta i+1}^j = \frac{2\pi \Delta r}{c r_j} \sum_{n=1}^j r_n \left(J_{bz i+1}^n + \sigma_{i+1}^n E_{zi+1}^n \right) \quad (4-5)$$

$$\begin{aligned}
E_{ri+1}^j & = E_{ri}^j \exp(-X) \\
& + t_{ci+1/2}^j \frac{(B_{\theta i+1}^j - B_{\theta i}^j)}{\beta \Delta t} [1 - \exp(-X)] \quad (4-6)
\end{aligned}$$

$$\text{where } X = \frac{\Delta t}{t_{ci+1/2}^j}$$

$$n_{ei+1}^j = \frac{n_{ei}^j (1-q) + K_{ri+1/2}^j \Delta t}{1+q} \quad (4-7)$$

$$q = (\alpha_a - \alpha_I - \beta_r n_e^j) \Delta t$$

$$w_{i+1}^j = w_i^j e^{-v_\epsilon \Delta t} + \left(\frac{e^2 E^2}{m v_m v_e} + \epsilon \right) (1 - e^{-v_\epsilon \Delta t}) \quad (4-8)$$

$$\epsilon_{i+1}^j = \epsilon_i^j e^{-v_\epsilon \Delta t} + \frac{1}{v_\epsilon} \left[f \left(\frac{dT}{dt} \right) + v_\epsilon w \frac{n_{i+1}^j}{N} \right] (1 - e^{-v_\epsilon \Delta t}) \quad (4-9)$$

$$\sigma_{i+1}^j = \frac{e^2 n_{i+1}^j}{m v_m} \quad (4-10)$$

Boundary conditions are taken as:

$$E_z(\tau, r = r_w) = 0 \quad (4-11)$$

$$\frac{\partial E_z(\tau, r = 0)}{\partial r} = 0 \quad (4-12)$$

where r_w is the distance to a conducting wall, and

$$E_r(\tau, r = 0) = 0, \quad (4-13)$$

$$B_\theta(\tau, r = 0) = 0, \quad (4-14)$$

$$n_e(\tau = 0, r) = 100/\text{cm}^3, \quad (4-15)$$

$$W(\tau = 0, r) = 0.025 \text{ eV}, \quad (4-16)$$

$$\epsilon(\tau = 0, r) = 0.025 \text{ eV}. \quad (4-17)$$

V. SAMPLE CASE

Parameters of the sample case examined are:

Beam electron energy = 1 MeV

Effective beam radius = 100 cm

Wall radius = 500 cm

Peak beam current = 100 kA

Number of axial data points = 1000

Number of radial data points = 101

Beam time history
$$= \begin{cases} 0, & t < 0 \\ \exp [\alpha t (1 - t/t_p) - 1], & 0 < t < t_p \\ 0, & t > t_p \end{cases}$$

$$t_p = 4 \times 10^{-8} \text{ sec}$$

$$\alpha = 10^9 \text{ sec}^{-1}$$

Radial profile (Bennett)
$$= [1 + (r/r_a)]^{-2}$$

Figure 5-1 is a radial slice of the three field components taken at $t = 1.5 \times 10^{-8}$ seconds. Figures 5-2 through 5-5 show a longitudinal slice taken at $r = r_a = 100$ cm. Finally, Figure 5-6 shows the integrated (net) current and beam current along the pulse.

Fig. 1. Radial profile of fields at $T = 1.5 \times 10^{-8}$ seconds.

Fig. 2. Longitudinal profile of electric fields at $R = 100$ cm.

Fig. 3. Longitudinal profile of magnetic field at $R = 100$ cm.

Fig. 4. Longitudinal profile of plasma electron energy at $R = 100$ cm.

Fig. 5. Longitudinal profile of plasma electron density and conductivity at $R = 100$ cm.

Fig. 6. Beam and net current.

