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TOPICAL REPORT
THE FORMATION OF DOUBLE SHEATHS
AND THE J-V CHARACTERISTICS
IN THE
OBSTRUCTED REGION

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GLOSSARY OF SYMBOLS

a	Dimensionless Ion Cutoff Energy
c	Dimensionless Electron Temperature
f_e	Electron Distribution Function
f_i	Ion Distribution Function
J	Current Density
J_s	Richardson Current Density
j	Dimensionless Current Density
k	Boltzmann Constant
m	Electron Mass
M	Ion Mass
N_{eo}	Density of Emitter Electron
N_{ep}	Density of Half-Maxwellian Distribution Electron from Plasma
N_{ip}	Density of Half-Maxwellian Distribution Ion from Plasma
n_e	Electron Density
n_i	Ion Density
n_{plasma}	Plasma Charged Particle Density
v	Velocity of Charged Particles
V	Electrostatic Potential
V_c	Cutoff Energy of Half-Maxwellian Distribution Ions
ψ	Dimensionless Potential
ψ_p	Dimensionless Plasma Potential
ρ_e	Dimensionless Electron Density
ρ_i	Dimensionless Ion Density

1. INTRODUCTION

The arc drop of the vapor thermionic converter is determined by the difference of sheath heights and the potential difference across the plasma. The J-V characteristics of cesium vapor diodes operating in the ignited mode show a knee corresponding to near optimum power. When the diode is operating in the region above the knee, the dominant contributor to the arc drop is the difference of the sheath heights (i.e., the potential difference across the plasma produces either a very small loss or may, in fact, reverse sign and reduce the arc drop). In this region the power decreases because the increase in output current is usually not able to compensate for the loss of output voltage. When the diode is operating in the region below the knee, the potential difference in the plasma becomes the dominant contributor to the arc drop. For these reasons, the shape of the J-V characteristics below the knee is different from that above the knee. The location of the knee is a good indicator of the maximum power and the maximum efficiency points. Under the fully optimized conditions, the knee also indicates the barrier index of the diode. Therefore, how and where the knee will appear is of great theoretical and practical interest. Some workers in the thermionic field have speculated that the appearance of the knee is due to the formation of the double sheath. However, this study indicates that the formation of the double sheath takes place significantly below the knee. In addition, this study evaluates J-V characteristic in the obstructed region.

2. MATHEMATICAL FORMULATION

Because of its simplicity, the monoenergetic beam approximation has been used to study the space charge problem in sheath coupling the electrode to the plasma. In Langmuir's basic model of a double sheath,¹ he considered only electrons and ions that enter the sheath with zero incident velocities at the sheath boundaries. He also assumed a monotonic variation of potential inside the sheath, which accelerates the electrons and ions. By the conservation of energy and the boundary conditions for the electric field, Langmuir solved the Poisson's equation and obtained the relation:

$$j_e/j_i = (M/m)^{1/2} \quad (1)$$

which states that the electron current density drawn across the double sheath is limited by the ion current density flowing in the opposite direction. Although by definition there must be quasi-neutrality, this model predicts infinite space charge density at the boundaries. Therefore, the model is not physically self-consistent.

2.1 MONOENERGETIC BEAM MODEL

Prewett and Allen² have investigated the double sheath associated with a hot cathode. They assumed that beam electrons produced by thermionic emission at cathode are accelerated by the sheath field and make no collisions in the sheath. The initial emission energy of the beam electrons is small compared with the energy they gain in their transit across the sheath; one has by energy conservation:

$$1/2 m v_e^2 = eV + eV_o, \quad (2)$$

where V_o is the equivalent potential energy of emitted electrons, and

$$eV_o = 1/2 m v_{eo}^2 = kT_E. \quad (3)$$

In the absence of ionization in the sheath, the current of electron beams must be continuous across the sheath. Thus, one has

