MASTER

Inelastic Strong Interactions at High Energies

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Abstract

Certain classical solutions of the field equations, hereafter sections, were shown to dominate inclusive and semi-inclusive cross sections which are proportional to the absolute square of the Fourier transform of these solutions in zeroth order of a model independent approximation scheme. Higher order approximations are obtained by the loop expansion method, or by a perturbation expansion around the classical solutions.

Sections were shown to be associated with nonvanishing values of two conserved topological quantum numbers (that vanish for solitons).

An application to ϕ^4 field theory gave an inclusive cross section with Feynman scaling, a total cross section with a power dependence on the primary energy and a Poisson-like multiplicity distribution.

An SU(2) invariant Reggeon Field Theory containing three reggeon multiplets, one with negative mass, has been studied in the classical approximation with zero transverse dimensions. A solution has been found, qualitatively similar to a solution of supercritical Pomeron theory, giving a constant local cross section and decreasing cross sections for quantum number exchanges.

A method, that allows the calculation of critical parameters of a lattice based on the strong coupling expansion of the finite renormalization group equations was worked out and shown to give excellent results for a two dimensional lattice.

1. The Theory of Sections

Past investigations of strong interaction theories have failed to produce a systematic method permitting the model independent calculation of various cross sections. Methods based on perturbation theory were unsuccessful due to the divergence of perturbation series and the large value of the coupling constant; on the other hand nonperturbative calculations have always been combined with various model assumptions.

We proved in a series of papers¹⁻³ that the calculation of inclusive, semi-inclusive (and possibly exclusive) cross sections is possible since these cross sections are closely related to certain classical solutions (which we call sections) of the field equations.

The simplest way to clarify the relation of classical solutions to cross sections is to make use of the representation of cross sections by functional integrals and to perform the asymptotic evaluation of these integrals by the method of steepest descent (or loop expansion⁴). The generating functional of cross sections is defined as follows:

$$\sigma[g] = \sum_{n=2} \int d^3 p_1 \dots d^3 p_n \sigma_n(p_a, p_b; p_1 \dots p_n) g(p_1) \dots g(p_n) , \quad (1)$$

where $\sigma_n(p_a, p_b; p_1 \dots p_n)$ is the differential cross section for an n particle production process. $\sigma[g]$ was shown to have the following representation¹:

$$\begin{aligned} \sigma[g] = m^2 [(p_a p_b)^2 - m^4]^{-1/2} 64 \int d^4 v d^4 v' \exp\{i(v - v')(p_a - p_b)\} \\ \int D\phi D\bar{\phi} K_V \phi(v) K_V \bar{\phi}(-v) K'_V \bar{\phi}(v') K'_V \phi(-v') \int dz \exp\{-A + iz(p_a + p_b)\} , \end{aligned} \quad (2)$$

where the action is given by

$$A = -\int d^4 x d^4 y D_+(q, z + x - y) K_x \phi(x) K_y \bar{\phi}(y) - i \int d^4 x [L(\phi) - L^\dagger(\bar{\phi})] , \quad (3)$$

and the function $D_+(g, x)$ is defined as

$$D_+(g, x) = (2\pi)^{-3} \int d^3 p (2E)^{-1} g(p) \exp\{-ixp\} ,$$

and $L(\phi)$ is the Lagrangian of the field theory in question.

In Eq. (1) p_a , p_b and m denote the momenta and mass of the initial state particles, respectively. $D\phi$ and $D\bar{\phi}$ imply functional integrals over the fields ϕ and $\bar{\phi}$. The Klein-Gordon operator K_x is defined as $K_x = -\Delta_x + m^2$, while $E = (p^2 + m^2)^{1/2}$. As is well known, the inclusive and exclusive cross sections are the functional derivatives of $\sigma[g]$ at $g=1$ and $g=0$, respectively. The multiplicity distribution can be obtained from $\sigma[g]$ if $g(p) = g = \text{constant}$ is set and the n th order term in g is found either by differentiation or with the help of a contour integral. One obtains the semi-inclusive cross sections if before calculating the n th order term one takes functional derivatives of $\sigma[g]$ with respect to $g(p)$.