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APPENDIX

CBEAM.2-D SOURCE CODE LISTING

```

595 REM*****
596 REM Function definitions
597 REM*****
700 DEF FN A1(W)=NN*(3.76E-15*(W^8.7/(1+.000292*(W^5)^1.412
710 DEF FN F1(W)=NN*(1.25E-07*(W^9.93*(1+.093*(W^1.5)^.405/(1+.26*(W^1.67)^.373
720 DEF FN F2(W)=1.46E-06*(NE/W^1.5)*LOG(1.55E+10*(W^3/NE)^.5)
730 DEF FN F3(W)=FN F1(W)+FN F2(W)
740 DEF FN FE(W)=NN*(1.7E-11*(1+44000*(W^5)^.32/(1+1930*(W^10)^.138+4.13E-10*(W^5
.32/(1+.06*(W^7)^.456/(1+.00168*(W^4)^.097)
745 DEF FN AX(W)=NN*(2.78E-31/(1+12.5*(W)
750 DEF FN AZ(W)=FN AX(W)*NN
760 DEF FN A2(W)=NN*(4.85E-12/(1+3.45/(W^4.8)^1.29
770 DEF FN B1(W)=2.1E-07*(1.026/(W)^.63
780 DEF FN HT(W)=WX*EXP(-FE*DT)+(X1*EP^2/(FM*(1.6E-12)))*(1-EXP(-FE*DT))/FE-W
790 DEF FN FW(E)=.025*(1+.11*(E^3.6)^.458/(1+.000056*(E^5)^.37
800 DEF FN FT(T)=NF*(EXP(A*(1-(1-T/TP))-1)*(>0)*(T<TP)
810 DEF FN RV(R)=RV/(1+(R/RA)^2)^2
820 GOTO 1000
900 IE=1: IO=100: RA=100: RW=100: TF=4E-08: TA=1E-09: A=1/TA: TE=.0000001: PR=1:
NZ=1000: NR=21: SI=100: GOTO 1150
905 REM*****
910 REM Data enter routine
915 REM*****
1000 CLS: LOCATE 2: X$="Program CBEAM-2-D": PRINT TAB(40-LEN(X$)/2) X$
1010 TB=50
1020 PRINT: PRINT "Do you wish to restart?" TAB(TB);: INPUT A$: IF A$="y" OR A$=
"Y" THEN 9000
1030 PRINT: PRINT "Enter electron energy (MeV)" TAB(TB);: INPUT TE
1040 PRINT "Enter peak beam current (kA)" TAB(TB);: INPUT IO
1050 PRINT "Enter effective beam radius (cm)" TAB(TB);: INPUT RA
1060 PRINT "Enter wall radius (cm)" TAB(TB);: INPUT RW
1070 PRINT "Enter pulse width (sec)" TAB(TB);: INPUT TP
1080 PRINT "Enter exponentiating time (sec)" TAB(TB);: INPUT TA: A=1/TA
1090 PRINT "Enter calculation time (sec)" TAB(TB);: INPUT TC
1100 PRINT "Enter air pressure (atmos)" TAB(TB);: INPUT PR
1110 PRINT "Enter number of axial data points" TAB(TB);: INPUT NZ
1120 PRINT "Enter number of radial data points" TAB(TB);: INPUT NR
1130 PRINT "Enter number of axial data points to save" TAB(TB);: INPUT SI
1150 IF SI<>0 THEN NP=INT(NZ/SI)
1160 IF SI=0 THEN NP=NZ+10
1170 JM=1
1185 REM*****
1190 REM Array dimensioning routine
1195 REM*****
1200 DIM ER(2,101), EZ(2,101), BT(2,101), W(2,101), EG(2,101), NE(2,101), SI(2,1