$$J_e = N_e v_e = J_{eo} = N_{eo} v_{eo}. \quad (4)$$

The combination of Equation 2 and Equation 4 gives the number density of beam electrons as a function of potential and density:

$$N_e = N_{eo} / (1 + \psi)^{1/2} \quad (5)$$

where:

$$\psi = eV / kT_E. \quad (6)$$

Positive ions entering this sheath are assumed to be monoenergetic with an initial energy $1/2 M v_{ip}^2$ at the boundary of entry. This kinetic energy may be defined as an equivalent potential energy V_{ip} :

$$1/2 M v_{ip}^2 = e V_{ip}. \quad (7)$$

The ions are accelerated by the field of the sheath, therefore, at a particular plane

$$1/2 M v_i^2 = 1/2 M_i v_{ip}^2 + e(V_p - V). \quad (8)$$

Substituting Equation 7 into Equation 8 one has:

$$1/2 M v_i^2 = e(V_{ip} + V_p - V) \quad (9)$$

and

$$v_i = \left(\frac{2e}{M} \right)^{1/2} (V_{ip} + V_p - V)^{1/2}. \quad (10)$$

In the absence of collisions, the current of ions must be continuous across the sheath so that

$$J_i = N_i v_i = J_{ip} = N_{ip} v_{ip} \quad (11)$$

then

$$N_i = N_{ip} \left(\frac{\psi_{ip}}{\psi_{ip} + \psi_p - \psi} \right)^{1/2} \quad (12)$$

where ψ_{ip} , ψ_p , and ψ are the dimensionless potential corresponding to V_{ip} , V_p , and V .

The last group of charged particles in the sheath is the thermalized plasma electrons that enter the sheath with their random velocities obeying the Boltzmann distribution law. Therefore, their number density, N_p , decays exponentially into the sheath from the plasma boundary

$$N_p = N_{ep} \exp - \left[\frac{(V_p - V)}{kT_e} \right] \quad (13)$$

or

$$N_p = N_{ep} \exp [-c (\psi_p - \psi)] \quad (14)$$

where c is the ratio between electrode temperature, T_E , and plasma electron temperature, T_e . Similarly, the current density of the thermalized plasma electron flowing toward the electrode is:

$$J_p = J_{ep} \exp (-c \psi_p) \quad (15)$$

where J_{ep} is the random current of Maxwellian distribution, which can be expressed as:

$$J_{ep} = N_{ep} \left(\frac{kT_e}{2\pi m} \right)^{1/2} \quad (16)$$

Summing up all contributions, one obtains

$$N = \frac{N_{eo}}{(1 + \psi)^{1/2}} + N_{ep} \exp [-c (\psi_p - \psi)] - N_{ip} \left(\frac{\psi_{ip}}{\psi_{ip} + \psi_p - \psi} \right)^{1/2} \quad (17)$$

$$J = N_{eo} \left(\frac{2kT_e}{m} \right)^{1/2} + N_{ip} \left(\frac{2eV_{ip}}{M} \right)^{1/2} - N_{ip} \left(\frac{kT_e}{2\pi m} \right)^{1/2} \exp (-c \psi_p) \quad (18)$$

2.2 HALF-MAXWELLIAN DISTRIBUTION MODEL

The problem of space charge in the sheath is considered for the case where electrons with a half-Maxwellian distribution are introduced into the sheath at the electrode and at the plasma sheath boundary, and the ions are introduced into the sheath from the plasma-sheath boundary, with a cutoff half-Maxwellian distribution. The inelastic and short-range elastic

collisions are being neglected, but each charged particle interacts with the coulomb field of all other charged particles. These assumptions allow for exact calculation of the self-consistent space charge potential distribution.