For large values of $s = (p_a + p_b)^2$ the functional integral and the integral over z are evaluated by the saddle point method. The extrema of the expression $-A + iz(p_a + p_b)$ are at the points where the fields satisfy the following integral equation (taking $g(p) = g = \text{constant}$)

$$\psi(x) = -ig \int d^4y D_+(x-y) \partial L_{\text{int.}}(\bar{\psi}) / \partial \bar{\psi}(y) - \int d^4y D_F(x-y) \partial L_{\text{int.}}(\psi) / \partial \psi(y), \quad (4)$$

and another equation obtained by exchanging ψ and $\bar{\psi}$ and complex conjugating the kernels. Proceeding one obtains the constraint

$$i(p_a + p_b)_\mu = g \int d^4x d^4y \partial D_+(x-y) / \partial x_\mu K_x \psi(x) K_y \bar{\psi}(y). \quad (5)$$

We have introduced the notation $L_{\text{int.}}$ for the interaction part of the Lagrangian L , and the new fields $\psi(x) = \phi(x-z/2)$ and $\bar{\psi}(x) = \bar{\phi}(x+z/2)$.

Any function that satisfies Eq. (4) satisfies the classical field equation as well. If $g=1$, Eq. (4) and the corresponding equation for $\bar{\psi}$ can be simultaneously solved by a function $\psi = \bar{\psi}$ provided ψ satisfies the equation

$$\psi(x) = -\int d^4y D_{\text{adv.}}(x-y) \partial L_{\text{int.}}(\psi) / \partial \psi(y). \quad (6)$$

The calculation of cross sections beyond the zeroth order approximation can proceed either by the loop expansion method⁴ or by a field theoretic

approach, by expanding the Green's functions of the field theory defined by action A of Eq. (3) around the classical solutions of Eq. (4) or (6).⁶

We succeeded in showing that the classical solutions in question are associated with two conserved topological charges, Q and P_μ .²

These charges can easily be expressed with the help of the field ψ , provided ψ satisfies Eq. (4). It turns out that²

$$P_\mu = \int d^4x d^4y \partial D_+(x-y) / \partial x_\mu K_X \psi(x) K_Y \bar{\psi}(y) , \quad (7)$$

while

$$Q = \int d^4x d^4y D_+(x-y) K_X \psi(x) K_Y \bar{\psi}(y) . \quad (8)$$

We have also shown that soliton solutions of the field equation⁶ give identically zero topological charges Q and P_μ ,² underlining the difference between solitons and sectons.

We found that the solution of Eq. (4) has a very simple relation to measurable physical quantities. Our semiclassical expression for the total cross section reads for large s (provided ψ satisfies Eq. (6))

$$\sigma \approx |j(p_a)|^2 |j(p_b)|^2 e^{Q/s} , \quad (9)$$

where $j(p)$ is defined by

$$j(p) = \int d^4x \exp\{ipx\} K_X \psi(x) . \quad (10)$$

The inclusive cross section is given by

$$d\sigma/d^3p = \sigma (2E)^{-1} (2\pi)^{-3} |j(p)|^2 . \quad (11)$$

Eqs. (10) and (11) imply the physical interpretations

$$Q = \int d^3p \, d\sigma/d^3p = \langle n \rangle \quad (12)$$

and

$$P_\mu = \int d^3p \, p_\mu d\sigma/d^3p = (p_a + p_b)_\mu \quad (13)$$

for the topological charges Q and P_μ . At the same time it becomes clear that Eq. (5) for $g=1$ is nothing else but the first sum rule for inclusive cross sections.

The only field theory we have studied in detail to date is that of a single Hermitean scalar field with $L_{\text{int.}} = -(f/4!) \phi^4$.² The only nontrivial solution of Eq. (4) we found generated an inclusive distribution with Feynman scaling and an exponential cutoff of the transverse momentum distribution. Since an inclusive distribution with Feynman scaling gives $\langle n \rangle \sim \log s$ Eqs. (9) and (12) imply that the total cross section has a power dependence on s .

The multiplicity distribution turned out to be Poisson-like and the semi-inclusive cross section also showed the scaling property that was found earlier in general multiperipheral models:⁷

$$d\sigma_n/d^3p \approx \sigma_n(2E) {}^1f(n/\langle n \rangle, p_T, p_{||}/s^{1/2}).$$

Research presently performed or planned for the future in the theory of sections will be described in the Proposal for the Renewal of the Contract.