01), R(101), FM(101), FE(101), XA(101), XB(101), XC(101), XD(101), CS(101), DS(1
01), XI(101), FR(101), JB(2), JP(2)
1210 IF JM=1 THEN RETURN
1235 REM*****
1240 REM Data initializing
1245 REM*****
1250 FOR J=1 TO NR: NE(1,J)=100: NE(2,J)=100: W(1,J)=.025: W(2,J)=.025: EG(1,J)=
.025: EG(2,J)=.025: NEXT J
1260 PJ=4*ATN(1):C=3E+10: NN=2.69E+19*PR: DT=TC/NZ: IND=0: GOSUB 13000
1270 IC=.5*1: GM=1+TE/TC: BE=SDR(1-1/GM^2): EC=4.8E-10: F=.5: BEC=2.733E-17
1285 REM*****
1290 REM Electron-impact ionization rate
1295 REM*****
1300 X1=2*(1+(EC^2*NN)/14.4*(BE*C*BEC)/(34.4*(1.6E-12)): X2=EC^2*(LOG(10*TE*(GM^
2-1)*1E+12/(84^2)-BE^2)
1310 RQ=1*X2

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1385 REM*****
1390 REM Calculations begin here
1395 REM*****
1400 FOR I=1M TO N: T=DT*I: XP=FN FT(T): XM=FN FT(T-DT): XH=FN FT(T-DT/2)
1410 FOR J=1 TO NR
1420 NB=ID*3E+12*XP*NJ*FR(J)/(EC*BE*C): ER=ER(2,J): EZ=EZ(2,J): ND=NE(1,J): NE=N
E(2,J): WK=W(1,J): EH=EG(1,J): FM=FM(J): FE=FE(J)
1430 GOSUB 4000
1440 NE(2,J)=NE: SI(2,J)=SI: W(2,J)=W: FM(J)=FM: FE(J)=FE: EG(2,J)=EG: NEXT J
1450 GOSUB 3000: GOSUB 7000
1460 FOR J=1 TO NR
1470 ER(1,J)=ER(2,J): EZ(1,J)=EZ(2,J): BT(1,J)=BT(2,J): NE(1,J)=NE(2,J): SI(1,J)
=SI(2,J): EG(1,J)=EG(2,J): W(1,J)=W(2,J): NEXT J
1480 LA=LA+1: IF LA=NP THEN LA=0: GOSUB 8000
1490 NEXT I
1500 END
2985 REM*****
2990 REM Field solver
2995 REM*****
3000 Z=1
3005 REM*****
3010 REM Compute EZ. Construct tridiagonal matrix for inversion.
3015 REM*****
3020 N=NR-1: FOR J=2 TO N
3030 PP=5*(1/SI(2,J)+1/SI(2,J+1))/(4*PI): PM=5*(1/SI(2,J)+1/SI(2,J-1))/(4*PI)
3040 IF SI(1,J)=0 OR SI(1,J-1)=0 OR SI(1,J+1)=0 THEN MM=PM: MP=PP: GOTD 3060
3050 MP=5*(1/SI(1,J)+1/SI(1,J+1))/(4*PI): MM=5*(1/SI(1,J)+1/SI(1,J-1))/(4*PI)
3060 RP=(1+.5*DR/R(J))/DR^2: RM=(1-.5*DR/R(J))/DR^2
3070 XA(J)=-RM*(F*DT+PM): XB(J)=RP*(F*DT+PP)+RM*(F*DT+PM)+4*PI*SI(2,J)/C^2: XC(J)
=-RP*(F*DT+PP)
3080 XD(J)=-4*PI*3E+12*ID*(XP-XM)*FR(J)/(PI*RA^2)/C^2
3090 XD(J)=XD(J)+EZ(1,J+1)*RP*((1-F)*DT-MP)
3100 XD(J)=XD(J)+EZ(1,J)*(-RP*((1-F)*DT-MP)-RI*((1-F)*DT-MM)+4*PI*SI(1,J)/C^2)
3110 XD(J)=XD(J)+EZ(1,J-1)*RM*((1-F)*DT-MM)
3120 NEXT J