The one-dimensional, collisionless Boltzmann equations for charged particles with external force replaced by the gradient of electrostatic potential with appropriate signs are:

$$u \frac{\partial f_e}{\partial x} + \frac{e}{m} \frac{dV}{dx} \frac{\partial f_e}{\partial u} = 0 \quad (19)$$

$$u \frac{\partial f_i}{\partial x} - \frac{e}{M} \frac{dV}{dx} \frac{\partial f_i}{\partial u} = 0 \quad (20)$$

where f_e and f_i are electron and ion distribution functions, u is the velocity of the charged particles, and V is the electrostatic potential. The distribution functions, f_e and f_i , are subjected to the boundary conditions that the particles are introduced into the sheath with half-Maxwellian or cutoff, half-Maxwellian distributions. The solution satisfying these assumptions are in the form of

$$f_e(V, u) = 2 N_e \left(\frac{m}{2\pi kT_e} \right)^{1/2} \exp \left[-\frac{m}{2 kT_e} \left(u^2 - \frac{2 eV}{m} \right) \right] \quad (21)$$

$$f_i(V, u) = 2 N_i \left(\frac{M}{2\pi kT_i} \right)^{1/2} \exp \left[-\frac{M}{2 kT_i} \left(u^2 - \frac{2 e(V + V_c)}{M} \right) \right] \quad (22)$$

where N_e and N_i are the densities of the electrons and ions at the boundary, T is the temperature, and V_c is the cutoff energy of ions. It will be convenient to introduce the dimensionless variables

$$\psi = \frac{eV}{kT_e} \quad (23)$$

$$n = \frac{\int_{-\infty}^{\infty} f(V, u) du}{N_{eo}} \quad (24)$$

where T_E is the electrode temperature and N_{eo} is the density of the emitted electrons. With these new variables and for a monotonic accelerating field, the density function of electrons and ions in the sheath can be formulated as

$$n_e(\psi) = H(\psi) + \frac{N_{ep}}{N_{eo}} e^{c(\psi - \psi_p)} [1 + E(c\psi)] \quad (25)$$

$$n_i(\psi) = \frac{N_{ip}}{N_{eo}} \frac{H(\psi_p + a - \psi)}{H(a)} \quad (26)$$

$$a = \frac{e V_c}{k T_E}, \quad \psi_p = \frac{e V_p}{k T_E} \quad (27)$$

$$E(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \quad (28)$$

$$H(x) = e^x [1 - E(x)]. \quad (29)$$

N_{ep} and N_{ip} are the electron and ion densities at the plasma-sheath boundary due to the introduction of charged particles from the plasma. The dimensionless sheath height is given by ψ_p , a is the dimensionless cutoff energy of the ions, and c is the ratio between electron and ion temperature. The ion temperature is assumed to be equal to the electrode temperature. With these distribution functions, the current density, J , across the sheath can be expressed as:

$$J = J_s \left[1 - \frac{N_{ep}}{N_{eo}} \left(\frac{T_e}{T_E} \right)^{1/2} e^{-c\psi_p} + \frac{N_{ip}}{N_{eo}} \left(\frac{m}{M} \right)^{1/2} H(a)^{-1} \right] \quad (30)$$

where J_s is the Richardson current from the electrode. By using the dimensionless variables:

$$\begin{aligned} j &= \frac{J}{J_s} \\ \rho_e &= \frac{N_{ep}}{N_{eo}} \\ \rho_i &= \frac{N_{ip}}{N_{io}} \end{aligned} \quad (31)$$

one obtains:

$$n_e(\psi) = H(\psi) + \rho_e e^{c(\psi - \psi_p)} [1 + E(c\psi)] \quad (32)$$

$$n_i(\psi) = \frac{\rho_i H(\psi_p + a - \psi)}{H(a)} \quad (33)$$

$$j = 1 - \frac{\rho_e}{\sqrt{c}} \exp(c\psi_p) + \rho_i \left(\frac{m}{M}\right)^{1/2} H(a)^{-1} \quad (34)$$