II. Reggeon Field Theory with an Internal Symmetry

Considerable progress has been made towards a satisfactory theory of high energy scattering and an understanding of the Pomeron since the formulation of Reggeon field theory. It has become clear within this framework that high energy scattering is to be regarded as a critical phenomenon with different asymptotic cross section limits emerging depending on whether the bare Pomeron coupling is below, at or above its critical value.⁸ Below the critical value σ_{tot} is found to fall with energy and above the critical value, the supercritical regime, two solutions have recently been found, one⁹ where $\sigma_{\text{tot}} \sim (\ln s)^2$ and the other¹⁰ where σ_{tot} again falls with increasing energy. These investigations, however, have shed little light on the dynamical origin of the Pomeron.

We have made a study of a Reggeon field theory with the addition of an internal symmetry.¹¹ The motivation is twofold. A suggestion has been made¹² that the zero mass Pomeron might be a Goldstone particle resulting from the spontaneous symmetry breaking of a theory with negative mass (intercept > 1) bare reggeons. In addition the specific model chosen has been motivated and suggested by an analysis of the high energy behavior of spontaneously broken non-abelian gauge theories¹³.

The model has an internal SU(2) symmetry, called I-spin and has three multiplets of bare reggeons of $I = 0, 1, 2$ with trajectories

$$\alpha_i(t) = 1 - \Delta_i + \alpha'_i t$$

where $i = 1, 2, 3$, and $\Delta_0, \alpha'_0, \Delta_1, \alpha'_1$ are positive and most importantly Δ_2, α'_2 are negative; i.e. the $I = 2$ field has negative mass and slope. The Lagrangian in zero transverse dimensions where we work is schematically

$$\begin{aligned} L &= 1/2 (p_0 \overleftrightarrow{\partial}_y q_0 + p_1 \overleftrightarrow{\partial}_y q_1 + p_2 \overleftrightarrow{\partial}_y q_2) - H(p, q) \\ H &= -\mu_0 p_0 q_0 - \mu_1 p_1 q_1 - \mu_2 p_2 q_2 + \lambda_{bc}^a (p_a q_b q_c + q_a p_b p_c) \end{aligned} \quad (14)$$

where the triple Regge interactions are SU(2) symmetric and $\mu_i = -\Delta_i = \alpha_i(0) - 1$.

The Hamiltonian as a function of I, H_I , corresponding to the overall exchange of isotopic spin I in the t channel was first derived. The scattering amplitude generated by the exchange of Reggeons interacting according to Eq. (14) was then calculated in the classical approximation using path integral techniques. Stable fixed points of the classical equations of motion were determined and permissible classical solutions connecting these fixed points were qualitatively discussed. These solutions exist for a large range of the coupling and mass parameters of the Lagrangian. These parameters were not regarded as fixed by the underlying gauge theory except as to sign for Δ_i and α'_i and reality for the λ_{bc}^a . Two distant classical solutions were found corresponding to those of Ref. (9) and (10) in the supercritical Pomeron theory.

The important result is found that for the solution analogous to that of Ref. (9),

$$S_I = \exp\{-[A + B(y_2 - y_1)I^2]\}$$

in the limit of large rapidity difference $y_2 - y_1$, where S_I is the S-matrix for exchange of isotopic spin I and A and B are positive (non-zero) constants. Therefore the correct cross section behavior is obtained, with the $I=0$ amplitude behaving as

$$s^{-1} \text{Im} T_0 = \sigma_{\text{tot}} = \text{const.}$$

and the other amplitudes behaving as

$$s^{-1} \text{Im} T_I \sim s^{-BI^2}.$$

The quantum number exchange cross sections vanish at large energy with $I=2$ suppressed more than $I=1$. This behavior is totally different from the behavior of the bare input trajectories which had an $I=2$ intercept greater than 1 and $I=1$ and $I=2$ intercepts < 1 . Further study of this Reggeon field theory will be discussed in the Proposal for the Renewal of the Contract.

III. Finite Cell Renormalization Group on Lattices

Niemeijer and van Leeuwen proposed a method of discussing critical phenomena on lattices¹⁴ without appealing to Wilson's method of continuous spin.¹⁵ The method realizes Kadanoff's original ideas¹⁶ by defining a finite rescaling of the original lattice, a lattice of cells of spins. The Hamiltonians of the original and rescaled lattices can be related to each other through the renormalization group equation¹⁴

$$\exp\{H_N(s', K') + g_N(K)\} = \sum_{\sigma} \exp\{H_N(s', \sigma, K)\} \quad (15)$$

where H is the Hamiltonian, K and K' represent the coupling constants on the original (L) and rescaled lattices (lattice of cells, L'), respectively. The quantities s and s' represent the configuration of spins in the two lattices; the label s can be substituted by labels s' and σ , where σ is a label indexing the states of L with fixed s' . The function $g_n(K)$ is the self energy of the cells.