3160 XB(1)=1: XC(1)=-1: XD(1)=0
3170 GOSUB 15000: FOR J=1 TO N: EZ(2,J)=XX(J): NEXT J
3185 REM*****
3190 REM Compute BT
3195 REM*****
3200 SUM=0: BT(2,1)=0: I1=0: FOR J=2 TO NR: DR=R(J)-R(J-1): JB=ID*3E+12*NJ*FR(J)
*XP: JP=SI(2,J)*EZ(2,J)
3210 CO=2*PI*(JB+JP): I2=R(J)*CO: SUM=SUM+.5*DR*(I2+I1): BT(2,J)=2*SUM/(C*R(J)): I
1=I2: NEXT J
3235 REM*****
3240 REM Compute ER
3245 REM*****
3250 FOR J=2 TO NR: D=4*PI*SI(2,J)*DT: IF D<B0 THEN EX=EXP(-D)
3255 IF Q<B0 THEN ER(2,J)=ER(1,J)*EX+(1-EX)*(BT(2,J)-BT(1,J))/(BE*Q)
3260 IF D<B0 THEN ER(2,J)=(BT(2,J)-BT(1,J))/(BE*Q)
3270 NEXT J
3285 REM*****
3290 REM Compute beam, net, and plasma currents
3295 REM*****
3300 SB=0: SP=0: FOR J=2 TO NR: X=ID*3E+12*XP*NJ
3310 JB(2)=X*FR(J): JB(1)=X*FR(J-1): JP(2)=SI(2,J)*EZ(2,J): JP(1)=SI(2,J-1)*EZ(2
,J-1)
3320 SB=SB+(R(J)-R(J-1))*PI*(R(J)*JB(2)+R(J-1)*JB(1))
3330 SP=SP+(R(J)-R(J-1))*PI*(R(J)*JP(2)+R(J-1)*JP(1))
3340 NEXT J
3350 IB=SB/3E+12: IP=SP/3E+12: IN=IB+IP
3360 RETURN
3985 REM*****
3990 REM Subroutine to evaluate plasma parameters
3995 REM*****
4000 X1=25290BB91#: W=WX: IF W<EG THEN W=EG: IF W<.025 THEN W=.025
4005 RI=RD*NB
4010 EP=SG*(ER^2+EZ^2): EX=EP/PR

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4014 REM*****
4015 REM Solve for W using Newton's method of iteration
4016 REM*****
4020 DW=.000001: M=1
4030 FM=FN FM(W): FE=FN FE(W): FX=FN HT(W)
4040 WA=W*(1+DW): FM=FN FM(WA): FE=FN FE(WA): FA=FN HT(WA)
4050 IF FA=FX THEN M=N1: GOTO 4080
4060 W2=W-FX*(WA-W)/(FA-FX): CK=2*ABS(W2-W)/(W2+W): IF CK<=TL THEN M=N1
4070 W=W2: IF W<E6 AND EX<=.1 THEN W=E6: M=N1
4080 M=M+1: IF M<N1 THEN 4030
4100 G1=7*RI/NN: G2=FE*NO*WX/NN: EG=EH+(G1+G2)*DT: IF EG<.025 THEN EG=.025
4105 IF W<E6 THEN W=E6
4110 A1=FN AI(W): A2=FN A2(W): A3=FN A3(W): BR=FN BR(W): AL=A2+A3+BR*(NE+NO)/2-A
I
4120 C=-.5*AL*DT: DF=1+Q: DM=1-Q
4130 NE=(NO*DF+R.*DT)/DM
4140 S1=X1*NE/FM
4150 RETURN
6985 REM*****
6990 REM Screen display subroutine
6995 REM*****
7000 CLS: LOCATE 3: GOSUB 7120: GOSUB 7120
7010 PRINT "I = "I SPC(5) "Time = "T
7020 GOSUB 7120
7030 PRINT "Position" TAB(24) "Fields" TAB(51) "Plasma": GOSUB 7120
7040 PRINT "J" TAB(7) "R" TAB(17) "AXIAL" TAB(27) "RADIAL" TAB(37) "MAGNETIC" TA