2.3 BOUNDARY CONDITIONS

1. Quasi-neutrality is generally assumed for the plasma, therefore, at the boundary between the sheath and plasma:

$$n_{\text{plasma}} = n_i(\psi_p) = n_e(\psi_p). \quad (35)$$

2. A sheath boundary condition first used by Allen and Thonemann has later been derived by Andrew and Allen,³ namely:

$$\frac{dn(\psi)}{d\psi} = 0 \quad \text{at} \quad \psi = \psi_p. \quad (36)$$

This condition gives

$$\frac{dn_e(\psi)}{d\psi} = \frac{dn_i(\psi)}{d\psi} \quad \text{at} \quad \psi = \psi_p. \quad (37)$$

3. It is usually assumed that the gradient of the potential vanishes at the plasma-sheath boundary. When a double sheath starts to form, the gradient of potential also vanishes at the electrode boundary. Poisson's Equation states that:

$$\frac{d^2V}{dx^2} = 4\pi n(x) \quad (38)$$

then

$$\int \frac{d^2V}{dx^2} \cdot \frac{dV}{dx} \cdot dx = \frac{1}{2} (E_1^2 - E_2^2) = 4\pi \int n(x) \frac{dV}{dx} dx. \quad (39)$$

Since E vanishes at both boundaries, the integral must be zero. That is,

$$\int_{V_2}^{V_1} n(V) dV \quad (40)$$

which implies:

$$\int_0^{\psi_p} [n_e(\psi) - n_i(\psi)] d\psi = 0. \quad (41)$$

The application of the above boundary conditions to the monoenergetic beam model resulted in the following equations:

$$j = 1 + \rho_i \left(\frac{m}{M}\right)^{1/2} a^{1/2} - \frac{\rho_e}{\sqrt{4\pi c}} e^{-c\psi_p} \quad (42)$$

$$\psi_p^{1/2} + \frac{\rho_e}{2c} \left(1 - e^{-c\psi_p}\right) + \rho_i a \left[1 - \left(1 + \frac{\psi_p}{a}\right)^{1/2}\right] = 0 \quad (43)$$

$$\rho_e = \rho_i - \psi_p^{-1/2} \quad (44)$$

$$a = \frac{\rho_i}{2c\rho_e - \psi_p^{-3/2}} \quad (45)$$

Similarly, for the half-Maxwellian model, one obtains:

$$\rho_e = \frac{[\rho_i - H(\psi_p)]}{[1 + E(c\psi_p)]} \quad (46)$$

$$\rho_i \left[\frac{1}{\sqrt{\pi a} H(a)} - 1 \right] - c\rho_e \left[1 + E(c\psi_p) + \frac{e^{-c\psi_p}}{\sqrt{c\pi\psi_p}} \right] - H(\psi_p) + \frac{1}{\sqrt{\pi\psi_p}} = 0 \quad (47)$$

$$G(\psi_p) - 1 + \frac{\rho_e}{c} e^{-c\psi_p} [L(c\psi_p) - 1] + \frac{\rho_i [G(a) - G(\psi_p + a)]}{H(a)} = 0 \quad (48)$$

where

$$G(x) = e^x [1 - E(x)] + 2\sqrt{\frac{x}{\pi}} \quad (49)$$

$$L(x) = e^x [1 + E(x)] - 2\sqrt{\frac{x}{\pi}} \quad (50)$$

and

$$j = 1 - \rho_i [1 + E(c\psi_p)]^{-1} - \left(\frac{m}{M}\right)^{1/2} H(a)^{-1} + \frac{H(\psi_p)}{\sqrt{c} \exp(c\psi_p) [1 + E(c\psi_p)]} \quad (51)$$

The match between the sheath and the plasma is based on the continuities of potential, current, and charged particle densities. These values are obtained independently from the sheath and the plasma calculations.