Eq. (15) can be regarded as a system of equations expressing the coupling constants K' with the help of K . The fixed point of this equation determines the critical point and the eigenvalues of the derivative matrix $\partial K'/\partial K$ at the fixed point determine the critical exponents.

Niemeijer and van Leeuwen expanded Eq. (15) around the symmetric phase to obtain a finite number of equations to determine an approximate value for the critical parameters.¹⁴

We worked out a different approach for finding the approximate solutions of Eq. (15). Our method relies on the strong coupling expansion (expansion around the ordered phase) of these equations.¹⁷ Since lattice field theories are becoming more and more important tools for calculating various parameters of strong interactions,¹⁸ and often the strong coupling limits of these field theories have special physical significance (as in quantum chromodynamics),¹⁹ special significance is attached to this approach.

Our method is based on taking only a finite number of equations of the type (15) corresponding to some of the lowest lying states and taking the same number of nearest neighbor coupling constants. At the same time we truncate the right-hand side of the equations to retain the first few terms of the low temperature expansion contributing to it only. In this way we obtain a finite system of equations for the coupling constants which allow the calculation of the critical parameters in the previously described way. The needed relation between the number of equations (or coupling constants) and the number of terms retained in the low temperature expansion can be established by making use of a cluster decomposition property of the renormalization group equations.

We applied our method to the calculation of the critical parameters of the plane triangular lattice and obtained results in a third order approximation that were within a few percent of the exactly known values.¹⁷

Further applications of our method to lattices of higher dimensions are in progress.

IV. General Remarks

The work performed during the present report period is at variance with that proposed in our last proposal. The reason for this discrepancy is - as discussed in our Topical Report³ - the achievement of significant new results in the theory of sectons. We judged that these results are much more important than the ones originally proposed, so we postponed the investigations proposed there to a later time.

The principal investigator and faculty associated devoted approximately 50% (100% during academic recesses) of their time to the investigations described in the present report; they are expected to spend the same fraction of their time in the continuation of these investigations during the rest of the current reporting period.

While both of the faculty investigators spent most of their time with research related to the first two sections of this report, two graduate students were occupied by the application of the strong coupling approach to the renormalization group on lattices.

References

1. P. Suranyi, Phys. Rev. D17, to be published.
2. P. Suranyi, to be published.
3. P. Suranyi, Topical Report COO-2978-2, unpublished.
4. F. Iliopoulos, C. Itzykson and A. Martin, Rev. Mod. Phys. 47,
165 (1975).
5. L.S. Brown, Phys. Rev. D5, 748 (1972).
6. R. Jackiw, Rev. Mod. Phys. 49, 681 (1977).
7. P. Suranyi, Phys. Letters, 36B, 47 (1971); *ibid.* 40B, 481 (1972);
Z. Koba, H.B. Nielsen and P. Olesen, Phys. Letters 38B, 25 (1972).
8. H.D.I. Abarbanel, J.B. Bronzan, R.L. Sugar and A.R. White, Phys.
Reports 21C, 119 (1975).
9. D. Amati, L. Caneschi and R. Jengo, Nucl. Phys. B101, 397 (1975)
and V. Alessandrini, D. Amati and M. Ciafaloni, CERN Th. 2349 (1977).
10. A.R. White, CERN Th. 2259 (1977).
11. J. Bartels and J. Scanio, CERN preprint, submitted to Nucl. Phys.
12. H.D.I. Abarbanel, Phys. Letters, 62B, 308 (1976).
13. J. Bartels, Phys. Letters 68B, 258 (1977) and in preparation.
14. Th. Niemeijer and J.M.J. van Leeuwen, Phys. Rev. Lett. 31, 1412 (1973),
Physica (Utrecht) 71, 17 (1974).
15. K. Wilson, Phys. Rev. B4, 3174 (1971).
16. L.P. Kadanoff, Physics (New York) 2, 263 (1966).
17. P. Suranyi, Phys. Rev. Letters, 38, 1436 (1977).
18. K.G. Wilson, Phys. Rev. D10, 2445 (1974).
19. J.B. Kogut and L. Susskind, Phys. Rev. D11, 395 (1975),
T. Banks et al., Phys. Rev. D15, 111 (1977).