B(47) "W" TAB(57) "NE" TAB(67) "COND"
7050 SP=INT(NR/11)+1: FOR J=1 TO NR STEP SP
7060 PRINT USING "###";J: PRINT TAB(5): PRINT USING "+#.#####";R(J): PRINT T
AB(15): PRINT USING "+#.#####";EZ(2,J): PRINT TAB(25): PRINT USING "+#.#####";
ER(2,J):
7070 PRINT TAB(35): PRINT USING "+#.#####";BT(2,J): PRINT TAB(45): PRINT USING
"+#.#####";W(2,J): PRINT TAB(55): PRINT USING "+#.#####";NE(2,J): PRINT TAB
(65): PRINT USING "+#.#####";SI(2,J)
7080 NEXT J
7090 PRINT "Beam current = ": PRINT USING "+#.#####";IB:PRINT " (kA)": PRIN
T "Plasma current = ": PRINT USING "+#.#####";IP: PRINT " (kA)"
7100 PRINT "Net current = ": PRINT USING "+#.#####";IN: PRINT " (kA)"
7110 RETURN
7120 H$="": FOR L=1 TO 79: H$=H$+" ": NEXT L: PRINT H$: RETURN
8000 I=1
8005 Y$=STR$(I): X$="DATA"+RIGHT$(Y$,LEN(Y$)-1)
8010 XX(1)=TE: XX(2)=IQ: XX(3)=RA: XX(4)=TP: XX(5)=TA: XX(6)=TC: XX(7)=PR: XX(8)
=NZ: XX(9)=NR: XX(10)=S1: XX(11)=I: XX(12)=T: XX(13)=IB: XX(14)=IN: XX(15)=RW
8020 OPEN "O",#1,X$
8030 NX=15: WRITE #1,NX: FOR J=1 TO NX: WRITE #1,XX(J): NEXT J
8040 WRITE #1, NR: FOR J=1 TO NR: WRITE #1, ER(2,J): NEXT J
8050 WRITE #1, NR: FOR J=1 TO NR: WRITE #1, EZ(2,J): NEXT J
8060 WRITE #1, NR: FOR J=1 TO NR: WRITE #1, W(2,J): NEXT J
8070 WRITE #1, NR: FOR J=1 TO NR: WRITE #1, NE(2,J): NEXT J
8080 WRITE #1, NR: FOR J=1 TO NR: WRITE #1, SI(2,J): NEXT J
8090 WRITE #1, NR: FOR J=1 TO NR: WRITE #1, BT(2,J): NEXT J
8100 WRITE #1, NR: FOR J=1 TO NR: WRITE #1, FM(J): NEXT J
8110 WRITE #1, NR: FOR J=1 TO NR: WRITE #1, FE(J): NEXT J
8120 CLOSE #1
8130 RETURN
9000 IX=1: GOSUB 1200: CLS: LOCATE 3: PRINT "Enter axial grid point": INPUT IM
9005 Y$=STR$(IM): X$="DATA"+RIGHT$(Y$,LEN(Y$)-1)
9010 OPEN "I",#1,X$: IM=IM+1: INPUT #1,NX: FOR J=1 TO NX: INPUT #1,XX(J): NEXT J
9020 TE=XX(1): IQ=XX(2): RA=XX(3): TP=XX(4): TA=XX(5): TC=XX(6): PR=XX(7): NZ=XX
(8): NR=XX(9): S1=XX(10): I=XX(11): T=XX(12): IB=XX(13): IN=XX(14): RW=XX(15)
9025 A=1/YA

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9030 INPUT #1,NR: FOR J=1 TO NR: INPUT #1,ER(2,J): ER(1,J)=ER(2,J): NEXT J
9040 INPUT #1,NR: FOR J=1 TO NR: INPUT #1,EZ(2,J): EZ(1,J)=EZ(2,J): NEXT J