3. CONDITIONS FOR DOUBLE-SHEATH FORMATION

With c and a held as constants, the relationship among j , ψ_p , and ρ_i can be obtained from Equation 51; they are plotted in Figure 1. The current that flows through the sheath and across the plasma must be the same so that j is a common variable. In addition to the potential, ψ_p , and the plasma density, ρ_i , there are other factors which limit the amount of current that can flow through the sheath and plasma. The flexibility in the values of ψ_p and ρ_i for the same amount of current gives the possibility that the sheath and the plasma can be matched at their boundary. Because of the ratio of masses between electron and ion, the contribution of ion current to the total current is small. The value of cutoff energy affects only the ion current, therefore, the relation among J , ψ_p , and ρ_i is not sensitive to the value of a . The value of c affects the electron current from the plasma toward the electrode. Electron current is the main component of the total current, therefore, the value of c has a profound effect on the relation between j , ψ_p , and ρ_i , as shown in Figure 2. For example, with the same values of ψ_p and ρ_i , a normalized current density, j , of 0.3 or 0.8 can flow through the sheath, depending on the value of c .

Boundary conditions 2 and 3 are the additional conditions to be satisfied in the sheath. Condition 2 is generally accepted for any form of sheath. Condition 3 holds just as the sheath is starting to become double valued.

The relation between ψ_p and ρ_i (ρ_i is the ratio between the plasma density and the electron density corresponding to saturation current density) for the half-Maxwellian model can be obtained by solving Equations 47 and 48. The results are plotted in Figure 3. These results are not sensitive to the values of c or a . For any fixed value of c , the value of a is almost constant. The most interesting result indicated by these curves is that the double sheath can only be formed with the value of ρ_i

Figure 1. Relation Between ψ_p and ρ_i as Function of j

Figure 2. Normalized Potential, ψ as a Function Normalized Ion Density, ρ_i - Parametric in "C" ρ

Figure 3. Condition for the Formation of a Double Sheath

less than 1.7. To limit the value of ρ_i to less than 1.7 rules out the possibility of forming a double sheath on the collector side of the thermionic converter.

On substituting the foregoing results into Equation 51, one obtains the relation between j and ψ_p . These relations are plotted in Figure 4. The results of two sample plasma calculations are also plotted in this figure. The intersection of plasma and sheath solutions indicates the matching conditions between the sheath and the plasma. For Case 1, the double sheath is formed at $j = 0.48$ and $\psi_p = 0.9$, while for Case 2, $j = 0.44$ and $\psi_p = 0.8$ (based on the results of plasma calculations, the value of $c \sim 6$).

The results shown in Figures 1 and 2, and the results for the same sample plasma conditions are combined in Figure 5 and 6.

Figure 5 shows that the results of the sheath calculation intersects those of the plasma calculations at $\psi_p = 1$ and $\rho_i = 1.55$. The values of the currents for the plasma calculation are indicated on the plasma curve. At the intersection, $j = 0.48$, the expression of j , from sheath analysis (Equation 51), is plotted as the dashed curve. The intersection of the dashed curve and the sheath solutions is not exactly the same as the intersection of plasma solution and the sheath solution. This slight discrepancy indicates that the match is not perfect. However, considering all the assumptions that have been used in the sheath and the plasma regions, this slight discrepancy can be tolerated. The matching conditions for another case are given in Figure 6. The discrepancy is also quite small.

The J-V characteristics for the plasma calculations are given in Figure 7. The matching conditions for the formation of the double sheath are indicated on the curves. They show that the formation of the double sheath is below the appearance of the knee.

Similar calculations for the monoenergetic beam model have also been carried out. The results are compared with the half-Maxwellian model

Figure 4. Matching Conditions of the Sheath and the Plasma Parameters for the Formation of Double Sheath

Figure 5. Matching Conditions of the Sheath and the Plasma Parameters for the Formation of the Double Sheath

Figure 6. Matching Conditions Between the Sheath and the Plasma Parameters for the Formation of Double Sheath

Figure 7. J-V Characteristics of Plasma Calculations

in Figures 8 and 9. The results of half-Maxwellian sheath model and the plasma calculation intersect at $\psi_p = 0.8$, $\rho_i = 1.6$ and $\psi_p = 0.7$, $j = 0.43$. The intersection points of monoenergetic beam sheath model and the plasma calculations are $\psi_p = 1.6$, $\rho_i = 2.26$ and $\psi_p = 0.35$, $j = 0.395$. The half-Maxwellian sheath model predicts a good match between the sheath and the plasma. The monoenergetic beam model fails to predict the mismatch between the sheath and the plasma.

Figure 8. Comparison of Half-Maxwellian Model and Monoenergetic Beam Model Matching Parameters ψ_p and ρ_i

Figure 9. Comparison of Half-Maxwellian Model and Monoenergetic Beam Model Matching Parameters ψ_p and ρ_i

4. CURRENT-VOLTAGE CHARACTERISTICS IN OBSTRUCTED REGION

After the formation of a double-value sheath between the electrode and plasma, the transmitted current due to thermal emission is reduced by the presence of an electron repelling potential barrier, V_m , near the electrode (Figure 10 shows the motive of diode with double sheath). This potential barrier can be referred to as a "virtual electrode." The calculation of the current-voltage characteristics of a thermionic converter can easily be extended to the obstructed region by using the concept of a virtual emitter which appears after the formation of a double-value sheath. With this approach, the boundary conditions and the governing equations of the plasma in the obstructed region are the same as those in the saturated region.

A typical, calculated J-V curve is shown in Figure 11. It indicates that after the formation of a double sheath, the J-V curve behaves as the Boltzmann line (i.e., an exponential function). However, most of the experimental data indicate that the slope of the J-V curve in this region is steeper than the Boltzmann line. Thus, this analytical result is not fully consistent with experimental data. The analyses also indicate that in the obstructed region 1) the potential and electron temperature distributions are almost independent of current density, and 2) the charged particle density decreases as the current density decreases.

Since the electron temperature distribution does not change, the heat energy carried by each electron to the collector is constant. A typical result of a heat flux measurement is shown in Figure 12. These data indicate that the auxiliary heater power to the collector is a linear function of current density, which implies that the average heat energy carried by each electron from the plasma to the collector is a constant. Thus, the experimental observations are consistent with the analytical calculations.

Figure 10. Calculated Current Density-Voltage Characteristic

Figure 11. Motive Diagram of a Thermionic Converter with a Double Sheath at the Emitter

Figure 12. Heat Flux Data as a Function of Current Density at an Emitter Temperature of 1700 K and a Collector Temperature of 850 K

Calculated potential distributions of the emitter sheath by integrating the Poisson equation are shown for typical values in Figure 13. These show that the plasma potential measured from the virtual electrode is insensitive to change after the double sheath is formed. Therefore, both plasma and sheath analyses indicate that the potential distribution near the electrode does not change in the obstructed region.

The contraction in glow discharges is a well-known phenomenon. For example, contraction of the positive column of an oxygen glow discharge has been observed at pressures higher than about 5 torr, where radial variation of the gas temperature in the column becomes large.⁴ Similarly, the contraction of the plasma in a thermionic diode has also been suggested to explain the steep J-V characteristics of the obstructed region.⁵ Spatially resolved spectroscopic measurements of the cesium plasma taken over the entire J-V characteristics of a typical diode indicate that the plasma, if it contracts, does so only at low current densities. However, for the experimental diode with a guarded collector, the plasma in the collector region should not be affected by the contraction.

Figure 13. Potential Profiles of Emitter Sheath with Different Current Ratio

5. CONCLUSIONS

The following conclusions can be drawn from this analysis:

1. The formation of the double sheath and the match between the sheath and the plasma is predicted by the theory presented herein.
2. The double sheath will be formed when $P_i < 1.7$. This constraint rules out the possibility of the formation of a double sheath on the collector side of the plasma.
3. The double sheath is formed below the knee of J-V characteristic.
4. The theoretically predicted J-V characteristic in the obstructed region behaves as the Boltzmann line. This result is not completely consistent with experimental observations.

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