9050 INPUT #1,NR: FOR J=1 TO NR: INPUT #1,W(2,J): W(1,J)=W(2,J): NEXT J
9060 INPUT #1,NR: FOR J=1 TO NR: INPUT #1,NE(2,J): NE(1,J)=NE(2,J): NEXT J
9070 INPUT #1,NR: FOR J=1 TO NR: INPUT #1,S1(2,J): S1(1,J)=S1(2,J): NEXT J
9080 INPUT #1,NR: FOR J=1 TO NR: INPUT #1,BT(2,J): BT(1,J)=BT(2,J): NEXT J
9090 INPUT #1,NR: FOR J=1 TO NR: INPUT #1,FM(J): NEXT J
9100 INPUT #1,NR: FOR J=1 TO NR: INPUT #1,FE(J): NEXT J
9110 CLOSE #1
9120 IF S1<>0 THEN NF=INT(NZ/S1)
9130 IF S1=0 THEN NF=NZ+10
9140 GOSUB 7000: GOTO 1260
13000 IF IND=1 THEN 13100
13010 DR=RW/(NR-1): FOR KM=1 TO NR: R(KM)=DR*(KM-1): NEXT KM: GOTO 13200
13100 RETURN
13200 RN=1: FOR I=1 TO NR: FR(I)=FN RV(R(I)): NEXT I
13210 MX=-1E+35: FOR I=1 TO NR: IF FR(I)>MX THEN MX=FR(I)
13220 NEXT I
13230 KN=1/MX: FOR J=1 TO NR: FR(I)=RN*FR(I): NEXT I
13240 IG=0: FOR I=1 TO NR-1: RX=.5*(R(I+1)-R(I))*R(I)*FR(I)+R(I+1)*FR(I+1): IG
=IG+RX: NEXT I: NJ=1/(2*PI*IG)
13250 MX=-1E+35: NF=1: FOR I=1 TO NZ: T=DT*I: FX=FN FT(T): IF FX>MX THEN MX=FX
13260 NEXT I
13270 NF=1/MX
13280 RETURN
14750 REM *****
14760 REM *** Subroutine TRIDIAGONAL ***
14770 REM *** by Dr. K. A. Dreyer ***
14780 REM *** April, 1982 ***
14790 REM *** IBM Version ***
14800 REM *****
14810 REM This routine solves a set of simultaneous, tridiagonal linear
14820 REM equations of dimension N.
14830 REM *****
14840 REM Input variables -
14850 REM XA(N) - Left diagonal coefficient matrix
14860 REM XB(N) - Right diagonal coefficient matrix
14870 REM XC(N) - Center diagonal coefficient matrix
14880 REM XD(N) - Constant matrix
14890 REM N - Number of equations
14900 REM *****
14910 REM Output variable -
14920 REM XX(N) - Solution matrix
14930 REM *****
14940 REM Other variables used -
14950 REM CS(N), DS(N), ZX, ZZ
14960 REM *****
15000 CS(1)=XC(1)/XB(1): DS(1)=XD(1)/XB(1): FOR ZX=2 TO N: ZZ=XB(ZX)-XA(ZX)*CS(Z
X-1): IF ZZ=0 THEN 15020
15010 CS(ZX)=XC(ZX)/ZZ: DS(ZX)=(XD(ZX)-XA(ZX)*DS(ZX-1))/ZZ
15020 NEXT ZX
15030 XX(N)=DS(N): FOR ZX=N-1 TO 1 STEP -1: IF CS(ZX)<>0 THEN XX(ZX)=DS(ZX)-CS(Z
X)*XX(ZX+1): GOTO 15050
15040 XX(ZX)=(XD(ZX-1)-XA(ZX-1)*DS(ZX-2))/XC(ZX-1)
15050 NEXT ZX
15060 RETURN

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